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ORIGINAL RESEARCH

Production-based pollution versus deforestation: optimal policy with state-independent and-dependent environmental absorption efficiency restoration process

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Abstract
An important yet largely unexamined issue is how the interaction between deforestation and pollution affects economic and environmental sustainability. This article seeks to bridge the gap by introducing a dynamic model of pollution accumulation where polluting emissions can be mitigated and the absorption efficiency of pollution sinks can be restored. We assume that emissions are due to a production activity, and we include deforestation both as an additional source of emissions and as a cause of the exhaustion of environmental absorption efficiency. To account for the fact that the switching of natural sinks to a pollution source can be either possible, and in such a case even reversible, or impossible, we consider that restoration efforts can be either independent from or dependent on environmental absorption efficiency, i.e., state-independent versus state-dependent restoration efforts. We determine (i) whether production or deforestation is the most detrimental from environmental and social welfare perspectives, and (ii) how state-dependent restoration process affects pollution accumulation and deforestation policies and the related environmental and social welfare consequences.

Keywords Optimal pollution · Deforestation · Environmental absorption efficiency · Restoration process · History dependence

1 Introduction

It is well known that deforestation reduces the carbon storage efficiency in the biosphere, thereby causing an increase in the stock of carbon persisting in the atmosphere (e.g., Canadell and Raupach 2008). However, it is less known that, due to rapidly increased oxidation of soil organic matter created by land use change, deforestation is also a significant carbon source (Baccini et al. 2012; Harris et al. 2012; Houghton et al. 2012). According to Canadell et al. (2007), deforestation was responsible for up to 1.5 billion metric tons of carbon emissions

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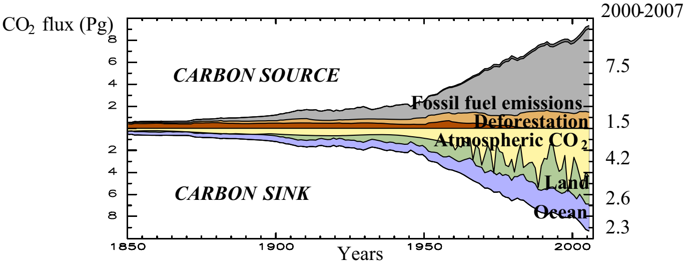

Fig. 1 Anthropogenic perturbation of the global carbon budget (Canadell et al. 2007)

between 2000 and 2007 (Fig. 1), which represents 17% of global emissions. Since the carbon accumulation has in turn a destructive impact on the assimilative capacity of oceans, deforestation contributes to switching the biosphere from a carbon sink to a source of carbon in the long run (Cox et al. 2000; Cramer et al. 2001; Joos et al. 2001; Lenton et al. 2006; Piao et al. 2008).

This phenomenon is already patent in the case of tropical forests that have partly switched from sinks to sources of carbon (Baccini et al. 2017). As noted by Gramling (2017), “tropical forests contribute more carbon dioxide to the atmosphere than they remove”.

Although the impact of deforestation on climate change is at the heart of international environmental policies, as in the multilateral REDD + vehicle (Angelsen and Rudel 2013), the crucial role that deforestation plays in the dynamics of pollution is most often overlooked in the environmental economics literature. This literature has mostly treated pollution control and deforestation as two separate issues so that the interaction between deforestation and pollution and its impact on the economic and environmental sustainability remains yet largely unexamined. One of the main reasons is that possible degradation of environmental absorption efficiency has not been considered in pollution control models for decades. Since the seminal paper by Keeler et al. (1972), pollution control models have aimed to determine an optimal emissions time-path in the presence of negative externalities of the pollution stock, and the related shadow price-based tax scheme necessary to implement this time-path. Among the various issues raised by these models, one strand of the literature in the wake of Forster (1973)’s intuition has questioned the assumption of constant environmental absorption efficiency. Various authors have considered the possibility for its degradation and possible irreversible exhaustion by formulating it as a function of the pollution stock (Tahvonen and Withagen 1996; Prieur 2009; Hediger 2009). These improvements did not however capture the effect of inertia on the carbon absorption efficiency degradation process (Leandri and Tidball 2019), hence the need to model environmental absorption efficiency as a state variable in its own right, either for pollution flows (Leandri 2009) or the pollution stock (El Ouardighi et al. 2014). As these last models grasp more realistically the dynamics of environmental absorption efficiency, in particular for greenhouse gases accumulation, they can better assess whether the resulting optimal emissions time-path are *environmentally sustainable*, i.e., whether they preserve a positive absorption efficiency in the long run (Leandri and Tidball 2019). What’s more, they have in turn allowed to considering the restoration of environmental absorption efficiency as another control variable in these models (El Ouardighi et al. 2014, 2016, 2018a, b).

In contrast, the issue of deforestation has emerged from the need of a trade-off between agriculture and forestry activities in forest-owners’ countries (Southgate 1990; Barbier and Burgess 1997), which involves a negative externality dimension from a global perspective

(Van Soest 1998; Van Soest and Lensink 2000) and the need for sustainable international mechanisms for conservation policies (Stähler 1996; Fredj et al. 2006). A distinction is further suggested between forest owners that draw revenues from timber and agricultural use of deforested land, and a non-forest-owner group that pollutes and suffers the negative externality of carbon accumulation (Andrés-Domenech et al. 2015). Relatedly, conditions for jointly profitable emissions abatement and/or deforestation net reduction are analyzed.

The present article further bridges the gap between these two streams of literature. Our approach differs from the existing studies in that we incorporate a number of essential, empirically established mutual influences between deforestation and pollution accumulation (see Table 1 for the essential features of our approach covered by the existing research in pollution accumulation and/or deforestation), that is:

- Deforestation reduces the efficiency of carbon sinks (e.g., Canadell and Raupach 2008)
- Deforestation generates polluting emissions (e.g., Baccini et al. 2012),
- Pollution accumulation damages the efficiency of carbon sinks (e.g., Raupach et al. 2014),
- The biosphere can switch from carbon sink to a source (e.g., Canadell et al. 2007),
- Restoration of carbon sinks can suffer from inertia (e.g., Liebsch et al. 2008).

Given that pollution accumulation and deforestation both accelerate and intensify global warming and constitute major threats for the earth's future potential of carbon biosequestration, the inclusion of these important mutual influences may enable more accurate emissions and deforestation policies than what can be found in the existing literature.

In this regard, this article introduces a dynamic model of pollution accumulation where polluting emissions can be mitigated and the absorption efficiency of pollution sinks can be restored. In the case of greenhouse gases (GHG) accumulation, the absorption efficiency of carbon sinks stems from the aggregated properties of land, notably forests, and oceans to absorb carbon emissions (see Fig. 1).[1] The terrestrial absorption efficiency is mechanically reduced by deforestation but also by climate change that can turn tropical forests (Baccini et al. 2017), permafrost and peatlands (Schuur et al. 2015) into carbon sources. The absorption efficiency of oceans is degraded by feedback loops between climate and marine carbon cycle, through either surface water warming, water stratification or thermohaline currents modification (Le Queré et al. 2007; Raupach et al. 2014).

Our second contribution resides in the improved modeling of the environmental absorption efficiency restoration process. In the case of GHG accumulation, efforts to restore this crucial ecosystem service consist mainly in promoting afforestation and reforestation and maintaining other terrestrial carbon sinks such as peatlands (Amesburry et al. 2019). Geo-engineering solutions specifically dedicated to mitigating the negative effects of carbon accumulation on the oceans' carbon absorption, are currently explored (Gattuso et al. 2018). Their net impact as well as their possible secondary effects must still be assessed before including them as actual restoration levers. Meanwhile, the potential for additional tree cover on the planet has been estimated to 0.9 billion hectares that would increase by 25% the current atmospheric carbon pool (Bastin et al. 2019). However, biological studies have shown that in the case of reforestation, replanting trees does not allow for a full recuperation of the initial absorption efficiency of forests and that the more severe the depletion of raw forest vegetation the slower the succession time and the more incomplete the recuperation (Liebsch et al. 2008). According to Martin et al. 2013), the carbon pool capacity of a secondary forest "could be expected to be 77–81% of those of undisturbed forests approximately 80 years after disturbance". Therefore, the actual result of restoration spending on the global carbon absorption

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[1] A similar concept of sink efficiency was introduced by Gloor et al. (2010) and operationalized by Raupach et al. (2014) as the carbon uptake rate by land and ocean sinks.

Table 1 Basic features of research on pollution accumulation and deforestation

Research	Deforestation decreases carbon sinks	Deforestation generates polluting emissions	Destructive impact of pollution on carbon sinks	Possible switch from carbon sink to a source	Restoring carbon sinks can suffer from inertia
Southgate (1990)	–	–	+	–	–
Stähler (1996)	–	–	+	–	–
Tahvonen and Salo (1996)	–	–	+	–	–
Tahvonen and Withagen (1996)	–	–	+	–	–
Barbier and Burgess (1997)	–	–	–	–	–
Van Soest (1998)	+	+	–	–	–
Van Soest and Lensink (2000)	–	–	–	–	–
Sohngen and Mendelsohn (2003)	+	–	–	–	+
Fredj et al. (2006)	–	–	–	–	–
Hediger (2009)	–	–	+	–	–
Leandri (2009)	–	–	+	–	–
Prieur (2009)	–	–	+	–	–
Boucekkine et al. (2013)	–	–	+	–	–
El Ouardighi et al. (2014)	–	–	+	+	–
Moser et al. (2014)	–	–	+	–	–
Andrés-Domenech et al. (2015)	+	–	–	–	–
El Ouardighi et al. (2016)	–	–	+	+	–
El Ouardighi et al. (2018a)	–	–	+	+	–
El Ouardighi et al. (2018b)	–	–	+	+	–
Leandri and Tidball (2019)	–	–	+	–	–
This paper	+	+	+	+	+

efficiency can be either independent (the most basic case of isolated afforestation) or dependent on the current state of the forest under restoration. Each assumption has non-trivial consequences on the resulting optimal paths as we will show later. Indeed, if the restoration results are assumed to be state-independent, as they have been until now in the literature (El Ouardighi et al. 2014, 2016, 2018a, b), it dissociates the actual restoration achieved by a given effort and the current state of the absorption efficiency, thus ignoring the ecological conditions in play in the restoration process. It implies in particular that if the natural carbon sinks have turned to sources, they can be restored back to sinks, which is a rather optimistic take on carbon potential recuperation. Meanwhile, the state-dependent case is better fitted to reflect how the history of absorption efficiency degradation can influence the effectiveness of restoration spending since the carbon potential added by a given effort will depend on the current conditions of the global absorption efficiency, or at least of its forest component. What’s more, under this assumption it is structurally impossible to turn exhausted carbon sinks to sinks again. It is thus a more conservative assumption that could provide a stronger case for actual environmental policies. Hence our proposition to consider alternatively both hypothesis and compare the subsequent results to capture the effect of restoration *inertia* on optimal pollution/restoration paths.

The dynamic tradeoff between revenues drawn from production of economic goods and deforestation, on the one hand, and costs incurred by pollution externalities and restoration efforts, on the other hand, are analyzed in the setup of a dynamic first-best central planner model. State-independent *versus* state-dependent restoration configurations are considered and their impact on the nature of long term equilibrium are compared. It is notably shown how key parameters as production-based versus deforestation-based revenues and the discounting rate affect the optimal policy-mix. The environmental sustainability of these optimal policies, i.e. the long-term maintenance of aggregated carbons sinks against a transient shift to carbon sources, are also scrutinized. Furthermore, the behaviors of the optimal paths in the case of state-dependent restoration process highlight the existence of history dependency and the importance of Skiba thresholds (Grass et al. 2008), and thus strengthen our take on the importance of initial conditions in optimal pollution control and deforestation/restoration policies.

The following issues are investigated:

(i) Is production-based pollution or deforestation more detrimental from environmental and social welfare perspectives?
(ii) How does state-dependent restoration process affect pollution accumulation and deforestation policies and the related environmental and social welfare consequences?

To address these issues, we characterize an optimal policy successively for state-independent and state-dependent restoration process and respectively compare in each case its behavior and sustainability with the respective optimal policy mix of mono-activity solutions (production-only and deforestation-only). Depending on the incentive structure in place, this comparison will shed light on the respective impact in the long run of each activity, as well as on the leverage effect that state-dependency induces on the restoration process.

The article is organized as follows. In the next section, we formulate an optimal control model where a social planner seeks to determine the optimal policy in terms of production-based emissions, deforestation rate and restoration efforts. Section 3 characterizes the general solution of the problem. Section 4 analyzes the model with numerical means, and Sect. 5 concludes the article.

2 Model formulation

Our model is an extended version of the model developed in El Ouardighi et al. (2014, 2016). In a dynamic continuous framework, the social planner seeks to maximize intertemporal social welfare by weighing benefits from polluting economic activities, the costs of restoration efforts and environmental damages caused by stock pollution accumulation subject to endogenous variations of the environmental absorption efficiency.

We consider two welfare-relevant activities: forest exploitation and production of economic goods. Both activities, i.e., deforestation and production, generate polluting emissions. The emissions rate resulting from production, that is, production-based emissions, is denoted by $u(t) \geq 0$. On the other hand, the deforestation rate, denoted by $v(t) \geq 0$, instantaneously generates polluting emissions at a proportional rate (e.g., Baccini et al. 2012; Harris et al. 2012; Houghton et al. 2012; Canadell et al. 2007), that is, $\alpha v(t)$, where $\alpha > 0$.

Therefore, the evolution of the pollution stock, denoted by $P(t) \geq 0$, is described as:

$$\dot{P}(t) = u(t) + \alpha v(t) - A(t)P(t) \qquad P(0) = P_0 \geq 0 \tag{1}$$

In (1), the pollution stock decreases at an environmental absorption efficiency rate $A(t)$, which obeys the transition equation:

$$\dot{A}(t) = w(t)A(t)^{\beta} - v(t) - \gamma P(t) \qquad A(0) = A_0 > 0 \tag{2}$$

where the units of α and $v(t)$ are scaled to fit both (1) and (2). The environmental absorption efficiency, $A(t)$, reflects the aggregated properties of oceans and forests as pollution sinks. Other studies where the environmental absorption efficiency is modeled as a state variable are El Ouardighi et al. (2014, 2016, 2018a, b). According to (2), environmental absorption efficiency decreases with both deforestation and the destructive impact of the pollution stock on oceans. The parameter $\gamma > 0$ inversely reflects the internal capacity of absorption efficiency, that is, oceans, to resist the destructive impact of pollution. It is clear that this simple linear degradation mechanism does not capture all the complex ecological processes at stake in the differentiated reactions of oceans', peatlands' and forests' carbon uptake to accumulated GES. But our main concern is to make sure that the pollution stock feedback loops are accounted for without overwhelming technical complexity.

Further, the environmental absorption efficiency increases with forest restoration efforts, denoted by $w(t) \geq 0$. This assumption is consistent with, e.g., Canadell and Raupach (2008), who assert that reforestation policies can increase the potential for carbon biosequestration. However, to account for the possibility of *inertial* restoration process in (2), restoration efforts can be either independent from or linearly dependent on the magnitude of absorption efficiency. The restoration efforts' state-dependency is such that β takes the value 0 if the restoration process suffers from inertia, or 1 if it is impossible. Thus, the actual impact of restoration efforts $w(t)$ on the environmental absorption efficiency will amount to $w(t)A(t)^{\beta}$ with, as explained above, β capturing whether the restoration process is state-independent ($\beta = 0$) or linearly state-dependent ($\beta = 1$). We aggregate in $A(t)$ the various carbon sinks of the biosphere, assuming that if $\beta = 0$, restoration can increase $A(t)$, through afforestation for example, independently of the current level of $A(t)$. In particular, even if $A(t)$ is negative, it can be restored back to a positive value. Conversely, if $\beta = 1$, the restoration achieved will depend upon the current state of environmental absorption efficiency and thus on its past evolution. Under this condition, restoration efforts can reflect a densification policy within an existing forest: the carbon sequestration potential is increased by different species mix or lengthening the rotation period (Sohngen and Mendelsohn 2003) but depends on the actual size and state of the forest already in place. In that case, if the forest has turned to a source, i.e., if $A(t)$ is negative, restoration cannot raise it back to a positive level, i.e., the shift is irreversible. To avoid diverging solution in the case of state-dependent restoration efforts, we assume a strictly positive initial absorption efficiency, that is, $A_0 > 0$. Overall, expressions (1) and (2) describe the joint dynamics of the pollution stock and environmental absorption efficiency.

We now define an objective criterion. Revenues are supposed to be drawn from production-based emissions and deforestation, whereas costs are incurred from pollution negative externalities and restoration efforts. We assume an increasing concave revenue function for the production-based emissions and use a standard specification (see Wirl 2007), that is, $u(t)(a - u(t)/2)$, where $a \geq 0$ is the revenue-maximizing production level, if any. The revenues drawn from deforestation are related to the amount of wood retrieved from clear-felling, if any, and to land use change (urbanization, agriculture, ranching, etc.). For simplicity, we omit revenues from selective logging, and consider that the revenues are drawn from deforestation. In all cases, we assume that deforestation generates only short-term rather than long-lasting revenues. This is in contrast with the optimistic assumption of Andrés-Domenech et al. (2015) that revenues can be drawn from deforested land over a significant time interval, i.e., 50 years. In fact, deforestation is a main cause of reduced water cycling (and rainfall) and of local climate change, including major occurrence of droughts (e.g., Sheil and Murdiyarso 2009; Makarieva et al. 2014), which makes deforested lands more eligible for ephemeral rather than long-lasting beneficial agricultural exploitation and economic development (Rodrigues et al. 2009).

In this regard, as we did for production-based emissions, we choose to assume an increasing concave revenue function for deforestation, that is, $v(t)(b - v(t)/2)$, where $b \geq 0$ is the revenue-maximizing deforestation level, if any. One merit of this prudent assumption is that it lets us avoid treating deforestation as a privileged source of revenues. The tipping point $2b$ interprets as an upper bound for the deforestation rate above which the magnitude of deforestation is so extreme that it engenders costs rather than revenues. Further, the negative externalities of pollution are valued as an increasing convex function of the pollution stock (e.g., Michel and Rotillon 1995), that is, $cP(t)^2/2$, $c > 0$. Finally, due to decreasing returns, the restoration efforts generate an increasing quadratic cost, $dw(t)^2/2$, $d > 0$, (see El Ouardighi et al. 2016). Without loss of generality, we assume that the cost coefficient of restoration efforts, d, is normalized to 1, $d = 1$, which implies that d can be interpreted relative to c, the cost coefficient of the pollution stock.

Assuming an infinite planning horizon, and denoting the discounting rate by $r > 0$, the optimal control problem writes:

$$\text{Max } W = \int_0^\infty e^{-rt}\left[u(t)(a - u(t)/2) + v(t)(b - v(t)/2) - cP(t)^2/2 - w(t)^2/2\right]\mathrm{d}t \qquad (3)$$

under the constraints (1)–(2) and $u(t) \geq 0$, $v(t) \geq 0$, $w(t) \geq 0$.

Overall, the time paths that can be generated by the problem decision defined by (3) under the transition Eqs. (1)–(2), are tightly dependent upon the value of β. In the case where restoration efforts are independent from the environmental absorption efficiency, i.e., $\beta = 0$, a negative value of $A(t)$ is possible over a finite time interval. However, in the case where restoration efforts depend upon the environmental absorption efficiency, i.e., $\beta = 1$, it results that $A(t) \geq 0$ for all t.

Table 2 Qualification constraint

Arcs	1	2	3	4	5	6	7	8
ρ_1	0	0	–	0	–	–	0	–
ρ_2	0	–	0	0	–	0	–	–
ρ_3	0	0	0	–	0	–	–	–

3 Optimality conditions and stability

Skipping the time index for convenience, the current-value Hamiltonian is:

$$H = u(a - u/2) + v(b - v/2) - cP^2/2 - w^2/2 + \lambda(u + \alpha v - AP) + \varphi\big(wA^{\beta} - v - \gamma P\big) \tag{4}$$

and the extended Hamiltonian is:

$$L(P, A, u, v, w, \lambda, \varphi, \rho_1, \rho_2, \rho_3) := H(P, A, u, v, w, \lambda, \varphi) + \rho_1 u + \rho_2 v + \rho_3 w \tag{5}$$

where $\lambda(t)$ and $\varphi(t)$ are costate variables associated with the pollution stock, and the absorption rate, and ρ_1, ρ_2, ρ_3 are Lagrange multipliers corresponding to the non-negativity constraints imposed on u, v and w, respectively.

Then an optimal solution $(P^*, A^*, u^*, v^*, w^*)$ satisfies the Hamiltonian maximizing condition:

$$\big(u^*, v^*, w^*\big) = \mathrm{A}rgmax_{u\geq 0, v\geq 0, w\geq 0} H\big(P^*, A^*, uv, w, \lambda, \varphi\big) \tag{6}$$

and the costates $\lambda(t)$ and $\varphi(t)$ satisfy the adjoint equations:

$$\dot{\lambda} = r\lambda - H_P\big(P^*, A^*, u^*, v^*, w^*, \lambda, \varphi\big) \tag{7}$$

$$\dot{\varphi} = r\varphi - H_A\big(P^*, A^*, u^*, v^*, w^*, \lambda, \varphi\big) \tag{8}$$

Lemma 1 *The Hamiltonian is concave with respect to the control vector* (uv, w) *and guarantees a (local) maximum of the Hamiltonian.*

Proof See "Appendix A1".

Therefore, the Hamiltonian maximizing condition (6) allows us to distinguish eight different arcs, which are presented in the following table (Table 2).

The relevant arcs under study for optimal environmental policy will be arcs 1, 2 and 3. Indeed, arcs 4, 6 and 7 display nil restoration efforts and positive emissions, through either or both production and deforestation. According to (2), they will eventually result in diverging paths. They might thus be the initial or an intermediate part of the optimal path but none of them can be followed over an infinite time horizon. Engaging in restoration efforts will in fact always be necessary at some point along our optimal policy mix. As for arcs 5 and 8, they are characterized by no emissions from any source (i.e., uncontrolled model). As such, they are not relevant for actual environmental policies and could only reflect pre-industrial states. Let us then focus on the three arcs with restoration that can appear at any stage of the optimal path, that is, the production-based emissions and deforestation arc, the production-based emissions-only arc, and the deforestation-only arc. These arcs are the only candidates for the final stage when the path converges to equilibrium. In economic terms, we can thus assert that the optimal policy-mix will always include restoration but can rely either on production only (arc 2), deforestation only (arc 3) or production and deforestation combined (arc 1).

For each of arcs 1, 2 and 3, explicit functions $(u^\circ, v^\circ, w^\circ, \rho_1, \rho_2, \rho_3)$ can be determined such that the costate Eqs. (7) and (8) write:

$$\dot{\lambda} = (r + A)\lambda + \gamma\varphi + cP \tag{9}$$

$$\dot{\varphi} = r\varphi + \lambda P - \beta\varphi w^\circ A^{\beta-1} \tag{10}$$

The properties of the three arcs are successively analyzed in "Appendix A2".

Proposition 1 *Along arc 1, the optimal restoration effort increases linearly with the production based emission rate and decreases linearly with the deforestation rate, for any value of* β.

Proof See "Appendix A3".

Proposition 2 *Given a set of parameter values* $(a, b, c, r, \alpha, \beta, \gamma)$, *the number of interior steady states, if they exist, is determined by the number of feasible solutions to*:

$$\varphi_\infty\left(r - \beta\varphi_\infty A_\infty^{2\beta-1}\right) + \lambda_\infty P_\infty = 0 \tag{11}$$

where:

$$\varphi_\infty = \frac{c(b-\alpha a) + \left[\gamma(a+\alpha b) + bA_\infty\right](r + A_\infty)}{c + \gamma\left[\alpha r + \gamma\left(1+\alpha^2\right)\right] + (r + 2\alpha\gamma + A_\infty)A_\infty + \left[c\left(1+\alpha^2\right) + (r + A_\infty)A_\infty\right]A_\infty^{2\beta}}$$

$$\lambda_\infty = -\frac{ac + \gamma\left[\gamma(a+\alpha b) + bA_\infty\right] + c(a+\alpha b)A_\infty^{2\beta}}{c + \gamma\left[\alpha r + \gamma\left(1+\alpha^2\right)\right] + (r + 2\alpha\gamma + A_\infty)A_\infty + \left[c\left(1+\alpha^2\right) + (r + A_\infty)A_\infty\right]A_\infty^{2\beta}}$$

$$P_\infty = \frac{\gamma(\alpha a - b) + \left[a + (a+\alpha b)A_\infty^{2\beta}\right](r + A_\infty)}{c + \gamma\left[\alpha r + \gamma\left(1+\alpha^2\right)\right] + (r + 2\alpha\gamma + A_\infty)A_\infty + \left[c\left(1+\alpha^2\right) + (r + A_\infty)A_\infty\right]A_\infty^{2\beta}}$$

where the subscript '∞' *denotes the steady state.*

Proof See "Appendix A4".

Clearly, the number of feasible, interior steady states cannot be established analytically. Therefore, we resort to numerical means in the following section.

4 Numerical analysis and economic implications

To determine which among production and deforestation is the most detrimental in terms of welfare and environmental sustainability depending on whether the exhaustion of environmental absorption efficiency is possible or not, we compare the time paths related to three arcs: production-based emissions-deforestation, production-based emissions-only and deforestation-only.

For each case, the parameter space is divided into regions, and each region is characterized by the number of feasible steady states and by the transient behavior in the neighborhood of each steady state. In this regard, we select the values for the parameters as shown in Table 3. By varying parameters a and b, we show that solutions including (locally) stable equilibria with monotonic or oscillating convergence and Skiba behavior (Grass et al. 2008) are possible.

The set of parameter values related to a unique equilibrium with monotonic convergence in Table 3 corresponds to the base case. It reflects a situation characterized by relative patience

Table 3 Parameter values and related solutions

Description	r	a	b	c	γ	α	β
Unique equilibrium with monotonic convergence (Figs. 4, 12)	0.05	0.4	0.1	0.001	0.05	0.15	0, 1
Unique equilibrium with oscillating convergence (Fig. 7)	*	0.087	1	*	*	*	0
Skiba with two equilibria and monotonic convergence (Fig. 14)	*	0.053	0.1	0.01	*	*	1

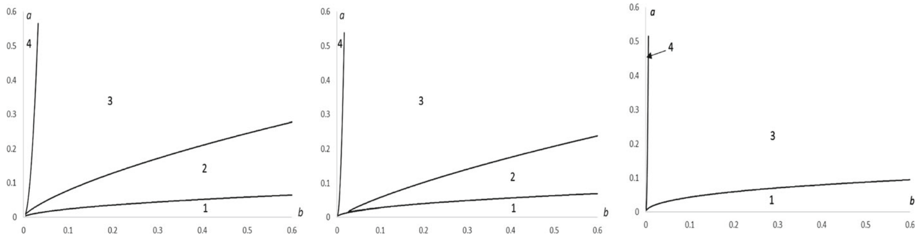
of the regulator with the discounting rate $r = 0.05$, limited destructive impact of pollution on environmental absorption efficiency with $\gamma = 0.05$, intermediate marginal incentives for production and deforestation with $a = 0.4 > b = 0.1$, relatively low marginal pollution cost (i.e., lower than marginal restoration cost) with $c = 0.001$, and a typical value for the marginal emissions induced by deforestation with $\dot{P}_v = \alpha = 0.15$ (see Houghton et al. 2012). Further, the unique equilibrium with oscillating convergence differs from the base case only because of a very low marginal incentive for production and a very high marginal incentive for deforestation with $a = 0.087 < b = 1$, and state-independent restoration efforts ($\beta = 0$). Regarding the case of Skiba behavior with two equilibria, it departs from the base case because of a very low marginal incentive for production ($a = 0.053 < b = 0.1$), a relatively higher marginal pollution cost ($c = 0.01$), and linearly state-dependent restoration efforts ($\beta = 1$).To compare the transient paths associated with the different cases and to assess their sensitivity to the initial conditions, we assume two initial values for each state variable, that is, small and large (i.e., smaller and larger than the base case steady state) values for the initial environmental absorption efficiency with $A_0 = (0.1, 2.5)$, and for the pollution stock with $P_0 = (0.05, 3)$. Although our numerical analysis includes *idyllic initial conditions* with initially small pollution stock and large absorption efficiency as a benchmark, i.e., $(A_0, P_0) = (2.5, 0.05)$, we are mainly interested in *relatively unfavorable* and *favorable* initial conditions, that is, $(A_0, P_0) = (0.1, 3)$ and $(A_0, P_0) = (2.5, 3)$.

For the numerical resolution, a time decomposition method (Maimon et al. 1998) is used to approximate the optimal paths (see "Appendix A5"). We successively study the cases with state-independent, and linearly state-dependent restoration process.

4.1 State-independent restoration process

In the case where restoration efforts are state-independent and can thus reverse the switching of natural sinks into a pollution source, the value of the sum of the principal minors of order 2 of the Jacobian matrix minus the squared discounting rate, $K_{|\beta=0}$, is negative for any feasible steady state, which rules out the occurrence of a limit-cycle (see details in "Appendix A2"). Therefore, the steady states, if they exist and are feasible, are categorized into saddle node, saddle focus, or unstable equilibrium.

Figure 2 shows the bifurcation diagram in the (b, a) parameter space for $\gamma = 0.05$, $\alpha = 0.15$, $r = 0.05$, and three values of c, $c = (0.001, 0.01, 0.1)$. Figure 2a, b predicts that a significantly unbalanced structure of incentives for production and deforestation (low a *vs.* high b and vice versa), leads in the long run to a unique (locally stable) steady state characterized by specialization of activity with higher revenues and withdrawal from the least remunerating activity. In the case of a prevalent incentive for deforestation (intermediate a



(a) $c = 0.001$ **(b)** $c = 0.01$ **(c)** $c = 0.1$

Region 1: one oscillating steady state with $u = 0$, Region 2: one oscillating interior steady state, Region 3: one monotonic interior steady state, Region 4: one monotonic steady state with $v = 0$

Fig. 2 Bifurcation diagram in the (b, a) parameter space with $c = (0.001, 0.01, 0.1)$

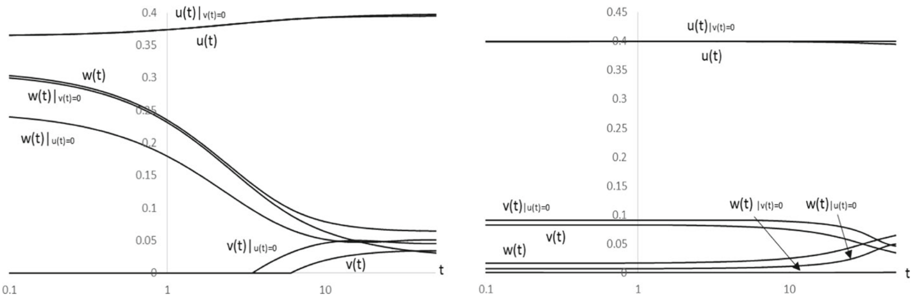
vs. high b), convergence to the unique (locally stable) steady state is oscillatory. Nonetheless, these patterns are affected by a higher pollution cost coefficient. It is interesting to study how these patterns are affected by the magnitude of the environmental damage factor, c. First, as the marginal environmental damage increases, by a factor 10 (Fig. 2b) then by a factor 100 (Fig. 2c), Region 2 respectively decreases and then disappears, in favor of Region 3, and to a lesser extent Region 1. Oscillations are thus eliminated by a large enough environmental damage when it comes to interior solutions. Second, a higher c increases slightly Region 1 and reduces Region 4. This means that when a unit of stock pollutant imposes a higher environmental damage, the specialization in deforestation is reinforced for wider range of (b, a) combinations while the specialization in production is less likely. The economic interpretation of this shift relies on the fact that an increase in c reduces drastically the restoration cost relatively to the marginal environmental damage, as d has been normalized to 1 with respect to c. Since we are dealing with state-independent restoration in this subsection, this implies that increasing or maintaining the environmental absorption efficiency through restoration becomes automatically relatively cheaper. Consequently, restoration efforts can be engaged to compensate the detrimental effort of deforestation on the environmental absorption efficiency, and we will show later that this compensation is systematic. Considering that deforestation adds significantly less pollution to the accumulated stock than production given that the deforestation emission factor is $\alpha = 0.15 < 1$, the trade-off between deforestation and production is now in favor of the former.

For the base case values ($a = 0.4$, $b = 0.1$, $c = 0.001$,$\gamma = 0.05$, $\alpha = 0.15$, $r = 0.05$), convergence to the unique locally stable steady-state is monotonic (Fig. 4). Table 4 reports the values of state and control variables at the steady state with both production and deforestation (interior solution) and with either production or deforestation (corner solutions). We observe that the steady-state production-based emissions are almost unaffected by eradication of deforestation. In contrast, the steady-state restoration efforts are much lower without deforestation than in the converse case. Nevertheless, the steady-state values of absorption efficiency and pollution stock are respectively higher and lower than in the case with deforestation. In other words, environmental sustainability improves with eradication of deforestation. In contrast, eradication of production-based emissions results in both lower environmental absorption efficiency and pollution stock in the long run, with a slightly higher deforestation rate and slightly lower restoration efforts. Overall, deforestation requires more restoration efforts than do production-based emissions in the long run.

Figure 4 shows the convergence to the interior steady state (black dot) from various initial conditions. The influence of production and deforestation is illustrated by comparing the

Table 4 Steady-state values with both production and deforestation and with either production or deforestation

	A_∞	P_∞	u_∞	v_∞	w_∞
Interior solution	0.664	0.602	0.395	0.035	0.065
Corner solution with $v(t) = 0$	0.721	0.551	0.398	–	0.028
Corner solution with $u(t) = 0$	0.063	0.108	–	0.046	0.051



(a) Relatively unfavorable initial conditions **(b)** Relatively favorable initial conditions

Fig. 3 Control policies from relatively unfavorable and favorable initial conditions (log. time scale)

interior steady state with the steady state that involves only production-based emissions (unfilled dot) or only deforestation (unfilled star). When starting from unfavorable initial conditions, the optimal path consists in first slightly increasing the absorption efficiency, and then rapidly decreasing the pollution stock while increasing the former until the steady state is reached. This is achieved with initially high and decreasing restoration efforts, and later on with initially intermediate and slightly increasing production-based emissions as shown in Fig. 3a. As for deforestation, it is postponed until the absorption efficiency is sufficiently restored to resist its impact. Even the deforestation-only path delays actual deforestation until sufficient restoration has been carried out.

The policy implications of these results are twofold. If the environmental absorption efficiency is already low, it is optimal to restore it at the earlier stage of the path, even if the restoration costs are high relatively to the environmental damage factor. This is true including for the production-only path, even if it converges towards a lower steady state restoration level than the other two. What's more, under such initial conditions it is always preferable to delay deforestation until the absorption efficiency has been at least partially restored. This result can be interpreted as a form of consistency between environmental sustainability and economic optimality but it holds under the assumption of this subsection that restoration is state-independent. If it were not the case, restoring the environmental absorption efficiency from a very low initial level could prove extremely costly.

From favorable initial conditions (Fig. 3), the optimal path consists first in rapidly decreasing the pollution stock and slowly decreasing the absorption efficiency, and then rapidly decreasing the latter and slowly increasing the former until the steady state is reached. This is implemented with initially low and increasing restoration efforts, and later on with on the one side initially high and slightly decreasing production and on the other side on initially high and decreasing deforestation, as shown in Fig. 3b. In that case, the initial absorption efficiency is sufficiently high to endure immediate deforestation, in the policy-mix as well as in the deforestation-only solution. (i.e., greater restoration efforts coincide with lower deforestation, and vice versa).

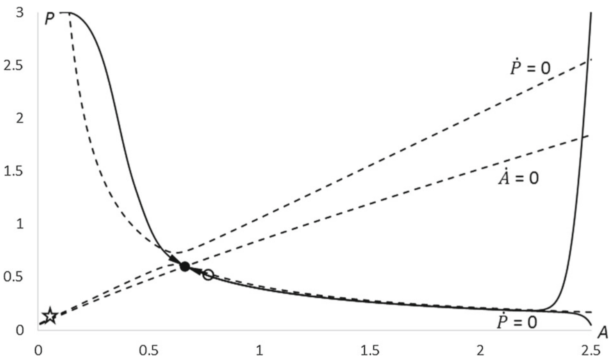


Fig. 4 Phase diagram in the state space with monotonic convergence (filled dot: interior solution, unfilled dot: corner solution with $v = 0$, unfilled star: corner solution with $u = 0$)

We note also the level of restoration efforts that is much higher in the interior solution and the deforestation-only case than in the production-only case. This results reflects the fact that in a production-only solution the environmental absorption efficiency will only be harmed by the feedback mechanisms (γ) while a policy involving deforestation will diminish the absorption efficiency much more rapidly. As a consequence heavier long-term restoration efforts are needed in the optimal policy-mix and in the deforestation-only solution. However, if the former succeeds in maintaining a significant level of environmental absorption efficiency, the latter ends up with an almost depleted absoprtion, while the production-only solution maintains the highest absorption efficiency (see Fig. 4). With the set of parameters of this subsection, it appears that deforestation is the activity that threatens most the environmental sustainability along an economic path as it impairs signifantly and indefintely the environmental absorption efficiency. In Fig. 4, the convergence towards A_∞ from low initial absorption efficiency levels highlights the crucial role of restoration to avoid the irreversible depletion of this essential ecosystem service (Leandri 2009).

As shown in Fig. 5, both under intermediate (high A_0 and P_0) and unfavorable initial conditions, the cumulative welfare associated to the deforestation-only solution is drastically lower than the welfare of the interior and the production-only paths. This obviously reflects the superior profitability of production relatively to deforestation set by the parameters, but it also translates the need to engage in additional costly restoration efforts if deforestation comes into in play. Although it also includes some deforestation, in the case of the interior solution this additional restoration cost is largely compensated through the exploitation at full rate of the production activity. As expected, the social welfare is lower along any paths when the initial conditions are less favorable, but we can note that the welfare differential between the optimal interior solution and the production-only solution is stronger for intermediate initial conditions. Indeed, in the short-term production only yields more welfare as the restoration efforts are lower (and the emissions are similar). However after some time, the additional profit from deforestation kicks in without threatening environmental sustainability and the interior solution's cumulative welfare exceeds the production-only solution. It is remarkable that under unfavorable initial conditions, both interior and production-only solutions achieve the same welfare, through a similar production/restoration initial mix until the interior solution activates deforestation (see Fig. 3a). Considering that the production-only path converges towards a more environmentally sustainable steady state (higher A_∞ in Table 4) and imposes less negative externalities (lower P_∞), we can conclude that in this base

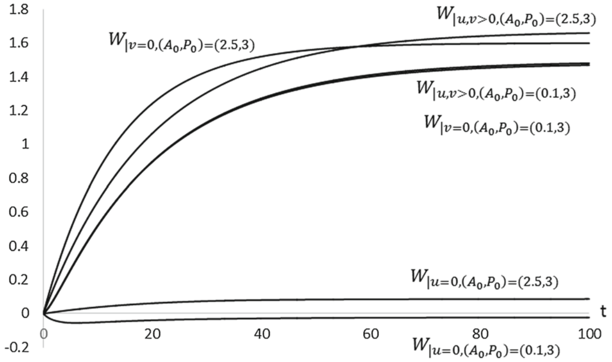
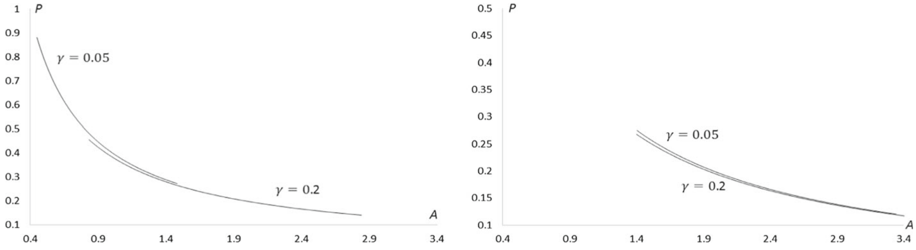


Fig. 5 Cumulative utility over time from relatively unfavorable and favorable initial conditions

case scenario (production more profitable and restoration costs relatively high) if the initial ecological conditions are unfavorable, the social planner should exclude deforestation from its policy-mix and resort to a production-only solution. This interesting result illustrates how (unfavorable) initial conditions can shape the content of an optimal policy when feedback effects are taken into account.

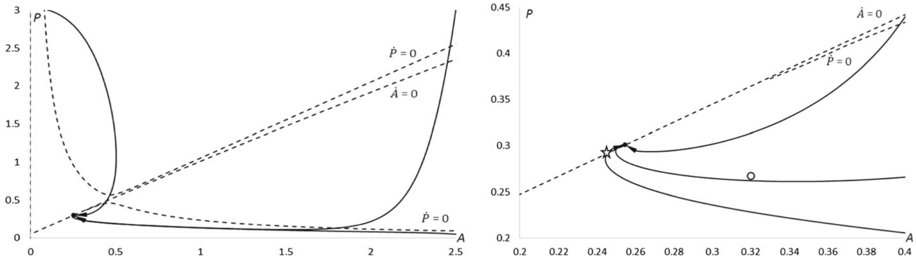
Figure 6 shows the sensitivity of the steady state with respect to the parameters c, r and γ. For $c = 0.001$ and r going from 0.01 to 0.1, the steady-state values of absorption efficiency and pollution stock are respectively lower and higher, as Fig. 6a illustrates. Figure 6b shows a similar dependence for $c = 0.1$. The steady-state absorption efficiency and pollution stock respectively decrease and increase monotonically with a higher discounting rate. Since the absorption efficiency is a service that allows to keeping pollution accumulation under control in the long run, its importance diminishes as the discounting rate increases. This sensitivity is less acute when the environmental damage factor is much higher. Indeed, despite a myopic social planner, future damages will nonetheless be taken into consideration when c is very high, thus limiting the increased (decrease) in accumulated pollution (absorption efficiency). The impact of γ on this discounting rate sensitivity is worth noting: when γ is equal to 0.05 as we assumed until now, i.e., when the pollution stock is less destructive to the absorption efficiency, the negative impact of a greater discounting rate is weaker, in the low c case (Fig. 6a). Keeping in mind that a low c captures a relatively higher restoration cost, this result can be seen as a reaction to this higher cost. If the pollution stock is more destructive than expected to the absorption efficiency (i.e., $\gamma = 0.2$), then loosing periodically a significant part of the absorption efficiency through these feedbacks will prove much more costly restoration-wise. As a result, the pollution stock will not be so drastically raised under an increased discounting rate. However, when restoration costs are lower (greater c), the value of γ does not affect the discounting rate sensitivity (Fig. 6b). This sensitivity with respect to γ is of crucial importance for environmental policies. Indeed, it warns us against an underestimation of γ that would justify an optimal pollution path threatening the environmental sustainability with low A and imposing high damages through a high P. Given the major uncertainties that still characterize the scientific understanding of these feedback loops, especially in the case of oceans (Leandri and Tidball 2019), it is crucial to dedicate monitoring efforts to better grasp the order of magnitude of γ and avoid over-optimistic parameters.

Figure 7 shows that oscillating convergence is obtained for $a = 0.087$ and $b = 1$, all things being equal. Due to the near-zero incentive for production, the steady state without



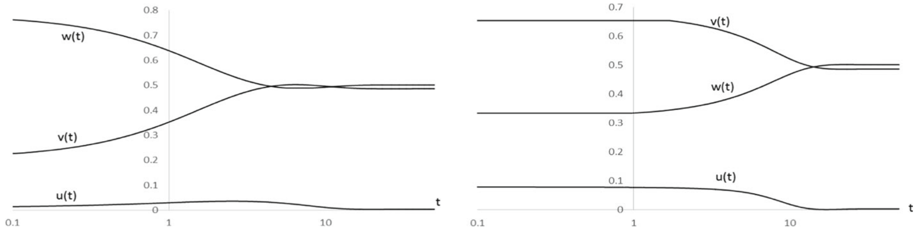
(a) $c = 0.001$ **(b)** $c = 0.1$

Fig. 6 Locus of the steady-state values for $r = (0.01, 0.1)$ and $c = (0.001, 0.1)$



(a) Phase-portrait diagram **(b)** Zoom in the vicinity of the steady point

Fig. 7 Phase-portrait diagram in the state space with oscillating convergence (filled dot: transiently interior solution, unfilled dot: corner solution with $v = 0$, unfilled star: corner solution with $u = 0$)



(a) Relatively unfavorable initial conditions **(b)** Relatively favorable initial conditions

Fig. 8 Control policies from relatively unfavorable and favorable initial conditions

production-based emissions is now very close to the transiently interior steady state (Fig. 7b). In contrast, the eradication of deforestation increases absorption efficiency and reduces pollution. This result suggests that environmental sustainability is always worse with deforestation alone than with production-based emissions alone, regardless of the incentive structures for the two activities.

Figure 8 confirms that deforestation and restoration efforts are mutual substitutes, and suggests that production-based emissions should be limited both in magnitude and duration for unfavorable initial conditions. In the long run, restoration efforts should always be greater than deforestation.

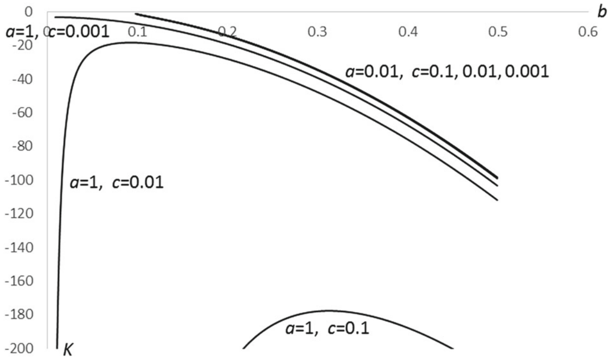


Fig. 9 Parameter $K_{|\beta=1}$

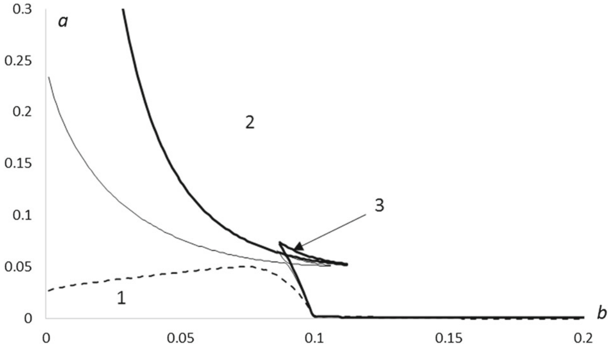


Fig. 10 Bifurcation diagram in the (b, a) parameter space Bold, thin and dashed lines for $c = (0.001, 0.01, 0.1)$. Region 1: no interior steady state. Region 2: one interior steady state with monotonic path. Region 3: two interior steady states with monotonic paths

4.2 Linearly state-dependent restoration process

In the case of linearly state-dependent restoration process, $\beta = 1$, and the sum of the principal minors of order 2 of the Jacobian matrix minus the squared discounting rate, $K_{|\beta=1}$, is negative for all combinations of parameters, which rules out the occurrence of a limit-cycle (see details in "Appendix A2"). Figure 9 shows the dependence of $K_{|\beta=1}$ on b for $\gamma = 0.05$, $\alpha = 0.15$, $r = 0.05$, $a = (0.01, 1)$ and $c = (0.001, 0.01, 0.1)$. Here also, for $a = 0.01$, the three curves corresponding to the three values of c cannot be distinguished.

On the other hand, Fig. 10 shows the bifurcation diagram in the (b, a) parameter space for steady states corresponding respectively to interior and corner solutions. The number of (locally) stable steady states is at most 2, with all stable paths being monotonic.

For the base case parameter values ($a = 0.4$, $b = 0.1$, $c = 0.001$,$\gamma = 0.05$, $\alpha = 0.15$, $r = 0.05$), there is a unique steady state with monotonic convergence (Fig. 12). Table 5 summarizes the value of the steady state and control variables with both production-based emissions and deforestation and with either production-based emissions or deforestation. Under linear state-dependent restoration process, we observe that the absorption efficiency in the interior solution is significantly greater than with the production-only corner solution. Long-run environmental sustainability is improved by the inclusion of deforestation in the

Table 5 Steady-state values with both production and deforestation and with either production or deforestation

	A_∞	P_∞	u_∞	v_∞	w_∞
Interior solution	2.001	0.205	0.399	0.078	0.044
Corner solution with $v(t) = 0$	0.957	0.416	0.398	–	0.022
Corner solution with $u(t) = 0$	1.128	0.0074	–	0.056	0.050

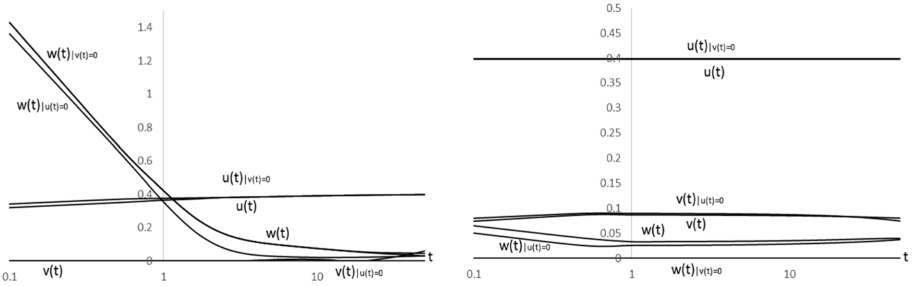


Fig. 11 Control policies from relatively unfavorable and favorable initial conditions (log. time scale)

optimal policy. This feature is distinct from the state-independent restoration case, where the inclusion of deforestation in the solution actually reduced the absorption efficiency (see Table 5). The reason is that the restoration efforts are now greater in the former than in the latter case at the steady state, which allows superior capitalization. These results can be explained by the fact that in the context of state-dependent restoration efforts, a high absorption efficiency (greater than 1) increases the actual restoration impact of a given restoration effort. The greater the current absorption efficiency is, the greater the effectiveness of the restoration efforts. As a result, the optimal interior solution can allow significantly more deforestation than in the state-independent restoration process case (roughly twice as much) and compensate for the subsequent loss in absorption efficiency with even less restoration efforts (0.044 *vs*. 0.065), since the latter are now much more effective.

Figure 11 confirms that production-based emissions are almost insensitive to deforestation. Restoration efforts are now lesser in the case of deforestation-only in the long run, except in the case of production-based emissions-only. From relatively unfavorable initial conditions, deforestation without production-based emissions is always near zero. Also, it is necessary to apply very high restoration efforts from the beginning that can decrease gradually as the absorption efficiency rise enough to increase the effectiveness of restoration spending. In contrast, from relatively favorable initial conditions, restoration without deforestation is always very low. Also, under the latter conditions, the optimal sequence is less obvious than in the cases of state-independent restoration efforts (Fig. 12).

In Fig. 13, welfare is now much lower without deforestation from relatively favorable initial conditions, and unchanged from relatively unfavorable initial conditions. Therefore, welfare is more sensitive to deforestation eradication than to initial conditions. Given that environmental sustainability is greater with than without deforestation, the eradication of deforestation is not economically advisable here. The robustness of this result is supported by our prudent assumption of short-term revenues drawn from deforestation. Finally, from relatively favorable (unfavorable) initial conditions, welfare is slightly greater (dramatically lower) without production-based emissions than without deforestation. This is due to the fact

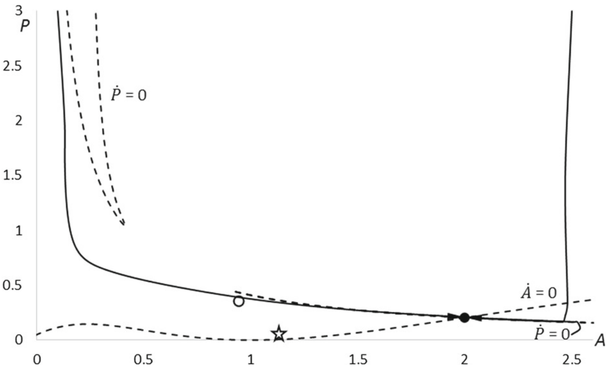


Fig. 12 Phase diagram in the state space with monotonic convergence (filled dot: interior solution, unfilled dot: corner solution with $v = 0$, unfilled star: corner solution with $u = 0$)

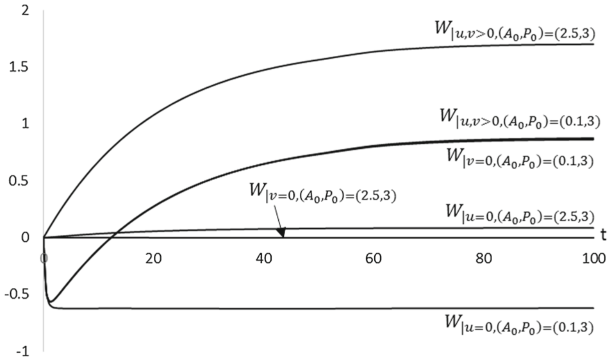


Fig. 13 Cumulative utility over time from relatively unfavorable and favorable initial conditions

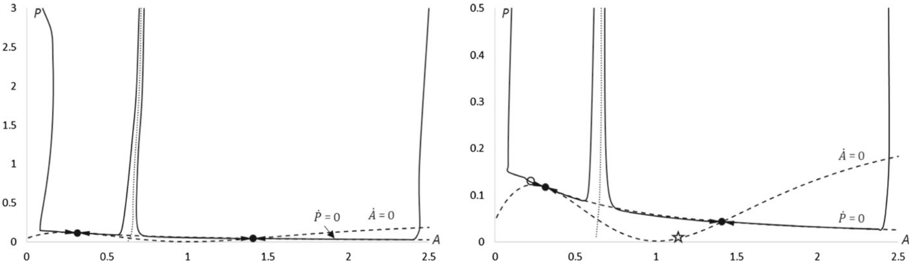
that from favorable conditions with state-dependent restoration a deforestation only path is made more profitable since restoration efforts are more effective in the long run, once we have settled at a higher than one absorption efficiency at the steady state. Conversely, from unfavorable conditions, the state-dependent property makes restoration much less effective, and the adverse effects of deforestation on absorption efficiency affects thus much more the social welfare, in part because they have to be compensated with heavy restoration spending at the beginning.

To conclude this analysis, we focus on a particularly interesting set of parameter values which reflects a scenario where production is significantly less profitable than deforestation, and that give rise to peculiar dynamics. For the set of parameter values such that $a = 0.053$, $b = 0.1$, $c = 0.01$,$\gamma = 0.05$, $\alpha = 0.15$, $r = 0.05$, there are two steady states for the interior solution (Table 6).

Figure 14 shows the steady states and the Skiba threshold (dotted line) that divides the state plane into two regions, each corresponding to the set of initial states that lead the system to either the left- or the right-hand steady-state value. The paths that start on the left-hand side of the Skiba threshold converge monotonically to $\left(A^1_\infty, P^1_\infty\right) = (0.315, 0.119)$, and those that start on the right-hand side converge monotonically to the more environmentally sustainable steady state $\left(A^2_\infty, P^2_\infty\right) = (1.409, 0.044)$.

Table 6 Steady-state values with both production and deforestation and with either production or deforestation

		A_∞	P_∞	u_∞	v_∞	w_∞
Interior solution	First steady state	0.315	0.119	0.037	0.003	0.030
	Second steady state	1.409	0.044	0.052	0.066	0.048
Corner solution with $v(t) = 0$		0.272	0.130	0.035	–	0.024
Corner solution with $u(t) = 0$		1.129	0.007	–	0.056	0.050



(a) Phase-portrait diagram **(b)** Zoom in the vicinity of the steady states

Fig. 14 Phase-portrait diagram in the state space with Skiba threshold (filled dot: interior solution, unfilled dot: corner solution with $v = 0$, unfilled star: corner solution with $u = 0$)

In a setting where deforestation revenues are significantly higher than production's, the optimal policy will depend on the initial conditions, thus reflecting history dependency. An economy starting from already deteriorated initial conditions, as on the left of the Skiba threshold in Fig. 15, will settle at the less desirable steady state $\left(A^1_\infty, P^1_\infty\right)$ whereas in presence more favorable initial conditions a more sustainable steady state $\left(A^2_\infty, P^2_\infty\right)$ will be attained. This particular property does not arise under state-independent restoration process, it is specific to the linear state-dependent restoration mechanism when deforestation is more profitable than production. Indeed, in that case, the economic tradeoff will be in favor of deforestation despite its more detrimental effect on absorption efficiency. But depending on the initial level of absorption efficiency, the leverage-effect highlighted previously will go one way or the other. If A_0 is already low, so will be the effectiveness of restoration efforts, and it will thus be optimal to implement deforestation with little restoration to compensate (Fig. 15). In this case, the only way to neutralize history dependency is to impose eradication of production-based emissions rather than deforestation, because this results in a steady state on the right-hand side of the Skiba threshold. Conversely, if A_0 is already large enough, highly effective restoration spending will allow to draw benefits from a higher level of deforestation (and production), while maintaining a sustainable level of absorption efficiency through higher and more effective restoration (Fig. 15).

5 Conclusions

This article extends the strand of literature on pollution accumulation by introducing deforestation and its impact on environmental absorption efficiency, and by allowing different restoration mechanisms. The objective is to determine an optimal policy-mix between pro-

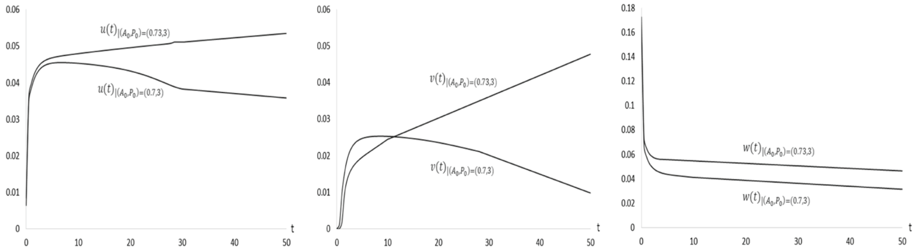


Fig. 15 Control policies from left- and right-hand side of the Skiba threshold

duction, deforestation and restoration of environmental absorption efficiency. In this respect, we suggest a first-best model where production and deforestation are the main pollution sources. Although the environmental absorption efficiency is negatively affected by both deforestation and pollution accumulation, it can be improved by restoration efforts, which can be either independent from or linearly dependent on the magnitude of pollution sinks. The economic tradeoff involves instantaneous revenues from production-based emissions and deforestation, and costs of pollution externalities and restoration efforts over an infinite time horizon.

Our conclusions are summarized below.

- Significantly unbalanced revenues from production and deforestation tend to lead to specialization in an activity with higher revenues and withdrawal from an activity with lower revenues.
- Restoration is used along optimal path to compensate the degradation of environmental absorption efficiency entailed directly by deforestation and indirectly by pollution accumulation in general.
- In general, a switching from a policy mix of production and deforestation to deforestation only leads to a higher level of deforestation. However, giving up deforestation does not lead to a significantly higher level of production-based emissions.
- In all cases, the optimal policy consists in *first restoring, then deforesting* from initially low environmental absorption efficiency, and *first deforesting, then restoring* from initially high environmental absorption efficiency.
- If the restoration mechanism is state-independent, the eradication of deforestation improves long-run environmental sustainability regardless of the structure of revenues from the two activities. In this case, however, stopping deforestation is economically justified only by initially low environmental absorption efficiency.
- If the restoration mechanism is linearly state-dependent, the effectiveness of restoration efforts is enhanced, which results in a positive influence on long-run environmental sustainability.
- If the restoration mechanism is linearly state-independent, stopping deforestation is neither environmentally nor economically desirable, as a greater effectiveness of restoration efforts can be achieved through the leverage effect in order to compensate the detrimental impact of deforestation on environmental absorption efficiency.
- If the restoration mechanism is linearly state-dependent, long-run environmental sustainability is greater along a deforestation-only path than along a production-only path, due to the leverage effect mentioned beforehand, regardless of the structure of revenues from the two activities.

- In the case of a low revenue-maximizing production level and an intermediate revenue-maximizing deforestation level, introducing linear state-dependency of restoration efforts leads to a Skiba point situation. As a result unfavorable initial conditions will lead to a long run solution with low environmental absorption efficiency and high pollution stock. In this scenario, the optimal policy displays a strong history-dependence that can be overcome by a deforestation-only path ensuring environmental sustainability with a more desirable steady state. This feature is quite noteworthy as it shows that under this specific incentive structure, there are initial situations from which it is not possible to revert to a sustainable long run solution. In doing so, our model echoes the concern for the *actual* state of 'initial' conditions when it comes to dwelling on theoretical models to enlighten actual environmental policies.

Most economic studies consider stopping deforestation the most effective way to improve both environmental sustainability and social welfare (e.g., Van Soest and Lensink 2000; Fredj et al. 2006; Stern 2006, 2015), yet our results, which are based on the prudent assumption that deforestation is a source of ephemeral revenues only, suggest that dropping deforestation from the policy mix is neither necessary nor sufficient. Instead, we conclude that a policy mix combining sustained levels of deforestation and significant restoration efforts can prove more efficient, both economically and environmentally, than a solution exclusively based on production. In addition, by permitting to consider the two kinds (state-independent and dependent) of restoration mechanism, our model sheds light of their importance on the optimal environmental policy at stake and thus calls for a more precise approach of environmental restoration, beyond the topical "*let's plant more trees*" perspective. Finally, the state-independency property generates over-confident results not only because, having disregarded that restoration of pollution sinks can suffer from inertia, it suggests that pollution sources to be restored back to sinks, but also because it systematically leads to overestimating the social welfare, especially under unfavorable initial conditions. This property is not compatible with the precaution principle and should be dropped in future research.

In this paper, the possibility of non-linear degradation of the absorption efficiency has not been explored. An accurate representation of a non-linear degradation mechanism in the absorption efficiency dynamics could be a concave-convex specification, as the one used for damage cost function in Moser et al. (2014). This specification, which would involve an inertial effect on the degradation mechanism side, might result in history-dependency and a multiple optimal long-run solution. We leave it as an extension in future research. Another interesting extension could consider the impact on the optimal policies of alternative revenue specification for production and deforestation, for example convex-concave functions, that would account for a wider range of revenue profiles for these activities. Finally, our results could be further generalized by accounting for the relative impact of forest-owners' and non-owners' non-cooperative strategies on welfare and environmental sustainability. An important issue related to this context is whether the onus for restoration of pollution sinks should fall on forest-owners or non-owners.

Acknowledgements The authors acknowledge helpful comments from one anonymous referee. This research was supported by ESSEC Business School (France) and Tel Aviv University (Israel). The first author dedicates this paper to the memory of Mohamed El Houari, a wonderful mentor and friend.

Appendix

A1. The Legendre-Clebsch condition holds for (4) since the Hessian is negative definite, that is:

$$\begin{bmatrix} H_{uu} & H_{uv} & H_{uw} \\ H_{vu} & H_{vv} & H_{vw} \\ H_{wu} & H_{wv} & H_{ww} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

The Hamiltonian is therefore concave with respect to (u, v, w), which guarantees a (local) maximum.

A2. The candidate arc segments of the optimal path are described in the following way.

Production-based emissions and deforestation arc

In this case, Eq. (7) yields $(u^*, v^*, w^*) = (u^\circ(\lambda), v^\circ(\lambda, \varphi), w^\circ(A^*, \varphi))$ and $\rho_1 = 0$, $\rho_2 = 0$, $\rho_3 = 0$, with:

$$\left(u^\circ v^\circ w^\circ\right)^t := \left(a + \lambda b + \lambda\alpha - \varphi\varphi A^\beta\right)^t \tag{A2.1}$$

being the solution of $H_u(P, A, u^\circ, v^\circ, w^\circ, \lambda, \varphi) = 0$, $H_v(P, A, u^\circ, v^\circ, w^\circ, \lambda, \varphi) = 0$ and $H_w(P, A, u^\circ, v^\circ, w^\circ, \lambda, \varphi) = 0$, respectively.

Plugging the expressions of u^*, v^* and w^* in (1)–(2) and (9)–(10), respectively, gives:

$$\dot{P} = a + \left(1 + \alpha^2\right)\lambda + \alpha(b - \varphi) - AP \tag{A2.2}$$

$$\dot{A} = \left(1 + A^{2\beta}\right)\varphi - b - \lambda\alpha - \gamma P \tag{A2.3}$$

$$\dot{\lambda} = (r + A)\lambda + \gamma\varphi + cP \tag{A2.4}$$

$$\dot{\varphi} = \left(r - \beta\varphi A^{2\beta-1}\right)\varphi + \lambda P \tag{A2.5}$$

To analyze the behavior of the canonical system (A2.2)–(A2.5) in the neighborhood of the steady state, if it exists, we compute the Jacobian matrix:

$$J = \begin{bmatrix} -A & -P & 1 + \alpha^2 & -\alpha \\ -\gamma & 2\beta\varphi A^{2\beta-1} & -\alpha & 1 + A^{2\beta} \\ c & \lambda & r + A & \gamma \\ \lambda & -\beta(2\beta - 1)\varphi^2 A^{2\beta-2} & P & r - \beta\varphi A^{2\beta-1} \end{bmatrix} \tag{A2.6}$$

where (P, A, λ, φ) are evaluated at their steady-state values, and whose determinant is given by:

$$\begin{aligned} |J| = {} & \beta\varphi^2 A^{2\beta-2} \left\{(1 - 2\beta)\left[(\alpha\gamma + A)(r + \alpha\gamma + A) + \gamma^2 + c\right] + (1 + 2\beta)A^{2\beta}\left[A(r + A) + c\left(1 + \alpha^2\right)\right]\right\} \\ & - 2\beta\varphi A^{2\beta-1}\left\{r\left[A(r + A) + c\left(1 + \alpha^2\right)\right] - (r + 2A)(\alpha\lambda + \gamma P) - 2\left[\gamma\lambda\left(1 + \alpha^2\right) - \alpha c P\right]\right\} \\ & - \alpha^2\lambda^2 A^{2\beta} + \left(1 + A^{2\beta}\right)\left[cP^2 - \lambda(\lambda + rP + 2AP)\right] \\ & - r\lambda\left[\gamma\left(1 + \alpha^2\right) + \alpha A\right] + P\left\{\alpha(rc - 2\gamma\lambda) + \gamma\left[\gamma P - r(2r + A)\right]\right\} \end{aligned}$$

and the sum of the principal minors of J of order 2 minus the squared discounting rate is:

$$K = -A(r + A) - c\left(1 + \alpha^2\right) + 2(\alpha\lambda - \gamma P) + \beta\varphi A^{2\beta-1}\left\{2r + \varphi A^{-1}\left[2\beta - 1 - (2\beta + 1)A^{2\beta}\right]\right\}$$

In the case of state-independent restoration efforts, $K_{|\beta=0} < 0$, which rules out the possibility of limit cycles (Dockner and Feichtinger 1991). For linearly state-dependent restoration efforts, the sign of $K_{|\beta=1} = -A(r + A) - c\left(1 + \alpha^2\right) + 2(\alpha\lambda - \gamma P) + \varphi\left[2rA + \varphi\left(1 - A^2\right)\right]$ is not clear.

Production-based emissions-only arc

In this case, the control constraint $v(t) \geq 0$ is active. Using (5), the maximizing condition (6) yields $(u^*, v^*, w^*) = (u^\circ(\lambda), 0, w^\circ(A^*, \varphi))$ and $\rho_1 = 0$, $\rho_2 \leq 0$, $\rho_3 = 0$, with:

$$\left(u^\circ w^\circ\right)^t := \left(a + \lambda\varphi A^\beta\right)^t \text{ and } \rho_2 = \varphi - b - \lambda\alpha \tag{A2.7}$$

being the solution of $L_u(P, A, u^\circ, 0, w^\circ, \lambda, \varphi, 0, \rho_2, 0) = 0$, $L_v(P, A, u^\circ, 0, w^\circ, \lambda, \varphi, 0, \rho_2, 0) = 0$ and $L_w(P, A, u^\circ, 0, w^\circ, \lambda, \varphi, 0, \rho_2, 0) = 0$, respectively. Plugging the expressions of u^* and w^* in (1)–(2) and (9)–(10) respectively gives the canonical system:

$$\dot{P} = a + \lambda - AP \tag{A2.8}$$

$$\dot{A} = \varphi A^{2\beta} - \gamma P \tag{A2.9}$$

$$\dot{\lambda} = (r + A)\lambda + \gamma\varphi + cP \tag{A2.10}$$

$$\dot{\varphi} = \left(r - \beta\varphi A^{2\beta-1}\right)\varphi + \lambda P \tag{A2.11}$$

The steady state, if it exists, is obtained by solving (11) where:

$$(\varphi_\infty \lambda_\infty A_\infty)^t = \left(\frac{\gamma a(r + A_\infty)}{\Phi} \; - \frac{a\left(\gamma^2 + cA_\infty^{2\beta}\right)}{\Phi} \; \frac{a(r + A_\infty)A_\infty^{2\beta}}{\Phi}\right)^t$$

with $\Phi = A_\infty^{2\beta}[c + A_\infty(r + A_\infty)] + \gamma^2$. It can be shown that the resolution of the system (A2.8)–(A2.11) for the case of state-independent restoration efforts ($\beta = 0$), leads to a steady state that is a saddle-point with either monotonic or spiraling convergence. On the other hand, if $\beta = 1$, we obtain $K_{|\beta=1} = -A\left[r + A\left(1 + 3\varphi^2\right)\right] - 2\gamma P - c + 2r\varphi A$. Therefore, the possibility of limit cycles cannot be ruled out for linearly state-dependent restoration efforts.

Deforestation-only arc

In this case, the control constraint $u(t) \geq 0$ is active. Using (5), the maximizing condition (6) yields $(u^*, v^*, w^*) = (0, v^\circ(\lambda, \varphi), w^\circ(A^*, \varphi))$ and $\rho_1 \leq 0$, $\rho_2 = 0$, $\rho_3 = 0$, with:

$$\left(v^\circ w^\circ\right)^t := \left(b + \lambda\alpha - \varphi\varphi A^\beta\right)^t \text{ and } \rho_1 = -\lambda - a \tag{A2.12}$$

being the solution of $L_u(P, A, 0, v^\circ, w^\circ, \lambda, \varphi, \rho_1, 0, 0) = 0$, $L_v(P, A, 0, v^\circ, w^\circ, \lambda, \varphi, \rho_1, 0, 0) = 0$ and $L_w(P, A, 0, v^\circ, w^\circ, \lambda, \varphi, \rho_1, 0, 0) = 0$, respectively. Plugging the expressions of v^* and w^* in (1)–(2) and (9)–(10) respectively gives the canonical system:

$$\dot{P} = \alpha(b + \lambda\alpha - \varphi) - AP \tag{A2.13}$$

$$\dot{A} = \varphi\left(1 + A^{2\beta}\right) - b - \lambda\alpha - \gamma P \tag{A2.14}$$

$$\dot{\lambda} = (r + A)\lambda + \gamma\varphi + cP \tag{A2.15}$$

$$\dot{\varphi} = \left(r - \beta\varphi A^{2\beta-1}\right)\varphi + \lambda P \tag{A2.16}$$

The steady state, if it exists, is obtained by solving (11) where:

$$(\varphi_\infty \lambda_\infty A_\infty)^t = \left(\frac{b(r + A_\infty)(\alpha\gamma + A_\infty)}{\Psi} \; - \; \frac{b\left[\gamma(\alpha\gamma + A_\infty) + \alpha c A_\infty^{2\beta}\right]}{\Psi} \; \frac{\alpha b(r + A_\infty) A_\infty^{2\beta}}{\Psi} \right)^t$$

with $\Psi = (A_\infty + \alpha\gamma)(r + A_\infty + \alpha\gamma) + A_\infty^{2\beta}\left[c\alpha^2 + A_\infty(r + A_\infty)\right]$. If $\beta = 0$, there can be no limit cycle. If $\beta = 1$, $K_{|\beta=1} = -A\left[r + A\left(1 + 3\varphi^2\right)\right] - \alpha^2 c + 2(\alpha\lambda - \gamma P) + \varphi(\varphi + 2rA)$, which implies that limit cycles are possible for linearly state-dependent restoration efforts.

A3. Using (A2.1) gives:

$$w^\circ = A^\beta\left(\alpha u^\circ - v^\circ + b - \alpha a\right)$$

A4. The steady state, if it exists, is obtained by solving (A2.8)–(A2.11) to zero. Equations (A2.8)–(A2.10) are linear in φ, λ and P, and after substituting their solutions into (A2.11), we get (11).

A5. The time decomposition method used for the numerical resolution follows the main steps below:

Step 1 Set a large enough time horizon $t \in [0, T]$. Set the initial states $P(0)$ and $A(0)$ at the given values. Set the terminal values of the costate variables at the steady-state values, $\lambda(T) = \lambda_\infty$ and $\varphi(T) = \varphi_\infty$.
Step 2 Guess feasible control functions, $u(t)$, $v(t)$ and $w(t)$.
Step 3 Integrate the state system from left to right.
Step 4 Integrate the costate system from right to left.
Step 5 If the optimality conditions are satisfied (the Hamiltonian is maximized at each t) with a required tolerance, stop. Otherwise, go to step 6.
Step 6 At each t, where the Hamiltonian is not maximized, change $u(t)$, $v(t)$ and $w(t)$ to $u(t) + \delta u(t)$, $v(t) + \delta v(t)$ and $w(t) + \delta w(t)$ where $\delta u(t)$, $\delta v(t)$ and $\delta w(t)$ are small enough positive/negative increments to make the Hamiltonian rise.
Step 7 Go to Step 3.

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