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(Q, r, L) model for stochastic demand with lead-time dependent partial backlogging

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Abstract The paper deals with an economic order quantity model for variable lead-time, order dependent purchasing cost, order size, reorder point and lead-time dependent partial backlogging. The average expected cost function is formulated by trading off setup cost, purchasing cost, lead-time crashing cost, inventory cost and costs of lost sale and partial backordering. In this cost function, order quantity, reorder point and lead-time are decision variables. The above average expected cost function is analysed by calculus method in light of both distribution-free and known distribution function. Numerical example is illustrated to justify our proposed model.

Keywords Order quantity · Reorder point · Lead-time · Backordering

1 Introduction

The economic order quantity (EOQ) model has a rich literature in inventory system. The (Q, r, L) model is more realistic and applicable in any business organisations, among several models. Our model is quite new and more appropriate in inventory literature, because order quantity dependent purchasing cost and lead-time dependent partial backlogging are newly introduced, unlike existing literature. Some noteworthy research works in this line of works are mentioned as follows.

The issue of lead time is common to all enterprises. It may be controlled adding crashing cost. Liao and Shyu (1991) discussed about lead time reduction in an inventory model considering lead time as a decision variable while order quantity is predetermined. Ben-Daya and Raouf (1994) investigated an inventory model both for variable lead time and order quantity.

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Thereafter, this model was studied extensively (Ouyang et al. 1996; Ouyang and Wu 1997, 1998) incorporating the lead time demand followed by Normal distribution or distribution free. Ouyang and Chuang (1999) developed an inventory model with quantity discount and partial backordering in which backordering rate was a random variable. Ouyang et al. (1999) considered a (Q, r, L) inventory model with defective items in an arrival lot, and asserted the objective function of expected total annual cost that is consisted of setup cost, non-defective holding cost, defective treatment cost, stock-out cost, inspecting cost, and lead time crashing cost. Wu (2000) studied the same model both for perfect and imperfect quantities of received order from the supplied, considering backorder and lost sales for variable lead time. Ouyang et al. (2003) investigated a stochastic periodic review inventory model involving controllable backorder discount. Chuang et al. (2004) generalized Ouyang et al. (1999) model by allowing setup cost as a decision variable in conjunction with order quantity, reorder point and lead time. Artalejo et al. (2006) studied extensively a continuous review (s, S) inventory system where the customers face the system out of stock, leave the service area and repeat their request after some random time. This stochastic model formulation was based on a bi-dimensional Markov process which was numerically solved to investigate the essential operating characteristics of the system. Yadavalli et al. (2006) derived long run total expected cost rate for a two commodity continuous review inventory system in which a buyer who intended to buy one particular commodity might also go for another commodity. Sivkumar and Gunaseelan (2009) established a continuous review perishable (s, S) inventory system in which the demands arrive according to a Markovian arrival process, In this model, the life time of each item in the stock and lead time of orders are considered to be independently distributed as exponential. Hsu and Lee (2009) discussed the optimal strategies of replenishment and lead-time reduction for integrated inventory system of single manufacturer and multiple-retailer in which the probability distribution of demand for each retailer is unknown but its mean and variance are known. Yue (2012) suggested a new direct proof of the tight range of the optimal solution range for the newsboy problem with known mean and standard deviation of demand distribution, using definition of the optimal solution only. Sivkumar et al. (2012) investigated a discrete-time inventory (s, S) model in which demands arrive according to a discrete Markovian process and the lead time is assumed to follow a discrete phase-type distribution. Yadavalli et al. (2012) studied a continuous review (s, S) policy inventory system with a finite source of customers and identical multiple servers in parallel, assuming the lead times for the orders to have independent and identical exponential distributions. Senapati et al. (2012) studied extensively literature review on lead time reduction in inventory control which is referred to the reader. Ma and Qiu (2012) investigated a joint decision problem of supplier and retailers' continuous review inventory system in which service level constrained is satisfied, based on the mean and the standard deviation of lead time demand. Sana (2013) developed a newsvendor type inventory model both for discrete and continuous cases incorporating two types of warehouses. Recently, Turgay et al. (2014) formulated a robust stochastic dynamic program to investigate the structural properties of a finite horizon, single product in discrete time inventory rationing problem, allowing random replenishment (production) opportunities. Olsson (2014) develop a new heuristic approach for evaluating and analyzing the proposed (R, Q) model for Poisson demand distribution of perishable items, assuming lifetimes of the products and lead times as fixed numbers.

In this model, the (Q, r, L) model is extended incorporating variable purchasing cost of the order quantity (Q), lead time (L) dependent partial backordering and lost sales. In stock out situation, patience of the customers depends on the waiting time of receiving their quantities. As a result, larger lead time increases the lost sale quantities. In this point of view, the partial backordering is dependent on lead time. The lead time is controlled here by adding crashing cost. The reorder point (r) and order quantity (Q) are decision variables. The demands of the customers during lead time and general inventory cycle follow the same distribution function. We first formulate the model for general distribution function. Then, the model is analysed for the case of distribution free. In numerical section, the model is tested for normal distribution and distribution free demand patterns.

2 Notation

The following notations are used to develop the proposed model.

$x =$ Random demand per unit time with p.d.f $f(x)$.
$\mu =$ Expected value (mean) of x.
$\sigma =$ Standard deviation of x.
$k =$ Safety stock risk factor.
$L_i =$ Length of lead time having i-components.
$a_i =$ Minimum duration of ith component.
$b_i =$ Normal duration of ith component.
$c_i =$ Crashing cost per unit time of ith component during $(b_i - a_i)$.
$A =$ Set up cost per cycle.
$h =$ Inventory holding cost per unit item per unit time.
$p(Q) =$ Purchasing cost per unit item.
$p^{Min} =$ Least cost of purchasing per unit item.
$p^{Max} =$ Maximum cost of purchasing per unit item.
$Q^{Max} =$ Maximum order quantity per order.
$\pi =$ Fixed penalty cost per unit item for shortages.
$\pi_0 =$ Marginal profit per unit item.
$L =$ Length of lead time, a decision variable.
$\beta(L) =$ Fraction of the demand backordered during shortage period. Here, $0 \leq \beta(L) \leq 1$.
$r =$ Level of reorder point, a decision variable.
$Q =$ Replenishment size, a decision variable.
$E(x) =$ Expected value of x.
$E(x-r)^+ = \int_r^\infty (x-r) f_x(x)\, dx$ where $f_x(x)$ is p.d.f with mean μL and standard deviation $\sigma\sqrt{L}$.

3 Formulation of the model

3.1 Model for known distribution function

In this model, an order of size Q is ordered when the inventory level reaches to the reorder point r. The lead time between placing and receiving of an order is L which is constituted by n mutually independent components. The minimum duration ith component is a_i and normal duration is b_i. Here, c_i $(i = 1, 2, \ldots, n)$ are crashing costs per unit time such that $c_1 \leq c_2 \leq \cdots \leq c_n$. Let $L_i$is the length of lead time with components $1, 2, \ldots,$ i and crashes to its minimum duration. In this situation, L_i can be expressed as $L_i = \sum_{j=1}^n b_j - \sum_{j=1}^i \left(b_j - a_j\right)$ with $L_0 = \sum_{j=1}^n b_j$. Lead time usually consists of the following components: order preparation, supplier lead time, order transit, delivery time, setup time and transportation time. This lead time is controllable. As for example, the effective communication media such as electronic data interchange system may speed up the order preparation and order transit. On the other hand, one may adopt special delivery by air instead of ordinary delivery by water to shorten the delivery time. For these actions to shorten lead time, extra money is certainly needed. Consequently, lead time is composed of n mutually independent components having different crashing cost of each component for reducing lead time. This cost is invested on equipment improvement, information technology, order expedite, or special shipping and handling to reduce the time span of lead time. More investment is needed to reduce the span of lead time that results in decreasing crashing cost with increases values of lead time (L).

Therefore, the lead time crashing cost, for $L \in \left[L_i, L_{i-1}\right]$, per cycle is

$$\alpha(L) = c_i\left(L_{i-1} - L\right) + \sum_{j=1}^{i-1} c_j\left(b_j - a_j\right) \tag{1}$$

During lead time stock out situation may occur because demand and lead time are unpredictable. Consequently, safety stock levels need to be determined to prevent stock-outs. To weight the risk of a stock out, we assign the desired probability of not running out of stock, using the Normal probability distribution. For example, if the value of safety stock risk factor (k) is 1.645 (90 % from the Normal probability table) then the risk of running out of stock during the lead time is 10 %(100 % − 90 % = 10 %). To formulate this policy into a specific safety stock level, the type of demand distribution during lead time is to be known. It is usually acceptable to consider that the demand during the lead time is normally distributed. Thus, safety stock is calculated by multiplying the risk factor (k) by the number of standard deviations (σ) and the square root of the lead time ($\sqrt{L}$). Therefore, *safety stock* $= k\sigma\sqrt{L}$. Thus, the reorder point $r =$ *expected demand during lead time* $+$ *safety stock* $= \mu L + k\sigma\sqrt{L}$, where k is the safety stock risk factor and μ is the mean of the distribution. The expected shortage at the end of L is given by

$$E\left(x-r\right)^+ = \int_r^{\infty}\left(x-r\right) f_x\left(x\right) dx$$

where $f_x(x)$ is p.d.f with mean μL and standard deviation $\sigma\sqrt{L}$. In this stock out situation, not all customers' demand are met. The demand of the customers who do have patience to wait is adjusted, i.e., $\beta(L)E\left(x-r\right)^+$ are backordered here. Now, $\beta\left(L\right) = e^{-\delta L}$ and lost sale quantity is $\{1-\beta(L)\}\,E(x-r)^+ = \left(1-e^{-\delta L}\right)E(x-r)^+$. When the lead time is too long, i.e., $L \to \infty$, then whole backordered quantity is unsold whereas all quantities of backordered are sold when $\delta \to 0$, i.e., the mean time of patience to wait $\left(\frac{1}{\delta}\right)$ tends to infinity. For normal distribution, $E\left(x-r\right)^+ = \sigma\sqrt{L}\psi(k)$ where $(k) = \phi\left(k\right) - k[1-\varphi\left(k\right)]$, ϕ is standard normal density function and ϕ is standard normal distribution function.

The expected inventory cost is (see Fig. 1)

$$\begin{aligned} I_c &= h\left[\int_{-\infty}^{\infty}\int_0^{\frac{Q}{\mu}}\left(Q-xt\right)dt\ f\left(x\right)dx + \frac{Q}{\mu}\left(r-\mu L\right) + \frac{Q}{\mu}\left(1-e^{-\delta L}\right)E\left(x-r\right)^+\right] \\ &= h\left[\frac{Q^2}{2\mu} + \frac{Q}{\mu}\left(r-\mu L\right) + \frac{Q}{\mu}\left(1-e^{-\delta L}\right)E\left(x-r\right)^+\right] \end{aligned} \tag{2}$$

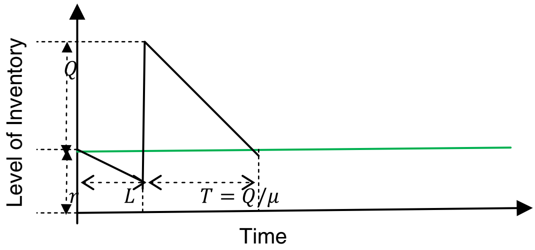

Fig. 1 Level of inventory versus time

The cost for shortages is

$$S_c = \left[\pi + \left(1 - e^{-\delta L}\right)\pi_0\right] E(x-r)^+$$
$$= \bar{\pi} E(x-r)^+, \quad \pi + \left(1 - e^{-\delta L}\right)\pi_0 = \bar{\pi}(\text{let}) \tag{3}$$

Purchasing cost per unit item is $p(Q) = p^{Max} - \varepsilon Q$. For feasibility of the model,

$$p^{Max} - \varepsilon Q \geq p^{Min} \overset{yields}{\longrightarrow} Q \leq \left(\frac{p^{Max} - p^{Min}}{\varepsilon}\right) = Q^{Max}.$$

Therefore, the total purchasing cost of order size Q is

$$w^p = \left(p^{Max} - \varepsilon Q\right) Q \tag{4}$$

Therefore, the average expected cost of the whole system is

$$AEC(Q, r|L) = \frac{A\mu}{Q} + \frac{w^p\mu}{Q} + \frac{I_c\mu}{Q} + \frac{S_c\mu}{Q} + \frac{\alpha(L)\mu}{Q} = \frac{A\mu}{Q} + \left(p^{Max} - \varepsilon Q\right)\frac{Q\mu}{Q}$$
$$+ \frac{h\mu}{Q}\left[\frac{Q^2}{2\mu} + \frac{Q}{\mu}(r - \mu L) + \frac{Q}{\mu}(1 - e^{-\delta L})E(x-r)^+\right]$$
$$+ \frac{\mu}{Q}\bar{\pi}E(x-r)^+ + \frac{\mu}{Q}\left[c_i(L_{i-1} - L) + \sum\nolimits_{j=1}^{i-1} c_j(b_j - a_j)\right]$$
$$= \frac{A\mu}{Q} + \left(p^{Max} - \varepsilon Q\right)\mu + h\left[\frac{Q}{2} + (r - \mu L) + \left(1 - e^{-\delta L}\right)E(x-r)^+\right]$$
$$+ \frac{\mu}{Q}\bar{\pi}E(x-r)^+ + \frac{\mu}{Q}\left[c_i(L_{i-1} - L) + \sum\nolimits_{j=1}^{i-1} c_j(b_j - a_j)\right] \tag{5}$$

Now, our objective is to minimize $AEC(Q, r|L)$subject to the constraints $Q > 0, r > 0$, $Q \leq Q^{Max}$, i.e.,

$$Min\, AEC(Q, r|L), \text{ such that } Q > 0,\ r > 0,\ Q \leq Q^{Max} \tag{6}$$

Now differentiating $AEC(Q, r|L)$ with respect to Q and r variables, keeping L as constant, we have

$$\frac{\partial}{\partial Q}\{AEC(Q,r|L)\} = -\frac{A\mu}{Q^2} + \frac{h}{2} - \varepsilon\mu - \frac{\mu}{Q^2}\bar{\pi}E(x-r)^+ - \frac{\mu\alpha(L)}{Q^2} \tag{7}$$

$$\frac{\partial}{\partial r}\{AEC(Q,r|L)\} = h\left\{1 + \left(1-e^{-\delta L}\right)\left(\frac{\partial E}{\partial r}\right)\right\} + \frac{\mu}{Q}\bar{\pi}\left(\frac{\partial E}{\partial r}\right) \tag{8}$$

$$\frac{\partial^2}{\partial Q^2}\{AEC(Q,r|L)\} = \frac{2\mu}{Q^3}\left[A + \bar{\pi}E(x-r)^+ + \alpha(L)\right] \tag{9}$$

$$\frac{\partial^2}{\partial r^2}\{AEC(Q,r|L)\} = \left[h\left(1-e^{-\delta L}\right) + \frac{\mu\bar{\pi}}{Q}\right]\left(\frac{\partial^2 E}{\partial r^2}\right) \tag{10}$$

$$\frac{\partial^2}{\partial r\partial Q}\{AEC(Q,r|L)\} = -\frac{\mu\bar{\pi}}{Q^2}\left(\frac{\partial E}{\partial r}\right) = \frac{\mu\bar{\pi}}{Q^2}\int_r^\infty f_x(x)\,dx \tag{11}$$

For optimum values of $AEC(Q, r|L)$,

$$\frac{\partial}{\partial Q}\{AEC(Q,r|L)\} = 0 = \frac{\partial}{\partial r}\{AEC(Q,r|L)\}$$

provide

$$Q = \sqrt{\frac{2\mu\left[A + \bar{\pi}E(x-r)^+ + \alpha(L)\right]}{(h-2\varepsilon\mu)}}, \quad h > 2\varepsilon\mu \text{ (for feasibility of the model)} \tag{12}$$

and

$$\left(\frac{\partial E}{\partial r}\right) = \frac{-hQ}{\left[hQ\left(1-e^{-\delta L}\right) + \mu\bar{\pi}\right]} \xrightarrow{yields} \int_r^\infty f_x(x)\,dx = \frac{hQ}{\left[hQ\left(1-e^{-\delta L}\right) + \mu\bar{\pi}\right]}. \tag{13}$$

Solving Eqs. (12) and (13), we have the optimal solution (Q^*, r^*) at which

$$\frac{\partial^2}{\partial Q^2}\{AEC(Q,r|L)\} = \frac{(h-2\varepsilon\mu)}{Q} > 0 \quad \text{for } h > 2\varepsilon\mu$$

and

$$\frac{\partial^2}{\partial r^2}\{AEC(Q,r|L)\}$$
$$= \left[h\left(1-e^{-\delta L}\right) + \frac{\mu\bar{\pi}}{Q}\right]\left(\frac{\partial^2 E}{\partial r^2}\right) = \left[h\left(1-e^{-\delta L}\right) + \frac{\mu\bar{\pi}}{Q}\right]f_x(r) > 0.$$

Now the optimal solution (Q^*, r^*) will give minimum value of $AEC(Q, r|L)$ if

$$\left[\frac{\partial^2}{\partial Q^2}\{AEC(Q,r|L)\}\right] \times \left[\frac{\partial^2}{\partial r^2}\{AEC(Q,r|L)\}\right] - \left[\frac{\partial^2}{\partial Q\partial r}\{AEC(Q,r|L)\}\right]^2 > 0,$$
$$\text{i.e.,} \quad \frac{(h-2\varepsilon\mu)}{Q}\left[h\left(1-e^{-\delta L}\right) + \frac{\mu\bar{\pi}}{Q}\right]f_x(r) - \frac{(\mu\bar{\pi})^2}{Q^4}\left[\int_r^\infty f_x(x)\,dx\right]^2 > 0$$

holds at $(Q^*,\ r^*)$, as

$$\frac{\partial^2}{\partial Q^2}\{AEC(Q,r|L)\} \quad \text{and} \quad \frac{\partial^2}{\partial r^2}\{AEC(Q,r|L)\}$$

are both positive at (Q^*, r^*). Here, the optimal solutions are obtained for $L = L_i (i = 1, 2, \ldots, n)$. Among these optimal solutions, the solution which provides minimum value of the objective function is our required optimal solution.

3.2 Distribution-free case

In many practical situations, distribution function of demand rate does not follow the known distribution functions. In such cases, mean and standard deviation are used to approximate the objective function. Now, using $E(x-r)^+ \leq \frac{1}{2}\left[\sqrt{\sigma^2 L^2 + (r-\mu L)^2} - (r-\mu L)\right]$ (see Ouyang and Chuang 2000) in the average expected cost function, we have the maximum value as follows

$$\begin{aligned} &Max\ AEC\,(Q, r, L) \\ &\quad = \frac{A\mu}{Q} + h\left[\frac{1}{2}Q + (r-\mu L) + \frac{1}{2}\left(1-e^{-\delta L}\right)\left\{\sqrt{\sigma^2 L^2 + (r-\mu L)^2} - (r-\mu L)\right\}\right] \\ &\qquad + \left(p^{Max} - \varepsilon Q\right)\mu + \frac{\mu\bar{\pi}}{2Q}\left\{\sqrt{\sigma^2 L^2 + (r-\mu L)^2} - (r-\mu L)\right\}. \end{aligned}$$

Our objective is to minimize $Max\ AEC\,(Q, r, L)$, i.e.,

$$Min\ Max\ AEC\,(Q, r, L)\ \text{ such that } Q > 0,\ r > 0,\ Q \leq Q^{Max} \tag{14}$$

Here,

$$\frac{\partial}{\partial Q}\left[Min\ Max\ AEC\,(Q, r, L)\right] = 0 = \frac{\partial}{\partial r}\left[Min\ Max\ AEC\,(Q, r, L)\right]$$

provide us

$$Q = \sqrt{\frac{2\mu\left[A + \frac{\bar{\pi}}{2}\left\{\sqrt{\sigma^2 L^2 + (r-\mu L)^2} - (r-\mu L)\right\} + \alpha(L)\right]}{h - 2\varepsilon\mu}} \tag{15}$$

and

$$\left[\frac{(r-\mu L)}{\sqrt{\sigma^2 L^2 + (r-\mu L)^2}} - 1\right]\left[\left(1-e^{-\delta L}\right)h + \frac{\mu\bar{\pi}}{Q}\right] + 2h = 0 \tag{16}$$

Solving Eqs. (15) and (16), we have the optimal solution (Q^*, r^*), At this value,

$$\frac{\partial^2}{\partial Q^2}\left[Min\ Max\ AEC\,(Q, r, L)\right] = \left(\frac{h - 2\varepsilon\mu}{Q}\right) > 0 \quad \text{as } h > 2\varepsilon\mu$$

and

$$\frac{\partial^2}{\partial r^2}\left[Min\ Max\ AEC\,(Q, r, L)\right] = \frac{\sigma^2 L^2}{2\left\{\sigma^2 L^2 + (r-\mu L)^2\right\}^{3/2}}\left\{h\left(1-e^{-\delta L}\right) + \frac{\mu\bar{\pi}}{Q}\right\} > 0$$

hold. Now, (Q^*, r^*) is our required optimal solution if

$$\frac{\partial^2}{\partial Q^2}\left[Min\ Max\ AEC\,(Q, r, L)\right] \times \frac{\partial^2}{\partial r^2}\left[Min\ Max\ AEC\,(Q, r, L)\right]$$

$$-\left\{\frac{\partial^2}{\partial Q \partial r}\left[Min\ Max\ AEC\,(Q, r, L)\right]\right\}^2 > 0,$$

$$\text{i.e.,} \left(\frac{h - 2\varepsilon\mu}{2Q\left\{\sigma^2 L^2 + (r-\mu L)^2\right\}^{3/2}}\right)\sigma^2 L^2 \left\{h\left(1 - e^{-\delta L}\right) + \frac{\mu\bar{\pi}}{Q}\right\}$$

$$-\frac{\mu^2\bar{\pi}^2}{2Q^2}\left\{\frac{r-\mu L}{\sqrt{\sigma^2 L^2 + (r-\mu L)^2}} - 1\right\}^2 > 0$$

holds. In this case, the optimal solutions are obtained for $L = L_i (i = 1, 2, \ldots, n)$. Among these optimal solutions, the solution which gives minimum value of the objective function is our required optimal solution. When the optimal solutions do not satisfy the constraints, then the constrained problem is solved by Kuhn–Tucker method.

4 Numerical example

The values of the parameters in appropriate units are considered as follows: $\mu = 15$ units/week, $\sigma = 7$ units/week, $Q^{Max} = 90$ units, $\delta = 0.8$, $\varepsilon = \$0.5$, $p^{Max} = \$75$, $p^{Min} = \$30$, $A = \$200$ per order, $h = \$20$, $\pi = \$50$, $\pi_0 = \$150$ and probability distribution of the demand is

$$f(x) = \left\{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \middle| -\infty < x < \infty\right\}.$$

The lead times with three components with its numerical data are shown in Table 1.

The optimal solutions for different lead times are shown in Table 2. Among these solutions, the required optimal solutions (Table 2) are ($L^* = 3$ weeks, $r^* = 50.1502$ units, $Q^* = 67.4888$ units, $AEC^* = \$1{,}614.40$) and ($L^* = 3$ weeks, $r^* = 34.0616$ units, $Q^* = 119.503$ units, $AEC^* = \$2{,}184.42$) for the cases of Normal distribution and distribution free respectively.

In Table 2, it is shown that the average expected cost decreases with decreases in lead time although the crashing cost increases with decreases in lead time. This fact occurs due to lead time dependent backlogging. In this case, lost sales quantity increases with increases in lead time. To avoid more cost for larger quantity of lost sales due to higher lead time, lead time decreases in spite of higher crashing cost. When the fraction β is independent on L, the expected average cost decreases (Table 3) with increases in β that results in higher

Table 1 Lead time data

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Table 2 Optimal solutions for different lead times (in week)

For normal distribution model					For distribution free model	
(i)	(L_i)	$\alpha(L_i)$	(r_i, Q_i)	$AEC(Q_i, r_i, L_i)$	(r_i, Q_i)	$AEC(Q_i, r_i, L_i)$
0	8	0	(127.198, 83.4983)	1,782.61	(51.5305, 262.171)	3,960.63
1	6	5.6	(97.0572, 76.5811)	1,726.07	(54.8133, 196.345)	3,119.49
2	4	22.4	(66.2422, 69.5432)	1,655.31	(43.2347,143.448)	2,476.99
3	3	57.4	(50.1502, 67.4888)	1,614.40	(34.0616, 119.503)	2,184.42

Table 3 Optimal solutions for different lead times (in week) when β is independent on L

β	For normal distribution model				For distribution free model			
	L	r	Q	$AEC(Q, r, L)$	L	r	Q	$AEC(Q, r, L)$
0.0	3	51.1308	66.3088	1,616.70	3	36.6316	115.5300	2,184.12
0.5	3	41.9614	80.1555	1,530.08	3	34.0456	100.4650	1,848.94
0.8	4	43.8196	97.5373	1,357.24	8	54.9179	164.6900	1,472.36
1.0	4	46.6248	66.3088	1,626.70	8	74.2402	116.1378	1,055.00

backordering. In such cases, lead time increases to reduce crashing cost. These features are quite realistic in business economics.

5 Conclusion

In a competitive marketing system, demand rate of the customers is uncertain in practice. The lead time, time gap between placing and receiving of an order, plays an important in EOQ modelling. Quite often, the demand of the customers who do not have patience to wait have are lost sales. Consequently, partial backordering varies with lead time. Moreover, larger lead time causes negative impression of the customers that reduces customers' demand. The objective of this paper is to find out optimal lead time, reorder point and order quantity in continuous review inventory model with a mixture of lead time dependent lost sales and backorders, considering the purchasing cost per unit item as a decreasing function of order quantity. The demand of the customers follows same type of distribution throughout the whole cycle. First, we formulate the model considering general distribution, then, it is studied extensively for normal distribution function. However, this model can be applied for any type of continuous distribution functions. This model helps to determine optimal safety stock, order quantity, reorder level and lead time such that the expected average cost by trading off purchasing cost, crashing cost, inventory cost, shortage costs of lost sale and backordering is minimized. Moreover, this model suggests a manager of a business organization how to obtain an optimal strategy when probability distribution of the customers' demand is unknown. In such cases, distribution free model provides an approximate optimal solution of the problem whereas the known distribution model provides better solution than the previous one. The new contribution of the proposed model is incorporation of order quantity dependent purchasing cost and lead time dependent backordering in the existing elegant (Q, r, L) model in continuous review inventory system.

The proposed model can be extended further for discrete type demand distribution, incorporating effect of different price settings demand over finite time horizon. In production-inventory management system, this model may be studied extensively incorporating delay in payment and supply disruption in future.

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