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Stochastic programming and the option of doing it differently

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Abstract Option theory and stochastic programming are tightly linked. Most options can be analyzed in both frameworks, and the two approaches support each other in many slightly more complex situations. But this similarity hides some central differences in perspective. This short note tries to focus on one of these, namely the fact that option theory can be applied only to options already identified, while stochastic programming is able to help us find options in contexts where it is not at all clear what they are, and where finding might be more important than valuing.

Keywords Programming: Stochastic, Relations to option theory

1 Background

The major question in option theory, from a modeling point of view, is "what is the option worth?" The modeler (decision-maker) observes an option, observes it has a price, and wonders what is its value. If the value is larger than the price, he buys the option. In financial option theory, this amounts to buying an actual financial option, while in real option theory it may amount to making an investment (in order to obtain the option), or simply buying the real option from someone else. If he is the writer of the option, or one considering selling a real investment opportunity, the option price reflects the minimal price he should ask for the option. Although there seems to be a general agreement that the concepts from option theory are sound, and that option theory presents an appropriate way of thinking about the value of flexibility in corporate investments (and other decisions under uncertainty for that matter), there is little agreement about how to apply the theory in practice. Borison (2005) discusses, rather critically, the different approaches to evaluating real options, with special emphasis on the applicability of the replication argument, and the distinction between public and private risk. As part of this he discusses how some approaches are in the financial option theory tradition, while others are more decision analysis focused. In particular, it is worth

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noting the work of Smith and Nau (1995), who elegantly combine the two depending on the nature of the risks.

In most of these settings, whether the authors rely on replication arguments, or revert to decision analysis, the focus is on one single predefined option, possibly complicated and compound, but still well defined. Even so, all these authors are of course aware that options normally occur within portfolios. An elegant setup for handling portfolio issues can be found in Wang and de Neufville (2004). They first distinguish between options "on" projects and options "in" projects. Options "on" projects are typically financial options which value depend on the development of a real project (financially or some technical aspect of the project). An option "in" a project is an option, which if exercised, will change the project itself. Such options "in" projects can be interdependent or parallel within the overall project. And their values will typically be path dependent. Wang and de Neufville set up a mixed-integer stochastic program that allows the evaluation of options "in" a project in a consistent way, taking dependencies into account. As part of their procedure, they have a phase where they use parametric optimization to find potential options. These options can be seen as potential parts of the final solution (i.e. the composition of the project). Different parameter settings give different ideas of what to do, but only within the stochastic program will it be determined which of these ideas (options) will be used under which scenarios (paths). So the total project will be path dependent, which is exactly the idea of investments under uncertainty.

Stochastic programming—see for example Kall and Wallace (1994) for an outline—will in this way play a role in the evaluation of a portfolio of options. The approach is strongly related to what Smith and Nau (1995) suggested for the private risks. At the same time, it fits with more traditional use of stochastic programming in portfolio management, see for example Ziemba and Mulvey (1998). If it is a portfolio of options, the relationship is strong.

In the above examples, stochastic programming plays the role of a tool within the decision analysis tradition. But stochastic programming has also been used to shed light on risk neutral probabilities within the replication argument based approaches, see King (2002). These ideas are later used by Zhao and Ziemba (2008). Therefore, although stochastic programming, in its direct appearance, fits within the decision analysis framework, it can play roles also within a replication argument setting.

Within the optimization community there is a discussion which is relevant to the above issues: Under what circumstances is it important to use stochastic rather than deterministic optimization models? Wallace (2000) and Higle and Wallace (2003) discuss this in detail. The important observation is that deterministic models, even if run repeatedly ("what-if"-sessions or parametric analyses) will never suggest buying any options, unless these happen to come for free (which they rarely do). The reason is that options are bought to help us handle the uncertain future in a better way, but in a deterministic world there is no uncertain future, and hence no need to pay to handle it. The union of option-free solutions is also option-free. Many deterministic instances will not substitute for one stochastic instance. Therefore, solutions coming from stochastic programs are *structurally different* from those coming from deterministic models because they contain options.

It is worth noting that the way Wang and de Neufville (2004) find possible options for their stochastic mixed-integer program potentially has a problem in light of the above discussion about deterministic models. Their "options" are wait-and-see optimal decisions (sites, reservoir storage capacities, and installed electricity generation capacities in the development plan for a river basin in China) for at least one scenario, and the stochastic program will find which of these will do best in a stochastic dynamic framework. But the approach will not discover possible options (decisions) "in" the project which are not part of optimal wait-and-see solutions. So their "options" are optimal decisions in some deterministic version of the world, and not options whose value come from being able to handle uncertainty well. That is, they do not end up with decisions made to open up for later decisions in light of uncertainty. That is meaningless in a deterministic world. And very often, these are the interesting ones, as pointed out in Wallace (2000) and Higle and Wallace (2003).

There is a certain mix of terms here. On one hand, the formal options, that is, those decisions (investments) that provide us with the possibility of observing the outcome of a random variable, and then do something (if we so wish), and the everyday use of "option", meaning "possible decision". The danger with Wang and de Neufville's approach is that they will miss the formal options, and end up with only "possible decisions", since the formal options are worthless in a deterministic world, and their search for "options" takes place in a deterministic environment.

So what can we do? In some cases, simply being aware of the fact that certain options will be missing if deterministic models are used in the search for options will be enough to understand how to remedy the weakness. Problem knowledge combined with knowledge of what might be missed may be enough.

2 Types of options

In the real options literature, it is common to distinguish among several types of options. See for example Table 1.1 in Trigeorgis (1996). Typical options are the option to defer, time-to-build options, the option to expand, the option to contract, to shut down and to restart, growth options and options to switch (inputs or outputs). And, of course, many real situations consist of several interdependent options. These are the types of options that are treated in the literature outlined above. Wang and de Neufville (2004), for example, will through their options identification phase find options "in" the project, that is, parts of the development of the river basin, which are of these types. Apart from the fact that good options normally are not part of the optimal wait-and-see solution for *any* scenario, there is a major shortcoming, from a modeling perspective, of all the approaches above: They consider only options of predefined types.

But to make good decisions in a stochastic environment, we need to have a broader understanding of what options might be. Let us briefly repeat how options are handled. From some starting point, we face the possibility of paying some amount (possibly zero, but normally a positive amount) to obtain an option. The purpose of this option is to make the future easier to handle—we wish to be more robust with respect to future shocks. The tradeoff is between a present deterministic cost and the expected future value of the option. This value can be in the form of increased expected profit (or decreased expected cost) as well as other types of goals based on expected utility or the probability of achieving a certain goal.

The future decisions are either not available, or only available at a higher cost, unless the option is bought now. In a stochastic dynamic model we might face a collection of such options, as outlined by Wang and de Neufville.

But there is a much more general type of option. And very often this option type is what we seek, but few tools can bring it about. We might call it *the option of doing it differently*.[1]

[1] This term is not in line with the formal definition of an option, as it refers to the action of obtaining the option, rather than the option itself.

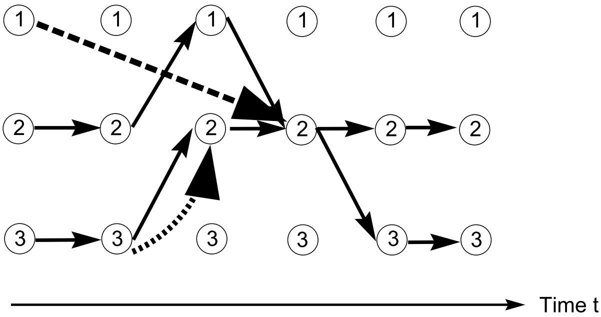

Fig. 1 Mean value solution with no consolidation, using two trucks

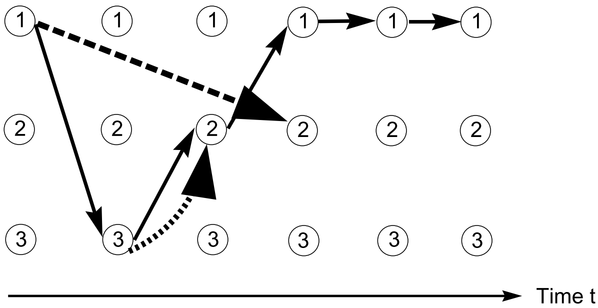

Fig. 2 Stochastic solution using consolidation between sites 3 and 2. Two trucks used

Let us start by a tiny example, taken from Lium et al. (2009). The example is a simple service network design problem, for a general overview, see Crainic (2003).

Consider Figs. 1 and 2. The staring point is a network representing three sites, which is then repeated over time. In the resulting time-space network, the three nodes to the left (in each figure) represent the three sites at time 0, and the last three nodes the sites at time 5.

In this tiny example, we have two commodities. One commodity (represented by a dashed arrow) becomes available at site 1 at time 0 and has to be delivered to site 2 within three periods. The demand here is 0, 2, or 4 units. The other commodity (represented by a dotted arrow) becomes available at site 3 at time 1 and has to be delivered to site 2 within one period. Also here the demand is 0, 2, or 4 units. The goal is to set up trucks that service these two needs. There is one cost associated with a truck that stands still, and another if it moves (full or empty). In this simple example all moves cost the same, and the number of trucks is a variable. Trucks must have repeatable schedules, so we must also consider their empty moves. Hence, for all solutions we present, trucks start and end in the same site. Rejection of flow is allowed at a rather high cost, in fact, so high that it is always profitable to set up a truck for a demand which is known by certainty. Each truck has a capacity of 2.

Under these simplifying assumptions optimal solutions can be found by inspection for the deterministic case. For the commodity represented by the dashed arrow, we shall get trucks that move twice between sites. Each truck will move from site 1 to site 2 in periods 0, 1 or 2 (in order to transport the commodity), and back to site 1 at any later point in time, so that it gets back to where it started. If the demand is 2, we shall get one truck, and if the demand is 4, we get two trucks. (Of course, with zero demand we get no truck.) For the "dotted" commodity, we similarly get trucks moving from site 3 to site 2 in period 1, and then back in some later period. Again the number of trucks will depend on the demand. In Fig. 1 we show one of the many alternative optimal solutions for expected demand, where there is one truck on each of the given paths of solid arcs. The "dashed" commodity in Fig. 1

will therefore wait at site 1 for two periods before being picked up by its truck and taken to site 2 just in time. We note that there will not be any consolidation in the deterministic solution (assuming just these two commodities) so that each commodity will use its own truck.

In the more expensive (also not unique) stochastic solution in Fig. 2, we see both trucks following the same path. That allows the two commodities to share the joint capacity of the two trucks from site 3 to site 2 in period 1. This is an example where commodity 1 is sent trough an intermediary breakbulk (site 3), before being consolidated with commodity 2. Having this kind of solution where two commodities share the common capacities of two trucks, makes expensive rejection less likely compared to if they had used one truck each as in the deterministic case. If we assume all scenarios equally likely, the expected rejection for the stochastic solution is only two thirds of that of the expected value solution. The two scenarios $(4,0)$ and $(0,4)$ can now be handled without rejection due to consolidation, that is, due to the ability to share transportation capacity. It is worth noting that this represents a reasonable choice of rejection cost. It is so high that any demand that occurs with certainty will pay for its own truck (if there is not available capacity on an existing truck). Further, the cost is high enough to make it profitable to add one move to each truck route to avoid rejecting two units with a probability of $\frac{2}{9}$. But the rejection cost is not so high that it allows an extra truck in the stochastic solution to help out with two units when the demand is $(2,4)$ and $(4,2)$ or four units when demand is $(4,4)$. So, it is better to reject than to have a truck with three moves carrying two units $\frac{1}{3}$ of the time.

The single path used in the stochastic solution does not appear in any scenario solution, because it is too expensive. This is typical. The most robust solution is not a wait-and-see optimal solution for any scenario. But for this paper the following is more important: Assume the starting point was the expected value solution in Fig. 1 (it is the best of the scenario solutions in terms of expected costs). We then pass to the stochastic solution. It costs more, it provides flexibility in the use of trucks between sites 2 and 3, and is therefore more robust with respect to changing demands. Hence, we have obviously bought an option. But the option is not explicit. We have not bought something on top of a starting point.

We may try to say that we have bought or sold a selection of options, but a close study of the solutions will show that this is not very fruitful, even for this tiny example. The different solutions in the example are simply *different*. Hence, although stochastic programming and option theory are very similar when applied to well defined options, this similarity hides important differences. Most of the interesting cases covered by stochastic programming are such that finding what the options are is a major research issue. Understanding what options amount to is crucial for understanding important decision problems. We may want to ask: What makes a schedule flexible? What is a flexible production system? How can we tell that one plan is more flexible or robust than another? In other words, finding what the options are is harder, and often more important, than valuing these options. Hence, stochastic programming is much more general than real option theory since its starting point is the decision context, and not a well defined option which is relatively easy to value. An important option is the *option of doing it differently*.

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