Ann Oper Res (2014) 220:181–204
DOI 10.1007/s10479-011-0844-0

Dynamically consistent collaborative environmental management with production technique choices

David W.K. Yeung

Published online: 19 January 2011

Abstract Adoption of environment-preserving production technique plays a key role to effectively solving the continual worsening global industrial pollution problem. Due to the global nature of environmental effects and trade, unilateral response on the part of one nation is often ineffective. Cooperation in environmental management holds out the best promise of effective action. For cooperation over time to be credible, a dynamic consistency condition which requires the agreed-upon optimality principle to remain in effect throughout the collaboration duration has to hold. In this paper, we present a cooperative dynamic game of collaborative environmental management with production technique choices. A dynamically consistent cooperative scheme is derived. It is the first time that dynamically consistent solution is obtained for dynamic games in collaborative environmental management with production technique choices. The analysis widens the scope of study in global environmental management.

Keywords Cooperative dynamic games · Time consistency · Production technique choices · Environmental management

1 Introduction

After several decades of rapid technological advancement and economic growth, alarming levels of pollutions and environmental degradation are emerging all over the world. Even

This research was supported by the European Union TOCSIN Research Grant and the HKSAR Research Grant Council (grant No. 32-07-028).

D.W.K. Yeung (✉)
SRS Consortium for Advanced Study in Cooperative Dynamic Games, Hong Kong Shue Yan University, Hong Kong, Hong Kong
e-mail: dwkyeung@hksyu.edu

D.W.K. Yeung
Center of Game Theory, Faculty of Applied Mathematics-Control Processes, St Petersburg State University, Petrodvorets, St Petersburg, 198904, Russia

substantial reduction in industrial output using conventional production technique would only slow down the rate of increase and not reverse the trend of continual pollution accumulation. Adoption of environment-preserving technique plays a central role to solving the problem effectively. Due to the geographical diffusion of pollutants and the globalization of trade, unilateral response on the part of one nation or region is often ineffective. Though cooperation in environmental control holds out the best promise of effective action, limited success has been observed. Conventional multinational joint initiatives like the Kyoto Protocol or the Copenhagen Accord can hardly be expected to offer a long-term solution because (i) the plans focus only on emissions reduction which is unlikely be able to offer an effective mean to halt the accelerating trend of environmental deterioration, and (ii) there is no guarantee that agreed-upon optimality principle could be maintained throughout the entire duration of cooperation.

Dynamic game theory provides an effective tool to study pollution control problems and to analyze the interactions between the participants' strategic behaviors and dynamic evolution of pollution. Applications of noncooperative differential games in environmental studies can be found in Yeung (1992), Dockner and Long (1993), Tahvonen (1994), Stimming (1999), Feenstra et al. (2001) and Dockner and Leitmann (2001). Dockner and Nishimura (1999) and Rubio and Ulph (2007) presented discrete-time dynamic game for pollution management and Dutta and Radner (2006) presented a discrete-time dynamic game to study global warming. Cooperative differential games in environmental control are studied by Dockner and Long (1993), Jørgensen and Zaccour (2001), Fredj et al. (2004), Breton et al. (2005, 2006), Petrosyan and Zaccour (2003), Yeung (2007, 2008) and Yeung and Petrosyan (2008).

In dynamic cooperative games, a credible cooperative agreement has to be dynamically consistent. For dynamic consistency to hold, a stringent condition on the cooperative agreement is required: The specific optimality principle chosen at the outset must remain in effect at any instant of time throughout the game along the optimal state trajectory. This condition is commonly known as time consistency. Cooperative differential games that have identified dynamically consistent solutions can be found in Jørgensen and Zaccour (2001), Petrosyan and Zaccour (2003), Yeung (2007), and Yeung and Petrosyan (2004, 2005, 2006a, 2006b, 2008).

In the case when discrete choices of production techniques are available, an analysis in discrete-time framework would in general be more effective. In this paper, we present a cooperative (discrete-time) dynamic game of collaborative environmental management with production technique choices. The policy instruments available include taxes and abatement efforts. The industrial sectors select the production technique and the amount of output to be produced. Under collaboration, nations will come up with a commonly agreed optimality principle which governs their policy instruments and ways to distribute their joint cooperative payoff among themselves. To ensure that the cooperative solution is dynamically consistent, this optimality principle has to be maintained throughout the period of cooperation. Crucial to the analysis is the formulation of a dynamically consistent solution so that the agreed-upon optimality principle will be maintained in the entire collaboration duration. In this paper a time consistent solution is derived. This analysis widens the application of cooperative dynamic cooperative game theory to environmental problems with technique selection.

The paper is organized as follows. Section 2 presents a dynamic game model with production technique choices. Noncooperative outcomes are characterized in Sect. 3. Cooperative arrangements, group optimal actions, solution state trajectories, and time consistent solution are examined in Sect. 4. A payment distribution mechanism bringing about the proposed time-consistent solution is derived and scrutinized in Sect. 5. A numerical illustration is provided in Sect. 6. Concluding remarks are given in Sect. 7.

2 A game model with production technique choices

In this section we present a dynamic game model of transboundary pollution with production technique choices. The game involves T-stages and n asymmetric nations (regions or jurisdictions).

2.1 The industrial sector

The industrial sectors of the n asymmetric nations form an international economy. The demand function for the output of nation $i \in \{1, 2, \dots, n\} \equiv N$ at stage $t \in \{1, 2, \dots, T\} \equiv \kappa$ is

$$P_t^i = \alpha_t^i - \sum_{j=1}^{n} \beta_j^i Q_t^j, \tag{2.1}$$

where P_t^i is the price of the output of nation i, Q_t^j is the output of nation j, α_t^i and β_j^i for $i \in N$ and $j \in N$ are positive parameters. The quantity of output $Q_t^j(s) \in [0, \bar{Q}^j]$ is nonnegative and bounded by a maximum capacity constraint $\bar{Q}^j$. Output price equals zero if the right-hand-side of (2.1) becomes negative. The demand system (2.1) shows that the economy is a form of differentiated products oligopoly. In the case when $\alpha_t^i = \alpha_t^j$ and $\beta_j^i = \beta_i^j$ for all $i \in N$ and $j \in N$, the industrial output is a homogeneous good. This type of model was first introduced by Dixit (1979) and later used in analyses in industrial organizations (see for example, Singh and Vives 1984) and environmental games (see for examples, Yeung 2007, 2008 and Yeung and Petrosyan 2008). Moreover, in this analysis α_t^i for $i \in N$ is allowed to change over time to reflect different growth rates in different nations.

There are two types of production techniques available to each nation's industrial sector: conventional technique and environment-preserving technique. Industrial sectors pay more for using environment-preserving technique. The amount of pollutants emitted by environment-preserving technique is less than that emitted by conventional technique.

We use q_t^j to denote the output of nation j if it uses conventional technique and $\hat{q}_t^j$ to denote the output of nation j if it uses environment-preserving technique. The average cost of producing a unit of output with conventional technique in nation j is c^j while that of producing a unit of output with environment-preserving technique is $\hat{c}^j$.

Let v_t^i denote the tax rate imposed by nation i on industrial output produced by conventional technique in stage t, and $\hat{v}_t^i$ denote the tax rate imposed on output produced by environment-preserving technique. Nation i's industrial sector will choose to use environment-preserving technique if $c^i + v_t^i > \hat{c}^i + \hat{v}_t^i$, otherwise it would choose to use conventional technique. In stage t, let the set of nations using conventional technique be denoted by S_t^1 and the set of nations using environment-preserving technique by S_t^2. The industrial sectors can switch their production techniques in any stage.

To reflect an industrial sector with a collection of heterogeneous firms which responds to current market signals, output is taken as an aggregate measure. In stage t, these firms react to the current market conditions and maximize their profits. Industry profit is an aggregate measure of the firms' profit. Profit maximization in this context is used to mimic the market outcome in the industrial sectors. The profit of industrial sector $i_t \in S_t^1$ and that of industrial sector $\ell_t \in S_t^1$ in stage t can be expressed respectively as

$$\pi_t^{i_t} = \left[\alpha_t^{i_t} - \sum_{j \in S_t^1} \beta_j^{i_t} q_t^j - \sum_{\zeta \in S_t^2} \beta_\zeta^{i_t} \hat{q}_t^\zeta\right] q_t^{i_t} - c^{i_t} q_t^{i_t} - v_t^{i_t} q_t^{i_t}, \quad \text{for } i_t \in S_t^1, \tag{2.2}$$

and

$$\hat{\pi}_t^{\ell_t} = \left[\alpha_t^{\ell_t} - \sum_{j \in S_t^1} \beta_j^{\ell_t} q_t^j - \sum_{\zeta \in S_t^2} \beta_\zeta^{\ell_t} \hat{q}_t^\zeta\right] \hat{q}_t^{\ell_t} - \hat{c}^{\ell_t} \hat{q}_t^{\ell_t} - \hat{v}_t^{\ell_t} \hat{q}_t^{\ell_t}, \quad \text{for } \ell_t \in S_t^2. \tag{2.3}$$

In each stage t the industrial sector of nation $i_t \in S_t^1$ seeks to maximize (2.2) and the industrial sector of nation $\ell_t \in S_t^1$ seeks to maximize (2.3). The first order condition for a Nash equilibrium in stage t yields

$$\begin{aligned} &\alpha_t^{i_t} - \sum_{j \in S_t^1} \beta_j^{i_t} q_t^j - \sum_{\zeta \in S_t^2} \beta_\zeta^{i_t} \hat{q}_t^\zeta - \beta_{i_t}^{i_t} q_t^{i_t} = c^{i_t} + v_t^{i_t}, \quad \text{for } i_t \in S_t^1; \quad \text{and} \\ &\alpha_t^{\ell_t} - \sum_{j \in S_t^1} \beta_j^{\ell_t} q_t^j - \sum_{\zeta \in S_t^2} \beta_\zeta^{\ell_t} \hat{q}_t^\zeta - \beta_{\ell_t}^{\ell_t} \hat{q}_t^{\ell_t} = \hat{c}^{\ell_t} + \hat{v}_t^{\ell_t}, \quad \text{for } \ell_t \in S_t^2. \end{aligned} \tag{2.4}$$

Condition (2.4) shows that the industrial sectors will produce up to a point where marginal revenue (the left-hand side of the equations) equals the cost plus tax of a unit of output produced (the right-hand-side of the equations).

2.2 Pollution dynamics

Industrial output creates long-term environmental impacts by building up existing pollution stocks like green-house-gas, CFC and atmospheric particulates. Each nation adopts its own pollution abatement policy to reduce pollutants in the environment. At the initial stage 1, the level of pollution is $x_1 = x^0$. The dynamics of pollution accumulation is governed by the difference equation:

$$x_{t+1} = x_t + \sum_{i_t \in S_t^1} a^{i_t} q_t^{i_t} + \sum_{\ell_t \in S_t^2} \hat{a}^{\ell_t} \hat{q}_t^{\ell_t} - \sum_{j=1}^{n} b_j u_t^j (x_t)^{1/2} - \delta x_t, \quad x_1 = x^0, \tag{2.5}$$

where a^{i_t} is the amount of pollution created by a unit of nation i_i's output using conventional technique, $\hat{a}^{\ell_t}$ is the amount of pollution created by a unit of nation ℓ_i's output using environment-preserving technique, u_t^j is the pollution abatement effort of nation j at stage t, $b_j u_t^j (x_t)^{1/2}$ is the amount of pollution removed by u_t^j units of abatement effort of nation j, δ is the natural rate of decay of the pollutants.

The damage (cost) of x_t amount of pollution to nation is $h^j x_t$. The cost of u_t^j units of abatement effort is $c_i^a (u_t^j)^2$.

The use of a discrete-time framework does not only provide an effective analytical device for discrete choices techniques study but also facilitates application of real data in empirical study and simulation in operations research analysis. Moreover, dynamics (2.5) is a discrete-time analogue of the continuous-time pollution dynamics in Yeung (2007) and Yeung and Petrosyan (2008). Similar theoretical underpinnings are shared by both the continuous system and the discrete system in the present analysis. Discrete dynamics in environmental analysis can also be found in Dockner and Nishimura (1999), Dutta and Radner (2006) and Rubio and Ulph (2007).

2.3 The nations' objectives

The nations have to promote business interests and at the same time bear the costs brought about by pollution. In particular, each nation maximizes the net gains in the industrial sector plus tax revenues minus the sum of expenditures on pollution abatement and damages from pollution. The payoff of nation $i_t \in S_t^1$ at stage t can be expressed as:

$$\left[\alpha_t^{i_t} - \sum_{j \in S_t^1} \beta_j^{i_t} q_t^j - \sum_{\zeta \in S_t^2} \beta_\zeta^{i_t} \hat{q}_t^\zeta\right] q_t^{i_t} - c^{i_t} q_t^{i_t} - c_{i_t}^a (u_t^{i_t})^2 - h^{i_t} x_t; \tag{2.6}$$

and the payoff of nation $\ell_t \in S_t^2$ at stage t can be expressed as:

$$\left[\alpha_t^{\ell_t} - \sum_{j \in S_t^1} \beta_j^{\ell_t} q_t^j - \sum_{\zeta \in S_t^2} \beta_\zeta^{\ell_t} \hat{q}_t^\zeta\right] \hat{q}_t^{\ell_t} - c^{\ell_t} \hat{q}_t^{\ell_t} - c_{\ell_t}^a \left(u_t^{\ell_t}\right)^2 - h^{\ell_t} x_t. \tag{2.7}$$

The nations' planning horizon is from stage 1 to stage T. It is possible that T may be very large. The discount rate is r. A terminal appraisal of pollution damage is $g^i(\bar{x}^i - x_{T+1})$ will be given to nation i at stage $T+1$, where $g^i \geq 0$. In particular, if the level of pollution at stage $T+1$ is higher (lower) than $\bar{x}^i$, nation i will receive a bonus (penalty) equaling $g^i(\bar{x}^i - x_{T+1})$. Each one of the n nations seeks to maximize the sum of the discounted payoffs over the T stages plus the terminal appraisal. In particular, nation i would seek to maximize the objective

$$\sum_{t=1}^{T} \left[\left[\alpha_t^i - \sum_{\substack{j \in S_t^1 \\ j \neq i}} \beta_j^i q_t^j - \sum_{\substack{\zeta \in S_t^2 \\ j \neq i}} \beta_\zeta^i \hat{q}_t^\zeta - \beta_i^i \bar{q}_t^i\right] \bar{q}_t^i - \bar{c}^i \bar{q}_t^i - c_i^a (u_t^i)^2 - h^i x_t\right] \left(\frac{1}{1+r}\right)^{t-1}$$
$$+ g^i(\bar{x}^i - x_{T+1}) \left(\frac{1}{1+r}\right)^T; \quad i \in N \tag{2.8}$$

where $\bar{q}_t^i = q_t^i$ and $\bar{c}_t^i = c_t^i$ if industrial sector i uses conventional technique; and $\bar{q}_t^i = \hat{q}_t^i$ and $\bar{c}_t^i = \hat{c}_t^i$ if industrial sector i uses environment-preserving technique.

Besides designing an optimal abatement policy, each of the nations also has to design a tax scheme which would lead to the level of output that maximizes its objective. The problem of maximizing objectives (2.8) subject to pollution dynamics (2.5) is a dynamic game between these n nations.

3 Noncooperative outcomes

In this section we discuss the solution to the noncooperative dynamic game (2.5) and (2.8). Since industrial output is in the objective of the nation in (2.8), each nation has to determine its preferred level of industrial output and use a tax scheme to motivate the industrial sector to produce the desired outputs. Hence output levels and abatement efforts become policy choices facing the nations. Under a noncooperative framework, pre-commitment is not possible, a feedback Nash equilibrium solution is sought. Such a solution can be characterized as follows.

Theorem 3.1 *A set of policies* $\{q_t^{i_t*}=\phi_t^{i_t}(x),\hat{q}_t^{\hat{i}_t*}=\hat{\phi}_t^{\hat{i}_t}(x),u_t^{i_t*}=\upsilon_t^{i_t}(x),u_t^{\hat{i}_t*}=\hat{\upsilon}_t^{\hat{i}_t}(x)$, *for* $t\in\kappa$ *and* $i_t\in S_t^1$ *and* $\hat{i}_t\in S_t^2\}$ *provides a feedback Nash equilibrium solution to the game* (2.5) *and* (2.8) *if there exist functions* $V^{i_t}(t,x):R\to R$ *and* $V^{\hat{i}_t}(t,x):R\to R$, *for* $t\in\kappa$ *and* $i_t\in S_t^1$ *and* $\hat{i}_t\in S_t^2$, *such that the following recursive relations are satisfied*:

$$\begin{aligned}
V^{i_t}(t,x)=\max_{q_t^{i_t},u_t}\Bigg\{\Bigg[\Bigg[&\alpha_t^{i_t}-\sum_{\substack{j\in S_t^1\\ j\neq i_t}}\beta_j^{i_t}\phi_t^j(x)-\sum_{\zeta\in S_t^2}\beta_\zeta^{i_t}\hat{\phi}_t^\zeta(x)-\beta_{i_t}^{i_t}q_t^{i_t}\Bigg]q_t^{i_t}-c^{i_t}q_t^{i_t}\\
&-c_{i_t}^a(u_t^{i_t})^2-h^{i_t}x\Bigg]\left(\frac{1}{1+r}\right)^{t-1}\\
&+V^{i_t}\Bigg[t+1,x+\sum_{\substack{j\in S_t^1\\ j\neq i_t}}a^j\phi_t^j(x)+\sum_{\zeta\in S_t^2}\hat{a}^\zeta\hat{\phi}_t^\zeta(x)+a^{i_t}q_t^{i_t}\\
&-\sum_{\substack{j\in S_t^1\\ j\neq i_t}}b_j\upsilon_t^j(x)x^{1/2}-\sum_{j\in S_t^2}^{n}b_j\hat{\upsilon}_t^j(x)x^{1/2}-b_{i_t}u_t^{i_t}x^{1/2}-\delta x\Bigg]\Bigg\},\quad \textit{for } t\in\kappa,
\end{aligned}$$

$$\begin{aligned}
V^{i_t}(T+1,x)&=g^{i_t}(\bar{x}^{i_t}-x)\left(\frac{1}{1+r}\right)^T,\quad \textit{if } i_t\in S_t^1;\quad \textit{and}\\
V^{\hat{i}_t}(t,x)=\max_{\hat{q}_t^{\hat{i}_t},u_t^{\hat{i}_t}}\Bigg\{\Bigg[\Bigg[&\alpha_t^{\hat{i}_t}-\sum_{j\in S_t^1}\beta_j^{\hat{i}_t}\phi_t^j(x)-\sum_{\substack{\zeta\in S_t^2\\ \zeta\neq\hat{i}_t}}\beta_\zeta^{\hat{i}_t}\hat{\phi}_t^\zeta(x)-\beta_{\hat{i}_t}^{\hat{i}_t}\hat{q}_t^{\hat{i}_t}\Bigg]\hat{q}_t^{\hat{i}_t}\\
&-\hat{c}^{\hat{i}_t}\hat{q}_t^{\hat{i}_t}-c_{\hat{i}_t}^a(u_t^{\hat{i}_t})^2-h^{\hat{i}_t}x\Bigg]\left(\frac{1}{1+r}\right)^{t-1}\\
&+V^{\hat{i}_t}\Bigg[t+1,x+\sum_{j\in S_t^1}a^j\phi_t^j(x)+\sum_{\substack{\zeta\in S_t^2\\ \zeta\neq\hat{i}_t}}\hat{a}^\zeta\hat{\phi}_t^\zeta(x)+\hat{a}^{\hat{i}_t}\hat{q}_t^{\hat{i}_t}\\
&-\sum_{j\in S_t^1}b_j\upsilon_t^j(x)x^{1/2}-\sum_{\substack{j\in S_t^2\\ j\neq\hat{i}_t}}^{n}b_j\upsilon_t^j(x)x^{1/2}-b_{\hat{i}_t}u_t^{\hat{i}_t}x^{1/2}-\delta x\Bigg]\Bigg\},\quad \textit{for } t\in\kappa,
\end{aligned}$$

$$V^{\hat{i}_t}(T+1,x)=g^{\hat{i}_t}(\bar{x}^{\hat{i}_t}-x)\left(\frac{1}{1+r}\right)^T,\quad \textit{if } \hat{i}_t\in S_t^2;\tag{3.1}$$

$$\begin{aligned}
&V_{x_{t+1}}^{i_t}(t+1,x_{t+1})>\frac{(\hat{c}^{i_t}-c^{i_t})}{(\hat{a}^{i_t}-a^{i_t})(1+r)^{t-1}},\quad \textit{for } i_t\in S_t^1\quad \textit{and}\\
&V_{x_{t+1}}^{\hat{i}_t}(t+1,x_{t+1})\le\frac{(\hat{c}^{\hat{i}_t}-c^{\hat{i}_t})}{(\hat{a}^{\hat{i}_t}-a^{i_t})(1+r)^{t-1}}\quad \textit{for } \hat{i}_t\in S_t^2;
\end{aligned}\tag{3.2}$$

where $V^{i_t}_{x_{t+1}}(t+1, x_{t+1})$ *is the short form for*

$$V^{i_t}_{x_{t+1}}\Bigg[t+1, x+\sum_{j\in S_t^1} a^j\phi_t^j(x)+\sum_{\zeta\in S_t^2}\hat{a}^\zeta\hat{\phi}_t^\zeta(x) - \sum_{j\in S_t^1} b_j\upsilon_t^j(x)x^{1/2} - \sum_{j\in S_t^2}^{n} b_j\hat{\upsilon}_t^j(x)x^{1/2} - \delta x\Bigg], \quad \textit{and}$$

$V^{\hat{i}_t}_{x_{t+1}}(t+1, x_{t+1})$ *is the short form for*

$$V^{\hat{i}_t}_{x_{t+1}}\Bigg[t+1, x+\sum_{j\in S_t^1} a^j\phi_t^j(x)+\sum_{\zeta\in S_t^2}\hat{a}^\zeta\hat{\phi}_t^\zeta(x) - \sum_{j\in S_t^1} b_j\upsilon_t^j(x)x^{1/2} - \sum_{j\in S_t^2}^{n} b_j\hat{\upsilon}_t^j(x)x^{1/2} - \delta x\Bigg].$$

Proof If nation $i_t \in S_t^1$ adopts conventional technique and nation $\hat{i}_t \in S_t^2$ adopts environment-preserving technique, the results in (3.1) satisfy the optimality conditions in dynamic programming and the Nash equilibrium. Hence a feedback Nash equilibrium is characterized. See Theorem 6.6 in Basar and Olsder (1999).

The inequalities in (3.2) yield the conditions justifying why nation $i_t \in S_t^1$ adopts conventional technique and nation $\hat{i}_t \in S_t^2$ adopts environment-preserving technique. To prove this we perform the indicated maximization in (3.1) and obtain:

$$\alpha_t^{i_t} - \sum_{j\in S_t^1}\beta_j^{i_t}\phi_t^j(x) - \sum_{\zeta\in S_t^2}\beta_\zeta^{i_t}\hat{\phi}_t^\zeta(x) - \beta_{i_t}^{i_t}\phi_t^{i_t}(x) = c^{i_t} - a^{i_t}V^{i_t}_{x_{t+1}}(t+1, x_{t+1})(1+r)^{t-1},$$
$$\text{for } i_t \in S_t^1; \tag{3.3}$$

and

$$\alpha_t^{\hat{i}_t} - \sum_{j\in S_t^1}\beta_j^{\hat{i}_t}\phi_t^j(x) - \sum_{\zeta\in S_t^2}\beta_\zeta^{\hat{i}_t}\phi_t^\zeta(x) - \beta_{\hat{i}_t}^{\hat{i}_t}\hat{\phi}_t^{\hat{i}_t}(x) = \hat{c}^{\hat{i}_t} - \hat{a}^{\hat{i}_t}V^{\hat{i}_t}_{x_{t=1}}(t+1, x_{t+1})(1+r)^{t-1},$$
$$\text{for } \hat{i}_t \in S_t^2. \tag{3.4}$$

In view of (2.4), the left-hand-side of (3.3) and that of (3.4) reflect the marginal revenues to the industrial sectors. To motivate the industrial sectors to produce outputs as given in (3.3) nation i_t has to impose a tax $\upsilon_t^{i_t}$ equaling $-a^{i_t}V^{i_t}_{x_{t+1}}(t+1, x_{t+1})(1+r)^{t-1}$ on a unit of output produced with conventional technique. Similarly, nation $\hat{i}_t$ has to impose a tax $\hat{\upsilon}_t^{\hat{i}_t}$ equaling $-\hat{a}^{\hat{i}_t}V^{\hat{i}_t}_{x_{t+1}}(t+1, x_{t+1})(1+r)^{t-1}$ on a unit of output produced with environment-preserving technique to arrive at (3.4).

To illustrate that (3.2) is indeed the criteria for technique choice, we consider the case where industrial sector i_t uses environment-preserving technique instead of conventional technique, condition (3.3) becomes

$$\alpha_t^{i_t} - \sum_{j\in S_t^1} \beta_j^{i_t} \phi_t^j(x) - \sum_{\zeta\in S_t^2} \beta_\zeta^{i_t} \hat{\phi}_t^\zeta(x) - \beta_{i_t}^{i_t} \hat{\phi}_t^{i_t}(x) = \hat{c}^{i_t} - \hat{a}^{i_t} V_{x_{t+1}}^{i_t\in S_t^2}(t+1, x_{t+1})(1+r)^{t-1}, \tag{3.5}$$

where $V_{x_{t+1}}^{i_t\in S_t^2}(t+1, x_{t+1})$ is the short form for

$$V_{x_{t+1}}^{i_t}\Bigg[t+1, x + \sum_{\substack{j\in S_t^1\\ j\neq i_t}} a^j \phi_t^j(x) + \sum_{\zeta\in S_t^2} \hat{a}^\zeta \hat{\phi}_t^\zeta(x) + \hat{a}^{i_t} \hat{\phi}_t^{i_t}(x) \\ - \sum_{\substack{j\in S_t^1\\ j\neq i_t}} b_j \upsilon_t^j(x) x^{1/2} - \sum_{j\in S_t^2}^{n} b_j \hat{\upsilon}_t^j(x) x^{1/2} - b_{\hat{i}_t} \hat{\upsilon}_t^{i_t} - \delta x\Bigg].$$

To motivate the industrial sectors to produce outputs as given in (3.5) nation i_t has to impose a tax $\hat{\upsilon}_t^{i_t}$ equaling $-\hat{a}^{i_t} V_{x_{t+1}}^{i_t\in S_t^2}(t+1, x_{t+1})(1+r)^{t-1}$.

The cost plus tax for a unit of output produced with conventional technique is $c^{i_t} + \upsilon_t^{i_t}$ and that for a unit of output produced with environment-preserving technique is $\hat{c}^{i_t} + \hat{\upsilon}_t^{i_t}$. Industrial sector i_t will choose to use conventional technique if $c^i + \upsilon_t^i < \hat{c}^i + \hat{\upsilon}_t^i$, otherwise it would choose to use environment-preserving technique. Using these terms one can show that if

$$c^{i_t} - a^{i_t} V_{x_{t+1}}^{i_t}(t+1, x_{t+1})(1+r)^{t-1} < \hat{c}^{i_t} - \hat{a}^{i_t} V_{x_{t+1}}^{i_t\in S_t^2}(t+1, x_{t+1})(1+r)^{t-1}, \tag{3.6}$$

industrial sector i_t will use conventional technique, otherwise it will use environment-preserving technique. Using the value functions in Proposition 3.1 below one can readily show that $V_{x_{t+1}}^{i_t\in S_t^2}(t+1, x_{t+1}) = V_{x_{t+1}}^{i_t}(t+1, x_{t+1})$. Therefore the condition reflected in (3.6) collapses to condition (3.2). Hence Theorem 3.1 follows. □

The term $-a^{i_t} V_{x_{t+1}}^{i_t}(t+1, x_{t+1})(1+r)^{t-1}$ reflects the marginal social cost to nation i_t brought about by a unit of output produced with conventional technique. The term $-\hat{a}^{i_t} V_{x_{t+1}}^{\hat{i}_t}(t+1, x_{t+1})(1+r)^{t-1}$ reflects the marginal social cost to nation $\hat{i}_t$ brought about by a unit of output produced with environment-preserving technique.

Rearranging (3.3) and (3.4) we obtain the system

$$\sum_{j\in S_t^1} \beta_j^{i_t} \phi_t^j(x) + \sum_{\zeta\in S_t^2} \beta_\zeta^{i_t} \hat{\phi}_t^\zeta(x) + \beta_{i_t}^{i_t} \phi_t^{i_t}(x) \\ = \alpha_t^{i_t} - c^{i_t} + a^{i_t} V_{x_{t+1}}^{i_t}(t+1, x_{t+1})(1+r)^{t-1}, \quad \text{for } i_t \in S_t^1; \tag{3.7}$$

$$\sum_{j\in S_t^1} \beta_j^{\hat{i}_t} \phi_t^j(x) + \sum_{\zeta\in S_t^2} \beta_\zeta^{\hat{i}_t} \hat{\phi}_t^\zeta(x) + \beta_{\hat{i}_t}^{\hat{i}_t} \hat{\phi}_t^{\hat{i}_t}(x) \\ = \alpha_t^{\hat{i}_t} - \hat{c}^{\hat{i}_t} + \hat{a}^{\hat{i}_t} V_{x_{t+1}}^{\hat{i}_t}(t+1, x_{t+1})(1+r)^{t-1}, \quad \text{for } \hat{i}_t \in S_t^2. \tag{3.8}$$

System (3.7)–(3.8) can be viewed as a set of equations linear in $\phi_t^{i_t}(x)$ for $i_t \in S_t^1$ and $\hat{\phi}_t^{\hat{i}_t}(x)$ for $\hat{i}_t \in S_t^2$, with $V_{x_{t+1}}^{i_t}(t+1, x_{t+1})(1+r)^{t-1}$ for $i_t \in S_t^1$ and $V_{x_{t+1}}^{\hat{i}_t}(t+1, x_{t+1})(1+r)^{t-1}$ for

$\hat{i}_t \in S_t^2$ being taken as a set of parameters. Solving (3.7)–(3.8) yields:

$$\begin{aligned}\phi_t^{i_t}(x) &= \bar{\alpha}_t^{i_t} + \sum_{j\in S_t^1} \bar{\beta}_t^{(i_t)j} V_{x_{t+1}}^j(t+1, x_{t+1})(1+r)^{t-1} \\ &\quad + \sum_{j\in S_t^2} \bar{\beta}_t^{(i_t)j} V_{x_{t+1}}^j(t+1, x_{t+1})(1+r)^{t-1}, \quad \text{for } i_t \in S_t^1; \\ \hat{\phi}_t^{\hat{i}_t}(x) &= \hat{\bar{\alpha}}_t^{\hat{i}_t} + \sum_{j\in S_t^1} \hat{\bar{\beta}}_t^{(\hat{i}_t)j} V_{x_{t+1}}^j(t+1, x_{t+1})(1+r)^{t-1} \\ &\quad + \sum_{j\in S_t^2} \hat{\bar{\beta}}_t^{(\hat{i}_t)j} V_{x_{t+1}}^j(t+1, x_{t+1})(1+r)^{t-1}, \quad \text{for } \hat{i}_t \in S_t^2;\end{aligned} \tag{3.9}$$

where $\bar{\alpha}_t^{i_t}$ and $\bar{\beta}_t^{(i_t)j}$ for $i_t \in S_t^1$, and $\hat{\bar{\alpha}}_t^{\hat{i}_t}$ and $\hat{\bar{\beta}}_t^{(\hat{i}_t)j}$, $\hat{i}_t \in S_t^2$, are constants involving the model parameters.

In addition, performing the maximization operator in (3.1) with respect to $u_t^{i_t}$ and $u_t^{\hat{i}_t}$ yields

$$\begin{aligned}\upsilon_t^{i_t}(x) &= -\frac{b_{i_t}}{2c_{i_t}^a} V_{x_{t+1}}^{i_t}(t+1, x_{t+1})(1+r)^{t-1} x^{1/2}, \quad \text{for } i_t \in S_t^1; \quad \text{and} \\ \hat{\upsilon}_t^{\hat{i}_t}(x) &= -\frac{b_{\hat{i}_t}}{2c_{\hat{i}_t}^a} V_{x_{t+1}}^{\hat{i}_t}(t+1, x_{t+1})(1+r)^{t-1} x^{1/2}, \quad \text{for } \hat{i}_t \in S_t^2.\end{aligned} \tag{3.10}$$

Proposition 3.1 *System (3.1)–(3.2) admits a unique solution*

$$V^i(t,x) = \begin{cases} V^{i_t}(t,x) & \text{if } i \in S_t^1, \\ V^{\hat{i}_t}(t,x) & \quad\; i \in S_t^2, \end{cases}$$

where

$$V^{i_t}(t,x) = (A_t^{i_t} x + C_t^{i_t})\left(\frac{1}{1+r}\right)^{t-1} \quad \text{and} \quad V^{\hat{i}_t}(t,x) = (\hat{A}_t^{\hat{i}_i} x + \hat{C}_t^{\hat{i}_t})\left(\frac{1}{1+r}\right)^{t-1},$$
$$\text{for } t \in \kappa; \tag{3.11}$$

with $A_t^{i_t}$, $C_t^{i_t}$, $\hat{A}_t^{\hat{i}_t}$ and $\hat{C}_t^{\hat{i}_t}$ being constants involving the model parameters.

Proof See Appendix A. □

Though conventional technique emits higher level of pollutants, nations have no incentive to switch to environment-preserving technique if the sum of marginal cost of producing the output and the nation's social cost resulted from using conventional technique is lower than that resulted from using environment-preserving technique.

Note that the nations' abatement efforts in (3.10) are direct policy strategies obtained from the equilibrium of the game. However, the nations' tax rules

$$v_t^{i_t} = -a^{i_t} V_{x_{t+1}}^{i_t}(t+1, x_{t+1})(1+r)^{t-1} \quad \text{on conventional output and}$$
$$\hat{v}_t^{\hat{i}_t} = -\hat{a}^{\hat{i}_t} V_{x_{t+1}}^{\hat{i}_t}(t+1, x_{t+1})(1+r)^{t-1} \quad \text{on environment-preserving}$$

are derived to motivate the industrial sectors to produce with the preferred techniques up to the desired output levels. Moreover, a desirable feature of feedback dynamic games is that nations can revise their policy as time goes by and the state changes to avoid the pitfalls of time inconsistent rigid tax rules and abatement schemes.

4 Cooperative arrangements in climate change control

Now consider the case when all the nations want to collaborate and tackle the pollution problem together. Cooperation suggests the possibility of socially optimal and group efficient solutions to decision problems involving interactive strategic actions. Formulation of optimal behavior for individual participants and imputations for sharing the joint payoffs constitute the most fundamental element in cooperative schemes. Two essential factors for collaborative scheme are group optimality and individual rationality. Group optimality ensures that all potential gains from cooperation are captured. Failure to fulfill group optimality leads to the condition where the participants prefer to deviate from the agreed-upon solution plan in order to extract the unexploited gains. Individual rationality is required to hold so that the payoff allocated to an economic agent under cooperation will be no less than its noncooperative payoff. Failure to guarantee individual rationality leads to the condition where the concerned participants would reject the agreed-upon solution plan and play noncooperatively.

On top of group optimality and individual rationality the solution has to be time consistent in the sense that the specific optimality principle chosen at the outset must remain in effect at any instant of time throughout the game along the optimal state trajectory. Hence none of the participating nations would have an incentive to depart from the collaborative scheme. Cooperation will cease if any of the nations refuses to act accordingly at any time within the game horizon.

To ensure group optimality, the nations would seek to maximize their joint payoff under cooperation. Since nations are asymmetric and the number of nations may be large, a reasonable optimality principle for gain distribution is to share the gain from cooperation proportional to the nations' relative sizes of noncooperative payoffs. Such sharing principle fulfills individual rationality.

4.1 Group optimality and cooperative state trajectory

Consider the case when all the nations agree to act cooperatively so that the joint payoff will be maximized. Since two technique choices are available they have to determine which nations would use which type of techniques over the T stages. Let M^γ be a matrix reflecting the pattern of technique choices by the n nations over the T stages. In particular, according to pattern M^γ, the set of nations that use conventional technique is $S_t^{M^\gamma[1]}$ and the set of nations that use environment-preserving technique is $S_t^{M^\gamma[2]}$ in stage $t \in \kappa$. To select the controls which would maximize joint payoff under pattern M^γ the nations have to solve the following optimal control problem:

$$\max_{u_t^{\ell_t}, q_t^{\ell_t}, \ell_t \in S_t^{M^\gamma[1]}; \hat{u}_t^{\hat{\ell}_t} \hat{q}_t^{\hat{\ell}_t}, \hat{\ell}_t \in S_t^{M^\lambda[2]}; t\in\kappa} \Bigg\{ \sum_{t=1}^{T} \sum_{i \in S_t^{M^\gamma[1]}} \Bigg[\Bigg[\alpha_t^i - \sum_{j \in S_t^{M^\gamma[1]}} \beta_j^i q_t^j - \sum_{\zeta \in S_t^{M^\gamma[2]}} \beta_\zeta^i \hat{q}_t^\zeta \Bigg] q_t^i$$
$$- c^i q_t^i - c_i^a (u_t^i)^2 - h^i x_t \Bigg] \left(\frac{1}{1+r} \right)^{t-1}$$

$$+\sum_{t=1}^{T}\sum_{i\in S_t^{M^\gamma[2]}}\Bigg[\Bigg[\alpha_t^i-\sum_{j\in S_t^{M^\gamma[1]}}\beta_j^i q_t^j-\sum_{\zeta\in S_t^{M^\gamma[2]}}\beta_\zeta^i\hat{q}_t^\zeta\Bigg]\hat{q}_t^i$$

$$-\hat{c}^{i(S_t^{M^\gamma[2]})}\hat{q}_t^i-c_i^a(u_t^i)^2-h^i x_t\Bigg]\left(\frac{1}{1+r}\right)^{t-1}+\sum_{i=1}^{n}g^i(\bar{x}^i-x_{T+1})\left(\frac{1}{1+r}\right)^{T}\Bigg\}, \quad (4.1)$$

subject to

$$x_{t+1}=x_t+\sum_{\ell_t\in S_t^{M^\gamma[1]}}a^{\ell_t}q_t^{\ell_t}+\sum_{\hat{\ell}_t\in S_t^{M^\gamma[2]}}\hat{a}^{\hat{\ell}_t}\hat{q}_t^{\hat{\ell}_t}-\sum_{\ell_t\in S_t^{M^\gamma[1]}}b_{\ell_t}u_t^{\ell_t}(x_t)^{1/2}$$

$$-\sum_{\hat{\ell}_t\in S_t^{M^\gamma[1]}}b_{\hat{\ell}_t}u_t^{\hat{\ell}_t}(x_t)^{1/2}-\delta x_t, \quad (4.2)$$

$$x_1=x^0.$$

The solution to the optimal control problem (4.1)–(4.2) can be characterized as follows.

Theorem 4.1 *A set of strategies* $\{q_t^{\ell_t*}=\psi_t^{(M^\gamma)\ell_t}(x),\hat{q}_t^{\hat{\ell}_t*}=\hat{\psi}_t^{(M^\gamma)\hat{\ell}_t}(x),u_t^{\ell_t*}=\varpi_t^{(M^\gamma)\ell_t}(x),$ $u_t^{\hat{\ell}_t*}=\hat{\varpi}_t^{(M^\gamma)\hat{\ell}_t}(x)$, *for* $t\in\kappa$ *and* $\ell_t\in S_t^{M^\gamma[1]}$ *and* $\hat{\ell}_t\in S_t^{M^\gamma[2]}\}$ *constitutes an optimal solution to the control problem* (4.1) *and* (4.2) *if there exist functions* $W^{M^\gamma}(t,x):R\to R$, *for* $t\in\kappa$, *such that the following recursive relations are satisfied*:

$$W^{M^\gamma}(t,x)=\max_{u_t^{\ell_t},q_t^{\ell_t},\ell_t\in S_t^{M^\gamma[1]};\hat{u}_t^{\hat{\ell}_t}\hat{q}_t^{\hat{\ell}_t},\hat{\ell}_t\in S_t^{M^\lambda[2]}}\Bigg\{\sum_{i\in S_t^{M^\gamma[1]}}\Bigg[\Bigg[\alpha_t^i-\sum_{j\in S_t^{M^\gamma[1]}}\beta_j^i q_t^j-\sum_{\zeta\in S_t^{M^\gamma[2]}}\beta_\zeta^i\hat{q}_t^\zeta\Bigg]q_t^i$$

$$-c^i q_t^i-c_i^a(u_t^i)^2-h^i x_t\Bigg]\left(\frac{1}{1+r}\right)^{t-1}$$

$$+\sum_{i\in S_t^{M^\gamma[2]}}\Bigg[\Bigg[\alpha_t^i-\sum_{j\in S_t^{M^\gamma[1]}}\beta_j^i q_t^j-\sum_{\zeta\in S_t^{M^\gamma[2]}}\beta_\zeta^i\hat{q}_t^\zeta\Bigg]\hat{q}_t^i$$

$$-\hat{c}^{i(S_t^{M^\gamma[2]})}\hat{q}_t^i-c_i^a(u_t^i)^2-h^i x_t\Bigg]\left(\frac{1}{1+r}\right)^{t-1} \quad (4.3)$$

$$+W^{M^\gamma}\Bigg[t+1,x+\sum_{\ell_t\in S_t^{M^\gamma[1]}}a^{\ell_t}q_t^{\ell_t}+\sum_{\hat{\ell}_t\in S_t^{M^\gamma[2]}}\hat{a}^{\hat{\ell}_t}\hat{q}_t^{\hat{\ell}_t}-\sum_{\ell_t\in S_t^{M^\gamma[1]}}b_{\ell_t}u_t^{\ell_t}(x_t)^{1/2}$$

$$-\sum_{\hat{\ell}_t\in S_t^{M^\gamma[1]}}b_{\hat{\ell}_t}u_t^{\hat{\ell}_t}(x)^{1/2}-\delta x\Bigg]\Bigg\}, \quad \textit{for } t\in\kappa;$$

$$W^{M^\gamma}(T+1,x)=\sum_{i=1}^{n}g^i(\bar{x}^i-x)\left(\frac{1}{1+r}\right)^{T}.$$

Proof The results in (4.3) satisfy the standard optimality conditions in discrete-time dynamic programming. □

Performing the indicated maximization in (4.3) yields the optimal controls under cooperation as:

$$\varpi_t^{(M^\gamma)\ell_t}(x) = -\frac{\hat{b}_{\ell_t}}{2c^a_{\ell_t}} W^{M^\gamma}_{x_{t+1}}(t+1, x_{t+1})(1+r)^{t-1} x^{1/2}, \quad \text{for } \ell_t \in S_t^{M^\gamma[1]}, \quad \text{and}$$
$$\hat{\varpi}_t^{(M^\gamma)\hat{\ell}_t}(x) = -\frac{\hat{b}_{\hat{\ell}_t}}{2c^a_{\hat{\ell}_t}} W^{M^\gamma}_{x_{t+1}}(t+1, x_{t+1})(1+r)^{t-1} x^{1/2}, \quad \text{for } \hat{\ell}_t \in S_t^{M^\gamma[2]}; \tag{4.4}$$

$$\alpha_t^{\ell_t} - \sum_{j\in S_t^{M^\gamma[1]}} \beta_j^{\ell_t} \psi_t^{(M^\gamma)j}(x) - \sum_{j\in S_t^{M^\gamma[2]}} \beta_j^{\ell_t} \hat{\psi}_t^{(M^\gamma)j}(x) - \sum_{\zeta\in S_t^{M^\gamma[1]}} \beta_{\ell_t}^{\zeta} \psi_t^{(M^\gamma)\zeta}(x)$$
$$- \sum_{\zeta\in S_t^{M^\gamma[2]}} \beta_{\ell_t}^{\zeta} \hat{\psi}_t^{(M^\gamma)\zeta}(x) = c^{\ell_t} - a^{\ell_t} W^{M^\gamma}_{x_{t+1}}(t+1, x_{t+1})(1+r)^{t-1},$$
$$\text{for } \ell_t \in S_t^{M^\gamma[1]}; \tag{4.5}$$

$$\alpha_t^{\hat{\ell}_t} - \sum_{j\in S_t^{M^\gamma[1]}} \beta_j^{\hat{\ell}_t} \psi_t^{(M^\gamma)j}(x) - \sum_{j\in S_t^{M^\gamma[2]}} \beta_j^{\hat{\ell}_t} \hat{\psi}_t^{(M^\gamma)j}(x) - \sum_{\zeta\in S_t^{M^\gamma[1]}} \beta_{\hat{\ell}_t}^{\zeta} \psi_t^{(M^\gamma)\zeta}(x)$$
$$- \sum_{\zeta\in S_t^{M^\gamma[2]}} \beta_{\hat{\ell}_t}^{\zeta} \hat{\psi}_t^{(M^\gamma)\zeta}(x) = c^{\hat{\ell}_t(S_t^{M^\gamma[2]})} - a^{\hat{\ell}_t} W^{M^\gamma}_{x_{t+1}}(t+1, x_{t+1})(1+r)^{t-1},$$
$$\text{for } \hat{\ell}_t \in S_t^{M^\gamma[2]}, \tag{4.6}$$

where $W^{M^\gamma}_{x_{t+1}}(t+1, x_{t+1})$ is the short form for

$$W^{M^\gamma}_{x_{t+1}}\Bigg[t+1, x + \sum_{j\in S_t^{M^\gamma[1]}} a^j \psi_t^{(M^\gamma)j}(x) + \sum_{j\in S_t^{M^\gamma[2]}} \hat{a}^j \hat{\psi}_t^{(M^\gamma)j}(x)$$
$$- \sum_{j\in S_t^{M^\gamma[1]}} b_j \varpi_t^{(M^\gamma)j}(x) - \sum_{j\in S_t^{M^\gamma[2]}} b_j \hat{\varpi}_t^{(M^\gamma)j}(x) - \delta x\Bigg].$$

System (4.5)–(4.6) can be viewed as a set of equations linear in $\psi_t^{\ell_t}(x)$ and $\hat{\psi}_t^{\hat{\ell}_t}(x)$ for $\ell_t \in S_t^{M^\gamma[1]}$ and $\hat{\ell}_t \in S_t^{M^\gamma[2]}$ with $W^{M^\gamma}_{x_{t+1}}(t+1, x_{t+1})(1+r)^{t-1}$ being taken a parameter. Solving (4.5)–(4.6) yields:

$$\psi_t^{(\mathrm{M}^\gamma)\ell_t}(x) = \tilde{\alpha}_t^{(M^\gamma)\ell_t} + \tilde{\beta}_t^{(M^\gamma)\ell_t} W^{M^\gamma}_{x_{t+1}}(t+1, x_{t+1})(1+r)^{t-1}, \quad \text{for } \ell_t \in S_t^{M^\gamma[1]};$$
$$\hat{\psi}_t^{(\mathrm{M}^\gamma)\hat{\ell}_t}(x) = \hat{\tilde{\alpha}}_t^{(M^\gamma)\hat{\ell}_t} + \hat{\tilde{\beta}}_t^{(M^\gamma)\hat{\ell}_t} W^{M^\gamma}_{x_{t+1}}(t+1, x_{t+1})(1+r)^{t-1}, \quad \text{for } \hat{\ell}_t \in S_t^{M^\gamma[2]}; \tag{4.7}$$

where $\tilde{\alpha}_t^{\ell_t}$ and $\tilde{\beta}_j^{\ell_t}$ for $\ell_t \in S_t^{M^\gamma[1]}$, and $\hat{\tilde{\alpha}}_t^{\hat{\ell}_t}$ and $\hat{\tilde{\beta}}_j^{\hat{\ell}_t}$, $\hat{\ell}_t \in S_t^{M^\gamma[2]}$, are constants involving the model parameters.

Proposition 4.1 *System* (4.3) *admits a solution*

$$W^{M^\gamma}(t, x) = (A_t^{M^\gamma} x + C_t^{M^\gamma})\left(\frac{1}{1+r}\right)^{t-1}, \quad t \in \kappa, \tag{4.8}$$

where $A_t^{M^\gamma}$ and $C_t^{M^\gamma}$ are constants involving the model parameters.

Proof See Appendix B. □

The technique pattern M^γ which yields the highest joint payoff $W^{M^\gamma}(t,x)$ will be adopted in the cooperative scheme. Let us denote the technique pattern that yields the highest joint payoff by M^*. If the marginal joint social cost of pollution is sufficiently higher complete switching to environment-preserving technique would be realized.

Using (4.4), (4.7) and (4.8), the control strategy under cooperation with technique pattern M^* can be obtained accordingly. To induce the industrial sector to produce the socially optimal levels of output with the desired technique, we first substitute the social optimally outputs into (2.4) to obtain the optimal tax rates

$$v_t^{(M^*)i_t} = \alpha_t^{i_t} - \sum_{j\in S_t^1} \beta_j^{i_t} \psi_t^{(M^*)j} - \sum_{\zeta\in S_t^2} \beta_\zeta^{i_t} \hat{\psi}_t^{(M^*)\zeta} - \beta_{i_t}^{i_t} \psi_t^{(M^*)i_t} - c^{i_t}, \quad \text{for } i_t \in S_t^1; \quad \text{and}$$

$$\hat{v}_t^{(M^*)\ell_t} = \alpha_t^{\ell_t} - \sum_{j\in S_t^1} \beta_j^{\ell_t} \psi_t^{(M^*)j} - \sum_{\zeta\in S_t^2} \beta_\zeta^{\ell_t} \hat{\psi}_t^{(M^*)\zeta} - \beta_{\ell_t}^{\ell_t} \hat{\psi}_t^{(M^*)\ell_t} - \hat{c}^{\ell_t}, \quad \text{for } \ell_t \in S_t^2.$$

A salient feature of the optimal scheme is that each nation's optimal tax rate reflects the marginal social cost of its output. The tax rate on output produced with conventional techniques can be easily shown to be higher than that with environment-preserving technique for each nation. Since even environment-preserving techniques emit pollution the socially optimal tax rate cannot be zero. However, the tax rate differential acts as an indication for technique switching or technology transfer to nations (especially developing nations). Guided by the tax rates each industrial sector will adopt techniques leading to a social optimum. Finally a lump-sum levy/subsidy will be given to each industrial sector to guarantee that the same profit level as that under a noncooperative equilibrium is maintained.

Substituting the optimal control strategy into (4.2) yields the dynamics of pollution accumulation under cooperation as:

$$\begin{aligned} x_{t+1} = &\sum_{j\in S_t^{M^*[1]}} a^j \big[\tilde{\alpha}_t^{(M^*)j} + \tilde{\beta}_t^{(M^*)j} A_{t+1}^{M^*}(1+r)^{-1}\big] \\ &+ \sum_{j\in S_t^{M^*[2]}} \hat{a}^j \big[\hat{\tilde{\alpha}}_t^{(M^*)j} + \hat{\tilde{\beta}}_t^{(M^*)j} A_{t+1}^{M^*}(1+r)^{-1}\big] \\ &+ \left[1 + \sum_{j=1}^{n} \frac{(b_j)^2}{2c_j^a} A_{t+1}^{M^*}(1+r)^{-1} - \delta\right] x_t, \quad x_1 = x^0. \end{aligned} \tag{4.9}$$

Equation (4.9) is a linear difference equation with time varying coefficients. We use $\{x_1^*, x_2^*, \ldots, x_n^*\}$ to denote the solution path satisfying (4.9). Solving (4.9) gives

$$x_t^* = \left(\prod_{\zeta=1}^{t} S_\zeta^1\right) x^0 + \sum_{k=1}^{t} \left(\prod_{\zeta=k+1}^{t} S_\zeta^1\right) S_k^2, \tag{4.10}$$

where

$$S_\zeta^1 = 1 + \sum_{j=1}^{n} \frac{(\hat{b}_j)^2}{2c_j^a} A_{\zeta+1}^{M^*}(1+r)^{-1} - \delta, \quad \text{and}$$

$$S_\zeta^2 = \sum_{j \in S_t^{M^*[1]}} a^j [\tilde{\alpha}_\zeta^{(M^*)j} + \tilde{\beta}_\zeta^{(M^*)j} A_{\zeta+1}^{M^*}(1+r)^{-1}] + \sum_{j \in S_t^{M^*[2]}} \hat{a}^j [\hat{\tilde{\alpha}}_t^{(M^*)j} + \hat{\tilde{\beta}}_t^{(M^*)j} A_{t+1}^{M^*}(1+r)^{-1}].$$

4.2 Time consistent collaborative solution

To achieve dynamic consistency the agreed-upon optimality principle must be maintained at every stage of collaboration. The agreed-upon optimality principle requires the nations to share the gain from cooperation proportional to the nations' relative sizes of noncooperative payoffs. In a dynamic framework this condition has to be maintained at every stage. Let $\xi^\ell(t, x_t^*)$ denote nation ℓ's imputation (payoff under cooperation) covering the stages t to T under the agreed-upon optimality principle along the cooperative trajectory $\{x_k^*\}_{k=t}^T$. Hence the solution imputation scheme has to satisfy:

Condition 4.1

$$\xi^\ell(t, x_t^*) = \frac{V^\ell(t, x_t^*)}{\sum_{j=1}^n V^j(t, x_t^*)} W^{M^*}(t, x_t^*), \tag{4.11}$$

for all $\ell \in N$ and all $t \in \kappa$, where $V^\ell(t, x_t^*) = V^{\ell_t}(t, x_t^*)$ in Proposition 3.1 if $\ell \in S_t^1$ and $V^\ell(t, x_t^*) = V^{\hat{\ell}_t}(t, x_t^*)$ if $\ell \in S_t^2$.

Hence a time-consistent solution has to satisfy Condition 4.1. Crucial to the derivation of a time-consistent solution is the formulation of a payment distribution mechanism that would lead to the realization of Condition 4.1. This will be done in the next section.

5 Payment distribution mechanism

To design a payment distribution scheme over time so that the agreed-upon imputation in Condition 4.1 can be realized we apply the techniques developed in Petrosyan and Zenkevich (1996) and Yeung and Petrosyan (2010). In formulating a Payoff Distribution Procedure (PDP) we let $B_t^\ell(x_t^*)$ denote the payment that nation ℓ will received at stage t under the cooperative agreement given the state x_t^* at stage $t \in \kappa$.

The payment scheme involving $B_t^\ell(x_t^*)$ constitutes a PDP in the sense that along the optimal state trajectory $\{x_t^*\}_{t=1}^T$ the imputation to nation ℓ over the stages from t to T can be expressed as:

$$\xi^\ell(t, x_t^*) = \sum_{\zeta=t}^{T} B_\zeta^\ell(x_\zeta^*) \left(\frac{1}{1+r}\right)^{\zeta-1} + g^\ell(\bar{x}^\ell - x_{T+1}) \left(\frac{1}{1+r}\right)^T, \tag{5.1}$$

for $\ell \in N$ and $t \in \kappa$.

Theorem 5.1 *A payment*

$$B_t^\ell(x_t^*) = (1+r)^{t-1}[\xi^\ell(t, x_t^*) - \xi^\ell(t+1, x_{t+1}^*)], \quad \textit{for } \ell \in N,$$

given to nation $\ell \in N$ *at stage* $t \in \{1, 2, \dots, T-1\}$, *and a payment*

$$B_T^\ell(x_T^*) = (1+r)^{T-1}\left[\xi^\ell(T, x_T^*) - g^\ell(\bar{x}^\ell - x_{T+1}^*)\left(\frac{1}{1+r}\right)^T\right], \tag{5.2}$$

given to nation $\ell \in N$ *at stage* T *would lead to the realization of the imputation* $\{\xi^\ell(t, x_t^*)$, *for* $t \in \kappa$ *and* $\ell \in N\}$.

Proof Making use of (5.1), one can arrive at:

$$\xi^\ell(t, x_t^*) = \sum_{\zeta=t}^{h-1} B_\zeta^\ell(x_\zeta^*)\left(\frac{1}{1+r}\right)^{\zeta-1} + \xi^\ell(h, x_h^*), \tag{5.3}$$

for $\ell \in N$ and $t \in \kappa$ and $h \in \{t+1, t+2, \dots, T\}$.

From (5.3) one can obtain

$$B_t^\ell(x_t^*)\left(\frac{1}{1+r}\right)^{t-1} = \xi^\ell(t, x_t^*) - \xi^\ell(t+1, x_{t+1}^*),$$

for $\ell \in N$ and $t \in \kappa$.

Note that $B_t^\ell(x_t^*)(\frac{1}{1+r})^{t-1}$ is the present value (as from initial stage 1) of a payment $B_t^\ell(x_t^*)$ that will be given nation ℓ at stage t. Hence if a payment as specified in (5.2) is given to nation ℓ at stage $t \in \kappa$, the imputation $\{\xi^\ell(t, x_t^*)$, for $t \in \kappa$ and $\ell \in N\}$ can be realized by showing that

$$\sum_{\zeta=t}^{T} B_\zeta^\ell(x_\zeta^*)\left(\frac{1}{1+r}\right)^{\zeta-1} + g^\ell(\bar{x}^\ell - x_{T+1})\left(\frac{1}{1+r}\right)^T$$
$$= \sum_{\zeta=t}^{T}[\xi^\ell(\zeta, x_\zeta^*) - \xi^\ell(\zeta+1, x_{\zeta+1}^*)] = \xi^\ell(t, x_t^*), \tag{5.4}$$

for $\ell \in N$ and $t \in \kappa$, given that $\xi^\ell(T+1, x_{T+1}^*) = g^\ell(\bar{x}^\ell - x_{T+1}^*)(\frac{1}{1+r})^T$. □

Theorem 5.1 yields a payoff distribution mechanism which leads to a time-consistent solution. In particular, given a set of agreed-upon imputations $\xi^\ell(t, x_t^*)$, a payment $B_t^\ell(x_t^*)$ in (5.2) would lead to the realization of the imputation $\{\xi^\ell(t, x_t^*)$, for $t \in \kappa$ and $\ell \in N\}$. As demonstrated in (5.4),

$$\sum_{\zeta=t}^{T} B_\zeta^\ell(x_\zeta^*)\left(\frac{1}{1+r}\right)^{\zeta-1} + g^\ell(\bar{x}^\ell - x_{T+1})\left(\frac{1}{1+r}\right)^T = \xi^\ell(t, x_t^*),$$

which is the set of agreed-upon imputations.

According to Condition 4.1,

$$\xi^\ell(t, x_t^*) = \frac{V^\ell(t, x_t^*)}{\sum_{j=1}^n V^j(t, x_t^*)} W^{M^*}(t, x_t^*).$$

Invoking Theorem 5.1 the payment (in present value terms) to nation ℓ in stage $t \in \kappa$ can be obtained as:

$$\begin{aligned} B_t^\ell(x_t^*)\left(\frac{1}{1+r}\right)^{t-1} &= \xi^\ell(t, x_t^*) - \xi^\ell(t+1, x_{t+1}^*) \\ &= \frac{V^\ell(t, x_t^*)}{\sum_{j=1}^n V^j(t, x_t^*)} W^{M^*}(t, x_t^*) \\ &\quad - \frac{V^\ell(t+1, x_{t+1}^*)}{\sum_{j=1}^n V^j(t+1, x_{t+1}^*)} W^{M^*}(t+1, x_{t+1}^*), \end{aligned} \tag{5.5}$$

for $\ell \in N, t \in \kappa$, where $V^\ell(t, x_t^*) = V^{\ell_t}(t, x_t^*)$ if $\ell \in S_t^1$ and $V^\ell(t, x_t^*) = V^{\hat{\ell}_t}(t, x_t^*)$ if $\ell \in S_t^2$.

Thus the agreed-upon imputations are realized as the game proceeds. Hence the agreed-upon optimality principle is maintained at every stage of collaboration and a time-consistent solution results. Formula (5.5) indeed provides a payoff distribution procedure leading to the satisfaction of Condition 4.1 and hence a time-consistent solution will be obtained.

Under cooperation, nations would use the optimal cooperative strategies (4.4) and (4.7). Substituting the state x_t^* and these strategies into the nations' payoff at stage t in (2.6) and (2.7) with reference to the chosen technique pattern M^*, one can obtain the payoffs that these nations receive in stage t. We use $\zeta_t^i(x_t^*)$ to denote payoff at stage t that nations ℓ receives when nations are using the optimal cooperative strategies.

According to Theorem 5.1, the payoff that nation ℓ should receive under the agreed-upon optimality principle is $B_t^\ell(x_t^*)$. Hence a transfer payment

$$\chi_t^\ell(x_t^*) = B_t^\ell(x_t^*) - \zeta_t^i(x_t^*)$$

has to be given to nation ℓ in stage t, for $\ell \in N$ and $t \in \kappa$.

Moreover, note that there can be other optimality principles for gain distribution besides sharing the gain from cooperation other than sharing proportionally the nations' relative sizes of noncooperative payoffs. For instance, the nations may agree to share the gain from cooperation equally among themselves. The corresponding solution imputation scheme becomes:

Condition 5.1

$$\xi^\ell(t, x_t^*) = V^\ell(t, x_t^*) + \frac{1}{n}\left[W^{M^*}(t, x_t^*) - \sum_{j=1}^n V^j(t, x_t^*)\right], \tag{5.6}$$

for all $\ell \in N$ and all $t \in \kappa$, where $V^\ell(t, x_t^*) = V^{\ell_t}(t, x_t^*)$ in Proposition 3.1 if $\ell \in S_t^1$ and $V^\ell(t, x_t^*) = V^{\hat{\ell}_t}(t, x_t^*)$ if $\ell \in S_t^2$.

The nations may also agree to share the gain as various linear combinations of the imputations in Condition 4.1 and Condition 5.1. Applying Theorem 5.1 one can readily derive a payoff distribution procedure satisfying any agreed-upon imputation scheme[1] $\{\xi^\ell(t, x_t^*)$, for $t \in \kappa$ and $\ell \in N\}$ and obtain a time-consistent solution.

[1] The Shapley value is also an imputation scheme, but in a dynamic context, the super-addivity property of the corresponding characteristic functions does not hold in general, including in the case of the present analysis. Therefore nations would not choose the Shapley value as the imputation in an optimality principle.

6 Numerical illustration

As a numerical illustration we consider the case where there are 3 nations which have 3 stages of actions.

The demand functions of these nations are $P_t^1 = 50 - 2Q_t^1 - Q_t^2 - Q_t^3$, $P_t^2 = 72 - Q_t^1 - 4Q_t^2 - 2Q_t^3$ and $P_t^3 = 60 - 2Q_t^1 - Q_t^2 - 3Q_t^3$. The costs of producing output with conventional technique are $c^1 = 1$, $c^2 = 0.5$, $c^3 = 1$; and those of using environment-preserving technique are $\hat{c}^1 = 2.5$, $\hat{c}^2 = 2$, $\hat{c}^3 = 2$. The abatement costs are $c_1^a = 2$, $c_2^a = 2$, $c_3^a = 2.5$; and the abatement parameters are $b_1 = 1$, $b_2 = 1$, $b_3 = 1.5$. The pollution dynamics parameters are $a^1 = 2$, $\hat{a}^1 = 0.5$, $a^2 = 2$, $\hat{a}^2 = 0.5$, $a^3 = 2$, $\hat{a}^3 = 1$. The pollution decay rate $\delta = 0.05$ and the pollution damage parameters are $h_1 = 0.7$, $h_1 = 0.8$, $h_3 = 1.8$. The initial pollution stock is $x_1 = 4$ and the discount rate is $r = 0.04$. The terminal bonus (penalty) parameters are $g^1 = 0.5$, $g^2 = 0.4$, $g^3 = 1.7$; $\bar{x}^1 = 200$, $\bar{x}^2 = 500$, $\bar{x}^3 = 100$.

We first compute the outcome under non-cooperation. At stage $T + 1 = 4$, invoking Proposition 3.1 we have

$$A_4^1 = -0.5, \qquad A_4^2 = -0.4, \qquad A^3 = -1.7;$$

$$C_4^1 = 100, \qquad C_4^2 = 200 \quad \text{and} \quad C_4^3 = 170.$$

Using condition (3.2), one can show that industrial sectors 1 and 2 will use conventional technique and sector 3 will use environment-preserving technique in stage 3. Industrial outputs can be obtained as:

$$q_3^1 = 9.1169, \qquad q_3^2 = 6.3787, \qquad \hat{q}_3^3 = 5.2921.$$

Invoking (A.4) and (A.5) in Appendix A we obtain:

$$A_3^1 = -0.727968, \qquad A_3^2 = -0.817751, \qquad \hat{A}_3^3 = -2.398049,$$

$$C_3^1 = 252.8004, \qquad C_3^2 = 346.0104 \quad \text{and} \quad \hat{C}_3^3 = 194.6502.$$

Using condition (3.2), one can show that industrial sectors 1 and 2 will use conventional technique and sector 3 will use environment-preserving technique in stage 2. Industrial outputs can be obtained as:

$$q_2^1 = 9.0168, \qquad q_2^2 = 6.3074, \qquad \hat{q}_2^3 = 5.2255.$$

Invoking (A.4) and (A.5) we obtain:

$$A_2^1 = -0.43983, \qquad A_2^2 = -0.516227, \qquad \hat{A}_2^3 = -1.937482,$$

$$C_2^1 = 393.1945, \qquad C_2^2 = 473.5493 \quad \text{and} \quad \hat{C}_2^3 = 196.2488.$$

According to condition (3.2), industrial sectors 1 and 2 will use conventional technique and sector 3 will use environment-preserving technique in stage 1. Industrial outputs can be obtained as:

$$q_1^1 = 9.1361, \qquad q_1^2 = 6.3587, \qquad q_1^3 = 5.2510.$$

Invoking (A.4) and (A.5) we obtain:

$$A_1^1 = -0.672388, \qquad A_1^2 = -0.772149, \qquad \hat{A}_1^3 = -2.360774,$$

$$C_1^1 = 537.4093, \qquad C_1^2 = 605.3887 \quad \text{and} \quad \hat{C}_1^3 = 211.4722.$$

The noncooperative state path can be obtained as:

$$x_1 = 4, \qquad x_2 = 42.0106, \qquad x_3 = 27.0039, \qquad x_4 = 41.0968.$$

Now consider the case that the 3 nations agree to collaborate so that they would maximize their joint payoff and share the gain from cooperation proportional to the nations' relative sizes of noncooperative payoffs. The joint payoff maximizing pattern of technique choices is that all 3 nations will adopt environment-preserving technique.

First consider stage 4, from (B.2) in Appendix B we obtain

$$A_{T+1}^{M^*} = -2.6 \quad \text{and} \quad C_4^{M^*} = 470.$$

Invoking (B.3) we obtain:

$$A_3^{M^*} = -2.70625, \qquad A_2^{M^*} = -2.555709 \quad \text{and} \quad A_1^{M^*} = -2.766075.$$

The nations' outputs in the 3 stages under cooperation are

$$\hat{q}_3^1 = 6.2344, \qquad \hat{q}_3^2 = 5.8281, \qquad \hat{q}_3^3 = 3.2186; \qquad \hat{q}_2^1 = 6.2325, \qquad \hat{q}_2^2 = 5.8275,$$
$$\hat{q}_2^3 = 3.2050; \qquad \hat{q}_1^1 = 6.2363, \qquad \hat{q}_1^2 = 5.8288, \qquad \hat{q}_1^3 = 3.2225.$$

Invoking (B.3) again we obtain:

$$C_3^{M^*} = 885.7551, \qquad C_2^{M^*} = 1284.575 \quad \text{and} \quad C_1^{M^*} = 1684.066.$$

Solving the optimal cooperative trajectory yields:

$$x_1^* = 4, \qquad x_2^* = 3.7169, \qquad x_3^* = 3.5777, \qquad x_4^* = 4.1516.$$

The joint payoffs in stages 1 to 4 along the optimal cooperative trajectory can be obtained as:

$$W^{M*}(1, x_1^*) = 1673.002, \qquad W^{M*}(2, x_2^*) = 1226.035,$$
$$W^{M*}(3, x_3^*) = 809.9787 \quad \text{and} \quad W^{M*}(4, x_4^*) = 408.2323.$$

The individual payoffs for the 3 nations along the optimal cooperative trajectory are

$$V^1(1, x_1^*) = 534.7198, \qquad V^2(1, x_1^*) = 602.3001, \qquad V^3(1, x_1^*) = 202.0292,$$
$$V^1(2, x_2^*) = 376.4998, \qquad V^2(2, x_2^*) = 453.4909, \qquad V^3(2, x_2^*) = 182.0357,$$
$$V^1(3, x_3^*) = 231.3202, \qquad V^2(3, x_3^*) = 317.2011, \qquad V^3(3, x_3^*) = 172.0328,$$
$$V^1(4, x_4^*) = 87.0542, \qquad V^2(4, x_4^*) = 176.323, \qquad V^3(4, x_4^*) = 144.8551.$$

To summarize the results we first present the technology patterns, national outputs and levels of pollution stock under non-cooperation and collaboration in Table 1. The technique pattern under noncooperation involves nation 3 adopting environment-preserving techniques in stages 2 and 3. While under cooperation all 3 nations adopt environment-preserving techniques in all the 3 stages. The levels of pollution under collaboration are below those with no cooperation.

Table 1 Technology patterns, national outputs and levels of pollution stock under non-cooperation and collaboration

Stage	Non-cooperation				Collaboration			
t	Nation 1	Nation 2	Nation 3	x_t	Nation 1	Nation 2	Nation 3	x_t^*
1	q_1^1 9.1361	q_1^2 6.3587	q_1^3 5.2510	4	$\hat{q}_1^1$ 6.2363	$\hat{q}_1^2$ 5.8288	$\hat{q}_1^3$ 3.2225	4
2	q_2^1 9.0168	q_2^2 6.3074	$\hat{q}_2^3$ 5.2255	42.0106	$\hat{q}_2^1$ 6.2325	$\hat{q}_2^2$ 5.8275	$\hat{q}_2^3$ 3.2050	3.7169
3	q_3^1 9.1169	q_3^2 6.3787	$\hat{q}_3^3$ 5.2921	27.0039	$\hat{q}_3^1$ 6.2344	$\hat{q}_3^2$ 5.8281	$\hat{q}_3^3$ 3.2186	3.5777
4				41.0968				4.1516

Table 2 Total collaborative payoff and nations' imputations

t	Total collaborative payment	Nation 1's imputation	Nation 2's imputation	Nation 3's imputation
	$W^{M^*}(t, x_t^*)$	$\xi^1(t, x_t^*)$	$\xi^2(t, x_t^*)$	$\xi^3(t, x_t^*)$
1	1673.002	668.0765	752.5111	252.4143
2	1226.035	456.1164	549.3885	220.5299
3	809.9787	260.0283	356.5674	193.383
4	408.2323	87.0542	176.323	144.8551

Table 3 Payments incurred to nations in each stage—present value and current value

t	Stage cooperative payment (in current value)			Stage cooperative payment (in present value)		
	$B_t^1(x_t^*)$	$B_t^2(x_t^*)$	$B_t^3(x_t^*)$	$R_t B_t^1(x_t^*)$	$R_t B_t^2(x_t^*)$	$R_t B_t^3(x_t^*)$
1	211.9601	203.1226	31.8844	211.9601	203.1226	31.8844
2	203.9317	200.5339	28.2327	196.0882	192.8211	27.1469
3	187.0887	194.9524	52.4879	172.974	180.2445	48.528
4	97.9242	198.3393	162.9422	87.0542	176.323	144.8551

Note: $R_t = (1+r)^{-(t-1)}$

Then we proceed to compute the imputations to the nations under collaboration using Condition 4.1 and these figure are given in Table 2 along with the joint payoff under cooperation.

Finally, payoff distribution procedures leading the realization of the imputations in Table 2 are derived using Theorem 5.1 and displayed in Table 3. Note that both the current value and present values of these payment in various stages are provided.

7 Concluding remarks

Adoption of environment-preserving production technique plays a key role to effectively solving the continual worsening global industrial pollution problem. In this analysis, we present a dynamic game of collaborative pollution management with production technique choices. This is the first time that technique choices are allowed in cooperative dynamic game analysis. In dynamic cooperation, a credible cooperative agreement has to be time consistent. Time consistent cooperative solution and analytically tractable payoff distribution procedures are derived in the analysis. This approach widens the application of cooperative differential game theory to environmental problems with technique choice.

Various extensions can be incorporated into the analysis readily. First, there could be more than two choices of techniques leading to different degrees of pollution. Second, the natural rate of decay may be related to the pattern of technique choice. Thirdly, the number of industrial products produced by a nation could be more than one. Moreover, one may introduce costs of technique switching. Sufficiently high costs for switching new techniques back to conventional techniques would yield the outcome of irreversibility of the new techniques once they have been installed. Using the optimal technique pattern searching method in Sect. 4.1, one can determine when a nation will switch to new technologies permanently.

Finally, a research project involving large-scale computer simulation to generate a practicable multi-national collaborative scheme based on this analysis by the author is under way. Since this is the first time dynamically consistent cooperative dynamic games are applied in collaborative environmental management, further research along this line is expected.

Appendix A: Proof of Proposition 3.1

From (3.11) we can obtain $V^{i_t}_{x_{t+1}}(t+1, x_{t+1})(1+r)^{t-1}$ as $A^{i_t}_{t+1}(1+r)^{-1}$ and $V^{\hat{i}_t}_{x_{t+1}}(t+1, x_{t+1})(1+r)^{t-1}$ as $\hat{A}^{\hat{i}_t}_{t+1}(1+r)^{-1}$. Substituting these results into the game equilibrium strategies (3.9) and (3.10), and then into (3.1) yield:

$$\begin{aligned}
A^{i_t}_t x + C^{i_t}_t = \Bigg[&\bigg(\alpha^{i_t}_t - \sum_{j\in S^1_t} \beta^{i_t}_j \phi^j_t(x) - \sum_{j\in S^2_t} \beta^{i_t}_j \hat{\phi}^j_t(x)\bigg)\phi^{i_t}_t(x) \\
&- c^{i_t}\phi^{i_t}_t(x) - \frac{(b_{i_t})^2}{4c^a_{i_t}}[A^{i_t}_{t+1}(1+r)^{-1}]^2 x - h^{i_t}x\Bigg] \\
&+ (1+r)^{-1}\Bigg[A^{i_t}_{t+1}\bigg(x + \sum_{j\in S^1_t} a^j\phi^j_t(x) + \sum_{j\in S^2_t}\hat{a}^j\hat{\phi}^j_t(x) \\
&+ \sum_{j\in S^1_t}\frac{(b_j)^2}{2c^a_j}A^j_{t+1}(1+r)^{-1}x + \sum_{j\in S^2_t}\frac{(b_j)^2}{2c^a_j}\hat{A}^j_{t+1}(1+r)^{-1}x - \delta x\bigg) + C^{i_t}_{t+1}\Bigg], \\
&\text{for } t\in\kappa \text{ and } i_t\in S^1_t,
\end{aligned}$$

$$A^{i_t}_{t+1}x + C^{i_t}_{t+1} = g^{i_t}(\bar{x}^{i_t} - x), \quad i_t \in S^1_t;$$

$$\begin{aligned}
\hat{A}_t^{\hat{i}_t} x + \hat{C}_t^{\hat{i}_t} = {} & \Bigg[\Bigg(\alpha_t^{\hat{i}_t} - \sum_{j\in S_t^1} \beta_j^{\hat{i}_t} \phi_t^j(x) - \sum_{j\in S_t^2} \beta_j^{\hat{i}_t} \hat{\phi}_t^j(x)\Bigg)\hat{\phi}_t^{\hat{i}_t}(x) \\
& - \hat{c}^{\hat{i}_t}\hat{\phi}_t^{\hat{i}_t}(x) - \frac{(b_{\hat{i}_t})^2}{4c_{\hat{i}_t}^a}\big[\hat{A}_{t+1}^{\hat{i}_t}(1+r)^{-1}\big]^2 x - h^{\hat{i}_t} x\Bigg] \\
& + (1+r)^{-1}\Bigg[\hat{A}_{t+1}^{\hat{i}_t}\Bigg(x + \sum_{j\in S_t^1} a^j \phi_t^j(x) + \sum_{j\in S_t^2} \hat{a}^j \hat{\phi}_t^j(x) \\
& + \sum_{j\in S_t^1} \frac{(b_j)^2}{2c_j^a} A_{t+1}^j (1+r)^{-1} x + \sum_{j\in S_t^2} \frac{(b_j)^2}{2c_j^a} \hat{A}_{t+1}^j (1+r)^{-1} x - \delta x\Bigg) + \hat{C}_{t+1}^{\hat{i}_t}\Bigg], \\
& \text{for } t\in\kappa \text{ and } \hat{i}_t \in S_t^2,
\end{aligned}$$

$$\hat{A}_{t+1}^{\hat{i}_t} x + \hat{C}_{t+1}^{\hat{i}_t} = g^{\hat{i}_t}(\bar{x}^{\hat{i}_t} - x), \quad \hat{i}_t \in S_t^2; \tag{A.1}$$

$$A_{t+1}^{i_t}(1+r)^{-1} > \frac{(\hat{c}^{i_t} - c^{i_t})}{(\hat{a}^{i_t} - a^{i_t})}, \quad \text{for } i_t \in S_t^1 \quad \text{and}$$

$$\hat{A}_{t+1}^{\hat{i}_t}(1+r)^{-1} \le \frac{(\hat{c}^{\hat{i}_t} - c^{\hat{i}_t})}{(\hat{a}^{\hat{i}_t} - a^{\hat{i}_t})}, \quad \text{for } \hat{i}_t \in S_t^2. \tag{A.2}$$

where

$$\phi_t^j(x) = \Bigg[\bar{\alpha}_t^j + \sum_{\zeta\in S_t^1} \bar{\beta}_t^{(j)\zeta} A_{t+1}^\zeta (1+r)^{-1} + \sum_{\zeta\in S_t^2} \bar{\beta}_t^{(j)\zeta} \hat{A}_{t+1}^\zeta (1+r)^{-1}\Bigg] \quad \text{and}$$

$$\hat{\phi}_t^k(x) = \Bigg[\hat{\bar{\alpha}}_t^k + \sum_{\zeta\in S_t^1} \hat{\bar{\beta}}_t^{(k)\zeta} A_{t+1}^\zeta (1+r)^{-1} + \sum_{\zeta\in S_t^2} \hat{\bar{\beta}}_t^{(k)\zeta} \hat{A}_{t+1}^\zeta (1+r)^{-1}\Bigg].$$

First consider the stage $T+1$, from (A.1) we obtain

$$\begin{aligned}
& A_{T+1}^{i_T} = -g^{i_T}, \qquad C_{T+1}^{i_T} = g^{i_T}\bar{x}^{i_T}, \qquad \hat{A}_{T+1}^{\hat{i}_T} = -g^{\hat{i}_T} \quad \text{and} \quad \hat{C}_{T+1}^{\hat{i}_T} = g^{\hat{i}_T}\bar{x}^{\hat{i}_T}, \\
& \text{for } i_T \in S_T^1 \text{ and } \hat{i}_T \in S_T^2.
\end{aligned} \tag{A.3}$$

At stage T, invoking (A.2), industrial sector i which has $A_{x_{T+1}}^i(1+r)^{-1} > \frac{(\hat{c}^i - c^i)}{(\hat{a}^i - a^i)}$ would use conventional technique, otherwise it would use environment-preserving technique.

Note that on the left-hand-side of (A.1) the expressions are $A_t^{i_t}x + C_t^{i_t}$ and $\hat{A}_t^{\hat{i}_t}x + \hat{C}_t^{\hat{i}_t}$. On the right-hand-side there are expressions which are linear in x with coefficients involving the terms $A_{t+1}^{i_t}$, $C_{t+1}^{i_t}$, $\hat{A}_{t+1}^{\hat{i}_t}$ and $\hat{C}_{t+1}^{\hat{i}_t}$. The values of $A_T^{i_T}$, $C_T^{i_T}$, $\hat{A}_T^{\hat{i}_T}$ and $\hat{C}_T^{\hat{i}_T}$ for $i_t \in S_t^1$ and $\hat{i}_t \in S_t^2$ can be obtained using the values of $A_{T+1}^{i_T}$, C_{T+1}^{i}, $\hat{A}_{T+1}^{\hat{i}_T}$ and $\hat{C}_{T+1}^{\hat{i}_T}$ in (A.3). Moreover, the linearity structures guarantee that this set of values is unique.

For industrial sector $i_T \in S_T^1$ which chooses to adopt conventional technique at stage T, one can invoke (A.1) to obtain the explicit solutions of $A_t^{i_t}$ and $C_t^{i_t}$ for $t = T$ as:

$$C_t^{i_t} = \Bigg(\alpha_t^{i_t} - \sum_{j\in S_t^1} \beta_j^{i_t}\phi_t^j(x) - \sum_{j\in S_t^2} \beta_j^{i_t}\hat{\phi}_t^j(x)\Bigg)\phi_t^{i_t}(x) - c^{i_t}\phi_t^{i_t}(x)$$

$$+(1+r)^{-1}\left[A_{t+1}^{i_t}\left(\sum_{j\in S_t^1} a^j\phi_t^j(x)+\sum_{j\in S_t^2}\hat{a}^j\hat{\phi}_t^j(x)\right)+C_{t+1}^{i_t}\right], \quad \text{and}$$

$$A_t^{i_t}=-\frac{(b_{i_t})^2}{4c_{i_t}^a}[A_{t+1}^{i_t}(1+r)^{-1}]^2-h^{i_t}$$
$$+(1+r)^{-1}A_{t+1}^{i_t}\left(1+\sum_{j\in S_t^1}\frac{(b_j)^2}{2c_j^a}A_{t+1}^j(1+r)^{-1}+\sum_{j\in S_t^2}\frac{(b_j)^2}{2c_j^a}\hat{A}_{t+1}^j(1+r)^{-1}-\delta\right). \tag{A.4}$$

For industrial sector $\hat{i}_T\in S_T^2$ which chooses to adopt environment-preserving technique at stage T, one can invoke (A.1) to obtain the explicit solutions of $\hat{A}_t^{\hat{i}_t}$ and $\hat{C}_t^{\hat{i}_t}$ for $t=T$ as:

$$\hat{C}_t^{\hat{i}_t}=\left(\alpha_t^{\hat{i}_t}-\sum_{j\in S_t^1}\beta_j^{\hat{i}_t}\phi_t^j(x)-\sum_{j\in S_t^2}\beta_j^{\hat{i}_t}\hat{\phi}_t^j\right)\hat{\phi}_t^{\hat{i}_t}(x)-\hat{c}^{\hat{i}_t}\hat{\phi}_t^{\hat{i}_t}(x)$$
$$+(1+r)^{-1}\left[A_{t+1}^{\hat{i}_t}\left(\sum_{j\in S_t^1} a^j\phi_t^j(x)+\sum_{j\in S_t^2}\hat{a}^j\hat{\phi}_t^j(x)\right)+\hat{C}_{t+1}^{\hat{i}_t}\right], \quad \text{and}$$

$$\hat{A}_t^{\hat{i}_t}=-\frac{(b_{\hat{i}_t})^2}{4c_{\hat{i}_t}^a}[\hat{A}_{t+1}^{\hat{i}_t}(1+r)^{-1}]^2-h^{\hat{i}_t}$$
$$+(1+r)^{-1}\hat{A}_{t+1}^{\hat{i}_t}\left(1+\sum_{j\in S_t^1}\frac{(b_j)^2}{2c_j^a}A_{t+1}^j(1+r)^{-1}+\sum_{j\in S_t^2}\frac{(b_j)^2}{2c_j^a}\hat{A}_{t+1}^j(1+r)^{-1}-\delta\right). \tag{A.5}$$

Now consider the situation at stage $T-1$. At stage $T-1$, the industrial sector i which has $A_{x_T}^i(1+r)^{-1}>\frac{(\hat{c}^i-c^i)}{(\hat{a}^i-a^i)}$ would use conventional technique, otherwise it would use environment-preserving technique. A unique set of values of $A_{T-1}^{i_T}$, C_{T-1}^i, $\hat{A}_{T-1}^{\hat{i}_T}$ and $\hat{C}_{T-1}^{\hat{i}_T}$ can be obtained using (A.4) and (A.5).

Repeating the process, $A_t^{i_t}$, $C_t^{i_t}$, $\hat{A}_t^{\hat{i}_t}$ and $\hat{C}_t^{\hat{i}_t}$. A_t^i and C_t^i, for $i_t\in S_t^1$ and $\hat{i}_t\in S_t^2$ and $t\in\{1,2,\dots,T-2\}$, can be explicitly obtained from (A.4) and (A.5). □

Appendix B: Proof of Proposition 4.1

From (4.8) we can obtain $W_{x_{t+1}}^{M^\gamma}(t+1,x_{t+1})(1+r)^{t-1}$ as $A_{t+1}^{M^\gamma}(1+r)^{-1}$. Substituting this result into the optimal controls in (4.4) and (4.7), and then into (4.3) yields

$$A_t^{M^\gamma}x+C_t^{M^\gamma}=\sum_{i\in S_t^{M^\gamma[1]}}\Bigg[\left(\alpha_t^i-\sum_{j\in S_t^{M^\gamma[1]}}\beta_j^i\psi_t^{(\mathrm{M}^\gamma)j}(x)-\sum_{\zeta\in S_t^{M^\gamma[2]}}\beta_\zeta^i\hat{\psi}_t^{(M^\gamma)\zeta}\right)\psi_t^{(\mathrm{M}^\gamma)i}(x)$$
$$-c^i\psi_t^{(\mathrm{M}^\gamma)i}(x)-\frac{(\hat{b}_i)^2}{4c_i^a}[A_{t+1}^{M^\gamma}(1+r)^{-1}]^2x-h^ix\Bigg]$$
$$+\sum_{i\in S_t^{M^\gamma[2]}}\Bigg[\left(\alpha_t^i-\sum_{j\in S_t^{M^\gamma[1]}}\beta_j^i\psi_t^{(\mathrm{M}^\gamma)j}(x)-\sum_{\zeta\in S_t^{M^\gamma[2]}}\beta_\zeta^i\hat{\psi}_t^{(M^\gamma)\zeta}(x)\right)\hat{\psi}_t^{(\mathrm{M}^\gamma)i}(x)$$

$$
\begin{aligned}
&- \hat{c}^{i(S_t^{M^\gamma[2]})} \hat{\psi}_t^{(\mathrm{M}^\gamma)i}(x) - \frac{(\hat{b}_i)^2}{4c_i^a}\left[A_{t+1}^{M^\gamma}(1+r)^{-1}\right]^2 x - h^i x \Big] \\
&+ (1+r)^{-1}\Bigg[A_{t+1}^{M^\gamma}\Bigg(x + \sum_{j\in S_t^{M^\gamma[1]}} a^j \psi_t^{(\mathrm{M}^\gamma)j}(x) + \sum_{j\in S_t^{M^\gamma[2]}} \hat{a}^j \hat{\psi}_t^{(\mathrm{M}^\gamma)j}(x) \\
&+ \sum_{j=1}^{n} \frac{(\hat{b}_i)^2}{2c_i^a} A_{t+1}^{M^\gamma}(1+r)^{-1}x - \delta x\Bigg) + C_{t+1}^{M^\gamma}\Bigg], \quad \text{for } t\in\kappa;
\end{aligned}
\tag{B.1}
$$

$$
A_{T+1}^{M^\gamma} x + C_{T+1}^{M^\gamma} = \sum_{i=1}^{n} g^i(\bar{x}^i - x),
$$

where

$$
\begin{aligned}
\psi_t^{(\mathrm{M}^\gamma)j}(x) &= \tilde{\alpha}_t^{(M^\gamma)j} + \tilde{\beta}_t^{(M^\gamma)j} A_{t+1}^{M^\gamma}(1+r)^{-1} \quad \text{and} \\
\hat{\psi}_t^{(\mathrm{M}^\gamma)\zeta}(x) &= \hat{\tilde{\alpha}}_t^{(M^\gamma)j} + \hat{\tilde{\beta}}_t^{(M^\gamma)j} A_{t+1}^{M^\gamma}(1+r)^{-1}.
\end{aligned}
$$

First consider the stage $T+1$, from (B.1) we obtain

$$
A_{T+1}^{M^\gamma} = \sum_{i=1}^{n} -g^i \quad \text{and} \quad C_{T+1}^{M^\gamma} = \sum_{i=1}^{n} g^i \bar{x}^i. \tag{B.2}
$$

Now we consider the stage T. Note that the left-hand-side of (B.1) consists of the expression $A_T^{M^\gamma} x + C_T^{M^\gamma}$. On the right-hand-side there is an expression which is linear in x with coefficients involving $A_{T+1}^{M^\gamma}$ and $C_{T+1}^{M^\gamma}$. The values of $A_T^{M^\gamma}$ and $C_T^{M^\gamma}$ can be obtained using $A_{T+1}^{M^\gamma}$ and $C_{T+1}^{M^\gamma}$ in (B.2). Using (B.1) yields the explicit solution of $C_t^{M^\gamma}$ and $A_t^{M^\gamma}$ for $t=T$ as:

$$
\begin{aligned}
C_t^{M^\gamma} = &\sum_{i\in S_t^{M^\gamma[1]}} \Bigg[\Bigg(\alpha_t^i - \sum_{j\in S_t^{M^\gamma[1]}} \beta_j^i \psi_t^{(\mathrm{M}^\gamma)j}(x) - \sum_{\zeta\in S_t^{M^\gamma[2]}} \beta_\zeta^i \hat{\psi}_t^{(M^\gamma)\zeta}\Bigg)\psi_t^{(\mathrm{M}^\gamma)i}(x) \\
&- c^i \psi_t^{(\mathrm{M}^\gamma)i}(x)\Bigg] + \sum_{i\in S_t^{M^\gamma[2]}} \Bigg[\Bigg(\alpha_t^i - \sum_{j\in S_t^{M^\gamma[1]}} \beta_j^i \psi_t^{(\mathrm{M}^\gamma)j}(x) \\
&- \sum_{\zeta\in S_t^{M^\gamma[2]}} \beta_\zeta^i \hat{\psi}_t^{(M^\gamma)\zeta}(x)\Bigg)\hat{\psi}_t^{(\mathrm{M}^\gamma)i}(x) - \hat{c}^{i(S_t^{M^\gamma[2]})}\hat{\psi}_t^{(\mathrm{M}^\gamma)i}(x)\Bigg] \\
&+ (1+r)^{-1}\Bigg[A_{t+1}^{M^\gamma}\Bigg(\sum_{j\in S_t^{M^\gamma[1]}} a^j \psi_t^{(\mathrm{M}^\gamma)j}(x) + \sum_{j\in S_t^{M^\gamma[2]}} \hat{a}^j \hat{\psi}_t^{(\mathrm{M}^\gamma)j}(x)\Bigg) + C_{t+1}^{M^\gamma}\Bigg],
\end{aligned}
$$

and

$$
\begin{aligned}
A_t^{M^\gamma} = &\sum_{i=1}^{n}\Bigg[-\frac{(b_i)^2}{4c_i^a}\left[A_{t+1}^{M^\gamma}(1+r)^{-1}\right]^2 - h^i\Bigg] \\
&+ (1+r)^{-1}\Bigg[A_{t+1}^{M^\gamma}\Bigg(1 + \sum_{j=1}^{n}\frac{(b_i)^2}{2c_i^a}A_{t+1}^{M^\gamma}(1+r)^{-1} - \delta\Bigg)\Bigg].
\end{aligned}
\tag{B.3}
$$

Now consider stage $T-1$. One can obtain $A_{T-1}^{M\gamma}$ and $C_{T-1}^{M\gamma}$ as in (B.3) by setting $t = T-1$. Repeating the process, $A_t^{M\gamma}$ and $C_t^{M\gamma}$ for $t \in \{1, 2, \ldots, T-2\}$ can be explicitly obtained. □

References

Basar, T., & Olsder, G. J. (1999). *Dynamic noncooperative game theory* (2nd ed.). Philadelphia: SIAM.

Breton, M., Zaccour, G., & Zahaf, M. (2005). A differential game of joint implementation of environmental projects. *Automatica*, *41*, 1737–1749.

Breton, M., Zaccour, G., & Zahaf, M. (2006). A game-theoretic formulation of joint implementation of environmental projects. *European Journal of Operational Research*, *168*, 221–239.

Dixit, A. K. (1979). A model of duopoly suggesting a theory of entry barriers. *Bell Journal of Economics*, *10*, 20–32.

Dockner, E. J., & Leitmann, G. (2001). Coordinate transformation and derivation of open-loop Nash equilibria. *Journal of Economic Dynamics and Control*, *110*, 1–15.

Dockner, E. J., & Long, N. V. (1993). International pollution control: cooperative versus noncooperative strategies. *Journal of Environmental Economics and Management*, *25*, 13–29.

Dockner, E., & Nishimura, K. (1999). Transboundary pollution in a dynamic game model. *The Japanese Economic Review*, *50*(4), 443–456.

Dutta, P.-K., & Radner, R. (2006). Population growth and technological change in a global warming model. *Economic Theory*, *29*(2), 251–270.

Feenstra, T., Kort, P. M., & De Zeeuw, A. (2001). Environmental policy instruments in an international duopoly with feedback investment strategies. *Journal of Economics Dynamics and Control*, *25*, 1665–1687.

Fredj, K., Martín-Herrán, G., & Zaccour, G. (2004). Slowing deforestation pace through subsidies: a differential game. *Automatica*, *40*, 301–309.

Jørgensen, S., & Zaccour, G. (2001). Time consistent side payments in a dynamic game of downstream pollution. *Journal of Economic Dynamics and Control*, *25*, 1973–1987.

Petrosyan, L., & Zaccour, G. (2003). Time-consistent Shapley value allocation of pollution cost reduction. *Journal of Economic Dynamics and Control*, *27*, 381–398.

Petrosyan, L. A., & Zenkevich, N. A. (1996). *Game theory*. Singapore: World Scientific Publishing Co. Pte. Ltd.

Rubio, S., & Ulph, A. (2007). An infinite-horizon model of dynamic membership of international environmental agreements. *Journal of Environmental Economics and Management*, *54*(3), 296–310.

Singh, N., & Vives, X. (1984). Price and quantity competition in a differentiated duopoly. *RAND Journal of Economics*, *15*, 546–554.

Stimming, M. (1999). Capital accumulation subject to pollution control: open-loop versus feedback investment strategies. *Annals of Operations Research*, *88*, 309–336.

Tahvonen, O. (1994). Carbon dioxide abatement as a differential game. *European Journal of Political Economy*, *10*, 685–705.

Yeung, D. W. K. (1992). A differential game of industrial pollution management. *Annals of Operations Research*, *37*, 297–311.

Yeung, D. W. K. (2007). Dynamically consistent cooperative solution in a differential game of transboundary industrial pollution. *Journal of Optimization Theory and Applications*, *134*(1), 143–160.

Yeung, D. W. K. (2008). Dynamically consistent solution for a pollution management game in collaborative abatement with uncertain future payoffs. In: D. W. K. Yeung & L. A. Petrosyan (Guest Eds.) Special issue on frontiers in game theory: in honour of John F. Nash. *International Game Theory Review*, *10*(4), 517–538.

Yeung, D. W. K., & Petrosyan, L. (2004). Subgame consistent cooperative solutions in stochastic differential games. *Journal of Optimization Theory and Applications*, *120*, 651–666.

Yeung, D. W. K., & Petrosyan, L. (2005). Subgame consistent solution of a cooperative stochastic differential game with nontransferable payoffs. *Journal of Optimization Theory and Applications*, *124*(3), 701–724.

Yeung, D. W. K., & Petrosyan, L. (2006a). Dynamically stable corporate joint ventures. *Automatica*, *42*, 365–370.

Yeung, D. W. K., & Petrosyan, L. (2006b). *Cooperative stochastic differential games*. New York: Springer.

Yeung, D. W. K., & Petrosyan, L. (2008). A cooperative stochastic differential game of transboundary industrial pollution. *Automatica*, *44*, 1532–1544.

Yeung, D. W. K., & Petrosyan, L. (2010). Subgame consistent solutions for cooperative stochastic dynamic games. *Journal of Optimization Theory and Applications*, *145*(3), 579–596.