Acta Math. Hungar., **152** (**2**) (2017), 464–472
DOI: 10.1007/s10474-017-0701-y
First published online March 20, 2017

APPROXIMATE CONVEXITY WITH RESPECT TO A SUBFIELD

Z. BOROS[1,*,‡] and N. NAGY[2,†]

[1]Institute of Mathematics, University of Debrecen, 4002 Debrecen, Pf. 400, Hungary
e-mail: zboros@science.unideb.hu

[2]Department of Applied Mathematics, University of Miskolc, 3515 Miskolc-Egyetemváros, Hungary
e-mail: matnagyn@uni-miskolc.hu

(Received January 4, 2017; revised January 31, 2017; accepted January 31, 2017)

Abstract. Let $\mathbb{F}$ be a subfield of $\mathbb{R}$ and X be a linear space over $\mathbb{F}$. Let $D \subseteq X$ be a nonempty $\mathbb{F}$-convex set, $D^* := D - D := \{x - y : x, y \in D\}$, and $\alpha\colon D^* \to \mathbb{R}$ be a nonnegative even function. The function $f\colon D \to \mathbb{R}$ is called $(\alpha, \mathbb{F})$-convex, if it satisfies the inequality

$$f(tx + (1-t)y) \le tf(x) + (1-t)f(y) + t\alpha((1-t)(x-y)) + (1-t)\alpha(t(y-x))$$

for all $x, y \in D$ and for all $t \in \mathbb{F} \cap [0,1]$. In this paper we characterize $(\alpha, \mathbb{F})$-convex functions by comparison of modified difference ratios and support properties. If α satisfies some additional conditions, we obtain the differentiability of $(\alpha, \mathbb{F})$-convex functions in the appropriate sense.

1. Introduction

A real valued function f, defined on a convex subset of a real linear space, is called convex if it satisfies the inequality

$$(1) \qquad f(tx + (1-t)y) \le tf(x) + (1-t)f(y)$$

for every $t \in [0,1]$ and for all x, y taken from the domain of f. We call f mid-convex or Jensen convex if (1) holds for $t = 1/2$. We may consider Jensen's paper [6], in which the author proved that any mid-convex function f satisfies (1) for all rational $t \in [0,1]$, as the beginning of the investigation of

* Corresponding author.

† This research was (partially) carried out in the framework of the Center of Excellence of Mechatronics and Logistics at the University of Miskolc.

‡ Research of Z. Boros has been supported by the Hungarian Scientific Research Fund (OTKA) grant K-111651.

Key words and phrases: approximately convex function, convexity with respect to a subfield, differentiation of convex function.

Mathematics Subject Classification: 26A51, 39B62.

convex functions. A few years later Bernstein and Doetsch [1] proved, supposing that the domain is a convex subset of a finite dimensional Euclidean space, that a mid-convex function which is bounded above on a non-void open set has to be continuous (and thus convex). Since then, a huge literature of this topic has been published. In what follows, we list only a couple of papers and monographs in order to indicate some aspects of these investigations.

When the domain is an open interval, one may characterize the convexity of a function in terms of inequalities fulfilled by various difference ratios as well as by the existence of supporting lines below its graph at each point of the domain. These observations lead to one-sided differentiability and continuity of convex real functions. These results can be extended to the investigations of convex functions on more general domains [12,13] as well as for convexity with respect to a subfield [2] (which is well motivated by Jensen's result and by the existence of discontinuous mid-convex functions).

Numerous articles are devoted to the investigations of approximately convex functions, that satisfy (1) with some reasonably small error. Many of these articles establish the stability of convexity in the sense that if the error of the convexity of f is estimated by a constant or by a multiple of the norm of the difference of the arguments (or by other similar expressions), then f is close (in an appropriate sense) to a convex function [3, 5,11]. Rolewicz extended support theorems and differentiability properties for continuous, approximately convex functions with sufficiently small errors of convexity [14,15]. A couple of authors extended Jensen's theorem and the Bernstein–Doetsch theorem to approximately mid-convex functions [4,10,16–18].

Makó and Páles [8] considered the inequality

$$f(tx+(1-t)y) \leq tf(x)+(1-t)f(y)+t\varphi((1-t)|x-y|)+(1-t)\varphi(t|x-y|) \tag{2}$$

in order to provide a common generalization of several previous results on this topic. They considered the cases $t = 1/2$ and $t \in [0, 1]$ with various assumptions concerning the non-negative error function φ. This approach made it possible, among others, to provide characterizations of (2) via inequalities involving modified difference ratios as well as in terms of a modified support property.

The concept of strong convexity refers to functions satisfying an inequality that is stronger than (1). Some basic properties of strongly convex functions are encountered by Merentes and Nikodem [9]. Mixing this concept with some of the above mentioned investigations [2,8], Makó, Nikodem and Páles [7] considered the inequality

$$f(tx+(1-t)y) \leq tf(x)+(1-t)f(y)-t\alpha((1-t)(x-y))-(1-t)\alpha(t(y-x)) \tag{3}$$

with a non-negative, symmetric function α, in the case when $t \in [0,1]$ is taken from a subfield of the reals. They proved various characterizations of strong convexity in this restricted sense.

The purpose of the present communication is to combine the approach of approximate convexity by Makó and Páles [8] with the concept of restricted strong convexity by Makó, Nikodem and Páles [7], in order to provide characterizations and regularity properties for functions that are approximately convex with respect to a subfield of the reals.

Throughout this paper, let $\mathbb{F}$ be a subfield of the field $\mathbb{R}$ of real numbers and X be a linear space over $\mathbb{F}$. Let $\mathbb{F}_+$ denote the set of positive elements of $\mathbb{F}$. Moreover let $\overline{\mathbb{R}}_+$ denote the set of nonnegative real numbers. First we have to define $\mathbb{F}$-convex and $\mathbb{F}$-algebraically open sets as well as $\mathbb{F}$-convex functions that are considered in [2].

Definition 1. A subset D of the space X is called $\mathbb{F}$-convex if $tx + (1-t)y \in D$ for every $x, y \in D$ and $t \in \mathbb{F} \cap [0,1]$. A subset D of the space X is called $\mathbb{F}$-algebraically open if, for all $x \in D$ and $u \in X$, there exists a $\delta > 0$ such that $x + ru \in D$ for every $r \in \mathbb{F} \cap \,]-\delta, \delta[$.

Definition 2. Let $D \subseteq X$ be a nonempty $\mathbb{F}$-convex set. A function $f \colon D \to \mathbb{R}$ is called $\mathbb{F}$-convex if

$$f(tx + (1-t)y) \le tf(x) + (1-t)f(y) \tag{4}$$

holds for every $x, y \in D$ and $t \in \mathbb{F} \cap [0,1]$.

In the following section we introduce the aforementioned concept of approximate $\mathbb{F}$-convexity and establish its characterizations by comparison of modified difference ratios and an appropriate support property. In a subsequent section we apply this support property to prove certain regularity properties for approximately $\mathbb{F}$-convex functions with sufficiently small errors, in the spirit of Rolewicz [14,15].

2. Characterizations of approximate $\mathbb{F}$-convexity

Definition 3. Let $D \subseteq X$ be a nonempty $\mathbb{F}$-convex set,

$$D^* := D - D := \{x - y : x, y \in D\},$$

and $\alpha \colon D^* \to \overline{\mathbb{R}}_+$ be an even function. The function $f \colon D \to \mathbb{R}$ is called $(\alpha, \mathbb{F})$-convex, if it satisfies the inequality

$$\begin{aligned} f(tx + (1-t)y) &\le tf(x) + (1-t)f(y) \\ &+ t\alpha((1-t)(x-y)) + (1-t)\alpha(t(y-x)) \end{aligned} \tag{5}$$

for all $x, y \in D$ and for all $t \in \mathbb{F} \cap [0,1]$.

THEOREM 1. *Let $D \subseteq X$ be a nonempty, $\mathbb{F}$-algebraically open, $\mathbb{F}$-convex set, $\alpha\colon D^* \to \overline{\mathbb{R}}_+$ be an even function, and let $f\colon D \to \mathbb{R}$ be a function. Then the following statements are equivalent*:

(i) *f is $(\alpha, \mathbb{F})$-convex on D*;

(ii) *the inequality*

$$\frac{f(u) - f(u - sh) - \alpha(-sh)}{s} \le \frac{f(u + rh) - f(u) + \alpha(rh)}{r} \tag{6}$$

is satisfied for all $r, s \in \mathbb{F}_+$, $u \in D$, $h \in X$, where $u - sh, u + rh \in D$;

(iii) *there exists a function $A\colon D \times X \to \mathbb{R}$ such that*

$$f(u + rh) - f(u) \ge rA(u, h) - \alpha(rh) \tag{7}$$

for all $u \in D$, $r \in \mathbb{F}$, $h \in X$, where $u + rh \in D$.

PROOF. (i) $\Rightarrow$ (ii). Suppose that $u \in D$, $h \in X$ and $r, s \in \mathbb{F}_+$ such that $u - sh, u + rh \in D$. Let us substitute $u - sh$ and $u + rh$ in the place of x and y, resp. Moreover, let $t = \frac{r}{r+s}$. Then $t \in \mathbb{F}$, $0 < t < 1$, $1 - t = \frac{s}{r+s}$, $tx + (1 - t)y = u$, and from inequality (5) we get that

$$\begin{aligned} f(u) \le \frac{r}{r+s} & f(u - sh) + \frac{s}{r+s} f(u + rh) \\ &+ \frac{r}{r+s}\alpha\Big(\frac{s}{r+s}((u - sh) - (u + rh))\Big) \\ &+ \frac{s}{r+s}\alpha\Big(\frac{r}{r+s}((u + rh) - (u - sh))\Big), \end{aligned}$$

i.e.,

$$f(u) \le \frac{r}{r+s} f(u - sh) + \frac{s}{r+s} f(u + rh) + \frac{r}{r+s}\alpha(-sh) + \frac{s}{r+s}\alpha(rh).$$

From this inequality it follows that

$$(r + s)f(u) \le rf(u - sh) + sf(u + rh) + r\alpha(-sh) + s\alpha(rh) \tag{8}$$

and thus

$$r\big[f(u) - f(u - sh) - \alpha(-sh)\big] \le s\big[f(u + rh) - f(u) + \alpha(rh)\big].$$

This yields that inequality (6) holds.

(ii) $\Rightarrow$ (iii). Assume that (ii) is true, and for $u \in D$, $h \in X$ let us define the set

$$R(u, h) = \Big\{\frac{f(u + rh) - f(u) + \alpha(rh)}{r} : r \in \mathbb{F}_+ \text{ such that } u + rh \in D\Big\}.$$

Due to our assumptions on D, the set $R(u,h)$ is non-void and we may apply (6) to verify that it is bounded from below. Let $A(u,h) := \inf R(u,h)$. Moreover if we reformulate the left hand side of (6), we obtain that

$$\frac{f(u-sh)-f(u)+\alpha(-sh)}{-s} \le \frac{f(u+rh)-f(u)+\alpha(rh)}{r}$$

and by putting $-s$ in the place of s (in this case $s<0<r$ and $s,r\in\mathbb{F}$) we get

$$\frac{f(u+sh)-f(u)+\alpha(sh)}{s} \le \frac{f(u+rh)-f(u)+\alpha(rh)}{r}.$$

Hence

$$\frac{f(u+sh)-f(u)+\alpha(sh)}{s} \le A(u,h) \le \frac{f(u+rh)-f(u)+\alpha(rh)}{r} \tag{9}$$

and from the left side of (9) we get for all $s<0$, $s\in\mathbb{F}$ that

$$f(u+sh)-f(u)+\alpha(sh) \ge sA(u,h),$$

thus

$$f(u+sh)-f(u) \ge sA(u,h)-\alpha(sh),$$

and from the right side of (9) we get for all $0<r$, $r\in\mathbb{F}$ that

$$f(u+rh)-f(u) \ge rA(u,h)-\alpha(rh).$$

We may conclude the verification of (7) by observing that it obviously holds for $r=0$ as well.

(iii) $\Rightarrow$ (i). Assume that (iii) is satisfied and let $u\in D$, $h\in X$. Then in the case $r<0$, $r\in\mathbb{F}$ replace r with $-s$ (then $s>0$):

$$f(u-sh)-f(u) \ge -sA(u,h)-\alpha(-sh), \text{ whenever } u-sh\in D. \tag{10}$$

Let $0\le r\in\mathbb{F}$ such that $u+rh\in D$.

If we multiply (10) by $\frac{r}{r+s}$ and (7) by $\frac{s}{s+r}$ and add these two inequalities we obtain

$$\begin{aligned}&\frac{-rs}{r+s}A(u,h)-\frac{r}{r+s}\alpha(-sh)+\frac{rs}{r+s}A(u,h)-\frac{s}{r+s}\alpha(rh)\\ &\le \frac{r}{r+s}f(u-sh)-\frac{r}{r+s}f(u)+\frac{s}{r+s}f(u+rh)-\frac{s}{r+s}f(u),\end{aligned}$$

which can be obviously reduced to

$$f(u) \le \frac{r}{r+s}f(u-sh)+\frac{s}{r+s}f(u+rh)+\frac{r}{r+s}\alpha(-sh)+\frac{s}{r+s}\alpha(rh). \tag{11}$$

Let $x, y \in D$ and $t \in \mathbb{F} \cap [0,1]$. Performing the substitutions

$$u := tx + (1-t)y,\ r := t,\ s := 1-t,\ \text{and } h := y - x,$$

we get that inequality (11) can be written as

$$f(tx + (1-t)y)$$
$$\leq tf(x) + (1-t)f(y) + t\alpha((1-t)(x-y)) + (1-t)\alpha(t(y-x)). \quad \square$$

3. Differentiability of approximately $\mathbb{F}$-convex functions

THEOREM 2. *Let $D \subseteq X$ be a nonempty, $\mathbb{F}$-algebraically open, $\mathbb{F}$-convex set and $\alpha\colon D^* \to \overline{\mathbb{R}}_+$ be an even function such that, for every $h \in X$, the mapping $r \mapsto \alpha(rh)$ (defined for all $r \in \mathbb{F}_+$ that fulfils $rh \in D^*$) is continuous and satisfies*

$$\lim_{\mathbb{F}_+ \ni r \to 0} \frac{\alpha(rh)}{r} = 0\,. \tag{12}$$

If $f\colon D \to \mathbb{R}$ is $(\alpha, \mathbb{F})$-convex and $A\colon D \times X \to \mathbb{R}$ is the mapping described in statement (iii) *and the proof of Theorem* 1, *then*

$$A(u,h) = \lim_{s \to 0,\, s \in \mathbb{F}_+} \frac{f(u+sh) - f(u)}{s}$$

for all $u \in D$ and $h \in X$. Moreover, the mapping $h \mapsto A(u,h)$ $(h \in X)$ is positively $\mathbb{F}$-homogeneous and subadditive for every $u \in D$.

PROOF. From (8), which is equivalent to the inequality of $(\alpha, \mathbb{F})$ convexity, we get

$$\frac{f(u) - f(u-sh)}{s} \leq \frac{f(u+rh) - f(u-sh)}{r+s} + \frac{r}{s(r+s)}\alpha(-sh) + \frac{1}{r+s}\alpha(rh)$$

for all $u \in D$, $h \in X$ and $r, s \in \mathbb{F}_+$ such that $u - sh, u + rh \in D$. If we sub stitute a in the place of $u - sh$, we have

$$\frac{f(a+sh) - f(a)}{s} \leq \frac{f(a+(r+s)h) - f(a)}{r+s} + \frac{r}{s(r+s)}\alpha(-sh) + \frac{1}{r+s}\alpha(rh)$$

for all $a \in D$, $h \in X$ and $r, s \in \mathbb{F}_+$ such that $a + rh + sh \in D$. Finally, re placing a with u and $r + s$ with q we get that

$$\frac{f(u+sh) - f(u)}{s} \leq \frac{f(u+qh) - f(u)}{q} + \frac{q-s}{sq}\alpha(-sh) + \frac{1}{q}\alpha((q-s)h)$$

holds for all $u \in D$, $h \in X$ and $q, s \in \mathbb{F}$ such that $0 < s < q$ and $u + qh \in D$.

From (12) we obtain that, for every $\varepsilon > 0$, there exists an $r_\varepsilon \in \mathbb{F}_+$ such that $\frac{\alpha(rh)}{r} < \frac{\varepsilon}{3}$ for all $r \in \mathbb{F} \cap]0, r_\varepsilon[$.

Due to the definition of $A(u,h)$ there exists a value $q \in \mathbb{F}_+$ such that $u + qh \in D$ and

$$\frac{f(u+qh) - f(u) + \alpha(qh)}{q} < A(u,h) + \frac{\varepsilon}{3}\,.$$

Moreover there exists a $\delta > 0$ such that in the case $t \in \mathbb{F}_+$, $|t - q| < \delta$, we have that

$$|\alpha(th) - \alpha(qh)| < \frac{\varepsilon q}{3}.$$

If $s \in \mathbb{F}_+$ satisfies $s < \min\{r_\varepsilon, q, \delta\}$, then

$$\begin{aligned} A(u,h) - \frac{\varepsilon}{3} &< A(u,h) - \frac{\alpha(sh)}{s} \le \frac{f(u+sh) - f(u)}{s} \\ &\le \frac{f(u+qh) - f(u)}{q} + \frac{(q-s)\alpha(-sh)}{qs} + \frac{\alpha((q-s)h)}{q} \\ &= \frac{f(u+qh) - f(u) + \alpha(qh)}{q} + \frac{\alpha((q-s)h) - \alpha(qh)}{q} + \Big(1 - \frac{s}{q}\Big)\frac{\alpha(sh)}{s} \\ &< A(u,h) + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = A(u,h) + \varepsilon. \end{aligned}$$

Hence we get

$$A(u,h) = \lim_{s \to 0,\ s \in \mathbb{F}_+} \frac{f(u+sh) - f(u)}{s}\,. \tag{13}$$

Using (13) we can show that the map $h \mapsto A(u,h)$ is positively $\mathbb{F}$-homogeneous: Let $\lambda \in \mathbb{F}_+$. Then, for all $u \in D$, $h \in X$ we have

$$\begin{aligned} A(u,\lambda h) &= \lim_{s \to 0,\, s \in \mathbb{F}_+} \frac{f(u+s\lambda h) - f(u)}{s} = \lim_{s \to 0,\, s \in \mathbb{F}_+} \lambda \frac{f(u+\lambda s h) - f(u)}{\lambda s} \\ &= \lambda \lim_{r \to 0,\, r \in \mathbb{F}_+} \frac{f(u+rh) - f(u)}{r} = \lambda A(u,h). \end{aligned}$$

We can prove also that the map $h \mapsto A(u,h)$ is subadditive: For $u \in D$ and $h, k \in X$, we obtain

$$A(u,h+k) = \lim_{s \to 0,\ s \in \mathbb{F}_+} \frac{f(u+s(h+k)) - f(u)}{s}$$

$$= \lim_{s\to 0,\ s\in\mathbb{F}_+} \frac{f(u+sh+sk)-f(u)}{s}$$

$$= \lim_{s\to 0,\ s\in\mathbb{F}_+} \frac{f\big(\frac{1}{2}(u+2sh)+\frac{1}{2}(u+2sk)\big)-f(u)}{s}$$

$$\le \lim_{s\to 0,\ s\in\mathbb{F}_+} \frac{f(u+2sh)+f(u+2sk)+\alpha\,(s(h-k))+\alpha(s(k-h))-2f(u)}{2s}$$

$$= \lim_{s\to 0,\ s\in\mathbb{F}_+} \frac{f(u+2sh)-f(u)+\alpha\,(s(h-k))}{2s}$$

$$+ \lim_{s\to 0,\ s\in\mathbb{F}_+} \frac{f(u+2sk)-f(u)+\alpha\,(s(k-h))}{2s} = A(u,h)+A(u,k). \qquad \square$$

Example 1. Let X denote a real normed space, $p>1$, $c>0$, and $\alpha(x)=c\|x\|^p$ $(x\in X)$. If $D\subseteq X$ is open and convex, then the restriction of α to D^* satisfies the assumptions of Theorem 2 with $\mathbb{F}=\mathbb{R}$. In fact,

$$\lim_{0<r\to 0}\frac{\alpha(rh)}{r} = \lim_{0<r\to 0}\frac{c\|rh\|^p}{r} = \lim_{0<r\to 0} c\,r^{p-1}\|h\|^p = 0.$$

In this case the error term can be written in the form

$$\begin{aligned} E(t) &:= t\,\alpha\big[(1-t)(x-y)\big] + (1-t)\,\alpha\big[t(y-x)\big] \\ &= c\,t\big\|(1-t)(x-y)\big\|^p + c\,(1-t)\big\|t(y-x)\big\|^p \\ &= c\,t(1-t)\big[(1-t)^{p-1}+t^{p-1}\big]\,\|x-y\|^p, \end{aligned}$$

hence we have

$$c\big(\min\{t,1-t\}\big)^p\|x-y\|^p \le E(t) \le 2\,c\,t(1-t)\|x-y\|^p.$$

Remark 1. Let X denote a real normed space, $D\subseteq X$ be a nonempty $\mathbb{F}$-convex set, $d=\operatorname{diam}(D^*)$, $\varphi:[0,d[\,\to\overline{\mathbb{R}}_+$ be an increasing function, and $f\colon D\to\mathbb{R}$ such that

$$f(tx+(1-t)y) \le t\,f(x)+(1-t)f(y)+\varphi(t(1-t)\|x-y\|)$$

holds for all $x,y\in D$ and $t\in[0,1]\cap\mathbb{F}$. Then f is $(\alpha,\mathbb{F})$-convex with

$$\alpha(u)=\varphi(\|u\|) \quad (u\in D^*).$$

In fact, we have

$$\varphi\big(t(1-t)\|x-y\|\big) = \big[t+(1-t)\big]\,\varphi\big(t(1-t)\|x-y\|\big)$$

$$
\begin{aligned}
&= t\varphi\big(t(1-t)\|x-y\|\big) + (1-t)\varphi\big(t(1-t)\|y-x\|\big)\\
&\le t\varphi\big((1-t)\|x-y\|\big) + (1-t)\varphi\big(t\|y-x\|\big).
\end{aligned}
$$

References

[1] F. Bernstein and G. Doetsch, Zur Theorie der konvexen Funktionen, *Math. Ann.*, **76** (1915), 514–526.

[2] Z. Boros and Zs. Páles, $\mathbb{Q}$-subdifferential of Jensen-convex functions, *J. Math. Anal. Appl.*, **321** (2006), 99–113.

[3] J. W. Green, Approximately convex functions, *Duke Math. J.*, **19** (1952), 499–504.

[4] A. Házy and Zs. Páles, On approximately midconvex functions, *Bull. London Math. Soc.*, **36** (2004), 339–350.

[5] D. H. Hyers and S. M. Ulam, Approximately convex functions, *Proc. Amer. Math. Soc.*, **3** (1952), 821–828.

[6] J. L. W. V. Jensen, Sur les fonctions convexes et les inégualités entre les valeurs moyennes, *Acta Math.*, **30** (1906), 175–193.

[7] J. Makó, K. Nikodem and Zs. Páles, On strong $(\alpha,\mathbb{F})$-convexity, *Math. Inequal. Appl.*, **15** (2012), 289–299.

[8] J. Makó and Zs. Páles, On ϕ-convexity, *Publ. Math. Debrecen*, **80** (2012), 107–126.

[9] N. Merentes and K. Nikodem, Remarks on strongly convex functions, *Aequationes Math.*, **80** (2010), 193–199.

[10] C. T. Ng and K. Nikodem, On approximately convex functions, *Proc. Amer. Math. Soc.*, **118** (1993), 103–108.

[11] Zs. Páles, On approximately convex functions, *Proc. Amer. Math. Soc.*, **131** (2003), 243–252.

[12] A. W. Roberts and D. E. Varberg, *Convex Functions*, Academic Press (New York–London, 1973).

[13] R. T. Rockafellar, *Convex analysis*, Princeton University Press, (Princeton, 1970).

[14] S. Rolewicz, On $\alpha(\cdot)$-paraconvex and strongly $\alpha(\cdot)$-paraconvex multifunctions, *Control Cybernet.*, **29** (2000), 367–377.

[15] S. Rolewicz, Paraconvex analysis, *Control Cybernet.*, **34** (2005), 951–965.

[16] Ja. Tabor and Jó. Tabor, Generalized approximate midconvexity, *Control Cybernet.*, **38** (2009), 656–669.

[17] Ja. Tabor and Jó. Tabor, Takagi functions and approximate midconvexity, *J. Math. Anal. Appl.*, **356** (2009), 729–737.

[18] Ja. Tabor, Jó. Tabor and M. Żołdak, Optimality estimations for approximately midconvex functions, *Aequationes Math.*, **80** (2010), 227–237.