Ann Oper Res (2017) 255:9–28
DOI 10.1007/s10479-015-1831-7

Likelihood of environmental coalitions and the number of coalition members: evidences from an IAM model

Zili Yang

Published online: 11 March 2015

Abstract To hold the grand coalition voluntarily in an economy with detrimental externalities, the allocations should be in the core. Identifying the scope or 'size' of the core allocations is of vital importance for understanding such a coalition. Furthermore, the relationship between the number of agents and the 'size' of the core reveals some crucial characteristics of coalition formation. In this paper, a cooperative game of stock externality provision is constructed to study its core properties of an economy with detrimental externality. Particularly, methods and algorithms for testing the shrinking core hypothesis are developed in the RICE model, an integrated assessment model of climate change. The calculation results show that the size of the core shrinks as the number of regions increases in RICE. The paper also evaluates the policy implications of the shrinking core phenomenon with respect to the environmental coalitions.

Keywords Detrimental externalities · International environmental agreement (IEA) · Coalition theory · The core properties · Integrated assessment modeling (IAM)

1 Introduction

International cooperation is crucial for dealing with global environmental issues effectively. Studies of international environmental agreement (IEA) reflect the efforts aimed at better understanding cooperation mechanisms behind complicated global environmental challenges, such as climate change and other trans-boundary pollution. Global environmental problems are detrimental externalities without authoritative governance. Voluntary partic-

I thank for the financial support from Chair Lhoist Berghmans at Center for Operations Research (CORE), Université Catholique de Louvain, Belgium, for this research. I am grateful to Professor Henry Tulkens and two anonymous referees for their valuable comments on previous versions of this paper.

Z. Yang (✉)
State University of New York at Binghamton, Binghamton, NY 13850-6000, USA
e-mail: zlyang@binghamton.edu

ipation in the grand IEA is necessary to achieve efficiency or full internalization of the externality.

Game theoretic approach is a proper tool to study IEA issues. Scholars have examined IEA issues from different angles with game-theoretic modeling methods. For example, Barrett (1994) raised the notion of "self-enforcing" IEA and expanded the exposition later (Barrett 2003). Carraro and Siniscalco (1993) proposed the concept of "internal" and "external" stability concepts, followed the tradition of d'Aspremont et al. (1983), for IEAs and discussed the relationship between the numbers of coalition members and the stability of IEAs. Chander and Tulkens (1995, 1997) formulated the γ-core concept in the economy of detrimental externalities and applied it to IEA studies. Finus (2001) also critically assessed the coalition issues in length. In a theoretic treatment of coalition formation, Ray (2007) discusses the coalition of public good provisions.

Game theoretic treatments of IEA issues have been applied to empirical modeling works. Buchner and Carraro (2009) analyze stable coalitions in the FEEM-RICE model. Eyckmans and Tulkens (2003) identify the γ-core in the CWI model using the transfer scheme developed by Chander and Tulkens (1995, 1997), Finus et al. (2014) analyze the sequential coalition formation process in climate change. Yang (2008) conducts comprehensive coalitional analysis in the RICE model. The above studies show promising potentials of applying game theoretic concepts in empirical models of environmental issues.

The core of an economy is an important solution concept in general equilibrium theory and cooperative game theory. It is well documented in the literature that the existence of externality complicate the core allocations. Global environmental issues like climate change are detrimental stock externality phenomena.[1] In an economy with externality, the core allocations are needed to hold a grand coalition together because of their incentive properties. Therefore, examining the core properties of the cooperative games in international environmental issues and the relationship between the core and incentives of IEAs are important. Thus far, most studies related to the core concepts in the economies with detrimental externalities (the theoretical background for most environmental problems) are qualitative rather than quantitative.

For the Arrow–Debreu economy consisting of only private goods, there is a famous "shrinking core" theorem states that the "size" of the core reduces as the number of agents increasing in a "replica" economy. In asymptotic case where the number of agents goes to infinity, the core collapses onto the unique Walrasian equilibrium (Debreu and Scarf 1963). An intuitive interpretation of the shrinking core theorem is as follows. A grand coalition is easier to be blocked by some sub-coalitions as the number of agents increases. In other words, the grand coalition with large number of agents is more difficult to sustain than those with fewer agents, *ceteris paribus*.

In an economy with public goods or externalities, there is no definitive conclusion about the shrinking core hypothesis. Muench (1972) provided a counter-example where the size of the core is always larger than the Lindahl equilibrium allocations when the numbers of agents goes to infinity. Champsaur et al. (1975) illustrated a scenario in which the core of a public good economy increases along with the number of agents. Shapley and Shubik (1966) provided an example of non-existence of the core. As pointed out by Starrett (1973), the example in Shapley and Shubik (1966) is due to non-convexity preference and *ad hoc* blocking rule in defining the core.

Despite these inconclusive results, it is still interest to know whether the shrinking core hypothesis holds or not in "well-behaved" models under a set of clearly defined blocking

[1] "Stock externality", instead of "stock pollution", is used in this paper to emphasize spillovers among agents.

rules.[2] From policy perspective, we may want to know the core properties of the economy with externalities in lower dimensions and with a few players. In other words, we would like to know whether the core shrinks or not in non-asymptotic cases. Such a consideration reflects the appropriate background of IEAs: finite regions (total numbers of sovereign countries are no more than hundreds and key players are fewer than a dozen) negotiate the terms of international cooperation in dealing with global environmental externalities. In addition, the attitudes of a few key regions determine the outcome of an IEA. In this paper, we try to test a version of shrinking core hypothesis in low dimension: given a fixed-size aggregate economy with externality, do more agents (still small number) lead to shrinking of the core?

Scholars have examined the relationship between the number of agents and formation of coalition in environmental issues. For example, in a specific modeling setting, Carraro and Siniscalco (1993) concluded that maximum number of agents ensuring the stability of environmental coalition is 3. In an early paper, Roumasset (1979) calculated the shrinking cores of a sharecropping economy with production externality in a stylized numerical model. Despite these promising works, the literatures on the core properties of the grand coalition in the economy with externalities in empirical environmental studies need fresh additions. The core concepts in empirical environmental models call for further studies.

In empirical and policy related studies, economists strive to quantify the useful theoretic notions in numerical simulations, such as calculating estimated amounts of "double-dividends" of correctional pollution taxes and deadweight losses of trade barriers in general equilibrium models. Although the quantitative results of "double dividends" or "dead-weight losses" are model-specific, collective contributions from various models enable us to have better understanding of these notions in empirical contexts.

This paper aims at the similar target, i.e., quantifying the relationship between the scope of the core and the number of agents through carefully-designed numerical methods. As summarized above, the core concept is highly relevant to IEA studies. Calculating the scope, domain, and other quantitative properties of the core of cooperative games in environmental externality models offers a useful perspective to IEA research.

In this paper, a cooperative game of providing stock externalities is constructed. Its numerical solutions are sought in the RICE model (Nordhaus and Yang 1996)—a well-known integrated assessment model of climate change. Through meticulously designed algorithms, the scope ("size") of the core of this cooperative game is identified in simple metrics. Furthermore, the "sizes" of the cores are calculated in 2, 3, 4, 5, and 6-region RICE model respectively. The results from numerical simulations clearly show that the core of the cooperative game of providing stock externalities embedded in the RICE model shrinks as the number of regions increases, relative to the "size" of the entire efficiency frontier. In addition, the "size" of core, even in the 2-region RICE model, is a small portion of the efficiency frontier.

The remaining parts of this paper are as follows. Section 2 contains the construction of a cooperative game in a generic model of stock externality provision. It also outlines relevant assumptions, definitions, and solution concepts. Section 3 introduces the RICE model and its game theoretic solutions. Section 4 details the algorithms for calculating the metrics of the core. Calculation results of the "sizes" of the cores and other relevant outputs are also presented here. Section 5 discusses the policy implications of the shrinking core in RICE. Section 6 is the concluding remark.

[2] "Well-behaved" models refer to those models with twice differentiable functions, convex utility functions, and calibrated with real data. The blocking rules used in this paper will be explained in subsequent sections.

2 A cooperative game of stock externality provision[3]

Most global environmental problems can be formulated as a stock externality provision problem. A generic model of efficient provision of externality can be framed as a social planner's problem in which externalities are fully internalized:

$$\underset{\{x_i\}}{\text{Max}}\, V = \sum_{i=1}^{N} \varphi_i W_i^E = \sum_{i=1}^{N} \varphi_i \int_0^{\infty} U^i\,(x_i(t), B(t))\, e^{-\delta t} dt, \quad \sum_{i=1}^{N} \varphi_i = N \tag{1}$$

$$s.t. \quad F^i\,(x_i(t), b_i(t)) = 0, \quad i = 1, 2, \ldots, N. \tag{2}$$

$$\dot{B}(t) = \sum_{i=1}^{N} b_i(t) - \sigma B(t), \quad \sigma > 0. \tag{3}$$

$$B(0) = B_0, \quad x_i(0) = x_{i,0}, \tag{4}$$

$$\frac{\partial U^i}{\partial x_i} > 0, \quad \frac{\partial U^i}{\partial B} < 0, \quad \frac{\partial^2 U^i}{\partial x_i^2} < 0, \quad \frac{\partial^2 U^i}{\partial B^2} < 0, \quad \frac{\partial F^i}{\partial b_i} \Big/ \frac{\partial F^i}{\partial x_i} > 0 \tag{5}$$

In the above system, social planner maximizes the *weighted* sum of N agents' present-value utilities (regions in international environmental issues). U^i, the instantaneous utility function of agent i, is a function of private good $x_i(t)$ and B(t), the *stock* of externalities faced by all agents. Each agent has a transformation function (F_i) between private good $x_i(t)$ and the *flow* of externalities $b_i(t)$ as in (2). (2) describes the relationship between private good and externality generation implicitly. The control variable in this optimal control problem is $\{x_i(t)\}$. Through transformation function (2), control variable $\{x_i(t)\}$ determines the optimal flows of externality $\{b_i(t)\}$. The relationship between the flows and the stocks of the externality is determined by motion equation (3). Both private good $\{x_i(t)\}$ and stock level of externality B(t) are arguments of individual payoff function. In this framework, the budget constraints of agents and other exogenous parameters are implicit in transformation functions (2). The signs of derivatives in (5) and initial conditions (4) ensure that this social planner's problem is a well-behaved system.

This model of stock externality provision encompasses many dynamic environmental problems in abstract. Most importantly, it characterizes the totality of efficient solution set of stock externality provision. Solutions of (1) under different social welfare weights $\{\varphi_i\}$, the key parameter in this research, correspond to the entire efficiency frontier of externality provisions. Namely, simplex $S = \{\{\varphi_i\} \big| \sum \varphi_i = N\}$ has a one-to-one correspondence with the efficiency frontier of system (1). The core, the solution concept to be examined here, is a connected subset of the efficiency frontier. Therefore, the optimal solutions of system (1) under a subset of simplex S map to the core allocations.

Before proceeding with defining the cooperative game and its core, we need to specify another important solution concept first. It is the "open-loop" Nash equilibrium defined by the solution of the following non-cooperative game of stock externality provisions:

$$\underset{\{x_i(t)\}}{Max} \quad W_i^M = \int_0^{\infty} U^i\,(x_i(t),\ B(t))\ e^{-\delta t} dt, \quad i = 1, 2, \ldots, N. \tag{6}$$

$$s.t. \quad F^i\,(x_i(t), b_i(t)) = 0, \quad i = 1, 2, \ldots, N. \tag{7}$$

[3] The cooperative game adopted here is originated from Yang (2008).

$$\dot{B}(t) = b_i(t) + \sum_{j \neq i}^{N} \bar{b}_j(t) - \sigma B(t), \quad i = 1, 2, \ldots, N. \quad \sigma > 0. \tag{8}$$

The inefficient Nash equilibrium is the simultaneous solution of N systems of single agent's optimization problem specified by (6), (7), and (8), as well as the same initial conditions and sign requirements (4) and (5) as in system (1). The Nash equilibrium is a very important benchmark. In a non-cooperative scheme, each agent receives W_i^M payoffs. Because the Nash equilibrium is inefficient in the economy with externalities, we can find the Pareto improvement opportunities by internalizing the externalities as in social optimum (1). The cooperative game to be constructed is set up on such rationales. The game is to reach certain efficient outcome in (1) through bargaining, negotiations, and cooperation among agents.

2.1 A cooperative game of stock externality provision

The cooperative game is defined by a triplet $V(\varphi_i, x_i(t), W_i^C)$ for i = 1, 2, …, N. Here φ_i and $x_i(t)$, as defined in system (1), are decision variables for agent i. W_i^C is the pay-off function for agent i. W_i^C is determined in (1) and is *not* weighted by φ_i. Superscript C stands for cooperation.

The game is played in two stages. In the first stage, agents negotiate their respective weights $\{\varphi_i\}$ in system (1). Such a bargaining process is to decide an efficient solution of externality provision collectively. As a complete information game model, agents know the consequence of bargained $\{\varphi_i\}$ on his and others' real variable paths. In other words, they know the outcome of mapping $\{\varphi_i\} \to \{x_i(t)\}$ for all i in system (1). In subsequent discussions and numerical calculations, we can see that $\{\varphi_i\}$ reflects relative mitigation burdens shouldered by each agent in the efficient solutions. The first-stage of the game can be treated as a multi-agent bargaining game conducted in an abstract space $S = \{\{\varphi_i\} \big| \sum \varphi_i = N\}$. Nevertheless, every agent knows material consequence of such bargaining.

After $\{\varphi_i\}$ is agreed upon, the game moves to the second stage. In the second stage, all agents cooperate to achieve efficiency in system (1) and each agent will fulfill its own obligations by executing the optimal path of control variable $\{x_i^*(t)\}$ according to the optimal solution of system (1) under negotiated $\{\varphi_i\}$. Each agent generates $\{b_i^*(t)\}$ externalities and receives

$$W_i^c = \int_0^\infty U^i\left(x_i^*(t), B^*(t)\right) e^{-\delta t} dt \tag{9}$$

as the payoff in the grand coalition by solving (1).[4]

The cooperative game defined here connects the efficient provision of stock externalities with the bargaining of individual burden sharing. When φ_i is high, agent i's burden for mitigating externality is low in the grand coalition, compared with a lower φ_i, *ceteris paribus*. In addition, agent i would benefit more from other agents' mitigation efforts. On simplex $S = \{\{\varphi_i\} \big| \sum \varphi_i = N\}$, bargaining $\{\varphi_i\}$ is a "zero-sum" negotiation process: one's gain is somebody else's loss and vice versa. The first stage of the game bears similarities with the "cost-sharing equilibrium" notion for the efficient externality provision in Mas-Colell and Silvestre (1998).

Most solutions of (1) cannot be the outcomes from voluntary cooperation, or the results of collective bargaining. For an optimal solution of (1) under a particular set $\{\varphi_i\}$ to be an

[4] Different penalty rules can be introduced to support this cooperative game. Because they are not relevant to the core properties examined here, these possible penalty rules are not elaborated.

outcome of the cooperative game defined above, it is necessary that the solution under this set $\{\varphi_i\}$ is in the core.[5] The following is the definition of the core concept in this study.

2.2 The core allocation relative to the initial endowments

An optimal solution of system (1) $\{x_i^*(t)\}$ under a specific set $\{\varphi_i\}$ is in the core, if it cannot be blocked by any sub-coalitions. Particularly, it cannot be blocked by the Nash equilibrium solution (6).

We assume that the generic model of stock externality provision [system (6)] has a unique Nash equilibrium.[6] This unique equilibrium depends on the initial endowments and unspecified parameters of system (1). If these initial endowments and parameters change, the Nash equilibrium also shifts. The core allocations have the similar relationship with the initial endowments. With the fixed initial endowments, the domain of the core is determined. Following the tradition in Bergstrom (1976), we call the core in the above definition "the core relative to the initial endowments".

In the above definition, the notions of "blocking" and "sub-coalitions" need to be clarified. In the core, all agents are better off than in any sub-coalitions. To form a grand coalition, *all* agents have to be better off than their respective reserved payoffs, namely, the payoffs in the Nash equilibrium. This is the well-known incentive criterion—individual rationality (IR). To hold the grand coalition together, all agents should agree with the burden sharing arrangement under negotiated $\{\varphi_i\}$. If some agents can form sub-coalitions to increase their payoffs compared with the grand coalition under $\{\varphi_i\}$, the grand coalition cannot be sustained. Therefore, the members in a stable grand coalition should not be able to exploit such opportunities (no blocking property). Particularly, when such grand coalition has to be based on voluntary participation (as all IEAs), the core properties are of vital importance.

The "sub-coalitions" here have to be specified inductively. Inductive definition of sub-coalitions has some complications in the economy with externalities. Strategic behaviors of the agents outside the sub-coalition affect the actions inside the sub-coalition because of external effects.[7] We assume that a sub-coalition achieves internal efficiency under the same $\{\varphi_i\}$ as in the grand coalition and plays the non-cooperative Nash game with the agents outside the sub-coalition as a single agent. A blocking, if taken place, is under such strategic environment. In a framework that allows transfers, the core defined with such sub-coalition structure is called γ-core in Chander and Tulkens (1997). In fact, this definition is a more reasonable way to define the core in the economy with externalities than traditional approach, such as in Foley (1970).

Finally, the core allocations do not require cross agents transfers *ex ante*. Transfers alter the endowments *ex post*. Therefore, the post-transfer endowments and the Nash equilibrium are different from the initial one. Different Nash equilibriums correspond to different sets of core allocations. The cores in different transfer schemes do not coincide. Once transfers taken place, we are not dealing with "the core relative to the initial endowments". Allowing transfer schemes is "chasing moving targets" in finding the core allocations. Here, we are only interested in the core relative to the initial endowments. Specifically, we want to answer

5 Optimal solutions of (1) under arbitrary $\{\varphi_i\}$ are often conveniently called a "cooperative solution", in contrast to non-cooperative Nash solution. This is a misnomer. Solution of (1) is always an efficient solution. It is a necessary condition for a cooperative game solution but not sufficient.

6 Holding this assumption requires that the system of differential equations behind the differential game (6) has a unique solution under given initial conditions.

7 Such a phenomenon is a major departure from the assumption adopted in traditional cooperative game theory (see Osbrne and Rubinstein 1994, p. 258).

the questions related to this core: "What is the 'size' of the core at given number of agents?" "Whether the 'size' of the core shrinks or not as number of agents increases?"

To describe the cooperative game of stock externality provision requires more elaborations on the model and its solution concepts (see Yang 2008). Here we focus on the basic properties of the core allocations and try to identify the scope of the core in the model similar to (1). Discussions of other characteristics of this cooperative game are beyond the scope of this paper. Moreover, we answer the above two key questions in the context of the RICE model—a stock externality provision model like (1) on climate change with economic and climatic details.

3 The RICE model with different regional aggregations

The RICE model is a well-known integrated assessment model (IAM) developed by Nordhaus and Yang (1996). The RICE model is a multi-region optimal growth model with climate change externalities. Each region's economic growth [equivalent to $\{x_i(t)\}$ in (1)] generates greenhouse gas (GHG) emissions [equivalent to $\{b_i(t)\}$ in (1)]. Aggregate GHG emissions from regions lead to atmospheric temperature increases [equivalent to $B(t)$ in (1)]. In short, regional GDP co-generates GHG emissions (flows of detrimental externality) that lead to climate change (stock of detrimental externality) in RICE. The temperature increase causes the damages to regions. By incurring certain mitigation costs, each region can mitigate a portion of its own GHG emissions. The tradeoffs between mitigation costs and avoided climate damages can be assessed as a social optimum (an efficient solution) or as a decentralized Nash equilibrium (an inefficient solution). In essence, the RICE model is a special case of systems (1) and (6). We can define and find the non-cooperative Nash equilibrium, the cooperative game solutions, and the core allocations in this model.[8] In fact, most empirical works on coalition studies through IAM surveyed in Sect. 1 are based on the modified RICE models.

For the exercises here, the model is updated with new data. The base period is 2005. The model is solved numerically in discrete form in GAMS language (Brooke et al. 2004). The model runs for 50 periods in 5-year steps. The base model contains six regions: the United States (USA), European Union (EU), other high-income countries (OHI), China (CHN), Eastern European countries and former Soviet Union (EEC), and the rest of world (ROW).

To investigate the relationship between the core and number of regions in RICE, we aggregate the number of regions from six to five, four, three, and two regions. The aggregation is as follows (merged regions in brackets): (1) five-regions: USA, [EU + OHI], CHN, EEC, and ROW; (2) four-regions: [USA + EU + OHI], CHN, EEC, and ROW; (3) three-regions: [USA + EU + OHI], [CHN + EEC], and ROW; (4) two-regions: [USA + EU + OHI] and [CHN + EEC + ROW]. In the process of aggregating, regions' initial condition values are added together, such as population, GDP, GHG emissions in the base year, etc. The exogenous growth trend parameters are averaged across merged regions. In the aggregating processes, the global population and capital in the base year are fixed. Simply speaking, the models represent the same global economy that consists of different number of regions. Therefore, the RICE models with different number of regions are consistent for the purpose of investigating the relationship between the number of regions and the core properties. Grouping process is to generate a RICE model with 6, 5, 4, 3, and 2 regions, respectively to test the relationship

[8] Nordhaus and Yang (1996), Nordhaus and Boyer (2000), and Yang (2008) contains more detailed features of the RICE model. For quick reference, the algebraic description of RICE model is presented in the "Appendix".

Table 1 Design matrix for 5-region coalitions (1 = in, 0 = out)

No.	USA	OHI	CHN	EEC	ROW
1	1	1	1	1	1
2	1	1	1	1	0
3	1	1	1	0	1
4	1	1	1	0	0
5	1	1	0	1	1
6	1	1	0	1	0
7	1	1	0	0	1
8	1	1	0	0	0
9	1	0	1	1	1
10	1	0	1	1	0
11	1	0	1	0	1
12	1	0	1	0	0
13	1	0	0	1	1
14	1	0	0	1	0
15	1	0	0	0	1
16	**1**	**0**	**0**	**0**	**0**
17	0	1	1	1	1
18	0	1	1	1	0
19	0	1	1	0	1
20	0	1	1	0	0
21	0	1	0	1	1
22	0	1	0	1	0
23	0	1	0	0	1
24	**0**	**1**	**0**	**0**	**0**
25	0	0	1	1	1
26	0	0	1	1	0
27	0	0	1	0	1
28	**0**	**0**	**1**	**0**	**0**
29	0	0	0	1	1
30	**0**	**0**	**0**	**1**	**0**
31	**0**	**0**	**0**	**0**	**1**
32	0	0	0	0	0

between number of regions and "size of the core" in RICE. We do not investigate various possibilities of grouping or aggregating from 6 regions to 2 regions.

For convenience, we label the RICE model with m regions as RICE-m (m = 2, 3, 4, 5, 6). Here we try to identify the core allocations for all RICE-m. To do so, we strictly follow the definition of the core stated in Sect. 2. Namely, an efficient solution of RICE-m under a certain set $\{\varphi_i\}$ is a core allocation if it cannot be blocked by any sub-coalitions.

To check whether a solution is in the core of RICE-m, we follow the procedures below with a set of auxiliary tools for all RICE-m. Here, we use RICE-5 as an example to illustrate the procedure. First, we enlist all coalitions through a design matrix (Table 1), with 1 representing in a coalition and 0 representing outside the coalition. The first row (1, 1, 1, 1, 1) represents the grand coalition; the last row (0, 0, 0, 0, 0) represents the Nash equilibrium. All sub-coalition possibilities are represented by the second row (1, 1, 1, 1, 0) to 31st rows (0, 0, 0, 0, 1). As in Yang (2008), they are called "hybrid" Nash equilibriums in this exercise. Note that the bolded rows (rows 16, 24, 28, 30, 31) represent the cases where one region is in the coalition and other 4 regions are out. Conceptually, they are identical to the Nash equilibrium represented by row 32. They are kept in the calculation for algorithmic purpose.

Then we solve all $2^5 = 32$ "hybrid" Nash equilibriums in RICE-5 one by one under a given set of social welfare weight $\{\varphi_i\}$.[9] In each "hybrid" Nash equilibrium, regions cooperate within the coalition to achieve internal efficiency and interact strategically (non-cooperative) with the regions outside the coalition. Whole spectrum of "hybrid" Nash equilibrium solutions are obtained by executing a GAMS program with multi-layer loops that run the model from row 1 to row 32 in the design matrix. For each $\{\varphi_i\}$, the program runs 32 "hybrid" Nash equilibriums and ends at the same Nash equilibrium (row 32). If the algorithm is robust, the five interim cases (rows 16, 24, 28, 30, 31) should always be identical to the Nash equilibrium (row 32). The algorithm demonstrates this property.[10]

The third step is to record and compare the regional welfares (payoffs) in the 5×32-table format like Tables 2 and 3. Table 2 calculates the regional welfare differences (payoff difference) between the grand coalition (row 1) and all sub-coalitions. If a region is better off in a sub-coalition than in the grand coalition, the relevant cell records a "+" sign; if it is worse off, the cell contains a "−" sign; if the region is outside of the sub-coalition, the cell is empty. Table 2 is an example showing the results of $\varphi_i \equiv 1$. Table 3 calculates the regional welfare differences between the Nash equilibrium (row 32) and all sub-coalitions, including the grand coalition. If a region is better off in a sub-coalition than in the Nash equilibrium, the relevant cell records a "+" sign; if it is worse off, the cell contains a "−" sign; if the region is outside of the sub-coalition, the cell is empty. Again, Table 3 is an example showing the results of $\varphi_i \equiv 1$. We claim that the tables like Tables 2 and 3 for RICE-m contain complete information on the core properties of the RICE solutions.

Table 2 reveals the possibilities of sub-coalition blockings. If all signs are "−" under certain $\{\varphi_i\}$ in Table 2, no sub-coalition can block the grand coalition. If a row contains all "+" signs, this sub-coalition clearly blocks the grand coalition. For example, in row 6 all members in (USA, OHI, EEC) sub-coalition are better off than the grand coalition. It blocks 5-member grand coalition. If a row has mixed signs, the sub-coalition may or may not block the grand coalition, pending on whether the winners can sufficiently compensate the losers in this sub-coalition.

The signs in Table 3 inform us on the IR property of the grand coalition. Under the same $\{\varphi_i\}$, the grand coalition satisfies the IR condition, if all signs are "+" in the first row of Table 3. The "+" signs in the first row imply that all members in the grand coalition are better off than their respective payoffs in the Nash equilibrium, i.e. their reserve payoffs.

Therefore, a *sufficient condition* for a grand coalition under a specific set $\{\varphi_i\}$ to be in the core relative to the initial endowments is that all cells in Table 2 are "−" signs and all cells in the first row of Table 3 are "+" signs. The two tables are criteria for identifying the core allocations of the RICE model in this research.

Using those criteria, we find out that the popular utilitarian solution under $\varphi_i \equiv 1$ is not in the core. Table 3 shows that the solution is not compatible with 3 out of 5 regions' IR. They are USA, OHI, and EEC. These three regions rather play the non-cooperative Nash game

9 RICE-6 has $2^6 = 64$ "hybrid" Nash equilibriums, RICE-4 has $2^4 = 16$ "hybrid" Nash equilibriums, etc.

10 The robustness of the algorithm for searching the Nash equilibrium has been tested in other ways. By choosing different starting points, or changing the order of iterations, the algorithm converges to the same Nash equilibrium.

Table 2 Welfare change from grand coalition ($\varphi_i \equiv 1$)

No.	USA	OHI	CHN	EEC	ROW
1					
2	+	+	−	+	
3	+	−	−		−
4	+	+	−		
5	+	+		+	−
6	+	+		+	
7	+	+			−
8	+	+			
9	+		−	−	−
10	+		−	+	
11	+		−		−
12	+		−		
13	+			+	−
14	+			+	
15	+				−
16	+				
17		−	−	−	−
18		+	−	+	
19		−	−		−
20		+	−		
21		+		−	−
22		+		+	
23		+			−
24		+			
25			−	−	−
26			−	+	
27			−		−
28			−		
29				−	−
30				+	
31					−
32					

than join the grand coalition under $\varphi_i \equiv 1$. In addition, row 6 in Table 2 is a clear blocking of the grand coalition by {USA, OHI, EEC}.

Tables 2 and 3 are convenient tools for assessing the core properties. They also provide guidance for probing core allocations through searching algorithms. For example, USA is not content with the grand coalition under $\varphi_i \equiv 1$ (see Tables 2, 3). By increasing its social welfare weight relative to the "happy campers" in the grand coalition (such as CHN and ROW), USA will gain more in the grand coalition, others may concede a portion of their potential gains in the grand coalition. After adjusting social welfare weights, the whole spectrum of "hybrid" Nash equilibriums are solved again and resulted payoffs are placed in Tables 2 and 3 for incentive checking. Through such tatonnement searching routine on simplex $S = \{\{\varphi_i\} \big| \sum \varphi_i = m\}$, we can locate grand coalitions that possess the core properties. Under

Table 3 Welfare change from the Nash equilibrium ($\varphi_i \equiv 1$)

No.	USA	OHI	CHN	EEC	ROW
1	−	−	+	−	+
2	−	+	+	+	
3	−	−	+		+
4	−	+	+		
5	−	−		−	+
6	−	+		+	
7	−	−			+
8	−	+			
9	−		+	−	+
10	−		+	+	
11	−		+		+
12	−		+		
13	−			−	+
14	−			+	
15	−				+
16					
17		−	+	−	+
18		−	+	−	
19		−	+		+
20		−	+		
21		−		−	+
22		+		+	
23		−			+
24					
25			+	−	+
26			+	−	
27			−		+
28					
29				−	+
30					
31					
32					

such $\{\varphi_i\}$, all regions are better off in the grand coalition than in the Nash equilibrium and no sub-coalitions can block this grand coalition.[11]

4 Shrinking cores of the RICE model

Using the procedure outlined in the previous section, we identify a single core allocation in each RICE-m. The social welfare weights that map the optimal solution of RICE into the

[11] To focus on analyzing the core properties, simulation outputs related to the paths of all control and state variables of RICE are not presented in this paper. Please refer Yang (2008) for the algorithm details.

Table 4 Inner points of the core in RICE-m

	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	$\sum \varphi_i$
m = 2	1.802	0.198					2
m = 3	2.033	0.745	0.222				3
m = 4	1.737	0.353	1.752	0.158			4
m = 5	1.922	1.376	0.238	1.359	0.105		5
m = 6	1.574	1.805	1.094	0.202	1.229	0.096	6

core are reported in Table 4. Under these social welfare weights, the solutions of RICE-m have the properties that all signs in Table 2 are "−" and all signs in Table 3 are "+".[12] When obtaining these sets of weights, we look for the core allocations with stronger incentives in Table 3 such that regions are better off in *all* sub-coalitions than in the Nash equilibrium (noting that IR only requires "+" signs in the first row). By doing so, these allocations are "inner" or "central" points of respective cores.

In a well-behaved model of stock externality provision, the core relative to the initial endowments is not empty, just as Table 4 shows. In addition, the set of core allocations is not a singleton. In the simplex of social welfare weights, the area that maps to the core is assumed to be connected.[13] After one inner point of the core is identified, the remaining task is to measure the "size" or "volume" of the core that surrounds this point. For the purpose, we need to probe the boundary or circumference of the core on the simplex. Because the measurements are on simplex $S = \{\{\varphi_i\} | \sum \varphi_i = m\}$, the terms such as "size", "volume", and "boundary" are simply geometric.

To probe the boundary of the core, we use the following intuitive method (again, using RICE-5 as the example). We have identified an inner point of the core at $(\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5) = (1.922, 1.376, 0.238, 1.359, 0.105)$ in Table 4. Now we keep φ_2 to φ_5 at the current values and increase φ_1 (of USA) incrementally. Higher φ_1 increases the payoff of USA relative to other regions in the social optimum. After such an increase, the whole spectrums of "hybrid" Nash equilibriums are re-solved under this set of new weights and the regional welfare changes are assessed in Tables 2 and 3. If the core properties are intact, φ_1 is increased further for the next round of testing; if the core properties are violated, φ_1 is reduced slightly for the next round of testing. The probing accuracy is to the third decimal point. In RICE-5, the probing result is $\varphi_{1,up} = 2.751$. If φ_1 increases to 2.752, at least one other region would be unhappy in Tables 2 or 3. Namely, "+" signs appear in Table 2 and /or "−" signs appear in the first row of Table 3. Through such probing, a boundary point of the core is identified. Finally, this value, along with other fixed values of φ_2 to φ_5, are re-scaled to simplex $S = \{\{\varphi_i\} | \sum \varphi_i = m\}$, resulted in the second row of Table 5.

Similarly, lower the value of φ_1 while keeping other φ_i fixed will reduce the welfare of USA relative to other regions in the social optimum. We want to find the toleration limit of USA where it would be rather stay outside the grand coalition if its weight reduces any further. The probing result is $\varphi_{1,down} = 1.445$. If lowered to $\varphi_1 = 1.444$, USA prefers some sub-coalition or the Nash equilibrium to the grand coalition. Again, this value along with other φ_i is re-scaled to S, resulted in the third row of Table 5.

The coordinates of the above two boundary points are in "1 up" and "1 down" rows in Table 5. These two points are on the "opposite" sides of core boundary along the first axis.

[12] Due to homogenous signs, the tables of incentive checking do not reveal additional useful information. They are not presented to save space.

[13] This assertion is a hypothesis. No proofs have been given in the literature.

Table 5 Vertexes of the core in RICE-5

	φ_1	φ_2	φ_3	φ_4	φ_5
Inner P.	1.922	1.376	0.238	1.359	0.105
1 up	2.360	1.180	0.204	1.166	0.090
1 down	1.597	1.521	0.263	1.502	0.116
2 up	1.821	1.566	0.226	1.288	0.099
2 down	2.217	0.819	0.275	1.568	0.121
3 up	1.894	1.356	0.308	1.339	0.103
3 down	1.964	1.406	0.135	1.388	0.107
4 up	1.784	1.277	0.221	1.621	0.097
4 down	2.238	1.602	0.277	0.760	0.122
5 up	1.893	1.355	0.234	1.339	0.178
5 down	1.926	1.379	0.239	1.362	0.094

Table 6 Vertexes of the core in RICE-6

	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6
Inner P.	1.574	1.805	1.094	0.202	1.229	0.096
1 up	1.993	1.634	0.990	0.183	1.113	0.087
1 down	1.229	1.946	1.179	0.218	1.325	0.104
2 up	1.479	2.057	1.028	0.190	1.155	0.090
2 down	1.876	1.000	1.304	0.241	1.465	0.114
3 up	1.500	1.720	1.325	0.193	1.171	0.092
3 down	1.724	1.977	0.628	0.221	1.346	0.105
4 up	1.552	1.780	1.079	0.283	1.212	0.095
4 down	1.593	1.827	1.107	0.132	1.244	0.097
5 up	1.514	1.737	1.053	0.194	1.410	0.092
5 down	1.772	2.032	1.231	0.227	0.630	0.108
6 up	1.553	1.781	1.079	0.199	1.213	0.175
6 down	1.577	1.808	1.096	0.202	1.231	0.086

We can acquire 8 more such boundary points along 4 other axis. Numerical results of these boundary points from the probing are in Table 5. Using the same method, we obtain 12 boundary points in RICE-6; 8 boundary points in RICE-4; and 6 boundary points in RICE-3. They are reported in Tables 6, 7, and 8, respectively. RICE-2 composes a special case. Increasing φ_1 is equivalent to decreasing φ_2 and vice versa. The simplex of a two-region model is a one-dimension segment in two-dimension plane. It is AB in Fig. 1. The probing procedure can identify points C and D on AB. The inner point of the core is between C and D. In RICE-2, C = (1.841, 0.159) and D = (1.738, 0.262), according to the probing procedure.

Locating 2m points on boundaries of the core in $(m - 1)$-dimensional simplex provides useful information. Because these 2m boundary points are pair-wise orthogonal along the coordinates, they indicate the scope of the core. We use these 2m points as vertexes to span a convex hull H(m) on (m-1)-dimensional simplex $S = \{\{\varphi_i\} \big| \sum \varphi_i = m\}$.[14] The volume of this convex hull is an *approximate* measurement of the core size in the simplex. 2m vertexes

[14] S is an $(m - 1)$-dimension sub-space in m-dimension Euclidean space.

Table 7 Vertexes of the core in RICE-4

	φ_1	φ_2	φ_3	φ_4
Inner P.	1.737	0.353	1.752	0.158
1 up	2.056	0.303	1.505	0.136
1 down	1.441	0.399	1.981	0.179
2 up	1.645	0.546	1.659	0.150
2 down	1.811	0.198	1.826	0.165
3 up	1.246	0.253	2.387	0.113
3 down	2.165	0.440	1.198	0.197
4 up	1.702	0.346	1.717	0.235
4 down	1.752	0.356	1.767	0.124

Table 8 Vertexes of the core in RICE-3

	φ_1	φ_2	φ_3
Inner P.	2.033	0.745	0.222
1 up	2.348	0.502	0.150
1 down	1.940	0.816	0.243
2 up	1.924	0.866	0.210
2 down	2.310	0.438	0.252
3 up	1.994	0.731	0.275
3 down	2.077	0.761	0.161

Table 9 Volumes of the cores in RICE-m

	m = 2	m = 3	m = 4	m = 5	m = 6
V(H)	0.025797	1.40493E−3	3.45035E−5	1.71412E−7	1.58499E−9
V(S)	0.5	1.66667E−2	4.16667E−2	8.33333E−3	1.38889E−3
CS(m)	5.1594E−2	8.429E−3	8.2808E−4	2.0569E−5	1.1412E−6

are obtained through expansion from an inner point of the core. Therefore, the internal points of H(m) possess the core properties. If any point in H(m) is not on the core at all, it is on the surface of this convex hull spanned by two or more vertexes. The volume metric of such areas is 0 because they all collapse into lower dimensions. Volume of H(m) is a conservative measure of the core in the simplex because we only use the sufficient conditions of the core properties. We do not know how precise the metrics of H(m) in measuring the size of the cores. Nevertheless, the measurements are consistent across m. Therefore, the subsequent comparisons across m are consistent too.

We used QHull, a share-ware for computational geometry developed by Barber et al. (1996), to calculate the volumes of H(m). More specifically, we calculate the ratios between volumes of convex hull H(m) and simplex S(m):

$$\mathrm{CS(m)} = \mathrm{Vol}\left(\mathrm{H(m)}\right)/\mathrm{Vol}\left(\mathrm{S(m)}\right) \tag{10}$$

Metric CS(m) measures the size of the core relative to the entire efficiency set. Volumes are defined as the space inside the convex hull spanned by vertexes and the origin. It overcomes

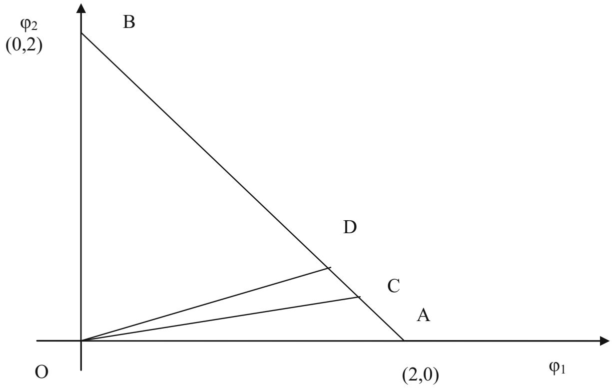

Fig. 1 Illustration of "size" of core in 2-dimension

the dimensionality problems and is comparable across all dimensions. Figuratively speaking, CS(m) is the volume ratio between a "thin crystal cone" (H(m)) and a "thick pyramid" (S(m)).

For example, the volume of S(5) is the volume inside the unit simplex (1, 0, 0, 0, 0), (0, 1, 0, 0, 0), (0, 0, 1, 0, 0), (0, 0, 0, 1, 0), (0, 0, 0, 0, 1), and the origin (0, 0, 0, 0, 0); the volume of H(5) is the volume inside normalized (coordinate values add up to 1, not to 5) convex hull spanned by the 10 points in Table 5, and the origin (0, 0, 0, 0, 0). When m = 2, CS(2) is the ratio between areas of triangles ΔOCD and ΔOAB in Fig. 1.[15]

The results of volume calculations are in Table 9. The outcome is clear-cut: *the core is shrinking as the number of regions increases*. In addition, the shrinking scale is roughly 10^{-1} as the number of regions increases by 1 in the range m $=$ 2, 3, 4, 5, and 6. However, we should point out that the results do not infer any shrinking patterns beyond m $>$ 6. Particularly, no conclusion should be drawn on the asymptotic scenario from the calculations here.

5 Policy implications of shrinking core on IEA negotiation

Several important policy implications can be drawn from testing of the shrinking cores in the RICE model empirically. First, the core is small in RICE. It is small in aggregate models with small numbers of regions; it would be even smaller in the RICE model with more regions, if we could induce the shrinking core hypothesis into higher dimensions. One can visualize such smallness by conducting the following thought experiment: what is the chance for you to hit the core when throwing a "dart" randomly onto the simplex of efficiency frontier? The answer would be "it is highly unlikely." Decades ago, Starrett (1973) pointed out that "core is a very 'small' concept" in the economy with externalities when discussing various blocking rules and restrictions of coalitions. Therefore, the small core in the presence of externality is not a new discovery. In this paper, the smallness of the core is exemplified and quantified in a well-known empirical model of externality provisions.

The smallness of core in RICE implies that consensus to form a grand coalition is intrinsically difficult for climate change. Post-Kyoto debates on climate change, often futile, reflect such dilemma. The "size" of the core depends partially on the location of the Nash equilibrium. We use Fig. 2 to illustrate the argument. In a two-agent economy of externality, N_1 is a

[15] Calculations show that normalization and without normalization have the same CS ratio. Normalization is used for safety checking when using unfamiliar shareware.

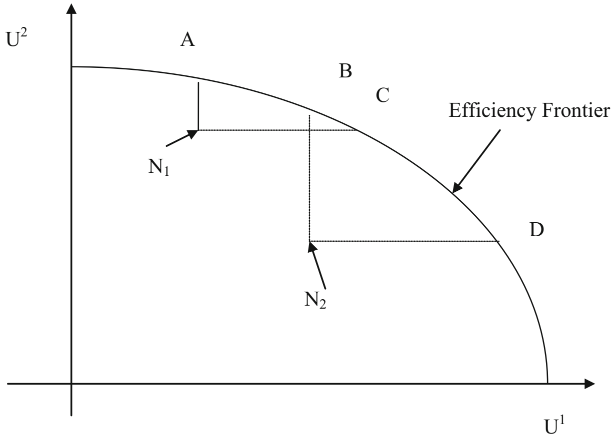

Fig. 2 Illustration of core and efficiency frontier in 2-dimension

Nash equilibrium allocation inside the efficiency frontier. AC on the efficiency frontier is the core with respect to N_1. If the Nash equilibrium is at N_2, BD on the efficiency frontier is the core. Apparently, BD is greater than AC. If the room for welfare improvement between the Nash equilibrium and efficiency allocations is large, the “size” of the core is relatively large. Efficiency improvements are incremental or marginal in the small core. In climate change, GHG mitigation costs are sizeable and avoided climate damages are highly uncertain. The gains by adopting optimal GHG mitigation policies are relatively small in terms of a region’s GDP level. Therefore, the “gap” between the Nash equilibrium and efficiency allocations is small. The small core in the RICE model is an evidence of the above argument.

In complicated systems with externalities, the number of agents also contributes to the smallness of the core. It is easier to find gaining opportunities by forming sub-coalitions among agents, as m increases. The core properties of the grand coalition require that they cannot be blocked by *any* sub-coalition. Such properties are precious and rare when the number of agents is large. Burden-sharing arrangement in the core has to please all agents. As shown here, an agent’s burden shift, reflected by changing φ_i, affect remaining m-1 agents’ burdens on $S = \{\{\varphi_i\} \big| \sum \varphi_i = m\}$ through many channels in the system.

However, when the number of the agents is low (such as in the range of this exercise), the smallness of the core might be helpful in the negotiation of IEA. The payoffs in the Nash equilibrium are the reserve payoffs for all agents. They are common knowledge. In IEA negotiations, a rational agent would put forward burden-sharing offers near his Nash equilibrium position and be considerate of other agents’ reserve payoffs. Because the room for welfare improvements is small, the distance between the initial proposal and unanimous final agreement is not far apart. In a recent study (Yang and Sirianni 2010), policy suggestions are similar to this notion.

The second implication is directly from the shrinking core observation. Large number of regions is a hindrance to IEA negotiation. When there are too many regions, the core allocations that all regions are happy with would be extremely difficult to find. Although we have successful precedent like the Montreal Protocol on phasing out CFC, it is rather an exception than a norm. The costs of stop using CFC are low when the substitute technologies are readily available. The damage of ozone layer depletion is imminent and severe. In the language used in this research, either the size of the core relative to the initial endowments is large, or the Nash equilibrium is very close to the core allocations (as Barrett 2003 inferring), in the case of ozone layer depletion. Despite numerous regions, common grounds can still be found (after the core has been shrunk). Nonetheless, climate change is a different story.

The shrinking core observations here have connection with other scholars' studies on coalition stability and number of players in IEAs. Difficulties in forming the grand coalition lead to empirical studies of stable sub-coalitions, such as Buchner and Carraro (2009). In the literature, no conclusions on the coalition stability have been drawn for the grand coalition with n heterogeneous agents. The shrinking core property is one of the reasons preventing stable coalitions.

The third implication comes from the properties of the core. Throughout our construction and analysis, transfers are not allowed or required. In fact, they are not mentioned until now because transfers in NTU game setting are generally not allowed. Given the nature of environmental externalities, regions do not need to provide or receive transfers to achieve efficiency and to be better off in the grand coalition. Nevertheless, the chances to find such common ground, i.e., the core property, are small. To promote the formation of grand IEA, we may ask some regions to sacrifice in order to "expand" the core. Expanding the core in the neighborhood of its boundary would hurt some regions' payoffs in the grand coalition. In reality, such sacrifices might be the promises of additional GHG mitigation unilaterally; *ex post* transfers from rich to poor; joint implementation (JI) arrangements, etc. In the modeling context, the sufficient condition we imposed on Table 2 can be relaxed. We can allow some mixed signs in the grand coalition and sub-coalitions (but still no sweeping "+" signs in any rows). Through compensations (transfers) inside the coalition, the boundary of the core expands.[16]

The smallness of the core and the shrinking core call for policy intervention in IEA formation (successful or not is a different matter). Consensus ground (the core allocations) can be substantially larger, if some regions concede portion of their efficiency gains *ex ante*. The room for aggregate gains is always there. The prerequisite for gaining is full participation in the grand IEA. Otherwise, all regions lose by holding the non-cooperative Nash equilibrium position.

6 Concluding remarks

The core is a very important solution concept in cooperative game and coalition theory. In empirical works, identifying the set of the core allocations is useful for both economists and policy makers, as this study has shown. The methods in this paper transform the issues of measuring the scope of the core into examining the social welfare weights of social optimum of externality provisions. Elusive core properties in a complex dynamic game turn into transparent geometric characteristics of a simplex. By examining the core quantitatively and constructively in a cooperative game embedded in the RICE model, this paper measures the "sizes" of the core in RICE with different number of regions. The shrinking core hypothesis for an economy with externalities is proven true in RICE through numerical simulations. The findings imply that it is difficult to form a voluntary global coalition in dealing with climate change when there are many regions involved.

Quantifying the measurements of the core in complicated environmental systems has many potential extensions and broader applications. Interesting issues includes: the relationship between the core and mitigation costs/environmental damages; altruism and the core;

[16] In NTU game, such internal transfers may not necessarily "dollar for a dollar."

renegotiations and the core in closed-loop dynamics, etc. Those tasks can be developed from the methodologies adopted in this paper.

This paper also indicates the policy implications of the shrinking core properties shown in RICE. The grand environmental coalitions are more and more difficult to bind together if the number of participants increases. Nevertheless, the grand coalition is not a "mission impossible." Especially if rich countries concede with moderate sacrifices to enlarge the core, all-inclusive environmental coalitions are more likely to form.

Appendix

The Description of the RICE-m Model.

1. The RICE model as a social planner's problem (an optimal control problem):

$$\underset{\{I_i(t),\, \mu_i(t)\}}{\text{Max}} W = \sum_{i=1}^{m} \varphi_i U_i = \sum_{i=1}^{m} \int_0^T \varphi_i L_i(t)\, Log\left(\frac{C_i(t)}{L_i(t)}\right) e^{-\delta t} dt,$$

$$\sum_{i=1}^{m} \varphi_i = m, \quad m = 2, 3, 4, 5, 6. \quad 0 < \delta < 1. \tag{11}$$

$$s.t. \quad Q_i(t) = A_i(t) K_i(t)^{\gamma} L_i(t)^{1-\gamma} \tag{12}$$

$$Y_i(t) = \Omega_i(t) Q_i(t) \tag{13}$$

$$C_i(t) = Y_i(t) - I_i(t) \tag{14}$$

$$\dot{K}_i(t) = I_i(t) - \delta_K K_i(t), \quad 0 < \delta_K < 1. \tag{15}$$

$$E_i(t) = (1 - \mu_i(t)) \sigma_i(t) Q_i(t), \quad 0 \le \mu_i(t) \le 1. \tag{16}$$

$$\Omega_i(t) = \frac{1 - b_{1,i} \mu_i(t)^{b_{2,i}}}{1 + a_{1,i} T_1(t)^{a_{2,i}}}, \quad \text{(In (12) to (17), i = 1, ..., m.)} \tag{17}$$

$$\dot{M}(t) = \beta_{11} M(t) + \beta_{13} U_U(t) + \sum_{i=1}^{m} E_i(t) \tag{18}$$

$$\dot{M}_L(t) = \beta_{22} M_L(t) + \beta_{23} M_U(t) \tag{19}$$

$$\dot{M}_U(t) = \beta_{31} M(t) + \beta_{32} M_L(t) + \beta_{33} M_U(t) \tag{20}$$

$$\dot{T}_1(t) = \varepsilon_{11} T_1(t) + \varepsilon_{12} T_2(t) + \varepsilon_3 F(t) \tag{21}$$

$$\dot{T}_2(t) = \varepsilon_{22} (T_2(t) - T_1(t)) \tag{22}$$

$$F(t) = \eta_1 Log(M(t)) - \eta_2 + O(t) \tag{23}$$

Definitions of variables:

U_i	Present value of intertemporal utility of region i;
$Q_i(t)$	Production function of region i;
$Y_i(t)$	Adjusted production function of region i;
$C_i(t)$	Consumption function of region i;
$K_i(t)$	Capital stock level of region i;
$I_i(t)$	Investment function of region i (control variable);
$E_i(t)$	GHG emission of region i;
$\mu_i(t)$	GHG emission control rate of region i (control variable);
$\Omega_i(t)$	Adjustment function of GDP;

M(t)	GHG concentration (atmospheric);
M_L(t)	GHG concentration (deeper ocean);
M_U(t)	GHG concentration (upper ocean);
T_1(t)	Atmospheric temperature;
T_2(t)	Deep ocean temperature;
F(t)	Radiative forcing function.

Definitions of time-variant parameters:

L_i(t)	Labor (population) trend of region i;
A_i(t)	Total factor productivity trend of region i;
σ_i(t)	Exogenous trend of GHG emission/output ratio of region i;
O(t)	Exogenous radiative forcing;
φ_i(t)	Social welfare weight of region i.

2. The RICE-m model as an open-loop differential game:

$$\underset{\{I_i(t),\mu_i(t)\}}{Max} \int_0^T L_i(t)Log(C_i(t)/L_i(t))\, e^{-\delta t} dt, \quad 0 < \delta < 1, \quad i = 1, \ldots, m. \tag{24}$$

s.t. (12) to (17) and (19) to (23)

$$\dot{M}(t) = \beta_{11}M(t) + \beta_{13}U_U(t) + \sum_{j \neq i}^{m} \overline{E}_j(t) + E_i(t) \tag{18'}$$

References

Barber, C., Dobkin, D., & Huhdanpaa, H. (1996). The Quickhull algorithm for convex hulls. *ACM Transactions on Mathematical Software, 22*(4), 469–483. http://www.qhull.org.

Barrett, S. (1994). Self enforcing international environmental agreements. *Oxford Economic Papers*, *46*, 878–894.

Barrett, S. (2003). *Environment and statecraft: The strategy of environmental treaty-making*. Oxford: Oxford University Press.

Bergstrom, T. (1976). Regulations of externalities. *Journal of Public Economics*, *5*, 131–138.

Brooke, A., Kendrick, D., Meeraus, A., & Raman, R. (2004). *GAMS: A user's guide*. Washington DC: GAMS Development Corporation.

Buchner, B., & Carraro, C. (2009). Parallel climate blocs, incentives to cooperation in international climate negotiations. In R. Guesnerie & H. Tulkens (Eds.), *The design of climate policy*. Cambridge, MA: The MIT Press.

Carraro, C., & Siniscalco, D. (1993). Strategies for the international protection of the environment. *Journal of Public Economics*, *52*, 309–328.

Champsaur, P., Roberts, J., & Rosenthal, R. (1975). On the cores in economies with public goods. *International Economic Review*, *16*, 269–288.

Chander, P., & Tulkens, H. (1995). A core-theoretic solution for the design of cooperative agreements on transfrontier pollution. *International Tax and Public Finance*, *2*, 279–294.

Chander, P., & Tulkens, H. (1997). The core of an economy with multilateral environmental externalities. *International Journal of Game Theory*, *26*, 379–401.

d'Aspremont, C., Jacquemin, J., Gabszeweiz, J., & Weymark, J. A. (1983). On the stability of collusive price leadership. *Canadian Journal of Economics*, *16*, 17–25.

Debreu, G., & Scarf, H. (1963). A limit theorem on the core of an economy. *International Economic Review*, *4*, 235–246.

Eyckmans, J., & Tulkens, H. (2003). Simulating coalitionally stable burden sharing agreements for the climate change problem. *Resource and Energy Economics*, *25*, 299–327.

Finus, M. (2001). *Game theory and international environmental cooperation*. Cheltenham: Edward Elgar.

Finus, M., Rundshagen, B., & Eyckmans, J. (2014). Simulating a sequential coalition formation process for the climate change problem: First come, but second served? *Annals of Operations Research*, *220*(1), 5–23.

Foley, D. (1970). Lindahl's solution and the core of an economy with public goods. *Econometrica*, *38*, 66–72.

Muench, T. (1972). The core and the Lindahl equilibrium of an economy with a public good: An example. *Journal of Economic Theory*, *4*, 241–255.

Mas-Colell, A., & Silvestre, J. (1998). Cost share equilibria: A Lindahlian approach. *Journal of Economic Theory*, *47*, 239–256.

Nordhaus, W., & Yang, Z. (1996). A regional dynamic general equilibrium model of alternative climate change strategies. *American Economic Review*, *86*, 741–765.

Nordhaus, W. D., & Boyer, J. (2000). *Warming the world: Economic models of global warming*. Cambridge, MA: The MIT Press.

Osbrne, M., & Rubinstein, A. (1994). *A course in game theory*. Cambridge, MA: The MIT Press.

Ray, D. (2007). *A Game-theoretic perspective on coalition formation*. Oxford: Oxford University Press.

Roumasset, J. (1979). Sharecropping, production externalities, and the theory of contract. *American Journal of Agricultural Economics*, *61*(4), 640–647.

Shapley, L., & Shubik, M. (1966). Quasi-cores in a monetary economy with nonconvex preferences. *Econometrica*, *34*, 805–827.

Starrett, D. (1973). A note on externalities and the core. *Econometrica*, *41*(1), 179–183.

Yang, Z. (2008). *Strategic bargaining and cooperation in greenhouse gas mitigations: An integrated assessment modeling approach*. Cambridge, MA: The MIT Press.

Yang, Z., & Sirianni, P. (2010). Balancing contemporary fairness and historical justice: A 'quasi-equitable' proposal for GHG mitigations. *Energy Economics*, *32*, 1121–1130.