

Optimal throughput analysis of multiple channel access in cognitive radio networks

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Abstract In this paper we consider a time slotted cognitive radio (CR) network with N wireless channels and M secondary users (SUs). We focus on a multiple channel access policy where each SU stochastically decides whether to access idle channels or not based on the given access probability (AP) that is adapted to the channel state information (CSI), if possible. The AP plays an important role in the random access policy because it can control the number of SUs who can access idle channels in a simple manner and hence alleviate packet collisions among SUs. We assume that each SU can access at most L idle channels simultaneously at a time slot whenever possible. We consider three cases—(a) all SUs have full CSI, (b) all SUs have no CSI, and (c) all SUs have partial CSI. We analyze the throughput of an arbitrary SU for the three cases, and rigorously derive a closed-form expression of the optimal AP values that maximize the throughput of an arbitrary SU for the three cases. From the analysis, we show the impact of multiple channel access and the acquisition of CSI on throughput performance.

Keywords Cognitive radio network · Random access policy · Throughput performance · Optimal access probabilities · Multiple channel access

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1 Introduction

As the demands of wireless communication services increase, the utilization of radio spectrum resources becomes more important. However, most of the radio spectrums have been already allocated to licensed users, and recent studies have revealed that they are not occupied for most of the time (Spectrum policy task force 2002; Facilitating opportunities 2003; McHenry 2003). To improve the efficiency of the radio spectrum usage, the concept of Cognitive Radio (CR) has been introduced. In a CR network, there are a set of primary users (PUs) and a set of secondary users (SUs). PUs are licensed users having absolute priority to occupy their designated radio spectrums, called channels. SUs are unlicensed users who are allowed to use the temporarily unoccupied licensed radio spectrums (Haykin 2005; Mitola and Maguire 1999).

We consider a time slotted CR network with N channels and M SUs and focus on a multiple channel access policy. To utilize idle channels by SUs in CR, SUs first have to acquire the channel state information (CSI) by individually or cooperatively sensing the channels and sharing the CSI as much as possible. To investigate the impact of the multiple channel access policy on network performance, we assume that the channel sensing is perfect in this paper. After acquiring the CSI, each SU selects multiple idle channels and transmits its packets according to a pre-determined channel access policy. Here, multiple channel access can be implemented by the channel aggregation (CA) scheme with discontinuous orthogonal frequency division multiplexing (DOFDM) defined in IEEE 802.22 (Poston and Horne 2005; Park et al. 2016). With this CA scheme, an SU can aggregate multiple channels and transmit data through them. Thus, it is important to choose appropriate channel sensing and accessing policies to improve the efficiency of spectrum usage and network performance.

The design challenge in CR is to optimize the performance of SUs, and in this regard a number of channel sensing and accessing policies in CR have been recently proposed in the open literature, e.g., Ma et al. (2005), Sankaranarayanan et al. (2005), Jia et al. (2008), Cordeiro and Challapali (2007), Sabharwal et al. (2007), Hsu et al. (2007), Zhao et al. (2007), Su and Zhang (2008), Zhao et al. (2008), Li et al. (2011), Hwang and Roy (2012), Cho and Hwang (2012), Urkowitz (1967), Yucek and Arslan (2009), Gardner (1988). A good survey on the design issues and recent works in CR is provided in Cormio and Chowdhury (2009), Hossain et al. (2009), Akyildiz et al. (2006), Qing and Adler (2007). Interested readers may refer to them and the references therein.

Since the already proposed channel sensing/accessing policies are different from each other in modeling assumption, analytic techniques, and having different implementation issues, it is not an easy task to determine which one is superior to the others. However, one of good candidates for CR is the random access policy considered in this work because it is a generalization of the conventional p -persistent CSMA which is easy to implement in practice. In the conventional p -persistent CSMA, a node senses a channel for busy or idle. If the channel is idle, the node transmits a packet with probability p and does not transmit a packet with probability $1 - p$. On the other hand, if the channel is busy, the node senses it continuously until it becomes idle (Kwon et al. 2009). Note that the conventional p -persistent CSMA is already considered for CR (Hu and Mao 2009; Wang et al. 2010). In addition, as discussed in Hwang and Roy (2012) the random access policy is closely related with the IEEE 802.11 Distributed Coordination Function (DCF) which is also considered for CR. A practical example of the random channel access policy is a sensor network equipped with CR where a fixed number of sensors send out measurement data of the environment sporadically and

opportunistically over empty PU channels (Wang et al. 2010). Due to the above-mentioned reasons, we focus in this work on the *random access policy* for SUs.

In the random access policy considered in this work, each SU stochastically decides whether to access idle channels at each slot based on the common access probabilities (APs). When an SU decides to access idle channels, it is called an *active* SU. To analyze the performance of the random access policy, we consider the following two extreme cases and an intermediate case which combines the two extreme cases:

- one extreme case where all SUs have full CSI;
- the other extreme case where all SUs have no CSI;
- an intermediate case where all SUs have partial CSI.

In the first case where all SUs have full CSI, we assume that an active SU is allowed to access at most $L (\geq 1)$ idle channels. That is, when the number of idle channels is k , an active SU selects $\min(L, k)$ idle channels randomly and transmits its packets over the selected idle channels, one packet over each idle channel. We call this model *the L-interface CR network with full CSI*. If two or more SUs select the same idle channel, it leads to a packet collision. Obviously, it is desirable to adapt the AP value according to the information on the idle channels so as to alleviate packet collisions. More importantly, we need to find the optimal AP values which are functions of the number of idle channels, which maximize the throughput of an arbitrary SU. Note that the full CSI can be acquired by all SUs via a suitable channel sensing such as the negotiation based sensing (Su and Zhang 2008).

In the second case where all SUs have no CSI, SUs cannot adapt the AP to the number of idle channels and accordingly use a fixed AP, irrespective of the number of idle channels. We call this model *the L-interface CR network with no CSI*. In this case, there is still the possibility of packet collision when two or more SUs select the same idle channel. So it is also important to find the optimal unified AP that maximizes the throughput of an arbitrary SU.

In the last case where all SUs have partial CSI, SUs have full CSI on N_1 channels and have no CSI on the remaining $N_2 (= N - N_1)$ channels. In this case, when the number of idle channels among N_1 channels is k_1 , an active SU selects $\min(N_2 + k_1, L)$ idle channels randomly among the k_1 idle channels with full CSI and the N_2 channels with no CSI, and then, it transmits its packets over the selected channels. We call this model *the L-interface CR network with partial CSI*. This case is a compromise between the first and second cases and is more practical than them because the sensing may not be completed for all channels within a short time interval in practice.

In this work each SU is allowed to access multiple idle channels simultaneously at a slot. To the best of the authors' knowledge there have been limited works on this issue. From the multiple channel access viewpoint, the most relevant works to ours include Wang et al. (2010), Xu et al. (2008). In Wang et al. (2010) consider a CR network with the conventional p -persistent CSMA and derive the optimal AP value under the assumption that no CSI is available and each SU can access two channels simultaneously, which is a special case of our work. In Xu et al. (2008) consider unslotted CR networks and derive the optimal SU throughput and the optimal value of the number of SU's interfaces, but the analysis on the case with multiple SUs is based on an approximation.

The contributions of this work are summarized as follows.

- We derive a closed-form expression of the optimal AP values that maximize the throughputs of an arbitrary SU in the L -interface CR network with full/no/partial CSI. Since our result is of closed-form, we can easily investigate the relation between the optimal AP values and the network parameters such as the numbers of PU channels and SUs as well

as the maximum number of idle channels allowed for SUs. One interesting fact that we find from the closed-form expression is that the optimal AP value is a function of the total number of idle channels *counting multiplicity* that all SUs can access simultaneously. This implies that each SU behaves as if there were multiple *virtual* SUs accessing a single idle channel as explained in the main context.

- In this work, we rigorously analyze three cases for the CR network with multiple channel access for SUs - having full CSI, no CSI, and partial CSI. Even though the no CSI case is the most practical due to less implementation cost, the analyses on two extreme cases are important because we can compare them to investigate the impact of the acquisition of CSI on throughput performance. In fact, we show that, when the network is not crowded, the acquisition on CSI can improve the optimal throughput performance. On the other hand, when the network is crowded, the acquisition on CSI cannot improve the optimal throughput performance in the L -interface CR network. Moreover, the analysis on the no CSI case becomes possible in this work with the help of the analytic result on the full CSI case.
- We also show that the optimal throughput performance can be improved as we increase L only when $LM \leq N$. This provides an important insight on the design of a CR network that, when $LM > N$, the optimal throughput performance cannot be improved any more even though we increase L .

Note that this paper is an extended version of the work by [Park et al. \(2016\)](#), and the extension is summarized as follows.

- While rigorous proofs of theorems are omitted in [Park et al. \(2016\)](#), we provide them in this paper.
- We also provide a mathematical analysis on the optimal access probability and the optimal throughput for the partial CSI case.
- Our analysis on the partial CSI case is validated through numerical and simulation studies.

The rest of this paper is organized as follows. In Sect. 2, we explain the L -interface CR network and mathematically model the PU's activity and the random access policy. In Sect. 3, we analyze the throughput of an arbitrary SU in the L -interface CR network, and derive the optimal APs that maximize the throughput of an arbitrary SU. In the analysis we consider three cases where full/no/partial CSI is available to SUs. In Sect. 4, we validate our analysis through numerical and simulation studies. In Sect. 5, we give our conclusions.

2 System model

We consider a time slotted cognitive radio (CR) network with N wireless channels. There are N primary users (PUs) and $M (\geq 2)$ secondary users (SUs). Each primary user has its own designated channel. At time slot t , we let $N(t)$ denote by the number of idle channels that are unoccupied by PUs. SUs are allowed to access only idle channels to transmit their packets at each time slot.

2.1 Wireless channel occupancy model

For each channel, the channel occupancy process by a PU is modeled by a 2-state Markov chain with state space $\{0, 1\}$, where 0 (1) denotes a busy (an idle) channel. Let p be the transition probability that the state of a channel is changed from busy to idle and q be the

transition probability that the state of a channel is changed from idle to busy. Then the transition probability matrix Q of the channel occupancy process of a channel is given by

$$Q = \begin{pmatrix} 1 - p & p \\ q & 1 - q \end{pmatrix}.$$

Let $\pi = (\pi_0, \pi_1)$ be the stationary probability vector of the matrix Q , i.e., $\pi Q = \pi$, $\pi_0 + \pi_1 = 1$. Note that π_0 is the stationary probability of being a busy channel and π_1 is the stationary probability of being an idle channel. Then, we obtain

$$\pi_0 = \frac{q}{p + q}, \quad \pi_1 = \frac{p}{p + q}. \tag{1}$$

Note that $N(t)$ is the number of idle channels at time slot t . We assume that the channel occupancies by PUs are independent from channel to channel. Then $N(t)$ is a discrete time Markov Chain (DTMC) with state space $\{0, 1, \dots, N\}$. The steady state probability of $N(t)$ is then given by

$$P\{N(t) = k\} = \binom{N}{k} \pi_1^k \pi_0^{N-k}, \quad 0 \leq k \leq N, \tag{2}$$

where π_0 and π_1 are given by (1).

2.2 Random access policy

Each SU is equipped with the capability of accessing L channels simultaneously at each time slot. So each SU can transmit at most L packets, one for each idle channel, if possible, at each time slot. Clearly, $L \leq N$. We consider and analyze three cases in this paper.

2.2.1 Case 1: full CSI

SUs have perfect information on the states of all N channels at each time slot, i.e., which channels are idle and which channels are busy.

The random access policy in this case is performed as follows. Each SU decides stochastically whether to access channels with a common access probability (AP) based on the number of idle channels $N(t)$, independent of all other SUs. When $N(t) = k$, the AP is a_k . When an SU decides to access channels at time slot t , it is called an *active* SU. Each active SU then randomly selects $\min(L, N(t))$ channels among idle channels and transmits packets over the selected channels at time slot t . We call this model *the L-interface CR network with full CSI*. Note that, when two or more active SUs select the same idle channel at a time slot, the transmitted packets over the same channel at the time slot collide and hence are assumed lost. From this viewpoint, it is important to optimize the APs $\{a_1, a_2, \dots, a_N\}$ that maximize the throughput. Figure 1 demonstrates the channel access in this case.

2.2.2 Case 2: no CSI

SUs have no CSI in this case. Then, all SUs cannot adapt the AP according to the number of idle channels, and hence use a common unified AP, say a , to decide whether they are active or not at each time slot, independently of all other SUs. When an SU becomes active, it randomly selects L channels among the N channels. The active SU then checks at the beginning of the time slot which selected channels are idle. We assume that the channel sensing is perfect. Then, the active SU transmits only through the idle channels among the

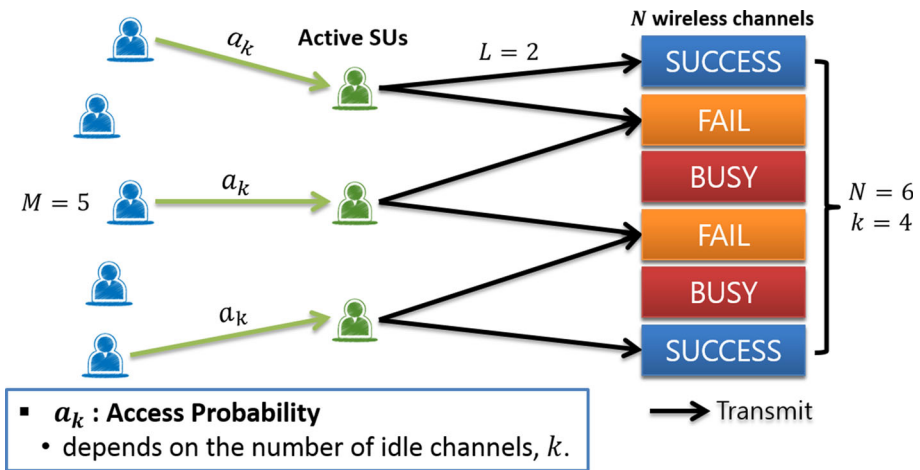


Fig. 1 L -interface CR network with full CSI

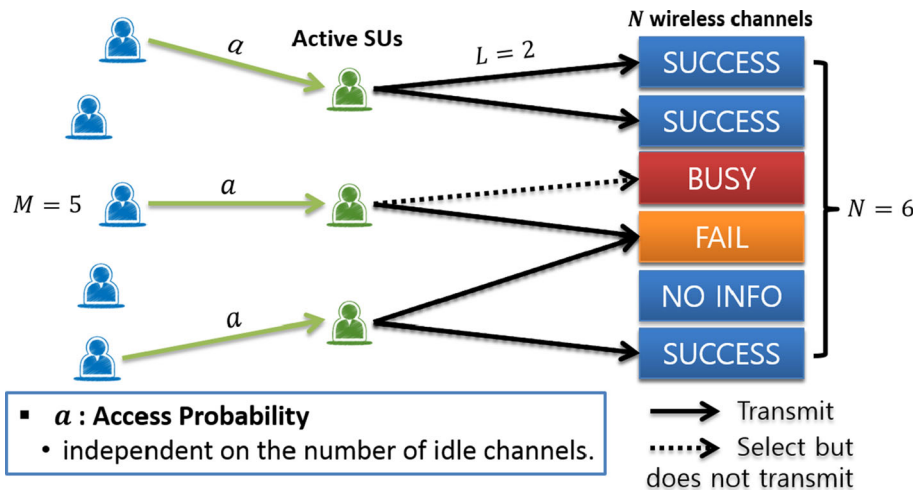


Fig. 2 L -interface CR network with no CSI

selected L channels. Obviously, there is the possibility of packet collision when two or more active SUs select the same idle channel. So, as in the previous case, the common unified AP a needs to be optimized. The benefit of this policy is that each active SU senses only L (randomly selected) channels at each time slot and does not need any information about the other channels, reducing the sensing overhead. This model is called *the L -interface CR network with no CSI*. Figure 2 demonstrates the channel access in this case.

2.2.3 Case 3: partial CSI

In the partial CSI case considered in this paper, wireless channels are divided into the following two groups, say \mathcal{N}_1 and \mathcal{N}_2 : (1) group \mathcal{N}_1 consists of N_1 channels of which CSI is perfectly known to all SUs; and (2) group \mathcal{N}_2 consists of the remaining $N_2 (= N - N_1)$

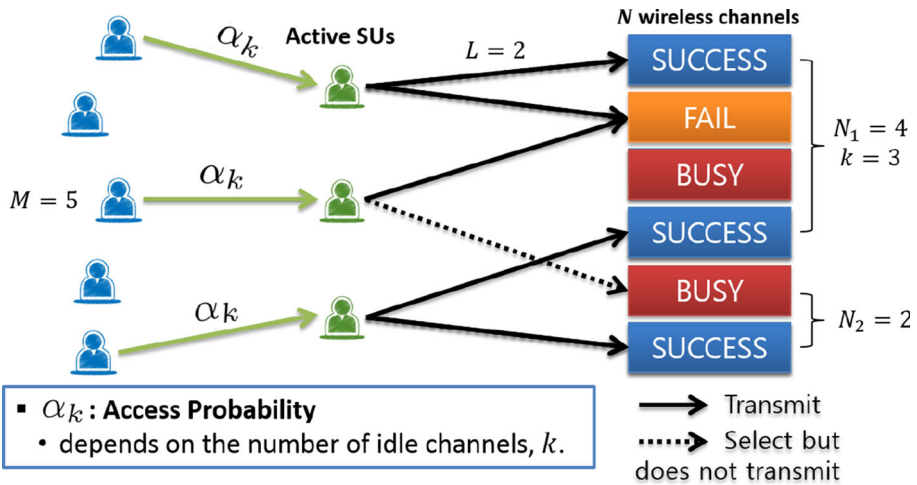


Fig. 3 L -interface CR network with Partial CSI

channels of which CSI is not known to all SUs. For convenience, define $\mathcal{N}_1^{\text{idle}}(t)$, $N_1(t)$, $\mathcal{N}_2^{\text{idle}}(t)$ and $N_2(t)$ as follows: $\mathcal{N}_1^{\text{idle}}(t) = \{\omega \in \mathcal{N}_1 \mid \omega \text{ is idle at time slot } t\}$ with $N_1(t) = |\mathcal{N}_1^{\text{idle}}(t)|$; $\mathcal{N}_2^{\text{idle}}(t) = \{\omega \in \mathcal{N}_2 \mid \omega \text{ is idle at time slot } t\}$ with $N_2(t) = |\mathcal{N}_2^{\text{idle}}(t)|$ where $|A|$ denotes the number of elements in set A . Each SU decides stochastically whether to access channels with an AP based on $N_1(t)$. Thus, when $N_1(t) = k$, the AP depends on the value of k , denoted by α_k .

Now, after an SU becomes active with AP α_k when $N_1(t) = k$, the active SU selects $\min(N_2 + k, L)$ channels among $\mathcal{N}_1^{\text{idle}}(t) \cup \mathcal{N}_2$ randomly. At the beginning of the time slot, the active SU checks the selected channels in \mathcal{N}_2 , since there is no CSI for the channels in \mathcal{N}_2 . After sensing, the active SU transmits only through the idle channels among the selected channels. The benefit of this policy is that each active SU senses up to L channels at each time slot using the already known CSI for the channels in \mathcal{N}_1 . This is why this model is called *the L -interface CR network with partial CSI*. Figure 3 demonstrates the channel access in this case.

3 Performance analysis

In this section, we analyze the throughput performance of an arbitrary SU in the L -interface CR network for the three cases; with full CSI, with no CSI and with partial CSI. To this end, we tag an arbitrary SU and call it the tagged SU. For analysis, we assume that all SUs are saturated, that is, all SUs always have packets to transmit. We let $c_L(t)$ denote by the number of packets transmitted *successfully* by the tagged SU at time slot t . Hence, $c_L(t) = j$ if the tagged SU successfully transmits j packets at time slot t . Obviously, $c_L(t) \leq L$.

Moreover, since each SU independently decides to access idle channels with the common APs, the probability $P_i^m(x)$ that there are i active SUs among the total of m SUs when the AP is x , is given by

$$P_i^m(x) = \binom{m}{i} x^i (1 - x)^{m-i}, \tag{3}$$

which will be used later.

3.1 The full CSI case

Consider the L -interface CR network with full CSI. We first define the throughput performance of the tagged SU, denoted by T_L^{full} . Assuming that the network is in steady state, T_L^{full} is defined by

$$T_L^{\text{full}} := E[c_L(t)] = \sum_{k=1}^N P\{N(t) = k\} E[c_L(t) | N(t) = k]$$

where $P\{N(t) = k\}$ is given by (2). We define the conditional throughput of the tagged SU in the L -interface CR network with full CSI, given that there are k idle channels, by $T_{L,k}^{\text{full}} = E[c_L(t) | N(t) = k]$, so that

$$T_L^{\text{full}} = \sum_{k=1}^N P\{N(t) = k\} T_{L,k}^{\text{full}}. \tag{4}$$

To compute T_L^{full} , observe that the throughput of the tagged SU depends on the number of idle channels that the tagged SU selects but all active untagged SUs do not select. So, from the tagged SU’s perspective the *effective* channels are defined by the idle channels that the tagged SU selects but all active untagged SUs do not select. Note that the tagged SU transmits one packet successfully through each effective channel. We now need the following two lemmas.

Lemma 1 *When $N(t) = k (\geq 1)$, assume that there are i ($0 \leq i \leq M - 1$) active untagged SUs and that the tagged SU becomes active and hence transmits its packets at time slot t . Then the probability $G_{L,k}^i(t; j)$ that there are j effective channels at time slot t , is given by, for $j \geq 1$,*

$$G_{L,k}^i(t; j) = \begin{cases} 1, & \text{if } i = 0 \text{ and } j = L_k; \\ \binom{L}{j} \sum_{u=0}^{L_k^- - j} (-1)^u \binom{L-j}{u} \left(\frac{\binom{k-j-u}{L}}{\binom{k}{L}} \right)^i, & \\ 0, & \text{otherwise} \end{cases}$$

where $L_k = \min(L, k)$ and $L_k^- = \min(L, k - L)$.

Proof Note that the tagged SU always chooses $L_k = \min(L, k)$ idle channels when $N(t) = k$. In the proof, we consider the following cases.

Case 1 There is no active untagged SU, i.e., $i = 0$:

In this case, it is obvious that the tagged SU successfully transmits its packets through all L_k idle channels that it selects and packet collisions do not occur. This implies that $G_{L,k}^i(t; j) = 1$ if $j = L_k$ and $G_{L,k}^i(t; j) = 0$ if $j \neq L_k$.

Case 2 There is at least one active untagged SU, i.e., $i > 0$, and $k \leq L$:

In this case, since $k \leq L$, all k idle channels are selected by the tagged SU. However, any active untagged SU also selects all k idle channels, which implies that there are no effective channels for the tagged SU. Hence, $G_{L,k}^i(t; j) = 0$.

Case 3 There is at least one active untagged SU, i.e., $i > 0$, and $k > L$:

In this case, for the tagged SU to have j effective channels, each active untagged SU must choose L idle channels from the $k - j$ idle channels that do not become the effective channels. If $L > k - j$, then any active untagged SU has to select at least one of the j effective channels, which is not possible. This implies that $G_{L,k}^i(t; j) = 0$ if $L > k - j$ in this case.

It remains to consider the case where $L \leq k - j$, i.e., $j \leq k - L$. Since $j \leq L$, we actually consider the case where $j \leq L_k^-$. Note that the tagged SU can choose L idle channels in this case, so we focus on the L idle channels selected by the tagged SU. To compute the probability that there are only j effective channels among the selected L channels, suppose we select j channels among the selected L channels and consider them as the effective channels. The number of selecting such j effective channels among the selected L channels is $\binom{L}{j}$. Now, we order the selected L channels in such a way that the j effective channels are numbered as channel 1, 2, ..., j and the remaining $L - j$ channels are numbered as channel $j + 1, j + 2, \dots, L$.

Define $E_l, 1 \leq l \leq L$ by the event that channel l is *not* selected by any of active untagged SUs. We need to compute

$$P\{E \cap E_{j+1}^c \cap \dots \cap E_L^c\}$$

where $E := E_1 \cap E_2 \cap \dots \cap E_j$ and E_l^c is the complement of E_l . Recall that, for any sequence of events $\{C_1, C_2, \dots, C_m\}$, the inclusion-exclusion formula for the sequence is given by

$$\begin{aligned} &P\{\cup_{l=1}^m C_l\} \\ &= \sum_{l=1}^m P\{C_l\} - \sum_{1 \leq l_1 < l_2 \leq m} P\{C_{l_1} \cap C_{l_2}\} + \sum_{1 \leq l_1 < l_2 < l_3 \leq m} P\{C_{l_1} \cap C_{l_2} \cap C_{l_3}\} \\ &\quad - \dots + (-1)^{m+1} P\{C_1 \cap C_2 \cap \dots \cap C_m\}. \end{aligned} \tag{5}$$

Using (5), we obtain

$$\begin{aligned} &P\{E \cap E_{j+1}^c \cap \dots \cap E_L^c\} \\ &= P\{E \cap (E_{j+1} \cup \dots \cup E_L)^c\} \\ &= P\{E\} - P\{E \cap (E_{j+1} \cup \dots \cup E_L)\} \\ &= P\{E\} - P\{(E \cap E_{j+1}) \cup \dots \cup (E \cap E_L)\} \\ &= P\{E\} - \sum_{l=1}^{L-j} P\{E \cap E_{j+l}\} + \sum_{1 \leq l_1 < l_2 \leq L-j} P\{E \cap E_{j+l_1} \cap E_{j+l_2}\} \\ &\quad - \sum_{1 \leq l_1 < l_2 < l_3 \leq L-j} P\{E \cap E_{j+l_1} \cap E_{j+l_2} \cap E_{j+l_3}\} \\ &\quad + \dots + (-1)^{L-j} P\{E \cap E_{j+1} \cap E_{j+2} \cap \dots \cap E_L\}. \end{aligned} \tag{6}$$

Note that, if $u > L_k^- - j$,

$$P\{E \cap E_{j+l_1} \cap \dots \cap E_{j+l_u}\} = 0 \tag{7}$$

because each active untagged SU selects L idle channels in this case, and hence the maximum of $j + u$ should be L_k^- . Furthermore, we see that, for $0 \leq u \leq L_k^- - j$,

$$P\{E \cap E_{j+l_1} \cap \dots \cap E_{j+l_u}\} = \left(\frac{\binom{k-j-u}{L}}{\binom{k}{L}}\right)^u, \tag{8}$$

and

$$\begin{aligned} & \{ \{l_1, \dots, l_u\} : 1 \leq l_1 < l_2 < \dots < l_u \leq L - j \} \\ & = \binom{L-j}{u} \end{aligned} \tag{9}$$

where $|C|$ denotes the number of elements in set C .

Combining (6), (7), (8), and (9), we finally have

$$P\{E \cap E_{j+1}^c \cap \dots \cap E_L^c\} = \sum_{u=0}^{L_k^- - j} (-1)^u \binom{L-j}{u} \left(\frac{\binom{k-j-u}{L}}{\binom{k}{L}} \right)^i. \tag{10}$$

Now, recall that there are $\binom{L}{j}$ cases of selecting j channels (that become the effective channels) from the selected L channels by the tagged SU. Multiplying it by the probability obtained above, we finally get the probability $G_{L,k}^i(t; j)$ in this case as follows:

$$G_{L,k}^i(t; j) = \binom{L}{j} \sum_{u=0}^{L_k^- - j} (-1)^u \binom{L-j}{u} \left(\frac{\binom{k-j-u}{L}}{\binom{k}{L}} \right)^i.$$

Hence, we prove our lemma. □

Lemma 2 *When $N(t) = k$, assume that there are i ($0 \leq i \leq M - 1$) active untagged SUs and that the tagged SU becomes active and hence transmits its packets at time slot t . Then the conditional throughput $T_{L,k}^{\text{full}}$ of the tagged SU, given that $N(t) = k$, is given by*

$$T_{L,k}^{\text{full}} = L_k a_k \left(1 - \frac{L_k}{k} a_k \right)^{M-1}.$$

Proof At time slot t , let $A_a(t)$ be the number of active untagged SUs ($0 \leq A_a(t) \leq M - 1$) and $A_e(t)$ be the number of effective channels for the tagged SU ($0 \leq A_e(t) \leq L_k$). We define $D(t)$ by 1 if the tagged SU is active and by 0 otherwise at time slot t . Then the conditional throughput $T_{L,k}^{\text{full}}$ of the tagged SU when $N(t) = k$, is given by

$$\begin{aligned} T_{L,k}^{\text{full}} &= E[c_L(t) | N(t) = k] \\ &= P\{D(t) = 1 | N(t) = k\} \cdot E[c_L(t) | N(t) = k, D(t) = 1] \\ &= a_k \sum_{j=1}^{L_k} j P\{A_e(t) = j | N(t) = k, D(t) = 1\} \\ &= a_k \sum_{j=1}^{L_k} j \sum_{i=0}^{M-1} P\{A_a(t) = i | N(t) = k, D(t) = 1\} \\ &\quad \times P\{A_e(t) = j | N(t) = k, D(t) = 1, A_a(t) = i\} \\ &= a_k \sum_{i=0}^{M-1} \sum_{j=1}^{L_k} j P_i^{M-1}(a_k) G_{L,k}^i(t; j) \\ &= a_k \sum_{j=1}^{L_k} j P_i^{M-1}(a_k) G_{L,k}^i(t; j) \Big|_{i=0} + a_k \sum_{i=1}^{M-1} \sum_{j=1}^{L_k} j P_i^{M-1}(a_k) G_{L,k}^i(t; j) \\ &= a_k L_k (1 - a_k)^{M-1} + a_k \sum_{i=1}^{M-1} \sum_{j=1}^{L_k} j P_i^{M-1}(a_k) G_{L,k}^i(t; j) \end{aligned} \tag{11}$$

where P_i^{M-1} is given by (3) and $G_{L,k}^i(t; j)$ is given by Lemma 1.

To further simplify $T_{L,k}^{\text{full}}$ in (11) we consider two cases — $k \leq L$ and $L < k$. First, when $k \leq L$, by Lemma 1, we know the second term in (11) vanishes in this case. So the conditional throughput $T_{L,k}^{\text{full}}$ when $k \leq L$, is given by

$$T_{L,k}^{\text{full}} = k a_k (1 - a_k)^{M-1}. \tag{12}$$

Second, when $L < k$, again by Lemma 1, the second term in (11) is reduced as follow using binomial theorem:

$$\begin{aligned} & a_k \sum_{i=1}^{M-1} \sum_{j=1}^{L_k} j P_i^{M-1}(a_k) G_{L,k}^i(t; j) \\ &= a_k \sum_{i=1}^{M-1} \sum_{j=1}^{L_k} j \binom{M-1}{i} a_k^i (1 - a_k)^{M-1-i} \binom{L}{j} \sum_{u=0}^{L_k^- - j} (-1)^u \binom{L-j}{u} \frac{\binom{k-j-u}{L}^i}{\binom{k}{L}^i} \\ &= a_k \sum_{j=1}^{L_k} \sum_{u=0}^{L_k^- - j} (-1)^u j \binom{L}{j} \binom{L-j}{u} \sum_{i=1}^{M-1} \binom{M-1}{i} \left(a_k \frac{\binom{k-j-u}{L}}{\binom{k}{L}} \right)^i (1 - a_k)^{M-1-i} \\ &= a_k \sum_{j=1}^{L_k} \sum_{u=0}^{L_k^- - j} (-1)^u j \binom{L}{j} \binom{L-j}{u} \left[\left(1 - a_k + a_k \frac{\binom{k-j-u}{L}}{\binom{k}{L}} \right)^{M-1} - (1 - a_k)^{M-1} \right]. \end{aligned} \tag{13}$$

Let now F be a function of $j + u$, defined by

$$F(j + u) = \left(1 - a_k + a_k \frac{\binom{k-j-u}{L}}{\binom{k}{L}} \right)^{M-1} - (1 - a_k)^{M-1}. \tag{14}$$

By replacing F in (13) and letting $p = j + u$, the second term in (11) is given by

$$\begin{aligned} & a_k \sum_{j=1}^{L_k} \sum_{u=0}^{L_k^- - j} (-1)^u j \binom{L}{j} \binom{L-j}{u} F(j + u) \\ &= a_k \sum_{p=1}^{L_k^-} \sum_{j=1}^p \frac{(-1)^{p-j} L!}{(p-j)!(j-1)!(L-p)!} F(p) \\ &= a_k \sum_{p=1}^{L_k^-} \frac{F(p)L!}{(L-p)!(p-1)!} \sum_{j=1}^p \frac{(-1)^{p-j} (p-1)!}{(j-1)!(p-j)!} \\ &= a_k L F(1) + a_k \sum_{p=2}^{L_k^-} \frac{F(p)L!}{(L-p)!(p-1)!} \sum_{j=1}^p \frac{(-1)^{p-j} (p-1)!}{(j-1)!(p-j)!} \end{aligned}$$

$$\begin{aligned}
 &= a_k L F(1) + a_k \sum_{p=2}^{L_k^-} F(p) \frac{L!}{(L-p)!(p-1)!} \cdot 0 \\
 &= L a_k \left[\left(1 - \frac{L}{k} a_k\right)^{M-1} - (1 - a_k)^{M-1} \right]. \tag{15}
 \end{aligned}$$

Hence, from (11) and (15) $T_{L,k}^{\text{full}}$ is given by

$$\begin{aligned}
 T_{L,k}^{\text{full}} &= L a_k (1 - a_k)^{M-1} + L a_k \left[\left(1 - \frac{L}{k} a_k\right)^{M-1} - (1 - a_k)^{M-1} \right] \\
 &= L a_k \left(1 - \frac{L}{k} a_k\right)^{M-1}, \tag{16}
 \end{aligned}$$

which is the conditional throughput for $L < k$.

Combining the two cases, we finally get the conditional throughput performance, given that $N(t) = k$, as follows:

$$T_{L,k}^{\text{full}} = L_k a_k \left(1 - \frac{L_k}{k} a_k\right)^{M-1}.$$

□

Using Lemmas 1, 2, and (4), we obtain the throughput T_L^{full} in the following theorem.

Theorem 1 *For the L -interface CR network with full CSI, the throughput T_L^{full} of an arbitrary SU is given by*

$$T_L^{\text{full}} = \sum_{k=1}^N P\{N(t) = k\} a_k L_k \left(1 - \frac{L_k}{k} a_k\right)^{M-1}$$

where $L_k = \min(L, k)$.

From Theorem 1, we can obtain the optimal AP values $\{a_1^*, a_2^*, \dots, a_N^*\}$ that maximize the throughput T_L^{full} in the L -interface CR network with full CSI as follows.

Theorem 2 *For the L -interface CR network with full CSI, the optimal AP values $\{a_1^*, a_2^*, \dots, a_N^*\}$ that maximize the throughput of an arbitrary SU, are given by*

$$a_k^* = \min\left(\frac{k}{L_k M}, 1\right), \quad 1 \leq k \leq N$$

where $L_k = \min(L, k)$.

Proof Note that all terms on the right hand side of (4) are nonnegative. So, if we maximize the conditional throughput $T_{L,k}^{\text{full}}$ for each k , then the throughput T_L^{full} is maximized. From Lemma 2, the conditional throughput $T_{L,k}^{\text{full}}$ is given by

$$T_{L,k}^{\text{full}} = L_k a_k \left(1 - \frac{L_k}{k} a_k\right)^{M-1}.$$

To find the optimal AP value a_k , we differentiate the above equation and solve the following equation

$$\frac{dT_{L,k}^{\text{full}}}{da_k} = L_k \left(1 - \frac{L_k}{k} a_k\right)^{M-2} \left(1 - \frac{L_k M}{k} a_k\right) = 0.$$

Since $L_k = \min(L, k) \leq k$ and $0 \leq a_k \leq 1$, it is easy to show that $T_{L,k}^{\text{full}}$ is maximized when,

$$a_k^* = \min\left(\frac{k}{L_k M}, 1\right).$$

□

An interesting fact that we find from Theorem 2 is that the optimal AP value a_k^* is a function of the total number of idle channels *counting multiplicity* that all SUs can access simultaneously in the L -interface CR network with full CSI. When $L = 1$, we see from Theorem 2 that the optimal AP value is a function of M , the number of SUs. This implies that, if we allow SUs to access L_k channels simultaneously, the L -interface CR network in the optimal case can be considered as a CR network where there are $L_k M$ virtual SUs and virtual SUs independently access one channel.

We are now ready to compute the optimal throughput, denoted by $(T_L^{\text{full}})^*$, of an arbitrary SU when the optimal APs $\{a_1^*, a_2^*, \dots, a_N^*\}$ given in Theorem 2 are used.

Theorem 3 *The optimal throughput $(T_L^{\text{full}})^*$ of an arbitrary SU in the L -interface CR network with full CSI satisfies*

$$(T_L^{\text{full}})^* = \sum_{1 \leq k \leq LM} P\{N(t) = k\} \frac{k}{M} \left(1 - \frac{1}{M}\right)^{M-1} + \sum_{LM < k \leq N} P\{N(t) = k\} L \left(1 - \frac{L}{k}\right)^{M-1}.$$

Moreover, $(T_L^{\text{full}})^*$ is an increasing function in L .

Proof We first need to compute the maximum conditional throughput performance $(T_{L,k}^{\text{full}})^*$. Bearing in mind that $a_k^* = \min(\frac{k}{L_k M}, 1)$, we consider two cases: $k > LM$ and $0 \leq k \leq LM$.

When $k > LM$, we see that $k > L$. Consequently, $L_k = \min(L, k) = L$ and $k > LM = L_k M$. Since $\frac{k}{L_k M} > 1$ in this case, the optimal AP for $(T_{L,k}^{\text{full}})^*$ is given by $a_k^* = 1$. Hence, $(T_{L,k}^{\text{full}})^*$ is given by

$$(T_{L,k}^{\text{full}})^* = L \left(1 - \frac{L}{k}\right)^{M-1}. \tag{17}$$

When $0 \leq k \leq LM$, we further have two subcases. If $k > L$, then $L_k = \min(L, k) = L$ and $k \leq LM = L_k M$. If $0 \leq k \leq L$, then $L_k = \min(L, k) = k$ and $k \leq kM = L_k M$. So we always have that $0 \leq k \leq L_k M$ in this case and $a_k^* = \frac{k}{L_k M}$. Hence, $(T_{L,k}^{\text{full}})^*$ is given by

$$(T_{L,k}^{\text{full}})^* = \frac{k}{M} \left(1 - \frac{1}{M}\right)^{M-1}. \tag{18}$$

Combining two Eqs. (17) and (18), we finally obtain the optimal throughput $(T_L^{\text{full}})^*$, given by

$$(T_L^{\text{full}})^* = \sum_{1 \leq k \leq LM} P\{N(t) = k\} \frac{k}{M} \left(1 - \frac{1}{M}\right)^{M-1} + \sum_{LM < k \leq N} P\{N(t) = k\} L \left(1 - \frac{L}{k}\right)^{M-1}.$$

Now, to prove that $(T_L^{\text{full}})^*$ is an increasing function in L , we prove that

$$(T_{L+1}^{\text{full}})^* - (T_L^{\text{full}})^* > 0.$$

Observe that

$$(T_{L+1}^{\text{full}})^* - (T_L^{\text{full}})^*$$

$$\begin{aligned}
 &= \sum_{0 \leq k \leq (L+1)M} P\{N(t) = k\} \frac{k}{M} \left(1 - \frac{1}{M}\right)^{M-1} \\
 &+ \sum_{(L+1)M < k \leq N} P\{N(t) = k\} (L+1) \left(1 - \frac{L+1}{k}\right)^{M-1} \\
 &- \sum_{0 \leq k \leq LM} P\{N(t) = k\} \frac{k}{M} \left(1 - \frac{1}{M}\right)^{M-1} - \sum_{LM < k \leq N} P\{N(t) = k\} L \left(1 - \frac{L}{k}\right)^{M-1} \\
 &= \sum_{LM < k \leq (L+1)M} P\{N(t) = k\} \times \left[\frac{k}{M} \left(1 - \frac{1}{M}\right)^{M-1} - L \left(1 - \frac{L}{k}\right)^{M-1} \right] \\
 &+ \sum_{(L+1)M < k \leq N} P\{N(t) = k\} \times \left[(L+1) \left(1 - \frac{L+1}{k}\right)^{M-1} - L \left(1 - \frac{L}{k}\right)^{M-1} \right].
 \end{aligned}$$

Thus, it suffices to show the following two claims.

Claim 1 When $LM < k \leq (L+1)M$,

$$\frac{k}{M} \left(1 - \frac{1}{M}\right)^{M-1} > L \left(1 - \frac{L}{k}\right)^{M-1},$$

that is,

$$\frac{k}{LM} \left[\frac{k(M-1)}{M(k-L)} \right]^{M-1} > 1.$$

Claim 2 When $(L+1)M < k \leq N$,

$$(L+1) \left(1 - \frac{L+1}{k}\right)^{M-1} > L \left(1 - \frac{L}{k}\right)^{M-1},$$

that is,

$$\frac{L+1}{L} \left(\frac{k-L-1}{k-L} \right)^{M-1} = \frac{L+1}{L} \left(1 - \frac{1}{k-L} \right)^{M-1} > 1.$$

For the proof of Claim 1, let $f(k)$ be defined by

$$f(k) = \frac{k}{LM} \left[\frac{k(M-1)}{M(k-L)} \right]^{M-1}$$

for $k \geq LM$. We can easily show that $f(LM) = 1$. The derivative of f is given by

$$f'(k) = \frac{(k-LM)}{Lk(M-1)} \left[\frac{k(M-1)}{M(k-L)} \right]^M.$$

It is clear that $f'(k) > 0$ for $k > LM$, that is, f is strictly increasing for $k > LM$. Thus, we conclude that $f(k) > 1$ for $k > LM$, as desired.

For the proof of Claim 2, let $g(k)$ be defined by

$$g(k) = \frac{L+1}{L} \left(1 - \frac{1}{k-L} \right)^{M-1}$$

for $k \geq (L + 1)M$. Obviously, g is strictly increasing for $k \geq (L + 1)M$. It remains to show $g((L + 1)M) > 1$ to complete the proof.

$$\begin{aligned}
 g((L + 1)M) &= \frac{L + 1}{L} \left(1 - \frac{1}{LM + M - L} \right)^{M-1} \\
 &\geq \frac{L + 1}{L} \left(1 - \frac{M - 1}{LM + M - L} \right) \\
 &= \frac{L + 1}{L} \left(\frac{LM - L + 1}{LM + M - L} \right) \\
 &> 1.
 \end{aligned}$$

□

From Theorem 3, when $LM \geq N$, the optimal throughput $(T_L^{\text{full}})^*$ is invariant in L because the second term in the right hand side of the equation in the theorem disappears. This implies that the optimal throughput $(T_L^{\text{full}})^*$ can be improved only when $LM < N$. Moreover, when $LM \geq N$, the optimal throughput $(T_L^{\text{full}})^*$ in the theorem becomes

$$\begin{aligned}
 (T_L^{\text{full}})^* &= \sum_{1 \leq k \leq N} P\{N(t) = k\} \frac{k}{M} \left(1 - \frac{1}{M} \right)^{M-1} \\
 &= \sum_{1 \leq k \leq N} \binom{N}{k} \pi_1^k \pi_0^{N-k} \frac{k}{M} \left(1 - \frac{1}{M} \right)^{M-1} \\
 &= \pi_1 \frac{N}{M} \left(1 - \frac{1}{M} \right)^{M-1}.
 \end{aligned} \tag{19}$$

We will use (19) later for comparison between the optimal throughputs of the three cases.

3.2 The no CSI case

In this subsection we consider the L -interface CR network with no CSI. We first define the throughput of the tagged SU, denoted by T_L^{no} , in this case. Assuming that the network is in steady state, T_L^{no} is defined by

$$\begin{aligned}
 T_L^{\text{no}} &:= E[c_L(t)] \\
 &= \sum_{k=1}^N P\{N(t) = k\} E[c_L(t) | N(t) = k]
 \end{aligned}$$

where $P\{N(t) = k\}$ is given by (2). We also define the conditional throughput of the tagged SU in the L -interface CR network with no CSI, given that there are k idle channels, by

$$T_{L,k}^{\text{no}} = E[c_L(t) | N(t) = k],$$

so that

$$T_L^{\text{no}} = \sum_{k=1}^N P\{N(t) = k\} T_{L,k}^{\text{no}}. \tag{20}$$

We then obtain the throughput T_L^{no} in the following theorem.

Theorem 4 For the L -interface CR network with no CSI, the throughput T_L^{no} of an arbitrary tagged SU is given by

$$T_L^{\text{no}} = \pi_1 L a \left(1 - \frac{L}{N} a \right)^{M-1}.$$

Proof For the L -interface CR network with no CSI, first observe that each active SU always randomly selects L channels and transmits packets through the idle channels among the selected L channels. This implies that, regardless of the number of idle channels ($N(t)$), each active SU acts almost the same as in the L -interface CR network with full CSI as if all channels were idle.

Let $A_e^N(t)$ be the number of effective channels in the L -interface CR network with full CSI when all channels are idle. For each effective channel, the tagged SU transmits one packet successfully through the effective channel if it is actually idle. Hence, the throughput $T_{L,k}^{\text{no}}$ is computed as follows.

$$\begin{aligned} T_{L,k}^{\text{no}} &= E[c_L(t) | N(t) = k] \\ &= \sum_{l=1}^{L_N^-} E[c_L(t) | N(t) = k, A_e^N(t) = l] P\{A_e^N(t) = l | N(t) = k\} \\ &= \sum_{l=1}^{L_N^-} E[c_L(t) | N(t) = k, A_e^N(t) = l] P\{A_e^N(t) = l\}. \end{aligned}$$

where $L_N^- = \min(L, N - L)$. For each effective channel considered above, the probability that it is actually idle is $\frac{k}{N}$. This implies that

$$E[c_L(t) | N(t) = k, A_e^N(t) = l] = \frac{lk}{N},$$

from which we get

$$\begin{aligned} T_{L,k}^{\text{no}} &= \sum_{l=1}^{L_N^-} \frac{lk}{N} P\{A_e^N(t) = l\} \\ &= \frac{k}{N} \sum_{l=1}^{L_N^-} l P\{A_e^N(t) = l\} \\ &= \frac{k}{N} L a \left(1 - \frac{L}{N} a \right)^{M-1}. \end{aligned} \tag{21}$$

From (20) and (21), we finally derive the throughput of the tagged SU in the L -interface CR network with no CSI as follows.

$$\begin{aligned} T_L^{\text{no}} &= a \sum_{k=1}^N P\{N(t) = k\} \frac{kL}{N} \left(1 - \frac{L}{N} a \right)^{M-1} \\ &= \left[\sum_{k=1}^N k P\{N(t) = k\} \right] \frac{L}{N} a \left(1 - \frac{L}{N} a \right)^{M-1} \\ &= N \pi_1 \frac{L}{N} a \left(1 - \frac{L}{N} a \right)^{M-1} \end{aligned}$$

$$= \pi_1 L a \left(1 - \frac{L}{N} a \right)^{M-1}.$$

□

From Theorem 4, by differentiating T_L^{no} with respect to a and solving $\frac{d}{da} T_L^{no} = 0$, we obtain the optimal AP a^* in the L -interface CR network with no CSI as follows.

Theorem 5 *For the L -interface CR network with no CSI, the optimal AP value a^* that maximizes the throughput of an arbitrary SU, is given by*

$$a^* = \min \left(\frac{N}{LM}, 1 \right). \tag{22}$$

From Theorem 5 we finally obtain the optimal throughput $(T_L^{no})^*$ of an arbitrary SU in the no CSI case as follows.

Theorem 6 *The optimal throughput $(T_L^{no})^*$ of an arbitrary SU in the L -interface CR network with no CSI satisfies that*

$$(T_L^{no})^* = \begin{cases} \pi_1 \frac{N}{M} \left(1 - \frac{1}{M} \right)^{M-1}, & \text{if } L \geq \frac{N}{M}; \\ \pi_1 L \left(1 - \frac{L}{N} \right)^{M-1}, & \text{if } L < \frac{N}{M}. \end{cases}$$

Moreover, $(T_L^{no})^*$ is an increasing function in L .

Proof Using the optimal AP a^* in (22), we can easily obtain the expressions for $(T_L^{no})^*$ in the theorem. The increasing property of $(T_L^{no})^*$ follows from the fact that $h(x) = x(1 - x/N)^{M-1}$ is an increasing function in x for $0 \leq x \leq N/M$. □

From Theorem 6 we see that, when $LM < N$, the optimal throughput $(T_L^{no})^*$ is a function of L and proportional to L . On the other hand, when $LM \geq N$, the optimal throughput $(T_L^{no})^*$ is not a function of L and invariant in L . This implies that the optimal throughput $(T_L^{no})^*$ can be improved by the increase in L only when $LM \leq N$, which is the same as in the full CSI case. Moreover, by comparing the optimal throughputs $(T_L^{full})^*$ and $(T_L^{no})^*$ in the two extreme cases, we find an interesting observation that, when $LM \geq N$, the optimal throughputs of an arbitrary SU in the two extreme cases are the same. On the other hand, when $LM < N$, we show that $(T_L^{no})^*$ is always less than or equal to $(T_L^{full})^*$. This implies that the acquisition of CSI is useful only when $LM \leq N$, i.e., the network is less crowded.

3.3 The partial CSI case

In this subsection, we consider the L -interface CR network with partial CSI. The throughput of the tagged SU in the partial CSI case is denoted by T_L^{part} .

Theorem 7 *For the L -interface CR network with partial CSI, the throughput T_L^{part} of an arbitrary SU is given by*

$$T_L^{part} = \sum_{k=0}^{N_1} \left[P\{N_1(t) = k\} \alpha_k L_{N_2+k} \left(1 - \frac{L_{N_2+k}}{N_2+k} \alpha_k \right)^{M-1} \left(\frac{\pi_1 N_2 + k}{N_2 + k} \right) \right] \tag{23}$$

where $L_{N_2+k} = \min(L, N_2 + k)$.

Proof First recall that $\mathcal{N}_1^{\text{idle}}(t) = \{\omega \in \mathcal{N}_1 \mid \omega \text{ is idle at time slot } t\}$ with $N_1(t) = |\mathcal{N}_1^{\text{idle}}(t)|$ and $\mathcal{N}_2^{\text{idle}}(t) = \{\omega \in \mathcal{N}_2 \mid \omega \text{ is idle at time slot } t\}$ with $N_2(t) = |\mathcal{N}_2^{\text{idle}}(t)|$. For convenience, we denote $(N_1(t), N_2(t))$ by $N(t)$. For the L -interface CR network with partial CSI, each active SU selects up to L channels among $\mathcal{N}_1^{\text{idle}}(t) \cup \mathcal{N}_2$ and transmits packets through the actually idle channels among the selected L channels. To this end, we define two conditional throughputs of the tagged SU as follows:

$$T_{L,k}^{\text{part}} = E [c_L(t) \mid N_1(t) = k],$$

$$T_{L,k,k_2}^{\text{part}} = E [c_L(t) \mid N(t) = (k, k_2)].$$

Then the throughput performance of the tagged SU is given by

$$T_L^{\text{part}} = \sum_{k=0}^{N_1} P\{N_1(t) = k\} T_{L,k}^{\text{part}}$$

$$= \sum_{k=0}^{N_1} P\{N_1(t) = k\} \sum_{k_2=0}^{N_2} P\{N_2(t) = k_2\} T_{L,k,k_2}^{\text{part}}. \tag{24}$$

Now suppose that $N_1(t) = k$. In this case, we only focus on all channels in \mathcal{N}_2 and k idle channels in \mathcal{N}_1 . So, from now on in the analysis, we assume that there are only $N_2 + k$ channels in the network. Similar to the analysis of the no CSI case, let $A_e^{N_2+k}(t)$ be the number of effective channels in the L -interface CR network with full CSI when all channels ($N_2 + k$ channels) are idle. Note that the tagged SU transmits one packet successfully through each effective channel if it is actually idle; if the selected channel is in $\mathcal{N}_1^{\text{idle}}(t)$, then it is obviously idle, however, if the selected channel is in \mathcal{N}_2 , then it can be busy. So, the conditional throughput of the tagged SU $T_{L,k,k_2}^{\text{part}}$ is computed as follows:

$$T_{L,k,k_2}^{\text{part}} = E [c_L(t) \mid N(t) = (k, k_2)]$$

$$= \sum_{l=1}^{L_{N_2+k}^-} E [c_L(t) \mid N(t) = (k, k_2), A_e^{N_2+k}(t) = l]$$

$$\times P\{A_e^{N_2+k}(t) = l \mid N(t) = (k, k_2)\}$$

$$= \sum_{l=1}^{L_{N_2+k}^-} E [c_L(t) \mid N(t) = (k, k_2), A_e^{N_2+k}(t) = l]$$

$$\times P\{A_e^{N_2+k}(t) = l \mid N_1(t) = k\}.$$

where $L_{N_2+k}^- = \min(L, N_2 + k - L)$. In order to get $T_{L,k,k_2}^{\text{part}}$, we need to compute the probability that a selected effective channel is actually idle. For an arbitrary effective channel selected by the tagged SU, denoted by $\tilde{\omega}$, we obtain

$$P\{\tilde{\omega} \text{ is idle} \mid N(t) = (k, k_2)\}$$

$$= P\{\tilde{\omega} \text{ is idle} \mid \tilde{\omega} \in \mathcal{N}_1^{\text{idle}}(t), N(t) = (k, k_2)\} P\{\tilde{\omega} \in \mathcal{N}_1^{\text{idle}}(t) \mid N(t) = (k, k_2)\}$$

$$+ P\{\tilde{\omega} \text{ is idle} \mid \tilde{\omega} \in \mathcal{N}_2, N(t) = (k, k_2)\} P\{\tilde{\omega} \in \mathcal{N}_2 \mid N(t) = (k, k_2)\}$$

$$= 1 \cdot \frac{k}{N_2 + k} + \frac{k_2}{N_2} \cdot \frac{N_2}{N_2 + k} = \frac{k}{N_2 + k} + \frac{k_2}{N_2 + k}$$

This implies that

$$E \left[c_L(t) | N(t) = (k, k_2), A_e^N(t) = l \right] = l \left(\frac{k}{N_2 + k} + \frac{k_2}{N_2 + k} \right),$$

from which we obtain

$$\begin{aligned} T_{L,k,k_2}^{\text{part}} &= \sum_{l=1}^{L_{N_2+k}^-} l \left(\frac{k}{N_2 + k} + \frac{k_2}{N_2 + k} \right) P \{ A_e^{N_2+k}(t) = l | N_1(t) = k \} \\ &= \left(\frac{k}{N_2 + k} + \frac{k_2}{N_2 + k} \right) \sum_{l=1}^{L_{N_2+k}^-} l P \{ A_e^{N_2+k}(t) = l | N_1(t) = k \} \\ &= \left(\frac{k}{N_2 + k} + \frac{k_2}{N_2 + k} \right) L_{N_2+k} \alpha_k \left(1 - \frac{L_{N_2+k}}{N_2 + k} \alpha_k \right)^{M-1}. \end{aligned}$$

Then, the conditional throughput $T_{L,k}^{\text{part}}$ is given by

$$\begin{aligned} T_{L,k}^{\text{part}} &= \sum_{k_2=0}^{N_2} P \{ N_2(t) = k_2 \} \left(\frac{k}{N_2 + k} + \frac{k_2}{N_2 + k} \right) L_{N_2+k} \alpha_k \left(1 - \frac{L_{N_2+k}}{N_2 + k} \alpha_k \right)^{M-1} \\ &= L_{N_2+k} \alpha_k \left(1 - \frac{L_{N_2+k}}{N_2 + k} \alpha_k \right)^{M-1} \left(\frac{k}{N_2 + k} + \frac{\sum_{k_2=0}^{N_2} P \{ N_2(t) = k_2 \} k_2}{N_2 + k} \right) \\ &= \alpha_k L_{N_2+k} \left(1 - \frac{L_{N_2+k}}{N_2 + k} \alpha_k \right)^{M-1} \left(\frac{\pi_1 N_2 + k}{N_2 + k} \right). \end{aligned} \tag{25}$$

From (24) and (25), we finally obtain the throughput of the tagged SU in the L -interface CR network with partial CSI as follows.

$$T_L^{\text{part}} = \sum_{k=0}^{N_1} \left[P \{ N_1(t) = k \} \alpha_k L_{N_2+k} \left(1 - \frac{L_{N_2+k}}{N_2 + k} \alpha_k \right)^{M-1} \left(\frac{\pi_1 N_2 + k}{N_2 + k} \right) \right].$$

□

Theorem 8 For the L -interface CR network with partial CSI, the optimal AP values α_k^* , $1 \leq k \leq N_1$, that maximize the throughput of an arbitrary SU, are given by

$$\alpha_k^* = \min \left(\frac{N_2 + k}{L_{N_2+k} M}, 1 \right). \tag{26}$$

Proof To find the optimal AP value α_k , we solve the equation $dT_{L,k}^{\text{part}}/d\alpha_k = 0$. By differentiating $T_{L,k}^{\text{part}}$ in (25), we get

$$\left(1 - \frac{L_{N_2+k}}{N_2 + k} \alpha_k \right)^{M-2} \left(1 - \frac{L_{N_2+k} M}{N_2 + k} \alpha_k \right) = 0.$$

Since $L_{N_2+k} = \min(L, N_2 - k) \leq N_2 - k$ and $0 \leq \alpha_k \leq 1$, it is clear that $T_{L,k}^{\text{part}}$ is maximized when

$$\alpha_k^* = \min \left(\frac{N_2 + k}{L_{N_2+k} M}, 1 \right).$$

□

Theorem 9 *The optimal throughput $(T_L^{\text{part}})^*$ of an arbitrary SU in the L-interface CR network with partial CSI satisfies*

$$(T_L^{\text{part}})^* = \sum_{0 \leq k \leq LM - N_2} \left[P\{N_1(t) = k\} \frac{1}{M} \left(1 - \frac{1}{M}\right)^{M-1} (\pi_1 N_2 + k) \right] + \sum_{LM - N_2 < k \leq N_1} \left[P\{N_1(t) = k\} L \left(1 - \frac{L}{N_2 + k}\right)^{M-1} \left(\frac{\pi_1 N_2 + k}{N_2 + k}\right) \right].$$

Moreover, $(T_L^{\text{part}})^*$ is an increasing function in L .

Proof It is easy to show that $\alpha_k^* = 1$ when $k > LM - N_2$ and $\alpha_k^* = \frac{N_2+k}{LN_2+kM}$ when $k \leq LM - N_2$. Substituting it into (23), we obtain the expressions for $(T_L^{\text{part}})^*$ in the theorem. The increasing property is shown by a similar way as given in the proof of the full CSI case. \square

From Theorem 9, similar to the full CSI case, we see that the optimal throughput $(T_L^{\text{part}})^*$ is improved only when $LM < N (= N_1 + N_2)$. Moreover, when $LM \geq N$, it is clear that the optimal throughput in the partial CSI case becomes

$$(T_L^{\text{part}})^* = \sum_{0 \leq k \leq N_1} \left[P\{N_1(t) = k\} \frac{1}{M} \left(1 - \frac{1}{M}\right)^{M-1} (\pi_1 N_2 + k) \right] = \frac{1}{M} \left(1 - \frac{1}{M}\right)^{M-1} \left[\pi_1 N_2 \sum_{0 \leq k \leq N_1} P\{N_1(t) = k\} + \sum_{0 \leq k \leq N_1} k P\{N_1(t) = k\} \right] = \frac{1}{M} \left(1 - \frac{1}{M}\right)^{M-1} (\pi_1 N_2 + \pi_1 N_1) = \pi_1 \frac{N}{M} \left(1 - \frac{1}{M}\right)^{M-1}$$

which coincides with the optimal throughput for both full CSI case and no CSI case.

4 Numerical results

In this section, we provide numerical results to validate our analysis and investigate throughput performance behaviors in the three cases. We consider a CR network with N wireless channels and M SUs. The state transition probabilities of each wireless channel are given by $p = 0.9$ and $q = 0.1$. In this case the probability π_1 of being an idle channel is 0.9, which is a reasonable case in CR networks where the spectrum usage is very low. To validate our analysis, we use MATLAB to simulate the random behaviors of all users in the L -interface CR network. We perform $T_{max} = 10^7$ slot times in each simulation run to obtain the simulation results. We then compare our analytical results with simulation results.

4.1 Throughput performance when $LM \geq N$

In this subsection, we validate and investigate that (1) the optimal AP values $\{a_i^*\}_{1 \leq i \leq N}$ achieve the optimal throughput in the full CSI case, (2) the optimal AP value a^* achieves the

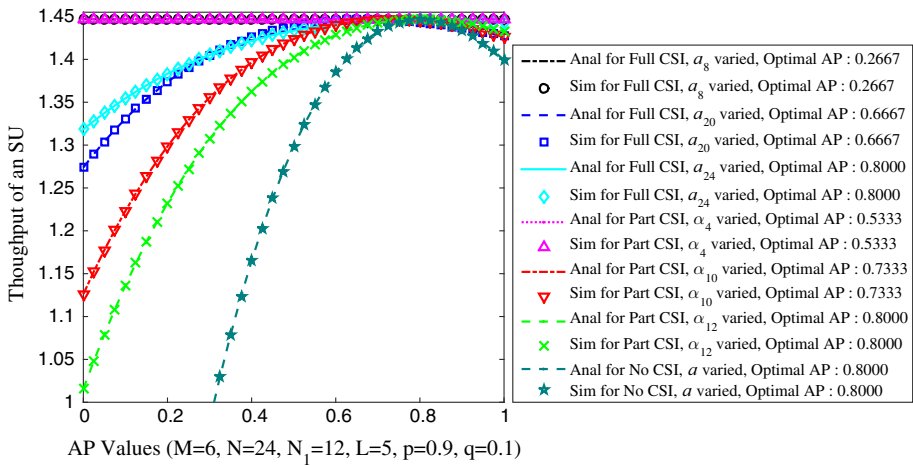


Fig. 4 Throughput of an arbitrary SU in the three cases versus AP values when $LM \geq N$

optimal throughput in the no CSI case and (3) the optimal AP value $\{\alpha_k^*\}_{1 \leq k \leq N}$ achieve the optimal throughput in the partial CSI case when $LM \geq N$. In the numerical examples, we use $M = 6, N = 24, N_1 = 12$ and $L = 5$ so that $LM (= 30) \geq N (= 24)$.

4.1.1 The full CSI case

In this case, the optimal AP values in Theorem 2 are given by

$$a_k^* = \begin{cases} 1/6, & k \leq 5; \\ k/30, & \text{otherwise.} \end{cases}$$

To validate whether our optimal AP values achieve the optimal throughput in this case, we select one AP a_k and change its value from 0 to 1 while all the other APs are set to their optimal values. Based on our observation that the AP values a_k for $k \leq 16$ do not affect the throughput significantly, we select the following APs $\{a_8, a_{20}, a_{24}\}$ and plot the results in Fig. 4. The index k and the optimal AP values $\{a_8^*, a_{20}^*, a_{24}^*\}$ are also given in Fig. 4. The x -axis is the value of a_k in the figure. As seen in the figure the optimal AP values achieve the optimal throughput. In addition, our analytic results are well matched with the simulation results in this case, which verifies the validity of our analysis.

4.1.2 The no CSI case

In this case, the optimal AP value in Theorem 5 is given by $a^* = 4/5$. To validate that our optimal AP value achieves the optimal throughput in this case, we change the value of AP from 0 to 1. The resulting throughput is also plotted in Fig. 4. The x -axis is the value of a in this case. From the figure we see that the optimal AP value achieves the optimal throughput. In addition, our analytic results are well matched with the simulation results in this case, which also verifies the validity of our analysis.

4.1.3 The partial CSI case

In this case, the optimal AP value in Theorem 8 is given by $\alpha_k^* = (12 + k)/30$. To validate our analysis, we select one AP α_k and change its value from 0 to 1 while the other APs are

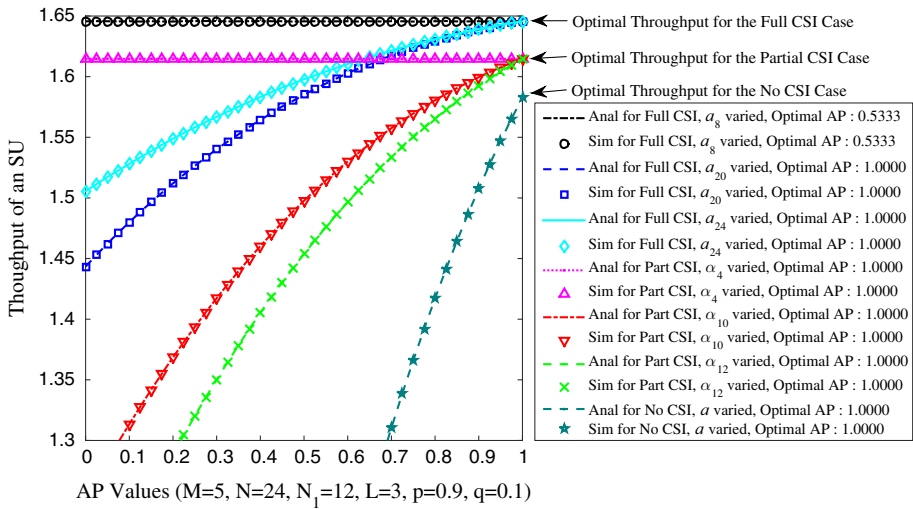


Fig. 5 Throughput of an arbitrary SU in the three cases versus AP values for $LM < N$

fixed to $\{\alpha_i^*\}_{1 \leq i \leq 12}$. The results are plotted in 7 and it shows that the optimal AP values in Theorem 8 achieve the optimal throughput. Moreover, we see that our analytic results are well matched with the simulation results, which verifies the validity of our analysis.

Furthermore, by comparing the optimal throughput for the three cases when $LM \geq N$, we see that the optimal throughputs in the three cases are the same, as we mentioned in our analysis. The optimal throughput is 1.45 in this example.

4.2 Throughput performance when $LM < N$

In this subsection, we validate our analytic results as in Sect. 4.1 when $LM < N$. That is, we use $M = 5, N = 24$ and $L = 3$ in this example, so that $LM (= 15) < N (= 24)$.

4.2.1 The full CSI case

In this case, the optimal AP values in Theorem 2 are given by

$$a_k^* = \begin{cases} 1/5, & k \leq 3; \\ k/15, & 3 < k \leq 15; \\ 1, & \text{otherwise.} \end{cases}$$

We select $\{a_8, a_{20}, a_{24}\}$ as in Sect. 4.1, and change each AP value from 0 to 1. The throughput results are plotted in Fig. 5. We see that the optimal AP values achieve the optimal throughput.

4.2.2 The no CSI case

In this case, the optimal AP value in Theorem 5 is given by $a^* = 1$. We change the AP value a from 0 to 1 and the resulting throughput is also plotted in Fig. 5. As we can see in the figure, the optimal AP a^* achieves the optimal throughput.

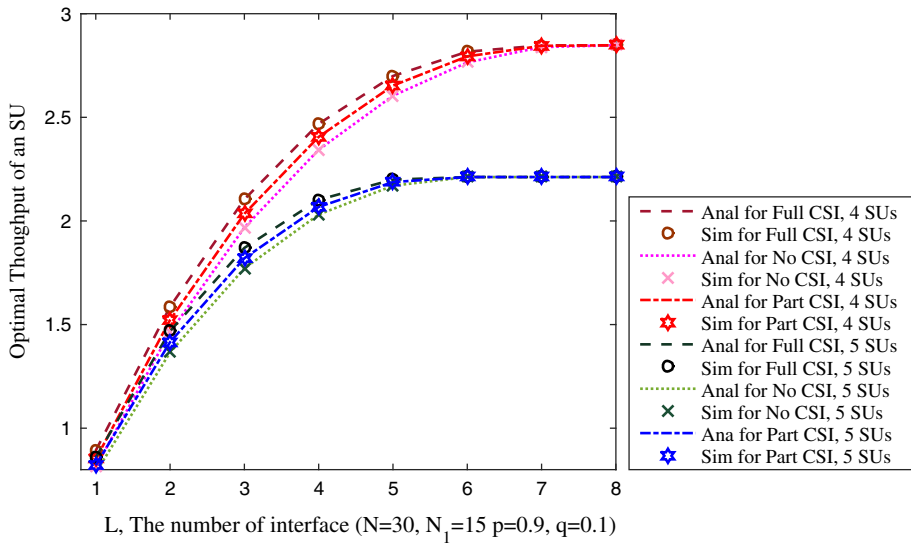


Fig. 6 The optimal throughput of an arbitrary SU in the three cases versus L

4.2.3 The partial CSI case

In this case, the optimal AP values in Theorem 8 are given by

$$\alpha_k^* = \begin{cases} (12 + k)/15, & k \leq 3; \\ 1, & \text{otherwise.} \end{cases}$$

For numerical analysis, we select $\{\alpha_8, \alpha_{20}, \alpha_{24}\}$ and change each AP value from 0 to 1. The throughput results are plotted in Fig. 5. The figure shows that the optimal AP values achieve the optimal throughput. It also shows that the analytic and simulation results are well matched

A comparison among the throughputs in the three cases when $LM < N$ shows that the optimal throughput for the no CSI case is less than that for the partial CSI case, and both of them are less than that for the full CSI case.

4.3 The optimal throughput performance

In this subsection, we investigate the optimal throughput performance behavior when $N = 30$ and $N_1 = 15$. So we assume that all SUs use the optimal AP values.

4.3.1 The impact of the maximum number of idle channels for SUs

First, we investigate the optimal throughput behavior of an arbitrary SU for the three cases by changing the maximum number of idle channels for SUs when $M = 4$ and $M = 5$. The results are plotted in Fig. 6. As seen in the figure, the optimal throughputs for the three cases increase as L increases when $L < N/M$. When $L \geq N/M$, it is observed that the optimal throughput becomes constant, as we showed in Theorems 3, 6 and 9. We can also observe that, only when $LM < N$, the optimal throughput for the full CSI case is bigger than that for the partial CSI case, and that for the partial CSI case is bigger than that for the no CSI case.

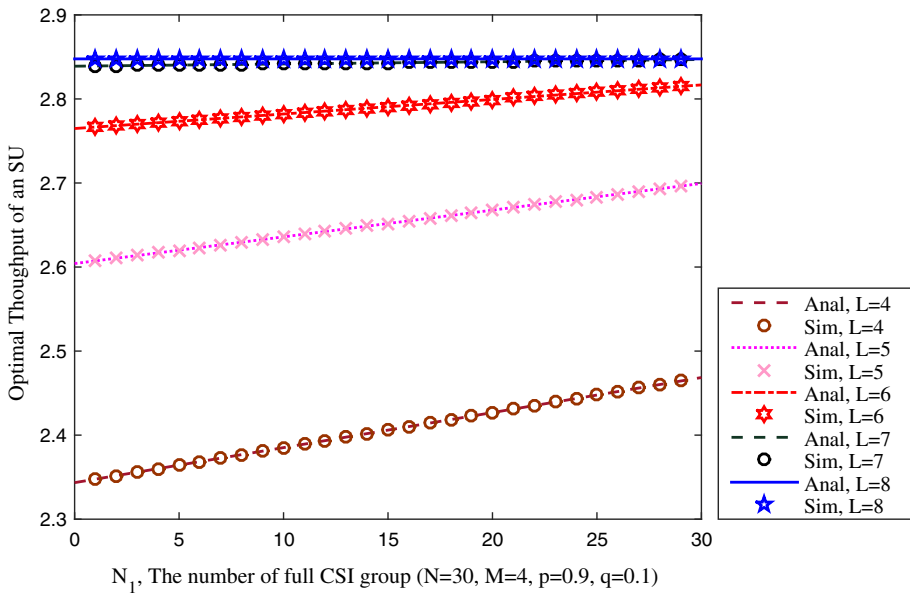


Fig. 7 The optimal throughput of an arbitrary SU for the partial CSI case versus N_1

4.3.2 The impact of the number in the full CSI group

The optimal throughput of an arbitrary SU for the partial CSI case is plotted in Fig. 7 when we change the number N_1 in the full CSI group. Note that if $N_1 = 0$, the L -interface CR network with partial CSI becomes the L -interface CR network with no CSI; if $N_1 = 30$, the L -interface CR network with partial CSI becomes the L -interface CR network with full CSI. In the figure, we see that the optimal throughput linearly increases as N_1 increases when $L = 4, 5, 6$, and 7 . However, when $L = 8$, the optimal throughput does not increase as N_1 increases; the reason is that $LM > N$ in this case, as we mentioned before.

5 Conclusions

In this paper, we considered a cognitive radio network where multiple SUs contend to access wireless channels. We analyzed the L -interface cognitive radio network with a random access policy where each secondary user stochastically decides whether to access L channels simultaneously based on a given access probability.

We considered three cases in the analysis - the case of full channel state information, the case of no channel state information, and the case of partial channel state information. For each case, to obtain the optimal access probabilities we analyzed the throughput of an arbitrary SU, defined by the average number of packets transmitted successfully per time slot. We then derived the optimal APs that achieve the optimal throughput of an arbitrary SU. We also investigated the impact of the number L on the optimal throughput for the three cases. Our analytic results were verified through numerical and simulation results.

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References

- Akyildiz, I. F., Lee, W.-Y., Vuran, M. C., & Mohanty, S. (2006). Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey. *Computer Networks*, *50*(13), 2127–2159.
- Cho, H. J., & Hwang, G. U. (2012). Throughput performance optimization in cognitive radio networks under rayleigh fading. In *IEEE Wireless Communications and Networking Conference (WCNC)*, (pp. 1410–1415).
- Cordeiro, C., Challapali, K. (2007). C-mac: A cognitive mac protocol for multi-channel wireless networks. In *2nd IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks, 2007. DySPAN 2007.*, pp. 147–157.
- Cormio, C., & Chowdhury, K. R. (2009). A survey on mac protocols for cognitive radio networks. *Ad Hoc Networks*, *7*(7), 1315–1329.
- FC Commission. (2003). Facilitating opportunities for flexible, efficient and reliable spectrum use employing cognitive radio technologies, notice of proposed rule making and order. *Federal Communications Commission, Washington, DC, Rep. ET Docket*, no. 03–222.
- Gardner, W. A. (1988). Signal interception: A unifying theoretical framework for feature detection. *IEEE Transactions on communications*, *36*(8), 897–906.
- Haykin, S. (2005). Cognitive radio: Brain-empowered wireless communications. *IEEE Journal on Selected Areas in Communications*, *23*(2), 201–220.
- Hossain, E., Niyato, D., & Han, Z. (2009). *Dynamic spectrum access and management in cognitive radio networks*. Cambridge: Cambridge University Press.
- Hsu, A. C.-C., Wei, D. S., & Kuo, C.-C. J. (2007). A cognitive mac protocol using statistical channel allocation for wireless ad-hoc networks. In *Wireless Communications and Networking Conference* (pp. 105–110).
- Hu, D., & Mao, S. (2009). Design and analysis of a sensing error-aware mac protocol for cognitive radio networks. In *IEEE Global Telecommunications Conference. GLOBECOM 2009.* (pp. 1–6).
- Hwang, G. U., & Roy, S. (2012). Design and analysis of optimal random access policies in cognitive radio networks. *IEEE Transactions on Communications*, *60*(1), 121–131.
- Jia, J., Zhang, Q., & Shen, X. S. (2008). Hc-mac: A hardware-constrained cognitive mac for efficient spectrum management. *IEEE Journal on Selected Areas in Communications*, *26*(1), 106–117.
- Kwon, H., Seo, H., Kim, S., & Lee, B. G. (2009). Generalized csma/ca for ofdma systems: protocol design, throughput analysis, and implementation issues. *IEEE Transactions on Wireless Communications*, *8*(8), 4176–4187.
- Li, X., Zhao, Q., Guan, X., & Tong, L. (2011). Optimal cognitive access of markovian channels under tight collision constraints. *IEEE Journal on Selected Areas in Communications*, *29*(4), 746–756.
- Ma, L., Han, X., & Shen, C. C. (2005). Dynamic open spectrum sharing mac protocol for wireless ad hoc networks. In *First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005.*, pp. 203–213.
- McHenry, M. (2003). Spectrum white space measurements. Presentation to *New America Foundation Broadband Forum*, Shared Spectrum Company, Technical Report, June.
- Mitola, J., & Maguire, G. Q. (1999). Cognitive radio: Making software radios more personal. *IEEE Personal Communications*, *6*(4), 13–18.
- Park, S., Hwang, G., & Choi, J. K. (2016). Optimal throughput analysis of random access policies for cognitive radio networks with multiple channel access. In *Proceedings of the 11th International Conference on Queueing Theory and Network Applications*.
- Park, S., Kim, J., Hwang, G., & Choi, J. K. (2016). Joint optimal access and sensing policy on distributed cognitive radio networks with channel aggregation. In *2016 Eighth International Conference on Ubiquitous and Future Networks (ICUFN)*, July 2016, (pp. 253–258).
- Poston, J. D., & Horne, W. D. (2005). Discontiguous ofdm considerations for dynamic spectrum access in idle tv channels. In *First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005.* (pp. 607–610).
- Qing, Z., & Adler, B. (2007). Survey of dynamic spectrum access: Signal processing, networking, and regulatory policy. In *Proceedings of ICASSP*, vol. 7.
- Sabharwal, A., Khoshnevis, A., & Knightly, E. (2007). Opportunistic spectral usage: Bounds and a multi-band csma/ca protocol. *IEEE/ACM Transactions on Networking (TON)*, *15*(3), 533–545.

- Sankaranarayanan, S., Papadimitratos, P., Bradley, A., & Hershey, S. (2005). A bandwidth sharing approach to improve licensed spectrum utilization. In *First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005.* (pp. 279–288).
- Spectrum policy task force. (2002). *Federal Communications Commission, Washington, DC, Rep. ET Docket*, no. 02–135.
- Standard for wireless regional area networks (wran)—specific requirements—part 22: Cognitive wireless ran medium access control (mac) and physical layer (phy) specifications: Policies and procedures for operation in the tv bands. In *The Institute of Electrical and Electronics Engineering, Std. IEEE 802.22.*
- Su, H., & Zhang, X. (2008). Cross-layer based opportunistic mac protocols for qos provisionings over cognitive radio wireless networks. *IEEE Journal on Selected Areas in Communications*, 26(1), 118–129.
- Urkowitz, H. (1967). Energy detection of unknown deterministic signals. *Proceedings of the IEEE*, 55(4), 523–531.
- Wang, S., Zhang, J., & Tong, L. (2010). Delay analysis for cognitive radio networks with random access: A fluid queue view. In *Proceedings of IEEE INFOCOM* (pp. 1–9).
- Xu, D., Jung, E., & Liu, X. (2008). Optimal bandwidth selection in multi-channel cognitive radio networks: How much is too much? In *3rd IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks, 2008. DySPAN* (pp. 1–11).
- Yucek, T., & Arslan, H. (2009). A survey of spectrum sensing algorithms for cognitive radio applications. *IEEE Communications Surveys & Tutorials*, 11(1), 116–130.
- Zhao, Q., Tong, L., Swami, A., & Chen, Y. (2007). Decentralized cognitive mac for opportunistic spectrum access in ad hoc networks: A pomdp framework. *IEEE Journal on Selected Areas in Communications*, 25(3), 589–600.
- Zhao, Q., Geirhofer, S., Tong, L., & Sadler, B. M. (2008). Opportunistic spectrum access via periodic channel sensing. *IEEE Transactions on Signal Processing*, 56(2), 785–796.