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DEA models for supply chain efficiency evaluation

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Abstract An appropriate performance measurement system is an important requirement for the effective management of a supply chain. Two hurdles are present in measuring the performance of a supply chain and its members. One is the existence of multiple measures that characterize the performance of chain members, and for which data must be acquired; the other is the existence of conflicts between the members of the chain with respect to specific measures. Conventional data envelopment analysis (DEA) cannot be employed directly to measure the performance of supply chain and its members, because of the existence of the intermediate measures connecting the supply chain members. In this paper it is shown that a supply chain can be deemed as efficient while its members may be inefficient in DEA-terms. The current study develops several DEA-based approaches for characterizing and measuring supply chain efficiency when intermediate measures are incorporated into the performance evaluation. The models are illustrated in a seller-buyer supply chain context, when the relationship between the seller and buyer is treated first as one of leader-follower, and second as one that is cooperative. In the leader-follower structure, the leader is first evaluated, and then the follower is evaluated using information related to the leader's efficiency. In the cooperative structure, the joint efficiency which is modelled as the average of the seller's and buyer's efficiency scores is maximized, and both supply chain members are evaluated simultaneously. Non-linear programming problems are developed to solve these new supply chain efficiency models. It is shown that these DEA-based non-linear programs can be treated as parametric linear programming problems, and best solutions can

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be obtained via a heuristic technique. The approaches are demonstrated with a numerical example.

Keywords Supply chain · Efficiency · Best practice · Performance · Data envelopment analysis (DEA) · Buyer · Seller

1. Introduction

Effective management of an organization's supply chains has proven to be a very effective mechanism for providing prompt and reliable delivery of high-quality products and services at the least cost. To achieve this, performance evaluation of the entire supply chain is extremely important. This means utilizing the combined resources of the supply chain members in the most efficient way possible to provide competitive and cost-effective products and services. However, lack of appropriate performance measurement systems has been a major obstacle to effective management of supply chains (Lee and Billington, 1992).

During the past decade, a number of supply chain research topics and methodologies have been identified and studied (see, e.g., Tayur, Ganeshan, and Magazine, 1998). Optimization criteria in supply chain models have included cost (Camm et al., 1997), inventory levels (Altiok and Ranjan, 1995), profit (Cohen and Lee, 1989), fill rate (Lee and Billington, 1993), stockout probability (Ishii, Takahashi, and R. Muramatsu, 1988), product demand variance (Newhart, Stott, and Vasko, 1993), and system capacity (Voudouris, 1996). Most deterministic and stochastic models deal with isolated parts of the supply chain system such as supply-production, production-distribution, or inventory-distribution systems. Some models are concerned with strategic issues for supply chains such as the most cost-effective location of plants and warehouses, flow of goods, etc., while others are concerned with operational issues such as order size, fill rate, inventory levels, etc.

However, until recently, measuring supply chain performance has not been considered an important source of competitive information. Even within corporations such as Sears and General Motors, which historically have had large company-owned supply chain systems, performance and measurement systems, in terms of their distribution networks, were not in existence (Ross, 1998). This is partially due to the fact that the tradeoffs/relationships between the measures/decision variables that characterize specific supply chain components are often not completely known. For example, stockout levels and inventory turns are two mutually dependent variables with performance tradeoffs. Technological and process innovations can shift the cost tradeoff curves by reducing the cost of achieving lower inventories at a particular stockout level, or the cost of achieving lower stockouts at a particular inventory level. Information on changes in tradeoffs may not be readily available.

Another reason for the absence of performance measurement tools is that the effective management of the supply chain requires knowing the performance of the overall chain rather than simply the performance of the individual supply chain members. Each supply chain member has its own strategy to achieve efficiency. However, what is best for one member may not work in favour of another member. Sometimes, because of the possible conflicts between supply chain members, one member's inefficiency may be caused by another's efficient operations. For example, the supplier may increase its raw material price to enhance its revenue and to achieve an efficient performance. This increased revenue means increased cost to the manufacturer. Consequently, the manufacturer may become inefficient unless it adjusts its current operating policy. Measuring supply chain performance becomes a difficult and challenging task because of the need to deal with the multiple performance measures related to the supply chain members, and to integrate and coordinate the performance of those members.

Data envelopment analysis (DEA) has proven to be an effective approach in estimating empirical tradeoff curves (efficient frontiers), and in measuring the relative efficiency of peer units when multiple performance measures are present. However, such an efficiency approach cannot be applied directly to the problem of evaluating the efficiency of supply chains, because some measures linked to supply chain members cannot be simply classified as "outputs" or "inputs" of the chain. In fact, with respect to those measures, conflicts between supply chain members are likely present. For example, the supplier's revenue is an output for the supplier, and it is in the supplier's interest to maximize it; at the same time it is also an input to the manufacturer who wishes to minimize it. Simply minimizing the total supply chain cost or maximizing the total supply chain revenue (profit) does not properly model and resolve the inherent conflicts.

Methods have been developed to estimate the exact performance of supply chain members based upon *single* performance measures (e.g., Cheung and Hausman, 2000). However, no attempts have been made to identify best practice in the case of supply chains. Other performance evaluations have been focused on studying the impact of management practices on supply chain performance (e.g., Forker, Mendez, and Hershauer, 1997). Within the context of DEA, there are a number of methods that have the potential to be used in supply chain efficiency evaluation. Seiford and Zhu (1999) and Chen and Zhu (2004) provide two approaches in modeling efficiency as a two-stage process. Along a related line, Färe and Grosskopf (2000) develop the network DEA approach to model general multi-stage processes with intermediate inputs and outputs. Golany, Hackman, and Passy (2003) provide an efficiency measurement framework for systems composed of two subsystems arranged in series that simultaneously compute the efficiency of the aggregate system and each subsystem. Zhu (2003), on the other hand, presents a DEA-based supply chain model to both define and measure the efficiency of a supply chain and that of its members, and yield a set of optimal values of the (intermediate) performance measures that establish an efficient supply chain.

Evaluation of supply chain efficiency, using DEA, has its advantages. In particular, it eliminates the need for unrealistic assumptions inherent in typical supply chain optimization models and probabilistic models; e.g., a typical EOQ model assumes constant and known demand rate and lead-time for delivery. These conventional approaches typically fail, however, to consider the cooperation within the supply chain system. Using a seller-buyer supply chain as an example, the current paper develops two classes of DEA-based models for supply chain efficiency evaluation. The first assumes that the relationship between the buyer and the seller is modeled as a non-cooperative two-stage game, and the second assumes the buyer and seller act in a cooperative sense. In the non-cooperative two-stage game, we use the concept of a leader-follower structure. In the cooperative game, it is assumed that the members of the supply chain cooperate on the intermediate measures. The resulting cooperative game model is a non-linear DEA model which can be solved as a parametric linear programming problem.

It is important to emphasize that the primary contribution of this paper is to provide an analytical framework within which to study supply chain operations. While it is the case that at present in many organizations, data may not be complete enough to permit one to render such models operational, the models do serve two important purposes. *First*, they provide a vehicle for performing 'what if?' analyses on any supply chain. Well known games like the 'beer game', used to simulate supply chain operations, are used for this very purpose.

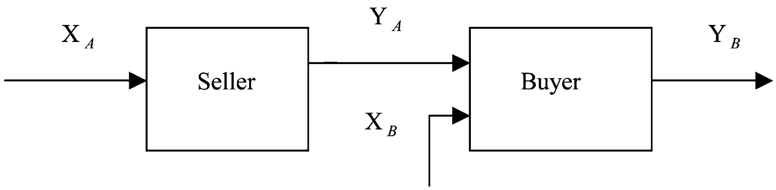

Fig. 1 Buyer-seller supply chain

Second, in those organizations where relevant data are relatively complete, the models can play a useful role. We discuss this more fully in the conclusions. Furthermore, the fact that the models can provide important insights into operations becomes an important motivator for larger organizations to strive to obtain needed data, and to work toward a more cooperative structure for managing their supply networks.

The reminder of the paper is organized as follows. The next section develops models for characterizing and measuring the efficiency of the overall supply chain, as well as that of its members. The relationship between overall supply chain efficiency and supply chain member efficiency is explored. The models are illustrated with a numerical example. Conclusions and directions for future research are given in the last section.

2. Models

Consider a buyer-seller supply chain as described in Fig. 1, where X_A is the input vector of the seller, and Y_A is the seller's output vector. Y_A is also an input vector of the buyer, along with X_B, with Y_B being the buyer's output vector.

Suppose there are n such supply chains or observations on one supply chain. The CCR DEA efficiency of the supply chain is measured as (Charnes et al., 1978; CCR)

$$\begin{aligned} &\text{Max } \frac{U^T Y_{B0}}{V^T(X_{A0}, X_{B0})} \\ &\text{s.t. } \frac{U^T Y_{Bj}}{V^T(X_{Aj}, X_{Bj})} \leq 1 \quad j = 1, 2, \ldots, n \\ &U^T, V^T \geq 0 \end{aligned} \tag{1}$$

Zhu (2003) shows that DEA model (1) fails to correctly characterize the performance of supply chains, because it only considers the inputs and outputs of the supply chain system and ignores measures Y_A associated with supply chain members. Zhu (2003) also shows that if Y_A are treated as both input and output measures in model (1), all supply chains become efficient. Zhu (2003) further shows that an efficient performance indicated by model (1) does not necessarily indicate efficient performance in individual supply chain members. Consequently, improvement to the best-practice can be distorted. i.e., the performance improvement of one supply chain member affects the efficiency status of the other, because of the presence of intermediate measures.

Alternatively, we may consider the average efficiency of the buyer and seller as in the following DEA model

$$
\begin{aligned}
&\text{Max } \frac{1}{2}\left[\frac{U_A^T Y_{A0}}{V_A^T X_{A0}} + \frac{U_B^T Y_{B0}}{V_B^T (X_{B0}, Y_{A0})}\right] \\
&\text{s.t. } \frac{U_A^T Y_{Aj}}{V_A^T X_{Aj}} \le 1 \qquad j = 1, 2, \ldots, n \\
&\frac{U_B^T Y_{Bj}}{V_B^T (X_{Bj}, Y_{Aj})} \le 1 \qquad j = 1, 2, \ldots, n \\
&U_A^T, V_A^T, U_B^T, V_B^T \ge 0
\end{aligned}
\tag{2}
$$

Although model (2) considers Y_A, it does not reflect the relationship between the buyer and the seller. The weights of Y_A as inputs of the buyer may not be equal to the weights of Y_A as outputs of the seller. Model (2) treats the seller and the buyer as two independent units. This does not reflect an ideal supply chain operation.

We next develop several models that can directly evaluate the performance of the supply chain as well as its members while considering the relationship between the buyer and the seller. Our modeling processes are based upon the concept of non-cooperative and cooperative games (see, e.g., Simaan and Cruz, 1973; Li, Huang, and Ashley, 1995; Huang, 2000).

2.1. The noncooperative model

Based upon Li, Huang and Ashley (1995), we suppose the seller-buyer interaction is viewed as a two-stage noncooperative game with the seller as the leader and the buyer as the follower. For example, in the case of non-cooperative advertising between the manufacture (seller) and the retailer (buyer), Toyota automobile company decides that it wants to promote sales of a particular model and directs and subsidizes its local dealers. The local dealers then react to Toyota's strategy by adjusting the amount they spend on advertising and promotion.

First, we use the CCR model to evaluate the efficiency of the seller, as the leader

$$
\begin{aligned}
&\text{Max } \frac{U_A^T Y_{A0}}{V_A^T X_{A0}} = E_{AA} \\
&\text{s.t. } \frac{U_A^T Y_{Aj}}{V_A^T X_{Aj}} \le 1 \quad j = 1, 2, \ldots, n \\
&U_A^T, V_A^T \ge 0
\end{aligned}
\tag{3}
$$

This model is equivalent to the following standard DEA multiplier model:

$$
\begin{aligned}
&\text{Max } \mu_A^T Y_{A0} = E_{AA} \\
&\text{s.t. } \omega_A^T X_{Aj} - \mu_A^T Y_{Aj} \ge 0 \quad j = 1, 2, \ldots, n \\
&\omega_A^T X_{A0} = 1 \\
&\omega_A^T, \mu_A^T \ge 0
\end{aligned}
\tag{4}
$$

Suppose we have an optimal solution of model (4) $\omega_A^{T^*}$, $\mu_A^{T^*}$, and E_{AA}^* and denote the seller's efficiency as E_{AA}^*. We then use the following model to evaluate the buyer's efficiency:

$$\begin{aligned}
&\text{Max } \frac{U_B^T Y_{B0}}{V_B^T X_{B0} + D \times \mu_A^T Y_{A0}} = E_{AB} \\
&\text{s.t. } \frac{U_B^T Y_{Bj}}{V_B^T X_{Bj} + D \times \mu_A^T Y_{Aj}} \leq 1 \qquad j = 1, 2, \ldots, n \\
&\mu_A^T Y_{A0} = E_{AA}^* \\
&\omega_A^T X_{Aj} - \mu_A^T Y_{Aj} \geq 0 \quad j = 1, 2, \ldots, n \\
&\omega_A^T X_{A0} = 1 \\
&\omega_A^T,\ \mu_A^T,\ U_B^T,\ V_B^T,\ D \geq 0
\end{aligned} \tag{5}$$

Note that in model (5), we try to determine the buyer's efficiency given that the seller's efficiency remains at E_{AA}^*. Model (5) is equivalent to the following non-linear model:

$$\begin{aligned}
&\text{Max } \mu_B^T Y_{B0} = E_{AB} \\
&\text{s.t. } \omega_B^T X_{Bj} + d\mu_A^T Y_{Aj} - \mu_B^T Y_{Bj} \geq 0 \quad j = 1, 2, \ldots, n \\
&\omega_B^T X_{B0} + d\mu_A^T Y_{A0} = 1 \\
&\mu_A^T Y_{A0} = E_{AA}^* \\
&\omega_A^T X_{Aj} - \mu_A^T Y_{Aj} \geq 0 \quad j = 1, 2, \ldots, n \\
&\omega_A^T X_{A0} = 1 \\
&\omega_A^T,\ \mu_A^T,\ \omega_B^T,\ \mu_B^T,\ d \geq 0
\end{aligned} \tag{6}$$

Note that $\omega_B^T X_{B0} + d\mu_A^T Y_{A0} = 1$ and $\mu_A^T Y_{A0} = E_{AA}^*$. Thus, we have $0 \leq d < \frac{1}{\mu_A^T Y_{A0}} = \frac{1}{E_{AA}^*}$, i.e, we have the upper and lower bounds on d. Therefore, d can be treated as a parameter and model (6) can be solved as a linear program. In computation, we set the initial d value as the upper bound, namely, $d_0 = \frac{1}{E_{AA}^*}$, and solve the resulting linear program. We then start to decrease d according to $d_t = \frac{1}{E_{AA}^*} - \varepsilon \times t$ for each step t, where ε is a small positive number.[1] We solve each linear program of model (6) corresponding to d_t and denote the optimal objective value as $E_{BA}^*(d_t)$.

Let $E_{BA}^* = \underset{t}{\text{Max}}\ E_{BA}^*(d_t)$. Then we obatin a best heuristic search solution E_{BA}^* to model (6).[2] This E_{AB}^* represents the buyer's efficiency when the seller is given the

[1] In the current study, we set $\varepsilon = 0.01$. If we use a smaller ε, the difference only shows in the fourth decimal point in the current study.

[2] The proposed procedure is a global solution approximation using a heuristic technique, as it searches through the entire feasible region of d when d is decreased from its upper bound to lower bound of zero. It is likely that estimation error exists. The smaller the decreased step, the better the heuristic search solution will be.

pre-emptive priority to achieve its best performance. The efficiency of the supply chain can then be defined as

$$e_{AB} = \frac{1}{2}(E^*_{AA} + E^*_{AB})$$

Similarly, one can develop a procedure for the situation when the buyer is the leader and the seller the follower. For example, in the October 6, 2003 issue of the Business Week, its cover story reports that Walmart dominates its suppliers and not only dictates delivery schedules and inventory levels, but also heavily influences product specifications.

We first evaluate the efficiency of the buyer using the standard CCR ratio model

$$\begin{aligned}
&\text{Max } \frac{U_B^T Y_{B0}}{V_B^T X_{B0} + V^T Y_{A0}} = E_{BB} \\
&\text{s.t.} \quad \frac{U_B^T Y_{Bj}}{V_B^T X_{Bj} + V^T Y_{Aj}} \leq 1 \quad j = 1, 2, \ldots, n \\
&U_B^T,\ V_B^T,\ V^T \geq 0
\end{aligned} \tag{7}$$

Model (7) is equivalent to the following standard CCR multiplier model

$$\begin{aligned}
&\text{Max } \mu_B^T Y_{B0} = E_{BB} \\
&\text{s.t.} \quad \omega_B^T X_{Bj} + \mu^T Y_{Aj} - \mu_B^T Y_{Bj} \geq 0 \quad j = 1, 2, ..., n \\
&\omega_B^T X_{B0} + \mu^T Y_{A0} = 1 \\
&\omega_B^T,\ \mu_B^T,\ \mu^T \geq 0
\end{aligned} \tag{8}$$

Let ω_B^{T*}, μ_B^{T*}, μ^{T*}, E^*_{BB} an optimal solution from model (8) where E^*_{BB} represents the buyer's efficiency score. To obtain the seller's efficiency given that the buyer's efficiency is equal to E^*_{BB}, we solve the following model

$$\begin{aligned}
&\text{Max } \frac{U \times \mu^T Y_{A0}}{V_A^T X_{A0}} = E_{BA} \\
&\text{s.t.} \quad \frac{U \times \mu^T Y_{Aj}}{V_A^T X_{Aj}} \leq 1 \quad j = 1, 2, \ldots, n \\
&\mu_B^T Y_{B0} = E^*_{BB} \\
&\omega_B^T X_{Bj} + \mu^T Y_{Aj} - \mu_B^T Y_{Bj} \geq 0 \quad j = 1, 2, \ldots, n \\
&\omega_B^T X_{B0} + \mu^T Y_{A0} = 1 \\
&\omega_B^T, \mu_B^T, \mu^T, V_A^T, U \geq 0
\end{aligned} \tag{9}$$

Model (9) is equivalent to the following non-linear program

$$\begin{aligned}
&\text{Max } u \times \mu^T Y_{A0} = E_{BA}\\
&\text{s.t.} \quad \omega_A^T X_{Aj} - u \times \mu^T Y_{Aj} \geq 0 \quad j = 1, 2, \ldots, n\\
&\omega_A^T X_{A0} = 1\\
&\mu_B^T Y_{B0} = E_{BB}^*\\
&\omega_B^T X_{Bj} + \mu^T Y_{Aj} - \mu_B^T Y_{Bj} \geq 0 \quad j = 1, 2, \ldots, n\\
&\omega_B^T X_{B0} + \mu^T Y_{A0} = 1\\
&\omega_B^T,\ \mu_B^T,\ \mu^T,\ \omega_A^T,\ u \geq 0
\end{aligned} \tag{10}$$

This model (10) is similar to model (6) and can be treated as a linear program with u as the parameter. We next show how to select the initial value of this parameter.

We first solve the following model.

$$\begin{aligned}
&\text{Max } \frac{U \times \mu^{T*} Y_{A0}}{V_A^T X_{A0}} = EF_{BA}\\
&\text{s.t.} \quad \frac{U \times \mu^{T*} Y_{Aj}}{V_A^T X_{Aj}} \leq 1 \quad j = 1, 2, \ldots, n\\
&V_A^T,\ U \geq 0
\end{aligned} \tag{11}$$

where μ^* is an optimal solution from model (8).

Model (11) is equivalent to the following linear program

$$\begin{aligned}
&\text{Max } u \times \mu^{T*} Y_{A0} = EF_{BA}\\
&\text{s.t.} \quad \omega_A^T X_{Aj} - u \times \mu^{T*} Y_{Aj} \geq 0 \quad j = 1, 2, \ldots, n\\
&\omega_A^T X_{A0} = 1\\
&\omega_A^T,\ u \geq 0
\end{aligned} \tag{12}$$

Let $\omega_A^{T*},\ u^*,\ EF_{BA}^*$ be an optimal solution from model (12). Note that the optimal value to model (12), EF_{BA}^*, may not be the maximum value for the seller because of possible multiple optima in model (8). We have $u \times \mu^T Y_{A0} \geq EF_{BA}^*$. Further, based upon $\omega_B^T X_{B0} + \mu^T Y_{A0} = 1$, we have $\mu^T Y_{A0} \leq 1$. Therefore, $u \geq EF_{BA}^*$. We then utilize EF_{BA}^* as the lower bound for the parameter u when solving for seller's efficiency using model (10). However, this lower bound can be converted into an upper bound as follows.

Let $u \times \mu^T = \nu^T$ and $g = \frac{1}{u}$, then model (10) is equivalent to the following model

$$\begin{aligned}
&\text{Max } \nu^T Y_{A0} = E_{BA} \\
&\text{s.t.} \quad \omega_A^T X_{Aj} - \nu^T Y_{Aj} \geq 0 \quad j = 1, 2, \ldots, n \\
&\omega_A^T X_{A0} = 1 \\
&\mu_B^T Y_{B0} = E_{BB}^* \\
&\omega_B^T X_{Bj} + g\nu^T Y_{Aj} - \mu_B^T Y_{Bj} \geq 0 \quad j = 1, 2, \ldots, n \\
&\omega_B^T X_{B0} + g\nu^T Y_{A0} = 1 \\
&\omega_B^T,\ \mu_B^T,\ \nu^T,\ \omega_A^T,\ g \geq 0
\end{aligned} \tag{13}$$

where $0 \leq g \leq \frac{1}{EF_{BA}^*}$ can be treated as a parameter.

In the current study, we solve model (13) for the seller's efficiency. The computational procedure is similar to the one used in model (6). Denote the heuristic search solution to (13) E_{BA}^*. Then the efficiency of the supply chain can be defined as

$$e_{BA} = \frac{1}{2}(E_{BA}^* + E_{BB}^*)$$

We now illustrate the above DEA procedures with ten supply chain operations (DMUs) given in Table 1. The seller has three inputs, X_{A1} (labor), X_{A2} (operating cost) and X_{A3} (shipping cost) and two outputs, Y_{A1} (number of product *A* shipped), Y_{A2} (number of product *B* shipped) and Y_{A3} (number of product *C* shipped). The buyer has another input X_B (labor) in addition to Y_{A1}, Y_{A2} and two outputs: Y_{B1} (sales) and Y_{B2} (profit).

Table 2 reports the efficiency scores obtained from the newly developed supply chain efficiency models. It can be seen from models (1) and (2) that the supply chain is rated as efficient while its two members are inefficient (e.g., DMUs 2, 3, 4, 7, 8 and 9). This is because the intermediate measures are ignored in model (1). This also indicates that supply chain efficiency cannot be measure by the conventional DEA approach.

Table 1 Numerical example

DMU	X_{A1}	X_{A2}	X_{A3}	Y_{A1}	Y_{A2}	Y_{A3}	X_B	Y_{B1}	Y_{B2}
1	9	50	1	20	10	5	8	100	25
2	10	18	10	10	15	7	10	70	20
3	9	30	3	8	20	2	8	96	30
4	8	25	1	20	20	10	10	80	20
5	10	40	5	15	20	5	15	85	15
6	7	35	2	35	10	5	5	90	35
7	7	30	3	10	25	8	10	100	30
8	12	40	4	20	25	4	8	120	10
9	9	25	2	10	10	5	15	110	15
10	10	50	1	20	15	9	10	80	20

Table 2 Leader-follower structure results

DMU	Model (1)	Model (2)	Model (4) (seller)	Model (6) (buyer)	e_{AB}	Model (8) (buyer)	Model (13) (seller)	e_{BA}
1	1	1	1	0.894	0.947	1	0.760	0.880
2	1	0.903	1	0.585	0.793	0.805	0.914	0.859
3	1	0.900	0.800	0.667	0.733	1	0.691	0.846
4	1	0.814	1	0.628	0.814	0.628	1	0.814
5	0.612	0.640	0.676	0.573	0.625	0.604	0.504	0.554
6	1	1	1	1	1	1	1	1
7	1	0.917	1	0.819	0.909	0.833	0.854	0.844
8	1	0.885	0.770	0.833	0.802	1	0.747	0.873
9	1	0.75	0.5	1	0.750	1	0.5	0.750
10	0.800	0.834	1	0.596	0.798	0.668	0.874	0.771

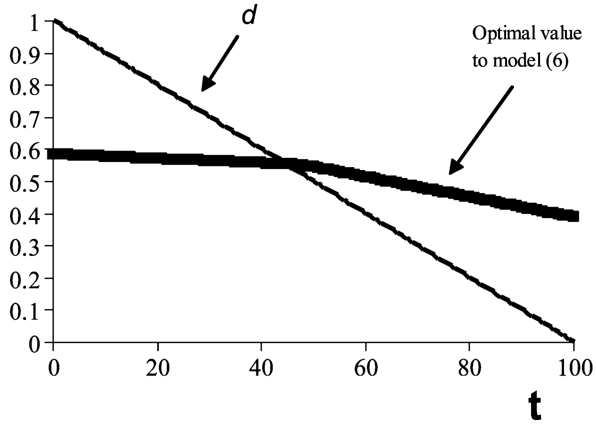

Fig. 2 Solving non-cooperative model for DMU2

When the seller is treated as the leader, six seller operations in DMUs 1, 2, 4, 6, 7 and 10 are efficient with only two efficient buyer operations in DMUs 6 and 9. This indicates that only DMU 6 is the efficient supply chain.

When the buyer is treated as the leader, model (8) shows that 5 buyer operations are inefficient and model (13) shows that only two seller operations are efficient. This also implies that only DMU6 is efficient.

Figure 2 shows how the best heuristic search is obtained when solving model (6) for DMU2. We set $d_t = \frac{1}{E^*_{AA}} - 0.01 \times t$, where $E^*_{AA} = 1$ and $t = 0, \ldots, 100$. Note that when $t = 100$, the parameter $d = 0$, the lower bound, and the optimal value to model (6) is 0.3889. Therefore, we have completed the search over the entire feasible region of d and the best solution is obtained at $t = 0$, that is $E^*_{AB} = 0.585$.

2.2. The cooperative model

In game theory, when the buyer-seller relation was treated as leader-follower, the buyer does not have the control over the seller, and the seller determines the optimal strategy (optimal weights for the intermediate measures). Recent studies however have demonstrated that many retailers (buyers) have increased their bargaining power relative to the manufactures' (sellers) bargaining power (Porter, 1974; Li, Huang, and Ashley, 1996). The shift of power from manufacturers to retailers is one of most significant phenomena in manufacturing and retailing. Walmart is an extreme case where the manufacturer becomes a "follower". Therefore, it is in the best interest of the supply chain to encourage cooperation. This section considers the case where both the seller and buyer have the same degree of power to influence the supply chain system. Our new DEA model seeks to maximize both the seller's and buyer's efficiency, subject to a condition that the weights on the intermediate measures must be equal.

$$\begin{aligned} &\text{Max } \frac{1}{2}\left[\frac{C^T Y_{A0}}{V_A^T X_{A0}} + \frac{U_B^T Y_{B0}}{C^T Y_{A0} + V_B^T X_{B0}}\right] \\ &\text{s.t.} \quad \frac{C^T Y_{Aj}}{V_A^T X_{Aj}} \le 1 \quad j = 1, 2, \ldots, n \\ &\frac{U_B^T Y_{Bj}}{C^T Y_{Aj} + V_B^T X_{Bj}} \le 1 \quad j = 1, 2, \ldots, n \\ &C^T,\ V_A^T,\ V_B^T,\ U_B^T \ge 0 \end{aligned} \tag{14}$$

We call model (14) the cooperative efficiency evaluation model, because model (14) maximize the joint buyer's and seller's efficiency and fεorces the buyer and the seller agree on a same set of weights on the intermediate measures.[3]

We apply the following Charnes-Cooper transformation to model (14)

$$\begin{aligned} &t_1 = \frac{1}{V_A^T X_{A0}}, \quad t_2 = \frac{1}{C^T Y_{A0} + V_B^T X_{B0}} \\ &\omega_A^T = t_1 V_A^T, \quad c_A^T = t_1 C^T \\ &\mu_B^T = t_2 U_B^T, \quad \omega_B^T = t_2 V_B^T, \quad c_B^T = t_2 C^T \end{aligned}$$

Note that in the above transformation, $c_A^T = t_1 C^T$ and $c_B^T = t_2 C^T$ imply a linear relationship between c_A^T and c_B^T. Therefore, we can assume $c_B^T = k c_A^T, \quad k \ge 0$. Then model (14) can be changed into:

$$\begin{aligned} &\text{Max } \frac{1}{2}\left(c_A^T Y_{A0} + \mu_B^T Y_{B0}\right) \\ &\text{s.t.} \quad \omega_A^T X_{Aj} - c_A^T Y_{Aj} \ge 0 \quad j = 1, 2, \ldots, n \\ &\omega_B^T X_{Bj} + c_B^T Y_{Aj} - \mu_B^T Y_{Bj} \ge 0 \quad j = 1, 2, \ldots, n \\ &\omega_A^T X_{A0} = 1 \\ &\omega_B^T X_{B0} + c_B^T Y_{A0} = 1 \\ &c_B^T = k c_A^T \\ &\omega_A^T,\ \omega_B^T,\ c_A^T,\ c_B^T,\ \mu_B^T,\ k \ge 0 \end{aligned} \tag{15}$$

[3] In cooperative game theory, a joint profit of seller and buyer is maximized.

Table 3 Cooperative structure results

DMU	θ_A^*	θ_B^*	Supply chain
1	1	0.894	0.947
2	0.924	0.801	0.862
3	0.691	1	0.846
4	1	0.628	0.814
5	0.676	0.573	0.625
6	1	1	1
7	1	0.819	0.909
8	0.747	1	0.873
9	0.5	1	0.75
10	1	0.596	0.798

Model (15) is a non-linear programming problem and can be converted into the following model

$$\begin{aligned}
&\text{Max } \frac{1}{2}\left(c_A^T Y_{A0} + \mu_B^T Y_{B0}\right) = V_P \\
&\text{s.t.} \quad \omega_A^T X_{Aj} - c_A^T Y_{Aj} \geq 0 j = 1, 2, \ldots, n \\
&\omega_B^T X_{Bj} + k \times c_A^T Y_{Aj} - \mu_B^T Y_{Bj} \geq 0 j = 1, 2, \ldots, n \\
&\omega_A^T X_{A0} = 1 \\
&\omega_B^T X_{B0} + k \times c_A^T Y_{A0} = 1 \\
&\omega_A^T,\ \omega_B^T,\ c_A^T,\ \mu_B^T,\ k \geq 0
\end{aligned} \tag{16}$$

Note that $\omega_B^T X_{B0} + k c_A^T Y_{A0} = 1$, $c_A^T Y_{A0} \leq 1$ and $\omega_B^T X_{B0} > 0$ in model (15). We have $k = \frac{1-\omega_B^T X_{B0}}{c_A^T Y_{A0}} < \frac{1}{c_A^T Y_{A0}}$. Note also that the optimal $c_A^{T*} Y_{A0}$ in model (15) will not be less than E_{BA}^* in model (10). Thus, we have $0 \leq k < \frac{1}{E_{BA}^*}$. That is, model (15) can be treated as a parametric linear program and we can obtain a heuristic search solution using the procedure developed for models (6) and (13).

At the optima, let $\theta_A^* = c_A^{T*} Y_{A0}$ and $\theta_B^* = \mu_B^{T*} Y_{B0}$ represent the efficiency scores for the seller and buyer respectively.

The following two remarks show that in general, the supply chain efficiency under the assumption of cooperation will not be less than the efficiency under the assumption of non-cooperation.

Remark 1. If we set $c_A^T Y_{A0} = E_{AA}^*$ as a constraint in model (15), then the feasible region of model (15) is the same as that of model (6). Therefore, $V_P^* = e_{AB}$.

Remark 2. If we set $\mu_B^T Y_{B0} = E_{BB}^*$ as a constraint in model (15), then the feasible region of model (15) is the same as that of model (13). Therefore, $V_P^* = e_{BA}$.

We consider again the numerical example in Table 1. Table 3 reports the results from model (15), where columns 2 and 3 report the efficiency scores for the seller and buyer respectively and the last column reports the optimal value to model (15), the supply chain efficiency.

Table 4 compares the efficiency scores for the cooperative and non-cooperative assumptions. In this numerical example, except for DMU2, one of the two leader-follower models

Table 4 Comparison of non-cooperative and cooperative results

DMU	e_{AB}	e_{BA}	Model (15)
1	0.947	0.880	0.947
2	0.793	0.859	0.862
3	0.733	0.846	0.846
4	0.814	0.814	0.814
5	0.625	0.554	0.625
6	1	1	1
7	0.909	0.844	0.909
8	0.802	0.873	0.873
9	0.750	0.750	0.75
10	0.798	0.771	0.798

Table 5 Solving cooperative model for DMU2

t	$k_t = \frac{1}{E^*_{BA}} - 0.01 \times t$	$V^*_P(k_t)$
0	1.094	0.8596
1	1.084	0.8620 **(best solution)**
2	1.074	0.8615
3–9	1.064–1.004	0.8604–0.8535
10–19	0.994–0.904	0.8523–0.8418
20–29	0.894–0.804	0.8407–0.8299
30–39	0.794–0.704	0.8287–0.8174
40–49	0.694–0.604	0.8162–0.8043
50–59	0.594–0.504	0.8029–0.7900
60–69	0.494–0.404	0.7885–0.7738
70–79	0.394–0.304	0.7720–0.7572
80–89	0.294–0.204	0.7544–0.7204
90–99	0.194–0.104	0.7248–0.7107
100–109	0.094–0.004	0.7092–0.6951
110	0	0.6944

achieves the efficiency under the cooperative assumption. This indicates that no better solution can be found to yield a higher efficiency in the cooperative assumption. However, in DMU2, the supply chain shows a better performance when assuming cooperative operation.

Finally, Table 5 shows the calculation steps for DMU2 using model (15). We set $k_t = \frac{1}{E^*_{BA}} - 0.01 \times t$ where $E^*_{BA} = 0.914$. The first column reports the t from 0 to 110. When $t = 110$, k approaches lower bound of 0. Note that in general, the optimal value to model (15) decreases as we decrease k. The best heuristic solution is found when $t = 2$, where $V^*_P = \underset{t}{\text{Max}}\ V^*_P(k_t) = 0.862$.

3. Conclusions

The current paper develops a number of DEA models for evaluating the performance of a supply chain and its members. The non-cooperative model is modelled as a leader-follower structure, where in our case, the leader is first evaluated by the regular DEA model, and then the follower is evaluated using the leader-optimized weights on the intermediate measures. Our cooperative model tries to maximize the joint efficiency of the seller and buyer, and imposes weights on the intermediate measures that are the same when applied to the measures as outputs of the supplier as when applied to those measures as inputs of the buyer. Although the models are nonlinear programming problems, they can be solved as parametric linear programming problems, and a best solution can be found using a heuristic technique.

We point out that although the current paper uses the concept of a cooperative game, it does not try to examine whether the members of a specific supply chain are behaving in a cooperative or non-cooperative manner. This is a topic for further research. Specifically, the current paper is not an empirical one and we, therefore, do not pursue these opportunities. Instead, a simple numerical example has been used to demonstrate the theoretical contributions of the current paper. Other useful theoretical developments include the idea that one echelon can use knowledge about another echelons (supplier or customer), to improve its own performance or the mutual performance of the members. This is consistent with the theory by games as it applies, for example, to the 'bullwhip effect' such as in the earlier-referenced beer game.

The current study develops new DEA-based models aimed at: (i) correctly characterizing multi-member supply chain operations, and (ii) calculating the efficiencies of the supply chain and its members. Because conventional DEA models cannot be directly applied to evaluating multi-member supply chain operations, our models become important tools for the managers in monitoring and planning their supply chain operations, and can significantly aid in making supply chains more efficient.

While the major contribution of the current study lies in its methodological developments, it has potential for application in various supply chain operations. For example, some types of supply chains either have or can acquire reasonably adequate data. One important class of supply chains involves goods entering ports from abroad and destined for a Canada/US border crossing. With the recent (since Sept 11) interest in the installation of security devices at various points in this international 'supply chain', government organizations such as Transport Canada, and the US Dept of Commerce can demand easy access to the relevant data that drive ports, rail yards and truck depots. This can aid the relevant agencies in making security allocation decisions, and can pinpoint weak links in the chain. A study of such supply chains is a subject for further research.

Finally, our models can also be applied to any multi-stage production systems (Seiford and Zhu, 1999; Golany, Hackman, and Passy, 2003). Note also that in the current paper, an average overall efficiency is used. We can use a weighted average to reflect the power relationships and importance of supply chain's members. Further, Chen, Liang, and Yang (2006) develop a game theory approach to address how to integrate the seller's and buyer's efficiency scores and obtain an efficiency score for the supply chain.

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