Ann Oper Res (2016) 247:549–580
DOI 10.1007/s10479-015-1947-9

Multi-criteria decision making for choosing socially responsible investment within a behavioral portfolio theory framework: a new way of investing into a crisis environment

Amelia Bilbao-Terol[1] · **Mar Arenas-Parra**[1] · **Verónica Cañal-Fernández**[2] · **Celia Bilbao-Terol**[2]

Published online: 31 July 2015

Abstract The current economic crisis fuels the financial social responsibility after an epoch of many excesses with damaging effects. This work tackles two emerging streams in the financial literature: the behavioral portfolio theory with mental accounting and the socially responsible investment (SRI). Promoting SRI is regarded by a lot of financial experts, policy-makers and researchers from the field of economic and social sciences, as one of the potential solutions in order to avoid future crises. Therefore, new models for this investment approach are necessary. We try to support the class of investors that select their investments under a mental accounting framework and also they want to achieve a certain level of SR quality in their portfolios. In order to reconcile the two choice frames, avoiding unnecessary sacrifices in financial performance, we have designed a model based on goal programming that integrates the two cornerstones of the investor. Furthermore, we propose a fuzzy inference system to determine the amount of money allocated to each mental account as well as the confidence level assigned to each mental account. This tool is based on expert knowledge modeled by fuzzy if–then rules.

Keywords Behavioral portfolio with mental accounting · Socially responsible investing · Goal programming · Conditional value at risk · Fuzzy inference system

✉ Amelia Bilbao-Terol
ameliab@uniovi.es

Mar Arenas-Parra
mariamar@uniovi.es

Verónica Cañal-Fernández
vcanal@uniovi.es

Celia Bilbao-Terol
cbilbao@uniovi.es

1 Department of Quantitative Economics, University of Oviedo, Oviedo, Spain

2 Department of Economics, University of Oviedo, Oviedo, Spain

1 Introduction

The aim of this work is to link the mental accounting (MA) framework for managing investments with the socially responsible investment style. We provide a methodology to build portfolios for behavioral investors that follow ethical, environmental and social considerations in their investment decisions. To do so, we use behavioral portfolio theory with mental accounting (BPT-MA) approach, goal programming (GP) modeling and fuzzy logic tools.

Socially responsible investment (SRI), now known as sustainable and responsible investment, is defined as an investment process that considers environmental, social and governance (ESG) consequences of investments, both positive and negative, within the context of financial analysis (EUROSIF 2012). Ethicality in financial business has never been as necessary as today because it is generally recognized that unethical behaviors conducted in recent years have led to the current crisis. The 2008–09 World Financial Crisis' impact on economic markets, international financial policies and society is indubitable.

Many voices consider SRI as a solution to the current crisis or at least a means to avoid it happening again. Financial social responsibility links the finance world and society through SRI, in which securities are primarily selected for ESG issues. Globalization, political changes and societal trends, but in particular the current world economy, have leveraged the societal demand for social responsibility in market systems and regain trust in the economy (Summers, in speech, April 2012, The Economist July 7th 2012). The recent crisis has demonstrated that decisions made by financial institutions have implications for entire economies. Thus the work of the PRI Initiative to promote better risk management, transparency and good governance has never been more important (PRI 2009). In the eye of the many negative consequences of the 2008–09 world financial downturn, the crisis appears to hold less acknowledged potentials to raise social responsibility in economic markets (Summers, in speech, April 2012). Former World Bank President Robert Zoellick describes the "new era of responsibility" as featuring "changed attitudes and co-operative policies" that promote responsible corporate conduct and socially conscientious investments to imbue trust in the global economy (Financial Times January 25th 2009).

Soros (in speech, April 2011, The Economist July 7th 2012) calls on the scientific community considering that the financial crisis clearly underlines how classical finance and economic theories do not truly capture human cognition in economic markets. He says that mainstream economics must be complemented by heterodox insights on socio-psychological notions of fallible market actors and pay attention to harmful contagion effects of their limited decisions. We have come along way from the ideas of Friedman (1962) that considered corporate responsibility solely as the quest for profit maximization, companies have nowadays adopted further responsibilities toward society in general and stakeholders in particular. In doing so, companies have increasingly acknowledged the need to conduct business responsibly and accountably (Moneva et al. 2006). An accurate understanding of socio-economic market behavior in the interaction of financial markets and real-world economic outcomes is needed (Hofmann et al. 2008).

On the other hand, a research stream considers that the main macroeconomic theory also might be a cause of some financial disasters. "The ideas at the heart of modern macroeconomics provided the intellectual justification of the economic policies of the past 10–15 years. It is these ideas which the financial crisis falsified" (Ormerod 2010). Several views criticize the model of rational economic agent and its consequences in the frame of financial decisions. Prospect theory (Kahneman and Tversky 1979) has emerged as a descriptive decision theory under uncertainty that studies the actual behavioral investor and puts into question, e.g. the investor's unbounded rationality, the normality of the assets' distributions and the variance as risk measure. The Basel accords already take into account that downside risk measures can avoid bankruptcy situations. In this work, the financial framework is within the behavioral finances with MA as a tool of decision making. The notion of MA was introduced by Thaler (1980). It refers to the tendency for people to separate their money into separate accounts based on a variety of subjective criteria. Thaler defines MA as the set of cognitive operations used by individuals to code financial activities as profits and losses, categorize them into various mental accounts, and then evaluate them (Thaler 1999). MA provides a foundation for the way decision makers (DMs) set reference points for the accounts that determine profits and losses. The main idea is that decision makers tend to segregate different types of gambles into separate accounts, and then apply the prospect theory to each account by ignoring possible interactions (Grinblatt and Han 2005). The break-down of the investment problem into mental accounts is interpreted as the result of framing the complex problem into simplex subproblems. The theory purports individuals assign different levels of risk to each asset group, which affects their consumption decisions and other behaviors (Hsieh 2011).

Behavioral portfolio theory (BPT), proposed by Shefrin and Statman (2000), is a positive theory of choice under uncertainty that essentially tries to provide a contrast to the fact that the ultimate motivation for investors is the maximization of the value of their portfolios. They develop a model of multi-layered portfolio construction in which each layer is associated with a particular aspiration level (goal) and the covariances between the layers are overlooked. Thus, each portfolio layer resembles a separate mental account. Shefrin and Statman suggest that investors have varied aims and create an investment portfolio that meets a broad range of goals. Most investors combine low aspirations with high ones; they want to avoid poverty, but they also want a shot at riches. Thus, BPT investors do not follow the same principles as the Capital Asset Pricing Model (Sharpe 1964), Modern Portfolio Theory (Markowitz 1952) and the Arbitrage Pricing Theory (Ross 1976).

In the current economic crisis we ask how operational research models can help to expand SRI. The aim is that SRI moves from a marginal investment option into a more mainstream one. Our contribution is focused on the design of models more tailored to actual decision making process in selecting portfolios with SRI products. As far as the authors know, the SRI approach has not been addressed in the BPT-MA framework. But we do want to highlight the work of Gärling et al. (2009) relating to prospect theory, SRI and the financial crises from the perspective of psychological science.

Multi-criteria decision making (MCDM) (Zeleny 1974) is devoted to the development of appropriate methodologies that can be used to support DMs in circumstances where multiple conflicting decision criteria have to be considered simultaneously. It has been usefully used in portfolio selection, both conventional portfolios (e.g., Lee and Chesser 1980; Ogryczak 2000; Bilbao et al. 2007; Steuer et al. 2007; Ballestero et al. 2009; Pérez and Gómez 2014) and socially responsible (SR) portfolios. Gupta et al. (2013) propose a comprehensive three-stage multiple criteria decision-making framework for portfolio selection based on using financial and ethical criteria simultaneously. Hirschberger et al. (2013) extend Markowitz's portfolio selection to an additional linear criterion (dividends, liquidity, sustainability etc.) and propose an example for SR investors, showing that their algorithm can outperform standard portfolio strategies for multi-criteria DMs. Bilbao et al. (2013) propose an approach for portfolio selection based on the market valuation of the social responsibility of financial assets and multi-objective programming tools in order to obtain an optimal SRI portfolio with a financial performance similar to an optimal portfolio without ESG considerations. Calvo et al. (2014) consider the social responsibility of the portfolio as an additional secondary non-financial goal in the mean-variance portfolio selection model. Zopounidis and Doumpos (2013) present an exhaustive literature review on the application of multi-criteria decision aid tools to financial problems.

Within MCDM approach, GP is introduced by Charnes and Cooper (1961). It can be said that GP is the most widely used multi-objective technique because of its inherent flexibility in handling decision-making problems with several conflicting objectives and incomplete or imprecise information. Many models concerning the portfolio selection using GP have been developed (e.g., Abdelaziz et al. 2007; Kaminski et al. 2009; Ballestero et al. 2012; Bilbao-Terol et al. 2012a, b; Aouni et al. 2013; Bahloul and Abid 2013). Our contribution designs a GP model with goals relative to each mental account. The preference structure is defined by setting the desired financial features for each account as well as a SR-value of the aggregate allocation across MA subportfolios. The financial preferences of the investor are specified directly in percentile terms, instead of utility functions. Approaches based on values limiting the worst losses provide efficient and flexible risk management tools in the current financial markets.

Since its introduction by Zadeh (1965) the fuzzy logic models employing fuzzy sets have been used by handling and describing imprecise and complex phenomena that often rise in business, financial and managerial systems involving a great number of interacting factors, some of socio-psychological nature. A fuzzy inference system (FIS) tries to formalize the reasoning process of human language by means of fuzzy logic. FIS applies fuzzy if-then rules that can model the qualitative aspects of human knowledge and reasoning processes (Bojadziev and Bojadziev 1996). It uses either prior experiences or knowledge within a set of constraints to obtain a good solution. Among many FIS models, the Mamdani model (Mamdani and Assilian 1975) and Takagi-Sugeno model (Takagi and Sugeno 1985) are the most commonly used fuzzy model. Adaptive neuro fuzzy inference system (ANFIS), first proposed by Jang (1993), combined the benefits of artificial neural networks (ANNs) and FIS. ANFIS can adapt the parameters of the membership functions quickly and optimize them depending on the input data. It provides a method to generate fuzzy rules from a set of input-output data pairs. In this work, we propose to utilize these models for determining the allocation between the different mental accounts according to the investor's profile and the expert knowledge provided by the financial manager.

The paper is organized as follows. Socially responsible investment is reviewed in Sect. 2. The next section briefly introduces our theoretical framework. Section 4 illustrates the GP model that integrates the financial objectives of each account with the SR objective of the total portfolio and provides a systematic procedure for evaluating the investor's risk profile by a FIS/ANFIS. Section 5 presents a real case study with four broad market indexes, two SR and two conventional ones. Section 6 discusses the main results of the model and provides our final remarks.

2 Socially responsible investment

The origins of SRI are quite old, before 1960 it was focused on faith-based exclusions (the first SR fund, Pioneer Fund, was launched in 1928), the social climate of the 1960s, fed by Vietnam war, the feminist movement, anti-nuclear weapons, raised concerns among some investors about civil rights, the environment, and militarism, but the turning point for SR investors came during the campaign to eliminate the institutionalized racial discrimination of Apartheid in South Africa. The major ecological disasters caused by man in the two following decades with immense damage to the environment and human health (e.g., the

Seveso incident in 1976; Three Mile Island nuclear emergency in 1979; Bhopal incident in 1984; Chernobyl nuclear emergency in 1986; Exxon Valdez oil spill in 1989; Brent Spar incident in 1995; the oil-tanker Prestige off the coasts of Galicia, in 2002; or more recently, the explosion and subsequent sinking of the Deepwater Horizon, an oil rig that was leased to the British company BP, at least 20 million gallons spilled into the Gulf of Mexico, affecting more than 110 km of Louisiana's coastline, in 2010) have motivated a substantial growth of the social responsibility awareness in the society in general and in the investors, in particular. Therefore, though SRI has a long history, the emerging global environmental crisis, legislative compulsion and stakeholder pressure (Wolff 2002) and the current financial crisis have put in the spotlight the SRI as a tool for addressing solutions to these crises. The global financial crisis has also led consumers and investors to pay attention to responsibility, transparency and accountability of market participants (EFAMA 2013). SRI has altered the perception of what a sound investment must consider in addition to traditional measures of financial performance (USSIF 2013).

Recently a global organization—Global Sustainable Investment Alliance (GSIA)—has been created with the purpose of establishing cooperation between seven regions around the world (Europe, Asia, excluding Japan, United States, Australia and New Zealand, Canada, and Africa) to increase the impact and visibility of SRI at a global level (GSIA 2013). According to this study, the current global market share of SRI amounts to US $ 13,6 billion which represents 21.8 % of the total universe of assets under management within the regions studied. The market for SRI is led by Europe, where almost two-thirds of the world's SRI assets are managed ($8,75 billion, 64.5 %). The US ($3,75 billion, 27.6 %) and Canada ($589 billion, 4.3 %) have also a significant proportion of those assets and the three of them combined account for 96 % of the assets covered by the mentioned report (GSIA 2013). Considering ESG issues, Europe is also the region with the highest proportion of SRI assets, a market share of 49 percent of total assets under management (EUROSIF 2012; GSIA 2013). On the other hand, in the US this proportion is of 11.2 % and in Asia, not more than 3 % (USSIF 2012). Canada and Australia fall in a middle-range with respectively 20 and 18 % of SRI assets among total assets.

The standard approach in SRI consists of screening methods which select the investable assets. There are two main ways of applying screens: the first generation of SRI involves the use of the negative screen whereby certain businesses are avoided (alcohol, tobacco, nuclear power, gambling, etc). In the second generation of SRI, the focus is more on adopting positive screening to select firms and a 'best-in-class' approach, with those identified as engaging in SR practices seen as more attractive for investors (Kinnel 2009; Radu and Funaru 2011; Barreda-Tarrazona et al. 2011). The combination of both positive and negative screens leads to the creation of the third generation of SRI. The fourth and most recent generation of SRI includes the shareholder advocacy strategy, by which the investor acquires shares in companies that would be rejected if employing the first strategy (social screening). The goal of such a strategy is to impact on the company's policies in order to make it more SR through shareholders resolutions and divestment campaigns. The biggest advantage of this strategy is that investors are allowed to benefit from the company's stock price appreciation and dividends together with changing company policy. However, the disadvantage is that this strategy generally requires a sizeable commitment in terms of time and capital (Cortez et al. 2009). Finally, with the community investing strategy, the investor directs capital to communities around the world that have limited access to traditional financial service institutions or funding.

The sustainability and responsibility of the companies with the social and natural environment are evaluated by extra-financial rating agencies such as Vigeo, KLD Research & Analytics, Inc (now MSCI), Ethical Investment Research Service (EIRiS), Sustainable Investment Research International Company (SiRi Co), Ethibel, Innovest, a division of RiskMetrics, and others. These agencies rate the sustainable and SR efforts of companies, so studies can be conducted to investigate the ways SR mutual funds do or do not actually incorporate social responsibility into their operations (Berry and Junkus 2013; Utz et al. 2013; Basso and Funari 2014).

The growing interest in the concept of responsible investment in general, both among individual and institutional investors, has led to an increasing volume of academic literature in this sector. Several SRI topics have been extensively investigated, with most of the existing empirical studies focusing on the financial performance of SRI assets (see, e.g., Statman 2000; Bauer et al. 2005; Barnett and Salomon 2006; Kempf and Osthoff 2007; Renneboog et al. 2008; Galema et al. 2008; Basso and Funari 2014). Also there exists research about the theoretical background of SRI (see, Bénabou and Tirole 2010; Dam and Heijdra 2011) and the flow-performance relation for SRI funds (see, e.g., Bollen 2007; Benson and Humphrey 2008; Renneboog et al. 2011). Islamic funds have received special attention in literature as a particular kind of ethical funds presenting several differences with the SRI investment. Islamic finance has very specific features such as investing in Shariah compliant assets. Four pillars represent the defining prohibitions of Islamic finance: Gharar (excessive uncertainty); Riba (usury); Maysir (speculation) and investing in prohibited activities. The fifth pillar is the encouragement of risk and return sharing (Shanmugam and Zahari 2009; Hayat and Kraeussl 2011). Islamic funds exclude investments in instruments with fixed income, such as certificates of deposit (CDs), corporate bonds, preferred stocks and some derivatives (e.g., options). While equity mutual funds represent the largest slice of Islamic funds, SRI mutual funds can freely choose between equity-bearing investments and debt-bearing investments, as long as the chosen stocks adhere to sustainable and responsible investment strategies and governance principles. In addition, Islamic funds apply further financial ratio filters on equity selected, such as leverage and percentage of interest paid or received (Abdelsalam et al. 2014). Although both SRI and Islamic investment instruments employ screening strategies in their investment process, there are distinct differences between them (Forte and Miglietta 2007). A large part of investment from SRI funds is directed toward small companies with lower dividend yields (Fowler and Hope 2007). On the opposite side, Islamic mutual funds carry out a more rigorous screening process in an attempt to select portfolios that meet both qualitative and quantitative criteria set by Shariah guidelines. The screening process eliminates companies engaged in prohibited activities under Shariah and companies whose capital structures rely heavily on debt financing to avoid dealing with interest. Furthermore, Shariah provides structural guidance regarding the governance and management structure of Islamic mutual funds.

An interesting topic that has not been developed in the SRI literature is concerning to the Arrow-Schumpeter debate[1] on the relationship between market structure and innovation incentives. The origins of SRI are of Arrovian flavour, the earliest SRI screening was applied by religious groups such as the Lutheran Brotherhood and the Quakers which excluded sin industries such as tobacco and alcohol (Schepers 2003). On the other hand, SRI instruments can be viewed as a variety of financial products to meet investors love for investments consistent with their personal value system and beliefs. So the launching of SRI products falls within the behaviour of multiproduct firms. In this sense Lambertini (2009) has taken a dynamic approach to the analysis of the optimal investment in research and development (R&D) activities aimed at enlarging the spectrum of varieties offered by a multiproduct

[1] This issue has been suggested by an anonymous referee of this paper. The authors greatly appreciate his/her thoughtful comment.

monopolist. The outcome of his work can be considered as a well-defined Arrovian result. Indeed, a profounder study is necessary in order to translate this conclusion to the SRI field.

With respect to the application of mathematical programming tools in order to construct SR portfolios, the literature is more scarce than for other SR topics. Drut (2010) proposes introducing social ratings in mean-variance optimization via linear constraints in order to explore the implications of considering a social responsibility threshold in the traditional Markowitz portfolio selection setting. Dorfleitner and Utz (2012) contribute to the literature on SRI by using the concept of stochastic sustainability returns in the context of a safety first portfolio choice. Barracchini and Addessi (2012) deal with the problem of how to quantify an "ethical portfolio size" within a dynamic algorithm as well as how to use such size in the portfolio selection criteria. The outcome of this work is the introduction of the ethical size as a third dimension in the same way as return and risk in the mean-variance method, thus becoming a three-dimensional mean-variance-ethics model. Simister and Whittle (2013) propose that each investment is seen as being on a continuum from "least ethical" to "most ethical", by using a conventional portfolio analysis (which focuses on risk and return) combined with analysis of principal components in order to minimize the risk of a portfolio. Ballestero et al. (2012) developed a financial-ethical bi-criteria model with absolute risk aversion coefficients and targets depending on the investors ethical aspirations. Bilbao et al. (2012a) develop models for selecting portfolios for conventional and SR mutual funds. The optimal portfolio selection problem is solved when the expected returns of the assets as well as the periodic returns are not precisely known. The multidimensional nature of the problem leads them to work with GP, and the incomplete information is handled by employing a fuzzy robust approach. Bilbao et al. (2012b) present an index called 'SRI-Attractiveness' that summarizes the SR characteristics of each SR mutual fund for a particular investor.

3 Behavioral portfolio construction with mental accounts

BPT is based on the SPA theory of Lopes (1987) with MA structure from Kahneman-Tversky's prospect theory (1979), and closely related to Roy's safety-first criterion (1952).

In SPA theory, the S stands for *security*, P for *potential*, and A for *aspiration*. Lopes' notion of security is analogous to safety in safety-first, (a general concern about avoiding low levels of wealth). Her notion of aspiration relates to a goal, and generalizes the safety-first concept of reaching a specific target value. Potential relates to a general desire to reach high levels of wealth.

Kahneman and Tversky explained several aspects of the investors' financial behavior in their prospect theory:

- People think in terms of profits and losses rather than in total wealth (mental accounting).
- Individuals are more averse to loss rather than profit (loss aversion).
- Whether a certain outcome is a profit or a loss depends on the individual's reference point.
- Typical reference points are the price at which an asset was bought, the initial wealth multiplied by the risk-free rate, or the wealth multiplied by the return of a benchmark.
- People have multiple reference points, below each reference point an investment is considered a loss.

A feature in BPT-MA is the observation that investors view their portfolios not as a whole, as prescribed by mean-variance portfolio theory, but as distinct mental account layers in a pyramid (see Fig. 1). These layers are associated with particular goals and the attitudes toward risk vary across layers. One mental account layer might be a downside protection

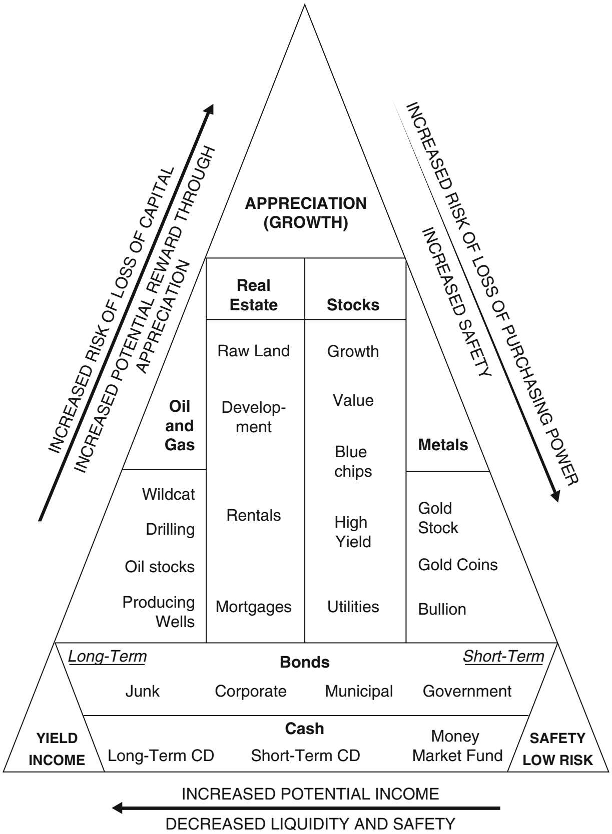

Fig. 1 Pyramid behavioral portfolio (*source*: Hens and Bachmann 2008, p. 83)

layer, designed to protect investors from being poor. Another might be an upside potential layer, designed to give investors a chance at being rich. Investors might behave as if they hate risk in the downside protection layer, while they behave as if they love risk in the upside potential layer. Therefore, this kind of design also incorporates the structure of prospect theory and Lopes' two factor theory (*fear*/*security* and *hope*/*potential*).

For BPT investors, the parameters that are relevant to asset allocation are the relative importance of the upside potential goal relative to the downside protection goal, and the reference points of the upside and downside goals. The features of the BPT-MA framework include the probability of failing to reach the threshold level in each mental account, and the attitudes toward risk that vary by account. One benefit of such a framework is that risk aversions are specified better (the research by Das et al. (2010) has found that the losses are in the range of 5–40 bps when investors mis-specify their risk aversion, and losses are higher for investors who are less risk averse). However, MA may result in a loss in portfolio efficiency because the aggregate of optimized subportfolios is not always mean-variance efficient. Beyond that, it is important to note that after investors specify their subportfolio threshold levels and probabilities, the issue may be a standard mean-variance problem with an implied risk-aversion coefficient (Das et al. 2010).

Mental accounts can be used for two different purposes: the separation of assets into different asset classes (risk free, save, moderately risky, capital appreciation) and also for allowing several investment goals. In the first case it is possible associate one subset of assets for each investment goal (risk free for capital protection, ..., stocks for long term growth) and therefore, the two ways of establishing mental accounts agree. One such example for the first case is given by the portfolio pyramid (see Fig. 1) where each layer is linked with a particular goal or attitude toward risk. According to Hens and Bachmann (2008), the clear link of assets to investment goals may, however, not be possible and all assets may be needed for each goal so that the two implementations of mental accounts do not agree. In this work we are interested in this second situation called goal-based portfolio selection.

3.1 Risk profiling in behavioral portfolio theory

One of the most important tasks of a financial manager is to identify the characteristics of her client. The financial circumstances as well as the psychological aspects are essential in determining the set of records that define the investor's risk profile. The level of application of the suitable indicators for scoring the personal risk will be to a large extent determining factor. The bad praxis in this area involves scandals and abuses as, e.g. the case of the preference shares in Spain in which, among other inappropriate actions, financial products with perpetual maturity and risk of illiquid market, were sold to investors with short-term liabilities, lack of financial literacy and great confidence in the banking staff (El País April 22th 2012; Cinco Días July 28th 2012; Expansión.com September 9th 2012). Indeed, the elaboration of good questionnaires in order to know risk profiles is a task of a manager team, the operational research modeler can help in the improvement of them from finding inconsistencies and/or regular patrons.

The risk profile of a BPT investor relies on three main features: risk ability, risk preferences and risk awareness (Hens and Bachmann 2008). The former represents a constraint for the optimization of the investor's utility. It is necessary to find an optimal asset allocation that ensures her capacity for financing her liabilities and to prioritize the investor's liabilities in 'hard' (that wealth which is necessary to keep up the lifestyle) and 'soft' (wealth to add plans and wishes that shall improve the lifestyle). It is important to know that liabilities shall not be risked in any case, or whether a certain small probability of risking them is acceptable. In the second case, it is possible to apply downside risk measures (Pla-Santamaria and Bravo 2013) such as the value-at-risk (VaR) or the conditional value-at-risk (CVaR) (Köksalan and Tuncer Sakar 2014; Krzemienowski and Szymczyk 2014). Below we summarize the main features of both downside risk measures:

- VaR_α of a portfolio is the lowest amount such that, with probability α, the loss will not exceed this value over a specified time period.
- When VaR_α is used as risk measure it involves accepting positions as safe when is not more than $(1-\alpha\,\%)$ of the cases one loses more money than one can afford.
- VaR has undesirable mathematical characteristics such as a lack of sub-additivity and convexity (see Artzner et al. 1999): VaR associated with a combination of two portfolios can be deemed greater than the sum of the risks of the individual portfolios.
- Using VaR for separate mental accounts is very problematic since adding up the VaR of different mental accounts may lead to erroneous conclusions with respect to the total risk exposure of the investor.
- For continuous distributions, $CVaR_\alpha$ is the conditional expectation of the loss variable L above the VaR_α level:

$$CVaR_\alpha = E[L|\, L \geq VaR_\alpha],$$

for general distributions, $CVaR_\alpha$ is defined as the weighted average of VaR_α and losses strictly exceeding VaR_α (see Rockafellar and Uryasev 2000).
- CVaR is a coherent risk measure (Pflug 2001).
- Minimizing CVaR of a portfolio is closely related to minimizing VaR, as already observed from the definition of these measures.
- The main advantages of CVaR as compared to VaR are that it takes into account the size of losses and does not distort the risk exposure at portfolio level.

The second feature for determining the risk profile of an individual investor are her risk preferences, that are referred to two aspects:

- risk aversion (curvature of the utility function): the higher the risk aversion, the higher the required expected return for a unit increase in risk, and
- loss aversion: the value function (Kahneman and Tversky 1979) is steeper for losses than for gains. Individuals making a loss of 100 m.u. need to gain more than 100 m.u. as compensation. The higher an individual's aversion to losses, the higher the required compensation.

Lastly, the risk awareness refers to the perception of probabilities for gains and losses (for details see Tversky and Kahneman 1992).

4 Our proposal

The aim of this work is to integrate the construction of a portfolio based on the BPT-MA framework with the SRI approach. Within MCDM approach, GP is a technique for solving multi-objective problems in which the DM expresses her preferences by determining an aspiration level (or target) for each objective. The preference functional is the average level of achievement of a set of goals and its algebraic expression is constructed by summation of the lack of achievement of the goals. Thus, the feasible solutions with minimal distance from the aspiration levels are sought.

Our proposal designs a GP model with goals relative to each mental account. So, the preference structure is defined by setting the desired financial features for each account as well as a SR-value of the aggregate allocation across MA subportfolios. In addition, preference weights can be attached to each goal. The financial preferences of the investor are specified directly in percentile terms, instead of the more classical approach, which defines risk preferences in terms of utility functions. According to Krokhmal et al. (2002) approaches based on values limiting the worst losses provide efficient and flexible risk management tools. Furthermore, the allocation among mental accounts should be determined by taking care of the investor's profile and to carry out this task we use a fuzzy inference system (FIS). We apply FIS for finding the allocated budget for each mental account and its confidence level according to needs/status/preferences of the investor and the expertise knowledge of the financial advisor team. Therefore, our methodological bases are: BPT-MA that is used as a choice theory tailored to the decision making process of actual investors, the GP approach that provides a modeling for handling conflicting objectives and fuzzy logic that is a tool for solving imprecise phenomena involving human judgement.

4.1 GP model for constructing mental-account portfolios with SRI features

4.1.1 Establishing the objectives

Our setting is that the investor wishes to find an optimal financial portfolio in each MA so that the obtained aggregated portfolio meets several SR characteristics. The optimality of each MA-portfolio is measured in terms of expected return and risk. We use the CVaR as a downside risk measure in each mental account, therefore two parameters define the risk performance of each mental account: the confidence level α and the value of the $CVaR_\alpha$.

For N investment instruments and assuming that the confidence level (α_j) for the mental account j has been chosen, we present the following multi-criteria model with $2k$ objectives concerned with the two financial features of the k mental accounts: CVaR and expected return (ER), and the maximizing objective of SR-value of the aggregated portfolio:

$$\left.\begin{array}{l} min\ \{CVaR_{\alpha_j}(MA_j);\quad j=1,2,\ldots,k\} \\ max\ \{ER(MA_j);\quad j=1,2,\ldots,k\} \\ max\ SR\left(\sum_{j=1}^{k} MA_j\right) \\ s.t. \\ \sum_{i=1}^{N} x_i^{MA_j} = b_j,\quad j=1,2,\ldots,k, \\ x_i^{MA_j} \geq 0,\quad j=1,2,\ldots,k;\quad i=1,2,\ldots,N, \end{array}\right\} \tag{1}$$

where $MA_j = (x_1^{MA_j}, \ldots, x_N^{MA_j})$ denotes the subportfolio of the mental account j, being $x_i^{MA_j}$ the subportfolio weights, and b_j is the proportion allocated to this mental account. The non-negativity of the variables is included in case the short sales are not allowed. The aggregated portfolio is obtained as

$$\sum_{j=1}^{k} MA_j = \left(\sum_{j=1}^{k} x_1^{MA_j}, \ldots, \sum_{j=1}^{k} x_N^{MA_j}\right). \tag{2}$$

Most of the literature about SR measures of financial portfolios (see Drut 2010; Bilbao-Terol et al. 2012a; Dorfleitner and Utz 2012) assumes that these measures are linear. This linearity hypothesis is often used by practitioners to SR-rate financial indices too. Consequently, the SR-value of a portfolio, is given by aggregating the SR-value of its component assets:

$$SR\left(\sum_{j=1}^{k} MA_j\right) = \sum_{i=1}^{N}\sum_{j=1}^{k} sr_i x_i^{MA_j} \tag{3}$$

where sr_i denotes the SR-value of asset i. Several proposals have been carried out for determining the SR measure of an asset, from simple measures, based on a binary variable showing the SR or non-SR character of the investment instrument, to more sophisticated approaches. In most of them the data provided by SR rating agencies (Inrate, Vigeo, KLD, etc.) are used. Dorfleitner and Utz (2012) propose stochastic sustainability returns. To compute the sustainability returns, they use annual ESG scores from the sustainability rating agency Inrate for all companies in their rating universe. These ESG scores consist of a high number of indicators, which score the efforts of each company in several fields of environmental protection, social issues and corporate governance issues. Bilbao et al. (2012b) present an index called 'SRI-Attractiveness' that summarizes the SR characteristics of each SR mutual fund according to the investor's preferences. Basso and Funari (2014) use information from Vigeo SRI Research (2009) in order to construct an ethical measure. Their approach is based on the number of negative and positive screening features presented by each fund and the (eventual) presence of an ethical committee. Indeed, our modeling allows us to work with any measure sr_i defined for evaluating the SR quality of investment instrument i.

Rockafellar and Uryasev (2000) proposed a methodology for minimizing CVaR that will be used in this work and briefly described below.

Let $L(x, y)$ be the loss associated with the portfolio $\boldsymbol{x}$ and the random vector y (e.g., market prices or rates of return of the instruments) and any specified confidence level α. Rockafellar and Uryasev (2000) have proven that:

$$CVaR_\alpha(x) = \min_{\xi} F_\alpha(x, \xi) \tag{4}$$

being $F_\alpha(x, \xi) = \xi + (1-\alpha)^{-1} \int_{y \in \Re^n} [L(x, y) - \xi]^+ p(y)dy$, $[t]^+ = max(t, 0)$ and $p(y)$ the density of vector y.

Rockafellar and Uryasev (2000) have also proven that:

$$\min_{\mathbf{x}} CVaR_\alpha(x) = \min_{\mathbf{x},\xi} F_\alpha(x, \xi) \tag{5}$$

The integral in $F_\alpha(x, \xi)$ can be approximated by sampling the distribution function of y. Let $y_1, \ldots, y_J$ be the collection generated by sampling, then the following approximation is obtained for the function $F_\alpha(x, \xi)$:

$$F_\alpha(x, \xi) \cong \xi + (1-\alpha)^{-1} \sum_{j=1}^{J} \pi_j z_j \tag{6}$$

with

$$\left.\begin{array}{l} z_j \geq L(x, y_j) - \xi, \quad j = 1, \ldots, J \\ z_j \geq 0, \quad \xi \in \Re \end{array}\right\} \tag{7}$$

where z_j are auxiliary variables and π_j the probabilities of scenarios j. Joining (5) with the approximation defined by (6) and (7) allows for minimizing $CVaR_\alpha(x)$ using linear techniques.

4.1.2 *Goal programming model*

The GP approach replaces each objective to be optimized for a goal to be attained. Model (1) is solved using the GP technique. To do so, the aspiration levels of ER and CVaR for each mental account should be set as well as the level of SR for the aggregated portfolio. The formulation of the GP model to be solved is:

$$\left.\begin{array}{l} min \quad \left(\sum_{j=1}^{k} u_j \frac{n_{er_j}}{f_{er_j}} + v_j \frac{p_{cvar_j}}{f_{cvar_j}} \right) + w_{sr} \frac{n_{sr}}{f_{sr}} \\ s.t. \\ \quad ER(MA_j) + n_{er_j} - p_{er_j} = ER_j^*, \quad j = 1, 2, \ldots, k, \\ \quad CVaR_{\alpha_j}(MA_j) + n_{cvar_j} - p_{cvar_j} = CVaR_j^*, \quad j = 1, 2, \ldots, k, \\ \quad SR\left(\sum_{j=1}^{k} MA_j \right) + n_{sr} - p_{sr} = SR^*, \\ \quad \sum_{i=1}^{N} x_i^{MA_j} = b_j, \\ \quad n_l, p_l \geq 0, \qquad l \in \{er_j, cvar_j, sr\} \\ \quad x_i^{MA_j} \geq 0, \;\; j = 1, 2, \ldots, k; \;\; i = 1, 2, \ldots, N, \end{array} \right\} \tag{8}$$

the asterisk denotes the target for each objective. The deviation variables are denoted by n_l and p_l, negative and positive, respectively, u_l, v_l and w_l are the preferential weights associated with the minimization of the corresponding unwanted deviation variable and f_l are normalizing factors. The preference weights are used to model the relative importance of the minimization of the associated deviation variable for the investor.

The k first constraints of this model stand for that the expected return of the subportfolio MA_j associated with mental account j should be greater than or equal to its target ER_j^*, hence the negative deviation variable n_{er_j}, representing the expected return below the target, should be minimized. Analogously, the k following constraints are referred to the desired risk level, which is measured by CVaR, in each mental account. CVaR is associated to an upper percentile of the loss distribution. Notice that according to BPT-MA investors have multiple reference points, below each reference point an investment is considered a loss. In model (8) it is possible to set a different reference point on each mental account, this involves a different loss variable on each mental account. The most usual reference point is the initial budget, (or return equal to 0), in this case the loss function over the period is defined as the difference between initial and final portfolio values, (or the opposite of the final return). Another possible reference point is the budget multiplied by the return of a benchmark being the loss the difference between the final wealth on benchmark and final portfolio value.

Up to here the model deals with local objectives associated with the mental accounts, but the $(2k + 1)$-objective refers to a global feature of the portfolio, its SR-value. As has been discussed previously, the SR-value could be the proportion of the budget invested in SR assets or a value obtained from aggregation of some SR quality index of the investment instruments.

4.1.3 Determining the aspiration levels for the financial objectives: efficient frontiers

For designing any GP-model the DM should establish the aspiration level for each objective. In model (8) the target for the SR quality of the portfolio is a subjective choice of the investor according to her ESG concerns. However, for establishing the aspiration levels for the financial objectives, in addition to the preferences of the investor, the performance of the given investment universe has to be analyzed. For example, high expected returns or very small values of risk tolerance could not be attained with the chosen set of instruments. Also, achieving particular combinations of values of the CVaR, confidence levels, and ER may not always be feasible with a given set of assets. In order to obtain the targets for the financial objectives, it is necessary to evaluate the trade-off between ER and CVaR on each

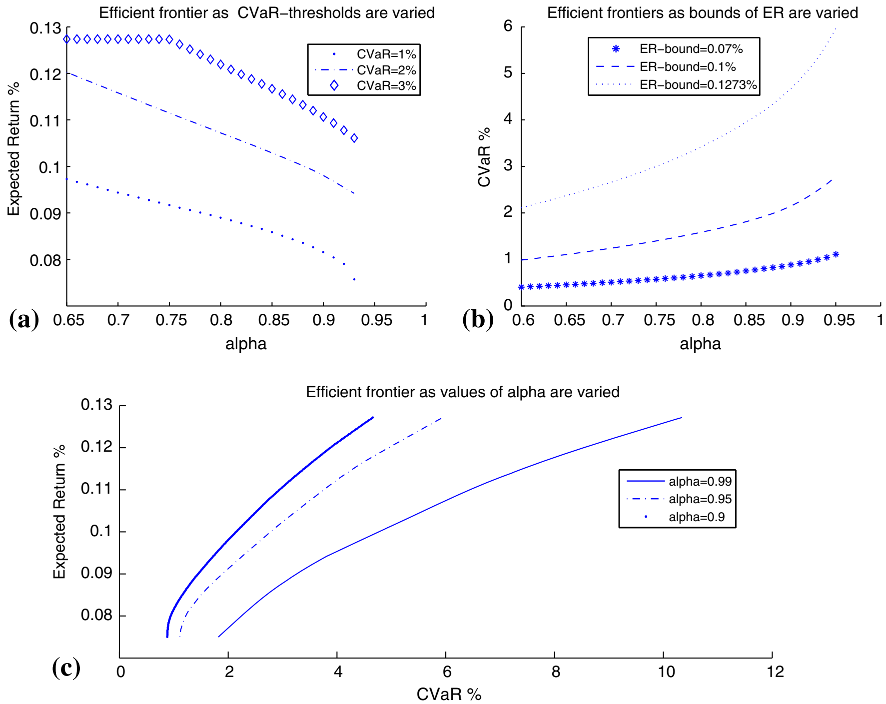

Fig. 2 Efficient frontier of a mental account

mental account. The efficient frontier concept was introduced in Modern Portfolio Theory (Markowitz 1952) for the trade-off between expected return and variance. Applied in BPT-MA, this tool admits several versions. The efficient frontier of one mental account, when a threshold level for the CVaR has been fixed, reflects the trade-off between the expected return (ER) and the confidence level (α) to reach this threshold. Also, an efficient frontier with the ER level fixed and reflecting the trade-off between the level confidence and the CVaR value could be used. Lastly, fixing the value for α it is possible to generate the efficient frontier between CVaR and ER.

Figure 2 shows the three forms of efficient frontiers varying in each case the values of the fixed parameter. The upper-left plot shows MA frontiers with fixed threshold levels for CVaR with the confidence level to reach the threshold on the x-axis, and the expected return on the y-axis. For each level of the threshold CVaR a different MA frontier is obtained. As thresholds of CVaR increase, we shift from the lowest frontier to the highest one. The same occurs for the upper-right plot where bounds for the ER have been fixed. However, the lower plot shows an opposite performance, the lowest frontier is obtained for the highest value of α. Furthermore, this plot displays how higher values of CVaR allow for achieving higher expected returns. With respect to the meaning of CVaR, in Fig. 2c, setting the value of CVaR equal to 1 % and $\alpha = 0.9$ implies that the average loss in 10 % of the worst cases must not exceed 1 % of the initial portfolio value.

In our proposal the efficient frontier between CVaR and ER fixing the value for α is used. Even though simple adaptations of model (8) can also give rise to the other variants of efficient frontiers. The efficient frontier of the j-th mental account with α_j fixed is obtained by solving the following bi-objective problem:

$$\left.\begin{array}{l} max\ ER(\mathbf{x}) = \sum_{i=1}^{N} \mu_i x_i \\ min\ CVaR_{\alpha_j}(\mathbf{x}) \\ s.t \\ \sum_{i=1}^{N} x_i = 1 \\ x_i \geq 0 \end{array}\right\} \tag{9}$$

where μ_i is the ER of asset i. This bi-objective problem can be handled by solving a collection of problems minimizing CVaR with constraint for the level of ER.

$$\left.\begin{array}{l} min\ CVaR_{\alpha_j}(\mathbf{x}) \\ s.t \\ \sum_{i=1}^{N} x_i = 1 \\ ER(\mathbf{x}) \geq h \\ x_i \geq 0 \end{array}\right\} \tag{10}$$

The knowledge of the efficient frontier allows the investor to choose the combination of CVaR and ER according to her preferences.

One last question to be solved in order to have specified all the elements of the model (8) is to determine the b_j, i.e., the proportion allocated to each mental account. To do so, we propose to use fuzzy inference systems (Mamdani and Assilian 1975; Takagi and Sugeno 1985; Jang 1993; Pais and Amaral 2012).

4.2 Model for risk profiling: fuzzy inference system

We design a support system for allocating the investment budget of the investor among the k mental accounts ($MA_1, MA_2, \ldots MA_k$) according to her risk ability, risk preferences and risk awareness.

Therefore, the system has three input variables related to the *Risk Ability*, the *Risk Preferences* and the *Risk Awareness* and k output variables (**MA1**, **MA2**, . . ., **MAk**) that represent the budget allocation to each mental account. A mental account is set by the confidence level and the threshold for the CVaR.

According to available information we will use Mamdani-FIS or an ANFIS with Sugeno-FIS (see Figs. 3, 4). The former will be used when a rule base from experts is available. Below we summarize the steps of fuzzy reasoning.

In the first step, often called fuzzification, the input variables (x and y in Fig. 3) are compared with the membership functions (A_i and B_i in Fig. 3) on the premise part to obtain the membership values (or compatibility measures) of each linguistic label.

In the second step, through a specific T-norm operator, usually multiplication or minimum, the membership values on the premise part are combined to get firing strength (weight) of each rule.

In the third step, the qualified consequent, fuzzy in Mamdani-type (C_1' and C_2' in Fig. 3) and crisp in Sugeno type (z_1 and z_2 in Fig. 4) of each rule is generated depending on the firing strength.

In the last step, the qualified consequents to produce a crisp output are aggregated (this step is called defuzzification).

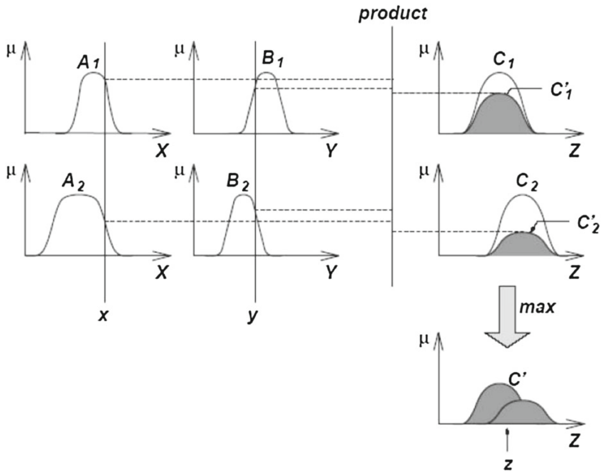

Fig. 3 Fuzzy inference systems (FIS): Mamdani type

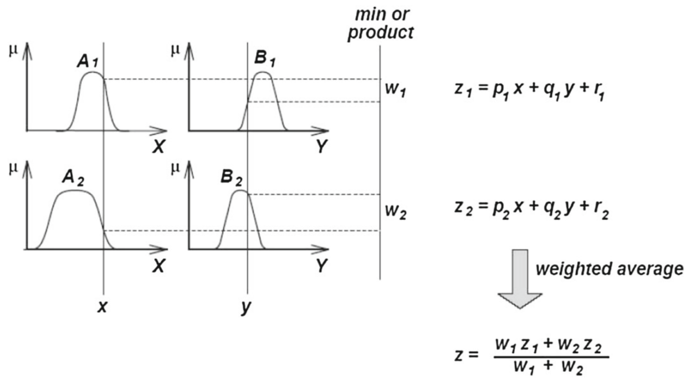

Fig. 4 Fuzzy inference systems (FIS): Sugeno type

Fig. 5 The proposed model scheme

In the case that the information is in the way of a set of examples we propose an Adaptive-Network-based on Fuzzy Inference System, or simply (ANFIS), with first order Sugeno fuzzy reasoning (Fig. 4). A suitable set of input-output data pairs allows us to identify the premise and the consequent parameters.

In short, the model works according to the following scheme (Fig. 5):

Firstly, the particular situation of the investor is obtained from a questionnaire elaborated by the financial manager. This information is gathered as scores for the three input variables determining the client's risk profile. Then, the chosen fuzzy inference system provides the required outputs: the share to be invested in each layer of investment. As mentioned, the expert knowledge could be supplied in two ways: (i) as if-then fuzzy rules from master portfolios, or (ii) from a well diversified set of examples for designing the parameters of the rules.

5 Example of application

For the sake of illustration, let us consider an example depicting the introduced methodology. The steps to be implemented are:

1. Set the investment universe.
2. Establish the characteristics of each Mental Account by:
    i) Constructing a FIS from expert financial knowledge. This FIS should be tested by the management team in order to validate its goodness. This process must be carried out only once and may be used by any investor to determine the invested proportion in each mental account and its corresponding confidence level.
    ii) Applying the obtained Inference System in i) to the specific investor.
    iii) Determining the risk profile of each MA taking into account both the special financial situation of the investor and the financial characteristics of the investment universe. In the case that the chosen set of investment instruments does not meet the desired pairs return-risk, such a set should be modified.
3. Solve the GP model with the parameters determined in the above steps and the desired SR level of the portfolio.
4. Feedback with the investor. The investor is required to accept the found solution, otherwise parameters in the GP model may be changed.

5.1 Setting the investment universe

As investment instruments we have chosen four broad market indexes, two SR and two conventional ones:

1. 'Ethical Euro Government Bond TR EUR' (SRI-Bonds). The Index tracks fixed-rate, investment grade EUR denominated sovereign bonds issued by European Monetary Union countries which are eligible investments according to ECPI Government and Supranational Screening Methodology.
2. 'FTSE All World TR USD' (Conventional-Stocks). The FTSE All-World Index Series is the Large/Mid Cap aggregate of 2,800 stocks from the FTSE Global Equity Index Series. It covers 90–95 % of the investable market capitalisation.
3. 'FTSE4Good Global 100 PR USD' (SRI-Stocks). The FTSE4Good Index Series has been designed to objectively measure the performance of companies that meet globally recognized corporate responsibility standards. Transparent management and criteria make FTSE4Good a valuable tool for consultants, asset owners, fund managers, investment banks, stock exchanges and brokers when assessing or creating responsible investment products.
4. 'FTSE Gilts All Stocks TR GBP' (Conventional-Bonds). This index is among the industry's most widely-used performance benchmarks for the UK Government bond market. They are sanctioned by the UK Actuaries and used for benchmarking pension benefits and obligations and mutual funds.

We have used weekly returns from December 26th 2004 to January 28th 2012, that have been obtained from the *Morgninstar Direct* web-based research platform. In this application the SR of the portfolio is measured in the most simple way as the total proportion invested in SR assets.

5.2 Establishing the characteristics of the mental accounts

In our modeling the investor allocates her investment budget among MAs. Here we have divided the amount of money to be invested in three MAs corresponding to three risk levels: low, medium and high. The low risk MA will have a high probability (usually between 99 and 90 %) of not exceeding the threshold of average loss along with a small value of this threshold. Decreasing the confidence level and increasing the corresponding threshold give the parameters of more risky MAs.

5.2.1 Constructing a fuzzy inference system (FIS)

In this application we have used two input variables: *Risk Ability* and *Risk Tolerance*, where the latter is the logical conjunction of *Risk Preferences* and *Risk Awareness*, handled with the minimum operator. First we have to design a support system for allocating the investment budget of the investor among the three mental accounts (MA_1, MA_2 and MA_3, sorted from lowest to highest risk) according to her risk ability and risk tolerance. To estimate the investor's risk profile we use a questionnaire proposed by the former Bank Leu, now Clariden Leu (see Hens and Bachmann 2008, pp. 110–113). This questionnaire comprises 6 questions that address the needs analysis, the investment horizon, the risk ability, the aspiration level and the loss aversion of the investor. The client's risk ability is checked by asking for events in which she would need to get the invested money back (questions 4 and 5) and her needs (question 6). The score for risk ability is obtained by the scheme contained in Fig. 6 and considering the minimum value.

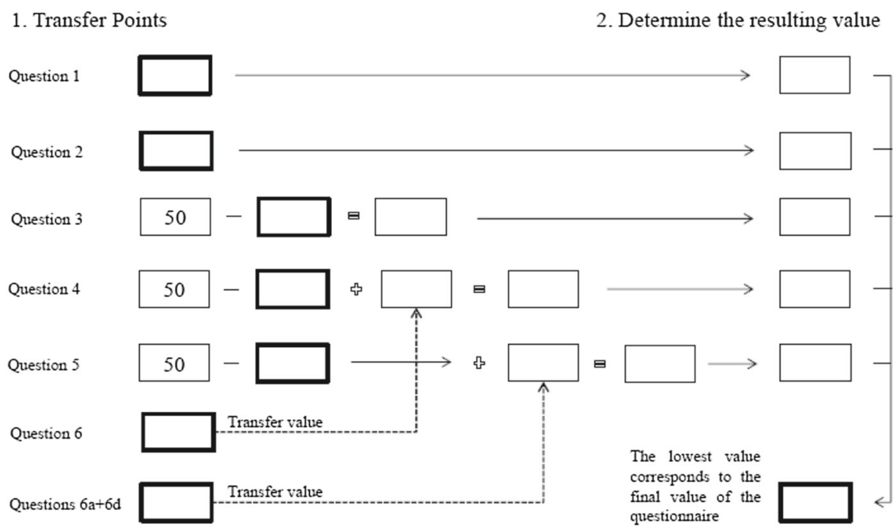

Fig. 6 Risk ability (*source*: Bank Leu)

The risk tolerance is evaluated by questions 1, 2 and 3 (see Hens and Bachmann 2008, pp. 110–113). The first one determines the risk awareness and the investor's aspiration level, question 2 is about the investor's loss aversion and the last question determines the investment horizon, an important issue for the valuation of the investor's risk tolerance. The scoring of this question is subtracted from 50. The minimum value of the questions is assigned to the investor's risk tolerance.

Each linguistic variable involved in the FIS will have associated a fuzzy partition of its domain representing the fuzzy set associated with each of its linguistic labels. Each fuzzy set defines a part of the domain of the variable. But this part is not uniquely defined. Fuzzy sets overlap as a natural consequence of their elastic boundaries. Such an overlap implements a realistic and functional semantic mechanism for defining the nature of a variable when it assumes various data values. The use of overlapped linguistic labels gives smoothed results with soft transitions. For the input variables, *Risk Ability* and *Risk Tolerance* we have set three linguistic labels (Low, Medium, High); the support set of these being [0, 80] and [0, 50], respectively. The linguistic labels are modeled by triangular or trapezoidal membership functions. In the case of *Risk Ability* we have the following:

$$\mu_{Low}(x) = \begin{cases} 1 & \textit{if} \quad x \leq 15 \\ \frac{24-x}{9} & \textit{if} \quad 15 \leq x \leq 24 \end{cases} \tag{11}$$

$$\mu_{Me}(x) = \begin{cases} \frac{x-20}{17} & \textit{if} \quad 20 \leq x \leq 37 \\ \frac{55-x}{18} & \textit{if} \quad 37 \leq x \leq 55 \end{cases} \tag{12}$$

$$\mu_{High}(x) = \begin{cases} \frac{x-50}{10} & \textit{if} \quad 50 \leq x \leq 60 \\ 1 & \textit{if} \quad 60 \leq x \end{cases} \tag{13}$$

Analogously, the triangular or trapezoidal membership functions are defined for the *Risk Tolerance* (see Fig. 8 in Appendix). For the output variables, **MA1**, **MA2** and **MA3**, the budget of each MA, we have chosen five linguistic labels (Very Low, Low, Medium, High, Very High) modeled by triangular or trapezoidal membership functions, in the same way as for the input variables. The support is the closed interval [0, 100] in all cases (see Fig. 9 in Appendix).

For eliciting the values of the three output variables we propose two alternative rule bases called *aggressive* and *cautious*. As can be seen in Table 1, in the aggressive case a *Risk Ability* 'High' can offset a *Risk Tolerance* 'Low' and give as a result of the inference a 'Low' investment in the mental account of low risk-low return (MA_1) and a 'High' investment in the mental account of high risk-high return (MA_3).

In the same way, in the cautious case (see Table 2) a *Risk Ability* 'High' and a *Risk Tolerance* 'Low' give as result of the inference a 'High' investment in the mental account of low risk-low return (MA_1) and a 'Low' investment in the mental account of high risk-high return (MA_3). In this case, knowledge based on a conservative view (minimum operator) has been used by the financial manager.

5.2.2 *Applying the FIS to a particular investor*

Now we are going to apply these rules to a particular investor with the following personal and financial characteristics.
With regard to *Risk Ability*:

- The investor is 35 years old and she is a salaried employee without children.
- She wants to purchase a house for an attractive price using the entire amount of the sum invested (10 points in question 4).

Table 1 Inference rules: aggressive case

	Low	*Medium*	*High*
Risk Ability and Risk Tolerance → *MA1*			
Low	**Very High**	**Very High**	**Very High**
Medium	**High**	**Medium**	**Medium**
High	**Low**	**Very Low**	**Very Low**
Risk Ability and Risk Tolerance → *MA2*			
Low	**Low**	**Low**	**Medium**
Medium	**Medium**	**Medium**	**Medium**
High	**Medium**	**High**	**High**
Risk Ability and Risk Tolerance → *MA3*			
Low	**Very Low**	**Very Low**	**Very Low**
Medium	**Low**	**Medium**	**Medium**
High	**High**	**Very High**	**Very High**

Table 2 Inference rules: cautious case

	Low	*Medium*	*High*
Risk Ability and Risk Tolerance → *MA1*			
Low	**Very High**	**Very High**	**Very High**
Medium	**High**	**Medium**	**Medium**
High	**High**	**Low**	**Very Low**
Risk Ability and Risk Tolerance → *MA2*			
Low	**Low**	**Low**	**Medium**
Medium	**Medium**	**Medium**	**Medium**
High	**Medium**	**High**	**High**
Risk Ability and Risk Tolerance → *MA3*			
Low	**Very Low**	**Very Low**	**Very Low**
Medium	**Low**	**Medium**	**Medium**
High	**Low**	**High**	**Very High**

- She owns property and the proportion of her financial investments to her total assets is less than 10 % and the proportion of her investment assets to her annual income is between 50 and 80 % (40 points in question 6).
- She does not need the sum invested for any reasons in everyday life (0 points in question 5).
- Hence she obtains a scoring of 80 points with respect to her *Risk Ability*.

The information about the *Risk Tolerance* is:

- Her investment horizon is 6 years (45 points in question 3).
- She has invested in a stock market, expecting to get 10 % p.a., she was aware that with the same probability she could have lost up to 5 % in over five years (10 points in question 1).
- Hence she obtains a scoring of 10 points with respect to her *Risk Tolerance*.

For the sake of illustration we present the running of Aggressive FIS when the input data (or readings) are *Risk Ability* $= 80$ and *Risk Tolerance* $= 10$.

– *First step: fuzzification* The input variables (*Risk Ability* = 80 and *Risk Tolerance* = 10) are compared with the membership functions μ_{Low}, μ_{Me} and μ_{High}, (in (11), (12) and (13), respectively, and Fig. 8 in Appendix) on the premise part to obtain the membership values (or compatibility measures) of each linguistic label.

$$\begin{aligned} &\mu_{Low}(80) = 0, \quad \mu_{Me}(80) = 0, \quad \mu_{High}(80) = 1, \\ &\mu_{Low}(10) = \frac{5}{7}, \quad \mu_{Me}(10) = 0, \quad \mu_{High}(10) = 0. \end{aligned} \tag{14}$$

– *Second step: firing strength of each rule*
Through a specific T-norm operator, in this case minimum, the membership values on the premise part are combined to get firing strength (weight) of each rule. For the rules included in Table 1 with consequence **MA1** (top decision table), we have:
Rule (1,1): "If Risk Ability is *Low* and Risk Tolerance is *Low* then **MA1** is **Very High**"

$$\alpha_{11} = \mu_{Low}(80) \wedge \mu_{Low}(10) = min\left\{0, \frac{5}{7}\right\} = 0 \tag{15}$$

Analogously, rules from (1, 2) to (2, 3) give values equal to zero because the membership value of the reading 80 is zero for the two labels *Low* and *Medium*.
Rule (3, 1): "If Risk Ability is *High* and Risk Tolerance is *Low* then **MA1** is **Low**"

$$\alpha_{31} = \mu_{High}(80) \wedge \mu_{Low}(10) = min\left\{1, \frac{5}{7}\right\} = \frac{5}{7} \tag{16}$$

The remainder rules in row 3 are not active because $\mu_{Me}(10) = 0$ and $\mu_{High}(10) = 0$.

– *Third step: obtaining qualified consequent*
The qualified consequence of each rule is generated depending on the firing strength:

$$\mu(z) = min\{5/7, \mu_{MA1-Low}(z)\} \tag{17}$$

where the membership function of the label *Low* for the consequent **MA1** is as follows:

$$\mu_{MA1-Low}(z) = \begin{cases} \frac{z-5}{10} & if \quad 5 \le z \le 15 \\ \frac{25-z}{10} & if \quad 15 \le z \le 25 \end{cases} \tag{18}$$

The membership function qualified is obtained by truncating of the shape of $\mu_{MA1-Low}(z)$ with the straight line parallel to z-axis $\mu = 5/7$:

$$\mu^q_{MA1-Low}(z) = \begin{cases} \frac{z-5}{10} & if \quad 5 \le z \le \frac{85}{7} \\ \frac{5}{7} & if \quad \frac{85}{7} \le z \le \frac{125}{7} \\ \frac{25-z}{10} & if \quad \frac{125}{7} \le z \le 25 \end{cases} \tag{19}$$

– *Four step: defuzzification.*
The qualified consequents to produce a crisp output are aggregated and a process of defuzzification is carried out. This process produces a nonfuzzy decision, z^*, that adequately represents the aggregated fuzzy decision. There are several existing methods for defuzzification (van Leekwijck and Kerre 1999), here we work with the center of gravity method:

$$z^* = \frac{\int z\mu(z)dz}{\int \mu(z)dz} \tag{20}$$

that produces a value equal to 15 when it is applied to $\mu^q_{MA1-Low}(z)$. Therefore, for the studied case the output resulting for **MA1** is equal to 15. Analogously, the outputs for

MA2 and **MA3** (in Table 1) are 35 and 60, respectively (for graphic details, see Fig. 10 in Appendix). A simple proportional sharing gives the following allocation between the three mental accounts for the client whose *Risk Ability* is 80 and *Risk Tolerance* is 10:

- 14 % of the budget should be allocated to the first mental account MA_1,
- 32 % should be allocated to the second one MA_2, and
- 54 % of the investor's wealth should be allocated to the third one MA_3.

Other results obtained applying the two proposed FISs are set out below.

In the cautious case, with the same input variables, the allocated wealth among the three mental accounts are 54, 32, and 14 %, respectively. And as can be seen, these allocations are dual with regard to MA_1 and MA_3.

– If we change the score for *Risk Tolerance* to 20 in the cautious case, the allocated wealth among the three mental account is 18, 41 and 41 %, that is to say, the investor has now a higher risk tolerance and therefore she is willing to invest more in the upper layers.

5.2.3 Determining the parameters of risk for each MA

Each one of the MAs has different risk profiles. Two desired features define each MA: the confidence level and the value of the CVaR. For eliciting the values for these two parameters it is necessary to take into account both the financial situation of the investor and the features of the set of investment instruments.

Once solved the question of the allocation among the three mental accounts, it is also possible to use a set of inference rules for determining the value of the confidence level of each MA. For example, working with the nine rules of the cautious rule base associated to the MA_1, the values α_1, α_2 and α_3 corresponding to MA_1, MA_2 and MA_3 can be obtained instead of the allocated proportion of each MA. To do this, the single modification that should be done is the one relative to the support of the value of α assigned to each MA (see details in Fig. 11 in Appendix), these parameters should be determined by the financial expert. The resulting confidence levels are 91.3, 80.5 and 70.3 % for the three MAs (rounded to 91, 80 and 70 %, respectively). Notice that calculating the value of confidence level follows the same patterns as calculating the proportion of MA_1, for example, a client with *Risk Ability* and *Risk Tolerance* low would need high confidence levels.

With the amount of money invested in each MA and its confidence level determined, what is just missing is the setting of a threshold for CVaR to define each MA. To do this, we approximate the CVaR-ER efficient frontier of each mental account setting the confidence level in MA_1, MA_2 and MA_3, equals to $\alpha = 0.91$, $\alpha = 0.8$ and $\alpha = 0.7$, respectively. For the first MA, the least risky one, a pair (CVaR, ER) on the low zone of its efficient frontier (Fig. 7a) should be chosen. For the MA_2 an intermediate zone of its efficient frontier (Fig. 7b) should be chosen and pairs (CVaR, ER) on the top portion of the efficient frontier (Fig. 7c) would be candidates for the third MA. In Table 3 appear the three triplets: confidence level, aspiration levels for the expected return and for the conditional value at risk for each MA. With these values the GP-model will be solved. Notice that according to the GP philosophy the aspiration levels are desired values for the objectives whose non-achievements do not make the model to be infeasible. Highly demanding values for the aspiration level of an objective produces a solution to the GP model with good performance on this objective, perhaps to the expense of others. In this numerical example, the maximum rate of return that can be achieved for the given set of instruments equals 0.1277 % over one week. Once the above steps are completed, the GP model can be formulated.

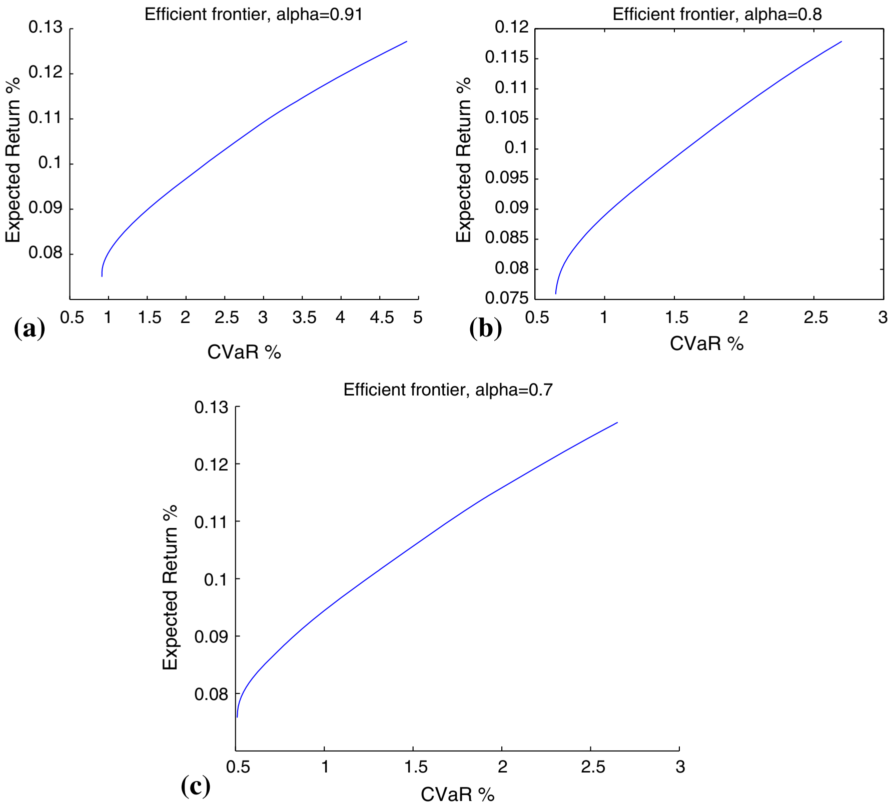

Fig. 7 Efficient frontier for MA_1, MA_2 and MA_3

Table 3 Parameters for the GP model

	α	CVaR* (%)	ER* (%)
MA_1	0.91	0.75	0.08
MA_2	0.8	1.5	0.1
MA_3	0.7	2.65	0.13

5.3 Solving the GP model

At this point we tackle the construction of the aggregated portfolio by joining the subportfolios designed according to the particular preferences of the investor on each mental account. We have solved the GP model[2] for three cases concerning the proportion of SR-assets (remember that the indexes I_1 and I_3 are SR ones), with allocations $b_1 = 18\,\%$, $b_2 = 41\,\%$, $b_3 = 41\,\%$ for the MAs and the parameters shown in Table 3 (the asterisk denotes the aspiration levels). In this example (see efficient frontiers around the targets Table 7 in Appendix), the targets fixed for MA_1 show a more demanding value of CVaR than the value of ER. The opposite occurs for MA_3 and the targets for MA_2 are balanced. In real situation, for choosing these targets a collaboration between the financial advisor and the investor is needed. The weighted goal programme is given as

[2] Using MATLAB R2013.

$$
\left.
\begin{array}{l}
min \quad \left(\sum_{j=1}^{3} \frac{n_{er_j}}{f_{er_j}} + \frac{p_{cvar_j}}{f_{cvar_j}} \right) + \frac{n_{sr}}{f_{sr}} \\
s.t. \\
\quad \sum_{i=1}^{4} \mu_i x_i^{MA_j} + n_{er_j} - p_{er_j} = ER_j^*, \quad j = 1, 2, 3, \\
\quad \xi_j + \frac{1}{J(1-\alpha_j)} \sum_{h=1}^{J} z_h^j + n_{cvar_j} - p_{cvar_j} = CVaR_{\alpha_j}^*, \quad j = 1, 2, 3, \\
\quad z_h^j \geq - \sum_{i=1}^{4} \mu_{ih} x_i^{MA_j} - \xi_j, \quad j = 1, 2, 3,\ h = 1, \ldots, J, \\
\quad \sum_{j=1}^{3} (x_1^{MA_j} + x_3^{MA_j}) + n_{sr} - p_{sr} = SR^*, \quad \sum_{i=1}^{4} x_i^{MA_j} = b_j \quad j = 1, 2, 3, \\
\quad x_i^{MA_j}, z_h^j \geq 0, \quad i = 1, \ldots, 4, \quad j = 1, 2, 3, \quad h = 1, \ldots, J, \\
\quad n_l, p_l \geq 0 \quad\quad l \in \{er_j, cvar_j, sr\}
\end{array}
\right\}
\tag{21}
$$

where J denotes the number of scenarios (in this case, 370 historical weekly periods), μ_i is the expected return on index i, μ_{ih} is the return on index i in the scenario h. The normalizing factors are fixed to their aspiration levels (Jones and Tamiz 2010). Here each deviation is turned into a proportion away from its target level. In this application, consider that the investor regards the penalization of all unwanted deviations as equally important, but, of course, it is easy to introduce preferential weights in the model. The reference points of all mental accounts have been chosen equal to 0.

In Table 4 the results of model (21) using the parameters in Table 3 are presented. Three cases have been solved:

– **without SR goal**, in this case the goals corresponding to CVaR in accounts MA_2 and MA_3 are satisfied. As expected the account with the highest expected return is MA_3 and also the riskiest investment,

Table 4 Portfolios (18:41:41) without and with SR-goal

Mental account	I_1	I_2	I_3	I_4	CVaR (%)	ER (%)
Without SR						
MA_1	0.1662	0.0138	0	0	0.915	0.0747
MA_2	0.1123	0.1730	0	0.1247	1.5	0.0988
MA_3	0	0.4080	0	0.0020	2.65	0.1275
SR $\geq$ 40 %						
MA_1	0.1662	0.0138	0	0	0.915	0.0747
MA_2	0.1662	0.0138	0	0	0.915	0.0747
MA_3	0.1662	0.0138	0	0	0.915	0.0747
SR $\geq$ 60 %						
MA_1	0.1662	0.0138	0	0	0.915	0.0747
MA_2	0.2858	0.1242	0	0	0.991	0.0877
MA_3	0.1480	0.2620	0	0	1.639	0.1070

Table 5 Portfolios (18:41:41) without and with SR-goal: financial performance

Mental Account	ER (annualized) Without SR (%)	SR $\geq$ 40 % (%)	SR $\geq$ 60 % (%)	Losses (bps)	
MA_1	3.958	3.958	3.958	0	0
MA_2	5.268	5.104	4.663	16	44
MA_3	6.850	6.817	5.718	3	110

Table 6 Portfolios (18:41:41) without and with SR-goal: aggregated *CVaR* and *ER*

	Aggregated ER (weekly) Without SR (%)	SR $\geq$ 40 % (%)	SR $\geq$ 60 % (%)
	0.1062	0.1047	0.09326

α	Aggregated CVaR Without SR (%)	SR $\geq$ 40 % (%)	SR $\geq$ 60 % (%)	Performance ER/CVaR without SR (%)	SR $\geq$ 40 % (%)	SR $\geq$ 60 % (%)
0.7	1.553	1.532	1.004	6.839	6.834	9.285
0.8	1.991	1.971	1.2871	5.334	5.314	7.246
0.91	2.842	2.811	1.837	3.737	3.726	5.078

- **at least the 40 % should be invested in SR assets** (I_1 and I_3), also here CVaR goals are satisfied in the accounts MA_2 and MA_3, but again the ER targets are not reached. The introduction of SR goal produces a decrease in expected returns of mental accounts MA_2 and MA_3 together with a decrease in their risks.
- **at least the 60 % should be invested in SR assets**, the decrease of risk is more pronounced when a more demanding SR goal has been imposed. However, the ER values have worsened.

In these three cases the mental account MA_1 is the same. For the accounts MA_2 and MA_3, the higher the SR target the higher the proportion invested in I_1 and the lower the one invested in I_2 (the asset with the highest expected return). The expected return of the portfolio without a SR-goal overcomes those of the portfolios with a SR-goal. However, the SR-portfolios are less risky (see Table 5). The ER and the CVaR for the aggregated portfolio according to the different values of confidence level (α) are displayed in Table 6. In this table the better performance of the ER per unit of risk for the SR portfolios is shown. Therefore, in our data base, the SR investment seems to give a better risk-adjusted performance. However, for risk-taking investors the regret due to the loss of return may be too large. Our results are according to the research by Drut (2010) that shows that the additional cost for responsible investing depends essentially on the investors' risk aversion.

6 Conclusions

In today's world of globalization and interdependence, and in times of environmental and financial crisis issues such as climate change, biodiversity, human rights, business ethics and corporate governance are at the forefront of public and political attention. In fact, they are heading the agendas of many countries and supranational initiatives to drive future policies and underpin economic development. So, the world of investment is changing: ESG issues are becoming a more important part of investors' decision-making in the effort to help identify the long-term opportunities and risks for companies. SRI is regarded by a lot of financial experts, policy makers and economic and social researchers, as one of the potential solutions in order to avoid future crises. Therefore, new models for this investment approach are necessary. We try to support the class of investors that select their investments under a BPT-MA framework and they also wish to follow SR principles in their investment decisions.

This work presents an approach based on GP for aiding a SR investor that wants to manage her investments by a mental accounting structure of portfolios. We have worked jointly with three bounded rationality theories: BPT-MA, fuzzy logic and GP modeling, in order to support financial advisors to address the SR investment process when her client feels comfortable with a BPT-MA asset allocation of her wealth. In this paper an implementation of the MA theory allowing a global objective about the SR performance of the portfolio is presented. The new methodology overcomes several drawbacks presenting in other approaches. So, it is not necessary to tackle parametric problems, interaction with the financial manager is possible, coherent measure of risk is used in each MA being possible different reference points that determine the associated loss in such MAs and utility functions are replaced by satisfaction behavior. Furthermore, the model can be formulated as a linear programming problem.

Our proposal runs in two stages; firstly, for each MA-subportfolio the financial parameters are chosen regardless of the social responsibility profile of the investor. In the proposed methodology, the percentage of money invested in each MA as well as the confidence levels are generated by a FIS. The advantage of using a risk profile is improved by its implementation into Expert Systems such as the FISs. This technique provides to the financial practitioners a tool for recommending asset allocations that match the investor's needs and investment objectives. The investor feels more comfortable with the received advice because the conclusions obtained from a FIS can be explained to users. Furthermore, the possibility of accumulating expert knowledge in order to design the suitable FIS could avoid biases of the financial manager and, therefore, the developed model provides more consensual advices. In the second stage, a GP model is proposed for reconciling the financial targets on each MA with the one associated to the SR of the aggregated portfolio. With the MA-GP framework developed here it is possible to impose one specified pair (CVaR, ER) goal with its associated confidence level in each MA. This allows us to shape the loss distribution according to the preferences of the investor on each MA. Therefore, the investor's preferences are expressed in terms of aspiration levels for the performance measures of her investment. In this way, the difficult of eliciting a proper utility function is overcome. Furthermore, the financial manager does not need to address the allocation of SR preferences among the different MAs. The presented procedure allows us, as far as possible, to keep the investor's financial preferences on each mental account simultaneously reaching a SR target.

A possible extension of the model will rely on the handling of the allocated proportions attached to each MA as goals of the model in addition to introduce goals relative to overall risk and return. In this case, the division of the investable wealth across the subportfolios by the FIS decision support would determine targets, and the model could be extended by relaxing the fulfillment of this partition. So, transforming from constraints into goals may introduce flexibility in the model.

A future research will tackle the Arrovian versus Schumpeterian character of the SRI. A deep study is necessary in order to translate the conclusions, about this debate, obtained in other fields to the SRI one.

We consider that the investors under Shariah guidelines could be interested in several features of BPT-MA approach. All referred differences between Islamic and standard SRI funds are very crucial in our model because the low aspiration level mental accounts usually are associated with debt-bearing instruments and very speculative instruments are used in the highest layer. However, we consider that it is possible to extend our proposal in order to work with Islamic funds. To do this, several modifications should be addressed due to mainly the standard low and high mental accounts could be not permitted by Shariah. We consider that the typical pyramid of the MA-subportfolios approach should be truncated and the middle mental account stratified by Shariah compliant equities of different levels of risk.

Acknowledgements We would like to thank the anonymous referees and the Editor for their valuable comments and suggestions. The authors wish to gratefully acknowledge the financial support provided by the Spanish Ministry of Education, Project ECO2011-26499.

Appendix

See Figs. 8, 9, 10, 11 and Table 7.

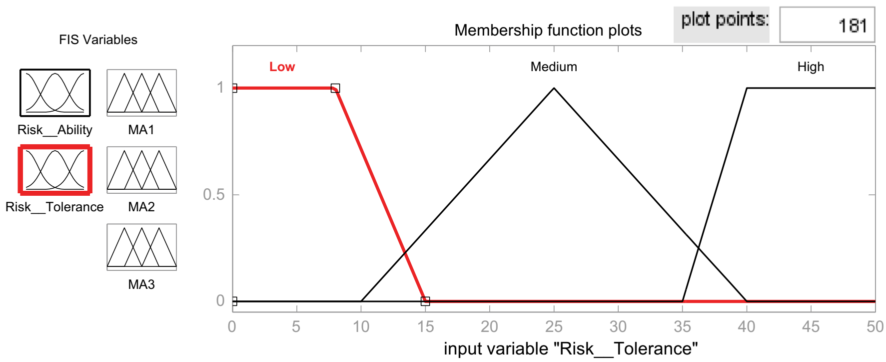

Fig. 8 Membership functions for risk tolerance

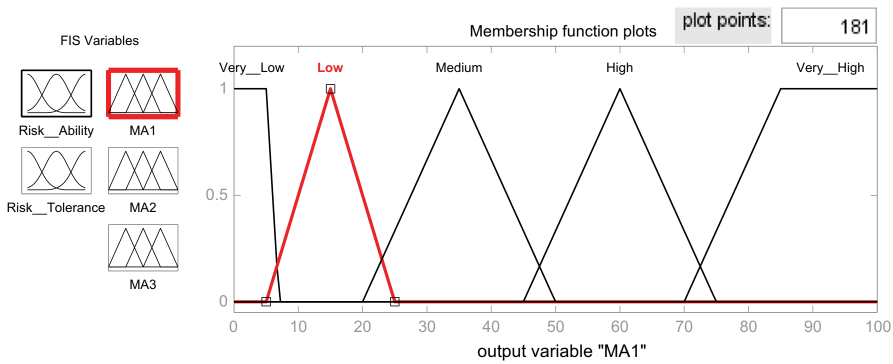

Fig. 9 Membership functions for MAs

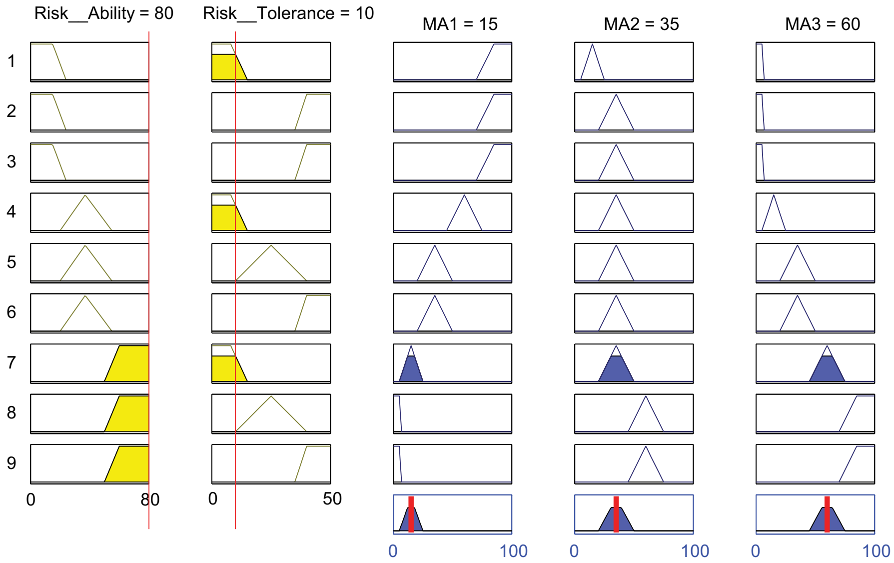

Fig. 10 Results of aggressive FIS

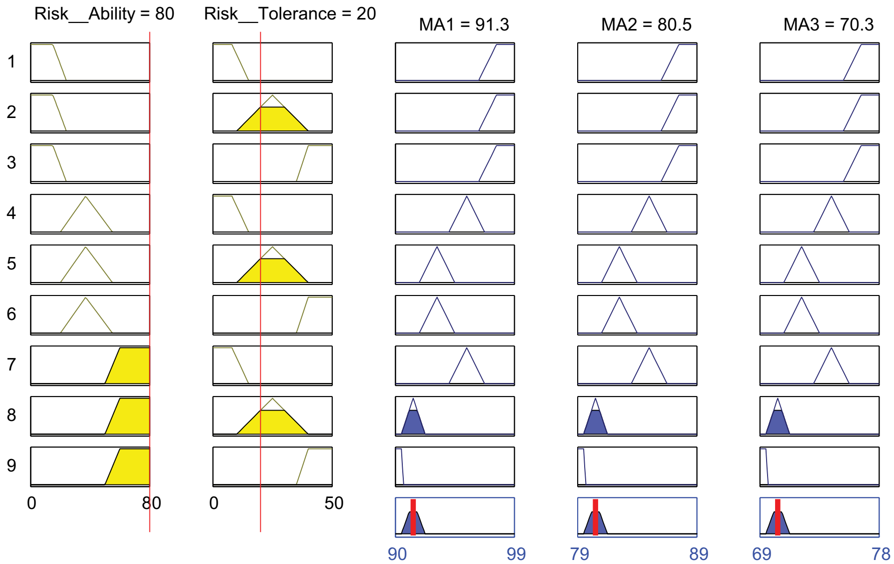

Fig. 11 Results of FIS for confidence levels

Table 7 Efficient frontier for $MA_1(\alpha = 0.91)$, $MA_2(\alpha = 0.8)$ and $MA_3(\alpha = 0.7)$

alpha = 0.91		alpha = 0.8		alpha = 0.7	
CVaR %	ER %	CVaR %	ER %	CVaR %	ER %
0.9155	0.0750	1.3012	0.0951	2.4916	0.1248
0.9159	0.0753	1.3172	0.0954	2.4976	0.1249
0.9164	0.0756	1.3332	0.0957	2.5036	0.1250
0.9170	0.0759	1.3493	0.0960	2.5096	0.1251
0.9181	0.0762	1.3655	0.0963	2.5156	0.1252
0.9198	0.0765	1.3819	0.0966	2.5216	0.1253
0.9218	0.0768	1.3983	0.0969	2.5277	0.1254
0.9242	0.0771	1.4147	0.0972	2.5337	0.1255
0.9271	0.0774	1.4312	0.0975	2.5397	0.1256
0.9304	0.0777	1.4476	0.0978	2.5458	0.1257
0.9341	0.0780	1.4641	0.0981	2.5518	0.1258
0.9381	0.0783	1.4807	0.0984	2.5579	0.1259
0.9426	0.0786	1.4974	0.0987	2.5639	0.1260
0.9486	0.0789	**1.5141**	**0.0990**	2.5699	0.1261
0.9556	0.0792	1.5307	0.0993	2.5760	0.1262
0.9637	0.0795	1.5474	0.0996	2.5820	0.1263
0.9721	0.0798	**1.5642**	**0.0999**	2.5880	0.1264
0.9812	**0.0801**	1.5810	0.1002	2.5941	0.1265
0.9915	0.0804	1.5978	0.1005	2.6002	0.1266
1.0026	0.0807	1.6146	0.1008	2.6063	0.1267
1.0137	0.0810	1.6314	0.1011	2.6124	0.1268
1.0251	0.0813	1.6482	0.1014	2.6184	0.1269
1.0368	0.0816	1.6651	0.1017	2.6245	0.1270
1.0485	0.0819	1.6820	0.1020	2.6306	0.1271
1.0605	0.0822	1.6990	0.1023	2.6367	0.1272
1.0731	0.0825	1.7160	0.1026	2.6428	0.1273
1.0870	0.0828	1.7330	0.1029	**2.6489**	**0.1274**
1.1012	0.0831	1.7502	0.1032	2.6551	0.1275
1.1157	0.0834	1.7677	0.1035	2.6614	0.1276
1.1303	0.0837	1.7851	0.1038	2.6678	0.1277

For each MA thirty efficient portfolios have been shown around its targets. The bold values represent the closest values to the fixed targets for each mental account on the efficient frontier

References

Abdelaziz, F. B., Aouni, B., & El Fayedh, R. (2007). Multiob-jective stochastic programming for portfolio selection. *European Journal of Operational Research*, *177*(3), 1811–1823.

Abdelsalam, O., Duygun, M., Matallín-Sáez, J.C. & Tortosa-Ausina, E. (2014). *Is ethical money sensitive to past returns? The case of portfolio constraints and persistence of Islamic and socially responsible funds*. Working Papers No 2014/19, Economics Department, Universitat Jaume I, Castelln (Spain). http://EconPapers.repec.org/RePEc:jau:wpaper:2014/19

Aouni, B., Colapinto, C., & La Torre, D. (2013). A cardinality constrained stochastic goal programming model with satisfaction functions for venture capital investment decision making. *Annals of Operations Research*, *205*, 77–88.

Artzner, P., Delbaen, F., Eber, J., & Heath, D. (1999). Coherent measures of risk. *Mathematical Finance*, *9*, 203–288.

Bahloul, S., & Abid, F. (2013). A combined analytic hierarchy process and goal programming approach to international portfolio selection in the presence of investment barriers. *International Journal of Multi-criteria Decision Making*, *3*(1), 1–20.

Ballestero, E., Bravo, M., Pérez-Gladish, B., Arenas-Parra, M., & Plá-Santamaría, D. (2012). Socially responsible investment: A multicriteria approach to portfolio selection combining ethical and financial objectives. *European Journal of Operational Research*, *216*(2), 487–494.

Ballestero, E., Pérez-Gladish, B., Arenas-Parra, M., & Bilbao-Terol, A. (2009). Selecting portfolios given multiple eurostoxx-based uncertainty scenarios. *INFOR Information Systems and Operational Research*, *47*(1), 59–70.

Barnett, M. L., & Salomon, R. S. (2006). The curvilinear relationship between social responsibility and financial performance. *Strategic Management Journal*, *27*, 1101–1122.

Barracchini, C., & Addessi, M. E. (2012). Ethical portfolio theory: A new course. *Journal of Management and Sustainability*, *2*(2), 35–42.

Barreda-Tarrazona, I., Matallín-Sáez, J., & Balaguer-Franch, M. (2011). Measuring investors socially responsible preferences in mutual funds. *Journal of Business Ethics*, *103*(2), 305–330.

Basso, A., & Funari, S. (2014). Constant and variable returns to scale DEA models for socially responsible investment funds. *European Journal of Operational Research*, *235*(3), 775–783.

Bauer, R., Koedijk, K., & Otten, R. (2005). International evidence on ethical mutual fund performance and investment style. *Journal of Banking & Finance*, *29*, 1751–1767.

Bénabou, R. & Tirole, J. (2010). Individual and corporate social responsibility. Economica, *77*, 1–19.

Benson, K. L., & Humphrey, J. E. (2008). Socially responsible investment funds: Investor reaction to current and past returns. *Journal of Banking & Finance*, *32*, 1850–1859.

Berry, T. C., & Junkus, J. C. (2013). Socially responsible investing: An investor perspective. *Journal of Business Ethics*, *112*(4), 707–720.

Bilbao, A., Arenas, M., Rodríguez, M., & Antomil, J. (2007). On constructing expert betas for single-index model. *European Journal of Operational Research*, *183*, 827–847.

Bilbao-Terol, A., Arenas-Parra, M., & Cañal-Fernández, V. (2012a). Selection of socially responsible portfolios using goal programming and fuzzy technology. *Information Sciences*, *189*, 110–125.

Bilbao-Terol, A., Arenas-Parra, M., & Cañal-Fernández, V. (2012b). A fuzzy multi-objective approach for sustainable investments. *Expert Systems with Applications*, *39*, 10904–10915.

Bilbao-Terol, A., Arenas-Parra, M., Cañal-Fernández, V., & Bilbao-Terol, C. (2013). Selection of socially responsible portfolios using hedonic prices. *Journal of Business Ethics*, *115*(3), 515–529.

Bojadziev, G., & Bojadziev, M. (1996). *Fuzzy sets, fuzzy logic, applications (advances in fuzzy systems—Applications & theory)*. Singapore: World Scientific Publishing Co. Pte. Ltd.

Bollen, N. P. B. (2007). Mutual fund attributes and investor behaviour. *Journal of Financial and Quantitative Analysis*, *42*, 689–708.

Calvo, C., Ivorra, C., & Liern, V. (2014). Fuzzy portfolio selection with non-financial goals: exploring the efficient frontier. *Annals of Operations Research*, doi:10.1007/s10479-014-1561-2

Cortez, M. C., Silva, F., & Areal, N. (2009). The performance of European socially responsible funds. *Journal of Business Ethics*, *87*(4), 573–588.

Charnes, A., & Cooper, W. W. (1961). *Management models and industrial applications of linear programming* (Vol. I). New York: Wiley.

Dam, L., & Heijdra, B. J. (2011). The environmental and macroeconomic effects of socially responsible investment. *Journal of Economic Dynamics and Control*, *35*, 1424–1434.

Das, S., Markowitz, H., Scheid, J., & Statman, M. (2010). Portfolio optimization with mental accounts. *Journal of Financial and Quantitative Analysis*, *45*(2), 311–334.

Dorfleitner, G., & Utz, S. (2012). Safety first portfolio choice based on financial and sustainability returns. *European Journal of Operational Research*, *221*(1), 155–164.

Drut, B. (2010). *Social responsibility and mean-variance portfolio selection, No. 10- 002, RS*. Working Papers CEB from Universit Libre de Bruxelles, Solvay Bruxelles School of Economics and Management, Centre Emile Bernheim (CEB).

EFAMA. (2013). *Annual report 2013*. European Fund and Asset Management Association. Brussels.

EUROSIF. (2012). *European SRI Study 2012*. European Sustainable and Responsible Investment Forum, Paris.

Forte, G., & Miglietta, F. (2007). Islamic mutual funds as faith-based funds in a socially responsible context. *SSRN*. doi:10.2139/ssrn.1012813

Fowler, S. J., & Hope, C. (2007). A critical review of sustainable business indices and their impact. *Journal of Business Ethics*, *76*(3), 243–252.

Friedman, M. (1962). *Capitalism and freedom*. Chicago, IL: The University of Chicago Press.

Galema, R., Plantinga, A., & Scholtens, B. (2008). The stocks at stake: Return and risk in socially responsable investment. *Journal of Banking and Finance*, *32*, 2646–2654.

Gärling, T., Kirchler, E., Lewis, A., & van Raaij, F. (2009). Psychology, financial decision making, and financial crises. *Psychological Science in the Public Interest*, *10*(Suppl 1), 1–47.
Grinblatt, M., & Han, B. (2005). Prospect theory, mental accounting, and momentum. *Journal of Financial Economics*, *78*, 311–339.
GSIA. (2013). *2012 Global sustainable investment review*. Global Sustainable Investment Alliance (GSIA). Available at: http://www.gsi-alliance.org/wp-content/uploads/2015/02/GSIA_Review_download.pdf.
Gupta, P., Mehlawat, M. K., & Saxena, A. (2013). Hybrid optimization models of portfolio selection involving financial and ethical considerations. *Knowledge-Based Systems*, *37*, 318–337.
Hayat, R., & Kraeussl, R. (2011). Risk and return characteristics of Islamic equity funds. *Emerging Markets Review*, *12*(2), 189–203.
Hens, T., & Bachmann, K. (2008). *Behavioural finance for private banking*. England: Wiley.
Hirschberger, M., Steuer, R. E., Utz, S., Wimmer, M., & Qi, Y. (2013). Computing the nondominated surface in tri-criterion portfolio selection. *Operations Research*, *61*, 169–183.
Hofmann, E., Hoelzl, E., & Kirchler, E. (2008). A comparison of models describing the impact of moral decision making on investment decision. *Journal of Business Ethics*, *82*(1), 171–187.
Hsieh, K. L. (2011). Employing a recommendation expert system based on mental accounting and artificial neural networks into mining business intelligence for study abroad's P/S. *Expert Systems with Applications*, *38*(12), 14376–14381.
Jang, J. S. R. (1993). ANFIS: Adaptive network-based fuzzy inference system. *IEEE Transactions on Systems Man and Cybernetics*, *23*, 665–685.
Jones, D., & Tamiz, M. (2010). *Practical goal programming*. New York: Springer.
Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, *47*(2), 263–292.
Kaminski, B., Czupryna, M., & Szapiro, T. (2009). On conditional value-at-risk based goal programming portfolio selection procedure. *Multiobjective Programming and Goal Programming. Lecture Notes in Economics and Mathematical Systems*, *618*, 243–252.
Kempf, A., & Osthoff, P. (2007). The effect of socially responsible investing on portfolio performance. *European Financial Management*, *13*, 908–922.
Kinnel, R. (2009). Investing in socially screened funds. *Morningstar Fund Investor*, *17*(7), 10–18.
Köksalan, M., & Tuncer Sakar, C. (2014). An interactive approach to stochastic programming-based portfolio optimization, *Annals of Operations Research*. doi:10.1007/s10479-014-1719-y
Krokhmal, P., Palmquist, J., & Uryasev, S. (2002). Portfolio optimization with conditional value-at-risk objective and constraints. *Journal of Risk*, *4*(2), 43–68.
Krzemienowski, A., & Szymczyk, S. (2014). Portfolio optimization with a copula-based extension of conditional value-at-risk. *Annals of Operations Research*. doi:10.1007/s10479-014-1625-3
Lambertini, L. (2009). Optimal product proliferation in monopoly: A dynamic analysis. *Review of Economic Analysis*, *1*, 8097.
Lee, S. M., & Chesser, D. L. (1980). Goal programming for portfolio selection. *The Journal of Portfolio Management*, *3*, 22–26.
Lopes, L. L. (1987). Between hope and fear: The psychology of risk. *Advances in experimental social psychology*, *20*, 255–295.
Mamdani, E. H., & Assilian, S. (1975). An experiment in linguistic synthesis with a fuzzy logic controller. *International Journal of Man-Machine Studies*, *7*(1), 1–13.
Markowitz, H. (1952). Portfolio selection. *Journal of Finance*, *7*, 77–91.
Moneva, J. M., Archel, P., & Correa, C. (2006). GRI and the camouflaging of corporate unsustainability. *Accounting Forum*, *30*(2), 121–137.
Ogryczak, W. (2000). Multiple criteria linear programming model for portfolio selection. *Annals of Operations Research*, *97*, 143–162.
Ormerod, P. (2010). The current crisis and the culpability of macroeconomic theory. *Revista de Economía Institucional*, *12*(22), 111–128.
Pais, T. C., & Amaral, P. (2012). Managing the tabu list length using a fuzzy inference system: An application to examination timetabling. *Annals of Operations Research*, *194*(1), 341–363.
Pérez, F., & Gómez, T. (2014). Multiobjective project portfolio selection with fuzzy constraints. *Annals of Operations Research*,. doi:10.1007/s10479-014-1556-z.
Pflug, G. (2001). Some remarks on the value-at-risk and the conditional value-at-risk. In S. Uryasev (Ed.), *Probabilistic constrained optimization: methodology and applications*. (pp. 272–281). Boston: Kluwer.
Pla-Santamaria, D., & Bravo, M. (2013). Portfolio optimization based on downside risk: A mean-semivariance efficient frontier from Dow Jones blue chips. *Annals of Operations Research*, *205*, 189–201.
PRI (2009). *Annual report of the PRI initiative. (2009). Principles for responsible investment*, UNEP Finance Initiative. Innovative financing for sustainability and UN Global Compact, UK

Radu, I., & Funaru, M. (2011). Socially reponsible investments in mutual funds. *Bulletin of the Transilvania University of Brasov. Series V: Economic Sciences*, *4*(1), 157–168.
Renneboog, L., Ter Horst, J., & Zhang, C. (2008). Socially responsible investments: Institutional Aspects, performance and investor behaviour. *Journal of Banking and Finance*, *32*, 1723–1742.
Renneboog, L., Ter Horst, J., & Zhang, C. (2011). Is ethical money financially smart? Nonfinancial attributes and money flows of socially responsible investment funds. *Journal of Financial Intermediation*, *20*, 562–588.
Rockafellar, R. T., & Uryasev, S. (2000). Optimization of conditional value-at-risk. *Journal of Risk*, *2*, 21–41.
Ross, S. (1976). The arbitrage theory of capital asset pricing. *Journal of Economic Theory*, *13*(3), 341–360.
Roy, A. D. (1952). Safety first and the holding of assets. *Econometrica*, *20*, 431–449.
Schepers, D. H. (2003). A critique of social investing's diversity measures. *Business and Society Review*, *104*(4), 487–508.
Shanmugam, B., & Zahari, Z. R. (2009). *A primer on Islamic finance*. Research Foundation Publications, Charlottesville: CFA Institute.
Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance*, *19*(3), 425–442.
Shefrin, H., & Statman, M. (2000). Behavioral portfolio theory. *Journal of Financial and Quantitative Analysis*, *35*, 127–151.
Simister, J., & Whittle, R. (2013). Ethical Investment and portfolio theory: Using Factor analysis to select a portfolio. *Journal of Mathematical Finance*, *3*(1A), 145–152. doi:10.4236/jmf.2013.31A014.
Statman, M. (2000). Socially responsible mutual funds. *Financial Analysts Journal*, *56*, 30–39.
Steuer, R. E., Qi, Y., & Hirschberger, M. (2007). Suitable-portfolio investors, non-dominated frontier sensitivity, and the effect of multiple objectives on standard portfolio selection. *Annals of Operations Research*, *152*, 297–317.
Takagi, T., & Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control. *IEEE Transactions on Systems, Man, Cybernetics*, *15*, 116–132.
Thaler, R. H. (1980). Toward a positive theory of consumer choice. *Journal of Economic Behavior and Organization*, *1*, 39–60.
Thaler, R. H. (1999). Mental accounting matters. *Journal of Behavioral Decision Making*, *12*, 183–206.
Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, *5*, 297–323.
USSIF. (2012). *Report on socially responsible investing trends in the United States*. US SIF, The Forum for Sustainable and Responsible Investment, Washington, DC.
USSIF. (2013). *The impact of sustainable and responsible investment*. US SIF, The Forum for Sustainable and Responsible Investment, Washington, DC.
Utz, S., Wimmer, M., Hirschberger, M., & Steuer, R. E. (2013). Tri-criterion inverse portfolio optimization with application to socially responsible mutual funds. *European Journal of Operational Research*,. doi:10.1016/j.ejor.2013.07.024.
van Leekwijck, W., & Kerre, E. E. (1999). Defuzzification: Criteria and classification. *Fuzzy Sets and Systems*, *108*(2), 159–178.
Vigeo SRI Research. (2009). *Green, social and ethical funds in Europe-2009 review*. Vigeo Report.
Wolff, M. (2002). Response to confronting the critics. *New Academy Review*, *1*, 230–237.
Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*, 338–353.
Zeleny, M. (1974). *Linear multiobjective programming*. Berlin: Springer.
Zopounidis, C., & Doumpos, M. (2013). Multicriteria decision systems for financial problems. *TOP*, *21*(2), 241–261.