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S.I.: RELIABILITY MODELING WITH APPLICATIONS BASED ON BIG DATA

Application of dynamic evidential networks in reliability analysis of complex systems with epistemic uncertainty and multiple life distributions

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Abstract
With the modernization and intelligent of industrial equipment and systems, the challenges of dynamic characteristics, failure dependency and uncertainties have aroused by the increasing of system complexity. Besides, various types of components may follow different life distributions which bring the multiple life distributions problem in systems. In order to model the impact of time dependency and epistemic uncertainty on the failure behavior of system, this paper combines the flexible dynamic modeling with the uncertainty expression. Its advantages are intuitively graphical representation and reasoning that brought by evidential network (EN). After that, the discrete time dynamic evidential network (DT-DEN) is introduced to analyze the reliability of complex systems, and the network inference mechanism is clearly defined. The evidence theory and original definition and inference mechanism of conventional EN is firstly recommended, and the DT-DEN is further presented. Furthermore, the multiple life distributions are synthesized into the DT-DEN to tackle the epistemic uncertainty and mixed life distribution challenges. Specifically, the dynamic logic gates are converted into equivalent DENs with distinguished conditional mass tables, and then the belief interval of system reliability can be calculated by network forward reasoning. Finally, the availability and efficiency of the proposed method is verified by some numerical examples.

Keywords Evidence theory · Dynamic evidential networks · Epistemic uncertainty · Multiple life distribution

List of symbols

Ω	Frame of discernment
2^{Ω}	Power set
$m(\cdot)$	Mass function
$Bel(\cdot)$	Belief function
$Pl(\cdot)$	Plausibility function

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ζ	An evidential network (EN)
N	Node set of EN
E	Edge set of EN
M	Belief mass set of EN
$\zeta_{\rightarrow}$	A dynamic evidential network (DEN)
$N_{\rightarrow}$	Node set of DEN
$E_{\rightarrow}$	Edge set of DEN
$M_{\rightarrow}$	Belief mass set of DEN
$M(X)$	Belief mass assignment of variable X
$\pi(\cdot)$	Set of parent nodes
Δ	Length of each time slice
$\mathbf{M}(\cdot)$	State transition matrix
BN	Bayesian network
CN	Credal network
EN	Evidential network
DT-DEN	Discrete time dynamic evidential network
BDD	Binary decision diagram
DFT	Dynamic fault tree
P-box	Probability box
BPA	Basic probability assignment
CMT	Conditional mass table
CBMT	Conditional belief mass table
DAG	Directed acyclic graph
CSP	Cold spare gate
HSP	Hot spare gate
FDEP	Functional dependent gate

1 Introduction

In modern complex industrial systems, increasing complexity and interaction of systems and components makes failure dependency and dynamic behavior have become the core issues that influence the reliability of systems (Peng et al. 2018; Zhang et al. 2018a). Some classical static and dynamic methods have been widely used to solve these issues. i.e. binary decision diagram (BDD) (Pliego Marugán et al. 2017; Bryant 2018) expands to sequential and ordered binary decision diagram (SBDD, OBDD) (Khadiev and Khadieva 2017), fault tree (FT) model (Kabir 2017) corresponds to dynamic fault tree (DFT) model (Volk et al. 2018; Li et al. 2015) and fuzzy DFT, Bayesian networks (BNs) (Zarei et al. 2017) extends to the dynamic Bayesian networks (DBN) (Khakzad et al. 2017). In the meantime, the lifetime distribution of different types of components will be various, some mixed extension methods should be induced to analyze the reliability and evaluate the lifetime of these systems.

Another significant issue that has plagued engineers is the uncertainties caused by insufficient fault data, small sample data, and lack of knowledge of systems which means unknown function and structure of systems precisely, this is also called epistemic uncertainty. The inevitable aleatory uncertainty can be conducted by classical probability theory, but there is no unified approach to deal with the epistemic uncertainty problem. In recent years, the interval-valued approach, fuzzy theory and evidence theory are commonly used theories that have been continually improved to adapt different kinds of engineering systems (Wei et al. 2015). Based on the structure of BN, Misuri et al. (2018) compared the evidence theory-based BN, which also called evidential networks (EN), with the generalized BN, called Credal networks (CN), for epistemic uncertainty and imprecision information assessment, and the methods are used to tackle uncertainty in security assessment of critical infrastructures. For hybrid uncertainty, Li and Mahadevan (2016) proposed a new framework for model-based sensitivity analysis which can deal with both parameter uncertainty and model uncertainty. Simon and Bicking (2017) combined the evidence theory and probability-boxes (P-boxes) with BN to present a hybrid method for system reliability assessment on consideration of aleatory and epistemic uncertainty. Sun et al. (2017) provided a linear-time algorithm to compute the state distribution of multi-state components, and the redundancy allocation problem is studied for multi-state series–parallel systems with consideration of interval-bounded epistemic uncertainties. Weber and Simon (2008) firstly proposed the DEN and used it in system reliability analysis. Xiahou et al. (2018) have taken some extension of DEN for multi-state system modeling and system reliability computing with consideration of epistemic uncertainty, the importance measure is also conducted by formulating it into optimization problems. A synthesis of DEN and improved multi-attribute decision making method was demonstrated by Duan et al. (2017) and used for fault diagnosis of complex systems. Rahman et al. (2018) used a deterministic sampling method in dynamic event tree to quantify the impact of aleatory and epistemic uncertainty for system probabilistic safety assessment. Through extension of belief rules and uncertainty measures, Deng and Jiang (2018) presented a new evidential network-based framework for system dependence assessment and human reliability analysis. All those evidence theory-based methods are facing a computational problem which caused by discrete uncertainty quantification mechanism of evidence variables. To solve these problems, Zhang et al. (2018b) described a continuous representation of epistemic uncertainty by using Johnson p-boxes method. For the unified expression of different types of epistemic uncertainties, Lv et al. (2018) proposed a definition of evidence-based fuzzy variables and established an uncertainty qualification model for unifying multiple types of epistemic uncertainty. The authors have done some contributions on the reliability analysis of complex multi-state systems with epistemic uncertainty based on EN and P-box theory (Mi et al. 2016, 2018; Li et al. 2018a, b).

Although the aforementioned methods have partially solved the existing problems in real industrial fields, it is also confronted with many mixed challenges, such as how to model and quantify the comprehensive influence for system reliability or failure behavior by synthetic impact of uncertainty, dynamic property, and dependency of systems. In this paper, a discrete time dynamic evidential network (DT-DEN) is induced to deal with the epistemic uncertainty and dependency analysis of system, the system dynamic characteristics and multiple life distributions are also considered in the presented method. The remainder of this paper is organized as follows. The conventional evidential network is introduced in Sect. 2. The DT-DEN is formulated in Sect. 3, and the inference mechanism of commonly used logic relationships between nodes in evidential networks is clearly redefined. Section 4 shows two illustration examples to investigate the efficiency and validity of the presented method. A brief conclusion is summarized in Sect. 5.

2 Conventional evidential networks and its application

2.1 Evidence theory

Evidence theory, also known as belief function theory, was first proposed by Dempster and further expanded and developed by Shafer (Mi et al. 2018). Three critical functions, i.e.

mass function, belief function and plausibility function are defined on a discrete frame of discernment Ω, which includes all the n possible states of variable X. The mass function $m(X)$ (also called basic probability assignment (BPA)) is defined based on the power set of Ω that is 2^{Ω} which includes all possible subsets of Ω. Each subset A^X on the power set satisfying $m\left(A^X\right) > 0$ is called a focal element on Ω. The mass function should satisfy those two conditions: (1) $m(\emptyset) = 0$ and (2) $\sum_{X\in 2^{\Omega}} m\left(A^X\right) = 1$ which means the sum of BPA of focal elements on power set 2^{Ω} is 1.

The belief function of event Y: $Bel(Y)$ is defined as the sum of all the masses that support event Y, and can be calculated from mass function m by

$$Bel(Y) = \sum_{A^X \subseteq Y} m\left(A^X\right) \tag{1}$$

The plausibility function of event Y: $Pl(Y)$ is defined as the sum of all the masses that not contradict event Y, and expressed as

$$Pl(Y) = \sum_{A^X \cap Y \neq \emptyset} m\left(A^X\right) \tag{2}$$

Then the epistemic uncertainty can be represented by the gap between $Bel(Y)$ and $Pl(Y)$, which can be expressed as an interval $[Bel(Y), Pl(Y)]$.

2.2 Basic definition of EN

Evidential network (EN) can express system uncertainty by using directed acyclic graph (DAG) in random and epistemic method. An EN is composed by a couple of sets, which can be denoted by ζ=$((N, E), M)$, where (N, E) is the DAG that composed by note set N and edge set E, M is a set of belief masses that corresponding to each note in DAG. The relationship between a node and its parent nodes can be quantified by conditional belief mass table (CBMT). Then, for an intermediate node, the belief probability assignment (BPA) can be obtained by the marginalization of CBMT. But for a root node, the prior BPA need to be defined.

2.2.1 Belief mass assignment of root node in EN

A discrete variable X can be represented by a node $X \in N$ which is on the basis of frame of discernment Ω_X, and there are q mutually exclusive and independent hypotheses $H_l \in \left\{H_1, \dots, H_q\right\}$ on the frame of discernment Ω_X. The power set 2^{Ω_X} has 2^q elements, each element is a focal element of Ω_X, then the set of focal elements can be shown as $A_1^X = \left\{H_1\right\}, \dots, A_q^X = \left\{H_q\right\}, A_{q+1}^X = \left\{H_1, H_2\right\}, \dots, A_{2^q-1}^X = \left\{\cup_i H_i\right\}$. Furthermore, a mass function $m(X)$ should be defined on Ω_X to describe the difference between focal elements, which can be expressed as $m(X) : 2^{\Omega} \rightarrow [0, 1]$. It is a mapping function that used to map each element on power set 2^{Ω_X} into a certain number $m(X)$ which belongs to interval [0, 1]. The certain $m(X)$ represents the accuracy belief degree of a focal element. Based on expert opinions, imprecise probabilities or objectively collected data, the belief mass assignment (BMA) $M(X)$ can be obtained and defined by the following equation,

$$M(X) = \left[m(X \subseteq \emptyset), m\left(X \subseteq A_1^X\right), \dots, m\left(X \subseteq A_i^X\right), \dots, m\left(X \subseteq A_{2^q-1}^X\right)\right] \tag{3}$$

where $m(\varnothing) = 0$ and $\sum_{A_i^X \in 2^\Omega} m\left(X \subseteq A_i^X\right) = 1$.

Based on the above definition, for a node X with n mutually exclusive and independent states, the BMA can be defined as,

$$M(X) = \left[m\left(X = x_1\right), m\left(X = x_2\right), \ldots, m\left(X = x_n\right)\right] \tag{4}$$

where $\sum_{x_i \in 2^\Omega} m\left(X \subseteq x_i\right) = 1$ and $m\left(X \subseteq x_i\right) \geq 0$.

2.2.2 Inference mechanism of EN

For a multi-state EN with n root nodes $X_1, X_2, \ldots, X_n$ as shown in Fig. 1, the corresponding states of nodes are $x_1, x_2, \ldots, x_n$. Suppose that node X_i has l_i states, the frame of discernment is Ω_{X_i}, the leaf node Y has l_y states, which defines the frame of discernment Ω_Y, and the set of its parents is defined as $\pi(Y)$. Based on Cartesian product, the CBMT is used to express the infer relationship between root nodes X_i and leaf node Y. The CBMT of leaf node Y can the inferred by the following equation when the BPAs of root nodes are known (Mi et al. 2018; Simon and Sallak 2018).

$$\mathbf{M}(Y|\pi(Y)) = \begin{bmatrix} m\left(Y = y^1 \middle| X_1 = x_1^1, \ldots, X_n = x_n^1\right), & \ldots, & m\left(Y = y^1 \middle| X_1 = x_1^{l_1}, \ldots, X_n = x_n^{l_n}\right) \\ m\left(Y = y^j \middle| X_1 = x_1^1, \ldots, X_n = x_n^1\right), & \ldots, & m\left(Y = y^j \middle| X_1 = x_1^{l_1}, \ldots, X_n = x_n^{l_n}\right) \\ \vdots & & \vdots \\ m\left(Y = y^{l_y} \middle| X_1 = x_1^1, \ldots, X_n = x_n^1\right), & \ldots, & m\left(Y = y^{l_y} \middle| X_1 = x_1^{l_1}, \ldots, X_n = x_n^{l_n}\right) \end{bmatrix} \tag{5}$$

where $1 \leq j \leq l_y$, $1 \leq i \leq n$ and $1 \leq k_i \leq l_i$.

The BPA of leaf node Y can be obtained by,

$$m\left(Y = y^j\right) = \sum_{\substack{X_1 \subseteq 2^{\Omega_{X_1}} \\ \vdots \\ X_n \subseteq 2^{\Omega_{X_n}}}} m\left(Y = y^j \middle| X_1 = x_1^{k_1}, \ldots X_i = x_i^{k_i}, \ldots, X_n = x_n^{k_n}\right) \cdot m\left(X_1 = x_1^{k_1}\right) \cdot \cdots \cdot m\left(X_i = x_i^{k_i}\right) \cdot \cdots \cdot m\left(X_n = x_n^{k_n}\right) \tag{6}$$

For an EN with 3 nodes, suppose the root nodes X_1 and X_2 have 3 states, and the state space is $\Lambda = \{0, 1, \{0, 1\}\}$, where state $\{0, 1\}$ represents an uncertain state, which means the state of $X_i (i = 1, 2)$ cannot be sure to be success "0" or failure "1". For a EN with logic AND and OR gates under evidence theory, the conditional mass tables (CMTs) of leaf node Y can be shown as Tables 1, 2.

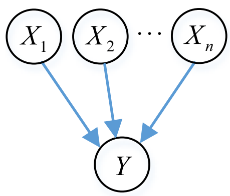


Fig. 1 An EN with n root nodes

Table 1 CMT of EN with "AND" logic relationship

X_1	X_2	Y(AND)			0		1	
		0	1	{0, 1}	*Bel*	*Pls*	*Bel*	*Pls*
0	0	1	0	0	1	1	0	0
0	1	1	0	0	1	1	0	0
0	{0, 1}	1	0	0	1	1	0	0
1	0	1	0	0	1	1	0	0
1	1	0	1	0	0	0	1	1
1	{0, 1}	0	0	1	0	1	0	1
{0, 1}	0	1	0	0	0	0	1	1
{0, 1}	1	0	0	1	0	1	0	1
{0, 1}	{0, 1}	0	0	1	0	1	0	1

Table 2 CMT of EN with "OR" logic relationship

X_1	X_2	Y(OR)			0		1	
		0	1	{0, 1}	Bel	Pls	Bel	Pls
0	0	1	0	0	1	1	0	0
0	1	0	1	0	0	0	1	1
0	{0, 1}	0	0	1	0	1	0	1
1	0	0	1	0	0	0	1	1
1	1	0	1	0	0	0	1	1
1	{0, 1}	0	1	0	0	0	1	1
{0, 1}	0	0	0	1	0	1	0	1
{0, 1}	1	0	1	0	0	0	1	1
{0, 1}	{0, 1}	0	0	1	0	1	0	1

Therefore, the relationship between basic probability assignment (BPA) of the states of leaf node Y and the states of root nodes can be expressed by,

$$m\left(Y = y^j \middle| X_1 = x_1^{k_1}, \dots X_i = x_i^{k_i}, \dots, X_n = x_n^{k_n}\right) = \frac{m\left(Y = y^j, X_1 = x_1^{k_1}, \dots, X_i = x_i^{k_i}, \dots, X_n = x_n^{k_n}\right)}{m\left(X_1 = x_1^{k_1}, \dots, X_i = x_i^{k_i}, \dots, X_n = x_n^{k_n}\right)} \tag{7}$$

For an EN that shown as Fig. 2 which has n root nodes X_i ($i = 1,\dots, n$) and m intermediate nodes Y_j ($j = 1,\dots, m$), the variables of root nodes X_i and intermediate nodes Y_i are represented by $x_i(i = 1, 2, \dots, n)$ and $y_j(j = 1, 2, \dots, m)$, then the k_i-th state of root nodes can be expressed as $x_{i,k_i}\left(1 \le k_i \le l_i\right)$. The k_j-th state of intermediate node is $y_{j,k_j}\left(1 \le k_j \le l_j\right)$. The variable of v-th state of leaf node is $T_v(v = 1, 2, \dots, q)$. Based on the previous inference mechanism shown in Eq. (7), the probability of leaf node T on failure state T_v can be expressed by a probability interval, which is,

$$[P]\left(T = T_v\right) = \left[Bel\left(T = T_v\right), Pl\left(T = T_v\right)\right] \tag{8}$$

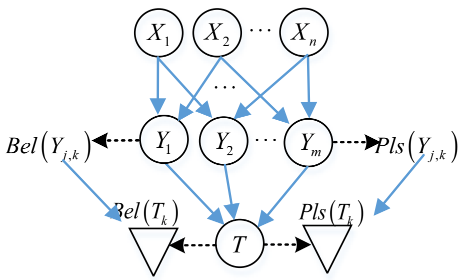


Fig. 2 The inference diagram of an EN

The plausibility probability of intermediate node that represents the upper bound of interval, is defined as Eq. (9),

$$Pl(Y = Y_V) = \sum_{x_1,\dots x_n \cap Y_V \neq \varnothing} m(y = y_V | x_1, x_2, \dots x_n) m(x_1) m(x_2) \cdots m(x_n) \tag{9}$$

The lower bound of interval is the belief reliability of intermediate node, and can be computed by using the following equation,

$$Bel(Y = Y_V) = \sum_{x_1,\dots x_n \subseteq Y_V} m(y = y_V | x_1, x_2, \dots x_n) m(x_1) m(x_2) \cdots m(x_n) \tag{10}$$

The plausibility probability of leaf node $Pl(T = T_v)$ can be calculated by Eq. (11),

$$Pl(T = T_V) = \sum_{y_1,\dots y_n \cap T_V \neq \varnothing} m(T = T_V | y_1, y_2, \dots y_n) m(y_1) m(y_2) \cdots m(y_n) \tag{11}$$

And the belief probability of leaf node $Bel(T = T_v)$ is

$$Bel(T = T_V) = \sum_{y_1,\dots y_n \subseteq T_V} m(T = T_V | y_1, y_2, \dots y_n) m(y_1) m(y_2) \cdots m(y_n) \tag{12}$$

3 DT- DEN models for system reliability analysis

3.1 DT-DEN modeling

DEN can be seen as a prolongation of EN on temporal dimension, then from the definition of EN in Sect. 2, A DEN can be represented as $\zeta_{\rightarrow} = \left(\left(N_{\rightarrow}, E_{\rightarrow}\right), M_{\rightarrow}\right) < E_0, E_{\rightarrow} >$, ξ_0 is the initial state or observation model with model state X^t which contains observation variables and latent variables. The obervation variable represents the leaf node in EN, and latent varialbes are root nodes and intermediate nodes. When $X^{(0)}$ represents the initial state of components and system, $\xi_{\rightarrow}$ is the transmit model of DEN. In a temporal slice of DEN, the observation variable is related to the latent variables which are correlated with the other variables in current temporal slice and the latent variables in previous temporal slice. There are 5 steps to built DT-DEN model.

Step 1 State definition. Assume that the system mission time is T, and it can be equally divided into K intervals, and each time slice has a length of $\Delta = T/K$. Then the timeline can be divided into $K+1$ intervals including $[(p-1)\Delta, p\Delta), \ldots, [T, \infty)(1 \le p < K)$ which are defined as the state space of nodes in BN. Let $X_{i,[(p-1)\Delta,p\Delta]}$ represents the failure state that system or component X_i is failed at time slice $[(p-1)\Delta, p\Delta)$, and $X_{i,[T,\infty)}$ means X_i will not fail (success) at mission time T.

Step 2 Building the dynamic fault tree model. By analyzing the function structure and failure process of practical engineering system, a DFT model can be built.

Step 3 Mapping the DFT model to DEN. The structure of DEN will be the same as BN, the differences are embodied in the CMTs on time slice. Therefore, based on the mapping rule of DFT to BN, the events in DFT can be mapped to the corresponding nodes in DEN.

Step 4 The mass assignment of node in DEN. After the modeling of DEN, the temporal dimension should be divided into K intervals, which means the state space of each root node will has $K+1$ state, and the failure probability distribution of these states will be the prior mass assignments of root nodes. And the CMTs can be gotten and used to express the failure logic relation between root nodes in DEN. The different CMTs corresponding to different logic gate in DFT will be discussed in the following sections.

Step 5 System reliability calculation by using DEN model inference.

When the failure probability density function (PDF) of X_i is $f_{X_i}(t)$, and node X_i is a root node with two states, where 1 represents failure state and 0 is working state. The prior mass assignment (probability distribution) can be expressed as the following equation,

$$m\left(X_{i,[(p-1)\Delta,p\Delta)} = 1\right) = \int_{(p-1)\Delta}^{p\Delta} f_{X_i}(t)\mathrm{d}t, 1 \le k \le n \tag{13}$$

$$m\left(X_{i,[T,\infty)} = 1\right) = \int_{T}^{\infty} f_{X_i}(t)\mathrm{d}t \tag{14}$$

When considering the epistemic uncertainty in system or component reliability parameter, an uncertain state {0, 1} is induced and it is transmitted through the time line on whole life cycle. The PDF of component can be represented by interval variable, that is $f_{X_i}(t)=\left[\underline{f}_{X_i}, \bar{f}_{X_i}\right]$. Then for each time interval, the error can be defined as,

$$m\left(X_{i,[(p-1)\Delta,p\Delta)} = \{0,1\}\right) = \int_{(p-1)\Delta}^{p\Delta} \bar{f}_{X_i}(t)\mathrm{d}t - \int_{(p-1)\Delta}^{p\Delta} \underline{f}_{X_i}(t)\mathrm{d}t \tag{15}$$

Then the prior mass assignment of node X in Eqs. (13) and (14) with consideration of epistemic uncertainty can be further expressed as,

$$m\left(X_{i,[(p-1)\Delta,p\Delta)} = 1\right) = \int_{(p-1)\Delta}^{p\Delta} \underline{f}_{X_i}(t)\mathrm{d}t \tag{16}$$

$$m\left(X_{i,[(p-1)\Delta,p\Delta)} = 0\right) = 1 - \int_{(p-1)\Delta}^{p\Delta} \bar{f}_{X_i}(t)\mathrm{d}t \tag{17}$$

The BMA on temporal dimension will be shown in Fig. 3, and the probabilistic inference of EN on temporal dimension can be shown in Fig. 3.

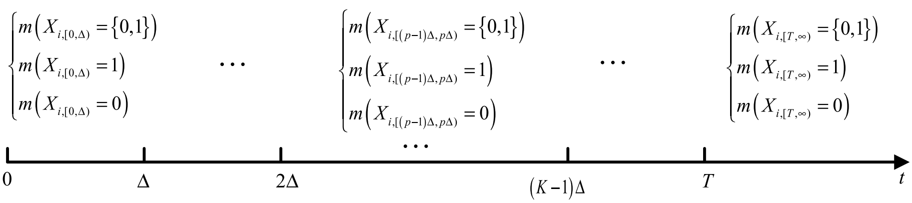


Fig. 3 The BMA on temporal dimension

3.2 DT-DEN with multiple life distribution

In terms of the epistemic uncertainty in system and the different failure distribution of basic events, the failure rates of root nodes of EN for complex system are expressed in interval variables. The typical life distributions of components and the corresponding failure rate function can be represented as the following equations,

(1) For component which lifetime obeys exponential distribution with failure rate λ which can be computed by using expert elicitation and fuzzy set theory (Duan, et al. 2017), the failure rate function is a constant interval, and

$$\left[\lambda_{\mathrm{exp}}\right](t) = \left[\underline{\lambda}_{\mathrm{exp}}, \bar{\lambda}_{\mathrm{exp}}\right] \tag{18}$$

The failure probability function of exponential distribution is

$$F_{\mathrm{exp}}(t) = 1 - \exp\left(-\left[\lambda_{\mathrm{exp}}\right]t\right) \tag{19}$$

(2) For component which lifetime obeys two-parameter Weibull distribution with shape parameter β and scale parameter $[\eta] = \left[\underline{\eta}, \bar{\eta}\right]$, the failure rate function and failure probability function can be represented as

$$\lambda_{\mathrm{Wb}}(t) = \left[\frac{\beta}{\eta}\right]\left(\frac{t}{[\eta]}\right)^{\beta-1} \tag{20}$$

$$F_{\mathrm{Wb}}(t) = 1 - \exp\left\{-\left(\frac{t}{[\eta]}\right)^{\beta}\right\} \tag{21}$$

(3) For component which lifetime obeys lognormal distribution with location parameter $[\mu] = \left[\underline{\mu}, \bar{\mu}\right]$ and shape parameter σ, the failure rate function and failure probability function will be

$$\lambda_{\mathrm{Logn}}(t) = \frac{\frac{1}{t\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln t-[\mu]}{\sigma}\right)^2\right]}{\int_t^{+\infty}\frac{1}{t\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln t-[\mu]}{\sigma}\right)^2\right]\mathrm{d}t} \tag{22}$$

$$F_{\text{Logn}}(t) = \int_0^t \frac{1}{t\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln t - [\mu]}{\sigma}\right)^2\right] dt \tag{23}$$

The parameters for Weibull distribution and lognormal distribution can be calculated by using the coefficient of variation (COV) method (Mi, et al. 2016).

As for the epistemic uncertainties are expressed by the interval parameters in multiple lifetime distributions, the system failure probability will be calculated by using the following equation,

$$[P_S] = \left[\underline{P}_S, \bar{P}_S\right] = \left[Bel(F_S), Pl(F_S)\right] \tag{24}$$

3.3 Inference mechanism of DT-DEN

3.3.1 The relationship between EN and Markovian behavior

For node X, corresponding to a component with the exponential distribution, X^k and X^{k+1} are used to express the random variable X being on time slice k and $k+1$, respectively. Assume that the failure rate of X at time t is $\lambda_X = \lambda_X(t)$, the epistemic uncertainty can be induced by failure rate interval $[\underline{\lambda}_X, \overline{\lambda}_X]$. Then the following three discussions about conditional confidence probability of X need to be conducted (Duan, et al. 2017).

(1) When the component is working at time k, which means the state of node X is "0", after the transition time it may fail on state "1", or with uncertain state {0, 1}. Then the transition mass distribution at time $k+1$ will become

$$m(X^{k+1} = 1|X^k = 0) = \underline{\lambda}_X \tag{25}$$

$$m(X^{k+1} = \{0,1\}|X^k = 0) = \overline{\lambda}_X - \underline{\lambda}_X \tag{26}$$

$$m(X^{k+1} = 0|X^k = 0) = 1 - \overline{\lambda}_X \tag{27}$$

(2) When the component failed at time k, the state of node X is "1", the transition mass distribution at time $k+1$ will be

$$m(X^{k+1} = 0|X^k = 1) = 0 \tag{28}$$

$$m(X^{k+1} = \{0,1\}|X^k = 1) = 0 \tag{29}$$

$$m(X^{k+1} = 1|X^k = 1) = 1 \tag{30}$$

(3) When the component is uncertain at time k, then the state of node X is {0, 1}, the transition mass distribution after the transition time will be

$$m(X^{k+1} = 1|X^k = \{0,1\}) = \underline{\lambda}_X \tag{31}$$

$$m(X^{k+1} = 0|X^k = \{0,1\}) = 0 \tag{32}$$

$$m(X^{k+1} = \{0,1\}|X^k = \{0,1\}) = 1 - \underline{\lambda}_X \tag{33}$$

According to the discussions from Eq. (25) to Eq. (33), the (state transition matrix) conditional belief mass table (CMT) of root nodes can be obtained and expressed as Eq. (34) (Duan et al. 2017)

$$\mathbf{M}\left(X^{k+1}\middle|X^{k}\right)=\begin{bmatrix} m(X^{k+1}=0|X^{k}=0) & m(X^{k+1}=1|X^{k}=0) & m(X^{k+1}=\{0,1\}|X^{k}=0) \\ m(X^{k+1}=0|X^{k}=1) & m(X^{k+1}=1|X^{k}=1) & m(X^{k+1}=\{0,1\}|X^{k}=1) \\ m(X^{k+1}=0|X^{k}=\{0,1\}) & m(X^{k+1}=1|X^{k}=\{0,1\}) & m(X^{k+1}=\{0,1\}|X^{k}=\{0,1\}) \end{bmatrix}$$
$$=\begin{bmatrix} 1-\overline{\lambda}_X & \underline{\lambda}_X & \overline{\lambda}_X-\underline{\lambda}_X \\ 0 & 1 & 0 \\ 0 & \underline{\lambda}_X & 1-\underline{\lambda}_X \end{bmatrix} \tag{34}$$

From Sect. 2.2.1, a priori belief mass assignment of X at time slice k can be defined as

$$\mathbf{m}\left(X^{k}\right)=\begin{cases} m\left(X^{k}=0\right)=\underline{P}_X \\ m\left(X^{k}=1\right)=1-\bar{P}_X \\ m\left(X^{k}=\{0,1\}\right)=\bar{P}_X-\underline{P}_X \end{cases} \tag{35}$$

Then the state mass distribution of component X after a time slice can be calculated by,

$$\mathbf{m}\left(X^{k+1}\right)=\mathbf{m}\left(X^{k}\right)\cdot\mathbf{M}\left(X^{k+1}\middle|X^{k}\right) \tag{36}$$

All those works are based on two hypothetical preconditions: (1) the topology of the DEN does not change over time, (2) the inference of DEN satisfies the condition of first-order Markov model, which means the current state of system is only related to the immediately preceding temporal slice. On the basis of these two assumptions, the DEN can be seen as the expansion of the EN on temporal dimension, and the node measures can be intuitively represented by Fig. 4.

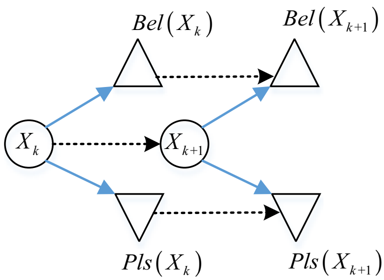


Fig. 4 Computing nodes of Bel and Pls measures

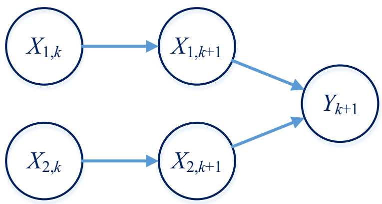


Fig. 5 The equivalent DEN of logic "AND" and "OR"

Table 3 CMT for AND logic structure

$X_{1,k}$	$X_{2,k}$	$X_{1,k+1}$	$X_{2,k+1}$	Y_k (AND)			Y_{k+1} (AND)			0		1		0		1	
				0	1	{0, 1}	0	1	{0, 1}	Bel_k	Pls_k	Bel_k	Pls_k	Bel_{k+1}	Pls_{k+1}	Bel_{k+1}	Pls_{k+1}
0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	0	{0, 1}	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	0	1	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	1	0	1	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	1	{0, 1}	1	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	1	1	1	1	0	0	0	1	0	1	1	0	0	0	0	1	1
0	{0, 1}	0	{0, 1}	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	{0, 1}	{0, 1}	{0, 1}	1	0	0	0	0	1	1	1	0	0	0	1	0	1
0	{0, 1}	1	{0, 1}	1	0	0	0	0	1	1	1	0	0	0	1	0	1
1	0	1	0	1	0	0	0	0	1	1	1	0	0	1	1	0	0
1	1	1	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
1	{0, 1}	1	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
{0, 1}	0	{0, 1}	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
{0, 1}	0	1	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
{0, 1}	1	0	1	0	0	1	1	0	0	0	1	0	1	1	1	0	0
{0, 1}	1	{0, 1}	1	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	1	1	1	0	0	1	0	1	0	0	1	0	1	0	0	1	1
{0, 1}	{0, 1}	0	{0, 1}	0	0	1	1	0	0	0	1	0	1	1	1	0	0
{0, 1}	{0, 1}	{0, 1}	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	{0, 1}	1	{0, 1}	0	0	1	0	0	1	0	1	1	1	0	1	0	1

Table 4 CMT for OR logic structure

$X_{1,k}$	$X_{2,k}$	$X_{1,k+1}$	$X_{2,k+1}$	Y_k (OR)			Y_{k+1} (OR)			0		1		0		1	
				0	1	{0, 1}	0	1	{0, 1}	Bel_k	Pls_k	Bel_k	Pls_k	Bel_{k+1}	Pls_{k+1}	Bel_{k+1}	Pls_{k+1}
0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	0	{0, 1}	0	1	0	0	0	0	1	1	1	0	0	0	1	0	1
0	0	1	0	1	0	0	0	1	0	1	1	0	0	0	0	1	1
0	1	0	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
0	1	{0, 1}	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
0	1	1	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
0	{0, 1}	0	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
0	{0, 1}	{0, 1}	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
0	{0, 1}	1	{0, 1}	0	0	1	0	1	0	0	1	0	1	0	0	1	1
1	0	1	0	0	1	0	0	1	0	0	0	1	1	0	0	1	1
1	1	1	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
1	{0, 1}	1	{0, 1}	0	1	0	0	1	0	0	0	1	1	0	0	1	1
{0, 1}	0	0	0	0	0	1	1	0	0	0	1	0	1	1	1	0	0
{0, 1}	0	{0, 1}	0	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	0	1	0	0	1	0	0	1	0	0	0	1	1	0	0	1	1
{0, 1}	1	0	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
{0, 1}	1	{0, 1}	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
{0, 1}	{0, 1}	0	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	{0, 1}	{0, 1}	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	{0, 1}	1	{0, 1}	0	0	1	0	1	0	0	1	0	1	0	0	1	1

3.3.2 CMTs determination for DEN with multiple dynamic logic relation

As for system dynamic characteristic in engineering systems, the commonly used modeling method is dynamic fault tree method which uses dynamic logic gates to represent the logic relation between components, i.e. cold spare gate (CSP), hot spare gate (HSP), functional dependent gate (FDEP), etc. From what have discussed in Sect. 3.1, the logic gates in FT and DFT can be translated to the equivalent DEN. For the static logic "AND" and "OR" gates, the equivalent DEN can be shown as Fig. 5. The CMTs at the k-th time interval are shown as Tables 3 and 4, respectively.

Based on the logic of CSP and HSP gates, the equivalent DEN can be obtained and shown in Fig. 6. The corresponding CMTs at the k-th time interval can be shown as Tables 5 and 6, respectively.

For the dynamic FDEP gate in DFT, the equivalent DEN and CMT are given in Fig. 7 and Table 7, respectively.

3.3.3 Calculation of system reliability

For a DT-DEN, $X^{(0)}$ represents the initial state of components and system, $E_{\rightarrow}$ is the transmit model of DEN. Suppose that the latent variable is X_k, and the observation variable is Y_k, for the initial slice to k-the slice, the mass assignment of output Y_i will be,

$$\mathbf{m}\left(Y_{1:k}\right) = \mathbf{m}\left(X_0\right)\prod_{i=1}^{k}\mathbf{M}\left(X_i|X_{i-1}\right)\mathbf{M}\left(Y_k|X_k\right) \tag{37}$$

where $m(X_i|X_{i-1})$ represent the transmit model, it is generally supposed to be not change over time. Then the mass assignment of Y_k from (k-1)-th temporal slice to k-th slice can be gotten by Eq. (37) and,

$$\mathbf{m}\left(Y_{k-1:k}\right) = \mathbf{m}\left(X_{k-1}\right)\mathbf{M}\left(X_k|X_{k-1}\right)\mathbf{M}\left(Y_k|X_k\right) \tag{38}$$

The upper limit of system failure probability interval $Pl(Y_i = Y_v)$ is the plausibility mass of leaf node, and can be calculated by the following Eq. (39),

$$Pls\left(Y_{1:k}\right) = \mathbf{m}\left(X_0\right)\prod_{i=1}^{k}\mathbf{M}\left(X_i|X_{i-1}\right)Pls\left(Y_k|X_k\right) \tag{39}$$

The lower limit of system failure probability interval $Bel(T = T_v)$ is the belief mass of leaf node, which can be computed by Eq. (40),

$$Bel\left(Y_{1:k}\right) = \mathbf{m}\left(X_0\right)\prod_{i=1}^{k}\mathbf{M}\left(X_i|X_{i-1}\right)Bel\left(Y_k|X_k\right) \tag{40}$$

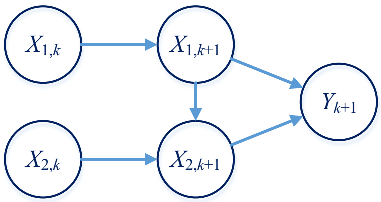


Fig. 6 The equivalent DEN of logic "CSP" and "HSP"

Table 5 CMT for CSP logic structure

$X_{1,k}$	$X_{2,k}$	$X_{1,k+1}$	$X_{2,k+1}$	Y_k (AND)			Y_{k+1} (AND)			0		1		0		1	
				0	1	{0, 1}	0	1	{0, 1}	Bel_k	Pls_k	Bel_k	Pls_k	Bel_{k+1}	Pls_{k+1}	Bel_{k+1}	Pls_{k+1}
0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	0	{0, 1}	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	0	1	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	1	0	1	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	1	{0, 1}	1	1	0	0	0	0	1	1	1	0	0	0	1	0	1
0	1	1	1	1	0	0	0	1	0	1	1	0	0	0	0	1	1
0	{0, 1}	0	{0, 1}	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	{0, 1}	{0, 1}	{0, 1}	1	0	0	0	0	1	1	1	0	0	0	1	0	1
0	{0, 1}	1	{0, 1}	1	0	0	0	0	1	1	1	0	0	0	1	0	1
1	0	1	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
1	1	1	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
1	{0, 1}	1	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
{0, 1}	0	{0, 1}	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
{0, 1}	0	1	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
{0, 1}	1	0	1	0	0	1	1	0	0	0	1	0	1	1	1	0	0
{0, 1}	1	{0, 1}	1	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	1	1	1	0	0	1	0	1	0	0	1	0	1	0	0	1	1
{0, 1}	{0, 1}	0	{0, 1}	0	0	1	1	0	0	0	1	0	1	1	1	0	0
{0, 1}	{0, 1}	{0, 1}	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	{0, 1}	1	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1

Table 6 CMT for HSP logic structure

$X_{1,k}$	$X_{2,k}$	$X_{1,k+1}$	$X_{2,k+1}$	Y_k (HSP)			Y_{k+1} (HSP)			0		1		0		1	
				0	1	{0, 1}	0	1	{0, 1}	Bel_k	Pls_k	Bel_k	Pls_k	Bel_{k+1}	Pls_{k+1}	Bel_{k+1}	Pls_{k+1}
0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	0	{0, 1}	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	0	1	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	1	0	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
0	1	{0, 1}	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
0	1	1	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
0	{0, 1}	0	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
0	{0, 1}	{0, 1}	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
0	{0, 1}	1	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
1	0	1	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
1	1	1	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
1	{0, 1}	1	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
{0, 1}	0	{0, 1}	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
{0, 1}	0	1	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
{0, 1}	1	0	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
{0, 1}	1	{0, 1}	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
{0, 1}	1	1	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
{0, 1}	{0, 1}	0	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	{0, 1}	{0, 1}	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	{0, 1}	1	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1

Table 7 CMT for FDEP logic structure

Tr_k	X_k	Tr_{k+1}	X_{k+1}	Y_k (FDEP)			Y_{k+1} (FDEP)			0		1		0		1	
				0	1	{0, 1}	0	1	{0, 1}	Bel_k	Pls_k	Bel_k	Pls_k	Bel_{k+1}	Pls_{k+1}	Bel_{k+1}	Pls_{k+1}
0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	0	0
0	0	{0, 1}	0	1	0	0	0	1	0	1	1	0	0	0	1	0	1
0	0	1	0	1	0	0	0	1	0	1	1	0	0	0	0	1	1
0	1	0	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
0	1	{0, 1}	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
0	1	1	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
0	{0, 1}	0	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
0	{0, 1}	{0, 1}	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
0	{0, 1}	1	{0, 1}	0	0	1	0	1	0	0	1	0	1	0	0	1	1
1	0	1	0	0	1	0	0	1	0	0	0	1	1	0	0	1	1
1	1	1	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
1	{0, 1}	1	{0, 1}	0	1	0	0	1	0	0	0	1	1	0	0	1	1
{0, 1}	0	0	0	0	0	1	1	0	0	0	1	0	1	1	1	0	0
{0, 1}	0	{0, 1}	0	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	0	1	0	0	0	1	0	1	0	0	1	0	1	0	0	1	1
{0, 1}	1	0	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
{0, 1}	1	{0, 1}	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
{0, 1}	1	1	1	0	1	0	0	1	0	0	0	1	1	0	0	1	1
{0, 1}	{0, 1}	0	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	{0, 1}	{0, 1}	{0, 1}	0	0	1	0	0	1	0	1	0	1	0	1	0	1
{0, 1}	{0, 1}	1	{0, 1}	0	0	1	0	1	0	0	1	0	1	0	0	1	1

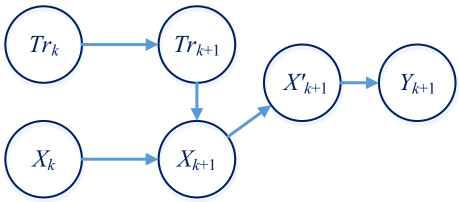


Fig. 7 The equivalent DEN of logic "FDEP"

4 Numerical examples: application of DT-DEN

4.1 Example 1: a simple DEN

For the EN in Fig. 8, the root nodes X_1 and X_2 are independent, the logic relationship with leaf node Y is shown in Table 1. Assume that the node has three states, including state 0, state 1 and state {0, 1}, which represent working state, failure state, and uncertain state respectively. Suppose that the root nodes obeys exponential distribution with failure rates of $\lambda_1 = 0.002$ and $\lambda_2 = 0.005$, from Sect. 2.2.2, when considering the epistemic uncertainty in system, the failure rates can be obtained and $\lambda_1' = [0.0019, 0.0021]$ and $\lambda_2' = [0.00475, 0.00525]$. The initial state probabilities of root nodes X_1 and X_2 are listed in Table 8.

By using the inference method of EN in Sect. 3.3, the belief probability and plausibility probability of leaf node Y can be obtained and listed in Table 9.

Suppose that the state transition matrix of root nodes X_1 and X_2 are A_1 and A_2, where

$$A_1 = \begin{bmatrix} 0.9979 & 0.0019 & 0.0002 \\ 0 & 1 & 0 \\ 0 & 0.0019 & 0.9981 \end{bmatrix} \tag{41}$$

$$A_2 = \begin{bmatrix} 0.99475 & 0.00475 & 0.0005 \\ 0 & 1 & 0 \\ 0 & 0.00475 & 0.99525 \end{bmatrix} \tag{42}$$

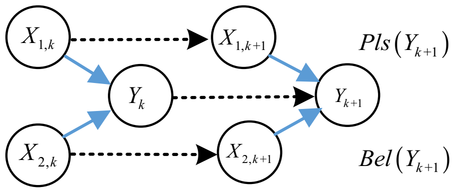
Then the state probability of root nodes X_1 and X_2 at $t = 2000$ h can be gotten by Eqs. (25)–(34), and listed in Table 10.

Then the belief probability and plausibility of leaf node Y at time $t + 1$ can be computed and listed in Table 11. And the evidential reliability curve of root nodes can be obtained and shown as Fig. 9.

4.2 Example 2: a DEN with multiple lifetime distributions

To further illustrate the application of this induced DT-DEN method, the reliability of sample system with dynamic characteristics and multiple lifetime distribution is implemented in this section. The DFT of the sample system can be built as Fig. 10, based on the DT-DEN modeling method in Sect. 3.1, the corresponding DEN can be built and shown in Fig. 10.

Fig. 8 Node mass inference of DEN on temporal dimension



In this numerical example, the lifetime of component X_1 is supposed to satisfy the following two scenarios: (1) X_1 follows exponential distribution with the failure rate $[\lambda_{X_1}] = [6.08e-5, 9.12e-5]/\text{h}$; (2) X_1 follows a two-parameter Weibull distribution with sharp parameter β and scale parameter η. Based on the general accelerated life test, the interval reliable life of X_1 is gotten as $t_{R=0.95}^{\text{Wb}} = 2100$ h and $t_{R=0.5}^{\text{Wb}} = 4200$ h. Then the reliable life of Weibull distribution is $t_R^{\text{Wb}} = \eta(-\ln R(t))^{1/\beta}$ (Mi, et al. 2016),

Table 8 Initial state probabilities of root nodes

Root node	State		
	0	1	{0, 1}
X_1	0.9916	0.0084	0
X_2	0.9954	0.0019	0.0027

Table 9 The state probability of leaf node

Leaf node	State			
	Bel		*Pls*	
	0	1	0	1
Y	0.99728	0.00002	0.99998	0.00272

Table 10 State probability of root nodes at t = 2000 h

Root node	State		
	0	1	[0, 1]
X_1	0.0148	0.9779	0.0073
X_2	2.6662e−05	0.9999	4.6376e−05

Table 11 State probability of leaf nodes at t = 2000 h

Leaf node	State			
	Bel		*Pls*	
	0	1	0	1
Y	0.0148	0.9778	0.0222	0.9851

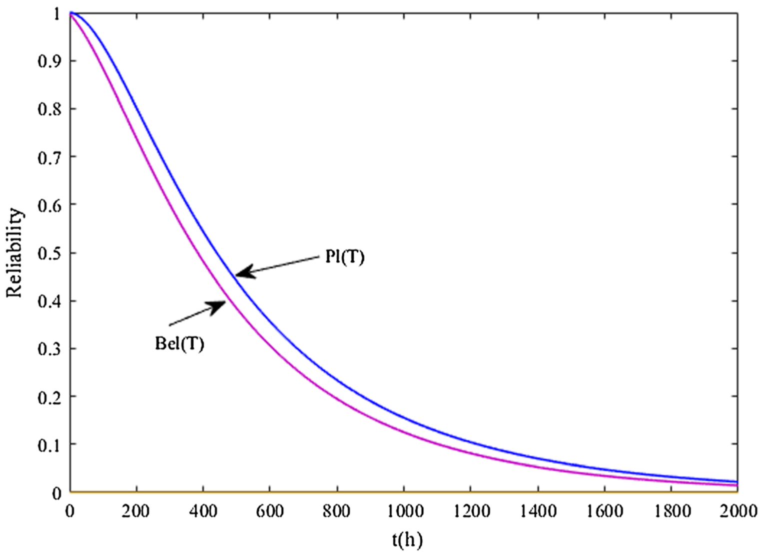


Fig. 9 The calculation result of evidential reliability

the parameters of X_1 can be calculated as $\beta = 3.76$ and $\eta = 4630.46$. When the components X_2 and X_3 follows exponential distributions, the failure rates are given as intervals $[\lambda_{X_2}] = [\lambda_{X_3}] = [6.08\mathrm{e}-5, 9.12\mathrm{e}-5]/\mathrm{h}$. Based on the multiple lifetime distribution inference mechanism and system reliability calculation method in Sect. 3, the belief and plausibility reliability of system can be computed. The reliability calculation results are compared with the directly sampling result, which are shown in Figs. 11 and 12.

From Fig. 11, the upper and lower bound of system reliability calculated by directly sampling are within the region of the results by DT-DEN, which verify the validity of this method. From Fig. 12, the lifetime distributions of components are supposed to be different, and the result has verified the feasibility of DT-DEN method to solve the multiple life distributions problem with the combination of uncertainty problems in system reliability analysis.

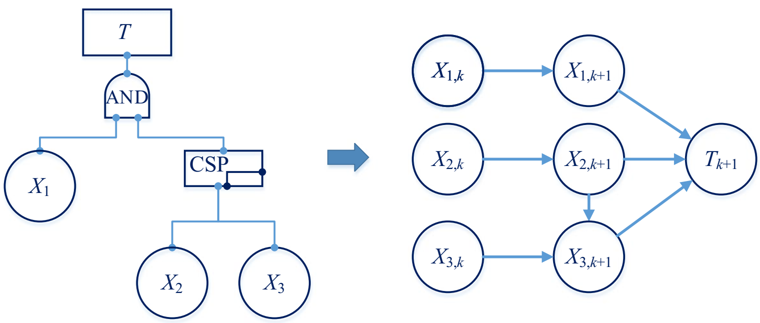


Fig. 10 The mapping relation of DFT to DEN

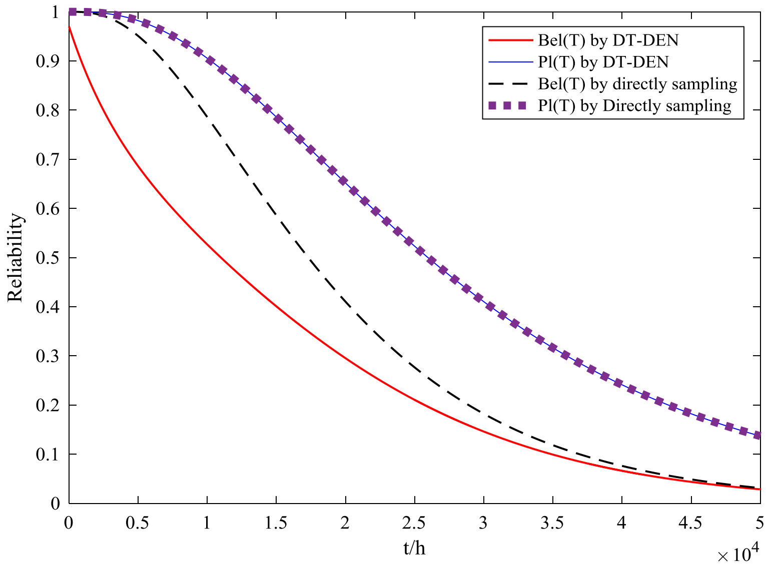


Fig. 11 System reliability (Exp distribution)

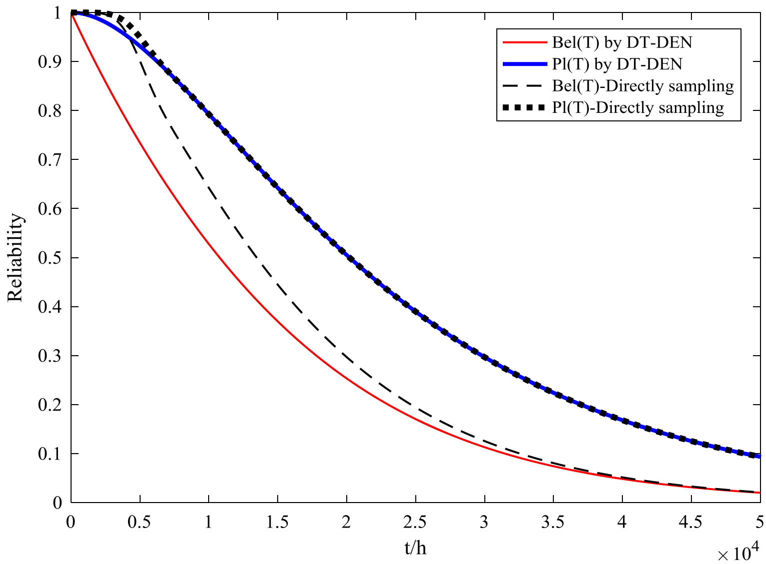


Fig. 12 System reliability (Exp and Wb distributions)

5 Conclusions

In this paper, through the discussion of evidence theory and conventional evidential network, a DT-DEN method is introduced to implement the reliability analysis of industrial complex systems that faces with the intricate problems, i.e. dependency and dynamic of failure behavior, components follow different life distributions, epistemic uncertainty induced by lack of data and imprecise knowledge, etc. The issues about epistemic uncertainty, dynamic failure behavior and mixed life distribution have received special attention in this paper. It has been proven that the evidence theory can express and quantify epistemic uncertainty in a simple and clear means, and the EN which converted by Bayesian network can demonstrate the intuitively graphical representation and probability reasoning. In terms of the advantages of EN, taking into account of the dynamic failure behavior and multiple life distribution problem, the DT-DEN is introduced and used to conduct reliability analysis of complex systems. The DT-DEN is transformed by DFT, and the probability inference of dynamic logic gates is expressed by a series of corresponding CMTs. The results indicate the validity and effectiveness of the presented method. In the future work, the optimization and application of the method in real industrial systems will catch our focus, and we also devote ourselves to find a better unified expression of hybrid uncertainties.

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