Ann Oper Res (2010) 173: 163–175
DOI 10.1007/s10479-008-0421-3

Context-dependent performance standards in DEA

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Published online: 4 September 2008
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Abstract Data envelopment analysis (DEA) is a mathematical approach to measuring the relative efficiency of peer decision making units (DMUs). It is particularly useful where no *a priori* information on the tradeoffs or relations among various performance measures is available. However, it is very desirable if "evaluation standards," when they can be established, be incorporated into DEA performance evaluation. This is especially important when service operations are under investigation, because service standards are generally difficult to establish. The approaches that have been developed to incorporate evaluation standards into DEA, as reported in the literature, have tended to be rather indirect, focusing primarily on the multipliers in DEA models. This paper introduces a new way of building performance standards directly into the DEA structure when context-dependent activity matrixes exist for different classes of DMUs. For example, two sets of branches, whose transaction times are known to be different from each other, usually have two different activity matrixes. We develop a procedure so that a set of standard DMUs can be generated and incorporated directly into the DEA analysis. The proposed approach is applied to a sample of 100 branches of a major Canadian bank where different sets of time standards exist for three distinct groups of branches.

Keywords Data envelopment analysis (DEA) · Performance standards · Efficiency · Context dependent

1 Introduction

Data Envelopment Analysis (DEA), as developed in Charnes et al. (1978), provides a measure of efficiency of a each of a set of decision making units (DMUs), *relative* to some

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comparison set of similar units. Several variations of this first (constant returns to scale) model have been presented since the appearance of that first paper, including the variable returns to scale model of Banker et al. (1984), the additive model of Charnes et al. (1985), and others. Cooper et al. (2000) and Zhu (2003) discuss various DEA models and software.

The relative efficiency approach has been applied in numerous settings over the past 25 years, including in particular, in the financial services sector. Since the first application of DEA to bank efficiency analysis by Sherman and Gold (1985), many subsequent studies have been conducted. These include those due to Barr and Siems (1994), Berger and Humphrey (1997), Cook et al. (2000), Cook and Hababou (2001), and others. In the articles by Cook et al. (2000), Cook and Hababou (2001), multiplier restrictions, in the form of assurance regions (AR) (Thompson et al. 1990) were imposed. These AR restrictions were based on available data extracted from time studies within a set of bank branches, and pertaining to unit processing times on all bank transactions. In this manner, a form of *production standards,* for bank branch transactions, were *indirectly* incorporated into the 'production model' for branch performance.

During the earlier study of Cook et al. (2000), Cook and Hababou (2001), it was expressed by bank management that there was a desire to set targets for branches that went beyond the 'relative' measures provided for in the DEA structure. Specifically, management was interested in knowing what *absolute* potential for improvement would be possible, within any given branch. The fact that ranges for transaction processing times are used in the earlier studies to restrict multipliers, is an important part of insuring that efficiency measures for branches reflect reality, at least in terms of comparing branches to one another. What that approach does not do, is to establish targets that define the *best,* or *benchmark* levels at which branches can hope to perform.

In earlier papers (see Cook and Zhu 2005, 2006), we introduced the idea of incorporating production standards *directly* into the DEA structure, rather than simply applying the indirect approach via multiplier restrictions. The idea of standard DMUs has been looked at previously; Golany and Roll (1994), for example, suggest adding 'standard DMUs' directly into the comparison group. It is not obvious, however, in many settings how such standard units would be generated. Cook and Zhu (2005, 2006) presented a methodology for using production standards as a direct means of generating such DMUs. Here we extend those ideas to situations wherein multiple sets of performance standards may be present.

In Sect. 2 we discuss the bank branch setting, and the development of 'production standards' therein. We note in particular that for this and related applications, different groups of DMUs may exhibit different standard requirements. Section 3 presents a model structure for developing 'standard' DMUs, from a set of activity matrices, to be incorporated within the DEA analysis. We examine first the case of a single activity matrix applicable to all DMUs; this is then extended to the case of group-specific, or *context-dependent* activity matrices. The new methodology is then applied, in Sect. 4 to a sample of 100 branches of a major Canadian bank. Conclusions are presented in Sect. 5.

2 Bank branch transactions and production standards

In the current competitive environment, banks are very conscious of the need to monitor branch performance. Thus, it is typical for the organization to attempt to develop 'standard processing times' for the wide variety of transactions performed by branch staff. The particular problem setting studied in this paper is that of a major Canadian bank, where such standards have been monitored over many years. The bank has chosen a small sample of branches in different branch-groupings, and has conducted time studies on a periodic basis. In this manner, estimated processing times for each transaction, within each group can be captured. Further, these estimates consist of two parts, namely that portion of the task performed by 'specialized' staff, and that part done by 'non-specialized' staff. In the case of sales transactions, an example task might be the opening of a mutual fund account. For such transactions, certain components must be completed by the specialist, namely the **Sales** employee. More routine components of that task, such as the photocopying of forms, and general filing and data entry, would be carried out by those in the **'Other staff',** or non-specialize category.

In an industry setting, where one is, for example, attempting to set labor standards for a product made on a machine, the time estimate would need to incorporate any 'allowances'—personal breaks, machine downtime allowance, etc. These allowances help to account for time when the employee is available, but cannot be producing units of the product. Unutilized (unproductive) time beyond these standards would contribute to any inefficiency. With a setting such as bank branch operations, the situation is more complex since many activities are not involved 'directly' in producing units of any specific product. Activities such as responding to customer inquiries, or conducting portfolio reviews, constitute important but 'non-volume related' activities.

In the current setting, branch staff members were asked to record the approximate portion of time spent on these non-volume related activities. This factor was then used to adjust total staff time to 'available' staff time. It is this available time that has been used to perform the efficiency evaluation within branches. It is this element of performance evaluation that renders the bank branch setting distinctly different from the traditional industry situation. In the latter, the job definition of the employee directly connects all activities of that person to tangible products produced. With the sales employee in the bank branch, many intangible activities are designed to support those tangible activities, but which cannot be attached to any *particular* product or volume.

In Sect. 4 we perform a detailed analysis of efficiency of a set of bank branches, and introduce developed time standards for different branch groupings, leading to standard branches. Using such standards, we wish to evaluate DMUs not only against best practice branches (the conventional approach), but as well against 'best possible' or standard branches. First, we develop the necessary methodology to augment the usual DEA model. This is the content of the following section.

3 Data envelopment analysis with standards

We assume that there are n DMUs, with each DMU_j ($j = 1, 2, \ldots, n$) consuming a vector of inputs, $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$ to produce a vector of outputs, $Y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})$. The efficiency of DMU_{j_o} ($= DMU_o$), *relative* to others, can be measured by way off the CCR ratio DEA model (Charnes et al. 1978), which in linear programming format is expressed as:

$$z^* = \max \sum_{r=1}^{s} \mu_r y_{ro}$$

subject to

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \omega_i x_{ij} \le 0 \quad j = 1, \ldots, n, \tag{1}$$

$$\sum_{i=1}^{m} \omega_i x_{io} = 1,$$
$$\omega_i, \mu_r \geq \varepsilon.$$

where ε is a non-Archimedean element defined to be smaller than any positive real number. Model (1) is called the CCR multiplier DEA model, where ω_i and μ_r are input and output multipliers, respectively.

In many service, and most manufacturing settings, *production standards* will have been developed, and specify amounts of various inputs required to produce a unit of various outputs. An example of such standards in a factory setting might be the number of minutes of machine time, finishing labor time, and time required in the paint spray booth, to produce one unit of a product. Such standards can generally be expressed in matrix format with rows indexed by the various resource or input requirements, and the columns by the different products or outputs generated. In general terms, consider the existence of an activity matrix $A = (a_{ir})$, where a_{ir} defines the amount of input i needed to produce one unit of output r. For simplicity of presentation, we assume first that the DMUs constitute a homogeneous group, and that a single uniform set of such values apply to all units. We examine the more general case later in this section. One means of incorporating standards into the DEA structure is to determine the maximum levels of outputs possible for each DMU, given the amounts of inputs consumed by those DMUs. In most cases, these maximum output levels will exceed the *observed* levels. Thus, two versions of each DMU can be specified—the *observed* and the *standard* (or best possible) versions.

The single group case We propose the following mathematical programming model for deriving the standard version of the DMU. Let w_r be a set of weights associated with the (unknown) outputs ($\hat{y}_{ro}$), and consider the math programming model:

$$\begin{aligned} &\max \sum_{r=1}^{s} w_r \hat{y}_{ro} \\ &\text{subject to} \\ &\quad \sum_{r=1}^{s} a_{ir} \hat{y}_{ro} \leq x_{io} \quad i = 1, \dots, m, \\ &\quad \hat{y}_{ro} \in \Gamma \quad r = 1, \dots, s, \end{aligned} \tag{2}$$

where Γ represents any imposed requirements/relations on the outputs. For example, Γ can represent lower bound restrictions $\hat{y}_{ro} \geq y_r^{\min}$ for all r, on the amounts of output y_{ro} across all DMUs.

Model (2) basically determines the optimal or possible output levels for DMU_0, given the current input levels x_{io} for that DMU. If the w_r are known, then model (2) is a linear programming problem. For a specific DMU, say, DMU_0 it would appear that reasonable candidates for the w_r are the optimal values of μ_r^* from model (1). That is, we rewrite model (2) as the following linear program

$$\max \sum_{r=1}^{s} \mu_r^* \hat{y}_{ro}$$
$$\text{subject to}$$
$$\sum_{r=1}^{s} a_{ir} \hat{y}_{ro} \le x_{io} \quad i = 1, \ldots, m, \tag{3}$$
$$\hat{y}_{ro} \in \Gamma \quad r = 1, \ldots, s.$$

The multi-group case We next assume that the n DMUs are arranged into K groups ($k = 1, \ldots, K$). Let a_{ir}^k denote the amount of input i needed to produce one unit of output r in group k. In this event it can be argued that in evaluating the efficiency of any given DMU, relative to *all* others, some groups may be unfairly treated. Consider the case wherein the standards in one group ($a_{ir}^{k_1}$) are uniformly larger than in another group ($a_{ir}^{k_2}$). In this instance, the members of group k_1 would be expected to under perform those of k_2. Hence, in order for the full set of DMUs to constitute a proper comparison group, there is a need to provide for some type of adjustment to be applied to outputs of one group to make them comparable to those of another group. In the above example, the members of group k_1 should be *upgraded* to make them comparable to those of group k_2.

In the simple case of a single input (i.e., $|I| = 1$, where $|I|$ denotes the cardinality of the input set), an appropriate way to adjust the outputs across groups would be to choose one of the groups to represent the standard or base line, say group $k = 1$, then for any $k \neq 1$, adjust the outputs y_{rj} for any $j \in J_k$ (J_k denotes the set of DMUs in group k), by the factor $\frac{a_{1r}^k}{a_{1r}^1}$; i.e. replace y_{rj} by $\tilde{y}_{rj}$, where

$$\tilde{y}_{rj} = \left(\frac{a_{1r}^k}{a_{1r}^1}\right) y_{rj}, \quad j \in J_k,\ k \neq 1. \tag{4}$$

The problem when $|I| > 1$ is that we have more than one ratio to consider. While there are many possible functions of the $\{a_{ir}^k\}_i$ that might be considered as an adjustment factor, we suggest using the geometric mean of the ratios across the members of inputs I. That is, define

$$\frac{a_r^k}{a_r^1} = \left[\prod_{i \in I} \left(\frac{a_{ir}^k}{a_{ir}^1}\right)\right]^{1/|I|}, \tag{5}$$

and let

$$\tilde{y}_{rj} = \left(\frac{a_r^k}{a_r^1}\right) y_{rj}, \quad j \in J_k,\ k \neq 1. \tag{6}$$

Property 1 The mean of the ratios is equal to the ratios of the means, specifically

$$\frac{a_r^k}{a_r^1} = \frac{[\prod_{i \in I} a_{ir}^k]^{1/|I|}}{[\prod_{i \in I} a_{ir}^1]^{1/|I|}}, \tag{7}$$

where $\frac{a_r^k}{a_r^1}$ is defined in (5).

Thus, the appeal of replacing the set of ratios by an average of these ratios in (7), is equivalent to taking the (geometric) average of the set of $|I|$ standard $\{a_{ir}^k\}_i$ in group k

and those $\{a^1_{ir}\}_i$ in group 1, and then simply using the ratio of those of two averages, and following (4) to adjust the y_{rj}.

It is noted that in the special case where there is the same percentage differences in a^k_{ir} versus a^1_{ir}, for every i (e.g., $\frac{a^k_{1r}}{a^1_{1r}} = c^k_r$, a constant), then we would expect that the proper way to adjust the outputs in group k, would be to scale them according to c^k_r. That is, the appropriate adjustment to the y_{rj} should be

$$\tilde{y}_{rj} = c^k_r y_{rj}, \quad j \in J_k,\ k \neq 1.$$

Property 2 If $\frac{a^k_{ir}}{a^1_{ir}} = c^k_r$, a constant for all i, then $c^k_r = \frac{a^k_r}{a^1_r}$, where $\frac{a^k_r}{a^1_r}$ is given by (5).

Note that Property 2 follows from $\frac{a^k_r}{a^1_r} = [\prod_{i\in I}(\frac{a^k_{ir}}{a^1_{ir}})]^{1/|I|} = [\prod_{i\in I}(c^k_r)]^{1/|I|} = c^k_r$.

It would appear from Properties 1 and 2, that the geometric mean of ratios of standards across the set of inputs is a reasonable adjustment coefficient, to render the DMUs in the various groups comparable to one another. To complete the picture, we recommend generating the standard DMUs by replacing a_{ir} in (3) by a^1_{ir}. We now prove the following theorem.

Theorem 1 *The new DMU* $= (x_{io}, \hat{y}^*_{ro})$ *obtained from* (3) *is efficient under model* (1).

Proof See Appendix. □

For each DMU_j ($j = 1, \ldots, n$), model (3) then generates a new "standard" DMU which can be used in the DEA analysis. These standard DMUs generate an *outer frontier*. Some of the standard DMUs are likely to be dominated by other standard DMUs, depending on the inputs used to generate these new units.

One may note that model (1) usually does not yield unique optimal solutions. However, this should not be a problem, because μ^*_r are only used as weights to aggregate the multiple outputs. A different set of optimal μ^*_r may lead to a different set of standard DMUs which are all efficient under DEA model (1). Our objective here is to obtain *one* set of standard DMUs.

Note that it may be desirable that μ^*_r are positive for all r. To achieve this, one can select a real value for ε in model (1). However, as indicated in Ali and Seiford (1993), caution should be exercised when selecting ε.

Alternatively, we may use the strong complementary slackness condition (SCSC) solutions from model (3) to obtain a set of positive multipliers. The SCSC states that there exists an optimal solution $(\lambda^*_j, s^+_r, s^-_i, \mu^*_r, \omega^*_i, t^*_j)$ to the DEA models for which, in addition to the complementary slackness condition, we have $s^+_r + \mu^*_r > 0$ ($r = 1, \ldots, s$), $s^-_i + \omega^*_i > 0$ ($i = 1, \ldots, m$), $\lambda^*_j + t^*_j > 0$ ($j = 1, \ldots, n$), where $t^*_j = -\sum_{r=1}^{s} \mu_r y_{rj} + \sum_{i=1}^{m} \omega_i x_{ij}$, and λ^*_j, s^+_r and s^-_i are optimal solutions to the dual program of model (1), namely

$$\min \quad \theta - \varepsilon\left(\sum_{i=1}^{m} s^-_i + \sum_{r=1}^{s} s^+_r\right)$$

subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s^-_i = \theta x_{io} \quad i = 1, 2, \ldots, m, \tag{8}$$

$$\sum_{j-1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad r = 1, 2, \dots, s,$$
$$\lambda_j, s_i^-, s_r^+ \geq 0.$$

For efficient DMUs, $s_i^- = s_r^+ = 0$ in all optimal solutions to model (8). Therefore, for efficient DMUs, if we obtain a set of SCSC solutions, we can always have positive μ_r^* from model (1). (See also Chen et al. (2003).)

Our approach is then summarized in the following three steps.

Step 1: Using the adjusted outputs $\tilde{y}_{rj}$ from (6), solve the following modification of model (1):

$$\begin{aligned}
z^* &= \max \sum_{r=1}^{s} \mu_r \tilde{y}_{ro} \\
&\text{subject to} \\
&\sum_{r=1}^{s} \mu_r \tilde{y}_{rj} - \sum_{i=1}^{m} \omega_i x_{ij} \leq 0 \quad j = 1, \dots, n, \\
&\sum_{i=1}^{m} \omega_i x_{io} = 1, \\
&\omega_i \geq \varepsilon > 0, \quad \mu_r \geq \varepsilon > 0.
\end{aligned} \tag{9}$$

Step 2: For each DMU_o, solve model (3) using a set of optimal μ_r^* (from model (9)), and with a_{ir} replaced by a_{ir}^1. Model (3) then yields a new set of n units that serve as standard DMUs.

Step 3: We denote this new set of DMUs as $\{\overline{DMU_k}\}$, where for $\overline{DMU_k}$, the ith input $\overline{x_{ik}} = x_{ik}$ and rth output $\overline{y_{rk}} = \hat{y}_{rk}^*$ $(k = 1, \dots, n)$, obtained from Step 2. Now, solve the following DEA model for each of the n original DMUs

$$\begin{aligned}
z^* &= \max \sum_{r=1}^{s} \mu_r \tilde{y}_{ro} \\
&\text{subject to} \\
&\sum_{r=1}^{s} \mu_r \tilde{y}_{rj} - \sum_{i=1}^{m} \omega_i x_{ij} \leq 0 \quad j = 1, \dots, n, \\
&\sum_{i=1}^{m} \omega_i x_{io} = 1, \\
&\sum_{r=1}^{s} \mu_r \overline{y_{rk}} - \sum_{i=1}^{m} \omega_i \overline{x_{ik}} \leq 0 \quad k = 1, \dots, n, \\
&\omega_i \geq \varepsilon > 0, \quad \mu_r \geq \varepsilon > 0.
\end{aligned} \tag{10}$$

Model (10) is actually model (1) with the n original DMUs, using adjusted outputs, *and* the n standard DMUs that have been generated.

In summary, in the multi-group case, one replaces the outputs y_{rj} by their adjusted versions $\tilde{y}_{rj}$, and solves problem (9) to derive a *best practice* efficiency score for each of the n DMUs. The solutions to these *n problems* yield, as well, a set of optimal output multipliers μ_r^* (for each DMU) that are used in model (3) to generate a 'standard' version of each of the n DMUs. Then using these standard DMUs together with their original (output-adjusted) counterparts, model (10) is solved to derive a final 'standard' efficiency rating for each of the original n DMUs relative to the full set of $2n$ DMUs.

We now apply the methodology developed herein to a problem on bank branch efficiency.

4 Bank branch transactions and production standards

We here apply our proposed approach to a set of 100 branches of a large Canadian bank. The data used to evaluate the sample of bank branches consists, on the input side, of three *staff types*, namely sales staff, service staff and other staff. On the output side, we use the most important transactions performed at the counter within the branch. Specifically, from the full set of counter transactions, the top *nine* of these were selected for use in evaluating performance. While these nine activities do not represent the full range of transaction types, they do account for over 80% of the volume of work carried out (in terms of time). For performance measurement purposes, it is felt by management that this large (80%) fraction of the activities is representative of the overall branch effort.

Outputs have been grouped under two general classes—*service outputs and sales outputs*. Under each, the data given represent the numbers of transactions during a recent fiscal year. The following is a brief description of the transactions:

Service Outputs

Deposits—all counter deposit transactions within the branch.
Account Openings—the number of personal accounts opened during the period.
Withdrawals—the number of withdrawals from interest-bearing accounts.
Passbook Updates—the number of updates of customer passbooks on accounts.
Transfers—the number of in-branch transfers of funds between accounts.
Visa Cash—the number of cash advances from Visa.

Sales Outputs

RRSPs—the number of registered retirement savings plan account openings.
Letters of Credit—the number of letters of credit issued.
Loans—the aggregation of all loan account openings, including mortgages.

To illustrate the application of the models of the previous section to a practical setting, these 100 branches are separated into three general groups. Table 1 reports the summary statistics of each group.

Each group has a specific activity matrix shown in Table 2, where standard hours needed to carry out one transaction are shown. For example, in the first group, one deposit takes on average 0.067 hours (about 4 minutes) to complete. Note that sales staff are assumed to participate only in sales transactions, service staff only in service transactions, while *other* staff members are involved in both. It is assumed that there are 35 working hours per week and that there are 48 weeks per year. Thus, each staff person translates to 35×48 (=1680) hours per year.

Table 1 Summary statistics for the 100 bank branches

	FTESales	FTESer	FTEOth	Deposits	OpenAcct	WD	UPD	TRF	VISA	RRSP	Let Cr.	Loans
Group 1 (40 branches)												
Average	4.19	4.59	5.31	44 710.75	779.80	2660.85	1735.68	204.93	443.63	817.50	1493.75	205.40
Standard Deviation	3.05	3.56	5.84	42 391.64	840.68	2101.29	1410.41	351.63	459.84	789.45	1117.97	247.02
Min	0.83	2.20	0.99	7251.00	191.00	574.00	393.00	1.00	27.00	120.00	50.00	2.00
Max	18.15	18.70	35.51	220 726.00	4731.00	12019.00	6653.00	1942.00	2973.00	4210.00	3920.00	1132.00
Group 2 (20 branches)												
Average	3.71	4.07	3.90	27 867.20	671.31	2600.23	1608.88	125.67	333.47	617.89	1927.38	71.94
Standard Deviation	2.97	2.23	2.91	24 715.03	471.67	1700.45	1153.28	154.12	255.57	617.22	1265.54	91.05
Min	0.83	2.20	1.32	4121.39	89.18	758.94	406.77	0.00	50.96	72.80	218.40	1.82
Max	13.20	9.90	13.03	113 285.00	1782.69	6240.78	4413.50	617.89	1029.21	2802.80	4349.80	410.41
Group 3 (40 branches)												
Average	2.85	3.03	3.09	17 298.24	404.34	1627.90	1023.17	78.96	238.54	383.25	1306.55	40.72
Standard Deviation	1.88	1.20	1.91	12 141.47	264.16	945.62	544.94	83.46	161.58	359.20	977.39	46.20
Min	0.83	2.20	0.99	3151.40	107.10	205.80	174.30	0.70	46.20	42.00	203.00	0.70
Max	9.90	6.60	9.24	58 607.50	1394.40	5631.50	2499.70	376.60	624.40	2163.00	4942.00	204.40

Table 2 Activity matrices with standard hours

Group 1	Deposits	OpenAcct	WD	UPD	TRF	VISA	RRSP	Let Cr.	Loans
FTESales	0	0	0	0	0	0	1.3	1.207	1.73
FTEServ	0.067	1.4	0.08	0.06	0.09	0.08	0	0	0
FTEOther	0.09	0.3	0.04	0.02	0.08	0.09	0.85	0.55	1.45
Group 2	Deposits	OpenAcct	WD	UPD	TRF	VISA	RRSP	Let Cr.	Loans
FTESales	0	0	0	0	0	0	1.417	1.316	1.885
FTEServ	0.073	1.526	0.087	0.065	0.098	0.087	0	0	0
FTEOther	0.098	0.327	0.043	0.021	0.087	0.098	0.926	0.599	1.580
Group 3	Deposits	OpenAcct	WD	UPD	TRF	VISA	RRSP	Let Cr.	Loans
FTESales	0	0	0	0	0	0	1.69	1.569	2.249
FTEServ	0.087	1.82	0.104	0.078	0.117	0.104	0	0	0
FTEOther	0.117	0.39	0.052	0.026	0.104	0.117	1.105	0.715	1.885

Based upon the three activity matrixes, we adjust the outputs of groups 2 and 3 branches using transformation (6). We then run model (1) for each of the adjusted 100 branches, i.e., we run model (9). Note that in this case, the group 1 branches remain unchanged.

Table 3 displays the CCR efficiency for the 100 branches under the heading "original efficiency" when the outputs are not adjusted. Table 3 also reports the efficiency scores from model (9) under the heading "adjusted efficiency". (We note that while we could impose AR constraints in this analysis, we have not done so herein.)

We also obtain a set of optimal multipliers (μ_r^*) for each DMU under evaluation for use in model (3). Model (3) hence generates 100 (new) standard DMUs that are efficient under the CCR model (1). With these 100 standard DMUs, we then apply model (10). Table 3 reports the efficiency scores under the heading "standard efficiency", where 100 newly generated branches are introduced. Note, however, that we report here only the efficiencies of the original DMUs (output-adjusted as per (6)), not the efficiencies of the standard DMUs, all of which are efficient in any event.

Note that based upon the original data, 53 branches are efficient. When the outputs of the groups 2 and 3 are adjusted based upon the three activity matrixes, 64 branches are efficient. When the standards are introduced, only 19 branches are efficient. Also, as expected, the efficiencies of inefficient branches drop when the standard DMUs are introduced.

5 Conclusions

While DEA has been proven an excellent method for performance evaluation, it only identifies *best practice*, and may not reflect the "true" frontier. The current paper develops an approach where efficient standards can be built into the DEA analysis. As a result, DEA is extended into situations where standards can be identified when multiple performance measures are present and their relations are not completely known. A possible future study is to develop an iterative procedure for moving from one set of established standard DMUs to a better set of such DMUs.

The identified standards form an outer layer of the efficient frontier, compared to the DEA best practice frontier. Consequently, a ranking of the entire set of DMUs is available.

Table 3 Efficiency scores

Branch	Original Efficiency	Adjusted Efficiency	Standard Efficiency	Branch	Original Efficiency	Adjusted Efficiency	Standard Efficiency
1	1.00000	1.00000	0.95035	51	0.88215	0.87168	0.77577
2	1.00000	1.00000	0.96606	52	0.99100	0.92214	0.76321
3	1.00000	0.99915	0.98609	53	1.00000	0.99834	0.90054
4	1.00000	1.00000	0.95995	54	1.00000	0.98403	0.82587
5	1.00000	1.00000	1.00000	55	0.96291	1.00000	1.00000
6	1.00000	1.00000	0.94566	56	0.98294	0.96587	0.76517
7	1.00000	1.00000	0.94668	57	1.00000	1.00000	0.91437
8	1.00000	1.00000	0.96437	58	1.00000	1.00000	1.00000
9	1.00000	1.00000	0.97928	59	1.00000	0.97000	0.91914
10	1.00000	1.00000	0.90807	60	1.00000	1.00000	0.96559
11	1.00000	1.00000	0.92802	61	0.65059	0.80130	0.73437
12	1.00000	1.00000	0.83404	62	0.89836	1.00000	1.00000
13	1.00000	1.00000	0.91892	63	1.00000	1.00000	1.00000
14	1.00000	1.00000	1.00000	64	0.64281	0.83816	0.80433
15	1.00000	1.00000	1.00000	65	1.00000	1.00000	1.00000
16	1.00000	1.00000	0.98203	66	0.75501	0.96617	0.94766
17	0.95889	0.93987	0.92385	67	0.86860	1.00000	1.00000
18	1.00000	1.00000	1.00000	68	0.85373	1.00000	1.00000
19	1.00000	1.00000	0.84442	69	1.00000	1.00000	0.94283
20	1.00000	1.00000	0.98073	70	0.97111	1.00000	1.00000
21	1.00000	1.00000	0.91902	71	1.00000	1.00000	0.97392
22	0.94927	0.84706	0.81408	72	1.00000	1.00000	0.95264
23	1.00000	1.00000	0.97883	73	0.83885	1.00000	0.81748
24	1.00000	1.00000	0.94965	74	0.89946	1.00000	0.94788
25	1.00000	1.00000	0.91182	75	0.78266	0.97985	0.88783
26	1.00000	1.00000	0.91086	76	0.72671	0.93444	0.87393
27	0.97939	0.94162	0.93287	77	0.83327	1.00000	0.98643
28	0.92925	0.89838	0.87837	78	0.92493	1.00000	1.00000
29	1.00000	0.98087	0.90814	79	0.81482	0.97773	0.96137
30	1.00000	0.94731	0.86600	80	0.87495	1.00000	0.95168
31	1.00000	0.98904	0.86836	81	1.00000	1.00000	0.85921
32	0.96743	0.93343	0.89142	82	1.00000	1.00000	1.00000
33	0.90145	0.83260	0.78758	83	0.67841	0.94551	0.93254
34	0.76184	0.70280	0.66959	84	0.92801	1.00000	1.00000
35	1.00000	1.00000	1.00000	85	0.97020	1.00000	1.00000
36	1.00000	1.00000	1.00000	86	0.78305	0.98769	0.92227
37	1.00000	1.00000	0.92041	87	0.92923	1.00000	0.94015
38	1.00000	1.00000	0.99939	88	0.97058	1.00000	0.76622
39	1.00000	1.00000	0.95801	89	1.00000	1.00000	0.82376
40	1.00000	1.00000	1.00000	90	0.90891	1.00000	0.95220
41	0.79611	0.81478	0.63600	91	1.00000	1.00000	0.82387
42	1.00000	1.00000	0.93274	92	0.86273	1.00000	1.00000
43	1.00000	0.99806	0.93493	93	0.67645	0.82911	0.75701

Table 3 (*Continued*)

Branch	Original Efficiency	Adjusted Efficiency	Standard Efficiency	Branch	Original Efficiency	Adjusted Efficiency	Standard Efficiency
44	0.97588	0.97536	0.95206	94	0.80332	1.00000	0.95220
45	1.00000	1.00000	0.95262	95	0.75241	0.94743	0.77550
46	0.93773	0.94679	0.93146	96	0.67500	0.85831	0.81123
47	0.89319	0.87388	0.82141	97	0.92977	1.00000	0.97396
48	0.96525	0.98233	0.96378	98	1.00000	1.00000	0.97762
49	1.00000	1.00000	0.90200	99	0.81696	1.00000	0.90722
50	0.88434	0.87768	0.80598	100	0.71645	0.96626	0.91712

We finally should point out that in the current application, we assume that it is only staff availability that is restricting the branch from creating more outputs. Other factors are, in fact, also playing a part in bank branch operations. These include available computer technology in the branch, maximum customers per day that the facility can physically handle (due to limitations on availability of parking lots, floor space, etc.), and most importantly, on the demand side, the maximum possible number of transactions that are reasonably available in the market. Therefore, a follow-up study may examine how the standard DMUs change when we consider these factors. This may involve the use of stochastic DEA, since we may not know what the market can bear in terms of number of additional customers who can be enticed into the branch.

Appendix

Proof of Theorem 1 With no loss of generality, we do not consider the constraints given by Γ in model (3). Now, consider the dual of (3)

$$\begin{aligned} &\min \quad \sum_{i=1}^{m} \tilde{\omega}_i x_{io} \\ &\text{subject to} \\ &\quad \sum_{i=1}^{m} \tilde{\omega}_i a_{ir} \geq \mu_r^* \quad r = 1, \dots, s, \\ &\quad \tilde{\omega}_i \geq 0, \quad i = 1, \dots, m. \end{aligned} \tag{11}$$

Let $\tilde{\omega}_i^*$ be a set of optimal solutions to (11). Then $\sum_{i=1}^{m} \tilde{\omega}_i^* x_{io} = \sum_{r=1}^{s} \mu_r^* \hat{y}_{ro}^*$, where $\hat{y}_{ro}^*$ is optimal in (3) and μ_r^* is optimal in (1).

Note that for any x_{ij} $(j = 1, \dots, n)$, $\tilde{\omega}_i^*$ is a feasible solution to (11), and for any μ_r^*, y_{rj} $(j = 1, \dots, n)$ is a feasible solution to (3). Therefore, by the weak duality theorem, we have

$$\sum_{i=1}^{m} \tilde{\omega}_i^* x_{ij} \geq \sum_{r=1}^{s} \mu_r^* y_{rj}.$$

Next, if we let

$$\omega_i = \frac{\tilde{\omega}_i^*}{\sum_{i=1}^m \tilde{\omega}_i^* x_{io}}, \quad \text{and} \quad \mu_r = \frac{\mu_r^*}{\sum_{i=1}^m \tilde{\omega}_i^* x_{io}},$$

we then have

$$\sum_{r=1}^s \mu_r \hat{y}_{ro}^* = \frac{\sum_{r=1}^s \mu_r^* \hat{y}_{ro}^*}{\sum_{i=1}^m \tilde{\omega}_i^* x_{io}} = 1,$$

$$\sum_{i=1}^m \omega_i x_{io} = \frac{\sum_{i=1}^m \tilde{\omega}_i^* x_{io}}{\sum_{i=1}^m \tilde{\omega}_i^* x_{io}} = 1,$$

$$\sum_{i=1}^m \omega_i x_{ij} = \frac{\sum_{i=1}^m \tilde{\omega}_i^* x_{ij}}{\sum_{i=1}^m \tilde{\omega}_i^* x_{io}} \geq \frac{\sum_{r=1}^s \mu_r^* y_{rj}}{\sum_{i=1}^m \tilde{\omega}_i^* x_{io}} = \sum_{r=1}^s \mu_r y_{rj}.$$

Thus, the new $DMU = (x_{io}, \hat{y}_{ro}^*)$ obtained from model (3) is efficient under model (1). □

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