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An approach to predictive-reactive scheduling of parallel machines subject to disruptions

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Abstract In this paper, a new predictive-reactive approach to a parallel machine scheduling problem in the presence of uncertain disruptions is presented. The approach developed is based on generating a predictive schedule that absorbs the effects of possible uncertain disruptions through adding idle times to the job processing times. The uncertain disruption considered is material shortage, described by the number of disruption occurrences and disruption repair period. These parameters are specified imprecisely and modelled using fuzzy sets. If the impact of a disruption is too high to be absorbed by the predictive schedule, a rescheduling action is carried out. This approach has been applied to solving a real-life scheduling problem of a pottery company.

1 Introduction

Production scheduling is typically defined as the optimal or near optimal allocation of scarce resources, usually machines, to tasks over time (Pinedo 2002). It has been a topic that has attracted a wide interest of both academics and practitioners in the last fifty years. Complexity of production scheduling problems is caused by a variety of machine configurations (e.g., single machine, parallel machines, flow shops, job shops), large scale dimensions (including the number of machines and the number of jobs to be scheduled), a wide range of parameters involved (job release dates, processing times, due dates, machine setup times, priorities of jobs, etc.), and uncertainty inherent in some parameters.

In addition, in most real life production environments, scheduling is an on going process where various disruptions in both external business and internal production conditions may occur dynamically and cause deviations from the initially generated schedule. Most often, these disruptions are uncertain. The importance of considering these uncertainties and developing rescheduling methods as a response to uncertain disruptions have been recognised

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mainly in the last decade. Vieira et al. (2003) reviewed rescheduling strategies, policies and methods. Aytug et al. (2005) classified different approaches to scheduling in the presence of uncertainty into three groups: reactive scheduling, robust scheduling and predictive-reactive scheduling.

O'Donovan et al. (1999) presented a single machine scheduling method that considered random machine breakdowns. In order to absorb the impact of the machine breakdowns, an idle time was added to the completion times of the scheduled jobs. Consequently, the jobs' sequence in both the original and predictive schedules remained the same, but the jobs' completion times were different. The objective was to maximise the predictability of the realised schedule by estimating the machine failure effects on the schedule and increasing the estimated job completion times. Mehta and Uzsoy (1999) developed an approach to a predictable scheduling of a single machine subject to random machine breakdowns with the objective to absorb disruptions without affecting planned activities. They defined different measures of schedule predictability that were based on comparisons of the predictive schedule and the realised schedule completion times. Li et al. (2000) identified and investigated four sources of production disturbances, including: (1) incorrect work, (2) machine breakdowns, (3) rework due to a quality problem and (4) rush orders. A new rescheduling method for a job shop under random disruptions was proposed by Abumaizar and Svestka (1997). Two measures of the rescheduling performance were considered simultaneously, including efficiency, measured by the makespan, and stability, measured as the deviation from the initial schedule. Rangsaritratsamee et al. (2004) developed and analysed a genetic algorithm based rescheduling method that considered both efficiency and stability criteria.

In this paper, a new predictive-reactive approach to a parallel machine scheduling in the presence of uncertain disruptions is presented. A parallel machine scheduling problem is common in practice and considers a number of jobs to be processed on parallel machines. The objective is to determine the jobs allocation to the machines and the sequence of the jobs on each machine in order to optimise certain criteria (Błażewicz et al. 1996). The developed approach is based on generating a predictive parallel machine schedule using dispatching rules. The predictive schedule is designed to absorb the effects of a possible disruption through adding idle times to the job processing times. The added idle time is equal to the approximated repair time needed to recover from a disruption during the processing of a certain job.

Usually, the idle time to be added to the job processing times is determined assuming that the disruptions are random. It is represented by a probability distribution that can be derived based on historical data. This requires a valid hypothesis that the data collected are complete and unbiased. However, very often in practice, there is no evidence of the events recorded, or there is lack of evidence, or lack of confidence in the evidence, or the evidence might have been recorded inconsistently, and therefore the use of concepts of probability theory might not be appropriate. In these situations, uncertain disruptions may be specified based on experience and managerial subjective judgement. It may be convenient to use natural language expressions in the specifications. It has been shown in a large body of literature that fuzzy sets theory provides a suitable framework for representing uncertainties in decision making problems where intuition and subjective judgements play an important role (Ruspini et al. 1998; Zimmermann 1996). In this paper, a new approach to dealing with uncertain disruptions is proposed where the disruption is specified imprecisely and modelled by fuzzy sets.

However, if the impact of a disruption is too high to be absorbed by the predictive schedule, a rescheduling action is needed. Reactive scheduling or rescheduling is defined as the process of modifying a schedule when a disruption occurs while executing the schedule on the shop floor (Alagöz and and Azizoğlu 2003). The developed predictive-reactive scheduling approach is applied to a real-life scheduling problem identified in collaboration with a manufacturing pottery company.

The paper is organised as follows. The problem of scheduling of parallel machines in the presence of uncertain disruptions is described in Sect. 1.1, while the new approach to a predictive-reactive scheduling is presented in Sect. 2. The real-life scheduling problem is presented in Sect. 3, and the analysis of the results obtained is given in Sect. 4. Finally, conclusions and directions for future work are outlined in Sect. 5.

1.1 Problem statement

A typical problem of identical parallel machines scheduling is stated as follows (Pinedo 2002): N jobs, J_j, $j = 1, \dots, N$, have to be scheduled on M machines M_i, $i = 1, \dots, M$, where each job is independent and nonpreemptive, and the machines are identical, i.e. they have the same speed. Hence, the jobs can be processed on any machine and their processing times are p_j, $j = 1, \dots, N$. The objective is to find the job's allocation for each machine that minimises the makespan, i.e. the time that is taken to complete all the jobs. The makespan is defined as follows:

$$C_{\max} = \max\{C_j | j = 1, \dots, N\} \tag{1}$$

where C_j is the completion time of job J_j.

In this paper, the typical scheduling problem defined above is generalised in the following way. Some of the jobs to be scheduled can be processed only on predetermined machines; formally stated, job J_j, $j = 1, \dots, N$, can be processed on subset M_j of the M machines only. In addition, it is assumed that the schedule will be realised in the presence of uncertain disruptions caused by material shortages. These disruptions have an adverse effect on the job processing times. In the case when the disruption is far too long or it cannot be repaired at all, the jobs affected are removed from the schedule. The remaining jobs are rescheduled.

The reactive scheduling problem is considered as a multi-criteria optimisation problem. In addition to the initial criterion of schedule efficiency (the makespan $C_{\max}$) used in generating the predictive schedule *PS*, a new criterion of stability/instability of the reactive schedule *RS* is considered. The instability *IST* of the reactive schedule *RS* is typically measured as the starting time deviations between the predictive schedule *PS* and the reactive schedule *RS* (Papoulis 1991). However, the pottery manufacturing process considered is sequential; after glazing, the products are transferred to the kiln section. Therefore, in the context of the pottery company, where the glazing scheduling affects the kiln operations, it is of importance to consider completion time deviations as follows:

$$IST(RS) = \sum_{j=1}^{N} |C_j(PS) - C_j(RS)|. \tag{2}$$

2 Predictive-reactive scheduling of identical parallel machines

A parallel machine schedule is generated following two steps. In the first step, a predictive schedule *PS* is determined, where idle times are added to jobs' processing times and two dispatching rules, namely the Least Flexible Job First (LFJ) and the Longest Processing Time (LPT), are combined. In the second step, two rescheduling methods, which modify the predictive schedule into a reactive schedule *RS* are applied including, Left-shift rescheduling and Building new schedules.

2.1 Predictive scheduling

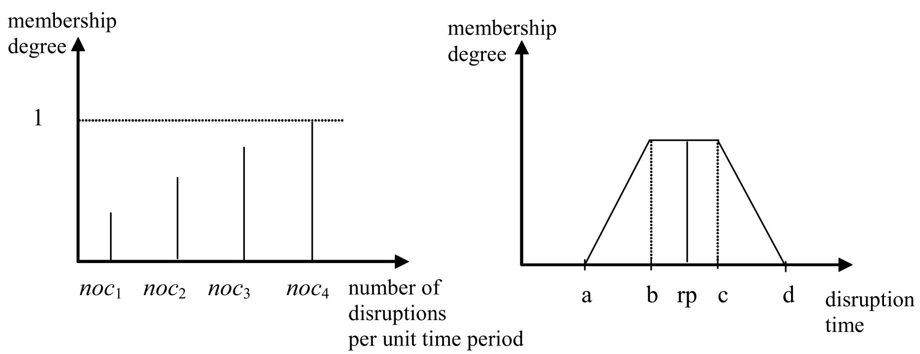
Predictive schedules are generated in order to absorb anticipated disruptions. Traditionally, uncertain disruptions have been modelled using probability distributions that concern the occurrence of well defined events. Corresponding probabilities are assessed or estimated taking into consideration repetition of the events. For example, an uncertain disruption that has most often been treated in the literature is machine breakdowns, typically described by two parameters, namely the mean time between failures and the mean time for repair. These two parameters may be determined based on maintenance records. However, historical data may not be available for other sources of disruptions. A typical example is the disruption caused by shortage of a raw material. In this case, practitioners may estimate the disruption occurrences based on vague or imprecise knowledge or accumulated experience. The corresponding data can be expressed using linguistic terms, such as 'the number of disruption occurrences is *much higher* than noc_1 times per unit time period' or 'the material is usually delivered in an *about rp* unit time periods'. They can be represented using fuzzy sets (see Appendix). Figure 1 shows the two linguistic terms modelled by fuzzy sets.

2.1.1 *Calculation of idle times*

In the new approach developed, the job processing times are extended by inserting idle times. Each job J_j requires one type of raw material and, therefore, might be affected by a shortage of that raw material, only. Adding an idle time enables the uncertain disruptions to be absorbed in the schedule. The extended processing time pd_j for job J_j, $j = 1, \ldots, N$ that includes the idle time id_j is defined as follows:

$$pd_j = p_j + id_j, \quad j = 1, \ldots, N \tag{3}$$

where id_j is the total disruption's repair time during the processing time of job J_j. The total disruption's repair time id_j is calculated as the product of the fuzzy number of disruption occurrences per unit time period and the fuzzy repair duration. The term 'disruption occurrence per unit time period' represents the number of disruptions that can occur while producing a certain, fixed number of products using a specific raw material.



(a) Fuzzy number of disruptions per unit time period (b) Fuzzy disruption time

Fig. 1 Fuzzy sets that represent an uncertain disruption

Having G different raw materials, m_g, $g = 1, \ldots, G$, as inputs to the production process and a discrete and finite set NOC_g that contains the possible numbers of shortage occurrences of material m_g, $NOC_g = \{noc_{g_1}, noc_{g_2}, \ldots, noc_{g_K}\}$, fuzzy set O_g, that represents the possible disruption occurrences per unit time period is defined as:

$$O_g = \sum_{k=1}^{K} \mu_{O_g}(noc_{g_k})/noc_{g_k} \tag{4}$$

where $\mu_{O_g}(noc_{g_k})$, $k = 1, \ldots, K$ is a possibility, subjectively determined, that there are noc_{g_k} disruption occurrences per unit time period, related to raw material m_g. The concept of possibility is defined in Appendix.

Fuzzy set $R_g(tr_g)$, $tr_g \in R^+$, that represents the repair duration, i.e. the delivery time of material m_g, has a continuous trapezoidal membership function $\mu_{Rg}(tr_g)$ defined as follows:

$$\mu_{Rg}(t_{rg}) = \begin{cases} 0 & \text{if } tr_g \le a_g, \\ \frac{tr_g - a_g}{b_g - a_g} & \text{if } a_g < tr_g \le b_g, \\ 1 & \text{if } b_g < tr_g \le c_g, \\ \frac{d_g - tr_g}{d_g - c_g} & \text{if } c_g < tr_g \le d_g, \\ 0 & \text{if } tr_g > d_g. \end{cases} \tag{5}$$

In order to calculate the total disruption repair time per unit time period, a product of discrete fuzzy set O_g and continuous fuzzy set R_g is determined using an approach suggested in Petrovic et al. (1996). The product is calculated as a level 2 fuzzy set $O_g \otimes R_g$, i.e. a fuzzy set whose elements are standard fuzzy sets. The elements of $O_g \otimes R_g$ are fuzzy sets $noc_{g_k} \tilde{\times} R_g$, $k = 1, \ldots, K$ calculated as a product of scalar noc_{g_k} and fuzzy set R_g. The product is determined using the Extension principle (Zadeh 1965), one of the most important principles in fuzzy sets theory, which allows the generalisation of classical mathematical concepts in the fuzzy sets framework.

In this case, the membership function $\mu_{noc_{g_k} \tilde{\times} R_g}$ has also a trapezoidal form and is calculated as follows:

$$\mu_{noc_{g_k} \tilde{\times} R_g}(t_{rg}) = \begin{cases} 0 & \text{if } tr_g \le a_g \cdot noc_{g_k}, \\ \frac{tr_g - (a_g \cdot noc_{g_k})}{(b_g - a_g) \cdot noc_{g_k}} & \text{if } a_g \cdot noc_{g_k} < tr_g \le b_g \cdot noc_{g_k}, \\ 1 & \text{if } b_g \cdot noc_{g_k} < tr_g \le c_g \cdot noc_{g_k}, \\ \frac{(d_g \cdot noc_{g_k}) - tr_g}{(d_g - c_g) \cdot noc_{g_k}} & \text{if } c_g \cdot noc_{g_k} < tr_g \le d_g \cdot noc_{g_k}, \\ 0 & \text{if } tr_g > d_g \cdot noc_{g_k}. \end{cases} \tag{6}$$

The possibility of the total disruption repair time per unit time period being $noc_{g_k} \tilde{\times} R_g$ is $\mu_{Og}(noc_{g_k})$.

Formally, the total disruption repair time per unit time period $O_g \otimes R_g$, can be represented as level 2 fuzzy set, $O_g \otimes R_g = \sum_{k=1}^{K} \mu_{Og}(noc_{g_k})/noc_{g_k} \tilde{\times} R_g$.

In order to determine a crisp idle time to add to the job processing time, it is necessary to transform this level 2 fuzzy set into a standard fuzzy set, and then to defuzzify it. The method used is the s-fuzzification proposed by Zadeh (1965). Using this method, the level 2 fuzzy sets $O_g \otimes R_g$ is transformed into the standard fuzzy sets s-fuzzif $(O_g \otimes R_g)$ with the membership function

$$\mu_{s\text{-}fuzzif(O_g \otimes R_g)}(tr_g) = \mathop{Sup}_{k=1,\ldots,K} \mu_{Og}(noc_{g_k}) \cdot \mu_{noc_{g_k} \tilde{\times} R_g}(tr_g). \tag{7}$$

The next step is to defuzzify the standard fuzzy set $s-fuzzif(O_g \otimes R_g)$, i.e. to find a scalar that represents the fuzzy set most appropriately (Ruspini et al. 1998). The defuzzification method applied is the centroid method that finds an element td_g in the support of the fuzzy set $s-fuzzif(O_g \otimes R_g)$ at which a line perpendicular to the axis passes through the centre of the area formed by the corresponding membership function. Value td_g, which is used as a crisp representation of the disruption's repair duration per unit time period, is calculated using the following expression:

$$td_g = \frac{\int_{tr_g \in R^+} tr_g \cdot \mu_{s-fuzzif(O_g \otimes R_g)}(tr_g) dtr_g}{\int_{tr_g \in R^+} \mu_{s-fuzzif(O_g \otimes R_g)}(tr_g) dtr_g}. \tag{8}$$

In order to calculate the idle time id_j to be added to the initial processing time p_j of job J_j that uses glaze m_g, the total disruption's repair time per unit time period is multiplied by the job processing time as follows:

$$id_j = td_g \cdot p_j/\text{unit time period}. \tag{9}$$

It is worth noting, that uncertain data represented by fuzzy sets, such as the number of material shortage occurrences and the shortage duration, can be effectively combined. On the other hand, probability theory is very restrictive on combining stochastic variables and the corresponding probability distributions.

2.1.2 Dispatching rules

Once the idle times are added to the initial processing times, the predictive schedule *PS* of the parallel machines is generated. According to Pinedo (2002), one of the most common methods used is dispatching or priority rules. These rules are heuristics that have low computational complexity and are easy to implement. For this problem, two dispatching rules, the Least Flexible Job First (LFJ) and the Longest Processing Time (LPT), are combined (Panwalkar and Iskander 1977). Both dispatching rules, when applied separately, generate good near optimal schedules for parallel machines when the objective is to minimise the makespan (see Pinedo 2002).

The LFJ rule allocates the job that can be processed on the smallest number of machines, to the machine that is freed. In order to break ties, the LPT rule is combined with the LFJ rule. The LPT rule assigns the job with the longest processing time. In other words, if a tie occurs when the LFJ rule is applied, the job that has the longest processing time is selected. In this way, the predictive schedule *PS* that takes into account uncertain disruptions is generated.

2.1.3 Predictive and actual schedules

The predictive schedule is released to the glazing shop floor. The actual schedule keeps the same machine allocation and jobs' sequences as the predictive schedule. However, the job completion times in the predictive and actual schedules might differ; if there is a glaze shortage on the shop floor, the initial processing times of the affected jobs are prolonged by the duration of the glaze shortage; otherwise, the job processing times remain the same as initially specified, without the idle times. A simulation tool has been developed to compare the actual schedule with the predictive schedule and to evaluate the performance of the predictive schedule (Duenas et al. 2005). The tests performed showed good performance of the predictive schedule.

2.2 Reactive scheduling

There are occasions when the predictive schedule is unable to absorb the impacts of disruptions. In other words, the disruption's repair time can be too long or the disruption cannot be recovered at all. When a disruption is caused by a material shortage that cannot be recovered, the jobs that use that specific material have to be removed from the sequence. Therefore, it is necessary to reschedule. In the developed approach, two rescheduling methods are applied and compared: 1. *Left-shift rescheduling*: The start times of the remaining jobs are left-shifted to the time when the high impact disruption occurs, keeping the same sequence of the jobs as in the predictive schedule *PS*. It is assumed that a high impact disruption affects one of the machines only, and the left-shift rescheduling is applied to that particular machine. Therefore, rescheduling in this case can be treated as a single machine scheduling, and 2. *Building new schedules*: Once a high impact disruption occurs, a new reactive schedule *RS* is generated considering the jobs that have not been processed. This new schedule is built using the same scheduling algorithm proposed for generating an initial identical parallel machines schedule, but in this case with a smaller number of jobs.

It is worth noting that in most of the rescheduling approaches proposed in the literature, rescheduling is applied on a periodic basis. However, in the approach presented in this paper, rescheduling is only applied when it is assumed that the disruption effect cannot be absorbed by the predictive schedule.

In addition, a fuzzy logic based decision support system has been developed to support rescheduling and to determine when to reschedule and which of the two rescheduling methods to apply (Petrovic and Duenas 2006).

3 A real-life scheduling problem

The predictive-reactive approach presented in the previous section is applied to a scheduling problem of the Denby Pottery Company Ltd. in the UK. One of the most important processes in the pottery industry is glazing. The glazing section is divided into three identical 'flow-lines' that are considered as identical parallel machines. A job is defined as the number of items of a specific product to be produced. The number of jobs can vary according to the production plan.

At present, the glazing section leader (decision maker) builds the schedule by hand dealing with approximately 30 jobs per production plan on a weekly basis. In the glazing section not all the jobs can be produced on any of the three machines. Therefore, the glazing scheduling problem is considered as an identical parallel machine scheduling problem with $N = 30$ jobs and $M = 3$ machines when job J_j, $j = 1, \ldots, 30$, can be processed only on a subset M_j of the 3 machines.

Table 1 presents a sample of the data used consisting of 10 jobs only, where jobs J_1, J_2, J_5, J_9 and J_{10} can only be processed on machines M_1 and M_3. The pattern refers to the glaze that is needed to process the job and the item refers to the shape of the product to be processed. The unit time period is different for each product and refers to the processing time of 100 items. The processing time p_j, $j = 1, \ldots, 10$ refers to the time required for processing job J_j, where the batch size of job J_j is not necessarily 100 items. The data given in the table are typical, however, hypothetical, as the real data cannot be presented due to confidentiality. In order to ensure that the production plan is feasible and can be done within a week, the chosen objective is the minimisation of the makespan. On the other hand, if the makespan is shorter then 40 hours (the whole week), the glazing section leader might decide to increase the productivity by increasing the number of jobs to be produced.

Table 1 Identical parallel machine scheduling data for 10 jobs

Job J_j	Pattern	Item	Unit time (hours)/100 items	Processing time p_j (hours)	M_1	M_2	M_3
1	Regency green	Large Jug	4	10	√	×	√
2	Greenwich	Large Jug	4	2	√	×	√
3	Marrakesh	Sauce Boat	6	5	√	√	√
4	Regency green	Teapot Base	4	7	√	√	√
5	Fire	Small Jug	5	2	√	×	√
6	Energy	Teapot (Classic)	3	3	√	√	√
7	Harlequin	Small Teapot Base	4	8	√	√	√
8	Spirit	Large Teapot Base	6	4	√	√	√
9	Fire	Large Jug	5	1	√	×	√
10	Blue Jetty	Sauce Jug	4	6	√	×	√

After observing the production process, it was concluded that one of the disruptions that had the greatest impact on the scheduling execution occurs when a machine runs out of a glaze. As previously defined, in order to generate a predictive schedule, it is necessary to add on idle time id_j to the initial processing time p_j of job J_j, $j = 1, \dots, 30$.

Historical data of numbers of glaze shortages and the time periods required to obtain the glazes are not available. However, the glazing section leader can specify imprecisely the number of shortage occurrences per unit time period for each glaze (i.e., the number of shortage occurrences per processing time of any 100 products that use the glaze) and the time that is usually needed to get the glaze from the glaze production department.

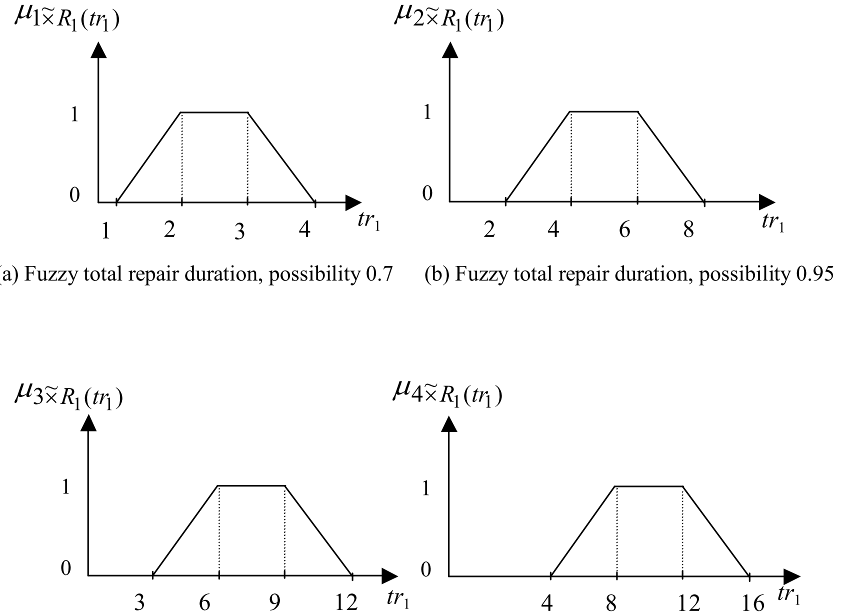
3.1 Predictive scheduling

In order to generate a predictive schedule *PS*, the total disruption's repair time per unit time period is calculated for each glaze. To illustrate this procedure, the Regency green glaze, enumerated as $g = 1$, is considered where the possible occurrences of glaze shortage per unit time period are 1, 2, 3 or 4 with possibilities 0.7, 0.95, 0.4 and 0.25, respectively, i.e. $O_1 = \{0.7/1 + 0.95/2 + 0.4/3 + 0.25/4\}$. Fuzzy set R_1 that represents the imprecise repair duration tr_1 of material m_1 (Regency green glaze) is modelled using a trapezoidal membership function (see Fig. 1(b)) with parameters $a_1 = 1$, $b_1 = 2$, $c_1 = 3$ and $d_1 = 4$.

Figure 2 shows the level 2 fuzzy set $O_1 \otimes R_1$ that represents the total disruption repair time per unit time period for glaze $m_1 = 1$, calculated as the product of the two fuzzy sets O_1 and R_1. Four fuzzy values of the total disruption repair time per unit time period with associated possibilities are presented in the figure.

Once the total repair duration time per unit time period is calculated as a level 2 fuzzy set, it is transformed into a standard set using s-fuzzification and then defuzzified, leading to the crisp total repair duration time $td_1 = 2$. In order to find the total disruption's repair time of Regency green glaze shortage during the processing of job J_1 that uses that specific glaze, td_1 is multiplied by the number of unit time periods within the processing time p_1 of the job and finally, $id_1 = 2 \times 10/4 = 5$ hours.

This procedure is repeated for each glaze and jobs that might be affected by the shortage of the glaze under consideration. Once, all the processing times are extended by adding the corresponding idle times, the schedule that minimises the makespan $C_{\max}$ is generated.



(a) Fuzzy total repair duration, possibility 0.7 (b) Fuzzy total repair duration, possibility 0.95

(c) Fuzzy total repair duration, possibility 0.4 (d) Fuzzy total repair duration, possibility 0.25

Fig. 2 The fuzzy values of the total disruption's repair time for Regency green glaze and the associated possibilities

3.2 Reactive scheduling

In practice, there will be occasions when the disruption cannot be absorbed. For example, if the glaze has low quality and it is impossible to apply it and the glaze production department informs the glazing section leader that the repair time, i.e. the time to deliver the glaze is too long. In this case, rescheduling has to be applied. Two rescheduling techniques are proposed: Left-shift rescheduling and Building new schedules as defined in Sect. 2.

4 Results analysis

Two analyses are carried out: (1) to evaluate the effects of the imprecisely specified disruptions on the jobs' sequence and completion times, and (2) to investigate the effects of rescheduling. A production plan that includes 30 jobs is considered.

4.1 Disruptions impact analysis

The aim of this analysis is to investigate the effects of changing the fuzzy repair duration time on the jobs' sequence in the predictive schedule and the corresponding completion times. The repair duration is linguistically specified in the glaze production section as '*about b to c unit time periods*' and modelled by a fuzzy set with a trapezoidal membership function specified by four parameters a, b, c, and d (see Fig. 1(b)).

In order to analyse the effects of changing the disruption duration period, the parameters a, b, c, and d are varied in two ways: (1) uncertainty in disruption duration period is increased by increasing the distance between b and c and (2) disruption duration period is increased by shifting the domain to the right, i.e. by increasing the values of parameters a, b, c and d.

4.1.1 Effect of increasing uncertainty in disruption duration period

Tables 2, 3 and 4 show the jobs' sequence, completion times and $C_{\max}$ obtained for three different sets of parameter values: (1) $a = 1$, $b = 2$, $c = 3$ and $d = 4$, (2) $a = 1$, $b = 2$, $c = 4$ and $d = 5$, and (3) $a = 1$, $b = 3$, $c = 6$ and $d = 8$, respectively. As expected, the results obtained showed that changing uncertainty in duration repair period has an adverse impact on the predictive schedule's makespan. For example, if the distance between b and c is increased from 1 hour ($b = 2$ and $c = 3$) to 2 hours ($b = 2$ and $c = 4$) the makespan increases from 88.5 to 98.9 hours, this represents and increment of 10.4 hours (12%). If the distance between b and c is increased from 1 hour ($b = 2$ and $c = 3$) to 3 hours ($b = 3$ and $c = 6$) the makespan increases from 88.5 to 130 hours, which represents an increment of 41.5 hours (47%). Consequently, it can be concluded that widening the distance between b and c does not have a linear impact on the makespan deterioration.

It may be interesting to analyse the effects of widening the distance between b and c on the glaze disruption's repair duration per unit time period td_g, $g = 1, \ldots, 8$. It can be

Table 2 Jobs' sequence, completion times and $C_{\max}$ when $a = 1$, $b = 2$, $c = 3$ and $d = 4$

Machine	Jobs' sequence										
M_1	20	21	11	22	12	25	5	9	4	23	13	6
M_2	27	17	7	24	28	18	3					
M_3	30	10	1	2	15	29	19	14	8	26	16	
Machine	**Completion times**											
M_1	18.0	31.5	45.0	48.0	51.0	54.0	57.0	58.5	69.0	76.5	84.0	88.5
M_2	16.0	32.0	48.0	58.5	68.5	78.5	86.0					
M_3	18.0	36.0	49.5	52.5	55.5	57.0	58.5	69.0	79.0	83.5	88.0	

$C_{\max} = 88.5$

Table 3 Jobs' sequence, completion times and $C_{\max}$ when $a = 1$, $b = 2$, $c = 4$ and $d = 5$

Machine	Jobs' sequence										
M_1	20	21	11	25	15	22	2	19	18	14	3
M_2	27	17	7	28	8	4	26	16			
M_3	30	10	1	5	12	29	9	24	23	13	6
Machine	**Completion times**										
M_1	21.0	35.6	50.2	53.5	56.7	60.0	63.2	64.9	76.4	87.7	95.9
M_2	18.0	36.0	54.0	65.5	77.0	88.4	93.2	98.1			
M_3	21.0	42.0	56.6	59.9	63.1	64.7	66.4	77.7	85.9	94.0	98.9

$C_{\max} = 98.9$

Table 4 Jobs' sequence, completion times and $C_{\max}$ when $a = 1, b = 3, c = 6$ and $d = 8$

Machine	Jobs' sequence										
M_1	20	21	11	25	22	15	12	2	9	24	14	3
M_2	27	17	7	28	8	23	13	6				
M_3	30	10	1	5	29	19	18	4	26	16		
Machine	**Completion times**											
M_1	30.0	48.0	66.0	70.0	74.0	78.0	82.0	86.0	88.0	102.0	116.0	126.0
M_2	24.0	48.0	72.0	88.0	104.0	114.0	124.0	130.0				
M_3	30.0	60.0	78.0	82.0	84.0	86.0	102.0	116.0	122.0	128.0		

$C_{\max} = 130$

Table 5 Glaze disruption's repair duration per unit time

Glaze m_g g	Pattern	td_g (hours) $a = 1, b = 2,$ $c = 3$ and $d = 4$	$a = 1, b = 2,$ $c = 4$ and $d = 5$	$a = 1, b = 3,$ $c = 6$ and $d = 8$
1	Regency green	2	2.5	4
2	Greenwich	2	2.5	4
3	Marrakesh	2	2.5	4
4	Fire	2	2.5	4
5	Energy	2	2.5	4
6	Harlequin	4	5	8
7	Spirit	6	7.5	12
8	Blue Jetty	8	10	16

seen, in Table 5, that the glaze disruption's repair duration per unit time period td_g, directly depends on the uncertainty in the repair duration; the more uncertainty in the disruption repair duration, the longer the idle time to be added to the processing time.

4.1.2 The effects of increasing the disruptions duration period

The jobs' sequence, completion times and $C_{\max}$ are obtained for three different disruption periods, specified by: (1) $a = 2$, $b = 3$, $c = 4$ and $d = 5$, (2) $a = 3$, $b = 4$, $c = 5$ and $d = 6$, and (3) $a = 4$, $b = 5$, $c = 6$ and $d = 7$ and the results obtained are presented in Tables 6, 7 and 8, respectively.

If possible disruption duration periods are increased by 1 hour, i.e. the domain of the corresponding fuzzy set is 1 hour right-shifted, starting from $a = 1$ to $a = 2$, the makespan $C_{\max}$ increases from 88.5 (Table 2) to 98.9 hours (Table 6); this represents an increment of 10.4 hours (12%). Similarly, if the possible disruption periods are increased by 2 hours, i.e. a, b, c, d are changed from $a = 1$ to $a = 3$, $b = 2$ to $b = 4$, $c = 3$ to $c = 5$ and $d = 4$ to $d = 6$ the makespan $C_{\max}$ increases from 88.5 to 108.7 hours, that represents an increment of 20.2 hours (23%). If the increment is 3 hours, i.e. a is changed from $a = 1$ to $a = 4$ and all other parameters correspondingly, the makespan $C_{\max}$ increases from 88.5 to 119.1 hours, and this is an increment of 30.6 hours (35%). By comparing these results with those found when the

Table 6 Jobs' sequence, completion times and $C_{\max}$ when $a = 2$, $b = 3$, $c = 4$ and $d = 5$

Machine	Jobs' sequence									
M_1	20	21	11	25	15	22	2	19	18	14	3
M_2	27	17	7	28	8	4	26	16			
M_3	30	10	1	5	12	29	9	24	23	13	6
Machine	Completion times										
M_1	21.0	35.6	50.2	53.5	56.7	60.0	63.2	64.9	76.4	87.7	95.9
M_2	18.0	36.0	54.0	65.5	77.0	88.4	93.2	98.1			
M_3	21.0	42.0	56.6	59.9	63.1	64.7	66.4	77.7	85.9	94.0	98.9

$C_{\max} = 98.9$

Table 7 Jobs' sequence, completion times and $C_{\max}$ when $a = 3$, $b = 4$, $c = 5$ and $d = 6$

Machine	Jobs' sequence											
M_1	20	21	11	25	15	5	12	29	9	24	14	3
M_2	27	17	7	28	8	23	13	6				
M_3	30	10	1	22	2	19	18	4	26	16		
Machine	Completion times											
M_1	24.0	39.7	55.5	59.0	62.5	66.0	69.5	71.2	73.0	85.2	97.5	106.2
M_2	20.0	40.0	60.0	73.0	86.0	94.7	103.5	108.7				
M_3	24.0	48.0	63.7	67.2	70.7	72.5	85.5	97.7	103.0	108.2		

$C_{\max} = 108.7$

Table 8 Jobs' sequence, completion times and $C_{\max}$ when $a = 4$, $b = 5$, $c = 6$ and $d = 7$

Machine	Jobs' sequence												
M_1	20	21	11	25	15	5	12	29	19	18	4	26	16
M_2	27	17	7	28	24	14	3						
M_3	30	10	1	22	2	9	8	23	13	6			
Machine	Completion times												
M_1	27.0	43.9	60.8	64.5	68.3	72.0	75.8	77.6	79.5	94.0	107.1	112.7	118.4
M_2	22.0	44.0	66.0	80.5	93.6	106.7	116.1						
M_3	27.0	54.0	70.9	74.6	78.4	80.3	94.8	104.1	113.5	119.1			

$C_{\max} = 119.1$

distance between b and c was increased (Table 2 to Table 4), it appears that the impact of increasing the uncertainty in the disruption duration is higher than the impact of increasing the possible disruptions durations. It can also be seen that the jobs' sequence and completion times when $a = 1$, $b = 2$, $c = 4$ and $d = 5$ are the same as for $a = 2$, $b = 3$, $c = 4$ and $d = 5$.

Table 9 Glaze disruption's repair duration per unit time

Glaze m_g	Pattern	td_g (hours)		
		$a=2, b=3,$ $c=4$ and $d=5$	$a=3, b=4,$ $c=5$ and $d=6$	$a=4, b=5,$ $c=6$ and $d=7$
1	Regency green	2.5	3	3.5
2	Greenwich	2.5	3	3.5
3	Marrakesh	2.5	3	3.5
4	Fire	2.5	3	3.5
5	Energy	2.5	3	3.5
6	Harlequin	5	6	7
7	Spirit	7.5	9	10.5
8	Blue Jetty	10	12	14

Table 10 Glazes considered

Glaze m_g g	Pattern	a	b	c	d	td_g (hours)
1	Regency green	2	5	7	10	9
2	Greenwich	1	2	3	4	4.5
3	Marrakesh	5	8	13	16	6.5
4	Fire	4	8	14	18	10
5	Energy	1	3	6	9	3
6	Harlequin	3	6	11	13	6
7	Spirit	8	12	16	20	9
8	Blue Jetty	1	2	5	6	12

It may be of interest to analyse the effects of increasing the possible disruption duration periods on the glazes' disruption's repair duration per unit time period td_g (see Table 9). If Table 5 and Table 9 are compared, one can see that the impact of increasing uncertainty in disruption duration period, i.e. increasing the distance between b and c is higher than the impact of increasing the possible disruption duration. In other words, increments in uncertainty in the disruption duration periods may cause more changes in the idle times to be added to the processing times than increments in the disruption duration.

4.2 Analysis of rescheduling impact

Two rescheduling methods are proposed including, Left-shift rescheduling and Building new schedules. The aim of this analysis is to determine which of these methods yields better results in terms of the two objectives considered, namely the efficiency and the stability.

In order to analyse the impact of rescheduling a scenario where high impact disruptions occur is considered. It is assumed that the section leader detects a bad quality in the Greenwich glaze at the moment of producing the first job that involves this glaze. However, the glaze production department is not able to deliver the glaze in an assumed time, and therefore it is necessary to remove all the jobs that use this glaze. Table 10 shows different glazes m_g, $g=1,\dots,8$ considered in this problem, as well as the values of the parameters a, b, c,

Table 11 Predictive schedule *PS*

Machine	Jobs' sequence									
M_1	11	30	20	25	5	12*	29	9	23	3	8
M_2	24	14	4	27	17	13	28	26	6		
M_3	21	1	10	15	22*	2*	19	7	18	16	
Machine	**Completion times**										
M_1	29.2	53.2	77.2	84.2	91.2	95.5*	99.0	102.5	115.6	128.7	141.7
M_2	22.7	45.5	68.2	88.2	108.2	121.4	134.4	139.6	144.9		
M_3	29.2	58.5	82.5	89.5	93.7*	98.0*	101.5	121.5	134.5	139.7	

$C_{\max} = 144.9$

*Jobs that use Greenwich glaze

Table 12 Reactive schedule *RS* obtained using Left-shifting rescheduling

Machine	Jobs' sequence									
M_1	11	30	20	25	5	29	9	23	3	8
M_2	24	14	4	27	17	13	28	26	6	
M_3	21	1	10	15	19	7	18	16		
Machine	**Completion times**									
M_1	29.2	53.2	77.2	84.2	91.2	94.7	98.2	111.3	124.4	137.4
M_2	22.7	45.5	68.2	88.2	108.2	121.4	134.4	139.6	144.9	
M_3	29.2	58.5	82.5	89.5	93	113	126	131.2		

$C_{\max} = 144.9$

and d that describe the repair duration. As described in Sect. 3, the disruption's repair time per unit time period td_g is calculated for each glaze. The jobs' sequence and completion times are obtained as presented in Table 11.

The jobs that use Greenwich glaze are J_2, J_{12} and J_{22}. In terms of starting times, J_2, J_{12} and J_{22} start processing at 93.7, 91.2 and 89.5 hours respectively. These jobs are cut off from the schedule. When Left-shifting rescheduling is applied, after the jobs are cut off, the machines where they are expected to be processed (M_1 and M_3) are treated as single machines. The schedule obtained after applying the Left-shifting rescheduling method is presented in Table 12.

As it can be seen, in this case the makespan was not affected since it was determined by machine M_2 and this machine was not affected by the disruption. In this case, the efficiency objective has not been affected.

The instability of the schedule is measured using formula (2) as $IST(RS) = \sum_{j=1}^{30} |C_j(PS) - C_j(RS)| = 342.7$.

Since jobs J_2, J_{12} and J_{22} are cancelled in the reactive schedule RS, in order to calculate $IST(RS)$, their completion times are considered to be equal to zero, i.e. $C_2(RS) = C_{12}(RS) = C_{22}(RS) = 0$.

Table 13 Reactive schedule *RS* obtained using Building new schedules method

Machine	Jobs' sequence								
M_1	11	30	20	25	5	19	7	28	26	16
M_2	24	14	4	27	17	13	18	6		
M_3	21	1	10	15	29	9	23	3	8	
Machine	Completion times									
M_1	29.2	53.2	77.2	84.2	91.2	94.7	114.7	127.7	132.9	138.2
M_2	22.7	45.5	68.2	88.2	108.2	121.3	134.3	139.6		
M_3	29.2	58.5	82.5	89.5	93	96.5	109.6	122.7	135.7	

$C_{\max} = 139.6$

Table 14 Added jobs

Job J_j	Pattern	Item	Unit time (hours)	Initial processing times p_k (hours)	M_1	M_2	M_3
31	Regency green	Large Jug	4	8	$\surd$	$\times$	$\surd$
32	Marrakesh	Sauce Boat	6	6	$\surd$	$\surd$	$\surd$

When the Building new schedules method is applied, the new schedule is generated with all the jobs that have not been processed yet, considering the moment of disruption to be the earliest starting time of all the affected jobs. In the predictive schedule *PS* (Table 11), the first job affected by the disruption is job J_{22} which starts processing at 89.5 hours, on machine M_3. Jobs J_5 and J_{17} have started being processed and have a completion time of 91.2 and 108.2 hours on machines M_1 and M_2, respectively. Therefore, the starting time for the new schedule to be built is 91.2 hours for machine M_1, 108.2 hours for machine M_2 and 89.5 for machine M_3. Table 13 shows the schedule obtained after applying the Building new schedules method.

In this case the makespan, $C_{\max} = 139.6$, is smaller than the one found by the Left-shifting rescheduling method, $C_{\max} = 144.9$. This occurs because the Building new schedule method applies the rules LFJ and LPT which minimise the makespan, while the Left-shifting rescheduling method does not consider the makespan at all. Additionally, in order to determine which method generates a more stable schedule, the instability of the new schedule is calculated; $IST(RS) = \sum_{j=1}^{30} |C_j(PS) - C_j(RS)| = 351.3$.

It can be seen that the rescheduling method that performs better for the efficiency objective (makespan) is the Building new schedules method, while the Left-shifting method might perform better for the stability/instability objective. It may be concluded that selection of the rescheduling method to be applied depends on which of the objectives considered is more important, the efficiency or stability/instability.

Another issue worth considering is the addition of new jobs. It is often the case in practice that, when a disruption occurs, the production planning department decides to produce some new jobs since some initially planned jobs cannot be produced. In this scenario, for illustrative purposes, two new jobs are considered with the data given in Table 14.

In the Left-shifting rescheduling method, the new jobs can only be added after all the jobs scheduled initially have been completed. Table 15 shows reactive schedule *RS* with two

Table 15 Reactive schedule *RS* obtained using Left-shifting rescheduling with two new jobs J_{31} and J_{32}

Machine	Jobs' sequence									
M_1	11	30	20	25	5	29	9	23	3	8	32
M_2	24	14	4	27	17	13	28	26	6		
M_3	21	1	10	15	19	7	18	16	31		
Machine	**Completion times**										
M_1	29.2	53.2	77.2	84.2	91.2	94.7	98.2	111.3	124.4	137.4	144.9
M_2	22.7	45.5	68.2	88.2	108.2	121.4	134.4	139.6	144.9		
M_3	29.2	58.5	82.5	89.5	93	113	126	131.2	149.2		

$C_{\max} = 149.2$

Table 16 Reactive schedule *RS* obtained using Building new schedules with two new jobs J_{31} and J_{32}

Machine	Jobs' sequence										
M_1	11	30	20	25	5	29	19	9	7	13	18	6
M_2	24	14	4	27	17	32	3	8				
M_3	21	1	10	15	31	23	28	26	16			
Machine	**Completion times**											
M_1	29.2	53.2	77.2	84.2	91.2	94.7	98.2	101.7	121.7	134.8	147.8	153.1
M_2	22.7	45.5	68.2	88.2	108.2	124	137.1	150.1				
M_3	29.2	58.5	82.5	89.5	115.5	128.6	141.6	146.9	152.1			

$C_{\max} = 153.1$

new jobs J_{31} and J_{32}, generated using the dispatching rules LFJ and LPT. The instability is calculated to be $IST(RS) = 636.8$. Since jobs J_{31} and J_{32} do not exist in the predictive schedule *PS*, in order to calculate $IST(RS)$ the completion times are considered to be equal to zero, i.e. $C_{31}(PS) = C_{32}(PS) = 0$.

When the Building new schedules method is applied, the new schedule is generated for all the jobs that have not been processed yet, including the two new jobs J_{31} and J_{32}, and considering, as the jobs starting time, the moment of disruption. Table 16 shows the schedule obtained after applying the Building new schedules method including the two new jobs. Makespan of the schedule $C_{\max}$ is 153.1, while instability is $IST(RS) = 626.9$.

It can be seen that in this scenario, when the two jobs are added, the rescheduling method that yields better results for the efficiency objective is the Left-shifting rescheduling, while for the instability objective it is the Building new schedules method. Hence, the analyses carried out show the importance of selecting the appropriate rescheduling method taking into consideration both the decision maker requirements and preferences, and the objectives to be optimised.

5 Conclusion

A predictive-reactive approach to an identical parallel machine scheduling problem in the presence of uncertain disruptions is presented. This approach is applied to a real life-scheduling problem identified in collaboration with a manufacturing pottery company. The use of fuzzy sets in modelling material shortage disruptions proved to be beneficial in the case when there are no historical data and subjective managerial judgement can be used. Two rescheduling methods namely Left-shifting and Building new schedules have been applied when high impact disruptions occur.

The results obtained show that uncertainty in disruptions caused by a glaze shortage or low quality of the glaze may have a higher impact on the schedule's execution than the repair time, i.e. the time needed for the glaze production department to deliver the glaze. Importance of selecting the appropriate rescheduling method, with respect to the decision maker requirements and preferences and the objectives to be optimised is demonstrated. The results obtained using the rescheduling methods are satisfactory. The developed model is flexible and can also be used when new jobs arrive.

Further work will be undertaken including: (a) investigation of different techniques to measure level of uncertainty, represented by fuzzy sets, in order to quantify the impact of uncertain disruptions on the schedule performance, (b) analysis of a large number of test cases in order to systematically quantify trade-offs between the two rescheduling methods, (c) development of a simulation model of realised schedules in order to measure the level of predictability of the predictive schedule, i.e. the quality of the schedule in terms of absorbing the effects of a disruption, and (d) analysis of possible improvements in scheduling that might be gained at the pottery company by using the method proposed.

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Appendix

Definition of a fuzzy set (Zadeh 1965) Let $\boldsymbol{X}$ denotes a universal set with elements denoted as x. A fuzzy set A in $\boldsymbol{X}$ is characterised by a membership function $\mu_A(x)$, where $\mu_A(x) : \boldsymbol{X} \to [0, 1]$ associates each element x with a degree of membership of x in A. In the case when x is discrete and finite $x = \{x_1, x_2, \dots, x_k\}$, fuzzy set A may be denoted by $A = \sum_{k=1}^{K} \mu_A(x_k)/x_k$.

Definition of a possibility distribution function Let X be a variable which takes values in a universal set $\boldsymbol{X}$ and let A be a fuzzy set of $\boldsymbol{X}$ with a membership function $\mu_A(x)$, interpreted as the compatibility of x with the concept represented by A. Fuzzy set A induces a possibility distribution function, denoted by π_X, which is defined to be numerically equal to μ_A, i.e. $\pi_X = \mu_A$. In other words, $\pi_X(x)$ represents the possibility that X takes value x and it is equal to membership degree $\mu_A(x)$.

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