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A NOTE ON STRONG β-I-SETS AND STRONGLY β-I-CONTINUOUS FUNCTIONS

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Abstract. In [6], we introduced and investigated the notions of strong β-I-open sets and strong β-I-continuous functions in ideal topological spaces. In this paper, we investigate further their important properties.

1. Introduction

Throughout the present paper, spaces always mean topological spaces on which no separation property is assumed unless explicitly stated. In a topological space (X, τ), the closure and the interior of any subset A of X will be denoted by $\mathrm{Cl}\,(A)$ and $\mathrm{Int}\,(A)$, respectively. An ideal is defined as a nonempty collection I of subsets of X satisfying the following two conditions: (1) If $A \in I$ and $B \subset A$, then $B \in I$; (2) If $A \in I$ and $B \in I$, then $A \cup B \in I$. Let (X, τ) be a topological space and I an ideal of subsets of X. An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I). For a subset $A \subset X$, $A^*(I) = \{x \in X \mid U \cap A \notin I$ for each neighbourhood U of $x\}$ is called the local function of A with respect to I and τ [11]. X^* is often a proper subset of X. The hypothesis $X = X^*$ [8] is equivalent to the hypothesis $\tau \cap I = \{\emptyset\}$ [14]. The ideal topological spaces which satisfy this hypothesis are called *Hayashi–Samuels spaces.* We simply write A^* instead of $A^*(I)$ in case there is no chance for confusion. For every ideal topological space (X, τ, I), there exists a topology $\tau^*(I)$, finer than τ, generated by $\beta(I, \tau) = \{U \backslash I \mid U \in \tau$ and $I \in I\}$, but in general $\beta(I, \tau)$ is not always a topology [10]. Additionally, $\mathrm{Cl}^*(A) = A \cup A^*$ defines a Kuratowski closure operator for $\tau^*(I)$.

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2. Preliminaries

Lemma 1 (Janković and Hamlett [10]). *Let (X,τ,I) be an ideal topological space and A, B subsets of X. Then the following properties hold:*

a) *If $A\subset B$, then $A^*\subset B^*$.*
b) $A^*=\mathrm{Cl}\,(A^*)\subset \mathrm{Cl}\,(A)$.
c) $(A^*)^*\subset A^*$.
d) *If $U\in\tau$, then $U\cap A^*\subset (U\cap A)^*$.*

Definition 1. A subset A of an ideal topological space (X,τ,I) is said to be

a) *I-open* [1] if $A\subset \mathrm{Int}\,(A^*)$,
b) *strong β-I-open* [6] $A\subset \mathrm{Cl}^*\big(\mathrm{Int}\,\big(\mathrm{Cl}^*(A)\big)\big)$.

The subset A is said to be strong β-I-closed if $(X-A)$ is strong β-I-open.

Lemma 2 (Hatir et al. [6]). *Let (X,τ,I) be an ideal topological space and $\{A_\alpha:\alpha\in\Delta\}$ a family of subsets of X. Then the following properties hold:*

a) *If $\{A_\alpha:\alpha\in\Delta\}\subset s\beta I(X,\tau)$, then $\bigcup\{A_\alpha:\alpha\in\Delta\}\in \beta I(X,\tau)$.*
b) *If $A\in s\beta I(X,\tau)$ and $U\in\tau$, then $(U\cap A)\in s\beta I(X,\tau)$.*

By $s\beta I(X,\tau)$, we denote the family of all strong β-I-open sets of (X,τ,I). We obtain a slight improvement of Lemma 2b) in Section 3.

Definition 2 (Husain [9]). A set X with a family $U\subset P(X)$ is called supratopological space, if U contains X, $\emptyset$ and is closed under arbitrary union.

3. Strong β-I-open sets

Theorem 1. *For a Hayashi–Samuels space (X,τ,I), the class $s\beta I(X,\tau)$ forms a supratopology.*

Proof. This follows by using Definition 2 and the fact that $\emptyset^*=\{x\in X\mid \emptyset\cap U\notin I$ for each neighbourhood U of $x\}$ (given in [10]) and by definition of Hayashi–Samuels space with Lemma 2. $\square$

Specially, if $I=N$, where N is the ideal of all nowhere dense sets, the above theorem holds for an ideal topological space (X,τ,N). Because, in the ideal topological space (X,τ,N), X coincides with its local function i.e. X^*. This is given in [10].

DEFINITION 3 (Dontchev [2]). A subset S of a space (X, τ, I) is a topological space with an ideal $I_S = \{I \in I \mid I \subseteqq S\} = \{I \cap S \mid I \in I\}$ on S.

LEMMA 3 (Dontchev et al. [3]). *Let (X, τ, I) be an ideal topological space and $A \subset S \subset X$. Then, $A^*(I_S, \tau \mid S) = A^*(I, \tau) \cap S$ holds.*

DEFINITION 4. Let (X, τ, I) be an ideal topological space and $A \subset S \subset X$. Then, $\mathrm{Cl}_S^*(A) = A \cup A^*(I_S, \tau \mid S)$ is a Kuratowski closure operator.

It is not difficult, however, to verify directly that Cl_S^*, where $\tau \mid S$ is the original topology on S, that is, $(\tau \mid S)^*(I_S) = \{U \subseteqq S \mid \mathrm{Cl}_S^*(S - U) = S - U\}$. When no ambiguity is present we will simply write $(\tau \mid S)^*$.

LEMMA 4. *Let A be a subset of an ideal topological space (X, τ, I).*
a) *If $U \in \tau$, then $U \cap \mathrm{Cl}^*(A) \subset \mathrm{Cl}^*(U \cap A)$.*
b) *If $A \subset S \subset X$, then $\mathrm{Cl}_S^*(A) = \mathrm{Cl}^*(A) \cap S$.*

PROOF. a) Since $U \in \tau$, by Lemma 1 we obtain $U \cap \mathrm{Cl}^*(A) = U \cap (A \cup A^*) = (U \cap A) \cup (U \cap A^*) \subset (U \cap A) \cup (U \cap A)^* = \mathrm{Cl}^*(U \cap A)$.

b) It is shown in [3, Lemma 2.7] that $A^*(\tau\mid_S, I_S) = A^*(\tau, I) \cap S$. Therefore, we obtain $\mathrm{Cl}_S^*(A) = A^*\big(\tau|_S, I_S\big) \cup A = (A^* \cap S) \cup A = (A^* \cap S) \cup (A \cap S) = (A \cup A^*) \cap S = \mathrm{Cl}^*(A) \cap S$ and hence $\mathrm{Cl}_S^*(A) = \mathrm{Cl}^*(A) \cap S$. □

PROPOSITION 1. *Let (X, τ, I) be an ideal topological space. If $U \in \tau$ and $A \in s\beta I(X, \tau)$, then $(A \cap U) \in s\beta I\big(U, \tau|_U\big)$.*

PROOF. By using Lemmas 1 and 4, we obtain the following. Since $U \in \tau$ and $A \in s\beta I(X, \tau)$, then $A \subset \mathrm{Cl}^*\big(\mathrm{Int}\big(\mathrm{Cl}^*(A)\big)\big)$. Therefore, we have

$$(U \cap A) \subseteqq \big(U \cap \mathrm{Cl}^*\big(\mathrm{Int}\big(\mathrm{Cl}^*(A)\big)\big)\big) \subset \mathrm{Cl}^*\big(U \cap \big(\mathrm{Int}\big(\mathrm{Cl}^*(A)\big)\big)\big) \cap U$$

$$= \mathrm{Cl}_U^*\big(U \cap \big(\mathrm{Int}\big(\mathrm{Cl}^*(A)\big)\big)\big) = \mathrm{Cl}_U^*(\mathrm{Int}_U\big(U \cap \big(\mathrm{Int}\big(\mathrm{Cl}^*(A)\big)\big)\big)$$

$$\subset \mathrm{Cl}_U^*\big(\mathrm{Int}_U\big(U \cap \mathrm{Cl}^*(A)\big)\big) \subset \mathrm{Cl}_U^*\big(\mathrm{Int}_U\big(\mathrm{Cl}^*(U \cap A) \cap U\big)\big)$$

$$= \mathrm{Cl}_U^*\big(\mathrm{Int}_U\big(\mathrm{Cl}_U^*(U \cap A)\big)\big).$$

Thus, $(A \cap U) \subset \mathrm{Cl}_U^*\big(\mathrm{Int}_U\big(\mathrm{Cl}_U^*(U \cap A)\big)\big)$ and hence $(A \cap U) \in s\beta I\big(U, \tau|_U\big)$. □

PROPOSITION 2. *Let (X, τ, I) be an ideal topological space. If $A \subset U \subset X$, $U \in \tau$ and $A \in s\beta I\big(U, \tau|_U\big)$, then $A \in s\beta I(X, \tau)$.*

Proof. Since $A \in s\beta I(U, \tau|_U)$, we have $A \subsetneqq \mathrm{Cl}_U^*\left(\mathrm{Int}_U\left(\mathrm{Cl}_U^*(A)\right)\right)$. Thus, by using Lemma 4, we obtain that

$$A \subsetneqq \mathrm{Cl}_U^*\left(\mathrm{Int}_U\left(\mathrm{Cl}_U^*(A)\right)\right) \subset \mathrm{Cl}^*\left(\mathrm{Int}_U\left(\mathrm{Cl}_U^*(A)\right)\right)$$

$$= \mathrm{Cl}^*\left(\mathrm{Int}\left(\mathrm{Cl}_U^*(A)\right)\right) \subset \mathrm{Cl}^*\left(\mathrm{Int}\left(\mathrm{Cl}^*(A)\right)\right).$$

Thus, we obtain $A \subset \mathrm{Cl}^*\left(\mathrm{Int}\left(\mathrm{Cl}^*(A)\right)\right)$ and hence $A \in s\beta I(X, \tau)$. □

Corollary 1. *Let (X, τ, I) be an ideal topological space and $A \subset U \in \tau$. Then $A \in s\beta I(X, \tau)$ if and only if $A \in s\beta I(U, \tau|_U)$.*

Proof. This is an immediate consequence of Propositions 1 and 2. □

A subset A of an ideal topological space (X, τ, I) is said to be an α-I-open set [7] if $A \subset \mathrm{Int}\left(\mathrm{Cl}^*\left(\mathrm{Int}\,(A)\right)\right)$. Every open set is α-I-open but the converse is not necessarily true as shown by the following simple example:

Let $X = \{a, b, c, \}$, $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$ and $I = \{\emptyset\}$. Then the subset $\{a, c\}$ is an α-I-open set which is not open. Therefore, the following proposition is an improvement of Lemma 2b).

Proposition 3. *Let (X, τ, I) be an ideal topological space. If U is α-I-open and $A \in s\beta I(X, \tau)$, then $(U \cap A) \in s\beta I(X, \tau)$.*

Proof. Let U be α-I-open and $A \in s\beta I(X, \tau)$. Then

$$U \subset \mathrm{Int}\left(\mathrm{Cl}^*\left(\mathrm{Int}\,(U)\right)\right) \quad \text{and} \quad A \subset \mathrm{Cl}^*\left(\mathrm{Int}\left(\mathrm{Cl}^*(A)\right)\right)$$

and hence by using Lemma 4, we have

$$(U \cap A) \subset \left[\mathrm{Int}\left(\mathrm{Cl}^*\left(\mathrm{Int}\,(U)\right)\right) \cap \mathrm{Cl}^*\left(\mathrm{Int}\left(\mathrm{Cl}^*(A)\right)\right)\right]$$

$$\subset \mathrm{Cl}^*\left[\mathrm{Int}\left(\mathrm{Cl}^*\left(\mathrm{Int}\,(U)\right)\right) \cap \left(\mathrm{Int}\left(\mathrm{Cl}^*(A)\right)\right)\right]$$

$$\subset \mathrm{Cl}^*\left(\mathrm{Int}\left[\mathrm{Cl}^*\left(\mathrm{Int}\,(U)\right) \cap \left(\mathrm{Int}\left(\mathrm{Cl}^*(A)\right)\right)\right]\right)$$

$$\subset \mathrm{Cl}^*\left(\mathrm{Int}\left(\mathrm{Cl}^*\left[\mathrm{Int}\,(U) \cap \left(\mathrm{Int}\left(\mathrm{Cl}^*(A)\right)\right)\right]\right)\right)$$

$$\subset \mathrm{Cl}^*\left(\mathrm{Int}\left(\mathrm{Cl}^*\left[\mathrm{Int}\,(U) \cap \left(\mathrm{Cl}^*(A)\right)\right]\right)\right)$$

$$\subset \mathrm{Cl}^*\left(\mathrm{Int}\left(\mathrm{Cl}^*\left[\mathrm{Cl}^*\left(\mathrm{Int}\,(U) \cap A\right)\right]\right)\right) \subset \mathrm{Cl}^*\left(\mathrm{Int}\left(\mathrm{Cl}^*(U \cap A)\right)\right).$$

Thus, we obtain that $(U \cap A) \subset \mathrm{Cl}^*\left(\mathrm{Int}\left(\mathrm{Cl}^*(U \cap A)\right)\right)$. This shows that $(U \cap A) \in s\beta I(X, \tau)$. □

DEFINITION 5. A subset A of an ideal topological space (X, τ, I) is said to be

a) semi-I-open [7] if $A \subset \mathrm{Cl}^*\big(\mathrm{Int}\,(A)\big)$,

b) pre-I-open [4] if $A \subset \mathrm{Int}\big(\mathrm{Cl}^*(A)\big)$.

REMARK 1. In Proposition 3, the assumption "α-I-open" on U cannot be replaced by semi-I-open nor by pre-I-open instead of α-I-open as the following examples show respectively.

EXAMPLE 1. Let $X = \{a, b, c, d, e\}$, $\tau = \big\{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\big\}$ and $I = \big\{\emptyset, \{b\}\big\}$. Then $A = \{a, d, e\}$ is a semi-I-open set and $B = \{c, d\}$ is a strong β-I-open set. However, $C = A \cap B = \{d\}$ is not a strong β-I-open set. For $A = \{a, d, e\}$, since $\mathrm{Int}\,(A) = \{a\}$ and $\big(\mathrm{Int}\,(A)\big)^* = \{a, b, d, e\}$, $\mathrm{Cl}^*\big(\mathrm{Int}\,(A)\big) = \mathrm{Int}\,(A) \cup \big(\mathrm{Int}\,(A)\big)^* = \{a, b, d, e\} \supset \{a, d, e\} = A$. Hence, A is a semi-I-open set. For $B = \{c, d\}$, since $B^* = \{c, d, e\}$ and $\mathrm{Cl}^*(B) = B \cup B^* = \{c, d, e\}$, we have $\mathrm{Int}\big(\mathrm{Cl}^*(B)\big) = \{c\}$ and $\big(\mathrm{Int}\big(\mathrm{Cl}^*(B)\big)\big)^* = \{c, d, e\}$. Therefore, we obtain $\mathrm{Cl}^*\big(\mathrm{Int}\big(\mathrm{Cl}^*(B)\big)\big) = \mathrm{Int}\big(\mathrm{Cl}^*(B)\big) \cup \big(\mathrm{Int}\big(\mathrm{Cl}^*(B)\big)\big)^* = \{c, d, e\} \supset \{c, d\} = B$. Hence, B is a strong β-I-open set. Consequently, for $C = A \cap B = \{d\}$, since $C^* = \big(\{d\}\big)^* = \{d, e\}$ and $\mathrm{Cl}^*(C) = C \cup C^* = \{d, e\}$, we have $\mathrm{Int}\big(\mathrm{Cl}^*(C)\big) = \mathrm{Int}\big(\{d, e\}\big) = \emptyset$. Since $\emptyset^* = \emptyset$, we have $\big(\mathrm{Int}\big(\mathrm{Cl}^*(C)\big)\big)^* = \emptyset$ and

$$\mathrm{Cl}^*\big(\mathrm{Int}\big(\mathrm{Cl}^*(C)\big)\big) = \big(\mathrm{Int}\big(\mathrm{Cl}^*(C)\big)\big) \cup \big(\mathrm{Int}\big(\mathrm{Cl}^*(C)\big)\big)^* = \emptyset$$

and hence $C \subsetneq \mathrm{Cl}^*\big(\mathrm{Int}\big(\mathrm{Cl}^*(C)\big)\big)$. This shows that C is not strong β-I-open.

EXAMPLE 2. Let $X = \{a, b, c, d\}$, $\tau = \big\{\emptyset, X, \{a, c\}, \{d\}, \{a, c, d\}\big\}$ and $I = \big\{\emptyset, \{b\}\big\}$. Then $A = \{b, c, d\}$ is a pre-I-open set and $B = \{a, b\}$ is a strong β-I-open set. However, $C = A \cap B = \{b\}$ is not a strong β-I-open set. For $A = \{b, c, d\}$, since $A^* = X$ and $\mathrm{Cl}^*(A) = A \cup A^* = X$, we have $\mathrm{Int}\big(\mathrm{Cl}^*(A)\big) = X \supset A$. Hence, A is a pre-I-open set. For $B = \{a, b\}$, since $B^* = \{a, b, c\}$ and $\mathrm{Cl}^*(B) = B \cup B^* = \{a, b, c\}$, we have $\mathrm{Int}\big(\mathrm{Cl}^*(B)\big) = \{a, c\}$. Therefore, we have $\big(\mathrm{Int}\big(\mathrm{Cl}^*(B)\big)\big)^* = \{a, b, c\}$ and $\mathrm{Cl}^*\big(\mathrm{Int}\big(\mathrm{Cl}^*(B)\big)\big) \supset B$ and hence B is a strong β-I-open set. Consequently, for $C = A \cap B = \{b\}$, since $C^* = \big(\{b\}\big)^* = \emptyset$ and $\mathrm{Cl}^*(C) = C \cup C^* = \{b\}$, we have $\mathrm{Int}\big(\mathrm{Cl}^*(C)\big) = \mathrm{Int}\big(\{b\}\big) = \emptyset$. Since $\emptyset^* = \emptyset$, we have $\big(\mathrm{Int}\big(\mathrm{Cl}^*(C)\big)\big)^* = \emptyset$ and $\mathrm{Cl}^*\big(\mathrm{Int}\big(\mathrm{Cl}^*(C)\big)\big) = \big(\mathrm{Int}\big(\mathrm{Cl}^*(C)\big)\big) \cup \big(\mathrm{Int}\big(\mathrm{Cl}^*(C)\big)\big)^* = \emptyset$ and hence $C \subsetneq \mathrm{Cl}^*\big(\mathrm{Int}\big(\mathrm{Cl}^*(C)\big)\big)$. This shows that C is not strong β-I-open.

4. Strongly β-I-continuous functions

In [6], we defined a function $f : (X, \tau, I) \to (Y, \sigma)$ to be strongly-β-I-continuous if for every $V \in \sigma$, $f^{-1}(V)$ is a strong β-I-open set of (X, τ, I).

Theorem 2. *For a function $f : (X, \tau, I) \to (Y, \sigma)$, the following properties are equivalent:*

a) *f is strongly-β-I-continuous.*

b) *The inverse image of each closed set in (Y, σ) is strong β-I-closed.*

c) *For each $x \in X$ and $V \in \sigma$ containing $f(x)$, there exists $U \in s\beta I(X, \tau)$ containing x such that $f(U) \subset V$.*

Proof. The proof is obvious from Lemma 2 and is thus omitted. $\square$

Theorem 3. *The restriction of a strongly-β-I-continuous function to an open set is also strongly-β-I-continuous.*

Proof. Let $f : (X, \tau, I) \to (Y, \sigma)$ be a strongly-β-I-continuous function and $U \in \tau$. We show that $f|_U$ is strongly-β-I-continuous. Let $V \in \sigma$, then since f is strongly-β-I-continuous, f^{-1}(V)$\in s\beta I(X, \tau)$. On the other hand, since $U \in \tau$, we have $\big(f|_U\big)^{-1}(V) = \big(U \cap f^{-1}(V)\big) \in s\beta I\big(U, \tau|_U\big)$ by using Proposition 1. Consequently, $f|_U$ is strongly-β-I-continuous. $\square$

Theorem 4. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is strongly-β-I-continuous if and only if for any open cover $\{U_\alpha : \alpha \in \Delta\}$ of X, the restriction $f|_{U_\alpha} : \big(U_\alpha, \tau|_{U_\alpha}, I_{U_\alpha}\big) \to (Y, \sigma)$ is strongly-β-I-continuous for each $\alpha \in \Delta$.*

Proof. This follows immediately by using Lemma 2, Proposition 2 and Theorem 3. $\square$

Theorem 5. *A function is strongly-β-I-continuous if and only if its graph function is strongly-β-I-continuous.*

Proof. *Necessity.* Let $f : (X, \tau, I) \to (Y, \sigma)$ be strongly-β-I-continuous, $x \in X$ and H an open set in $X \times Y$ containing $g(x)$. Then, there exist $U \in \tau$ and $V \in \sigma$ such that $g(x) = \big(x, f(x)\big) \in U \times V \subset H$. By hypothesis, there exists $W \in s\beta I(X, \tau)$ containing x such that $f(W) \subset V$. We have $x \in (U \cap W) \in s\beta I(X, \tau)$ by using Lemma 2b). So, $\big((U \cap W) \times V\big) \subset U \times V \subset H$ and hence $g(U \cap W) \subset H$. This shows that $g : (X, \tau, I) \to (X \times Y, \tau \times \sigma)$ is strongly-β-I-continuous by Theorem 2.

Sufficiency. Let $x \in X$ and $V \in \sigma$ containing $f(x)$, then $g(x) \in (X \times V) \in \tau \times \sigma$. Since g is strongly-β-I-continuous, there exists $W \in s\beta I(X, \tau)$ containing x such that $g(W) \subset X \times V$. Therefore, we obtain $f(W) \subset V$, because $g(W) = \big(W, f(W)\big)$. Hence f is strongly-β-I-continuous by Theorem 2. $\square$

PROPOSITION 4. *If $f : (X, \tau, I) \to (Y, \sigma, J)$ is a strongly-β-I-continuous function and $g : (Y, \sigma, J) \to (Z, \psi)$ is a continuous function, then $g \circ f : (X, \tau, I) \to (Z, \psi)$ is a strongly-β-I-continuous function.*

PROOF. Let $V \in \psi$. Since g is continuous, then $g^{-1}(V) \in \sigma$. On the other hand, since f is strongly-β-I-continuous, we have $f^{-1}\big(g^{-1}(V)\big) \in s\beta I(X, \tau)$. Since $(g \circ f)^{-1}(V) = f^{-1}\big(g^{-1}(V)\big)$, we obtain that $g \circ f$ is strongly-β-I-continuous. $\square$

5. Strongly β-I-compact spaces

DEFINITION 6. An ideal topological space (X, τ, I) is said to be strongly β-I-compact (resp. I-compact [12] and [13]) if for every strong β-I-open (resp. I-open) cover $\{W_\alpha : \alpha \in \Delta\}$, there exists a finite subset Δ_0 of Δ such that $\big(X - \bigcup\{W_\alpha : \alpha \in \Delta_0\}\big) \in I$.

LEMMA 5 (Hamlett and Janković [5]). *For any surjective function $f : (X, \tau, I) \to (Y, \sigma)$, $f(I)$ is an ideal on Y.*

THEOREM 6. *The image of a strongly β-I-compact space under a strongly-β-I-continuous surjection is $f(I)$-compact.*

PROOF. Let $f : (X, \tau, I) \to (Y, \sigma)$ be a strongly-β-I-continuous surjection and $\{V_\alpha : \alpha \in \Delta\}$ be an open cover of Y. Then, $\big\{f^{-1}(V_\alpha) : \alpha \in \Delta\big\}$ is a strong β-I-open cover of X. From the assumption, there exists a finite subset Δ_0 of Δ such that $\big(X - \bigcup\big\{f^{-1}(V_\alpha) : \alpha \in \Delta_0\big\}\big) \in I$. Therefore, $\big(Y - \bigcup\{V_\alpha : \alpha \in \Delta_0\}\big) \in f(I)$ which shows that $\big(Y, \sigma, f(I)\big)$ is $f(I)$-compact. $\square$

THEOREM 7. *Every strongly β-I-compact space is I-compact.*

PROOF. The proof is obvious since every I-open set is strong β-I-open. $\square$

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