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An integrated staff-sizing approach considering feasibility of scheduling decision

Yongjian Li · Jian Chen · Xiaoqiang Cai

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Abstract This paper presents an integrated staff-sizing system for analyzing and determining workforce management policies with consideration of staff flexibility in service organizations, which addresses and captures the integrated requirements between long-term manpower planning and short-term staff scheduling in the service sector. Multiple Objective Linear Programming (MOLP) is applied to optimize several diversified goals. Solution methods to the MOLP models for the staff planning and staff scheduling are developed respectively, then a solution approach is proposed to iteratively revise the unacceptable staff-sizing plan or scheduling plan. Finally, an example of nurse sizing is analyzed and computational studies are carried out to investigate managerial insights.

Keywords Staff-sizing · Planning · Scheduling · Staff flexibility · Multiple objective linear programming (MOLP)

Y. Li (✉)
Business School, Nankai University, Tianjin 300071, People's Republic of China
e-mail: liyongjian@nankai.edu.cn

J. Chen
College of Economics and Management, Tsinghua University, Beijing 100084, People's Republic of China
e-mail: chenj@sem.tsinghua.edu.cn

X. Cai
Department of System Engineering & Engineering Management, The Chinese University of Hong Kong, Shatin N.T., Hong Kong
e-mail: xqcai@se.cuhk.edu.hk

X. Cai
College of Information Technology Science, Nankai University, Tianjin 300071, People's Republic of China

1 Introduction

We propose an integrated approach to analyze and determine the manpower planning and scheduling decisions for service organizations. As it is well known, it is both important and difficult to allocate available staff to meet the service demands efficiently, particularly when the demands from customers (e.g., in hospitals, bank, post office, et al.) cannot be backlogged and must be satisfied promptly. This staffing problem is different from the workforce and production-planning problem found in production organizations (see, for example, Taubert 1968; Samuel and Reutzel 1981; Gen et al. 1992; Cheng et al. 2001). Production models rely upon the inventory and backorder capabilities. Furthermore, they typically exclude other considerations that must be included in an effective staff planning and scheduling models for service-sector applications. For example, they normally do not consider explicitly matching specific labor skills with job requirements at individual work centers. Yet this careful matching is important in organizations like hospitals, schools and banks, where the 'quality' of the service is crucial, personnel are highly specialized, and post *hoc* quality adjustment is not possible.

Two levels of staff planning decisions usually occur in the process of personnel management. One is *manpower planning*, which determines the number of employees needed for the planning period within budgetary constraints. The other is *staff assignment and scheduling*, which is conducted weekly or monthly within the boundary of the number of staff defined at the planning level. It has been argued that significant overstaffing often occurs in service organizations as a direct result of separate and independent decisions between long-term manpower planning and short-term staff assignment and scheduling. For example, in the nurse workforce problem, the planned staff capacity for each ward in some hospitals is held constant for the entire year at a level sufficient to meet the peak demand expected during the year, which may result in substantial under-utilization of personnel.

There are some published works describing the use of optimization techniques to tackle the manpower planning (staff-sizing) problem. Some of the previous results have been reviewed in (Bowey 1977; Price et al. 1980; Purkiss 1981; Edwards 1983; Ernst et al. 2004). The models for manpower planning can be classified into two categories (Purkiss 1981). The first is *exploratory models*, which can give the managers insights into the way his/her manpower system works and the way it would respond to different stimuli. These exploratory models range from the very simple ones that are applicable to almost every organization (though often dealing with one special feature such as career progression) to very comprehensive stochastic simulation models for examining individual movement in the manpower system (see for example, Zanakis and Maret 1981; McClean 1991; Georgiou and Vassiliou 1997). The second is the very powerful *normative models*, which can compute an optimal set of personnel decisions (on recruitment, promotion, training, etc.) against goals stated in some forms of objective function. A number of early attempts using linear programming have been described in Smith (1971). Within this general schema, practices vary. For instance, one important distinction is the way that movement between jobs is modelled (see for example, Price et al. 1980; Silverman et al. 1988). One class of problems is recruitment (see for example, Bres et al. 1980; Rao 1990; Gans et al. 2003; Cai et al. 2004; Li et al. 2005) with the selection of a recruitment schedule over multiple time periods, which best meets the goal pertaining to promotion opportunity, salary expenditure, desired levels of experience in the workforce and requirements for manpower in each planning period.

The staff scheduling problem has received more attention than staff-sizing; The latest annotated bibliography of personnel scheduling and rostering can be found in (Alfares 2004; Ernst et al. 2004). The studies in the staff scheduling problem have generally assumed that the workforce size and composition are predetermined and the staff-scheduling problem is defined in a very limited planning horizon (e.g., one week or one month). These assumptions ignore, however, the need to consider and address the changes in workforce size to respond to the dynamic fluctuations of the staff demands (Henderson et al. 1982).

While a significant amount of research has focused upon the staff-sizing and staff scheduling decisions individually, very little has considered the interaction between the two decisions. Perhaps this is due to the fact that staff-sizing decisions are often made by the manager at higher levels in an organization than scheduling decisions, and in a much longer planning time horizon than staff scheduling. Despite the difference in time horizon and decision level, staff-sizing and scheduling are clearly interdependent and should be examined as an integrated system.

A common problem associated with the integration of staff-sizing and scheduling decisions is that strict enforcement of aggregate staff-sizing decisions may lead to infeasibility of the scheduling policies (Venkataraman and Brusco 1996). This suggests the need for a recursive approach that enables the impact of the staff planning decisions made at the staff-sizing phase to be rapidly evaluated in a scheduling context. Abernathy et al. (1973) have described a recursive approach that begins with a simulation model for generating staffing parameters for nursing units. A chance-constrained staffing model was subsequently used to actually set the staffing levels. Venkataraman and Brusco (1996) have presented an integrated nurse staffing and scheduling system, where the staffing and scheduling models were described as a mixed-integer programs, respectively.

In the studies reviewed above, only a single objective concerning the cost incurred during the entire planning horizon was dealt with. However, service organizations are concerned generally about two types of objectives: service quality and cost. Service quality is represented through many indices, e.g., minimizing the quantity of employees augmented, minimizing the overtime incurred, ensuring the professional development of every employees, etc. So multi-objective planning problem needs to be considered to reflect the goals of service organizations. On the other hand, only one type of staff has been considered in the above references. Generally, there are, however, several types of staff existing in an organization, among which substitution may arise so as to reduce the manpower-related expenditure. So staff flexibility needs also to be considered.

The main purpose of this article is to develop an integrated approach for analyzing and determining workforce management policies in service organizations with consideration of staff flexibility and feasibility of the corresponding scheduling decisions. It addresses and captures the integrated requirements between long-term staff-sizing and short-term staff scheduling in the service sector. Multiple Objective Linear Programming (MOLP) approaches are applied to optimize several diversified goals in the staff-planning and the staff-scheduling problems respectively, which handle the conflicting objectives of costs and service levels. Our MOLP approaches examine the effects of staff flexibility on staff-sizing decisions at the aggregate and disaggregate levels respectively. An iterative procedure is proposed to iteratively improve the solutions generated by the MOLP approaches for the planning and scheduling problems.

The organization of this paper is as follows. In Sect. 2, we describe the general staff planning and scheduling system. In Sects. 3 and 4, we develop MOLP models to formulate the staff planning and scheduling problems, respectively. In Sect. 5, we propose our solution approach to compute the solutions for the planning and scheduling models. A recursive procedure is also developed to iteratively improve the planning and scheduling solutions. Numerical analysis is provided in Sect. 6. Section 7 gives a summary and some future research topics.

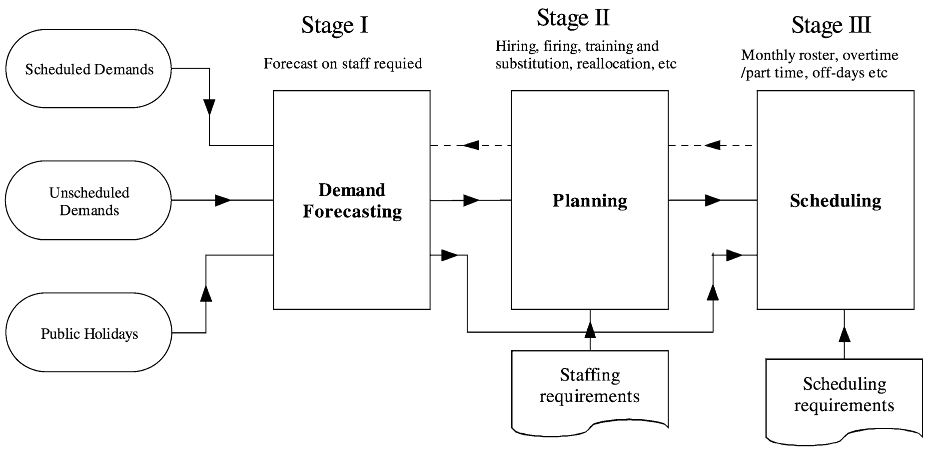


Fig. 1 An integrated staff decision model

2 Integrated staffing decision model

The general framework of an integrated staffing decision system is illustrated in Fig.1. We divide it into three stages (modules): demand forecasting, staff planning and staff scheduling.

In stage I, there are, in general, two kinds of manpower demands encountered by a service organization. One is the known demand—customers who make appointments with the service provider in advance. The other is the unknown demand—customers who walk in without appointments. In a real situation, demand uncertainty is inevitable due to causes such as customers who come without appointments or customers who do not show up after making appointments. The outputs in stage I are usually aggregate forecasts of monthly demand, which are the input to the staff planning model in stage II. These forecasts can subsequently be disaggregated to generate shorter-term forecasts for the staff scheduling problem in stage III.

Stage II concerns staff planning at the macro level, which generates the desirable size of the workforce for each skill class across the entire planning horizon. In addition to the forecast demands, the inputs to the staff planning model also include the staff planning requirements. Planning requirements are specified in terms of labor hours required in each period, as well as overtime policies, substitution policies, and training policies, etc., which will be presented in details in Sect. 3. The outputs of the staff planning model are the number of employees, the number of those being recruited or dismissed, the substitution number of higher-level staff to lower-level one and the amount of overtime hours during each period.

The outputs from the staff planning model comprise the inputs to staff scheduling in stage III. Specifically, the information extracted from the staff plan is the number of employees who are assigned to every demand type. In addition, the inputs to staff scheduling also include staff scheduling requirements, e.g., overtime policy, temporary employment policy and day-off policy, etc.

The existence of an effective coordination between the different subproblems (stages) is essential to the success of the integrated system. In this paper, we will mainly address the planning and scheduling problems, under the assumption that the manpower demands have been given by the forecast module. We will formulate the planning and scheduling models as multiple objective linear programs in the following two sections, respectively.

3 Staff planning model

The typical questions that will be asked in a service organization include: Do we have enough qualified staff to serve the customers? Is the system flexible enough to meet demand fluctuation? We will study these problems using a *multiple objectives staff planning model*, which considers a number of linear objectives and linear constraints, including staff task flexibility, staff development requirements, staff level change constraints, etc.

3.1 Notation

The variables and parameters used in the staff planning model are listed in the following.

Parameters

- T The number of planning periods under consideration. The length of each period may be a month or a year, depending on the application.
- n The number of job types and categories of full-time staff.
- D_{it}^1 The projected regular demand in staff hours for type-i job from customers who have made appointments beforehand during the period t.
- D_{it}^2 The projected irregular demand in staff hours for type-i job from customers who have made no appointments beforehand during the period t.
- H_i The number of regular hours available of a category-i employee each period, excluding the personnel time and nonbillable duties.
- f_i The maximal proportion of type-i jobs that can be performed by the employees at higher categories.
- o_i The maximum allowable overtime for a category-i employee as a proportion of regular hours.
- α_i The base salary per category-i employee.
- e_i The overtime cost per hour per category-i employee.
- β_i^+/β_i^- The recruitment/dismissal costs per category-i employee.
- s_{ji} $= 1$, if category-i staff can be substituted by category-j staff ($i \le j$); $= 0$, otherwise. Especially, $s_{ii} = 1$ means that the category-i staff can be assigned to do their own job.
- Q_i The target professional development hours over the planning horizon for category-i staff.
- S_i^- The shortfall in the achievement of the professional development target.

Variables

- X_{it} The number of category-i employees to be available during period t. X_{i0} is the initial value.
- u_{it}/v_{it} The number of category-i employees to be recruited/dismissed at the end of period t.
- Y_{jit} The number of category-j employees being assigned to do type-i job (category-j employees substitute category-i ones) during period t ($i \le j$). Especially, Y_{iit} is the number of category-i employees being assigned to do their own job.
- O_{it} The overtime hours of category-i employees during period t.
- E_{it} The actual number of category-i employees for professional development during period t.

3.2 Constraints

There are n types of jobs: type-i, $i = 1, 2, \ldots, n$, requiring pre-specified manpower resources respectively during period t, $t = 1, 2, \ldots, T$. Correspondingly, there are n categories of full-time staff: category-i, $i = 1, 2, \ldots, n$, available in the organization, where category-i employees can first be assigned to a type-i job ($i = 1, 2, \ldots, n$). Moreover, different categories of employees hold different skills, which can be assigned to a lower-level of job when needed. For example, category-1 employees have skill 1 and can be assigned to a type-1 job, while category-2 employees have better skills, then they can be assigned not only to a type-2 job, but also to a type-1 job (to substitute category-1 employees). In general, we assume that category-i employees have better skills than category-j employees for $i > j$.

The costs for using different categories of employees are different, and usually the salary for category-i employee is more expensive than that for category-j employee when $i > j$.

Constraints that must be satisfied in service staff planning include:

(1) *Staff supply-demand constraints*

An initial number of employees at each category is assumed. Hiring/firing decisions should be made to ensure that the manpower supply is always greater than or equal to the manpower demand. The manpower supply-demand constraints are given by the following dynamic equations:

$$X_{i(t+1)} = X_{it} + u_{it} - v_{it}, \quad \text{for } t = 0, 1, 2, \ldots, T-1, \forall i. \tag{1}$$

(2) *Staff utilization constraints*

Each staff member should be given a certain amount of regular work time each period, which should include allowances for personnel times (e.g., illness) and for nonbillable duties (e.g., administration, recruiting, practice development). In our problem, we have supposed that jobs can be done not only by staff of the scheduled category but also by some higher categories; other substitutions are not allowed. Over time may also be required to meet the demands.

$$\sum_{j \ge i} s_{ji} H_j Y_{jit} + O_{it} \ge D_{it} \quad \text{for } \forall i; t, \tag{2}$$

where $D_{it} = D_{it}^1 + D_{it}^2$ is the total demand of type-i job during period t that is the sum of the projected regular-demands and irregular-demands, where the irregular-demands are to be used to model the irregular manpower requirements due to public holidays, off-days, extra manpower requirements that may be introduced by multiple shifts, etc.

(3) *Staff gross constraints*

The number of category-i employees that can be assigned to every type-j job ($j \le i$) is limited to the total available number of employees at category-i, excluding those employees attending professional development.

$$\sum_{j \le i} Y_{ijt} + E_{it} = X_{it} \quad \text{for } \forall i; t. \tag{3}$$

(4) *Staff task flexibility constraints*

There is a limit on task flexibility and a limit on the amount of type-i job that can be performed by substitution.

$$H_i Y_{iit} \ge (1 - f_i) D_{it}, \quad \text{for } \forall i; t. \tag{4}$$

If the task substitution level were too high, the management would recruit some number of category-i employees for the type-i job instead of using substitution.

(5) *Overtime constraints*

The amount of overtime hours an employee can take has an upper limit.

$$0 \le O_{it} \le o_i H_i X_{it}, \quad \text{for } \forall i; t. \tag{5}$$

(6) *Professional development constraints*

Some employment time must be dedicated to professional development and similar activities, e.g., on-the-job training, seminars, conferences, etc. These requirements may vary by staff category.

$$\sum_t E_{it} H_i + S_i^- = Q_i \sum_t X_{it}, \quad \forall i. \tag{6}$$

Usually, an organization may have some targets for professional development. Meeting professional development targets may be formulated as an objective to be optimized.

If the professional development for category-i staff is constrained in one or several given periods, then constraints are similar to (6) above.

3.3 Objectives

Various objectives may be established to reflect the needs and goals of the organization. Generally, service organizations are concerned about two types of objectives: service quality and cost. For example, in the health service sector, the quality of service largely relies on proper assignment of the professional staff with the designated knowledge and skills, while the cost of service is heavily depending on the staff-sizing solution because 60 to 80% of budget of health service is occupied by staff cost (McConnell 2000). We consider the following five objectives:

(1) *Staffing cost*

$$\text{Min} \quad z_1 = \sum_{i=1}^{n} \left[\sum_{t=1}^{T} (\alpha_i X_{it} + e_i O_{it}) + \sum_{t=0}^{T-1} (\beta_i^+ u_{it} + \beta_i^- v_{it}) \right]. \tag{7}$$

This goal seeks to minimize the total cost arising from acquiring /maintaining the necessary level of staff supply to meet the demand.

(2) *Staff augmentation*

$$\text{Min} \quad z_2 = \sum_{i=1}^{n} \sum_{t=0}^{T-1} u_{it}. \tag{8}$$

The objective seeks to minimize the number of new employees recruited during the planning horizon. This criterion is particularly important for service organizations, because new employees are generally perceived as lack of the needed experience to provide high-level service.

(3) *Staff task substitution*

$$\text{Min} \quad z_3 = \sum_{i=1}^{n} \sum_{t=1}^{T} \left\{ \sum_{i<j} Y_{jit} \right\}. \tag{9}$$

The objective seeks to minimize the number of employees doing lower-level jobs. While substitution is an effective means to tackle the problem of temporary and unpredictable staff shortage in certain categories, it is generally regarded as an expensive solution because higher-level employees inevitably carry higher costs.

(4) *Overtime incurred*

$$\text{Min} \quad z_4 = \sum_{i=1}^{n} \sum_{t=1}^{T} O_{it}. \tag{10}$$

Overtime is undesirable because prolonged working time could cause fatigue and deficiency in service quality. In addition, there are also regulatory limits on the amount of overtime that an employee can be assigned to take.

(5) *Shortfall of professional development*

$$\text{Min} \quad z_5 = \sum_{i=1}^{n} S_i^-. \tag{11}$$

The objective seeks to minimize the unachieved professional development time during the planning horizon.

4 Staff scheduling model

Staff scheduling is usually conducted for a short period, e.g., one week or one month. An employee works on a shift on a workday, and can receive F off-days during the scheduling period. Labor demands are to be satisfied by regular time, overtime and temporary staff. In general, overtime is priced more than the wage rate for a same staff category and temporary staff (e.g., supplemental staff contracted through an outside agency) is assumed to be a last resort and is generally priced more than the corresponding overtime cost.

4.1 Notation

The variables and parameters used in the staff scheduling model are listed below.

Parameters

- S The number of days during a scheduling period under consideration.
- d_{it} The number of working hours of type-i job required on day t.
- h_i The number of regular hours per category-i employee per day.
- w_i The number of category-i employees who can be scheduled during the scheduling period. For example, assume the scheduling plan is made in a period s, then $w_i = X_{is} - E_{is}$ is the difference between states X_{is} and E_{is} obtained from the staff-planning model.
- d_{it}^+ The number of surplus category-i employees, who are unassigned to do any job, and not on leave, on day t.
- δ_i The maximum allowable rate of overtime for category-i staff as a proportion of regular hours during a period.
- F The number of off-days that an employee can receive during a scheduling period.
- c_i^o Hourly cost of overtime of category-i staff.
- c_i^a Salary of a category-i temporary employee per day.

Variables

$\bar{x}_{jit}$ The number of category-j employees who are assigned to do type-i job on day t.
T_{it} The amount of overtime hours provided on day t by the category-i staff.
A_{it} The number of temporary employees to be assigned to the type-i job on day t.
y_{it} The number of category-i employees who take day t off.

4.2 Constraints

The constraints in the scheduling stage can be classified into two categories (Miller et al. 1976): (i) *Feasibility set constraints, or hard constraints* (e.g., demand constraint, overtime constraint, day-off constraint, etc.), which define the sets of feasible staff schedules that must be satisfied; (ii) *Nonbinding constraints, or soft constraints* (e.g. staff preferences, preferential scheduling patterns, etc.), whose violation incurs certain degree of un-satisfaction of the staff concerned, but the solution is still usable by the organization. In our system, we only take into account the feasibility set constraints, and we suggest that the soft constraints be handled by managers using their experience and judgment, based on the actual situations and conditions.

(1) *Demand constraints*

$$\sum_{j\geq i} s_{ji}\bar{x}_{jit}h_j + T_{it} + A_{it}h_i \geq d_{it}, \quad \forall i, \forall t. \tag{12}$$

The requirements of type-i job at any time must be guaranteed and can be satisfied by category-i employees, category-j employees that satisfies $s_{ji} = 1$, overtime of category-i employees and temporary staff, with consideration of day-off for the full-time staff.

The number of full-time employees who are assigned each day is constrained by the number of employees during this scheduling period obtained from the staff planning model. This is the following constraint.

$$\sum_{i\leq j} s_{ji}\bar{x}_{jit} + y_{jt} + d_{jt}^{+} = w_j, \quad \forall j, \forall t. \tag{13}$$

(2) *Overtime constraints*

$$0 \leq T_{it} \leq \delta_i h_i w_i, \quad \forall i, \forall t. \tag{14}$$

It limits the use of overtime on each day during the scheduling period. This limit could create infeasibility if only regular time and overtime hours are permitted. To avoid potential infeasibility, temporary staff may have to be used to satisfy any excess demands. The number, A_{it}, of the temporary staff included in constraint (12) actually reflects this requirement.

(3) *Off-day constraints*

$$\sum_{t} y_{it} \geq F w_i, \quad \forall i. \tag{15}$$

Each full-time employee will receive at least F off-days during a scheduling period. We assume that overtime comes from extending the regular working time. Thus, overtime does not affect the day-off requirement. However, if overtime comes from working on a day-off, this constraint should be modified accordingly.

Remarks (1) Note that the scheduling model formulated here will determine the number of regular and temporary staff members assigned to each type of job on each day, the amount of overtime for each category of staff on each day given the workforce constraint imposed by the planning model, and also the day-off for each category of staff. It has not, however, addressed the issue of staff assignment down to the level of shifts on each day. After the daily assignment is determined that can meet the daily manpower demand, one may further divide the staff into shifts, based on the manpower demands and other practical constraints on the shifts. Note also that, when multiple shifts exist, the required number of employees is often increased. To address this issue, certain tolerance of manpower should be added to the manpower demand. In our model, this can be incorporated into the manpower demand D^2_{it}; see the definition below constraint (2).

(2) The solution that satisfies the hard constraints will be regarded as a feasible one. As we indicated above, there are also nonbinding constraints that exist in practice, which include certain desirable work patterns, and personnel preferences (e.g., weekend-off requests, minimum workstretch, etc., cf. Billionnet 1999; Uebe et al. 1990; Topaloglu and Ozkarahan 2004). It is not compulsory for an organization to satisfy every soft constraint. Instead, soft constraints are generally handled by management at the relevant levels, according to the situations and urgency/importance of the requests.

4.3 Objectives

We consider the following scheduling objectives: The objective of minimizing overtime and temporary staff costs subject to all scheduling constraints; and the objective of maximizing the surplus staff, in order to reduce the risk of underestimation of actual demands that may occur due to inaccurate forecast; see Cai and Li (2000).

(1) *Minimization of overtime and temporary staff cost*

$$\text{Min} \quad J_1 = \sum_i \sum_t (c_i^o T_{it} + c_i^a A_{it}). \tag{16}$$

(2) *Maximization of surplus staff*

$$\text{Max} \quad J_2 = \sum_i \sum_t d_{it}^+. \tag{17}$$

5 Solution techniques

5.1 Solving the staff planning model

We first convert the MOLP model as given in Sect. 3 into one with a single objective, and then determine the solution for the single-objective problem. The solution for the single-objective problem can be found by either a standard linear integer programming algorithm, or a specific algorithm (cf. Cai et al. 2004; Li et al. 2005). See Appendix 2 for more details. Overall, our solution procedure is described as follows.

Step 1. Set weights $\lambda_i (i = 1, 2, \dots, 5)$ for each objective according to their relative importance, by an Analytic Hierarchy Process (AHP) (see Appendix 5).

Step 2. Normalize the objectives as follows:

$$\overline{z}_i = \frac{z_i - z_i^{\min}}{z_i^{\max} - z_i^{\min}}, \tag{18}$$

where $z_i^{\max}$ and $z_i^{\min}$ are, respectively, the maximum and minimum values of the i-th objective z_i when we consider only this objective in the staff planning model. In Appendix 1 we give simple results to compute $z_i^{\max}$ and $z_i^{\min}$, which avoid the requirement to solve an optimization problem in order to obtain $z_i^{\max}$ or $z_i^{\min}$.

Step 3. Solve the following single objective problem

$$\begin{aligned} \text{IntP1:} \quad \min \quad & J = \textstyle\sum_{i=1}^{5} \lambda_i \overline{z}_i \\ \text{s.t.} \quad & \text{Constraints (1–6).} \end{aligned} \tag{19}$$

The solution approach to the model IntP1 is given in Appendix 2.

5.2 Solving the staff scheduling model

We also use a three-step approach to solve the staff scheduling problem. The procedure can be described as follows.

Step 1. Set weights λ_i $(i = 1, 2, \ldots, 5)$ for each objective according to their relative importance, by an AHP approach.

Step 2. Normalize the objectives as follows:

$$\overline{J}_1 = \frac{J_1 - J_1^{\min}}{J_1^{\max} - J_1^{\min}}, \tag{20}$$

$$\overline{J}_2 = \frac{J_2^{\max} - J_2}{J_2^{\max} - J_2^{\min}}, \tag{21}$$

where $J_i^{\max}$ and $J_i^{\min}$ are, respectively, the maximum and minimum values when we consider only the single objective J_i in the staff scheduling model. In particular, when we consider only the second objective J_2, we can show easily that $J_2^{\max} = \sum_{i=1}^{n} w_i$ and $J_2^{\min} = 0$. The computations for $J_1^{\max}$ and $J_1^{\min}$ are given in Appendix 3.

Step 3. Solve the following single-objective problem:

$$\begin{aligned} \text{IntS1:} \quad & \min J = \lambda_1 \overline{J}_1 + \lambda_2 \overline{J}_2 \\ & \qquad = \sum_{i=1}^{n} \left\{ \sum_t \overline{\lambda}_1 c_{it}^o T_{it} + c_{it}^a h_i A_{it}) - \overline{\lambda}_2 \sum_{i=1}^{n} \sum_{k \in \Theta_i} y_{ik} \right\} - C \\ & \text{s.t.} \quad \text{Constraints (12–14),} \end{aligned} \tag{22}$$

where $\overline{\lambda}_i = \frac{\lambda_i}{J_i^{\max} - J_i^{\min}}$, $i = 1, 2$ and $C = \frac{\lambda_1 J_1^{\min}}{J_1^{\max} - J_1^{\min}} - \frac{\lambda_2 J_2^{\max}}{J_2^{\max} - J_2^{\min}}$ is a constant.

The solution approach to the model IntS1 can be found by either a standard linear mixed integer programming algorithm, or a specific algorithm; See Appendix 4 for more details.

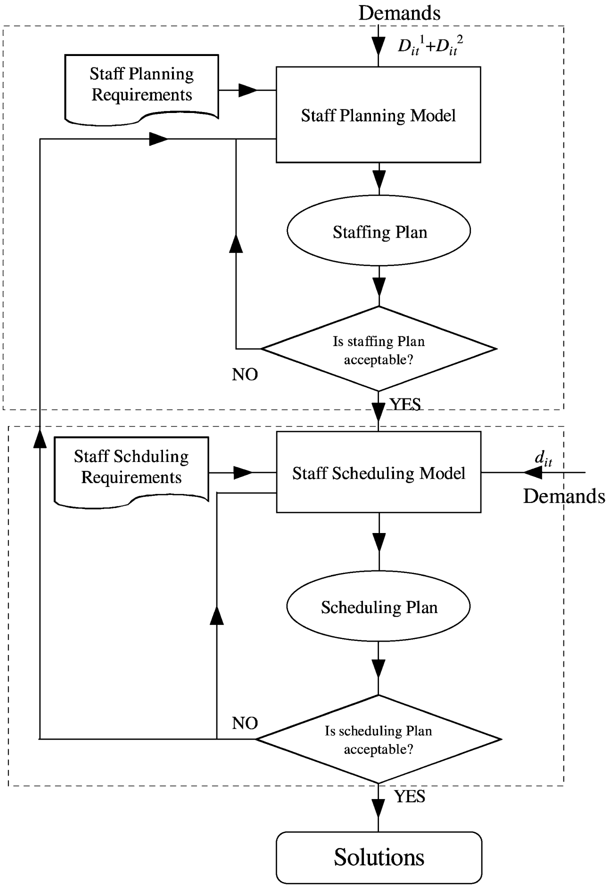


Fig. 2 The solution procedure of the integrated system

5.3 Solution procedure for the integrated model

5.3.1 The general solution framework

The general framework of the solution procedure we propose is illustrated in Fig. 2.

Basically, the solution procedure contains two stages, corresponding to the planning solution and the scheduling solution respectively. The two stages are, however, interrelated in our overall solution procedure; i.e., the solution of one stage may be fed back to revise the solution of the other stage. We have described, in the two subsections above, how to determine the planning and scheduling solutions. The remaining task now is how to revise (improve) the solutions based on the feedbacks. We propose to adjust those key weights/factors, within their practically allowable ranges, in the planning/scheduling models so as to achieve the desirable effects of solution improvements.

Table 1 lists the weights/factors that are to be adjusted in the solution procedure. Table 2 lists the revision actions we propose to address the relevant unacceptable planning/scheduling solutions. On the one hand, the planning solution (the number of staff at each category) will be direct input of the scheduling model to affect the scheduling solution. On the other hand, the scheduling solution will be checked and unacceptable solution will be fed back to the planning model so that the planning solution will be adjusted accordingly.

Table 1 Adjustable factors in the iterative solution procedure

Factor	Description
o_i	Overtime coefficient in the planning stage
δ_i	Overtime coefficient in the scheduling stage
F	Off-days factor
f_i	Substitution factor
Q_i	Professional development factor
OP_i	The weight of the i-th objective in the staff planning model
OS_i	The weight of the i-th objective in the staff scheduling model

Table 2 Adjustment schemes to revise/improve unsatisfactory solutions

Information feedback		Adjustment action
Staff planning stage	The number of overtime hours is too large. Specifically, there exists an i such that $\sum_{t=1}^{T} O_{it}/(h_i \sum_{t=1}^{T} X_{it}) \geq IF_1$, where IF_1 is the maximal ratio of total overtime allowed for category-i staff with respective to the total normal working time.	$OP_4 \uparrow$ $o_i \downarrow$
	The number of new employees being recruited is too large. Specifically, there exists an i such that $\sum_{t=1}^{T} u_{it} \geq IF_2$, where IF_2 is the maximal number of new category-i employees to be recruited.	$OP_2 \uparrow$ $o_i \uparrow$, $f_i \uparrow$
	There are too many staff task substitutions. Specifically, there exists an i such that $\sum_{t=1}^{T} \sum_{i<j} Y_{jit}/\sum_{t=1}^{T} X_{it} \geq IF_3$, where IF_3 is the maximal ratio of category-i employees who are assigned to take other jobs with respective to the total number of category-i employees.	$OP_3 \uparrow$ $f_i \downarrow$
	Unachieved professional development time is too high. Specifically, there exists an i such that $S_i^-/(Q_i \sum_{t=1}^{T} X_{it}) \geq IF_4$, where IF_4 is the maximal ratio of the unachieved professional development time with respective to the expected development time for category-i staff.	$OP_5 \uparrow$ $Q_i \uparrow$
	There are too many temporary staff. Specifically, there exists an i such that $\sum_{t=1}^{T} A_{it}/\sum_{t=1}^{T} X_{it} \geq IF_5$, where IF_5 is the maximal ratio of temporary category-i employees with respective to the total number of regular category-i employees.	$OS_1 \uparrow$, $OP_2 \downarrow$, $OP_4 \uparrow$, $\delta_i \uparrow$, $F \downarrow$, $o_i \uparrow$, $f_i \downarrow$
Staff scheduling stage	There are too many overtime hours. Specifically, there exists an i such that $\sum_{t=1}^{S} T_{it}/(Sh_i w_i) \geq IF_6$, where IF_6 is the maximal ratio of overtime of category-i staff with respective to their normal working time.	$OS_1 \uparrow$, $OP_2 \downarrow$, $OP_4 \uparrow$, $\delta_i \downarrow$, $F \downarrow$, $o_i \downarrow$, $f_i \downarrow$

$a \uparrow$ ($a \downarrow$) means that the value of a should be increased (decreased); IF_i $(i = 1, 2, \ldots, 6)$ is to be determined according to the situation under consideration

For example, if the scheduling stage generates too many temporary staff, then this implies that the size of the workforce determined by the planning stage is not sufficient (and consequently, temporary staff must be sought to meet the manpower demands). The following adjustments can be made. First, we may decrease, if possible, the weight OP_2 for the objective regarding staff augmentation. This is because if the objective of minimizing the staff augmentation is over emphasized, the quantity of new employees being recruited will be reduced and temporary staff may have to be sought in the scheduling stage to meet the unexpected fluctuation of the demands.

Secondly, we may increase the weight OP_4 for the overtime objective or decrease the value o_i of the corresponding category-i staff. This will discourage the use of overtime at the planning level and consequently increase the number of staff to be recruited.

Thirdly, we may decrease the substitution factor f_i of the corresponding category-i staff to restrict the flexibility of staff substitution. This will also increase the number of staff to be recruited at the planning level.

We propose to adopt the following methods to implement Table 2 for the adjustments of weights: OS_i and OP_i, and the parameters: o_i, δ_i, F, f_i, and Q_i.

5.3.2 Adjustment of the weights OS_i and OP_i

Recall that we use the AHP approach to set the weights of the objectives in the planning and the scheduling phases. Now, when these weights are to be adjusted, we will modify, accordingly, the corresponding components a_{ij} in the pairwise comparison matrix of the AHP approach (see Appendix 5). Specifically, if the adjustment action is $OP_i \uparrow$, then we will reset a_{ij} as follows:

For $i < j \leq 5$,

$$a_{ij} = \begin{cases} \min\{a_{ij}+1, 9\}, & \text{if } a_{ij} \geq 1, \\ \frac{a_{ij}}{1-a_{ij}}, & \text{if } a_{ij} < 1, \end{cases} \tag{23}$$

and for $1 \leq j < i$,

$$a_{ji} = \begin{cases} a_{ji}-1, & \text{if } a_{ji} > 1, \\ \max\{\frac{a_{ji}}{1+a_{ji}}, \frac{1}{9}\}, & \text{if } a_{ji} \leq 1. \end{cases} \tag{24}$$

If the adjustment action is $OP_i \downarrow$, then we will reset a_{ij} as follows:

For $i < j \leq 5$,

$$a_{ij} = \begin{cases} a_{ij}-1, & \text{if } a_{ij} > 1, \\ \max\{\frac{a_{ij}}{1+a_{ij}}, \frac{1}{9}\}, & \text{if } a_{ij} \leq 1, \end{cases} \tag{25}$$

and for $1 \leq j < i$,

$$a_{ji} = \begin{cases} \max\{a_{ji}+1, 1\}, & \text{if } a_{ji} \geq 1, \\ \max\{\frac{a_{ji}}{1-a_{ji}}, \frac{1}{9}\}, & \text{if } a_{ji} < 1. \end{cases} \tag{26}$$

If the adjustment action is $OS_1 \uparrow$, then there is only one component a_{i2} to modify (see Appendix 5: AHP):

$$a_{i2} = \begin{cases} \min\{a_{i2}+1, 9\}, & \text{if } a_{i2} \geq 1, \\ \frac{a_{i2}}{1-a_{i2}}, & \text{if } a_{i2} < 1. \end{cases} \tag{27}$$

5.3.3 Adjustment of the parameters o_i, δ_i, F, f_i, and Q_i

To adjust the parameters o_i, δ_i, F, f_i, and Q_i, we adopt the following heuristic idea.

Suppose p is the parameter to be adjusted and suppose its current value is p^k. Let $p^{\min}$ and $p^{\max}$ be its lower and upper bounds. Further, we let $\Delta p = (p^{\max} - p^{\min})/\gamma$, where $\gamma > 0$

is a scalar (e.g., $\gamma = 10$). Then, the new value for p will be computed as follows:

$$p^{k+1} = \begin{cases} p^{\min}, & \text{if } p^k \pm \Delta p \le p^{\min}, \\ p^{\max}, & \text{if } p^k \pm \Delta p \ge p^{\max}, \\ p^k \pm \Delta p, & \text{otherwise,} \end{cases} \tag{28}$$

where $+$ or $-$ will be used when the parameter is to be increased or decreased according to Table 2, respectively.

5.3.4 The algorithm

To summarize, our procedure is now described as follows.

Step 1. Compute the staff planning solution according to the procedure in Sect. 5.1.

Step 2. If the staffing plan is acceptable, then go to Step 3 directly. Otherwise,

- *Step 2.1* If the amount of overtime is too large, then return to Step 1, and increase the weight OP_4, or decrease the factor o_i of the corresponding staff category.
- *Step 2.2* If the number of new employees being recruited is too large, then return to Step 1, and increase the weight OP_2, or increase the factor o_i, or increase the factor f_i of the corresponding staff category.
- *Step 2.3* If there are too many staff task substitutions, then return to Step 1, and increase the weight OP_3, or decrease f_i of the corresponding staff category.
- *Step 2.4* If the unachieved professional development is unacceptable, then return to Step 1, and increase the weight OP_5, or increase Q_i of the corresponding staff category.

Step 3. Do loop from Step 4 to 5 with respect to each time period $t = 1, 2, \ldots, T$.

Step 4. Compute the staff scheduling solution for the period t according to the procedure in Sect. 5.2.

Step 5. If the staff scheduling plan is acceptable, then go to Step 3 with $t = t + 1$. Otherwise,

- *Step 5.1* If there are too many temporary staff, then return to Step 1, and increase the weight OP_4, or decrease the weight OP_2, or increase the factor o_i and/or decrease f_i of the corresponding staff category;
- *Step 5.2* If there are too much overtime hours, then return to Step 1, and increase the weight OP_4, or decrease the weight OP_2, or decrease the factor o_i and/or f_i of the corresponding staff category.

Step 6. An acceptable, integrated planning and scheduling solution is obtained.

6 Numerical results

6.1 An example

We consider the case with two categories of full-time (FT) nurses (category-1 and category-2) and two types of jobs (type-1 and type-2). The category-1 nurses have skill 1 (patient care assistants) and can be assigned to a type-1 job (common care), while the category-2 nurses have a higher level of skill set, and can be assigned not only to type-2 job (critical care), but also to type-1 job. The sample data are partly extracted from the example of Venkataraman and Brusco (1996). The planning horizon comprises six planning periods, each with

Table 3 Data for the nursing problem

Four-week period (t)	1	2	3	4	5	6
Demands for Job-1 labor hours (D_{1t})	4266	4221	4972	4310	4507	4442
Demands for Job-2 labor hours (D_{2t})	2145	2536	2141	2865	2206	2252
Maximum ratio of overtime to regular time	$o_i = 0.2, i = 1, 2$					
Maximum ratio of task flexibility	$f_1 = 0.3, f_2 = 0$					
Hours of professional development	$Q_1 = 5, Q_2 = 10$					
Initial number of employees	$X_1(0) = 20, X_2(0)=10$					
Cost coefficients	$\alpha_1 = \$10/\text{hour} \times 8\text{hours/day} \times 28\text{days} = \$2240$					
	$\alpha_2 = \$12/\text{hour} \times 8\text{hours/day} \times 28\text{days} = \$2688$					
	$\beta_1^+ = \alpha_1/2 = \$1120, \beta_1^- = \alpha_1 = \$2240$					
	$\beta_2^+ = \alpha_2/2 = \$1344, \beta_1 = \alpha_2 = \$2688$					
	$e_1 = \$10/\text{hour} \times 3/2 = \$15, e_2 = \$12/\text{hour} \times 3/2 = \$18$					
	$H_1 = H_2 = 8\text{hours/day} \times 20\text{days} = 160\text{hours}$					

4 weeks. The FT nurses are assumed to work 160 regular-time hours and have 8 off-days per 4-week period. It is assumed that nurses work an 8-hour shift on each workday of their schedule. Staff demands are to be satisfied from regular-time, overtime and temporary staff (supplemental staff contracted through an outside nursing agency). The hourly wage rates are set at \$12 and \$10 for FT category-2 and category-1 nurses, respectively. Overtime is priced at one-and-a-half times the corresponding FT wage rate. Temporary staff is assumed to be a last resort and is priced at twice the corresponding FT wage rate. Table 3 lists the nurse staffing requirements for a 6-month planning horizon.

Each planning period (4 weeks) gives a scheduling horizon of 28 days. The mean daily demand (in terms of working hours required) of category-i nurses is $a_i = D_{it}/28$. The sample demands d_{is} on each day s in the scheduling period t are generated through a Gaussian Distribution $N(a_i, 12)$. The maximum ratio of overtime to regular time on any day is $\delta_1 = \delta_2 = 0.25$ respectively.

Our iterative procedure in Sect. 5.3 was implemented in Matlab 6.5 on a PC with 1.8 G CPU. The staff planning model and staff scheduling model were solved by the Lingo 8.0 software package, respectively. The computation time required by the Lingo Tools to solve the planning model was less than 1 second, and that to solve the scheduling model was less than 20 seconds, and the computation time consumed in Matlab 6.5 was less than 1 second.

For the staff planning model, the relative importance of the objectives were set by a pairwise comparison matrix as follows:

$$A = \begin{bmatrix} 1 & 5 & 4 & 4 & 5 \\ \frac{1}{5} & 1 & 3 & 2 & 2 \\ \frac{1}{4} & \frac{1}{3} & 1 & 2 & 2 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 1 & 2 \\ \frac{1}{5} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix},$$

where the element a_{ij} indicates the relative importance of objective i as compared to objective j; cf. Table 22, Chapt. 14, Winston (1994).

Applying the Analytic Hierarchy Process, we generated a weight vector $\mathbf{w} = [0.4981, 0.1904, 0.1331, 0.1054, 0.0730]$ that satisfies the consistency requirement.

Table 4 Solution obtained

Four-week period (t)	0	1	2	3	4	5	6
No. of Category-1 staffs	20	24	24	24	24	24	24
No. of Category-1 staffs to be assigned to take Type-1 job	0	24	24	24	24	24	24
No. of Category-1 staffs to be hired	4	0	0	0	0	0	0
No. of Category-1 staffs to be fired	0	0	0	0	0	0	0
No. of Category-2 staffs	10	15	15	15	15	15	15
No. of Category-2 staffs to be assigned to take Type-2 job	0	15	15	13	15	15	15
No. of Category-2 staffs to be hired	5	0	0	0	0	0	0
No. of Category-2 staffs to be fired	0	0	0	0	0	0	0
Overtime hours of Category-1 staffs	0	426	381	812	470	667	602
Overtime hours of Category-2 staffs	0	462	453	378	182	423	469
Professional development hours of Category-1 staffs	0	0	0	0	0	0	0
Professional development hours of Category-2 staffs	0	0	0	0	0	0	0
No. of Category-2 staffs to be assigned to Type-1 job	0	0	0	2	0	0	0
Shortfall of professional development hours of Category-1 employees							2160
Shortfall of professional development hours of Category-2 employees							1800
Cost ($)	Category-1 nurses				Category-2 nurses		
Recruitment cost:	4928				7280		
Salary	354816				262080		
Overtime	55407				46156		
Totalcost: 730667							

Table 5 Parameters for the iterative solution procedure

IF_1	IF_2	IF_3	IF_4	IF_5	IF_6
0.15	10	0.1	0.1	0.1	0.2
$[o_i^{\min}, o_i^{\max}]$	$[\delta_i^{\min}, \delta_i^{\max}]$	$[f_2^{\min}, f_2^{\max}]$	$[Q_1^{\min}, Q_1^{\max}]$	$[Q_2^{\min}, Q_2^{\max}]$	$[F^{\min}, F^{\max}]$
[0.15, 0.25]	[0.2, 0.3]	[0.25, 0.35]	[4, 6]	[8, 12]	[6, 10]
$\boldsymbol{r}_1$	$\boldsymbol{r}_2$	$\boldsymbol{r}_3$	$\boldsymbol{r}_4$	$\boldsymbol{r}_5$	$\boldsymbol{r}_6$
10	10	5	8	8	4

r_i is the scalar corresponding to the parameter in the second row

With the input data in Table 3 and the weight vector **w**, we solved the staff planning model by the procedure of Sect. 5.1. Table 4 reports the solution obtained. The solution indicated that the professional development requirement was far from satisfaction. Moreover, too much overtime was adopted. These implied that the solution for the staff plan should be revised.

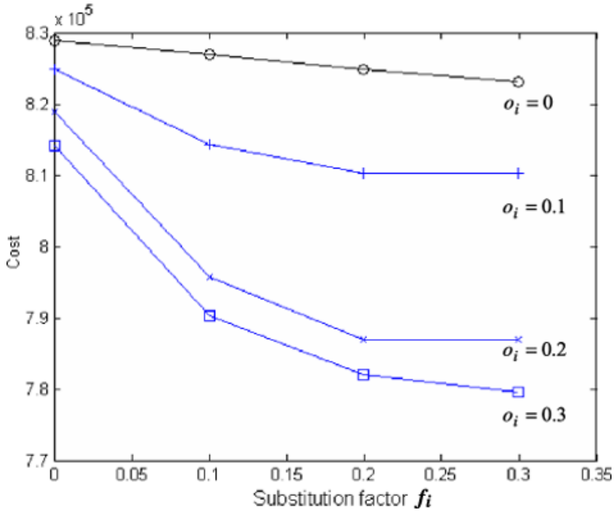
The iterative approach of Sect. 5.3 was then activated. Table 5 gives the values we used for the parameters in the iterative solution approach. The progress of this procedure in solv-

Table 6 Iterative improvement process

Itera-tions	Planning					Scheduling					Is the solution acceptable?	Adjustment
	z_1	z_2	z_3	z_4	z_5	Day	J_1	J_2	OT	TS		
1	730667	9	2	5725	**3920**			—			No. z_5 is too large: both the unachieved professional development of two categories of staff are too high	Increase the weight OP_5 according to Sect. 5.3.2
2	805803	12	1	7465	70	1	13479	0	816	**16**	No. There are	Increase the
						2	11010	0	684	**13**	too many	weight OP_4 and
						3	22104	0	513	**82**	temporary	decrease the
						4	37544	0	415	**77**	employees in	weight OP_2
						5	3991	0	247	**11**	most of	according to
						6	13710	0	805	**18**	periods	Sect. 5.3.2
3	816639	15	3	2718	30	1	11483	0	**730**	2	No. There are	Decrease the fac-
						2	8803	0	**865**	1	too many	tor o_1 from 0.2
						3	13589	0	**877**	2	overtime hours	gradually, each
						4	23394	1	**1189**	24	for category-1	time by a step-
						5	2356	0	78	0	staff in most of	size 0.01, to 0.16.
						6	10638	0	**693**	0	periods	
4	821271	17	3	1265	70	1	9457	0	341	**17**	No. There are	Increase the fac-
						2	9398	0	403	**15**	too many	tor δ_2 from 0.25
						3	9213	0	457	6	temporary	gradually, each
						4	9356	0	432	5	category-1 staff	time by a step-
						5	2129	1	75	2	in periods 1 and	size 0.01, to 0.29
						6	8925	0	237	1	2	in periods 1 and 2
5	821271	17	3	1265	70	1	9452	0	429	6	Improvement can	
						2	9343	0	491	4	no longer be	
						3	9213	0	457	6	made by the iterative solution ap-	
						4	9356	0	432	5	proach, so the solution procedure	
						5	2129	1	75	2	stops and an approximate	
						6	8925	0	237	1	solution is obtained.	

ing the planning and scheduling problems, together with the relevant solutions, is reported in Table 6, where the column 'OT' represents the amount of overtime required in each scheduling period, and the column 'TS' represents the number of temporary employees being em-

Fig. 3 The influence of o_i and f_i on the total cost



ployed in each scheduling period. After 5 iterations, a solution that is acceptable in terms of both planning and scheduling requirements was obtained.

6.2 Managerial insigts/observations

We have observed, in our computations above, that the substitution factor f_i and overtime coefficient o_i seem to have significant impacts on the planning and scheduling solutions, in particular the overtime and the number of temporary employees. We have consequently further examined these issues, in order to derive the necessary managerial insights, by conducting computational studies as follows.

We set $o_1 = o_2$, $f_1 = f_2$ and $\delta_1 = \delta_2 = o_1 + 0.05$, and tested values for o_1 and f_1 in the range from 0 to 0.3, respectively. Other parameters are same as those in the above example. The iterative approach of Sect. 5.3 was applied for each combination of o_1 and f_1. We examined the effects of f_i and o_i on: (1) the total manpower-related cost; (2) the number of new employees recruited; (3) the quantity of shortfall of professional development; (4) the total overtime used; and (5) the number of temporary employees employed. The detailed results are reported in Figs. 3–8.

We have the following observations:

(1) Increasing the staff flexibility or the maximum allowable proportion of overtime has a positive influence on the reduction of the total cost. Moreover, the effect of changing o_1 on the total cost is getting larger when f_1 becomes larger, since the slope of the line for o_1 increases when moving the value of f_1 from 0 to 0.3. However, the influence seems to have an upper limit, since even if one of f_1 and o_1 continues to increase, the effect line becomes horizontal gradually after $f_1 = 0.2$, as illustrated in Fig. 3.

(2) Increasing the staff flexibility or the maximum allowable proportion of overtime can decrease the number of new employees to be recruited; see Fig. 4. In particular, values of o_1 and f_1 in the ranges of [0.2, 0.3] seem to have very significant effects on the reduction of the number of new employees.

(3) Increasing the maximum allowable proportion of overtime may increase the number of staff to be substituted, as illustrated in Fig. 5.

(4) Changing o_1 or f_1 has effects on the shortfall of professional development; see Fig. 6. However, there does not seem to be a pattern detected.

Fig. 4 The influence of o_i and f_i on the number of employees being recruited

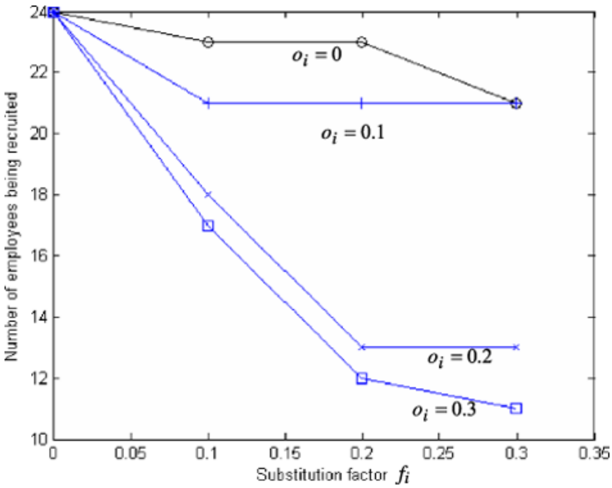
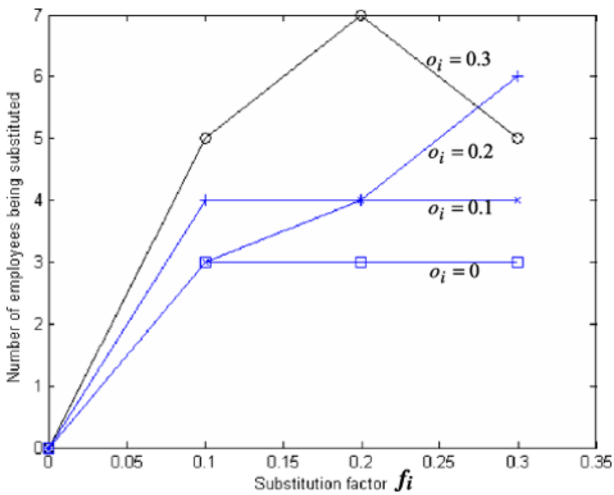


Fig. 5 The influence of o_i and f_i on the number of employees being substituted



(5) Figure 7 shows that increasing the value of f_1 has a negative influence on the reduction of overtime. The effect appears to become more obvious when o_1 becomes larger and larger.

(6) Figure 8 shows the effects of f_1 and o_1 on the number of temporary employees to be employed. First we found the effect is positive. It appears that increasing either f_1 or o_1 would increase the number of temporary employees that need to be hired in the scheduling stage.

We further examined the effects of staff flexibility, using 5 sets of demand data for each of the two scenarios: (i) Considering substitution; and (ii) Not considering substitution. Table 7 shows the results. We can see that: (a) Total cost can be reduced when substitution is considered; and (b) Fewer new employees need to be recruited when substitution is allowed. However, the amount of overtime and the number of temporary staff may be increased since the number of full time staff being employed is reduced.

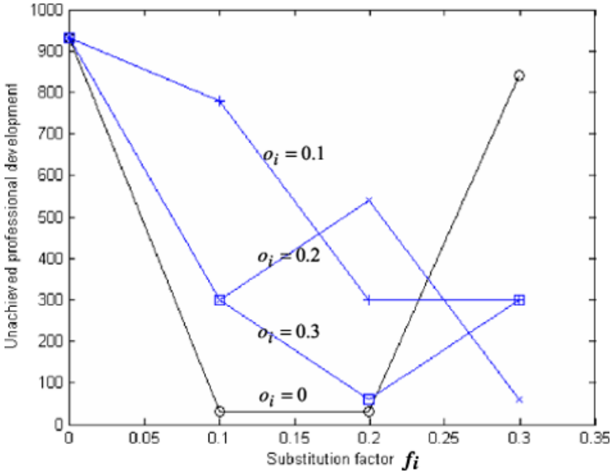


Fig. 6 The influence of o_i and f_i on the unachieved professional development

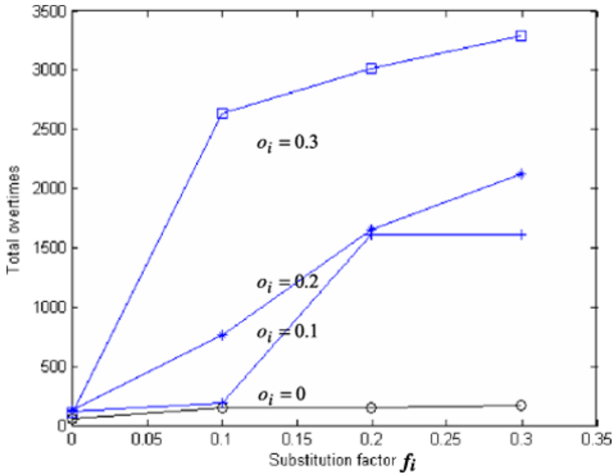


Fig. 7 The influence of o_i and f_i on the total overtime

7 Concluding remarks

In this study we have developed an integrated staffing decision system where impacts of planning and scheduling on each other are taken into account, and features and characteristics of service organizations are specifically addressed. Multiple-objective linear programs have been adopted to model the staff planning and scheduling requirements, respectively. Actions to iteratively improve the planning and scheduling solutions have also been suggested. Extensive computational studies have also been conducted, to evaluate the effectiveness of our proposed approach. Some managerial insights/observations have also been revealed from the numerical results.

Further research includes evaluation of the system in large applications. Various decision rules (adjustment schemes) need to be investigated further to provide the potential managerial implications. Further researches may also include sensitivity analysis on the effects of different pay scales as well as recruitment/dismissal costs on the overall cost reduction when staff flexibility is introduced.

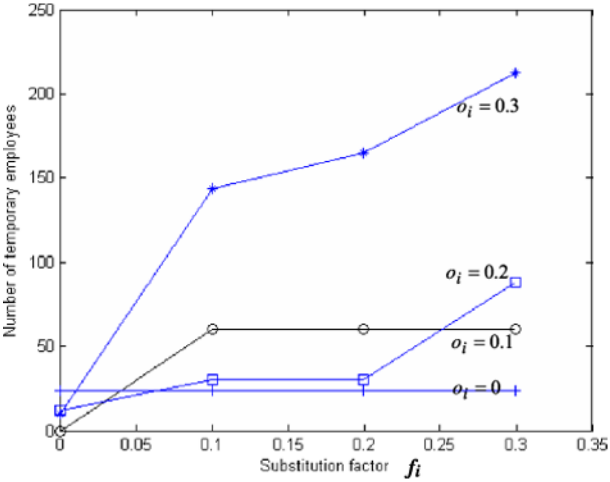


Fig. 8 The influence of o_i and f_i on the number of temporary employees

Table 7 Comparison of effects of substitution

Source	Substitution	No substitution	t-value
Total cost	\$5.2148e+005	\$5.4399e+005	−1251
Number of category-1 staff recruited	4	6.5	−2.0833
Number of category-2 staff recruited	4	5	−1.8333
Overtime of category-1 staff (hrs)	3100	2607	545.3
Overtime of category-2 staff (hrs)	2246	1690	753.1
Shortfall of professional development for category-1 staff (hrs)	0	0	No
Shortfall of professional development for category-2 staff (hrs)	0	0	No
Temporary staff of category-1	42	12	219.5
Temporary staff of category-2	21	12	121.5
Substitutions(hrs)	688	0	649

$P = 0.05$

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Appendix 1 Computing $z_i^{\max}$ and $z_i^{\min}$

Generally, the values $z_i^{\max}$ and $z_i^{\min}$ ($i = 1, 2, \ldots, 5$) could be computed respectively through solving the problem as follows

$$\min z_i / \max z_i \tag{29}$$

$$\text{s.t.} \quad \text{Constraints (1–6).}$$

In (18), we don't pay much attention to the exact values of $z_i^{\max}$ and $z_i^{\min}$, but to the value of the difference $z_i^{\max} - z_i^{\min}$. So it is enough in our computations, to get the near optimal values of $z_i^{\max}$ and $z_i^{\min}$ through the following methods for $i = 1, 2, \ldots, 5$.

Computing $z_1^{\min}$ and $z_1^{\max}$

We can easily show that when no substitutions are considered, the total cost to be derived is the upper bound of the objective z_1. Hence we set $z_1^{\max}$ the total cost of all n staff categories over the entire planning horizon without consideration of any substitutions. So, $z_1^{\max} = \sum_{i=1}^{5} z_{1i}^{S}$. The values z_{1i}^{S} $(i = 1, 2, \ldots, 5)$ are computed through the following models

SMP$_i$ $(i = 1, 2, \ldots, 5)$:

$$z_{1i}^{S} = \min \sum_{t=1}^{T} (\alpha_i X_{it} + e_i O_{it}) + \sum_{t=0}^{T-1} (\beta_i^+ u_{it} + \beta_i^- v_{it}) \tag{30}$$

s.t.

$$X_{i(t+1)} = X_{it} + u_{it} - v_{it}, \quad t = 0, 1, \ldots, T-1, \tag{31}$$

$$H_i X_{it} + O_{it} \geq D_{it}, \quad \forall t, \tag{32}$$

$$0 \leq O_{it} \leq o_i H_i X_{it}, \quad \forall t. \tag{33}$$

Since only the objective z_1 is considered, the constraints (4) and (6) are omitted. The constraint (3) is reduced into $Y_{iit} = X_{it}, \forall i, t$, so the constraint (2) is changed into the constraint (32).

Cai et al. (2004) and Li et al. (2005) have addressed the optimization approach to the models that are similar to SMP$_i$.

The value $z_1^{\min}$ is computed through the following model.

SMPS:

$$z_1^{\min} = \min \sum_{i=1}^{n} \left[\sum_{t=1}^{T} (\alpha_i X_{it} + e_i O_{it}) + \sum_{t=0}^{T-1} (\beta_i^+ u_{it} + \beta_i^- v_{it}) \right] \tag{34}$$

s.t.

$$X_{i(t+1)} = X_{it} + u_{it} - v_{it}, \quad \forall i, t = 0, 1, \ldots, T-1, \tag{35}$$

$$\sum_{j \geq i} s_{ji} H_j Y_{jit} + O_{it} \geq D_{it}, \quad \forall i, t, \tag{36}$$

$$0 \leq O_{it} \leq o_i H_i X_{it}, \quad \forall i, t. \tag{37}$$

Since only the objective z_1 is considered, the constraints (3), (4) and (6) are omitted. The model can be solved by commercial optimization software, e.g., Lingo tools.

Computing $z_2^{\min}$ and $z_2^{\max}$

We set $z_2^{\max} = \sum_{i=1}^{n} \sum_{t=1}^{T} (\lceil \frac{D_{it}}{H_i} \rceil - \lceil \frac{D_{i(t-1)}}{H_i} \rceil)^+$, since we can show that the value $\sum_{i=1}^{n} \sum_{t=1}^{T} (\lceil \frac{D_{it}}{H_i} \rceil - \lceil \frac{D_{i(t-1)}}{H_i} \rceil)^+$ is the upper bound of the objective z_2, where $\lceil x \rceil$ is the smallest integer that is greater than or equal to x.

We set $z_2^{\min} = \sum_{i=1,\nexists k>i,s_{ki}=1}^{n} \{\lceil \frac{\max D_i}{H_i} \rceil + \lceil \frac{\max \bar{D}_i - \max D_i}{H_i^*} \rceil\}$, where $\max \bar{D}_i = \sum_{j=1:s_{ij}=1}^{i} D_{jt}$ and $H_i^* = \max\{H_j, j = 1, 2, \ldots, i-1$ that satisfies $s_{ij} = 1\}$. It comes from the following two reasons.

(1) If no substitutions are considered, then obviously when there is only a recruitment activity and the recruitment activity occurs at the end of the initial period, the quantity of the staff augmentation of category-i staff is minimal and the value is $\lceil \frac{\max D_i}{H_i} \rceil$, where $\max D_i = \max\{D_{it}, \forall t\}$.

(2) It is true that $\max D_{ij} = \max\{D_{it} + D_{jt}, \forall t\} \le \max\{D_{it}, \forall t\} + \max\{D_{jt}, \forall t\} = \max D_i + \max D_j$. So $\max D_j \ge \max D_{ij} - \max D_i$. Then $\max D_{ij} - \max D_j$ category-i employees being recruited during some periods are only assigned to the type-j job. Thus, we get $\lceil \frac{\max D_i}{H_i} \rceil + \lceil \frac{\max D_j}{H_j} \rceil \ge \lceil \frac{\max D_i}{H_i} \rceil + \lceil \frac{\max D_{ij} - \max D_i}{H_i^*} \rceil$, where $H_i^* = \max\{H_i, H_j\}$.

Computing $z_3^{\min}$ and $z_3^{\max}$

We set $z_3^{\min} = 0$. The reason is that a feasible solution can be got even when no substitutions are considered, and $z_3 \ge 0$.

We set $z_3^{\max} = \sum_{t=1}^{T} \lceil \frac{\sum_{i=1:\exists j>i,s_{ji}=1}^{n} D_{it}}{H_j} \rceil$. The reason is as follows: since we only pay attention to the objective of maximizing the substitution quantity, it is optimal when all type-i jobs are assigned to a category-j staff over the entire planning horizon for $i < j$ and $s_{ji} = 1$. Then the quantity of jobs to be assigned to category-j is $\sum_{i=1:\exists j>i,s_{ji}=1}^{n} D_{it}$. Therefore, we get the value of $z_3^{\max}$.

Computing $z_4^{\min}$ and $z_4^{\max}$

We set $z_4^{\min} = 0$, since the lower bound of the overtime is 0.

We set $z_4^{\max} = \sum_{i=1}^{n} \sum_{t=1}^{T} o_i H_i \lceil \frac{D_{it}}{(1+o_i)H_i} \rceil$. Since we can show that the value of $z_4^{\max}$ can be reached only when no substitutions are considered. So, for a given type-i job, if we assume that there are x_{it} category-i employees during period t, to meet the demand of type-i job, then $H_i x_{it} + o_i H_i x_{it} \ge D_{it}$. Thus, we get $x_{it} = \lceil \frac{D_{it}}{(1+o_i)H_i} \rceil$ and the maximum of overtime being used for category-i staff during period t is $o_i H_i \lceil \frac{D_{it}}{(1+o_i)H_i} \rceil$.

Computing $z_5^{\min}$ and $z_5^{\max}$

Obviously, $z_5^{\min} = 0$, since the best instance takes place when the needs of professional development for all staff levels are satisfied.

We set $z_5^{\max} = \sum_{i=1}^{n} Q_{j_i} \lceil \frac{\sum_{t=1}^{T} D_{it}}{H_{j_i}} \rceil$, where $j_i = \operatorname{argmax}_j \{Q_j | s_{ji} = 1, j = 1, 2, \ldots, n\}$ for $i = 1, 2, \ldots, n$. From the constraint (6), we get $S_i^- \le Q_i \sum_t x_{it}, \forall i$. With consideration of substitution, it is possible that all type-i jobs, with the quantity of $\lceil \frac{D_{it}}{H_{j_i}} \rceil$, are assigned to category-j_i ($j_i > i$) staff. Then these category-j_i employees should receive the professional development hours with the quantity of $Q_{j_i} \lceil \frac{\sum_{t=1}^{T} D_{it}}{H_{j_i}} \rceil$.

Appendix 2 Solving the model IntP1

Let $\triangle z_i = z_i^{\max} - z_i^{\min}$, $C_i = \frac{z_i^{\min}}{\triangle z_i}$ and $\bar{a} = \frac{a}{\triangle_1}$, then

$$\bar{z}_1 = \sum_{i=1}^{n}\left[\sum_{t=1}^{T}(\bar{\alpha}_i X_{it} + \bar{e}_i O_{it}) + \sum_{t=0}^{T-1}(\bar{\beta}_i^{+} u_{it} + \bar{\beta}_i^{-} v_{it})\right] - C_1 \tag{38}$$

$$\bar{z}_2 = \sum_{i=1}^{n}\sum_{t=0}^{T-1}\frac{1}{\triangle z_2}u_{it} - C_2. \tag{39}$$

The objective z_3 is equivalent to $z_3' = \sum_{i=1}^{n}\sum_{t=1}^{T}\lceil(\frac{D_{it}}{H_i} - X_{it})^{+}\rceil$, since the optimal solution must meet the following two conditions,

(1) $E_{it} > 0$ occurs, only if $X_{it} > D_{it}, \forall i, t$;

(2) When $D_{it} > x_{it}$, there must be $Y_{ijt} = 0, \forall 1 \le j < i, i = 1, 2, \ldots, n$.

So, we get

$$\bar{z}_3 = \sum_{i=1}^{n}\sum_{t=1}^{T}\frac{1}{\triangle z_3}\left\lceil\left(\frac{D_{it}}{H_i} - X_{it}\right)^{+}\right\rceil - C_3, \tag{40}$$

$$\bar{z}_4 = \sum_{i=1}^{n}\sum_{t=1}^{T}\frac{1}{\triangle z_4}O_{it} - C_4, \tag{41}$$

From the constraint (6), we get $S_i^{-} = \sum_{t=1}^{T}(Q_i X_{it} - H_i E_{it}), \forall i$. Then,

$$\bar{z}_5 = \sum_{i=1}^{n}\sum_{t=1}^{T}\frac{1}{\triangle z_5}(Q_i X_{it} - H_i E_{it}) - C_5. \tag{42}$$

By replacing $\bar{z}_i$ in $\min J = \sum_{i=1}^{5}\lambda_i \bar{z}_i$ with the formulas (38–42), we get the following objective

IntP2:

$$J = \sum_{i=1}^{n}\left\{\sum_{t=1}^{T}\left[\left(\lambda_1\bar{\alpha}_i + \frac{\lambda_5 Q_i}{\triangle z_5}\right)X_{it} + \left(\lambda_1\bar{e}_i + \frac{\lambda_4}{\triangle z_4}\right)O_{it} + \frac{\lambda_3}{\triangle z_3}\left\lceil\left(\frac{D_{it}}{H_i} - X_{it}\right)^{+}\right\rceil \right.\right.$$
$$\left.\left. - \frac{\lambda_5 H_i}{\triangle z_5}E_{it}\right] + \sum_{t=0}^{T-1}\left[\left(\lambda_1\bar{\beta}_i^{+} + \frac{\lambda_2}{\triangle z_2}\right)u_{it} + \lambda_1\bar{\beta}_i^{-}v_{it}\right]\right\} - C \tag{43}$$

s.t. Constraints (1–5).

In (43), $C = \sum_{i=1}^{5}\lambda_i C_i$ is a constant since every value C_i is a constant and λ_i is predetermined. Constraint (6) is omitted in IntP2 since it has been included in the objective (43).

Further, let $\tilde{\alpha}_i = \lambda_1\bar{\alpha}_i + \frac{\lambda_5 Q_i}{\triangle z_5} - \frac{\lambda_5 H_i}{\triangle z_5}$, $\tilde{e}_i = \lambda_1\bar{e}_i + \frac{\lambda_4}{\triangle z_4}$, $\tilde{f}_i = \frac{\lambda_3}{\triangle z_3}$, $\tilde{p}_i = \frac{\lambda_5 H_i}{\triangle z_5}$, $\hat{E}_{it} = X_{it} - E_{it}$, $\tilde{\beta}_i^{+} = \lambda_1\bar{\beta}_i^{+} + \frac{\lambda_2}{\triangle z_2}$, and $\tilde{\beta}_i^{-} = \lambda_1\bar{\beta}_i^{-}$, then the model IntP2 is redescribed as the following formulation.

IntP3:

$$J=\sum_{i=1}^{n}\left\{\sum_{t=1}^{T}\left[\tilde{\alpha}_i X_{it}+\tilde{e}_i O_{it}+\tilde{f}_i\left\lceil\left(\frac{D_{it}}{H_i}-X_{it}\right)^{+}\right\rceil+\tilde{p}_i\hat{E}_{it}\right]+\sum_{t=0}^{T-1}[\tilde{\beta}_i^{+}u_{it}+\tilde{\beta}_i^{-}v_{it}]\right\} \tag{44}$$

s.t Constraints (1–5),

where the constraint (3) is changed into the formulation $\sum_{i=1}^{j-1} s_{ji}Y_{jit}=\hat{E}_{jt}$.

The model IntP3 can be solved by commercial optimization software packages, e.g., Lingo Tools.

Appendix 3 Computing $J_i^{\max}$ and $J_i^{\min}$

Computing $J_1^{\max}$ and $J_1^{\min}$

We can show that the objective J_1 reaches the upper bound $J_1^{\max}$ when no substitutions are considered. So, to meet the demand of type-i job on day t, it satisfies that $\{d_{it}-(w_i-y_{it})h_i\}^{+}\le T_{it}+A_{it}h_i\le(d_{it}-w_ih_i)^{+}$. Further, from the constraint (14), $T_{it}\le\min\{\delta_ih_iw_i,(d_{it}-w_iH_i)^{+}\}$. If $\delta_ih_iw_i<(d_{it}-w_iH_i)^{+}$, then the demand of type-i job can not be fulfilled only by both regular work hours and overtime of category-i staff. So temporary staff needs to be employed, and the number, A_{it}, of temporary staff is equal to $\lceil\frac{(d_{it}-w_ih_i-\delta_ih_iw_i)^{+}}{h_i}\rceil$ approximately. Thus,

$$J_1^{\max}=\sum_{i=1}^{n}\sum_{t=1}^{S}\left\{c_i^o\min\{\delta_ih_iw_i,(d_{it}-w_iH_i)^{+}\}+c_i^a\left\lceil\frac{(d_{it}-w_ih_i-\delta_ih_iw_i)^{+}}{h_i}\right\rceil\right\}.$$

The value of $J_1^{\min}$ needs to be computed through solving the following model.

MinJ1:

$$J_1^{\min}=\mathrm{Min}\sum_i\sum_t(c_i^oT_{it}+c_i^aA_{it}) \tag{45}$$

s.t.

$$\sum_{j=i}^{n}s_{ji}\bar{x}_{jit}h_j+T_{it}+A_{it}h_i\ge d_{it},\quad\forall i,\forall t, \tag{46}$$

$$\sum_{i=1}^{j}s_{ji}\bar{x}_{jit}+y_{jt}\ge w_j,\quad\forall j,\forall t, \tag{47}$$

$$0\le T_{it}\le\delta_ih_iw_i,\quad\forall i,t, \tag{48}$$

$$\sum_{t=1}^{S}y_{jt}\ge Fw_j,\quad\forall j. \tag{49}$$

Since we do not care for the variables d_{it}^{+} and $d_{it}^{+}\ge 0$ for all i and t, the constraint (13) is changed into the constraint (47).

The model MinJ1 can be solved by a similar method discussed in the Appendix 4, since the model MinJ1 is a special case of the model IntS1.

Computing $J_2^{\max}$ and $J_2^{\min}$

Since we do not care for the objective J_1, and there is no constraint for the quantity A_{it} of temporary staff, a special case occurs when all jobs will be assigned to temporary staff. Then, under this case, all Y_{jit} are equal to 0. From the constraint (13), we get $y_{jt}+d_{jt}^+=w_j$. So $\sum_{j=1}^n\sum_{t=1}^S y_{jt}=\sum_{j=1}^n\sum_{t=1}^S(w_j-d_{jt}^+)=S\sum_{j=1}^n w_j-\sum_{j=1}^n\sum_{t=1}^S d_{jt}^+$. Further, from the constraint (15), we get $\sum_{j=1}^n\sum_{t=1}^S y_{jt}\le\sum_{j=1}^n(S-F)w_j$. Therefore, we have $J_2^{\max}=\sum_{j=1}^n(S-F)w_j$.

On the other hand, for any feasible solution of the model IntS1, if we let $y_{jt}:=y_{jt}+d_{jt}^+$ and $d_{jt}^+:=0$, then it is still a feasible solution because the constraint $\sum_t y_{jt}\ge Fw_j$. So $\sum_j\sum_t d_{jt}^+=0$ is feasible. Further, since $d_{jt}^+\ge 0$ for all j and t, we get $J_2^{\min}=0$.

Appendix 4 Solving the model IntS1

The constraint matrix of IntS1 exhibits a block angular structure. The rows obtained from the constraint (15) contain all the variables y_{jt} for every index j, so they couple all variables y_{jt} for a same category staff. However, the remaining rows can be decomposed concerning the indices t into S sets with a same time index. Thus, if the constraint (15) is relaxed and incorporated into the objective function (22) with nonnegative weights κ_i $(i=1,2,\dots,n)$, the resulted model is as follows.

MIntS1:

$$v^*=\max_{\kappa}\min_{T_{it},A_{it},d_{it}^+}\sum_i\sum_t(\overline{\lambda}_1c_{it}^oT_{it}+\overline{\lambda}_1c_{it}^aA_{it}-\overline{\lambda}_2d_{it}^+-\kappa_iy_{it})-C_d \tag{50}$$

s.t.

$$\sum_{j=i}^n s_{ji}\bar{x}_{jit}h_j+T_{it}+A_{it}h_i\ge d_{it},\quad\forall i,\forall t, \tag{51}$$

$$\sum_{i=1}^j s_{ji}\bar{x}_{jit}+y_{jt}+d_{it}^+=w_j,\quad\forall j,\forall t, \tag{52}$$

$$0\le T_{it}\le\delta_ih_iw_i,\quad\forall i,t, \tag{53}$$

where $C_d=\sum_{j=1}^n\kappa_jFw_j$ is a constant and thus can be neglected in the following computation. The model MIntS1 can be decomposed into S time-level subproblems as follows.

SubIntSt $\quad(t=1,2,\dots,S)$:

$$v_t=\max_{\kappa}\min_{T_{it},A_{it},d_{it}^+}\sum_{i=1}^n(\overline{\lambda}_1c_{it}^oT_{it}+\overline{\lambda}_1c_{it}^aA_{it}-\overline{\lambda}_2d_{it}^+-\kappa_iy_{it}) \tag{54}$$

s.t.

$$\sum_{j=i}^n s_{ji}\bar{x}_{jit}h_j+T_{it}+A_{it}h_i\ge d_{it},\quad\forall i, \tag{55}$$

$$\sum_{i=1}^{j} s_{ji}\bar{x}_{jit} + y_{jt} + d_{it}^{+} = w_j, \quad \forall j, \tag{56}$$

$$0 \le T_{it} \le \delta_i h_i w_i, \quad \forall i, \tag{57}$$

The stepsizes κ_t can be computed iteratively by a subgradient optimization (see, Geoffrion 1974).

Every subproblem SubIntSt can be solved independently. First we assign only category-i staff to the type-i job. If overtime or temporary staff of category-i are needed, then we consider how to use category-j staff to substitute category-i, to reduce the amount of overtime or temporary staff of category-i, provided that there are redundant category-j employees for $j > i$ and $s_{ji} = 1$. In the objective function (54), both the coefficients of d_{it}^{+} and y_{it} are non-positive, which means that it can also reduce the value of objective function (54) when the redundant category-j employees take day t off (that is, y_{it} becomes larger) or do nothing on day t (that is, d_{it}^{+} becomes larger). So, there is a balance among the values $\overline{\lambda}_1 c_{it}^{o}$, $\overline{\lambda}_1 c_{it}^{a}$, $\overline{\lambda}_2$ and κ_i.

To summarize, the approach is described as follows.

Step 1. Compute $\overline{d}_{it} = d_{it} - w_i h_i$ for all i.
(*Compute the values T_{it}, A_{it} for every category of staff independently*)
If $\overline{d}_{it} > 0$, then $y_{it} = d_{it}^{+} = 0$, $T_{it} = \min\{\delta_i h_i w_i, \overline{d}_{it}\}$ and $A_{it} = \lceil \frac{\overline{d}_{it} - T_{it}}{h_i} \rceil$.
If $\overline{d}_{it} \le 0$, then let $\overline{w}_i = w_i - \lceil \frac{d_{it}}{h_i} \rceil$, $T_{it} = A_{it} = 0$.

Step 2. Do loop for $i = 1, 2, \dots, n$ that satisfies $\overline{d}_{it} > 0$.
(*Compute substitutions that can reduce the objective value of (54)*)

Step 2.1 If ($A_{it} \ge 0$ and $\overline{\lambda}_1 c_i^{a} \le \overline{\lambda}_2$) or ($\overline{\lambda}_1 c_i^{o} \le \overline{\lambda}_2 < \overline{\lambda}_1 c_i^{a}$ and $A_{it} = 0$), then no substitutions occur for the category-i staff and do the next loop with $i = i + 1$. Otherwise, do the next step with $k = 1$.

Step 2.2 Compute $j^{(k)} = \arg\min_j \{\kappa_j | s_{ji} = 1 \text{ and } \overline{w}_j > 0, j = i+1, \dots, n\}$.
If ($A_{it} \ge 0$ and $\overline{\lambda}_1 c_i^{a} \le \kappa_{j^{(k)}}$) or ($\overline{\lambda}_1 c_i^{o} \le \kappa_{j^{(k)}} < \overline{\lambda}_1 c_i^{a}$ and $A_{it} = 0$), then no substitutions occur for the category-i staff and do the next loop with $i = i + 1$.

Step 2.3 If $A_{it} > 0$ and $\overline{\lambda}_1 c_i^{a} > \max\{\overline{\lambda}_2, \kappa_{j^{(k)}}\} \ge \overline{\lambda}_1 c_i^{o}$, then let $\overline{w}_{j^{(k)}} = \max\{\overline{w}_{j^{(k)}} - A_{it}, 0\}$ and $A_{it} = \max\{A_{it} - \overline{w}_{j^{(k)}}, 0\}$.
If $A_{it} > 0$, then return to step 2.2 with $k = k + 1$.

Step 2.4 If $T_{it} > 0$ and $\overline{\lambda}_1 c_i^{o} > \max\{\overline{\lambda}_2, \kappa_{j^{(k)}}\}$, then let $\overline{w}_{j^{(k)}} = \max\{\overline{w}_{j^{(k)}} - \lceil \frac{T_{it}}{h_i} \rceil - A_{it}, 0\}$, $A_{it} = \max\{A_{it} - \overline{w}_{j^{(k)}}, 0\}$, and $T_{it} = \max\{T_{it} + A_{it} h_i - \overline{w}_{j^{(k)}} h_i, 0\}$.
If $T_{it} > 0$, then return to step 2.2 with $k = k + 1$.

Step 3. Do loop for $i = 1, 2, \dots, n$
(*Compute values d_{it}^{+} and y_{it}*)
If $\overline{w}_i > 0$ and $\overline{\lambda}_2 > \kappa_t$, then $d_{it}^{+} = \overline{w}_i$ and $y_{it} = 0$.
If $\overline{w}_i > 0$ and $\overline{\lambda}_2 \le \kappa_t$, then $d_{it}^{+} = 0$ and $y_{it} = \overline{w}_i$.
Otherwise, $d_{it}^{+} = 0$ and $y_{it} = 0$.

Obviously, the time requirements of both Steps 1 and 3 are $O(n)$. In the Step 2, the worst case occurs when at most $n - i$ computations are needed for some i ($i = 1, 2, \dots, n$), so the time requirement in Step 2 is at most (n^2). Thus, the computational complexity of each subproblem is $O(n^2)$ in the worst case.

Appendix 5 Introduction of the AHP approach

For details of the Analytical Hierarchy Process (AHP) approach, please refer to Sect. 14.3, (Winston 1994). The approach is described as follows briefly.

Suppose there are n objectives. We begin by writing down an $n \times n$ matrix (the pairwise comparison matrix) $A = (a_{ij})_{n\times n}$, where the entry a_{ij} indicates how much more important objective i is than objective j. "Importance" is to be measured on an integer-valued 1–9 scales. For all i, it is necessary that $a_{ii} = 1$, and for consistency, it is necessary that $a_{ji} = 1/k$ when $a_{ij} = k$.

Then, for each of A's columns, divide each entry in column i of A by the sum of the entries in column i and yield a new matrix (call it $A_{\text{norm}} = (\dot{a}_{ij})_{n\times n}$, for normalized). So we get the weight vector $\mathbf{w} = (w_i, i = 1, 2, \dots, n)$, where $w_i = \sum_{j=1}^{n} \dot{a}_{ij}/n$.

Finally, we should check the consistency of the decision maker's comparisons, which uses the following two-step procedure.

Step 1. Compute $CI = \frac{(\frac{1}{n}\sum_{i=1}^{n} \frac{i\text{th entry in } A\mathbf{w}^T}{i\text{th entry in } \mathbf{w}^T}) - n}{n-1}$;

Step 2. Compare CI to the random index (RI) for the approximate value of n, shown in Table 23 of Sect. 14.3 (Winston 1994). If $\frac{CI}{RI} < 0.1$, the degree of consistency is satisfactory; Otherwise, inconsistency may exist and the AHP may not yield meaningful results.

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