



# Multiple criteria ranking method based on functional proximity index: un-weighted TOPSIS

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## Abstract

The technique for order preference by similarity to ideal solution (TOPSIS) is a widely used ranking method which provides a composite index representing the relative proximity of each decision alternative to an ideal solution. The relative proximity index construction relays on the use of a single criterion aggregation approach. Its output, regardless the certainty or uncertainty nature of the problem's data, is usually a real number. In TOPSIS classical approach alternatives are ordered based on these numbers. The closer the number to 1, the higher the position of the alternative in the ranking. However, although the relative proximity index can be highly sensible to the weighting scheme, as far as the authors of this work know, the relative proximity index has never been treated as a function. In this work, a new TOPSIS approach is proposed in which weights are not fixed *a priori*. On the contrary, they are handled as decision variables in a set of optimization problems where the objective is to maximize the relative proximity of each alternative to the ideal solution. The only possible *a priori* information about the weights is that related to the existence of upper and lower bounds in their values. This information is incorporated into the optimization problems as constraints. The result is a new relative proximity index which is a function depending on the values of the weights. This feature of the proposed method could be useful in some decision situations in which the determination of subjective precise weights from decision makers could be problematic.

**Keywords** Weighting schemes · TOPSIS · Un-weighted TOPSIS

## 1 Introduction

Multiple Criteria Decision Analysis (MCDA) methods are widely used to solve real and complex decision-making problems characterized by the existence of multiple conflicting goals. Roy (1996) distinguishes four multiple criteria decision problematics: selection, sorting, ranking and description. Within each of these types of problematics, a large number of approaches exist which significantly differ in terms of complexity, type of data, kind

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of aggregation procedure and determination of their weighting schemes (Watröbski et al. 2019).

In this work, we will address the problem regarding the determination of weighting schemes reflecting the relative importance of the criteria in a widely used MCDA ranking method, TOPSIS. A common classification of weighting methods is the one that distinguishes among objective and subjective methods. A significant number of works can be found in the literature dealing with objective weights' determination in TOPSIS-based approaches (see Ouenniche et al. 2018). In this objective weighting schemes the relative importance of the criteria is mainly given by the nature of the data or by regulations or norms, for example, and not directly established by the decision maker based on expert knowledge or subjective preferences in most cases, difficult to be uphold, especially in those situations where public rankings are published.

The use of subjective weighting schemes is therefore, more controversial than the use of objective ones (see for example, Jacquet-Lagrèze and Siskos 1982; Watröbski et al. 2019). although is more common in the context of TOPSIS-based approaches. The subjective preference is usually assigned by the decision makers based on their own experiences, knowledge and perception of the problem. Barron and Barrett (1996), Hobbs (1980), Alemi-Ardakani (2016), Eshlaghy and Radfar (2006) and Németh et al. (2019) are some of the authors that have published reviews on subjective methods for the determination of weights in MCDA models.

The inherent difficulty of assigning reliable subjective weights is our main concern in this paper and has been well addressed by several authors (see Deng et al. 2000). Fisher (1995), Mareschal (1988) and Triantaphyllou and Sanchez (1997) use a sensitivity analysis approach to give decision makers' flexibility in assigning criteria weights trying to show how they could affect final decisions. Ribeiro (1996) proposes an interactive method with which decision makers are able to select the desirable preference elicitation technique. Yeh et al. (1999) develop a weighting approach which takes into account the specific characteristics of a decision situation in order to effectively assign weights and select the most suitable alternative. What the authors of this work wonder is, to what extent do the previous methods contribute to obtain a solution which cannot be obtained with a more general method not requiring the a priori establishment of subjective weights. A large number of ranking agencies, public and private, using multiple criteria to rank different alternatives, use subjective weights to express the different important of decision criteria. However, the obtained rankings are quite sensitive to changes in the weights of the criteria and therefore, subjective weighting schemes are subject to important criticisms.

In this work, a new approach will be proposed in which criteria weights are not fixed in an exact way a priori, overcoming in this way one of the main criticisms given to rankings obtained using subjective weights. In our approach, weights will be handled as decision variables in a set of optimization problems where the objective is to maximize the relative proximity of a set of decision alternatives to an ideal solution. The only possible a priori information about the weights is that related to the existence of upper and lower bounds in their values. This information is incorporated into the optimization problems as constraints. The result is a function depending on the values of the weights which measures the distance to an ideal solution. This feature of the proposed method could be useful in some decision situations in which the determination of precise subjective weights in advance from decision makers could be problematic. By means of two examples, we show how, under certain circumstances, the direct establishment of subjective weights at the beginning of the ranking process can be avoided, overcoming thus, one of the main problematics around subjective weighting schemes determination. The proposed method

provides a range of variation of the relative proximity index for each alternative. If we take the midpoint of this range as a reference for this index (for example), the result can be compared with that obtained with a choice of specific weights. With this, the decision maker can know if for a given alternative, the choice of those weights favours it or not. In real decision-making contexts, decision makers would like to avoid a situation in which an alternative is clearly favoured, preferring a classification of alternatives in which all of them have a relative proximity index with a low percentage of variation with respect to the average.

In the next section, we will briefly present the main steps of the classical TOPSIS approach which is the method selected to illustrate our approach. In Sect. 3, we will introduce our proposal of an un-weighted functional TOPSIS method (UW-TOPSIS). Two numerical examples will be discussed in Sect. 4 and a real case study will be presented in Sect. 5, where a sample of financial companies will be ranked based on their degree of diversity and inclusion. Finally, in Sect. 6 main conclusions will be presented.

## 2 Classical TOPSIS

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang and Yoon 1981) ranks decision alternatives based on their simultaneous distance to a positive ideal solution (PIS) and a negative ideal solution (NIS). The positive ideal solution maximizes criteria of the type “the more, the better” and minimizes criteria of the type “the less, the better”, whereas the negative ideal solution minimizes “the more, the better” criteria and maximizes “the more, the better” criteria. Distance to the PIS is minimized and distance to the NIS maximized. TOPSIS makes full use of the attribute information, provides a cardinal ranking of alternatives, and does not require the attribute preferences to be independent (Chen and Hwang 1992; Yoon and Hwang 1995).

TOPSIS allows total linear compensation among criteria using a single criterion aggregation approach (Roy 1996). The type of preferential information in a classical TOPSIS approach is deterministic and cardinal. Weights of the criteria in TOPSIS-based approaches may be quantitative, qualitative or relative; precise or uncertain and objectively or subjectively determined by one or more decision makers (Watröbski et al. 2019). In what follows we describe the main steps in the method:

*Step 1. Determine the decision matrix*  $D$ , where the number of criteria is  $m$  and the number of alternatives is  $n$ ,  $D = [x_{ij}]_{n \times m}$ .

*step 2. Construct the normalized decision matrix* Criteria are expressed in different scaling and therefore a normalizing procedure is necessary in order to facilitate comparison. Hwang and Yoon (1981) propose a vector normalization,<sup>1</sup>

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^n (x_{ij})^2}} \in [0, 1], \quad 1 \leq i \leq n, \quad 1 \leq j \leq m. \quad (1)$$

<sup>1</sup> In addition to the vector normalization proposed in the seminal paper by Hwang and Yoon, many other normalization processes have been used (Ouenniche et al. 2018).

*step 3. determine the weighted normalized decision matrix* It is well known that the weights of the criteria in decision making problems do not have the same mean and not all of them have the same importance. The weighted normalised value  $v_{ij}$  is calculated as:

$$v_{ij} = w_j r_{ij}, \quad 1 \leq i \leq n, \quad 1 \leq j \leq m, \tag{2}$$

where  $w_j$  is the weight associated to each criterion.

*step 4. Determine the positive ideal (PIS) and negative ideal solutions (NIS).* The positive ideal solution,  $A^+ = (v_1^+, \dots, v_m^+)$ , and the negative ideal solution,  $A^- = (v_1^-, \dots, v_m^-)$ , are determined as follows:

$$v_j^+ = w_j r_j^+ = \begin{cases} \max_{1 \leq i \leq n} v_{ij}, & j \in J \\ \min_{1 \leq i \leq n} v_{ij}, & j \in J' \end{cases} \quad 1 \leq j \leq m, \tag{3}$$

$$v_j^- = w_j r_j^- = \begin{cases} \min_{1 \leq i \leq n} v_{ij}, & j \in J \\ \max_{1 \leq i \leq n} v_{ij}, & j \in J' \end{cases} \quad 1 \leq j \leq m, \tag{4}$$

where  $J$  is associated with the criteria that indicate profits or benefits and  $J'$  is associated with the criteria that indicate costs or losses.

*step 5. Calculate the separation measures* Calculation of the separation of each alternative with respect to the PIS and NIS, respectively:

$$d_i^+ = \left( \sum_{j=1}^m (v_{ij} - v_j^+)^2 \right)^{1/2}, \quad d_i^- = \left( \sum_{j=1}^m (v_{ij} - v_j^-)^2 \right)^{1/2}, \quad 1 \leq i \leq n. \tag{5}$$

*step 6. Calculate the relative proximity to the ideal solution* Calculation of the relative proximity of each alternative to the PIS and NIS using the proximity index.

$$R_i = \frac{d_i^-}{d_i^+ + d_i^-}, \quad 1 \leq i \leq n. \tag{6}$$

The  $R_i$  value lies between 0 and 1. If  $R_i = 1$ , then  $A_i = A^+$  and if  $R_i = 0$ , then  $A_i = A^-$ . The closer the  $R_i$  value is to 1 the higher the priority of the  $i$ -th alternative.

*step 7. Rank the preference order.* Rank the best alternatives according to  $R_i$  in descending order.

**Remark 1** To facilitate the comparison of the classic TOPSIS with the UW-TOPSIS we express (6) in the following way:

$$R_i = \frac{\left( \sum_{j=1}^m (w_j r_{ij} - w_j r_j^-)^2 \right)^{1/2}}{\left( \sum_{j=1}^m (w_j r_{ij} - w_j r_j^+)^2 \right)^{1/2} + \left( \sum_{j=1}^m (w_j r_{ij} - w_j r_j^-)^2 \right)^{1/2}} \tag{7}$$

A large number of approaches can be found in the literature dealing with the previous questions and giving rise to different TOPSIS-based approaches (see for instance Behzadian et al. 2012; Zyoud and Fuchs-Hanusch 2017).

### 3 UW-TOPSIS approach

In what follows we will present the steps of the new algorithm proposed in this paper which does not require the introduction of a priori weights. Steps 1 and 2 remain the same than in the classical TOPSIS. However, the PIS and NIS solutions are determined now without taking into account the relative importance of the criteria. Weights are introduced as unknowns in step 4 when separation measures from the PIS and NIS are calculated. Their values are determined in step 5 solving two groups of mathematical programming problems which maximize and minimize the separation of each alternative to the PIS and NIS respectively, taking into account different constraints referred to the values of the weights. These constraints include the classical constraint in TOPSIS approaches which ensures all the weights are positive and sum up one and other constraints imposing lower and upper bounds on the weights. The resulting mathematical programming problems are, due to the nature of their objective, fractional mathematical programming problems. In what follows we describe the main steps of the method in detail.

*Step 1* Determine the decision matrix  $[x_{ij}]$ ,  $1 \leq i \leq n$ ,  $1 \leq j \leq m$ , where the number of alternatives is  $n$  and the number of criteria is  $m$ .

*Step 2* Construct the normalized decision matrix

$$[r_{ij}], r_{ij} \in [0,1], \quad 1 \leq i \leq n, \quad 1 \leq j \leq m. \quad (8)$$

*Step 3.* Determine the positive ideal  $A^+ = (r_1^+, \dots, r_m^+)$  and the negative ideal solutions  $A^- = (r_1^-, \dots, r_m^-)$ , where

$$r_j^+ = \begin{cases} \max_{1 \leq i \leq n} r_{ij}, & j \in J \\ \min_{1 \leq i \leq n} r_{ij}, & j \in J' \end{cases} \quad 1 \leq j \leq m, \quad (9)$$

$$r_j^- = \begin{cases} \min_{1 \leq i \leq n} r_{ij}, & j \in J \\ \max_{1 \leq i \leq n} r_{ij}, & j \in J' \end{cases} \quad 1 \leq j \leq m, \quad (10)$$

where  $J$  is associated with “the more, the better” criteria and  $J'$  is associated with “the less, the better” criteria.

*Step 4.* Let us consider  $\Omega = \left\{ w = (w_1, \dots, w_m) \in \mathbb{R}^m, w_j \in [0,1], \sum_{j=1}^m w_j = 1 \right\}$ . Given  $A^+, A^-$ , we define two separation functions,

$$D_i^+ : \Omega \times \mathbb{R}^m [0,1], D_i^- : \Omega \times \mathbb{R}^m [0,1], \quad 1 \leq i \leq n,$$

Given by

$$D_i^+(w) = d((w_1 r_{i1}, \dots, w_m r_{im}), (w_1 r_1^+, \dots, w_m r_m^+)), \quad 1 \leq i \leq n, \quad (11)$$

$$D_i^-(w) = d((w_1 r_{i1}, \dots, w_m r_{im}), (w_1 r_1^-, \dots, w_m r_m^-)), \quad 1 \leq i \leq n, \quad (12)$$

where  $d$  is a distance function in  $\mathbb{R}^m$ .

*Step 5.* Calculate the function of relative proximity to the ideal solution,  $R_i : \Omega \rightarrow [0,1]$ ,  $1 \leq i \leq n$ , as

$$R_i(w) = \frac{D_i^-(w)}{D_i^+(w) + D_i^-(w)}, \quad 1 \leq i \leq n. \tag{13}$$

*Step 6* For each  $i, 1 \leq i \leq n$ , we calculate the values  $R_i^L(w), R_i^U(w)$  solving the two following mathematical programming problems where decision variables are the criteria weights:

$$R_i^L = \text{Min} \left\{ R_i(w), \sum_{j=1}^m w_j = 1, l_j \leq w_j \leq u_j, 1 \leq j \leq m \right\}, \quad 1 \leq i \leq n, \tag{14}$$

$$R_i^U = \text{Max} \left\{ R_i(w), \sum_{j=1}^m w_j = 1, l_j \leq w_j \leq u_j, 1 \leq j \leq m \right\}, \quad 1 \leq i \leq n, \tag{15}$$

being  $l_j, u_j \geq 0$  lower and upper bounds for each criterion’s weight. Then, we obtain  $n$  relative proximity intervals,

$$R_i^I = [R_i^L, R_i^U], \quad 1 \leq i \leq n. \tag{16}$$

*Step 7* We rank the intervals  $R_1^I, R_2^I, \dots, R_n^I$  (see Remark 2).

**Remark 2** According to Canós and Liern (2008), given the intervals  $A = [a_1, a_2]$ , and  $B = [b_1, b_2]$  contained in, we will say that  $A$  is bigger than  $B$ , if and only if

$$A > B \Leftrightarrow \begin{cases} k_1 a_1 + k_2 a_2 > k_1 b_1 + k_2 b_2, & k_1 a_1 + k_2 a_2 \neq k_1 b_1 + k_2 b_2 \\ a_1 > b_1, & k_1 a_1 + k_2 a_2 = k_1 b_1 + k_2 b_2 \end{cases}$$

where  $k_1$  and  $k_2$  are two pre-established positive constants. In the context that concerns us, the values  $k_1$  and  $k_2$  inform us about the degree of confidence of the decision maker that the alternatives are in their best position or on the contrary (Canós and Liern 2008). When ordering the intervals  $[R_i^L, R_i^U], 1 \leq i \leq n$ , the relation  $k_2/k_1$  informs us about the importance (or truthfulness) given to the best situation of the alternatives  $R_i^U$  regarding of the worst situation  $R_i^L$ . In the following examples, since we do not have information that makes us opt for the best or worst situation, we have chosen to give the same importance to both, that is,  $k_1 = k_2 = 1$ .

By construction, UW-TOPSIS is a generalization of the classical TOPSIS approach. Indeed, as it is proven in the following result, if some conditions are added to the formulation of the UW-TOPSIS, this coincides with the classical TOPSIS approach.

**Proposition 1** *If in UW-TOPSIS the following conditions are verified:*

1. *a vector normalization is used in (8),*
2. *the Euclidean distance is used in (11) and (12),*
3. *bounds given in (14) and (15) verify  $l_j = u_j = w_j^0, 1 \leq j \leq m$ ,*

*then, UW-TOPSIS and classical TOPSIS coincide.*

**Proof** According to (a), normalization given in (8) is

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^n (x_{ij})^2}}, \quad 1 \leq i \leq n, 1 \leq j \leq m.$$

Taking into account (b), the Euclidean distance in (11) and (12) implies

$$R_i(w) = \frac{\left(\sum_{j=1}^m (w_j r_{ij} - w_j r_j^-)^2\right)^{1/2}}{\left(\sum_{j=1}^m (w_j r_{ij} - w_j r_j^-)^2\right)^{1/2} + \left(\sum_{j=1}^m (w_j r_{ij} - w_j r_j^-)^2\right)^{1/2}} \tag{17}$$

On the other hand, (c) implies that the weight for each criterion is the same in all the alternatives, i.e.  $\Omega$  has only one vector,  $\Omega = \{(w_1^0, w_2^0, \dots, w_m^0)\}$ . Taking into account the expressions of  $R_i$  and  $R_i(w)$  [see (7) and (17)], we have that  $R_i(w) = R_i$ . Besides,  $R_i^L = R_i^U = R_i(w_1^0, \dots, w_m^0)$ ,  $1 \leq i \leq n$ , and  $R_i^I = [R_i^L, R_i^U] = \{R_i\}$ . Therefore, UW-TOPSIS and classical TOPSIS coincide.

In addition, under certain conditions, optimization problems (14) and (15) appearing in UW-TOPSIS are linear programming problems.

**Proposition 2** *If and distance  $d$  in (11) and (12) is the Manhattan distance, then.*

(a) *The relative proximity function to the positive ideal solution can be calculated as.*

$$R_i(w) = \sum_{j=1}^m w_j r_{ij}, \quad w \in \Omega, \quad 1 \leq i \leq n. \tag{18}$$

(b) *Intervals  $R_i = [R_i^L, R_i^U]$ ,  $1 \leq i \leq n$ , given in (16) are obtained solving two linear programming problems,*

$$R_i^L = \text{Min} \left\{ \sum_{j=1}^m w_j r_{ij}, \sum_{j=1}^m w_j = 1, \quad l_j \leq w_j \leq u_j, \quad 1 \leq j \leq m \right\},$$

$$R_i^U = \text{Max} \left\{ \sum_{j=1}^m w_j r_{ij}, \sum_{j=1}^m w_j = 1, \quad l_j \leq w_j \leq u_j, \quad 1 \leq j \leq m \right\},$$

where decision variables are the criteria weights and the objective is to minimize and maximize, respectively, the relative proximity of each alterantive to the positive ideal solution

**Proof** In order to prove (a), let us consider and as in (8). As, we have that As  $d$  is the Manhattan distance, we know that = and Then,

$$D_i^+(w) = d(wr_i, wA^+) = \sum_{j=1}^m |w_j r_{ij} - w_j| = \sum_{j=1}^m (w_j - w_j r_{ij}) = \sum_{j=1}^m w_j - \sum_{j=1}^m w_j r_{ij} = 1 - \sum_{j=1}^m w_j r_{ij}.$$

$$D_i^-(w) = d(wr_i, wA^-) = \sum_{j=1}^m |w_j r_{ij}| = \sum_{j=1}^m w_j r_{ij}.$$

Replacing in (13), we have

$$R_i(w) = \frac{D_i^-(w)}{D_i^+(w) + D_i^-(w)} = \frac{\sum_{j=1}^m w_j r_{ij}}{1 - \sum_{j=1}^m w_j r_{ij} + \sum_{j=1}^m w_j r_{ij}} = \sum_{j=1}^m w_j r_{ij}, \quad 1 \leq i \leq n$$

Part (b) is immediate from (a) and expressions (14) and (15).

**Remark 3** The hypothesis given in Proposition 2, is not as restrictive as it may seem. In fact, it would be enough to transform all the criteria into the-more-the better criteria (Ouenniche et al. 2018) and to consider the PIS and NIS independent from the data, such that they do not need to be modified if new data are incorporated. On the other hand, if we normalize data taking into account their similarity with the ideal reference,

$$r_{ij}^- = \begin{cases} \frac{x_{ij} - A_j}{a_j - A_j}, & A_j \leq x_{ij} \leq a_j \\ 1, & a_j \leq x_{ij} \leq b_j \quad 1 \leq i \leq n, 1 \leq j \leq m, \\ \frac{B_j - x_{ij}}{B_j - b_j} & b_j \leq x_{ij} \leq B_j \end{cases} \tag{19}$$

where  $[A_j, B_j]$  is the range for the valuations of criterion  $j$  and  $[a_j, b_j] \subseteq [A_j, B_j]$  is the ideal fixed for criterion  $j$ , we can demonstrate that  $A^+$  and  $A^-$ , expressed in (9) and (10), will be  $A^+ = (1, \dots, 1), A^- = (0, \dots, 0)$  (see Acuña-Soto et al. 2018).

**Remark 4** In this work, we have only taken into account the case in which the decision matrix is expressed in terms of precise values. The generalization to the case in which the matrix is composed of uncertain data and expressed by intervals will depend on the selected departure model for the UW-TOPSIS approach.

If the relative proximity of each alternative is expressed using a real number  $R_i$  (Jahanshahloo et al. 2006), values  $R_i^L$  and  $R_i^U$  will be obtained similarly to (14) and (15).

- (a) If the relative proximity of each alternative is expressed using an interval  $[R_i^1, R_i^2]$  (León et al. 2019), values  $R_i^L$  and  $R_i^U$  are calculated in a similar manner than in (14) and (15) but in this case for  $R_i^1$  and  $R_i^2$ , that is

$$R_i^L = \text{Min} \left\{ \min_{w \in \Omega^*} R_i^1(w), \min_{w \in \Omega^*} R_i^2(w) \right\}, \tag{20}$$

$$R_i^U = \text{Max} \left\{ \max_{w \in \Omega^*} R_i^1(w), \max_{w \in \Omega^*} R_i^2(w) \right\}, \tag{21}$$

where  $\Omega^* = \{w \in \Omega, l_j \leq w_j \leq u_j, 1 \leq j \leq m\}$ .



**Table 1** Decision matrix *Source: Kao (2010)*

Alternatives	# Ref.	Criteria					
		C1	C2	C3	C4	C5	C6
		Maximal speed (km/h)	Horse power (CV)	Space (m <sup>2</sup> )	Consumption in town (lt/100km)	Consumption at 120km/h (lt/100km)	Price (Francs)
Peugeot 505 GR	1	173	10	7.88	11.4	10.01	49,500
Opel Record 2000 LS	2	176	11	7.96	12.3	10.48	46,700
Citroën Visa Super “E”	3	142	5	5.65	8.2	7.3	32,100
VW Golf 1300 GLS	4	148	7	6.15	10.5	9.61	39,150
Citroën CX 2400 Pallas	5	178	<b>13</b>	8.06	14.5	11.05	64,700
Mercedes 230	6	180	<b>13</b>	<b>8.47</b>	13.6	10.4	75,700
BMW 520	7	<b>182</b>	11	7.81	12.7	12.26	68,593
Volvo 244 DL	8	145	11	8.38	14.3	12.95	55,000
Peugeot 104 ZS	9	161	7	5.11	8.6	8.42	35,200
Citroën Dyane	10	117	3	5.81	<b>7.2</b>	<b>6.75</b>	<b>24,800</b>
Objective		Max	Max	Max	Min	Min	Min

## 4 Illustrative examples

Let us present in this section, two illustrative examples in order to show the advantages of the proposed method. To be able to compare the obtained results we will use two published examples from Jacquet-Lagrèze and Siskos (1982) and Alper and Basdar (2017), respectively. The authors of both papers used a classical TOPSIS approach to solve to different real problems. To apply UW-TOPSIS we have used the Euclidean distance and the optimization problems have been solved with LINGO.

**Example 1** In this section we will first present a real numerical example used by Kao (2010), previously proposed by Jacquet-Lagrèze and Siskos (1982). The example consisted of a real decision problem in which the best car is selected from a set of 10 cars taken into account 6 criteria. Table 1 displays the decision matrix of this problem. In the first column we have displayed the alternatives followed by a reference number which identifies them. Last six columns display the assessment of the cars with regards to six different decision criteria.

The three first criteria are of the type the-more-the-better (speed, power and space) and the three last ones of the type the-less-the-better for the decision maker (consumption in town, consumption at 120km/h and price). From a direct observation of data in Table 1, we can see how, for equal importance of the criteria, there is not a clear winner. We have used bold characters to identify the best value for each criterion.

The car with the highest speed is the model BMW 520; Citroën CX and Mercedes 230 are both the cars with more power being Mercedes 230 as well, the car with most space. With regards to the criteria to be minimized, there is a clear winner, Citroën Dyane, which has the minimum consumption in town and at 120 km/h and the minimum price.

**Table 2** Criteria Weights *Source: Kao (2010)*

	Maximal Speed (Km/h)	Horse power (CV)	Space (m <sup>2</sup> )	Consumption in town (lt/100km)	Consumption at 120 km/h (lt/100 km)	Price (Francs)
Weight	0.6346	0.01	0.01	0.01	0.01	0.3254

**Table 3** Weighted decision matrix

Alternatives	# Ref.	Criteria					
		Maximal speed (km/h)	Horse power (CV)	Space (m <sup>2</sup> )	Consumption in town (lt/100 km)	Consumption at 120 km/h (lt/100 km)	Price (Francs)
Peugeot 505 GR	1	109.786	0.100	0.079	0.114	0.100	16,107,300
Opel Record 2000 LS	2	111.690	0.110	0.080	0.123	0.105	15,196,180
Citroën Visa Super “E”	3	90.113	0.050	0.057	0.082	0.073	10,445,340
VW Golf 1300 GLS	4	93.921	0.070	0.062	0.105	0.096	12,739,410
Citroën CX 2400 Pallas	5	112.959	<b>0.130</b>	0.081	<i>0.145</i>	0.111	21,053,380
Mercedes 230	6	114.228	<b>0.130</b>	<b>0.085</b>	0.136	0.104	<i>24,632,780</i>
BMW 520	7	<b>115.497</b>	0.110	0.078	0.127	0.123	22,320,162
Volvo 244 DL	8	92.017	0.110	0.084	0.143	<i>0.130</i>	17,897,000
Peugeot 104 ZS	9	102.171	0.070	<i>0.051</i>	0.086	0.084	11,454,080
Citroën Dyane	10	<i>74.248</i>	<i>0.030</i>	0.058	<b>0.072</b>	<b>0.068</b>	<b>8,069,920</b>
Objective		Max	Max	Max	Min	Min	Min

Therefore, if the same importance was given to all the criteria and, from a simple direct observation of the decision matrix, a decision maker might select the model Citroën Dyane, as it is the apparent winner with regard to three different criteria.

Let us now suppose that criteria have different importance for the decision maker. Let us consider now the weights expressed in Table 2 which are the ones proposed by Kao (2010). These weights,  $w_i$ , are positive and they sum up one.

We can observe how the highest importance is given to speed, followed by the price having the rest of criteria the same importance. Speed, however, has almost double importance than price.

If we now observe the weighted decision matrix displayed in Table 3, we can see how the situation is the same than the previously obtained one. However, based on rational decision criteria such as the one highlighted in Ishizaka and Labib (2011), if a decision context with more than two criteria one criterion has an importance greater than 50% compared with the rest, the MCDM selection process would not be necessary as the selected alternative should be the one with the best assessment on that criterion.

In this example, speed is given an importance of more than 60% and thus, from the previous point of view, alternative 7, BMW 520, should be selected. However, this is not a car with a low price and this criterion has almost 30% of importance. A new decision might be then taken taking into account this fact.

In order to select a car taking into account all the considered criteria, even those criteria with only 1% of importance, power, space and consumption in town, the classical

**Table 4** Weighted and normalized decision matrix

Alternatives	# Ref.	Criteria					
		Maximal speed (km/h)	Horse power (CV)	Space (m <sup>2</sup> )	Consumption in town (lt/100 km)	Consumption at 120 km/h (lt/100 km)	Price (Francs)
Peugeot 505 GR	1	0.21497	0.00327	0.00345	0.00311	0.00313	0.09858
Opel Record 2000 LS	2	0.21870	0.00360	0.00348	0.00335	0.00328	0.09301
Citroën Visa Super “E”	3	0.17645	0.00164	0.00247	0.00224	0.00229	0.06393
VW Golf 1300 GLS	4	0.18391	0.00229	0.00269	0.00286	0.00301	0.07797
Citroën CX 2400 Pallas	5	0.22118	0.00426	0.00352	0.00395	0.00346	0.12886
Mercedes 230	6	0.22367	0.00426	0.00370	0.00371	0.00326	0.15076
BMW 520	7	0.22615	0.00360	0.00342	0.00346	0.00384	0.13661
Volvo 244 DL	8	0.18018	0.00360	0.00366	0.00390	0.00405	0.10954
Peugeot 104 ZS	9	0.20006	0.00229	0.00223	0.00234	0.00264	0.07010
Citroën Dyane	10	0.14538	0.00098	0.00254	0.00196	0.00211	0.04939
Objective		Max	Max	Max	Min	Min	Min

**Table 5** Positive and negative ideal solutions

	Maximal speed (km/h)	Horse power (CV)	Space (m <sup>2</sup> )	Consumption in town (lt/100 km)	Consumption at 120 km/h (lt/100 km)	Price (Francs)
Postive ideal solution	0.22615	0.00426	0.00370	0.00196	0.00211	0.04939
Negative ideal solution	0.14538	0.00098	0.00223	0.00395	0.00405	0.15076

**Table 6** Ranking obtained using classical TOPSIS

Alternatives	$R_i$	Alternatives	$R_i$
Peugeot 104 ZS	0.74473	Citroën Dyane	0.55642
Opel Record 2,000 LS	0.67828	Citroën CX 2400 Pallas	0.49787
Citroën Visa Super “E”	0.64012	BMW 520	0.48465
Peugeot 505 GR	0.63288	Mercedes 230	0.43589
VW Golf 1300 GLS	0.61731	Volvo 244 DL	0.41628

TOSPIS approach described in Sect. 2 will be applied. After normalization of all the data and taking into account the different weights of the criteria (see Table 4), a positive and a negative ideal solutions are identified. The positive ideal solution describes an imaginary car with best scores in all criteria. On the contrary the negative ideal solution represents an imaginary car with the worst scores. Table 5 displays the scores of both imaginary cars for each criterion.

The obtained ranking of the cars, based on the classical TOPSIS approach described in Sect. 2, is the one displayed in Table 6.

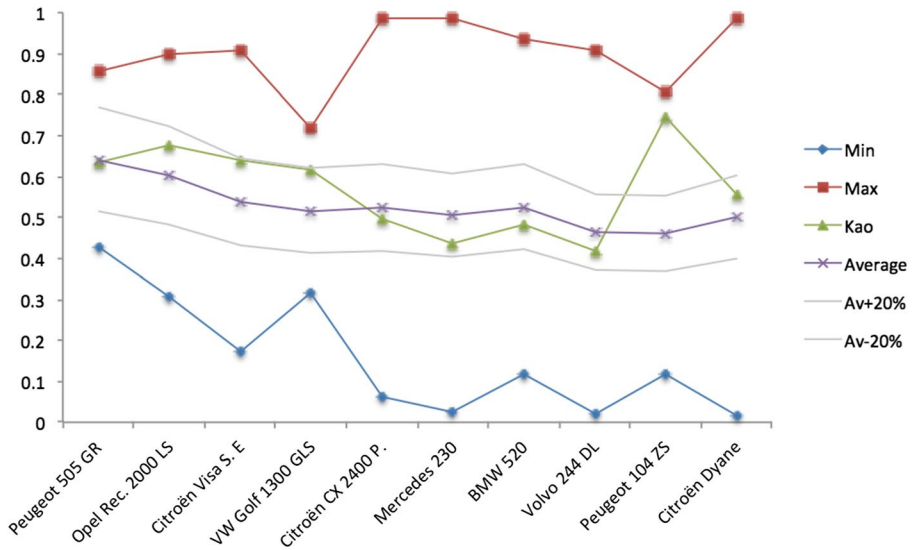


Fig. 1 Example 1: Similarity ratios

As we can observe, car number 9, Peugeot 104 ZS, is ranked first taking into account the weights fixed by the decision maker. However, if we focus only on the speed, which is a criterion with more than 60% importance, this car appears in the sixth position. The decision maker is giving the price almost a 30% of importance and with regards to this criterion, car number 9 occupies the third position. All the cars ranked on top of Peugeot with respect to speed have a sensible highest price, almost double or in some cases more than double the price of Peugeot 104 ZS. Figure 1 represents the relative position of the 10 cars with respect to the fictitious positive and negative ideal solutions.

Let us apply now the proposed method in this paper described in Sect. 3. With this purpose we will now consider the weights as unknown variables. The only a priori information regarding the weights could be, if considered necessary by the decision maker, the establishment of certain upper and/or lower bounds on the weights. Let us suppose that we want to ensure all the criteria are taken into account (e.g. all criteria have an importance of at least 1%) and that a criterion cannot be given a weight higher than 75%. The following constraints should be then included in the model:

$$l_j = 0.01, u_j = 0.75, 1 \leq j \leq 6.$$

The obtained results applying the steps of the proposed UW-TOPSIS algorithm are the ones displayed in Table 7. Second column shows the obtained intervals expressing the relative proximity of each alternative to the positive ideal solution. The lower and upper bounds are respectively, the minimum and maximum possible relative proximity values for a given set of optimal weights. This information could be very relevant for the decision maker as it let us know the possible range of variation of the relative proximity of an alternative depending on the relative importance given to each criterion.

**Table 7** Ranking using UW-TOPSIS

Alternative	Minimum	Maximum	Average	Kao (2010)
Peugeot 505 GR	0.4276476	0.8560963	0.64187195	0.632877911
Opel Record 2000 LS	0.3078024	0.8984179	0.60311015	0.678284923
Citroën Visa Super “E”	0.1715504	0.9056597	0.53860505	0.640119993
VW Golf 1300 GLS	0.3174212	0.7160294	0.51672530	0.617313555
Citroën CX 2400 Pallas	0.0624455	0.9849322	0.52368883	0.497873164
Mercedes 230	0.0263905	0.9850762	0.50573336	0.435888646
BMW 520	0.1172695	0.9353834	0.52632645	0.484645897
Volvo 244 DL	0.0225147	0.9068528	0.46468374	0.416284604
Peugeot 104 ZS	0.1156937	0.8062151	0.46095440	0.744727239
Citroën Dyane	0.0166673	0.9846395	0.50065342	0.556424329

**Table 8** Decision criteria *Source: Alper and Basdar (2017)*

Criteria	Ratios	Formula	Weights	Type
C1	Current ratio	Current assets/current liabilities	0.180	Max
C2	Leverage ratio	Total liabilities/total assets	0.130	Max
C3	Earnings per share	Net period profit/number of shares	0.200	Max
C4	Return on equity	Net profits/equity	0.160	Max
C5	Asset profitability	Net profits/total assets	0.190	Max
C6	Total assets turnover	Net sales/total assets	0.140	Max
	Total		1.000	

As the relative proximity value is used to determine the position of the alternatives in the ranking, with the proposed method we are able to know the possible worst and best situation of each alternative in the ranking which will depend on the set of weights assigned by the decision maker. The third column displays the values of the decision variables, the criteria weights.

On view of these results it would be easy to interact with the decision maker in order to obtain the solution with which he/she is more comfortable without asking him/her to fix the weights at the beginning of the process.

In Fig. 1, we have represented the similarity ratios obtained using the maximum, minimum and central point (in what follows, average), of the obtained intervals of weights and the values obtained by Jacquet-Lagrèze and Siskos (1982). We can observe, how alternative 9 is the car for which an anomaly can be observed.

**Example 2** Let us now consider a second example in which the criteria weights are not so much extreme, that is, in which all the criteria have an importance of less than 20%. In this example, taken from Alper and Basdar (2017) six factoring companies are ranked based on six financial statement ratios using as well a classical TOPSIS approach.

**Table 9** Original decision matrix  
Source: Alper and Basdar (2017)

Alternative	No	C1	C2	C3	C4	C5	C6
Başer	1	0.888	0.757	0.097	0.071	0.017	0.183
Creditwest	2	1.257	0.796	0.317	0.169	0.034	0.183
Garanti	3	1.063	0.936	0.248	0.106	0.007	0.084
Huzur	4	1.102	0.867	0.214	0.157	0.021	0.239
Lider	5	0.956	0.895	0.712	0.190	0.020	0.142
Yapı Kredi	6	1.074	0.930	1.182	0.175	0.012	0.063

**Table 10** Normalized and weighted decision matrix  
Source: Alper and Basdar (2017)

Alternative	C1	C2	C3	C4	C5	C6
Başer	0.061	0.046	0.013	0.031	0.065	0.065
Creditwest	0.087	0.049	0.044	0.073	0.130	0.065
Garanti	0.073	0.057	0.034	0.046	0.026	0.030
Huzur	0.076	0.053	0.029	0.068	0.079	0.085
Lider	0.066	0.055	0.098	0.082	0.075	0.050
Yapı Kredi	0.074	0.057	0.162	0.076	0.046	0.022
<b>Max</b>	0.087	0.057	0.162	0.082	0.130	0.085
<b>Min</b>	0.061	0.046	0.013	0.031	0.026	0.022

**Table 11** Positive (PIS) and negative ideal solutions (NIS)  
Source: Alper and Basdar (2017)

	C1	C2	C3	C4	C5	C6
PIS	0.087	0.057	0.162	0.082	0.130	0.085
NIS	0.061	0.046	0.013	0.031	0.026	0.022

**Table 12** Rank based on the relative proximity to the PIS obtained by Alper and Basdar (2017) Source: Alper and Basdar (2017)

Alternative	1	2	3	4	5	6
$R_i$	0.250 (5)	0.512 (3)	0.152 (6)	0.397 (4)	0.553 (2)	0.599 (1)

Numbers within parenthesis express the order of the alternative in the obtained ranking

Financial ratio analysis is commonly used to evaluate the financial performance of businesses. Table 8 displays the financial ratios selected by a group of experts as decision criteria and their relative importance determined as well by the group of experts. All the criteria are of the type “the more, the better”.

Table 9 displays the original decision matrix and Table 10 shows the normalized and weighted decision matrix. The positive and negative ideal solutions are shown in Table 11. Positive ideal solution (PIS) is composed of the maximum values in Table 10 and the Negative ideal solution (NIS) is composed of the minimum values in Table 10 (Table 12).

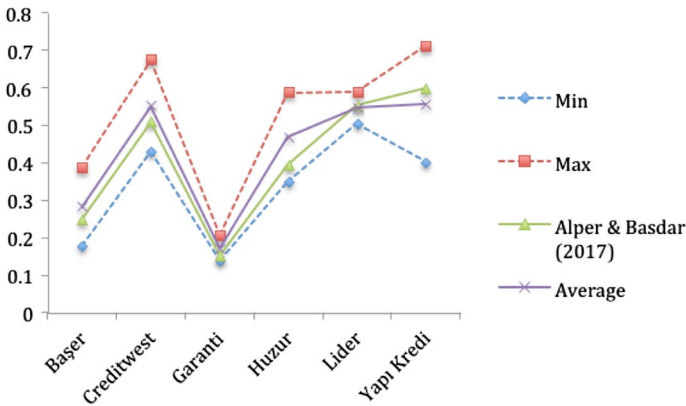


Fig. 2 Example 2: Similarity ratios

Table 13 Obtained relative proximity scores and ranking and comparison between results

Alternative	UW-TOPSIS			
	Minimum	Maximum	Average	Alper and Basdar (2017)
Başer	0.1768904	0.3888506	<b>0.28287050 (5)</b>	<b>0.24758 (5)</b>
Creditwest	0.4283943	0.6743262	0.55136025 (2)	0.50941 (3)
Garanti	0.1368155	0.2060412	<b>0.17142835 (6)</b>	<b>0.14989 (6)</b>
Huzur	0.3489264	0.5876978	<b>0.46831210 (4)</b>	<b>0.39434 (4)</b>
Lider	0.5030404	0.5891874	0.54611394 (3)	0.55208 (2)
Yapı Kredi	0.4017557	0.7128243	<b>0.55729000 (1)</b>	<b>0.59886 (1)</b>

Numbers within parenthesis represent the order in the ranking of the correspondent alternative. Bold characters show coincidence in the ranking

Let us consider now intervals for the weights letting the weights proposed by Alper and Basdar vary between 10% and 20%. The following results have been obtained applying the proposed method in this paper, using the same normalization technique and the same distance function than in the classical TOPSIS approach (see Fig. 2; Table 13).

Examples 1 and 2 show that the ranking using UW-TOPSIS (Average) seems to be different to the one obtained by Kao (2010) and similar to the one obtained by Alper and Basdar (2017). The reason for this difference lies in the choice of weights made in these papers.

While the weights shown in Alper and Basdar (2017) do not significantly favour the ranking position of any alternative (see Fig. 2), the weights in Kao (2010) favour the ranking position of alternative 9. Figure 1 shows how alternative 9 is the closest to the maximum similarity value that could be obtained. Detecting these anomalies may be another of the UW-TOPSIS utilities.

**Table 14** Selected European financial companies

Alternative	Country	Group
F <sub>1</sub>	DE	Insurance
F <sub>2</sub>	IT	Insurance
F <sub>3</sub>	GB	Insurance
F <sub>4</sub>	FR	Insurance
F <sub>5</sub>	ES	Banks
F <sub>6</sub>	GB	Banks
F <sub>7</sub>	FR	Banks
F <sub>8</sub>	FR	Banks

## 5 Case study: ranking firms based on their diversity and Inclusion degree

Lord Kelvin said “You can’t change what you can’t measure”. That is, in words of Diana van Maasdijk, CEO from Equileap, why Equileap collects data on gender equality and this is the purpose of this section. We aim at measuring the degree of gender equality in a sample of 8 companies from the financial sector in Europe, the biggest banks and insurance companies (see Table 14).

We will use data collected by Equileap, the leading organisation providing data and insights on gender equality in the corporate sector. This non-profit company researches and ranks over 3500 public companies in 23 developed countries using their Gender Equality Scorecard™ with 19 criteria (see Table 15), including the gender balance of the workforce, senior management and board of directors, as well as the pay gap, parental leave, and sexual harassment.

Equileap rank companies according to their overall Equileap gender equality score based on the 19 criteria listed in the Scorecard displayed in Table 15. Equileap use a two-fold research approach. First, they gather publicly available information provided by the companies themselves, including in their annual reports, sustainability reports and/or on their websites. Second, they engage with companies to allow them to send Equileap the latest publicly available data they have.

Companies are awarded points on a scale of 0-100 being all the criteria of the type “the more the better”. The first and second stages may produce a series of groups in which companies have the same total score. When two or more companies have the same score, Equileap break the ranking tie by determining which company performs best on criterion C<sub>5</sub>: Promotion and Career Development, and then, if necessary, on criteria C<sub>4</sub>, C<sub>3</sub>, C<sub>2</sub>, C<sub>1</sub>, in this order.

Table 16 displays comparison among Equileap’s ranking and the ranking obtained with our approach. To apply UW-TOPSIS we have used the Euclidean distance. The optimization problems have been solved with LINGO. In addition, we have assumed that each of the four blocks shown in Table 14 has a minimum weight of 5% and a maximum of 40%. On the other hand, since Equileap do not rank using the TOPSIS method, comparing only the scores obtained by both methods does not provide much information. For this reason, we have added, in columns 4 and 8 in Table 16, a score that depends on the order of the company and not on the ranking method used. The points assigned to each company have been calculated as follows:



**Table 15** Gender equality criteria *Source: Equileap (2019)*

Criteria	Description
<i>Gender balance in leadership and workforce</i>	
C <sub>1</sub> board of directors	Gender balance of the company's board of directors
C <sub>2</sub> executives	Gender balance of the company's executives
C <sub>3</sub> senior management	Gender balance of the company's senior management
C <sub>4</sub> workforce	Gender balance of the company's workforce
C <sub>5</sub> promotion and career development	Gender balance of the company's senior management compared to the gender balance of the company's workforce, signalling career progression opportunities
<i>Fair remuneration</i>	
C <sub>6</sub> living wage	Commitment to pay a living wage to all employees, even in those countries that do not legally require a minimum wage
C <sub>7</sub> gender pay gap	Transparency on the gender pay gap at company level and on multiple pay bands, commitment to close the pay gap
C <sub>8</sub> parental leave	Paid leave programs (at least 2/3 paid) for child care to both primary or secondary carers globally or at least in the country of incorporation
C <sub>9</sub> flexible work options	Option to employees to control and/or vary the start and end times and or vary the location from which employees
<i>Policies promoting gender equality</i>	
C <sub>10</sub> training and career development	Commitment to ensure equal access to training and career development irrespective of gender
C <sub>11</sub> recruitment strategy	Commitment to ensure non-discrimination against any type of demographic group and equal opportunities to ensure gender parity
C <sub>12</sub> freedom from violence, abuse and sexual harassment	Prohibit all forms of violence in the work place, including verbal, physical and sexual harassment
C <sub>13</sub> safety at work	Commitment to the safety of employees in the workplace, in travel to and from the workplace, and on company related business, and ensure the safety of vendors in the workplace
C <sub>14</sub> human rights	Commitment to ensure the protection of human rights, including employees' rights to participate in legal, civic and political affairs
C <sub>15</sub> social supply chain	Commitment to reduce social risks in its supply chain such as forbid business-related activities that condone, support, or otherwise participate in trafficking, force and child labour or sexual exploitation
C <sub>16</sub> supplier diversity	Commitment to ensure diversity in its supply chain, including procurement from women owned businesses in the supply chain
C <sub>17</sub> employee protection	Systems and policies for the reporting of internal ethical compliance complaints without retaliation or retribution, such as access to confidential third-party ethics hotlines or systems for confidential written complaints
<i>Commitment, transparency and accountability</i>	
C <sub>18</sub> commitment to women's empowerment	Signatory of the women's empowerment principles

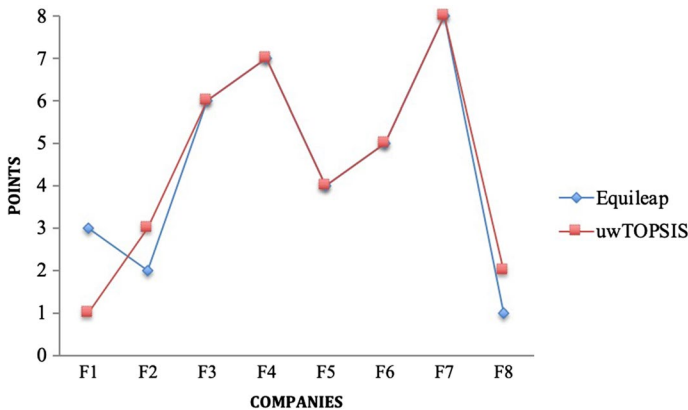
**Table 15** (continued)

Criteria	Description
C <sub>19</sub> audit	Undertaken and awarded an independent gender audit certificate by an Equileap recognised body

**Table 16** Equileap’s scores and ranking and UW-TOPSIS ranking

Firm	Equileap’s			UW- TOPSIS			
	Score	Score/100	Points	R <sub>i</sub> Min	R <sub>i</sub> Max	Average	Points
F <sub>1</sub>	47.3	0.473 (6)	3	0.023580847	0.382649508	0.203115178 (8)	1
F <sub>2</sub>	47.1	0.471 (7)	2	0.049045108	0.428874354	0.238959731 (6)	3
F <sub>3</sub>	56.5	0.565 (3)	6	0.545572756	0.937518003	0.741545379 (3)	6
F <sub>4</sub>	58.9	0.589 (2)	7	0.588540774	0.960165606	0.774353190 (2)	7
F <sub>5</sub>	51.3	0.513 (5)	4	0.095995168	0.489815814	0.292905491 (5)	4
F <sub>6</sub>	56.3	0.563 (4)	5	0.111141278	0.740122102	0.425631690 (4)	5
F <sub>7</sub>	63.0	0.630 (1)	8	0.783793256	0.989253923	0.886523589 (1)	8
F <sub>8</sub>	45.9	0.459 (8)	1	0.039834394	0.411459226	0.225646810 (7)	2

Numbers within parenthesis represent the order in the ranking of the alternative



**Fig. 3** Comparison between Equileap and UW-TOPSIS rankings

$$\text{Points } (F_i) = (\text{Number of companies} + 1) - (\text{order occupied by } F_i) \tag{22}$$

In Fig. 3, we have represented the scores obtained with (20). As we can observe, the representation of these values (columns 4 and 8 in Table 16) facilitates the comparison between rankings obtained with different methods.

The rankings obtained by Equileap and by the UW-TOPSIS are quite similar reflecting this fact the robustness of the ranking published by Equileap. Small differences in the global scores of some firms can be observed among both rankings. However, the tendency lines are almost the same in both cases (see Fig. 3).

## 6 Conclusions

In this work, a new TOPSIS approach is proposed in which weights are not fixed *a priori*. On the contrary, they are handled as decision variables in a set of optimization problems where the objective is to maximize the relative proximity of each alternative to the ideal solution. With this objective, we a new relative proximity index has been proposed which is a function depending on the values of the weights. If we take the maximum and minimum values of this function, given the considered constraints, an interval is obtained for the index of relative proximity of each alternative. When the decision maker considers it necessary or adequate, he/she can fix upper and lower bounds expressing the different importance of the criteria. With this the amplitude of the intervals is reduced.

The proposed method in this paper could be useful in those real decision making contexts in which the decision maker is not able or does not want to fix exact weights for the decision criteria. This is the case addressed in this paper, where a rating body ranks firms based on their diversity and inclusion degree without fixing *a priori* the relative importance of the decision criteria in an objective or subjective way.

The obtained rankings with both, Equileap's method and our approach are quite similar showing the results the robustness of the rankings proposed by Equileap. The main conclusion derived from the similarity of the rankings is that regardless the assigned weights to the different decision criteria, the order in the ranking would remain almost the same. This feature could be of great importance in those situations where reporting and measurement of diversity and inclusion practices are public and mandatory for the firms which is the tendency nowadays in most of the developed countries. In this situation, there would be no need to fix precise weights *a priori* reflecting the different relative importance of the decision criteria. The proposed method in this paper, would allow the ranking of the alternatives taking into account several decision criteria with no need to establish a precise subjective weighting scheme in advance. The ranking problem is carried out for an unknown set of weights and, an interval of relative proximity to the ideal solution is obtained for each decision alternative indicating its possible range of positions in the ranking. The alternative will occupy this range of positions in the ranking regardless the relative importance of the decision criteria.

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## Appendix

Tables 17, 18 and 19 show the values of the weights providing the maximum and minimum  $R_i$  in Examples 1, 2 and the Case Study, respectively. In all cases, the optimization problems have been solved with LINGO.

**Table 17** Weights for the maximum and minimum  $R_i$  values in Example 1

Alternative # Ref	$R_i$ Min weights						$R_i$ Max weights					
	C1	C2	C3	C4	C5	C6	C1	C2	C3	C4	C5	C6
1	0.0333	0.0100	0.0276	0.7500	0.1486	0.0305	0.7500	0.0100	0.2100	0.0100	0.0100	0.0100
2	0.0375	0.0100	0.0324	0.7500	0.1522	0.0179	0.7500	0.0177	0.2023	0.0100	0.0100	0.0100
3	0.0851	0.1349	0.7500	0.0100	0.0100	0.0100	0.0113	0.0100	0.0100	0.1625	0.7500	0.0562
4	0.1280	0.0397	0.7500	0.0344	0.0379	0.0100	0.0917	0.0100	0.0354	0.0568	0.0561	0.7500
5	0.0257	0.0100	0.0221	0.7500	0.1062	0.0860	0.1860	0.7500	0.0340	0.0100	0.0100	0.0100
6	0.0100	0.0100	0.0100	0.2016	0.0184	0.7500	0.0100	0.7500	0.2100	0.0100	0.0100	0.0100
7	0.0100	0.0100	0.0100	0.0521	0.7500	0.1679	0.7500	0.0357	0.1843	0.0100	0.0100	0.0100
8	0.0100	0.0100	0.0100	0.2100	0.7500	0.0100	0.0863	0.1264	0.7500	0.0120	0.0120	0.0133
9	0.1124	0.0504	0.7500	0.0322	0.0415	0.0135	0.0533	0.0100	0.0100	0.7500	0.0427	0.1340
10	0.2100	0.7500	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.2100	0.7500

**Table 18** Weights for the maximum and minimum  $R_i$  values in Example 2

Alternative	$R_i$ Min weights						$R_i$ Max weights					
	C1	C2	C3	C4	C5	C6	C1	C2	C3	C4	C5	C6
Başer	0.2000	0.2000	0.2000	0.2000	0.1000	0.1000	0.2000	0.2000	0.1000	0.1000	0.2000	0.2000
Creditwest	0.2000	0.2000	0.2000	0.1627	0.1000	0.1373	0.2000	0.1000	0.1000	0.2000	0.2000	0.2000
Garanti	0.1832	0.1126	0.2000	0.1042	0.2000	0.2000	0.2000	0.2000	0.1000	0.2000	0.1000	0.2000
Huzur	0.2000	0.2000	0.2000	0.1601	0.1399	0.1000	0.2000	0.2000	0.1000	0.2000	0.1000	0.2000
Lider	0.2000	0.2000	0.1000	0.1000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.1000	0.1000
Yapı Kredi	0.2000	0.2000	0.1000	0.1000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.1000	0.1000

**Table 19** Weights for the maximum and minimum  $R_i$  values in the case study

Alternative	$R_i$ Min weights				$R_i$ Max weights			
	A	B	C	D	A	B	C	D
$F_1$	0.0500	0.4000	0.1500	0.4000	0.4000	0.1900	0.3600	0.0500
$F_2$	0.4000	0.1363	0.0637	0.4000	0.2522	0.2978	0.4000	0.0500
$F_3$	0.4000	0.1500	0.4000	0.0500	0.2401	0.3099	0.0500	0.4000
$F_4$	0.1554	0.4000	0.3946	0.0500	0.4000	0.0500	0.1500	0.4000
$F_5$	0.1568	0.0821	0.3611	0.4000	0.4000	0.4000	0.1500	0.0500
$F_6$	0.4000	0.0866	0.1134	0.4000	0.1500	0.4000	0.4000	0.0500
$F_7$	0.4000	0.1500	0.4000	0.0500	0.1500	0.4000	0.0500	0.4000
$F_8$	0.4000	0.0500	0.1500	0.4000	0.1554	0.4000	0.3946	0.0500

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