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Conditions of reverse bullwhip effect in pricing for price-sensitive demand functions

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Abstract Supply chain mechanisms that exacerbate price variation needs special attention, since price variation is one of the root causes of the bullwhip effect. In this study, we investigate conditions that create an amplification of price variation moving from the upstream suppliers to the downstream customers in a supply chain, which is referred as the "reverse bullwhip effect in pricing" (RBP). Considering initially a single-stage supply chain in which a retailer faces a random and price-sensitive demand, we derive conditions on a general demand function for which the retail price variation is higher than that of the wholesale price. The investigation is extended to a multi-stage supply chain in which the price at each stage is determined by a game theoretical framework. We illustrate the use of the conditions in identifying commonly used demand functions that induce RBP analytically and by means of several numerical examples.

Keywords Pricing · Bullwhip effect · Supply chain management · Game theory

1 Introduction and scope

Pricing decisions are very important since they directly impact the performance of supply chains due to the direct linkage with the market demand, competition, procurement costs, revenues, and thus profits. One critical aspect of supply chain prices is the price variation, that has been proven to be one of the main causes of the bullwhip effect, which adversely affects the supply chain performance, causing excess inventories, backorders and inefficient use of resources (Lee et al. 1997). The bullwhip effect is the increase in order variation, and it starts at the downstream of the supply chain and is amplified from one supply chain

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stage to another as we move towards the upstream. In contrast, our primary concern in this study is the impact of upstream supply chain price variation on the downstream retail prices, and the conditions of demand mechanisms that induce an amplified price variation as we move from the upstream to the downstream of a supply chain. Due to the reverse direction of the price variation propagation (compared to the direction of the bullwhip effect in order quantity variation), this behavior is referred to as the "reverse bullwhip effect in pricing (RBP)" (Ozelkan and Cakanyildirim 2007).

As an empirical evidence on the existence of RBP, imagine a simple supply chain where we have a single supplier and a single retailer. The supplier is selling a product to the retailer at a wholesale price of w and the retailer is using a fixed percentage profit margin $m > 0$ to identify the retail price p, i.e., $p = (1 + m)w$. Letting σ_w and σ_p denote the standard deviation of the wholesale and retail prices, respectively, it is easy to verify that $\sigma_p > \sigma_w$ and thus, an amplification of price variation occurs since $\sigma_p^2 = (1 + m)^2\sigma_w^2$. Therefore, we can generalize that in supply chains where all stages apply fixed percentage margins, an RBP would occur with amplification from one stage to another. The question is whether, based on this simple supply chain scenario, one can expect price variation increase all the time when the retailer optimally determines the retail price. As previously discussed in Ozelkan and Cakanyildirim (2007) and further extended in this paper, RBP would not always occur when optimal pricing is employed for price sensitive demands (i.e., demand is dependent on the retail price).

Another empirical evidence of the existence of RBP can be drawn from price indices published by Bureau of Labor Statistics (2008). Consumer Price Index (CPI) is a measure of the average change over time in the prices paid by urban consumers for representative consumer goods and services. Similarly, Producer Price Index (PPI) measures the average change over time in the selling prices received by domestic producers of goods and services. Therefore, CPI represents retail prices while PPI can be considered as an indirect measure of upstream prices. Among the published price indices, index series coded as CUUR0000SAf and WPUSOP3110 correspond to the food products from January 1998 to February 2008 (see Bureau of Labor Statistics 2008, for raw data). Computing sample standard deviations of these indices yields 13.7 and 11.1, respectively. Since the variation of CPI is greater than the variation of PPI, we can conjecture the existence of RBP for food products.

Our main hypothesis in this study is that for RBP to occur, certain market conditions should hold. The market condition analyzed in this study is the demand generation mechanisms. Second, we hypothesize that the magnitude of RBP may be related to the number of stages in a supply chain. Third, we believe that measures used to quantify RBP can be important in our understanding of RBP itself. Therefore, the purpose of the presented study is to investigate several open RBP research questions, namely, investigation of RBP under random demand, differentiating between stronger and weaker RBP conditions, and analysis of commonly used demand functions to understand which demand generating mechanisms yield RBP. We show that RBP conditions for the deterministic case still apply to the random demand case for certain demand uncertainty structures, and while some demand functions exclusively yield either RBP or no RBP, others result in RBP only for partial ranges of the upstream supply price.

The paper is organized as follows. In Sect. 2, we present a brief review of the related work. Then, we discuss the price determination mechanism as well as the conditions of RBP for the single-stage supply chain in Sect. 3, which is extended to the multi-stage supply chain in Sect. 4. Relationship of RBP, variations of prices and the bullwhip effect in order quantity is discussed in Sect. 5. We describe specific conditions for commonly used demand functions in Sect. 6, and conclude with a summary of major findings in the final section.

2 Literature review

Since the popularization of the term "bullwhip effect" by Lee et al. (1997), there have been a number of studies to investigate the underlying causes (demand signal processing, order batching, price fluctuations and shortage gaming) of the bullwhip effect and to mitigate such order variation (see e.g. Chen et al. 2000; Disney and Towill 2003; Sheu 2005; Disney et al. 2006; Kim et al. 2006). We refer to Lee et al. (2004) for a comprehensive review of bullwhip effect literature, while providing a review of more relevant literature in this section. A similar term "reversed bullwhip effect" was introduced by Svensson (2003) under a slightly different context, who analyzed the bullwhip effect in intra-organizational echelons. In Svensson (2003), the reversed bullwhip effect (in order variation) occurs when there is a high degree of postponement in inbound logistics flows, and a high degree of speculation in outbound logistics flows. In our study, instead of analyzing the order variation as investigated in the mainstream bullwhip effect research, we consider the impact of upstream price variation on the variation of the downstream supply chain prices.

Pricing has been an ongoing area of research. The two main research directions have been supply chain coordination (see e.g. Jeuland and Shugan 1983; Lal 1990; Ingene and Parry 1995; Gerstner and Hess 1995; Iyer 1998), and identification of the best supply chain structure in terms of length, breadth and ownership (see e.g. Coughlan and Lal 1992; Tyagi 1999; Corbett and Karmarkar 2001; Chiang et al. 2003). On the other hand, RBP is a fairly new research area which has been recently introduced in Ozelkan and Cakanyildirim (2007). In their study, Ozelkan and Cakanyildirim (2007) analyzed price variation amplification for a multi-stage serial supply chain under deterministic and price-sensitive demand.

The work presented here is complementary to the pricing and bullwhip effect studies that are discussed in the literature. Here, we aim to extend the results of Ozelkan and Cakanyildirim (2007) to derive conditions for RBP under random price-sensitive demand. We also provide several different definitions of RBP where we differentiate between RBP versus strict and strong RBP cases. We derive RBP conditions for these different definitions, and also provide specific conditions of commonly used demand functions for RBP.

3 Single-stage supply chain model

Consider a single-stage supply chain where a retailer is subject to a random and price sensitive demand $q(p,\epsilon)$, where p denotes the retail price and $\epsilon \in [A, B]$ is a random variable with a probability density function $f(\cdot)$ and its mean μ_ϵ. Furthermore, let $w > 0$ denote the wholesale price that the retailer pays to the supplier, and let $E_q(p)$ denote the expected demand, $E_q(p) = E[q(p,\epsilon)]$. We assume that $E_q(p)$ is continuously twice differentiable and strictly decreasing in price ($E_q'(p) < 0$) for $p \in (0,\infty)$. While the former is made for the sake of theoretical analysis in the paper, the latter is a realistic assumption since when the retail prices go up, it is natural to expect that the customers will buy less, and vice versa. Furthermore, we assume that there exists an upper bound, say $u \in (0,\infty]$, on the retail price such that $E_q(u) = 0$. (We will use $u = \infty$ if $E_q(p) \to 0$ as $p \to \infty$.)

3.1 Uniqueness of retail price

Given the wholesale price $w > 0$, the retailer identifies the optimal retail price that maximizes the expected profit $E[\Pi_R(p,\epsilon)]$ as follows.

$$\sup_p \quad E[\Pi_R(p,\epsilon)] = \int_A^B (p-w)q(p,\epsilon)f(\epsilon)d\epsilon = (p-w)E_q(p) \tag{1a}$$

$$\text{s.t.} \quad w \le p \le u. \tag{1b}$$

We would like to note that representations of a supply chain similar to (1) have been extensively studied in the literature to derive managerial insights (see e.g. Coughlan and Lal 1992; Corbett and Karmarkar 2001; Chiang et al. 2003). As will be discussed in sequel, here we are only concerned with the case where the retail price is uniquely determined given a wholesale price so that we can have one-to-one relationship to investigate the impact of the wholesale price on the retail price. The retail price is trivially unique if $w = u$. To this end, we assume that $w < u$ in order to derive conditions for the uniqueness of the retail price given a wholesale price. Noting that the above problem is one dimensional optimization problem over the interval $[w, u]$, the solution can be identified as in the following proposition.

Proposition 1 *Let Λ denote the set of stationary points to (1a) defined as*:

$$\Lambda = \left\{ \lambda \in [w, u] : \left.\frac{dE[\Pi_R(p,\epsilon)]}{dp}\right|_{p=\lambda} = E_q(\lambda) + (\lambda - w)E_q'(\lambda) = 0 \right\}. \tag{2}$$

Let p^* denote the optimal solution to (1). Then, we have

$$p^* = \operatorname{argmax}\left\{E[\Pi_R(p,\epsilon)] : p \in \Lambda \cup \{u\}\right\}. \tag{3}$$

Proof The result directly follows the fact that the optimal solution will be either the lower or upper bounds of the feasible region or one of the feasible stationary points. The lower bound w is omitted since $(p - w)E_q(p) > 0$ for $p \in (w, u)$ whereas $E[\Pi_R(w,\epsilon)] = 0$. □

Corollary 1 *If $u < \infty$, then $\Lambda \neq \varnothing$ and*

$$p^* = \operatorname{argmax}\left\{E[\Pi_R(p,\epsilon)] : p \in \Lambda\right\}. \tag{4}$$

Proof From $E_q(w) > 0$, we have $\frac{dE[\Pi_R(p,\epsilon)]}{dp}|_{p=w} > 0$. Also, from $E_q(u) = 0$ and $E_q'(u) < 0$, we have $\frac{dE[\Pi_R(p,\epsilon)]}{dp}|_{p=u} < 0$. Since $\frac{dE[\Pi_R(p,\epsilon)]}{dp}$ is continuous on $[w, u]$, there exists $\lambda \in (w, u)$ such that $\frac{dE[\Pi_R(p,\epsilon)]}{dp}|_{p=\lambda} = 0$, and hence $\Lambda \neq \varnothing$. (4) directly follows since $E[\Pi_R(w,\epsilon)] = E[\Pi_R(u,\epsilon)] = 0$. □

Note that p^* in (3) is not necessarily unique. To see this, consider, e.g., a demand function with $E_q(p) = 2 + w - p$ for $p \in [w, w + 1)$ and $E_q(p) = (p - w)^{-1}$ for $p \ge w + 1$. It is easy to see that all the points in the interval $[w + 1, u]$ yield the same maximum expected profit of 1. We provide sufficient conditions of the uniqueness in the following propositions. First, we will discuss a corner case when Λ is an empty set.

Proposition 2 *If $\Lambda = \varnothing$, then the problem (1) has the unique solution $p^* = u$.*

Proof Note that $\frac{dE[\Pi_R(p,\epsilon)]}{dp}|_{p=w} > 0$. Since $\frac{dE[\Pi_R(p,\epsilon)]}{dp}$ is continuous, $\Lambda = \varnothing$ implies $\frac{dE[\Pi_R(p,\epsilon)]}{dp} > 0$ for $p \in [w, u]$. Since the profit function is monotonically increasing, $p^* = u$. □

A more interesting case is when Λ is nonempty which is investigated below.

Proposition 3 *If Λ is nonempty and $\lambda \in \Lambda$ implies $\frac{E_q(\lambda)E_q''(\lambda)}{(E_q'(\lambda))^2} < 2$, then Λ is a singleton $\Lambda = \{\lambda\}$ and the problem* (1) *has a unique solution $p^* = \lambda$.*

Proof Consider $\lambda \in \Lambda \neq \varnothing$. We know that if $\frac{d^2 E[\Pi_R(p,\epsilon)]}{dp^2}|_{p=\lambda} < 0$, then λ is a strict local maximum of the expected profit (see Bazaraa et al. 1993, Theorem 4.1.4). We can show that $\frac{d^2 E[\Pi_R(p,\epsilon)]}{dp^2}|_{p=\lambda} < 0$ if and only if $\frac{E_q(\lambda)E_q''(\lambda)}{(E_q'[\lambda)]^2} < 2$ as follows: From (2) and $E_q'(\lambda) < 0$, we have $\lambda - w = -\frac{E_q(\lambda)}{E_q'(\lambda)}$. Hence, $\frac{d^2 E[\Pi_R(p,\epsilon)]}{dp^2}|_{p=\lambda} = 2E_q'(\lambda) + (\lambda - w)E_q''(\lambda) = 2E_q'(\lambda) - \frac{E_q(\lambda)}{E_q'(\lambda)}E_q''(\lambda)$. If $2E_q'(\lambda) - \frac{E_q(\lambda)}{E_q'(\lambda)}E_q''(\lambda) < 0$, then we have $\frac{E_q(\lambda)E_q''(\lambda)}{(E_q'(\lambda))^2} < 2$ by multiplying $1/E_q'(\lambda)$ to both sides where $E_q'(\lambda) < 0$. Conversely, if $\frac{E_q(\lambda)E_q''(\lambda)}{(E_q'(\lambda))^2} < 2$, then $2E_q'(\lambda) - \frac{E_q(\lambda)}{E_q'(\lambda)}E_q''(\lambda) < 0$ by multiplying $E_q'(\lambda)$ to both sides. Next, we note that, since $\frac{dE[\Pi_R(p,\epsilon)]}{dp}$ is continuous, Λ must be a singleton. (Otherwise, we must have a stationary point local minimum between two local maximums.) Furthermore, we must have $\frac{dE[\Pi_R(p,\epsilon)]}{dp} > 0$ for $p \in [w, \lambda)$, and $\frac{dE[\Pi_R(p,\epsilon)]}{dp} < 0$ for $p \in (\lambda, u]$ since $\frac{dE[\Pi_R(p,\epsilon)]}{dp}|_{p=w} > 0$ and $\frac{dE[\Pi_R(p,\epsilon)]}{dp}|_{p=u} < 0$. Hence, λ is the unique global maximum. □

Note that since $\frac{E_q(\lambda)E_q''(\lambda)}{(E_q'(\lambda))^2} < 2$ identifies the local maximum point, we may consider this expression as the "concavity coefficient" (a similar use of this terminology can be found in Edwards 1950). In the previous propositions, the conditions of uniqueness depend on w since Λ needs to be identified. Instead, although it is more restrictive, the condition in the following proposition examines only the property of the expected demand function.

Proposition 4 *If $\frac{E_q(p)E_q''(p)}{[E_q'(p)]^2} < 2$ for $p \in (0, u)$, then the problem* (1) *has a unique solution.*

Proof If $\Lambda = \varnothing$, then $\frac{dE[\Pi_R(p,\epsilon)]}{dp} > 0$ for $p \in (0, u)$, and hence, the optimal solution is $p^* = u$. If $\Lambda \neq \varnothing$, then we have $\frac{E_q(\lambda)E_q''(\lambda)}{(E_q'(\lambda))^2} < 2$ for $\lambda \in \Lambda$. Thus, from Proposition 3, Λ is a singleton $\Lambda = \{\lambda\}$ and the unique solution is $p^* = \lambda$. □

A direct result of Proposition 4 is that when $u < \infty$, then Λ is a singleton $\Lambda = \{\lambda\}$, and the problem (1) has a unique solution $p^* = \lambda$. Note that this result is straightforward since $\frac{dE[\Pi_R(p,\epsilon)]}{dp}|_{p=w} > 0$ and $\frac{dE[\Pi_R(p,\epsilon)]}{dp}|_{p=u} < 0$ imply $\Lambda \neq \varnothing$.

3.2 Demand uncertainty structure

The demand $q(p,\epsilon)$ can take different forms. We will consider here both additive and multiplicative forms (see Petruzzi and Dada (1999) for a similar consideration). Letting $y(p)$ be the deterministic part of demand and ϵ be a random variable as defined before, the demand functions under additive and multiplicative uncertainties are defined as $q(p,\epsilon) = y(p) + \epsilon$ and $q(p,\epsilon) = \epsilon y(p)$, respectively, where $\epsilon > 0$ for multiplicative uncertainty. For the subsequent discussions, we will use y as a shorthand for $y(p)$, and y' and y'' to denote the first and second derivatives. Then, previous results yield the following corollary for the additive and multiplicative demand uncertainties.

Corollary 2 *Consider a single-stage supply chain where the demand uncertainty is additive. If $\frac{(y+\mu_\epsilon)y''}{y'^2} < 2$ for $p \in (0, u]$, then the retail price is uniquely determined. Furthermore, if* *there exists u such that $y(u) + \mu_\epsilon = 0$ for the additive uncertainty, then Λ is nonempty and a singleton $\{\lambda\}$, and $p^* = \lambda$ is the unique optimal solution to the optimization problem. For the multiplicative uncertainty, the two conditions are $\frac{yy''}{y'^2} < 2$ and $y(u) = 0$, respectively.*

Note that in Corollary 2, the multiplicative case result is independent of ϵ, since μ_ϵ can be eliminated from the stationary point equation. From Corollary 2, it is straightforward to see the next relationship between deterministic and probabilistic demands.

Corollary 3 *For a single-stage supply chain where the demand is additive with $\mu_\epsilon = 0$ or demand is multiplicative with $\mu_\epsilon > 0$, the retailer's optimality condition for maximizing expected profits is the same as the deterministic price-sensitive demand case.*

3.3 Conditions of reverse bullwhip effect in pricing

As presented earlier, the retail price is uniquely determined when certain conditions are satisfied. Now, assume that the optimal retail price is uniquely determined for each wholesale price w. Accordingly, let $p(w)$ be the function that maps the wholesale price w to the corresponding unique optimal retail price p. Noting that $E_q'(p) \neq 0$, we define a function $g(p) = \frac{E_q(p)}{E_q'(p)} + p$. Note that since $E_q(p)$ is continuously twice differentiable, $g(p)$ should be a differentiable function. From here, one can verify that $g(p)$ is monotonically increasing when the condition in Proposition 4 holds, thus the inverse $g^{-1}(w)$ exists, and identifies the retailer's stationary point optimal price given the wholesale price w. These observations are formally summarized in the following proposition and corollary:

Proposition 5 *If $\frac{E_q(p)E_q''(p)}{[E_q'(p)]^2} < 2$ for $p \in (0, u)$, then the inverse of $g(p)$ exists. Furthermore, the unique stationary point λ of (1a) is determined by $\lambda = g^{-1}(w)$.*

Proof Since $g'(p) = -\frac{E_q(p)E_q''(p)}{(E_q'(p))^2} + 2 > 0$, $g(p)$ is monotonically increasing. Hence, $g^{-1}(w)$ exists. From the definition of $g(p)$, $g^{-1}(w) \in \Lambda$. □

Note that the range identified by $g^{-1}(w) > u$ need not be considered when $g(p)$ (and hence $g^{-1}(w)$) is monotonically increasing since the optimal retail price is a constant u. In other words, when $g^{-1}(w)$ is monotonically increasing, $w \in (0, \bar{w})$ where $\bar{w} = g(u)$ is the only meaningful value. The next corollary specifies the (differentiable) function $p(w)$ for the meaningful range of w, i.e., $w \in (0, \bar{w})$.

Corollary 4 *Suppose that $\frac{E_q(p)E_q''(p)}{[E_q'(p)]^2} < 2$ for $p \in (0, u)$. Then, $p(w) = g^{-1}(w)$ for $w \in (0, \bar{w})$.*

Proof From Proposition 3, Proposition 5, and the fact that $g^{-1}(w)$ for $w \in (0, \bar{w})$ is the unique stationary point in $[w, u]$, $g^{-1}(w)$ is the optimal retail price. □

We remark that the condition $\frac{E_q(p)E_q''(p)}{(E_q'(p))^2} < 2$ not only provides the uniqueness of the retail price as in Proposition 4, but also guarantees a unique retail price value for each wholesale prices.

Based on the above results, our discussion can now be focused on RBP. Note that RBP indicates that the change in retail price is amplified as the wholesale price varies. In this context, we define RBP based on the value of the rate of price change $\frac{dp(w)}{dw}$ (also referred as the "cost-pass-through coefficient").

Definition 1 Consider a differentiable function $p(w)$ that determines the retail price. Then, RBP is said to exist for a given interval (a, b) if

$$\frac{dp(w)}{dw} \geq 1 \quad \text{for } w \in (a, b). \tag{5}$$

When the inequality is replaced by the strict inequality, a strict RBP is said to exist.

Note that this definition for RBP is more general than the one provided in Ozelkan and Cakanyildirim (2007), who considered only the strict inequality case. As we will discuss later for the multi-stage supply chains, differentiating between strict RBP versus RBP (or non-strict RBP) can help us characterize required conditions for RBP in multi-stage supply chains better. Observe that the wholesale price w varies over time and so does the retail price p. Let μ_w and μ_p denote respective expected values. Here, we are also interested in the relationship between normalized wholesale and retail prices (i.e., $\frac{dw}{\mu_w}$ and $\frac{dp}{\mu_p}$). Hence, in addition to the above RBP definition, we define a *strong* RBP (SRBP) case as follows.

Definition 2 An SRBP is said to exist for a given interval (a, b) if

$$\frac{dp(w)}{dw} \geq \frac{\mu_p}{\mu_w} \quad \text{for } w \in (a, b). \tag{6}$$

When the inequality is replaced by the strict inequality, a strict SRBP is said to exist.

As stated in the following proposition, SRBP requires a stronger condition than RBP does (hence, it is named as "strong").

Proposition 6 *If SRBP exists for $w \in (a, b)$, then there exists RBP for $w \in (a, b)$.*

Proof Given a wholesale price w, we always have the retail price such that $p \geq w$. Hence, we have $\mu_p \geq \mu_w$ or $\frac{\mu_p}{\mu_w} > 1$. □

An implication of Proposition 6 is that if RBP does not occur SRBP will not occur either. In what follows, we show that the concavity coefficient helps us derive sufficient conditions for the existence of RBP and SRBP. First, we analyze the RBP conditions as follows:

Proposition 7 *Consider a single-stage supply chain. Suppose that $\frac{E_q(p)E_q''(p)}{[E_q'(p)]^2} < 2$. Then, RBP exists for $(a, b) \subset (0, \bar{w})$ if and only if $\frac{E_q(p)E_q''(p)}{[E_q'(p)]^2} \geq 1$ for $p \in (p(a), p(b))$. Furthermore, strict RBP exists if and only if $\frac{E_q(p)E_q''(p)}{[E_q'(p)]^2} > 1$.*

Proof From Corollary 4 and $\frac{E_q(p)E_q''(p)}{[E_q'(p)]^2} < 2$, we have $p(w) = g^{-1}(w)$, which determines the unique stationary point. Since $g(p) = \frac{E_q(p)}{E_q'(p)} + p$, we have $\frac{dw}{dp} = g'(p) = \frac{[E_q'(p)]^2 - E_q(p)E_q''(p)}{[E_q'(p)]^2} +$

$1 = 2 - \frac{E_q(p)E''_q(p)}{[E'_q(p)]^2}$. Note that $\frac{dw}{dp} \leq 1$ (equivalently, $\frac{dp}{dw} \geq 1$) if and only if $\frac{E_q(p)E''_q(p)}{[E'_q(p)]^2} \geq 1$. The strict RBP result directly follows by deleting the equality. □

Based on Proposition 7, specific RBP conditions for additive and multiplicative random demand structures can be derived as follows:

Corollary 5 *Consider a single-stage supply chain where the demand is random but additive in structure. Suppose that $(y+\mu_\epsilon)y''(y')^{-2} < 2$. Then, RBP occurs for $(a,b) \subset (0,\bar{w})$ if and only if $(y+\mu_\epsilon)y''(y')^{-2} \geq 1$ for $p \in (p(a), p(b))$. Moreover, strict RBP exists if and only if $(y+\mu_\epsilon)y''(y')^{-2} > 1$.*

The proofs of this and several subsequent corollaries are straightforward, and therefore, they are omitted.

Corollary 6 *Consider a single-stage supply chain where the demand is random but multiplicative in structure. Suppose that $yy''(y')^{-2} < 2$. Then, RBP occurs for $(a,b) \subset (0,\bar{w})$ if and only if $yy''(y')^{-2} \geq 1$ for $p \in (p(a), p(b))$. Moreover, strict RBP exists if and only if $yy''(y')^{-2} > 1$.*

Similar to the RBP conditions, SRBP conditions can be derived as follows:

Proposition 8 *Consider a single-stage supply chain. Suppose that $\frac{E_q(p)E''_q(p)}{[E'_q(p)]^2} < 2$. Then, SRBP exists for $(a,b) \subset (0,\bar{w})$ if and only if $\frac{E_q(p)E''_q(p)}{[E'_q(p)]^2} \geq 2 - \frac{\mu_w}{\mu_p}$ for $p \in (p(a), p(b))$. Furthermore, strict SRBP exists if and only if $\frac{E_q(p)E''_q(p)}{[E'_q(p)]^2} > 2 - \frac{\mu_w}{\mu_p}$.*

Proof The proof is similar to that of Proposition 7. Again, we have $\frac{dw}{dp} = 2 - \frac{E_q(p)E''_q(p)}{[E'_q(p)]^2}$. Thus, $\frac{dw}{dp} \leq \frac{\mu_w}{\mu_p}$ (equivalently, $\frac{dp}{dw} \geq \frac{\mu_p}{\mu_w}$) if and only if $\frac{E_q(p)E''_q(p)}{[E'_q(p)]^2} \geq 2 - \frac{\mu_w}{\mu_p}$. The strict SRBP directly follows by deleting the equality. □

SRBP conditions for additive and multiplicative random demand structures follow directly from Proposition 8 as stated in the following corollaries:

Corollary 7 *Consider a single-stage supply chain where the demand is random but additive in structure. Suppose that $(y+\mu_\epsilon)y''(y')^{-2} < 2$. Then, SRBP occurs for $(a,b) \subset (0,\bar{w})$ if and only if $(y+\mu_\epsilon)y''(y')^{-2} \geq 2 - \frac{\mu_w}{\mu_p}$ for $p \in (p(a), p(b))$. Moreover, strict SRBP exists if and only if $(y+\mu_\epsilon)y''(y')^{-2} > 2 - \frac{\mu_w}{\mu_p}$.*

Corollary 8 *Consider a single-stage supply chain where the demand is random but multiplicative in structure. Suppose that $yy''(y')^{-2} < 2$. Then, SRBP occurs for $(a,b) \subset (0,\bar{w})$ if and only if $yy''(y')^{-2} \geq 2 - \frac{\mu_w}{\mu_p}$ for $p \in (p(a), p(b))$. Moreover, strict SRBP exists if and only if $yy''(y')^{-2} > 2 - \frac{\mu_w}{\mu_p}$.*

Comparison of the results of Corollaries 7 and 8 to those of Corollaries 5 and 6, once again confirms that SRBP requires stronger conditions than RBP. Next, we observe from Corollaries 5 through 8 that the deterministic case is a specific case of these results, which is stated formally in the following corollary:

Corollary 9 *For a single-stage supply chain where the demand is random but additive with $\mu_\epsilon = 0$ or multiplicative with $\mu_\epsilon > 0$, RBP and SRBP conditions are the same as the deterministic price-sensitive demand case.*

A closer look at the concavity coefficient indicates that not all demand functions will result in RBP and SRBP as summarized below:

Corollary 10 *RBP and SRBP do not occur when the expected value of the demand function is concave.*

Proof If the expected value of the demand function is concave, we have $d^2E_q(p)/dp^2 \le 0$, and in turn, satisfies the uniqueness condition $\frac{E_q(p)E''_q(p)}{[E'_q(p)]^2} < 2$. However, since $\frac{E_q(p)E''_q(p)}{[E'_q(p)]^2} < 1$, RBP does not exist due to Proposition 7. Next, based on Proposition 6, we know that if RBP does not occur SRBP will not occur either. □

The observation of Corollary 10 can further be streamlined for the additive and multiplicative uncertainty cases as follows:

Corollary 11 *For random demand with additive uncertainty* (*with* $y + \mu_\epsilon > 0$) *and multiplicative uncertainty* (*with* $\mu_\epsilon > 0$) *structure*, *RBP does not occur when the deterministic part of the demand function is concave.*

Proof This result follows from the fact that shifting a function by a constant (μ_ϵ) or multiplying with a positive constant does not change concavity. □

Results of Corollary 11 is interesting since it indicates that RBP and SRBP may occur only for ranges where the deterministic part of the demand function is strictly convex. In Sect. 6, we will provide examples for both concave and convex demand functions.

4 Multi-stage supply chains

In this section, we extend our analysis to a multi-stage supply chain that links more than two entities. This case requires playing a multi-step sequential game for the n-stage supply chain, where in each step one player makes a pricing decision based on the actions taken by other players in previous steps. Starting from the retailer S_n and moving upstream, each supplier S_i identifies their reaction function $p_i = r_i(p_{i-1})$ based on their objective function. For example, the reaction function of S_n is obtained by solving the following objective function:

$$\max_{p_n} \quad \Pi_{S_n}(p_n) = (p_n - p_{n-1})E_q(p_n) \tag{7a}$$

$$\text{s.t.} \quad p_n^L \le p_n \le p_n^U, \tag{7b}$$

where p_n^L and p_n^U denote the lower and upper bounds on the price, and $E_q(p_n)$ denotes the expected demand when the retail price is p_n. As in the single-stage case, we consider a stationary point optimal solution to this optimization problem as follows.

$$p_n = r_n(p_{n-1}) = \left\{ p_n | E_q(p_n) + (p_n - p_{n-1})\frac{dE_q(p_n)}{dp_n} = 0 \right\}, \tag{8}$$

where $r_n(p_{n-1})$ is used to denote the reaction function of the retailer S_n to the previous stages sales price p_{n-1}. Similarly, for supplier S_{n-1}, the reaction function can be obtained as

$$p_{n-1}=r_{n-1}(p_{n-2})=\left\{p_{n-1}|E_q[r_n(p_{n-1})]+(p_{n-1}-p_{n-2})\frac{dE_q[r_n(p_{n-1})]}{dp_{n-1}}=0\right\}. \tag{9}$$

Repeating this argument, we can write the reaction function for supplier S_i as

$$p_i=r_i(p_{i-1})=\left\{p_i|E_q[\psi_i(p_i)]+(p_i-p_{i-1})\frac{dE_q[\psi_i(p_i)]}{dp_i}=0\right\}, \tag{10}$$

where $\psi_i(p_i)=r_n(r_{n-1}(\ldots(r_{i+1}(p_i))\ldots))$. The price of the product (e.g., raw material) fed into the most upstream supplier, S_1, is given by p_0. Then, the order of events in an n-stage supply chain will be as follows: In Step 1, the most upstream supplier S_1 makes a pricing decision based on the reaction functions of $S_2\ldots S_n$ given p_0. Then in Step 2, S_2 makes pricing decision based on S_1's decision and the reaction functions of $S_3\ldots S_n$. In general, S_i makes a decision based on decisions taken by players $S_1,\ldots,S_{i-1}$, and the reaction functions of $S_{i+1},\ldots,S_n$. Under this circumstance, similar to the single-stage case, we define RBP and SRBP as follows.

Definition 3 Consider a differentiable function $\psi_0(p_0)$ that determines the retail price p_n via the above sequential decision-making process. Then, RBP is said to exist for a given interval (a,b) if

$$\frac{d\psi_0(p_0)}{dp_0}\geq 1 \quad \text{for } p_0\in(a,b). \tag{11}$$

Furthermore, SRBP is said to exist if

$$\frac{d\psi_0(p_0)}{dp_0}\geq \frac{\mu_n}{\mu_0} \quad \text{for } p_0\in(a,b), \tag{12}$$

where $\mu_0=E[p_0]$ and $\mu_n=E[p_n]$. When the inequality is replaced by the strict inequality, it is said that strict RBP and strict SRBP exist, respectively.

The following arguments summarize our observations for sequential games in an n-stage supply chain.

Proposition 9 *The retail price p_n is uniquely determined as a stationary point for each p_0 and there exists RBP if*

$$0<\prod_{i=1}^{n}\left[2-\frac{E_q[\psi_i(p_i)]\frac{d^2E_q[\psi_i(p_i)]}{dp_i^2}}{\left\{\frac{dE_q[\psi_i(p_i)]}{dp_i}\right\}^2}\right]\leq 1. \tag{13}$$

Proof Note that if $\frac{dp_0}{dp_n}\in(0,1]$, p_n is uniquely determined for p_0 and RBP exists. From the stationary point condition, we have $p_{i-1}=\frac{E_q[\psi_i(p_i)]}{dE_q[\psi_i(p_i)]/dp_i}+p_i$. Hence, $\frac{dp_{i-1}}{dp_i}=2-E_q[\psi_i(p_i)]\frac{d^2E_q[\psi_i(p_i)]}{dp_i^2}\{\frac{dE_q[\psi_i(p_i)]}{dp_i}\}^{-2}$. The chain rule, $\frac{dp_0}{dp_n}=\prod_{i=1}^{n}\frac{dp_{i-1}}{dp_i}$, results in (13). $\square$

Corollary 12 *The retail price p_n is uniquely determined as a stationary point for each p_0 and there exists RBP if*

$$1 \leq \frac{E_q[\psi_i(p_i)]\frac{d^2 E_q[\psi_i(p_i)]}{dp_i^2}}{\left\{\frac{dE_q[\psi_i(p_i)]}{dp_i}\right\}^2} < 2, \quad i = 1, \ldots, n. \tag{14}$$

Proof Since $2 - E_q[\psi_i(p_i)]\frac{d^2 E_q[\psi_i(p_i)]}{dp_i^2}\{\frac{dE_q[\psi_i(p_i)]}{dp_i}\}^{-2} \in (0, 1]$, (14) holds true. □

Corollary 12 is intuitive because the condition (14) results in RBP between S_i and S_{i-1}. From this corollary, we see that when there is a strict RBP between supply chain stages S_i and S_{i-1} (i.e., when the "$\leq$" sign in (14) is replaced with a "$<$" sign), the variation of prices as measured by the cost-pass-through coefficient will be amplified. Furthermore, the amplification increases as the number of stages in the supply chain increases. Another intuitive result implied by Corollary 12 is that to have a strict RBP in a supply chain (i.e., from most upstream S_1 to most downstream S_n), we need to have strict RBP between at least one of the supply chain stages. Next, using Proposition 9, additive and multiplicative uncertainties results in specific conditions summarized in the following corollaries, where we used the definition $\hat{y}_i = y[\psi_i(p_i)]$.

Corollary 13 *Consider an n-stage supply chain where the demand is random but additive in structure. Then, if* $0 < \prod_{i=1}^n [2 - \frac{(\hat{y}_i + \mu_\epsilon)\frac{d^2\hat{y}_i}{dp_i^2}}{\{\frac{d\hat{y}}{dp_i}\}^2}] \leq 1$, *RBP exists.*

Corollary 14 *Consider an n-stage supply chain where the demand is random but multiplicative in structure. Then, if* $0 < \prod_{i=1}^n [2 - \frac{\hat{y}_i \frac{d^2\hat{y}_i}{dp_i^2}}{\{\frac{d\hat{y}}{dp_i}\}^2}] \leq 1$, *RBP exists.*

Using a similar argument as in Proposition 9, we have a similar condition for SRBP as follows:

Proposition 10 *The retail price p_n is uniquely determined as a stationary point for each p_0 and there exists SRBP if*

$$0 < \prod_{i=1}^n \left[2 - \frac{E_q[\psi_i(p_i)]\frac{d^2 E_q[\psi_i(p_i)]}{dp_i^2}}{\left\{\frac{dE_q[\psi_i(p_i)]}{dp_i}\right\}^2}\right] \leq \frac{\mu_0}{\mu_n}. \tag{15}$$

Proof Noting that if $\frac{dp_0}{dp_n} \in (0, \frac{\mu_0}{\mu_n}]$, the proof is identical to that of Proposition 9. □

Again, specific SRBP results for the additive and multiplicative uncertainties follow directly from Proposition 10 as summarized below:

Corollary 15 *Consider an n-stage supply chain where the demand is random but additive in structure. Then, if* $0 < \prod_{i=1}^n [2 - \frac{(\hat{y}_i + \mu_\epsilon)\frac{d^2\hat{y}_i}{dp_i^2}}{\{\frac{d\hat{y}}{dp_i}\}^2}] \leq \frac{\mu_0}{\mu_n}$, *SRBP exists.*

Corollary 16 *Consider an n-stage supply chain where the demand is random but multiplicative in structure. Then, if* $0 < \prod_{i=1}^{n}[2 - \frac{\hat{y}_i \frac{d^2\hat{y}_i}{dp_i^2}}{\{\frac{d\hat{y}}{dp_i}\}^2}] \le \frac{\mu_0}{\mu_n}$, *SRBP exists.*

5 Relationship of RBP, SRBP, variance and coefficient of variation

While in the previous sections, we investigated the conditions that resulted in RBP, and SRBP, a related question of interest is how they relate to σ^2. Ozelkan and Cakanyildirim (2007) showed that RBP is related to σ^2 as follows:

Proposition 11 *If* $\frac{dp(w)}{dw} \ge c$ *for all* $w \ge 0$, *then* $\frac{\sigma_p}{\sigma_w} \ge c$, *where c is a positive constant.*

Corollary 17 *If* $\frac{dp(w)}{dw} = c$, *then* $\frac{\sigma_p}{\sigma_w} = c$.

For the proofs of Proposition 11 and Corollary 17, the reader can refer to Ozelkan and Cakanyildirim (2007). Proposition 11 also implies that if $\frac{dp(w)}{dw} \ge 1 + \delta$, then $\frac{\sigma_p}{\sigma_w} \ge 1 + \delta$, where $\delta \ge 0$. Thus, when an RBP takes place, an increase of price variance will take place. Furthermore, based on the results of Sect. 4, this increase in price variance may be amplified in multi-stage supply chains. The results of Proposition 11 can be expanded to the SRBP case as follows:

Proposition 12 *Let* $CV_p = \sigma_p/\mu_p$ *and* $CV_w = \sigma_w/\mu_w$ *denote the coefficient of variation* (*CV*) *for p and w, respectively. Then, SRBP implies* $\frac{CV_p}{CV_w} \ge 1$, *and strict SRBP implies* $\frac{CV_p}{CV_w} > 1$.

Proof The proof follows from Proposition 11 by setting $c = \frac{\mu_p}{\mu_w} + \delta$ where $\delta \ge 0$. $\delta = 0$ results in RBP, whereas $\delta > 0$ results in strict SRBP. □

Based on Corollary 17, we can find an exact estimate of the ratio of the retail price CV to whole sale price CV.

Corollary 18 *If* $\frac{dp(w)}{dw} = c$, *then* $\frac{CV_p}{CV_w} = c\frac{\mu_w}{\mu_p}$.

Proof If $\frac{dp(w)}{dw} = c$, then $\frac{\sigma_p}{\sigma_w} = c$. By dividing both sides by $\frac{\mu_p}{\mu_w}$ we get $\frac{\sigma_p/\mu_p}{\sigma_w/\mu_w} = \frac{CV_p}{CV_w} = c\frac{\mu_w}{\mu_p}$. □

Remark 1 Lee et al. (1997) identified variation in the retail price as one of the main sources of the information distortion that causes bullwhip effect (in order quantities) toward the upstream of a supply chain. Since RBP implies amplified variation in retail prices based on the above observation, it is not difficult to deduce that variations in the wholesale price would exacerbate the bullwhip effect in order quantities under the presence of RBP.

6 Illustration of RBP for common demand functions

In this section, we aim to analyze some of the most commonly used demand functions and identify which demand functions might result in RBP and which demand functions would not using the conditions presented in Sect. 3. Table 1 shows the demand functions investigated and summarizes the results. In this table, the "Concavity" is defined as $(y+\mu_\epsilon)y''(y')^{-2}$ for the additive uncertainty case as in Proposition 7, and as $yy''(y')^{-2}$ for the multiplicative uncertainty case. As discussed in Sect. 3.3, the "cost-pass-through" and "concavity" coefficients can help us identify if RBP would take place or not. In what follows, we will summarize the RBP results for each demand function.

Linear demand The concavity coefficient indicates that the optimal retail price is uniquely determined as the stationary point. However, since it is less than 1, RBP never occurs due to Proposition 7. (This is also confirmed by Corollary 10 since the expected demand function is concave.)

Logit demand Observing the negative concavity coefficient, RBP will not take place for either the additive or the multiplicative case. This is again confirmed by the concave expected demand function (see Corollary 10).

Iso-elastic demand For the iso-elastic demand function, we only consider the case that $l>1$. Otherwise, the optimal retail price is always ∞, which does not have a meaningful interpretation in practice. The multiplicative case results in RBP since the concavity coefficient lies in $[1,2)$. Indeed, it is strict RBP because $\frac{l+1}{l}>1$ for $l>1$. (Note that this is confirmed by $\frac{dp}{dw}=\frac{l}{l-1}>1$.) Furthermore, note that $p(w)=\frac{l}{l-1}w$. Hence, we have $\mu_p=\frac{l}{l-1}\mu_w$, or $\frac{\mu_p}{\mu_w}=\frac{l}{l-1}$, which results in SRBP. (However, strict SRBP does not exist because $\frac{dp}{dw}=\frac{\mu_p}{\mu_w}$.) For the additive case, if $\mu_\epsilon>0$, then the optimal retail price is always ∞, which lies in

Table 1 Demand functions and the reverse bullwhip effect conditions for a single-stage supply chain

Function name	Deterministic demand $y(p)$	Uncertainty	Cost-pass-through $\frac{dp}{dw}$	Concavity $\frac{E_q(p)E_q''(p)}{[E_q'(p)]^2}$	RBP	SRBP
Linear	$a-bp$,	A	$\frac{1}{2}$	0	N	N
	$a,b>0$	M	$\frac{1}{2}$	0	N	N
Logit	$a\frac{e^{u-p}}{1+e^{u-p}}$, $a>0$	A	$[2-(1-e^{u-p})(1+\frac{\mu_\epsilon}{y})]^{-1}$	$(1-e^{u-p})(1+\frac{\mu_\epsilon}{y})$	N	N
		M	$[1+e^{u-p}]^{-1}$	$1-e^{u-p}$	N	N
Iso-elastic	ap^{-l},	A	$[2-\frac{l+1}{l}(1+\frac{\mu_\epsilon}{y})]^{-1}$	$\frac{l+1}{l}(1+\frac{\mu_\epsilon}{y})$	Y/N	Y/N
	$a>0, l>1$	M	$\frac{l}{l-1}$	$\frac{l+1}{l}$	Y	Y
Logarithmic	$a\left(-\ln\frac{p}{u}\right)^b$, $a,b>0$	A	$[2-(1-\frac{\ln\frac{p}{u}+1}{b})(1+\frac{\mu_\epsilon}{y})]^{-1}$	$(1-\frac{\ln\frac{p}{u}+1}{b})(1+\frac{\mu_\epsilon}{y})$	Y/N	Y/N
		M	$[1+\frac{1+\ln\frac{p}{u}}{b}]^{-1}$	$1-\frac{\ln\frac{p}{u}+1}{b}$	Y/N	Y/N

Legend: A—Additive, M—Multiplicative, Y—Yes, N—No outside of our concern. Noting that the case of $\mu_\epsilon = 0$ is equivalent to the multiplicative case, assume that $\mu_\epsilon < 0$. Then, there exists $u = (-\frac{a}{\mu_\epsilon})^{1/l}$, where $E_q(u) = 0$. Hence, again we only consider $p \in (0, u]$. Note that since $\mu_\epsilon < 0$, the concavity coefficient $\frac{E_q(p)E_q''(p)}{[E_q'(p)]^2}$ is monotonically decreasing in p while its minimum value of 0 is attained at $p = u$. Furthermore, we have $\frac{E_q(p)E_q''(p)}{[E_q'(p)]^2} \to \frac{l+1}{l}$ as $p \to 0$. This implies that the concavity coefficient is always less than 2 and the retail price is uniquely determined by the corresponding stationary point for $p \in (0, u]$. Solving $\frac{E_q(p)E_q''(p)}{[E_q'(p)]^2} = 1$ yields $\hat{p} = [-\frac{a}{(l+1)\mu_\epsilon}]^{1/l}$. Therefore, we conclude that RBP exists for $w \in (0, g(\hat{p})]$ while it does not for $w \in (g(\hat{p}), g(u))$, where $g(p) = \frac{E_q(p)}{E_q'(p)} + p = (\frac{p}{l})[l - 1 - (\frac{\mu_\epsilon}{a})p^l]$ (see Proposition 7). A similar argument can be used for SRBP. Letting $\tilde{p}$ denote the solution of $\frac{E_q(p)E_q''(p)}{[E_q'(p)]^2} = 2 - \frac{\mu_w}{\mu_p}$ (although this is not always easy to analytically find), there exists SRBP for $w \in (0, g(\tilde{p})]$. The following example illustrates both RBP and SRBP conditions for the additive uncertainty structure.

Example of iso-elastic demand Consider an iso-elastic demand function with having $a = 1000$ and $l = 2$. If the uncertainty structure is additive with $\mu_\epsilon = -10$, then $\hat{p} = \sqrt{\frac{10}{3}} = 7.07$. The concavity coefficient is $\frac{3}{2}(1 - \frac{p^2}{100})$. Since $g(p) = \frac{p}{2} + \frac{p^3}{200}$, we have $g(\hat{p}) = 5.3$. Thus, there exists RBP for $w \in (0, 5.3)$. To verify this, note that $\frac{dp}{dw} = \frac{1}{\frac{3}{200}p^2 + \frac{1}{2}} \geq 1$ when $p \in (0, 7.07)$, which corresponds to $w \in (0, 5.3)$. Furthermore, note that RBP does not exists for $p \in (7.07, 10)$ (or $w \in (5.3, 10)$), where $u = 10$ is determined by setting $E_q(u) = 0$. For the existence of SRBP, suppose that w is uniformly distributed in $(0, 10)$ (i.e., the probability density function is $f_W(w) = \frac{1}{10}$). Then, $\mu_w = 5$. Furthermore, since $\frac{dp}{dw} = \frac{l+1}{l}(1 + \frac{\mu_\epsilon}{y})$, the probability density function of p is given by $f_P(p) = f_W(w)(1/\frac{dp}{dw}) = \frac{3}{2000}p^2 + \frac{1}{20}$. Hence, $\mu_p = \int_0^{10} p f_P(p)dp = 6.25$. Solving $\frac{3}{2}(1 - \frac{p^2}{100}) = 2 - \frac{\mu_w}{\mu_p} = 1.2$ yields $\tilde{p} = 4.47$. Since $g(4.47) = 2.68$, there exists SRBP for $w \in (0, 2.68)$.

Logarithmic demand First, consider the multiplicative demand uncertainty. Observe that the concavity coefficient of the logarithmic demand function is not always less than 2, which is a sufficient condition for a unique retail price. However, it is not difficult to see that the concavity coefficient is less than 2 for $w \in (0, u)$ (hence the retail price is uniquely determined) as follows. Solving $1 - \frac{\ln \frac{p}{u} + 1}{b} < 2$ yields $p > ue^{-b-1}$. (The right-hand-side is always less than u.) Note that the value of p such that $p > ue^{-b-1}$ (uniquely) corresponds to a wholesale price w since the function $g(p)$ is monotonically increasing. Furthermore, note that $g(ue^{-b-1}) = -\frac{ue^{-b-1}}{b} < 0$ and $g(u) = u$. Therefore, for each $w \in (0, u)$, a stationary point retail price is uniquely determined. To examine the existence of RBP, the solution of $1 - \frac{\ln \frac{p}{u} + 1}{b} \geq 1$ results in $p \leq ue^{-1}$. Hence, there exists RBP for $w \in (0, g(ue^{-1})]$ (and strict RBP for $w \in (0, g(ue^{-1})))$. Furthermore, noting that $g(ue^{-1}) \leq 0$ when $b > 1$, no RBP exists if $b \leq 1$. Similarly, given μ_w and μ_p, there exists SRBP when $w \in (0, g(\hat{p})]$, where $\hat{p} = ue^{-b(1-\mu_p/\mu_w)-1}$. As far as the additive demand uncertainty is concerned, we can derive similar conditions for RBP and SRBP. However, due to the complication of the equation, these conditions can be derived numerically as in the example below.

Example of logarithmic demand Consider a logarithmic demand function $y(p) = 5[-\ln \frac{p}{10}]^2$. Let the expected value of demand uncertainty be $\mu_\epsilon = -2$. Due to the negative value of μ_ϵ, the maximum value of p is now 5.3 at which the expected demand becomes 0.

(For the similar reason as in the iso-elastic demand case, a positive value of μ_ϵ is not considered here.) Then, a numerical search yields that the concavity coefficient is less than 2 when $p > 0.42$ (or, $w > -0.22 = g(0.42)$). Hence, the retail price is unique when $w > 0$. Together with this, a numerical search gives the range $w \in (0, 0.83)$ for which RBP exists. For SRBP, assume that the wholesale price uniformly varies in $(0, 5.3)$ (i.e., $\mu_w = 2.65$). A simulation yields $\mu_p = 3.58$. Hence, we have $\mu_p/\mu_w = 1.35$. This results in the interval $w \in (0, 0.133)$ for which SRBP exists.

7 Summary and conclusions

In this paper, we introduced several definitions of reverse bullwhip effect in pricing (RBP) differentiating between strict and strong cases (i.e., SRBP): More specifically, RBP is defined as the case that the "cost-pass-through" coefficient is greater than or equal to one, and SRBP is defined as the case that "cost-pass-through" coefficient is greater or equal to the ratio of the expected values of sales price to buying price, with the strict cases requiring strict inequalities. We analyzed several factors that may impact RBP such as the demand uncertainty structure, type of demand function, and number of stages in a supply chain. We derived conditions of demand functions for which RBP (or strict RBP) and SRBP (or strict SRBP) may occur in supply chains. Some of the major findings are summarized below.

From a managerial perspective, the results indicate that a retailer, who is making pricing decisions under additive demand uncertainty, should consider not only the deterministic part of the market demand but also the expected demand shift due to the uncertainty. The cases of additive uncertainty structure with an expected value of the uncertainty of zero, and multiplicative uncertainty structure with a positive expected uncertainty result in the same solution for RBP as in the deterministic demand case. Importance of demand function shifts on the price and profitability has been recently discussed in Cowan (2004), who showed that additive demand shifts result in price increases. Our results agree with these previous findings and extend the results to the RBP case.

Another managerial insight from our findings is that RBP may not occur in every supply chain. We have seen that the occurrence of RBP or SRPB is related to the shape of the expected profit function which can be represented by the "concavity" coefficient. One can conclude about the existence of RBP or SRPB by examining the magnitude of the "concavity" coefficient alone. For RBP to occur the coefficient needs to be greater than or equal to one, and for SRBP to occur the "concavity" coefficient needs to be greater than a threshold which is a function of the ratio of the expected values of buying price to sales price.

Analysis of different demand functions indicates that not all demand functions result in RBP. More specifically, RBP does not occur when the expected demand function is concave. Therefore, while the linear and logit demand functions do not result in RBP for additive and multiplicative uncertainty cases, iso-elastic and logarithmic demand functions may. The iso-elastic demand function does not result in strict SRBP for multiplicative uncertainty.

We have seen that in supply chains where pricing decisions are made sequentially starting from the most upstream supplier and then proceeding sequentially all the way down to the retailer, both RBP and SRBP conditions for a multi-stage supply chain is a multiplicative combination of each-stage's condition. Based on this finding, we conclude that both RBP and SRBP may be amplified as the number of stages in a supply chain increases.

We have also observed that the analysis of the "cost-pass-through coefficient" can reveal information about variability metrics such as price variances (in the context of an RBP) and price coefficients of variation (in the context of an SRBP). Based on these findings, we conclude that RBP and SRBP may increase "forward" bullwhip effect in order variation. From a managerial perspective, this is one of the main reasons why studying the RBP and SRBP is important. A natural question is what can be done to avoid or minimize RBP. While identifying optimal strategies to deal with RBP and SRBP is beyond the scope of the current study, we believe that longer term supplier relationships through win-win contracts will stabilize price fluctuations and will eliminate or at least minimize the root-cause of RBP and SRBP, i.e., the price variation in the upstream of a supply chain.

The results presented here provide valuable insights on when and how RBP or SRBP may occur in supply chains. In reality, supply chains may be more complex than what is modelled here, therefore the results can be extended to involve more complexities. We expect that RBP will occur for more complex supply chain systems as well, but the required conditions will be specific to the cases analyzed. Some of the future work may involve the investigation of price variation under coordination, varying supply chain structures (e.g., under competition), joint optimization of pricing and order quantity decisions, and consideration of future cost fluctuations. Finally, further empirical studies can shed more light into the reverse bullwhip effect in pricing for a variety of practical demand functions.

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