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ORIGINAL RESEARCH

# Influence of Red and Yellow cards on team performance in elite soccer

Llorenç Badiella[1,2] · Pedro Puig[2,3] · Carlos Lago-Peñas[4] · Martí Casals[5,6,7]

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## Abstract

The aim of the current study is to analyze the effects of red and yellow cards on the scoring rate in elite soccer. The sample was composed of 1826 matches in the top five European leagues. All events were structured in 5-min intervals and were analyzed by means of a Generalized Linear Mixed Model with Poisson distribution, considering the presence of correlated data, where the dependent variable is represented by scoring rate. Team strength and home advantage were considered implicitly by means of a transformation of the betting odds for each game. The model also took into account the goal difference and time evolution. Overall, we found that after a sending off, each team's scoring rate changes significantly, damaging the penalised team and favouring its opponent. When the player who is sent off belongs to the Away team, the impact of a red card is more or less maintained over time intervals. The red card effect, on the other hand, tends to fade over time when the affected team is stronger. The relative difference in scoring rates is also affected by the goal difference and the difference in booked players, being slightly lower for the team going ahead if it has more booked players. Our approach allows estimating the expected cumulative soring rate through time for various red card scenarios. Particularly if a red card is given with 30 min of remaining time, the expected impact is 0.39 goals if the guilty player is on the visiting team and 0.50 if he plays for the home team. Coaches and analysts could use this information

✉ Llorenç Badiella
badiella@mat.uab.cat

1 Servei Estadística Aplicada, Universitat Autònoma de Barcelona, Cerdanyola del Vallès, Barcelona, Spain

2 Departament de Matemàtiques, Universitat Autònoma de Barcelona, Cerdanyola del Vallès, Barcelona, Spain

3 Centre de Recerca Matemàtica, Cerdanyola del Vallès, Barcelona, Spain

4 Faculty of Education and Sports Sciences, Universidad de Vigo, Pontevedra, Spain

5 Sport and Physical Activity Studies Centre (CEEAF), Faculty of Medicine, University of Vic-Central University of Catalonia (UVic-UCC), Barcelona, Spain

6 Sport Performance Analysis Research Group, University of Vic-Central University of Catalonia (UVic-UCC), Barcelona, Spain

7 National Institute of Physical Education of Catalonia (INEFC), University of Barcelona, Barcelona, Spain

to establish objectives for players and teams in training and matches and to be prepared for these very different scenarios of numerical superiority or inferiority.

**Keywords** Soccer · Performance analysis · Red and yellow cards · Generalized linear mixed model · Poisson distribution

## 1 Introduction

The analysis of strategies plays a major role in soccer. Managers use scouting, video footage, and soccer data feeds to collect information about tactics and player performance. Performance analysis in soccer requires objective recording and examination of behavioral events involving one or more players during training or competition. The primary goal of performance analysis is to provide information to coaches and players about player and/or team performance in order to plan subsequent training sessions to improve performance or to support preparations for the next match (McGarry et al. 2013).

However, the validity of the data generated by most studies, especially regarding the prescriptive function of tactical analysis, may be questionable due to the lack of assessment of the role of the opposing team in their analysis (McGarry et al. 2013). Other critical issues related to the conceptual and methodological shortcomings of contemporary research that require attention in future research may include the development of a theoretical framework, research on critical behaviors, consideration of situational and game contexts, and the inclusion of spatial and temporal dynamics (Bornn et al. 2018).

At the highest level of competition, team performances are generally affected by the smallest of details that can imply considerable advantages in the quest for success. It is relatively common for referees to decide to send off soccer players at all levels and in all competitions and it is therefore likely that such events could change the outcome of matches. Red cards are a significant event that can influence the outcome of a game. Previous research demonstrates that advantaged teams exhibit better team performance after a player has been sent off and that teams modify their tactics and style of play in response to this new match scenario (Bar-Eli et al. 2006; Carling and Bloomfield 2010; Lago-Peñas et al. 2016). A sending off represents a disadvantage and can trigger negative momentum leading to decreased performance. Previous literature suggests it may reduce self-confidence, increase demoralization and reduce the team-cohesion effect (Bar-Eli and Tenenbaum 1989; Bar-Eli et al. 1996).

Team performances have been analyzed previously by different authors, using data from different countries or competitions and based on substantially different quantitative approaches guided by the type of data available (Anders and Rotthoff 2011; Bar-Eli et al. 2006; Carmichael and Thomas 2005; Chowdhury 2015; Gómez-Déniz et al. 2019; Greenberg 2015; Mattera 2021; Mechtel et al. 2011; Titman et al. 2015). Regarding the effects and occurrence of red cards, most of the literature finds, as might be expected, that red cards produce a decrease in the performance of the penalized team while causing an increase in that of the opposing team, either in terms of the probability of winning the game or in terms of expected goals, points achieved or scoring rate; although the magnitude of the effect can vary considerably. One study performed on 743 Bundesliga matches suggested that red cards weaken the affected team in terms of goals conceded and final score following the sending off (Bar-Eli et al. 2006). The study subsequently highlighted that the scoring or eventually winning the match chances of a team that has had a player sent off are substantially reduced.

Carling and Bloomfield (2010) examined the effects of an early dismissal (after 5-min play) on work-rate in a professional soccer match. Their results suggest that playing with ten men leads to a greater total distance being covered than normal (particularly in moderate-intensity activities) revealing shorter recovery times between high-intensity activities. Furthermore, this study suggests that in 11 versus 11 situations, players might not always use their full physical potential, because they are able to increase their overall work-rate when reduced to 10 players. The study also suggests that a team with 10 players should change its strategy and tactical set-up in order to minimize the effects of the higher levels of fatigue. Lago-Peñas et al. (2016) found that playing 11 versus 10 increases the time spent in possession, number of total passes, short passes, total touches and the percentage of successful passes compared with playing 11 versus 11. Advantaged teams also spent less time defending. The punished team performed worse in all variables after the dismissal. Sapp et al. (2019) also found similar rates of aggressive play in the top English soccer leagues, possibly due to a standardized refereeing style.

While everyone understands the circumstantial effect of red cards (the immediate loss of a player), statistical analysis helps us better understand the extent to which teams are affected when they receive one (Chowdhury 2015). The main methodological criticism of previous research is the use of aggregate data at match level (e.g. in Anders and Rotthoff (2011), Carmichael and Thomas (2005) and Chowdhury (2015) that only provides a partial view of the problem without considering the temporal coherence of the events. Other authors used a similar approach including information from the remaining time at the issuing of red cards (e.g. in Albanese et al. 2020; Červený et al. 2018; Mechtel et al. 2011).

In a similar manner, some other studies have compared the occurrence of post-card goals versus the pre-card situation (e.g. in Bar-Eli et al. 2006; Caliendo and Radic 2006; Ridder et al. 1994) or analyzed time to goal. These approaches do not suffer the aforesaid bias but the series analyzed are relatively small: only games with players sent off are considered and no other event data is included.

In much of the literature home/away effects are evaluated. Adjustments for team strength are also common, using the previous year's rankings (Anders and Rotthoff 2011; Mechtel et al. 2011), rankings provided by third parties (Červený et al. 2018; Albanese et al. 2020) or betting odds (Červený et al. 2018; Titman et al. 2015). Only in a small number of studies have authors examined the effect of yellow cards in combination with red cards (Anders and Rotthoff 2011; Titman et al. 2015).

Considering all previous limitations, the objective of this study is to propose a more valid statistical approach to analyze the consequences of red and yellow cards. Our proposal is based on the analysis of a large series of games using aggregated data per short time intervals from a dataset of event data. The response variable is the number of goals scored in an interval, whereas the explanatory variables are a series of variables that characterize the match, the team and the game situation at the beginning of the interval. Hence there is temporal coherence between the response variable and explanatory variables (including the goal difference, number of players that have received yellow or red cards, remaining time and time elapsed since a superiority scenario); injury time can be included as a special interval and Team strength is considered using information from betting odds. In this way, our method will allow us to estimate the scoring rate over time for specific red card scenarios and team characteristics, and in consequence will provide a framework to quantify the average impact of having a player sent off.

## 2 Data processing

The original database was taken from (Pappalardo et al. 2019), version 3. It consists of the 1826 games played in the 2017/2018 season in the five biggest European soccer leagues (French Ligue 1, Spanish La Liga, English Premier League, Italian Serie A and German Bundesliga). The dataset consists of spatio-temporal match events, from which we extracted goals, red and yellow cards and substitutions for each team and its opposing team. The sample was classified into 5-min periods. The response variable is the number of goals scored in a given interval for a particular team. In order to characterize the particularities of the match and the situation at each interval, we considered the following explanatory variables:

1. Home advantage (*HomeAdv*) and competition (*Competition*). Home advantage is a pervasive phenomenon in sport and soccer. The variable *HomeAdv* identifies whether a team is playing as a Home (*H*) or Away (*A*) team, whereas the variable *Competition* identifies each soccer league: French Ligue 1, Spanish La Liga, English Premier League, Italian Serie A and German Bundesliga.
2. Team strength and winning odds (*WinOdds*). To adjust the model more accurately, information has been included on team strength by employing a new variable constructed on the basis of betting odds. As gambling on professional soccer is a highly liquid market, it can be assumed that betting odds include the full combination of information about team strength and other game-related effects. To a certain extent, betting odds are much more accurate than any other measure of team strength as it can be assumed that these take into account, when relevant, such effects as (de)motivation prior to the game, form, stadium, rivalry between clubs, home advantage, competition, etc. Hence they are more sensitive than any other method for quantifying team strength, such as the Elo ratings or points at the end of the season. The Football-Data.co.uk (2020) database of European soccer leagues results was used to obtain the betting odds from different bookmakers (Bet and Win, Bet365 and William Hill) for each game. These values were transformed into probabilities by eliminating the bookmaker overround in accordance with the standard methodology described by Wunderlich and Memmert (2018), assuming a proportional margin for all outcomes of a game:
   Let $odds_{i,j}$ be the betting odds offered by bookmaker $i$ $(i = 1 \dots B)$ of outcomes $j = 1$ (home win), 2 (draw) and 3 (away win). Then the margin $m_i$ for the $i$th bookmaker can be calculated as:

$$m_i = \sum_{j=1}^{3} \frac{1}{odds_{i,j}} \tag{1}$$

   and the probabilities $p_{i,j}$ and $p_j$ for outcome $j$ are:

$$p_{i,j} = \frac{1}{m_i odds_{i,j}} \tag{2}$$

$$p_j = \sum_{i=1}^{B} \frac{p_{i,j}}{B} \tag{3}$$

   The resulting probabilities $p_j$ add up to 1. The probability of a draw is distributed equally to both teams, obtaining probabilities of a win/draw for each team and game. Thus, $p_W = p_1 + p_2/2$ for a team playing at home and $p_W = 1 - (p_1 + p_2/2)$ for a team playing away. These probabilities are transformed using a logit function to avoid floor/ceiling effects, and linearize the relation with the goal rate, ultimately obtaining a quantitative measure of the chances of a win or loss. Hereinafter this variable is called *WinOdds*, which is formally the log-odds of the probability of victory (contemplating part of the probabilities of a draw): $WinOdds = \log(p_W/(1-p_W))$. In fact, for any game between any two teams A and B, $WinOdds_A + WinOdds_B = 0$. This means that the models only require the specific term of the team in question, without the need to include information about the opponent.

3. Goal difference (*GoalDif*). The goal difference at the beginning of the interval has been included as a quantitative variable. The purpose of including this variable is to adjust the scoring rate by the current goal difference, and to evaluate the extent to which the effect of a red or yellow card depends on this variable.
4. Red cards (*RedDif*) and yellow cards (*YellowDif*). For each interval of time, we obtained the number of players to have received red or yellow cards. Thus, the superiority scenario at the beginning of the interval (0: 11 vs. 11, +1: 11 vs. 10, −1: 10 vs. 11, etc.) is considered using a categorical variable named *RedDif*. In the case of yellow cards, a quantitative variable measuring the difference in number of booked players between opposing teams has been included. A relevant consideration regarding this last variable is that the number of booked players has been corrected when one of the players on a yellow card is substituted or sent off.
5. Current clock time (*ClockTime*) and superiority scenario time (*ScenarioTime*). In order to take into account game evolution, the current clock time at the beginning of each interval has been considered. The elapsed time from the beginning of the red card scenario has also been included, i.e. the elapsed time from the last shown red card in either team. If no red cards are shown *ScenarioTime* is set to 0. Both variables are regarded as continuous variables and are recorded in minutes. However, for comparability and computational reasons, we standardized them to take values in the interval [0, 1] by dividing by the total match duration. Note that in practice, *Clocktime* provides the same information as the remaining time.
6. Interval length (*IntervalLength*) and injury time (*InjuryTime*). The database has included specific intervals for each of the periods of time added on for stoppages. The inclusion of this phase is more than relevant as this is when some of the most intense soccer gets played, generating a higher rate of goals and yellow and/or red cards, and it tends to be the tightest games that produce the longest injury times. The dataset for each team and match consists of 20 intervals, 18 for normal time and 2 for stoppage time. Although the intervals are generally five minutes long, injury times are of variable duration, from just a few seconds to periods sometimes greater than 10 min. In order to adequately weight the number of goals scored in these intervals, a weighting variable was created (*IntervalLength*), dividing the interval duration in minutes by 5. Finally, injury time has been taken into account by means of a categorical variable named *InjuryTime* with different values for regular intervals (*RT*) or injury times at the end of the first or second half (*I1* and *I2*).

We therefore obtained a database made up of 1826 games. For each game we have information about the two teams and 20 time intervals, with a total of 73,040 observations.

Table 1 shows the processed data corresponding to the English Premier League game between Chelsea and Burnley on 12/08/2017 that ended 3-2 to the visiting team. In this particular game, the home team had two players sent off after 13 and 81 minutes. The first was a direct red card and the latter was for a second bookable offense. It can be observed that the data on *WinOdds*, goal difference, red and yellow cards for one team are the same measure for the other team with the opposite sign.

**Table 1** Game details per time interval for the Premier League game between Chelsea and Burnley in the 2017/18 season

| Home Adv | WinOdds | Period | Injury time | Clock time | Interval duration | Goal | Goal Dif | Yellow Dif | Red Dif | Superiority Time |
|---|---|---|---|---|---|---|---|---|---|---|
| H | 1.74 | 1H | RT | 0.0 | 5.0 | 0 | 0 | 0 | 0 | 0.0 |
| H | 1.74 | 1H | RT | 5.0 | 5.0 | 0 | 0 | 1 | 0 | 0.0 |
| H | 1.74 | 1H | RT | 10.0 | 5.0 | 0 | 0 | 1 | 0 | 0.0 |
| H | 1.74 | 1H | RT | 15.0 | 5.0 | 0 | 0 | 1 | −1 | 2.0 |
| H | 1.74 | 1H | RT | 20.0 | 5.0 | 0 | 0 | 2 | −1 | 7.0 |
| H | 1.74 | 1H | RT | 25.0 | 5.0 | 0 | −1 | 2 | −1 | 12.0 |
| H | 1.74 | 1H | RT | 30.0 | 5.0 | 0 | −1 | 2 | −1 | 17.0 |
| H | 1.74 | 1H | RT | 35.0 | 5.0 | 0 | −1 | 2 | −1 | 22.0 |
| H | 1.74 | 1H | RT | 40.0 | 5.0 | 0 | −2 | 2 | −1 | 27.0 |
| H | 1.74 | 1H | I1 | 45.0 | 3.3 | 0 | −3 | 2 | −1 | 32.0 |
| H | 1.74 | 2H | RT | 48.3 | 5.0 | 0 | −3 | 2 | −1 | 37.0 |
| H | 1.74 | 2H | RT | 53.3 | 5.0 | 0 | −3 | 2 | −1 | 42.0 |
| H | 1.74 | 2H | RT | 58.3 | 5.0 | 0 | −3 | 2 | −1 | 47.0 |
| H | 1.74 | 2H | RT | 63.3 | 5.0 | 0 | −3 | 1 | −1 | 52.0 |
| H | 1.74 | 2H | RT | 68.3 | 5.0 | 1 | −3 | 0 | −1 | 57.0 |
| H | 1.74 | 2H | RT | 73.3 | 5.0 | 0 | −2 | 0 | −1 | 62.0 |
| H | 1.74 | 2H | RT | 78.3 | 5.0 | 0 | −2 | 0 | −1 | 67.0 |
| H | 1.74 | 2H | RT | 83.3 | 5.0 | 0 | −2 | 0 | −1 | 72.0 |
| H | 1.74 | 2H | RT | 88.3 | 5.0 | 1 | −2 | −1 | −2 | 2.7 |
| H | 1.74 | 2H | I2 | 93.3 | 4.5 | 0 | −1 | 0 | −2 | 7.7 |
| A | −1.74 | 1H | RT | 0.0 | 5.0 | 0 | 0 | 0 | 0 | 0.0 |
| A | −1.74 | 1H | RT | 5.0 | 5.0 | 0 | 0 | −1 | 0 | 0.0 |
| A | −1.74 | 1H | RT | 10.0 | 5.0 | 0 | 0 | −1 | 0 | 0.0 |
| A | −1.74 | 1H | RT | 15.0 | 5.0 | 0 | 0 | −1 | 1 | 2.0 |
| A | −1.74 | 1H | RT | 20.0 | 5.0 | 1 | 0 | −2 | 1 | 7.0 |
| A | −1.74 | 1H | RT | 25.0 | 5.0 | 0 | 1 | −2 | 1 | 12.0 |
| A | −1.74 | 1H | RT | 30.0 | 5.0 | 0 | 1 | −2 | 1 | 17.0 |
| A | −1.74 | 1H | RT | 35.0 | 5.0 | 1 | 1 | −2 | 1 | 22.0 |
| A | −1.74 | 1H | RT | 40.0 | 5.0 | 1 | 2 | −2 | 1 | 27.0 |
| A | −1.74 | 1H | I1 | 45.0 | 3.3 | 0 | 3 | −2 | 1 | 32.0 |
| A | −1.74 | 2H | RT | 48.3 | 5.0 | 0 | 3 | −2 | 1 | 37.0 |
| A | −1.74 | 2H | RT | 53.3 | 5.0 | 0 | 3 | −2 | 1 | 42.0 |
| A | −1.74 | 2H | RT | 58.3 | 5.0 | 0 | 3 | −2 | 1 | 47.0 |
| A | −1.74 | 2H | RT | 63.3 | 5.0 | 0 | 3 | −1 | 1 | 52.0 |
| A | −1.74 | 2H | RT | 68.3 | 5.0 | 0 | 3 | 0 | 1 | 57.0 |
| A | −1.74 | 2H | RT | 73.3 | 5.0 | 0 | 2 | 0 | 1 | 62.0 |
| A | −1.74 | 2H | RT | 78.3 | 5.0 | 0 | 2 | 0 | 1 | 67.0 |
| A | −1.74 | 2H | RT | 83.3 | 5.0 | 0 | 2 | 0 | 1 | 72.0 |
| A | −1.74 | 2H | RT | 88.3 | 5.0 | 0 | 2 | 1 | 2 | 2.7 |
| A | −1.74 | 2H | I2 | 93.3 | 4.5 | 0 | 1 | 0 | 2 | 7.7 |

**Table 2** Betting odds and corrected probabilities for the Premier League game between Chelsea and Burnley in the 2017/18 season

| | Odds | | | Corrected probabilities | | |
|---|---|---|---|---|---|---|
| Bookmaker | Home | Draw | Away | Home (%) | Draw (%) | Away (%) |
| BW | 1.22 | 6.5 | 12.5 | 77.80 | 14.60 | 7.60 |
| B365 | 1.25 | 6.5 | 15 | 78.40 | 15.10 | 6.50 |
| WH | 1.25 | 5.5 | 13 | 75.60 | 17.20 | 7.30 |

As for the *WinOdds* variable, the odds on a win, draw or loss for the home team offered by different bookmakers are shown in Table 2.

The estimated margin is 5.35%, 2.05% and 5.87% for each bookmaker respectively. The probabilities of a win/draw for the home team given by each bookmaker are 85.1%, 85.9% and 84.1%. The average $p_W$ is 85% and the *WinOdds* for that team is 1.74. So, for the visiting team, $p_W = 15\%$ and *WinOdds* = $-1.74$.

## 3 Methodological approach

To study the influence of red cards on the performance of the different teams, the number of goals scored was modeled in 5 min intervals, taking into account contextual and other variables that could be predictive of the rate of goals scored, including the variables described before, quadratic effects and interactions.

The most natural model for count data is a generalized linear model (GLM) with Poisson distribution using the logarithm as a link function between expected values and explanatory variables (McCullagh and Nelder 2019). This model assumes that events occur randomly at a constant rate over the observed time period conditionally on the explanatory variables. It can be formulated as follows:

$$\begin{aligned} \log(\mathrm{E}(Y_i)) &= X_i\beta + \log(w_i) \\ Y_i &\sim \mathrm{Poi}(\mu_i) \end{aligned} \tag{4}$$

where $i$ is the observation index, $Y$ is the response variable, $X$ is the design matrix for fixed effects, $\beta$ is the vector of model coefficients and $w$ is an offset variable that if considered, enables modulation of the expectation in relation to its magnitude (in our case, the interval length is included as an offset). This model can be fitted by restricted maximum likelihood. It assumes that observations are independent and that the distribution is equidispersed, i.e. $\mathrm{V}[Y_i] = \mathrm{E}[Y_i] = \mu_i$.

In the present case, given the repeated measures that appear in the data, observations of the same individuals (teams, match, etc.) may be correlated. In order to adapt the former model to the presence of correlated data it is common to include random effects (also called random intercepts) associated to experimental units that are sampled repeatedly. In longitudinal data, it is also common to evaluate whether there are random time trends associated to those units (also called random slopes). These models can be formulated as:

$$\begin{aligned} \log(\mathrm{E}[Y_i|\gamma]) &= X_i\beta + Z_i\gamma + \log(w_i) \\ Y_i|\gamma &\sim \mathrm{Poi}(\mu_i) \\ \mathrm{V}[Y_i|\gamma] &= R \end{aligned} \tag{5}$$

where $Z$ is the design matrix for the random effects, $\gamma$ is a vector of normally distributed random coefficients (or random effects) and $R$ is a matrix that contains the variance functions of the model (determined by the distribution of the data) and when necessary, other terms related to the correlation structure of the data. The matrix R allows to take into account other correlation structures that random effects cannot account for, such as negative correlations between groups of units. When there are no effects in R, these models can be fitted by maximum likelihood and are referred as conditional GLMMs (Lee et al. 2018). Alternatively they can also be fitted using a pseudo-likelihood approach (Molenberghs and Verbeke 2005). When part of the correlation structure is also specified through the matrix R, the models are referred as marginal GLMMs and can only be fitted by the pseudo-likelihood method.

On the other hand, the Poisson distribution has the property that the expectation is equal to the variance. The equidispersion assumption can be evaluated allowing for the presence of a free dispersion parameter, estimated using Pearson's $\chi^2$ statistic, so that $\mathrm{V}[Y_i] = \mu_i \phi$:

$$\hat{\phi} = \sum_i \frac{(y_i - \mu_i)^2}{\mu_i} \tag{6}$$

In order to select the most appropriate model, different steps were applied. First, in order to simplify the fixed part of the model (i.e. explanatory variables), a hierarchical backward stepwise selection procedure based on the smallest Akaike Information Criteria (AIC) index was performed. The selection was hierarchical in the sense that the main effects were not removed whenever a higher order term involving this variable was present. Secondly, the best variance and covariance structure was chosen from a set of proposals considering AIC (when available) and pseudo-AIC indexes. Conditional GLMMs can be compared using the AIC criterion, however marginal GLMMs can only be compared with other models when they share the same fixed and random effects (i.e. if they only differ in the R matrix) by means of a pseudo-AIC. Finally, in order to quantify the explained variability of the initial and final models a pseudo-$\mathrm{R}^2$ for GLMMs (Nakagawa and Holger 2013) and RMSE were also computed.

The model estimates presented in the results section are least square means (LSMEANS, also called empirical marginal means) estimations, i.e. point estimates of different levels of interest evaluated at the average of other explanatory variables or random effects. The inverse transformation of the link function was applied in order to provide the results in terms of scoring rates. In graphical representations, 95% confidence intervals are also shown.

After the initial exploration, we decided to exclude from the analysis such intervals where more than one player had been sent off from either team, since these scenarios are very infrequent and scoring rates cannot be estimated with enough precision. This affected 196 observations out of 73,040.

The model was validated by revising the lack of pattern in the residual plots against the predicted values. Moreover, we validated our proposal based on 5-min intervals using shorter lengths of 2 and 3 min. The results obtained from these models are essentially the same, but offer slightly more sensitivity related to a larger sample size and require much greater computational effort when evaluating random components. Since, by dividing games in 5-minutes intervals the count of goals scored per interval is very rarely 2 or more, We also used a GLMM with Bernoulli distribution and a logit link to validate the model (with response values greater than 1 replaced with 1). Given the low event rate, models fitted with a logit or a log link produce similar results.

**Table 3** Descriptive summary for goals, yellow, red cards and substitutions per match and match location for the different competitions in the dataset

| Venue | Competition | Goals | 1st Yellow cards | 2nd Yellow cards | Direct Red cards | Total Red cards | Substituted players |
|---|---|---|---|---|---|---|---|
| Home | England | 1.53 (1.34) | 1.51 (1.27) | 0.02 (0.15) | 0.03 (0.16) | 0.05 (0.23) | 2.73 (0.57) |
| | France | 1.53 (1.35) | 1.76 (1.21) | 0.04 (0.19) | 0.07 (0.26) | 0.11 (0.31) | 2.84 (0.43) |
| | Germany | 1.60 (1.28) | 1.56 (1.28) | 0.03 (0.17) | 0.04 (0.20) | 0.07 (0.26) | 2.89 (0.34) |
| | Italy | 1.46 (1.31) | 1.86 (1.27) | 0.04 (0.21) | 0.05 (0.24) | 0.10 (0.32) | 2.92 (0.27) |
| | Spain | 1.55 (1.38) | 2.27 (1.48) | 0.06 (0.24) | 0.05 (0.21) | 0.11 (0.32) | 2.89 (0.34) |
| Away | England | 1.15 (1.18) | 1.59 (1.28) | 0.03 (0.16) | 0.03 (0.18) | 0.06 (0.23) | 2.75 (0.52) |
| | France | 1.19 (1.13) | 2.01 (1.26) | 0.05 (0.21) | 0.07 (0.25) | 0.12 (0.32) | 2.83 (0.41) |
| | Germany | 1.19 (1.14) | 1.82 (1.22) | 0.04 (0.20) | 0.03 (0.16) | 0.07 (0.25) | 2.90 (0.34) |
| | Italy | 1.22 (1.19) | 2.11 (1.31) | 0.07 (0.26) | 0.07 (0.26) | 0.14 (0.37) | 2.93 (0.28) |
| | Spain | 1.15 (1.19) | 2.63 (1.53) | 0.05 (0.22) | 0.03 (0.17) | 0.08 (0.28) | 2.89 (0.33) |

Values represent mean and standard deviation

# 4 Results

Table 3 presents a descriptive summary of the number of goals, red and yellow cards and substitutions per game for the different competitions in the countries in the sample and depending on whether they are home or away teams. Although the data used are based on 5-min intervals, the diagrams show overall rates.

Figure 1 shows the scoring, card and substitution raw rates per interval of time depending on whether the teams are at home or away. The average number of players sent off per game is 0.18. Of these, 51.5% are direct red cards, whereas 48.5% correspond to second bookings. The average time when sendings off occur is at minute 67.4. In particular, second bookings occur on average at minute 74.4 and direct red cards at minute 60.8. An average of 3.84 players receive a yellow card per game, approximately 20% of whom are substituted before the game ends, while 2.3% receive a second booking and are sent off.

## 4.1 Model fitting

The following models were initially evaluated: a model considering all explanatory variables, quadratic terms for all quantitative variables, and all interactions involving *RedDif* or *YellowDif* (M1) and a simplified version of M1 after a hierarchical backward stepwise selection procedure (M2). Subsequently a set of models was fitted to evaluate M2 with different variance components: random intercepts for Team, Match or Match nested in Team (M3, M4, M5); random intercepts and time slopes for Team, Match or Match nested in Team (M6, M7, M8); including the covariance between opposing teams for the same interval (M9) and the final model (M10) that considers all the random components found to be relevant in the previous models.

Table 4 shows the RMSE, AIC and pseudo-AIC fit indexes for the different evaluated models. M2 is the model with the smallest AIC considering independent observations. The variance components fitted in models M4, M5, M7 and M8 are not relevant, essentially because the presence of *WinOdds* in the model eliminates any variability that may exist

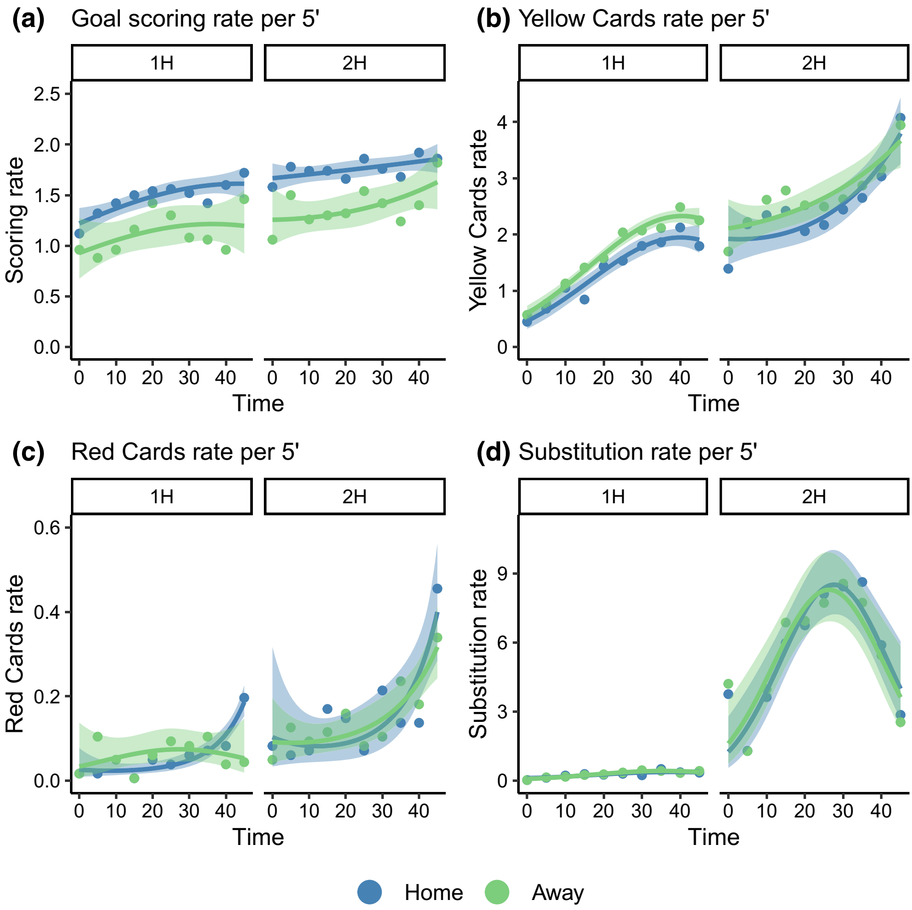



**Fig. 1** **a** Goals, **b** red cards, **c** yellow cards and **d** substitution rates for Home and Away teams over time and match period. Rates are adjusted to reflect the total number of events if the rate was constant for the full duration of the match. Smooth fits using the loess method are shown as well as 95% confidence intervals (Cleveland, 1979)

**Table 4** Fit indexes for different model proposals

| Model | Variance components | AIC | Pseudo-AIC | RMSE |
|---|---|---|---|---|
| M2: | None (independent observations) | 35475.1 | 411240.7[a] | 0.06479 |
| M3: | Team (random intercepts) | 35470.8 | 410817.2[b] | 0.06471 |
| M9: | Interval\|Match (random intercepts) | NA | 411204.4[a] | 0.06479 |
| M10: | Team (random intercepts) and Interval\|Match (random intercepts) | NA | 410750.5[b] | 0.06470 |

[a] Pseudo-AIC with different letters cannot be compared
[b] Models not presented have irrelevant variance components and do not improve the model fit

**Table 5** Results of the Poisson generalized mixed model, for the analysis of the 5-minute scoring rate

| Variable | M1 | | | M10 | | |
|---|---|---|---|---|---|---|
| | Estimate | Std. error | *P* value | Estimate | Std. error | *P* value |
| Intercept | −2.958 | 0.055 | < 0.001 | −2.958 | 0.043 | < 0.001 |
| HomeAdv—Home | −0.025 | 0.031 | 0.420 | | | |
| Competition—E | −0.045 | 0.045 | 0.318 | | | |
| Competition—F | 0.006 | 0.044 | 0.886 | | | |
| Competition—G | 0.061 | 0.047 | 0.196 | | | |
| Competition—I | −0.039 | 0.045 | 0.381 | | | |
| Extratime—I1 | 0.092 | 0.113 | 0.416 | | | |
| Extratime—I2 | 0.124 | 0.079 | 0.119 | | | |
| WinOdds | 0.421 | 0.020 | < 0.001 | 0.397 | 0.018 | < 0.001 |
| $\text{WinOdds}^2$ | 0.008 | 0.012 | 0.513 | | | |
| GoalDif | −0.038 | 0.014 | 0.006 | −0.039 | 0.013 | 0.003 |
| $\text{GoalDif}^2$ | 0.008 | 0.005 | 0.103 | 0.009 | 0.005 | 0.067 |
| RedDif -1 | −1.821 | 1.177 | 0.122 | −1.496 | 1.114 | 0.179 |
| RedDif +1 | 1.498 | 0.499 | 0.003 | 1.623 | 0.479 | 0.001 |
| YellowDif | 0.054 | 0.109 | 0.623 | 0.010 | 0.016 | 0.532 |
| $\text{YellowDif}^2$ | −0.007 | 0.008 | 0.350 | | | |
| ClockTime | 0.710 | 0.213 | 0.001 | 0.601 | 0.193 | 0.002 |
| $\text{ClockTime}^2$ | −0.354 | 0.221 | 0.109 | −0.227 | 0.193 | 0.239 |
| YellowDif*RedDif -1 | −0.207 | 0.126 | 0.101 | | | |
| YellowDif*RedDif +1 | −0.216 | 0.109 | 0.048 | | | |
| WinOdds*RedDif -1 | −0.001 | 0.138 | 0.995 | 0.089 | 0.125 | 0.476 |
| WinOdds*RedDif +1 | −0.309 | 0.088 | 0.001 | −0.267 | 0.086 | 0.002 |
| $\text{WinOdds}^2$ *RedDif -1 | 0.096 | 0.085 | 0.256 | | | |
| $\text{WinOdds}^2$ *RedDif +1 | 0.051 | 0.062 | 0.416 | | | |
| GoalDif*RedDif -1 | 0.034 | 0.076 | 0.656 | | | |
| GoalDif*RedDif +1 | 0.010 | 0.056 | 0.866 | | | |
| $\text{GoalDif}^2$ *RedDif -1 | 0.018 | 0.027 | 0.505 | | | |
| $\text{GoalDif}^2$ *RedDif +1 | 0.018 | 0.020 | 0.386 | | | |
| ClockTime*RedDif -1 | 1.836 | 3.689 | 0.619 | 1.125 | 3.547 | 0.751 |
| ClockTime*RedDif +1 | −4.021 | 1.764 | 0.023 | −4.382 | 1.658 | 0.008 |
| $\text{ClockTime}^2$ *RedDif -1 | −0.922 | 2.762 | 0.739 | −0.469 | 2.665 | 0.860 |
| $\text{ClockTime}^2$ *RedDif +1 | 3.107 | 1.417 | 0.028 | 3.554 | 1.340 | 0.008 |
| ScenarioTime*RedDif -1 | 5.017 | 2.106 | 0.017 | 5.194 | 2.056 | 0.012 |
| ScenarioTime*RedDif +1 | 0.102 | 1.115 | 0.927 | | | |
| $\text{ScenarioTime}^2$ *RedDif -1 | −8.072 | 3.142 | 0.010 | −8.062 | 3.071 | 0.009 |
| $\text{ScenarioTime}^2$ *RedDif +1 | 0.114 | 1.418 | 0.936 | | | |
| WinOdds*YellowDif | −0.003 | 0.019 | 0.882 | | | |
| $\text{WinOdds}^2$ *YellowDif | 0.003 | 0.012 | 0.790 | | | |
| GoalDif*YellowDif | −0.001 | 0.011 | 0.939 | −0.001 | 0.010 | 0.879 |

**Table 5** continued

| Variable | M1 | | | M10 | | |
|---|---|---|---|---|---|---|
| | Estimate | Std. error | *P* value | Estimate | Std. error | *P* value |
| GoalDif$^2$ *YellowDif | −0.007 | 0.004 | 0.060 | −0.008 | 0.004 | 0.034 |
| ClockTime*YellowDif | 0.120 | 0.565 | 0.832 | | | |
| ClockTime$^2$ *YellowDif | 0.152 | 0.477 | 0.750 | | | |
| ScenarioTime*YellowDif | −0.138 | 0.464 | 0.767 | | | |
| ScenarioTime$^2$ *YellowDif | −0.209 | 0.430 | 0.626 | | | |
| Covariance team | | | | 0.011 | 0.005 | |
| Covariance interval\|Match | | | | −0.025 | 0.005 | |
| Dispersion | 0.969 | | | 0.991 | | |
| Degrees of freedom | 72800 | | | | 72728 | |
| RMSE | | 0.06475 | | 0.06470 | | |
| $R^2_{GLMM(m)}$ | 6.16% | | | 6.02% | | |

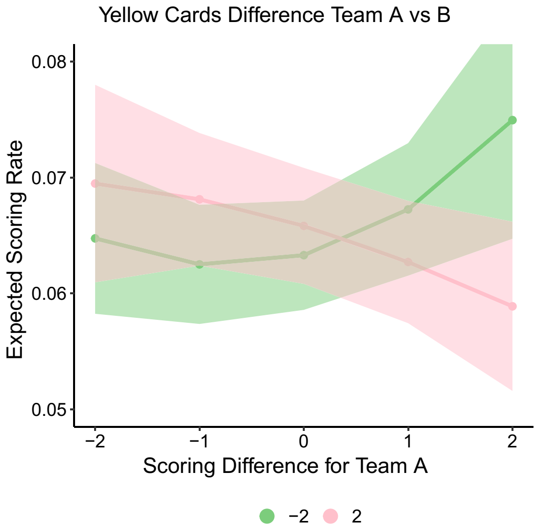
Model M1 includes the explanatory variables, quadratic terms and potential interactions of interest, without variance components. Model M10 consists of those variables remaining in the model after a hierarchical backward stepwise selection procedure including the variance components for Team and Interval|Match. For the categorical variable *RedDif*, the reference category is 0 (11 vs. 11)

between games whereas time variables capture the potential autocorrelation between consecutive observations of the same game. Model M3 considers a random effect associated to the Team that turns out to be relevant. This component might reflect playing styles and other team-related characteristics that *WinOdds* does not capture. However adding random slopes (M6) does not improve the fit. Model M9 includes the covariance between opposing teams for the same interval. This component is also relevant and has a negative sign, reflecting a negative correlation between both teams at the interval level. The final model M10 considers simultaneously both covariance components included in models M3 and M9. Adding a free dispersion parameter does not improve the model, since the empirical dispersion in model M10 is 0.991 (close to 1).

Table 5 presents the results of models M1 and the selected final model M10 obtained after fitting a Poisson Generalized Mixed Model with the variance components described earlier.

The final model detects a highly relevant effect of the *WinOdds* criterion. There is no effect associated to competition or playing as a home team as this information is in fact already reflected by the aforesaid variable. The model also depicts large differences associated to playing with a player less or having an extra player and several interactions between this variable and other terms: with *WinOdds*, *ClockTime*, *ScenarioTime* and their respective quadratic terms. As for Yellow cards, a slight interaction with the quadratic term for goal difference is detected. In average terms, the model reveals that the scoring rate is approximately 0.065 goals per 5-min interval. The difference between teams when *WinOdds* is 0.38 versus −0.38 is around 37%, these values represent the difference in favor of the home side, since the average *WinOdds* for Home teams is 0.38. This ratio is maintained throughout the game. However, the scoring rate does have an upward tendency, right at the start of the game the average scoring rate is 0.054 and reaches rates of 0.079. The relative performance between teams also depends on the goal difference and the difference in booked players. The winning team's performance drops if it has more booked players, see Fig. 2. With equal numbers of

**Fig. 2** Scoring rate for different number of players warned by Scoring difference



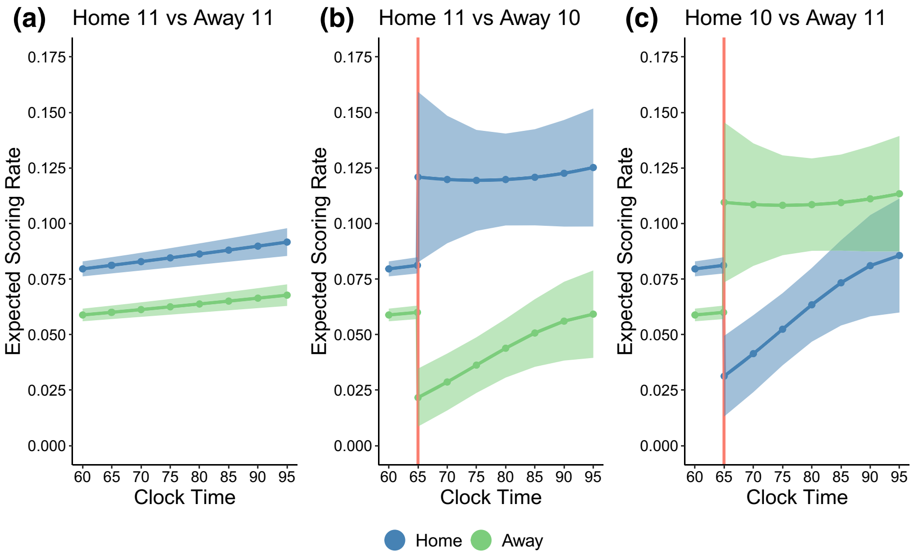




**Fig. 3** Scoring rate for different Red Card Scenarios for Home and Away teams through time **a** 11 versus 11, **b** 11 versus 10 and **c** 10 versus 11, respectively

booked players (not shown in the plot), the scoring rate is slightly lower (around 10%) when the team is one or two goals ahead.

The effect over time of sending off a player is presented graphically. To this end, Fig. 3 presents the expected scoring rates and their 95% confidence intervals at different times for the following situations: no players sent off and a player sent off at minute 65 for teams with either *WinOdds* = 0.38 or *WinOdds* = −0.38. The idea behind these particular estimates is again to reproduce the situation where a Home or an Away player is sent off, and taking into account that the average time for a player to be sent off is approximately at minute 67.5.

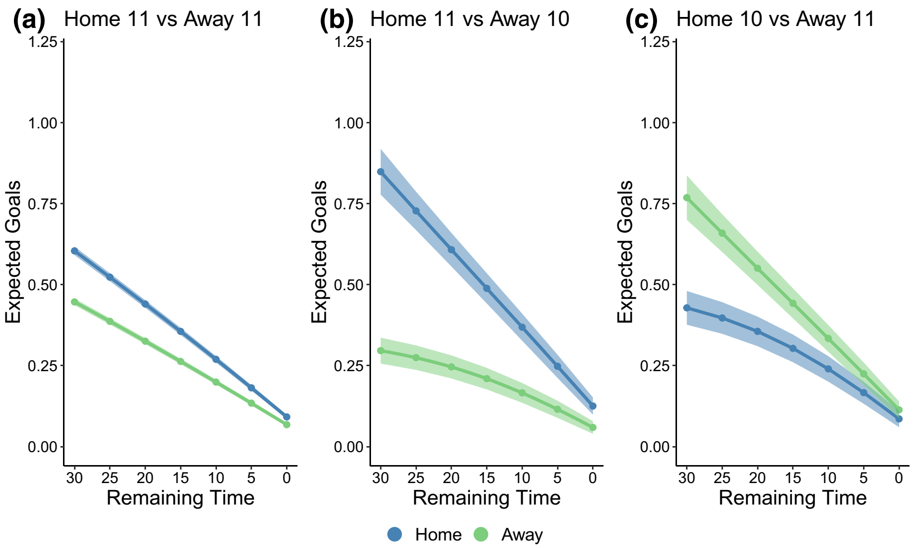



**Fig. 4** Cummulative scoring rates for different Red Card scenarios for Home and away teams through time **a** 11 versus 11, **b** 11 versus 10, **c** 10 versus 11, respectively

After a sending off, scoring rates undergo an abrupt change. To exemplify the situation, at minute 65 the scoring rates per interval are on average 0.081 and 0.060 for the Home and Away teams. If a red card is shown to the visiting team, the scoring rates become 0.121 and 0.022, whereas if the affected team is playing at home, the rates are 0.031 and 0.109 respectively. The relative difference in performance is more or less maintained when the player sent off belongs to the Away team. However, when the affected team is stronger, the relative difference tends to decrease over time.

The accumulated scoring rate presented in Fig. 4 allows us to quantify the consequences of a sending off. For instance, when the remaining time is 30 minutes, the difference in accumulated expected goals is 0.16 in favor of the Home team. If an Away player is sent off, the difference becomes 0.55, whereas if the player dismissed plays for the Home team, the difference would be −0.34. In this sense, the expected impact of a sending off at 30 min to finish, is approximately 0.39 or 0.50 goals if the guilty player is on the visiting team or on the home team.

## 5 Discussion

This study analyses the effect of red and yellow cards on performance, using the scoring rate from the perspective of cohesion over time. The analysis is based on Poisson distribution models, which are more appropriate for count data than the linear regression models used in other studies, such as (Carmichael and Thomas 2005) and (Mechtel et al. 2011). The analysis also takes into account the correlated nature of the data incorporating the pertinent variance components (random effects) that provide a more valid inference framework.

Events that happen in injury time are also included and given the appropriate weight. This consideration allows for more concise modeling of the scoring rate as the game progresses. The model also incorporates as an adjustment variable all variables associated to each game and team including in particular the home/away and team strength effects based on a transformation of betting odds. This latter criterion gives rise to a more efficient model that can be used to more clearly evaluate the study goals. The analysis also takes into account two time-related variables, the current clock time and the time elapsed from the red card scenario. This consideration enables differentiation of the effect of the remaining time from the effect of playing in numerical inferiority over time. Unlike the findings of (Caliendo and Radic 2006), but in agreement with (Lago-Peñas et al. 2016) and (Bar-Eli et al. 2006), this analysis does detect a relevant change in scoring rates associated to red cards. As suggested by Červený et al. (2018) our model detects an interaction with playing time, in particular the red card effect diminishes over time when the player sent off belongs to the strongest team.

The study by Mechtel et al. (2011) detected certain asymmetry depending on whether a team is playing at home or away, whereby ”sending-offs against home teams have a negative impact on their performance. However, for guest teams, the impact depends on the time remaining after the sending-off and can be positive if the sending-off occurs late in the game.” Our analysis does not detect such a pattern.

The number of players that have been cautioned with a yellow card has a slight effect on the scoring rate, in agreement with (Anders and Rotthoff 2011) and (Titman et al. 2015). In our case, the effect appears through an interaction with the goal difference. In particular, teams with a greater number of booked players have a lower scoring rate when they are winning. Since the first yellow card is a precursor to a second booking, the effect of yellow cards could be indirect. It would be interesting to conduct a more detailed analysis of the relationship between yellow and red cards, and how bookings affect a player's performance and how recommendable it is for booked players to be substituted.

As for playing time, a certain upward tendency has been detected as a game progresses, with the scoring rate increasing by around 2% per interval. Other variables measured at game level, such as competition, team strength, home advantage, attendance, etc. that are commonly included in similar studies (e.g. Chowdhury 2015; Mechtel et al. 2011) have been analyzed implicitly within the *WinOdds* variable, a transformation of the odds of a win, and thus cannot be discussed individually. Indeed, *WinOdds* constitutes a very relevant variable, and shows an interaction with the red card situation. It should be noted that there are some limitations to our study. While it is comprehensive in that it considers the top five European leagues, it would be interesting to compare our results to others for knockout competitions or weaker leagues (2nd divisions). On the other hand, the use of the *WinOdds* variable based on betting odds has the advantage of including all variables related to teams and the particular match; but it does not distinguish between the individual effects of each component. The current study found that a sending off is a significant event that has a dramatic influence on the outcome of a match and particularly produces a decrease in the scoring rate for the penalized team and an increase for the opposing team, and if taking place with 30 minutes of remaining time, it translates to more or less 0.5 goals. In some cases, red cards are received as a punishment for preventing an obvious scoring occasion, and it is therefore interesting to evaluate the extent to which it is better to concede a goal or receive a red card. It should be noted that in this situation, in addition to the sending off, the penalized team will also be punished with a penalty or a free kick.

For instance:

- At the 2010 World Cup in South Africa, in the final minute of overtime in the quarter final between Ghana and Uruguay and with the score at 1–1, Luis Suárez stopped a clear goal with his hand, leading to a red card and penalty. Ghanaian captain Asamoah Gyan took the penalty and the ball hit the crossbar. The game went to a penalty shootout and

Uruguay won. In this situation, Luis Suárez clearly made the right decision, but it would not have been such a clever move had he done so in the first minute of overtime: there would have been a penalty (the conversion rate of which is almost 80%) and his team would have played the remaining 30 min a man down.

- In the Spanish Liga game between Real Madrid and FC Barcelona in December 2017, with Barça leading 1-0 with 30 min to go, Real's Dani Carvajal stopped a goal with his hand: red card and penalty. Leo Messi's converted penalty as good as clinched the win in a game that ended 3-0.
- In the 2020 Spanish Super Cup Final between Real Madrid and Atlético Madrid, with the teams level at 1-1 with minutes to go before the end of overtime, Real's Federico Valverde fouled Álvaro Morata outside the penalty area when the latter was through on goalkeeper Thibaut Courtois. This was a clear goalscoring opportunity and the offender was sent off. The scoring rate in one-on-one situations is approximately 40% while for well-positioned direct free kicks it is between 10 and 20%. As the game was coming to an end, and the foul was outside the penalty area, Valverde would appear to have made the right decision. Real Madrid won the game in a penalty shootout.

Nowadays, thanks to Video Assistant Referee (VAR) technology, the possibility of a referee not spotting an offense is negligible. Hence, just at the end of the game, it will make sense to prevent a clear goalscoring opportunity and be punished with a red card plus the consequent penalty.

Coaches and players should be very cautious and try to avoid situations where players might receive a red card. Otherwise, teams need to be prepared for these scenarios of numerical inferiority.

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## Declarations

**Conflict of interest** The authors declare that they have no conflict of interest.



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