

# Costly information providing in binary contests

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# Abstract

Contests are commonly used as a mechanism for eliciting effort and participation in multiagent settings. Naturally, and much like with various other mechanisms, the information provided to the agents prior to and throughout the contest fundamentally influences its outcomes. In this paper we study the problem of information providing whenever the contest organizer does not initially hold the information and obtaining it is potentially costly. As the underlying contest mechanism for our model we use the binary contest, where contestants' strategy is captured by their decision whether or not to participate in the contest in the first place. Here, it is often the case that the contest organizer can proactively obtain and provide contestants information related to their expected performance in the contest. We provide a comprehensive equilibrium analysis of the model, showing that even when such information is costless, it is not necessarily the case that the contest organizer will prefer to obtain and provide it to all agents, let alone when the information is costly.

Keywords Contest design · Information disclosure · Mechanism design · Equilibrium

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# **1** Introduction

Many economic, political and social environments can be described as contests where parties expand some resources in order to win a prize or control in resources. In recent decades, contests are growingly being used as a mechanism for eliciting work (e.g., effort, ideas) from

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individuals rather than (or in addition to) a means for determining the best contestant. Well known examples include the Netflix prize (netflixprize.com), DARPA challenges [2] and the Hult prize (hultprize.org). As such, the study and analysis of contests have become prominent in mechanism design and multi-agent systems literature [3–8], focusing primarily on how to design the payoffs structure [9, 10]—that is, how to set the prize budget and its division among contestants to optimize expected performance. Another important aspect of contest design deals with information disclosure. In many cases the contest organizer is more informed than the potential participants in the contest, hence has some control on the information to be disclosed. As such, prior work has considered various information providing schemes [11–14], analyzing their influence on contest (e.g. Tullock), where performance is tightly correlated with the amount of effort exerted hence, naturally, the type of information considered related to one's cost of exerting effort in the contest or disambiguation nature's state.

In this paper we focus on the provision of information relating to contestants' own performance in the contest, whenever such information is a priori uncertain. This is typically the case in contests known as *binary contests*. In a binary contest prospective contestants need to decide whether to participate in the contest in the first place, as opposed to deciding how much effort to exert in the contest, where participation is costly [5, 6, 15–17]. The underlying assumption in binary contest is that if deciding to participate, a contestant will give its best in terms of effort, as it is the participation itself that incurs cost rather than the amount of effort exerted. This is the case in beauty contests, sport contests, applications for academic awards, applying for a post-doc, etc.

Most research on binary contests to date relates to two primary model variants: one where prospective contestants' performance in the contest (and consequently their winning probability if participating) are a priori unknown [6, 16, 18, 19], and the other where each contestant knows in advance its expected performance should participating [5, 9]. These are actually two specific extreme cases of our model, mapping to not providing any of the contestants information about their expected performance in the contest and obtaining (and providing) such information to all contestants. Our model postulates that information about a contestant's performance is not a priori available to the contest organizer, however she can acquire (or make an effort to obtain) it. Meaning that in order to provide such information can be obtained from an external source (i.e., expert) or when the organizer needs to collect and pre-process data about the contestant (rather than receive it directly from the contestant when participating in the contest) in order to predict its performance. The organizer therefore needs to decide on the subset of potential contestants, to whom she should acquire and provide the information.

In the following sections we review related work, formally describe the model, and provide a detailed description of a game-theoretic-based analysis of the model. We show that in equilibrium the participation decision of contestants that do not receive information is probablistic, whereas the decision of those receiving the information is threshold-based. This complicates the equilibrium calculation (compared to the above mentioned specific model variants), as both the participation probability to be used by the non-informed participants and the participation threshold to be used by the informed participants influence each other. We use numerical analysis to show that even when obtaining the information is costless, the contest organizer will often find it most rewarding to obtain (and provide) the information only for some of the participants, let alone when the information is costly. Furthermore, we show that as long as the organizer sets the reward rationally, providing such information to a subset of the participants increases the average (cross all participants) benefit and the overall social welfare. While prior work has shown that partial information providing often dominates full disclosure, this was usually demonstrated in effort-based contests, i.e., when the information provided relates to the cost of exerting effort or aim to reveal the true nature's state. When it comes to information related to contestants' performance, providing all contestants with such information (even if acquiring it is costless) is somehow non-intuitive. This is because the essence of the contest is in revealing the participant associated with the highest performance and luring it to take part in the contest. Hence such information which enables agents reason about their actual performance if taking part in the contest is intuitively beneficial.

### 2 Related work

Contests are organizational structures in which contestants spend costly efforts (e.g., time, resources) to win one or more prizes [20]. Much interest in literature focuses on contest design, i.e., the settings that define a contest [5, 21, 22], differing primarily in the assumptions made in the underlying contest model, e.g., offering several prizes [23–25], using more than a single stage (most commonly in the form of a tournament [26–28] or in the form of subcontests [29–31]) and the contest organizer's goals (e.g., maximizing overall effort, best effort, fairness) [32, 33].

Traditionally, contest models have considered the decision problem of contestants to be the amount of effort (or money) they exert in the contest, as this directly influences their performance in the contest (and consequently their chance of winning) [4, 22, 34–37]. The study of such effort-based contests constitutes the majority of literature on contest design. In recent years there is a growing interest in the study of binary contests, which are the focus of our paper, where contestants only strategize on participation itself (as participation incurs a cost) rather than the amount of effort to exert [5, 7, 16, 17, 38–43]. This captures settings where the "quality" of the participating agents does not depend on effort (e.g. beauty contests), or where performance is pre-determined by the time evaluation takes place (e.g. life-time achievement award). The binary contest model also maps well to settings where upon deciding to participate in the contest, a contestant will do its best to shine and exert the maximum possible effort (e.g., in sports contests).

Several prior works have studied information providing in the context of binary contests, typically focusing in two-players settings and considering information that does not relate to the agent's own performance and capabilities. For example, Dubey [16] studies a two-players contests, where each player has one of two abilities (which it knows) and can choose whether to shirk (zero effort) or work (maximal effort). Each player's output depends upon its innate ability (over which he has no choice), and the effort he chooses to undertake. The author explores the effect of the awarded prize over the preference of having a contestant know the ability of its rival, and finds that to inspire performance, it is often better for the contest organizer neither to reveal all nor to conceal all, but to follow a middle path of partial revelation. Building on a similar model, Ponce [17] studies a two-agent binary contest with incomplete information regarding the abilities of the players. The contest designer can send private messages to players regarding the abilities of their rivals, according to some information disclosure rule which is common knowledge. The author shows that the optimal disclosure rule is partial information disclosure. Somehow tangential to the current work, some prior work has dealt with the design of temporal information provision, wherein information is revealed over time during the course of the contest. For example, Hinnosaar [44] studies a sequential contest in which the efforts of earlier contestants may be disclosed to later contestants and proves that in the case of homogeneity, full information revelation scheme is optimal. Levy et al. [45, 46] investigate temporal disclosure in sequential binary contests where some of the agents know the performances of some preceding agents and examine the effect of such design on the organizer's profit. They conclude that while information disclosure in binary contests can be beneficial, it is not necessarily the case that disclosing all available information is always the best strategy. Other works on binary contests have considered aspects of optimal prize allocation [5, 7] and computational aspects of equilibrium-calculation [47, 48].

Unlike the above cited work, our model enables the contest organizer to obtain and provide information that disambiguate the uncertainty related to agents' *own* performance and offer it only to a subset of the agents. To the best of our knowledge, such mechanism has not been proposed in prior work. As we manage to show in the following paragraphs, the organizer does not necessarily find it beneficial to obtain and provide this kind of information information to all agents.

More generally, information provision, and in particular the possible superiority of providing contestants with partial information regarding different aspects of the contest, can be found in the rich literature on effort-based contests. Here, however, the focus is primarily on information relating to the types of contestants, commonly modeled by their cost of exerting effort. Fu et al. [13] study a two-players contest where players are allowed to send a (costly) public message of confidence in winning the contest (which may disclose their private valuation of the prize) prior to the contest. They show that such pre-contest communication may deter the rival's entry into the contest and strategically manipulate its belief. Morath and Munster [11] study incentives for information acquisition ahead of a contest, focusing on the effect of whether the decision to acquire information is observable or not. Denter et al. [12] study information policies of contestants who can decide and commit to acquire information about their rival valuation of the prize or disclose their own private information. They show that mandatory disclosure policy can decrease social welfare. Zhang and Zhou [49] study a two-player contest, where one contestant's valuation is commonly known and the other has private valuation. The organizer can commit to a signal about the second player's valuation, before the contest begins. They reach the conclusion that the organizer should fully reveal information about contestants if they are good enough but otherwise keep some uncertainties. Kovenock et al. [14] study the incentives to share private information (e.g., signals related to the value of winning in the contest) ahead of the contest. They show that with independent values of winning the contest, expected effort is lower with than without information sharing. Letina et al. [50] study the optimal design of contests when the principal can choose both the prize profile and how the prizes are allocated to the agents as a function of a possibly noisy signal about their efforts. They show that with perfect observability of effort, an appropriately chosen nested Tullock contest is optimal. When efforts are imperfectly observed, they derive an upper bound on the level of noise such that an all-pay contest is optimal. Aich et al. [51] study a two-player Tullock contest with information advantage, and show that when one player has information advantage over the other (in terms of the realization of the state of nature which affects their cost of effort), the total effort exerted by the players is smaller. Einy et al. [52] also study a two-player contest with asymmetric information, where one player has an information advantage over its opponent in terms of the nature's state. They show that the highest expected total effort is obtained when the difference in the players information is as small as possible. In a parallel research, Einy et al. [53] study a Tullock contest with incomplete information, where the uncertainty is associated with the players' common value of the prize and each player's cost function. They show that if players' cost of effort is state independent, players' effort decreases with the level of information.

While the bottom line in the latter reviewed works is similar to the conclusion reached in the current paper, in the sense that partial information disclosure can be (at times) beneficial, the nature of information addressed and the mechanics that lead to the similar conclusion differ: in effort-based contests, the contestant himself controls its performance in the contest based on the amount of effort he exerts. The information provided thus typically assists in realizing the cost of exerting different levels of effort and the competence (in terms of cost of exerting efforts) of others. In binary contests, contestants have no control over their performance, hence the purpose of information is in having them realize their performance as a means for guiding their decision whether to participate in the contest in the first place.

Finally, we note that the idea of (selective) information providing has been extensively studied in recent years in the field of psychology and behavioral economics [54, 55] and in multi-agent literature [56–61]. Nevertheless, the ideas provided there do not trivially carry over to contest design.

# 3 The contest model

We rely on the standard binary contest model which is commonly used in prior work [5–7, 15–17, 19, 43, 46, 62, 63]. The model considers a contest organizer (denoted "organizer" onward) and a set  $A = \{A_1, ..., A_n\}$  of n > 1 potential contestants (denoted "agents" onward). Each agent  $A_i$  can either participate in the contest, incurring some cost c, <sup>1</sup> or opt to avoid participating in the contest. The performance of an agent if taking part in the contest is a priori unknown and determined according to some probabilistic function f(x) (where F(x) is the corresponding cumulative distribution function). This latter assumption corresponds to settings where contestants are a priori typically alike (e.g., chess masters, gifted painters, professional Judo players, internationally renowned scientists) and their performance in the contest is mostly influenced by (commonly external) factors, such as weather, momentary focus, and subjective evaluation of referees.<sup>2</sup> The contest is executed in parallel, i.e., all agents make their participation decisions simultaneously, having no information related to their own performance, should they participate, or the expected performances of others.

Both the organizer and the agents are fully-rational and self-interested. To elicit participation, the organizer offers a prize M > 0 to the agent ranked first (performance-wise) in the contest.<sup>3</sup> If none of the agents participates, the prize is not awarded and the performance as perceived by the organizer is set to some preset fallback performance  $x_0$ , which, w.l.o.g. we normalize to zero. In line with previous work [5–7, 31], it is assumed that n, f(x), c and M are common knowledge, i.e., known to all agents and the organizer.

We extend the above model by enabling the contest organizer to obtain and consequently provide specific agents with some information that can help them disambiguate the uncertainty associated with their actual performance.<sup>4</sup> We assume that getting a hold of such information and providing it to the agents is costly. Specifically, the cost of obtaining and

<sup>&</sup>lt;sup>1</sup> e.g., time and money spent in getting to a contest, fee of participation, reputation loss if not winning etc.

 $<sup>^2</sup>$  As discussed later on, a modeling that uses a priori heterogeneous agents merely requires simple adaptation to the equations representing the agents' best response, and does not add much insight, qualitatively.

<sup>&</sup>lt;sup>3</sup> Due to the continuous nature of f(x), the chance of a tie is negligible and can be ignored.

<sup>&</sup>lt;sup>4</sup> This does not necessarily mean the organizer becomes a priori acquainted with the performance of the agents receiving the information—the model only requires that the information provided enables the receiving agent to extract its performance if taking part in the contest.

providing such information to a subset A' of agents is  $c^{org} : A' \subseteq A \rightarrow R_{\geq 0}$ ,<sup>5</sup> which is a monotone increasing function of |A'|. We assume the organizer is truthful and does not hold the information to herself once obtaining it [7, 44, 64–66]. This can be because only the agent receiving the information can use it to extract its own performance (i.e., the information complements other private information available to the agent, e.g., the information relates to the efficiency of some specific equipment, and only the agent knows if it is planning to use such equipment in the contest). However, even if the information enables the organizer calculating the actual performance of the agent, it is quite common that "full disclosure" regulatory requirements force the organizer to disclose to each participating individual all the information she holds related to that individual. Hence, if obtaining information related to a specific agent  $A_i$ , the organizer necessarily discloses that information to the agent (and only to that agent). We further assume that once the information was obtained and disclosed, all agents are aware of the identity of those that have become informed, though the information revealed to an agent remains unknown to others.

The goal of the organizer is to maximize her expected profit defined as the expected best performance of agents participating in the contest minus the prize awarded minus, if choosing to obtain information, the cost of obtaining the information. The goal of each agent is to maximize its own expected profit, defined as the expected prize it receives minus the cost incurred in case of participating in the contest.

A real-life example that maps to the above model is a National Renewable Energy Startup Grant Competition which focuses on innovative solutions in the renewable energy sector. Since this is a national contest, potential candidates, i.e., entrepreneurs developing new technologies for solar, wind, hydro, geothermal, and alike, are likely to be familiar with the list of prospective contestants from their community. The contest is binary - participants can either decide to participate (apply) or not participate. Applying involves participating in a pitch event, which incurs costs associated with preparation and traveling. Additionally, participants may need to invest in professional services like legal advice or consulting to refine their pitch. Naturally, participants do not know their true performance in the contest as they do not know how their ideas compare to other applicants. The contest organizer however can provide to some or all participants information that will reveal to them their expected performance in the contest. This can take the form of receiving an evaluation of their business plans and an estimate of their standing among the applicants. Producing such information is cosly in the sense that the information based on which the evaluation is made needs to be actively seek rather than receiving it from the contestant as part of the submission. Knowing the benefit of providing only some of the prospective applicants with such information, the committee can announce that it will provide such services only to a limited number of applicants which will be selected with a lottery (or the first N to register on the contest web-site). The list of participants receiving the information will become available on the web-site so that participants will know who received the information.

# 4 Analysis

We provide a comprehensive equilibrium analysis of the above contest model. This enables highlighting several interesting properties of the solution, in particular those relating to the

<sup>&</sup>lt;sup>5</sup> Since the agents are a priori homogeneous, a simpler representation is  $c^{org} : k \to R_{\geq 0}$ , as only the number (rather than identity) of agents becoming informed matters. Still, in Section 5 we extend the analysis to the case of heterogeneous agents, where the specific subset of informed agents matters, hence we leave  $c^{org}$  in its more general form.

subset of agents for which information should be obtained, and the expected profit of the organizer and the agents.

An equilibrium specifies the subset of agents the organizer chooses to inform (i.e., to obtain and disclose information to) and the participation decisions of all n agents (both informed and non-informed). From the organizer's point of view, this is a Stackelberg game, where she is the first mover, picking the number of informed agents k, and the agents are the responders, setting their strategies accordingly.

### 4.1 Agents' strategies

We use  $\{P, \neg P\}$  to denote the actions available to each agent, where *P* stands for participating in the contest and  $\neg P$  for not participating. Similarly, we use  $B_i^P$  and  $B_i^{\neg P}$  to denote the expected profit of agent  $A_i$  when participating and when not participating, respectively. An agent's best-response strategy in our model depends on the information available to it at the time of making the participation decision.

Consider agent  $A_i \in A$  and recall that the agent ranked first (performance-wise) in the contest is awarded a prize M > 0. We use  $S_{-i}$  to denote the strategies (in the form of participation decisions) of all other agents  $A_j \in A$ ,  $j \neq i$ . If not receiving information from the organizer then the agent's best response strategy is necessarily probabilistic and captured by some participation probability  $0 \leq p_i \leq 1$  (where  $p_i \in \{0, 1\}$  indicates a pure strategy). If receiving information, then the agent becomes "informed" and should rely on this information to make its participation decision. In this case, we turn to the expected profit of  $A_i$  if participating, given its performance x, denoted  $B_i^P(x)$ .<sup>6</sup> Here, the expected-profitmaximizing strategy is threshold-based as we show in the following proposition.

**Proposition 1** When information regarding the performance of an agent is disclosed to *it*, the agent's best-response strategy given the information received *t* is threshold-based. Meaning that the agent will set a threshold *T* and choose to participate in the contest only if  $t \ge T$ .

**Proof** Assume otherwise, i.e., given  $S_{-i}$ , the best-response strategy of an informed agent  $A_i$  is to participate when its performance is t and not participate when it is t' > t. Denote the winning probability of an informed agent  $A_i$ , given  $S_{-i}$  and its performance x, by  $p_i^{Win}(x)$ . In order to win the contest while associated with performance x, it is required that all other contestants that choose to participate will be associated with performance smaller than x. Meaning that  $p_i^{Win}(x)$  increases in x. Using  $p_i^{Win}(x)$  we can express  $B_i^P(x)$ :

$$B_i^P(x) = p_i^{Win}(x) \cdot M - c \tag{1}$$

If not participating, the expected profit of the agent is  $B_i^{\neg P} = 0$ ,  $\forall x$ . From t' > t we obtain that  $p_i^{Win}(t') \ge p_i^{Win}(t)$ , hence  $B_i^P(t') \ge B_i^P(t)$ . Since the best response strategy of the agent is to participate when its performance is t then the following must hold:  $B_i^P(t) \ge B_i^{\neg P}(t) = 0$ , and similarly for t':  $B_i^P(t') < B_i^{\neg P}(t') = 0$ . We obtain:  $B_i^P(t) \ge B_i^{\neg P}(t) = 0 > B_i^P(t')$  which is a contradiction.

Since the agents are a priori alike, we aim for the symmetric equilibrium, i.e., one where the informed agents use the same threshold T and the non-informed agents use the same

<sup>&</sup>lt;sup>6</sup> As opposed to  $B_i^P$ , which is the a priori expected profit, i.e., when the performance of the agent in the contest is still unknown.

participation probability p. Naturally this equilibrium can coexist alongside other equilibria, yet this type is the most natural and fair one and therefore the one we relate to in our analysis as well as in the numerical illustration.

Recall that k is the number of informed agents. We can formally express the probability for an informed agent  $A_i$  receiving information that its performance is t will win the contest and be awarded the prize M:

$$p_i^{Win}(t) = \begin{cases} F(T)^{(k-1)} \cdot \sum_{w=0}^{n-k} {n-k \choose w} p^w (1-p)^{n-k-w} F(t)^w & t \le T \\ F(t)^{(k-1)} \cdot \sum_{w=0}^{n-k} {n-k \choose w} p^w (1-p)^{n-k-w} F(t)^w & t > T \end{cases}$$
(2)

The above calculation is based on the fact that in order for  $A_i$  to win, all other k - 1 informed agents need to have a value smaller than t when t > T, or a value smaller than T if  $t \le T$  (in which case they opt not to participate). The probability of this event is  $F(T)^{k-1}$  in case  $t \le T$ , and  $F(t)^{k-1}$  otherwise. As for the remaining n - k non-informed agents, these are divided to  $w \le n - k$  agents that participate and n - k - w agents that do not participate, with probability  $\binom{n-k}{w}p^w(1-p)^{n-k-w}$ . In order for  $A_i$  to outrank them, we need the w agents that participate to be associated with performance values smaller than t, which happens with probability  $F(t)^w$ .

Now consider any non-informed agent  $A_i$ . Here, again, we can formally express the probability that the agent will win and be awarded the prize M should it participate and its performance (at retrospect) is realized to be t, denoted  $\hat{p}^{Win}(t)$ .<sup>7</sup>

$$\hat{p}^{Win}(t) = \begin{cases} F(T)^k \cdot \sum_{w=0}^{n-k-1} \binom{n-k-1}{w} p^w (1-p)^{n-k-w-1} F(t)^w & t \le T \\ F(t)^k \cdot \sum_{w=0}^{n-k-1} \binom{n-k-1}{w} p^w (1-p)^{n-k-w-1} F(t)^w & t > T \end{cases}$$
(3)

We note that our proposed model structure defines two extremes. The first is where k = n, i.e., the organizer obtains and discloses the performance of *all* agents, and so each agent is informed. The best-response strategy of agents in this case is purely threshold-based, and so the probability an agent with performance *t* will win the contest (given in (2)) is simply  $F(\max(T, t))^{n-1}$ , i.e.,  $A_i$  is awarded the prize *M* whenever all other agents' performances are either below *T* (in which case they opt not to participate) or below *t* (in which case they lose). A comprehensive analysis of this specific variant can be found in the work of Ghosh and Kleinberg [5].

The second extreme is where k = 0, i.e., information is not obtained and so at the time of making their participation decisions all agents rely only on their a priori distribution of performances f(x). The best-response strategy of agents in this case is purely based on the participation probability p ( $0 \le p \le 1$ ) and so the probability an agent will win (given in (3)) is simply  $(pF(y) + (1-p))^{n-1}$ . A comprehensive analysis of this specific case can be found in the work of Levy et al [6].

#### 4.2 Equilibrium in subgame

We move on to finding the equilibrium in the subgame resulting from the organizer's choice of the agents that will become informed,  $A \subseteq A$  (|A'| = k). Here, as discussed earlier, we are interested the symmetric equilibrium, i.e., one where the informed agents use the same threshold *T* and the non-informed agents use the same participation probability *p*. We begin with finding the value of the expected profit-maximizing threshold, denoted  $T_i^*$ , given  $S_{-i} = (p, T)$ . Recall that the expected profit of an informed agent  $A_i$  if not participating

 $<sup>^7</sup>$  The calculation is a modification of (2), hence we omit the explanation for how it was derived.

is zero, and that  $p_i^{Win}(x)$  (and consequently  $B_i^P(x)$ ) increases in x. The expected-profitmaximizing threshold  $T_i^*$  is therefore the one satisfying  $B_i^P(T_i^*) = B_i^{\neg P} = 0$ , i.e., the equilibrium participation-threshold T to be used by all agents receiving information is the one satisfying:

$$B_i^P(T) = 0 \tag{4}$$

The expected a priori profit of any agent  $A_i$ , denoted  $B_i$ , depends on whether or not it is about to receive information, and is given by:<sup>8</sup>

$$B_{i} = \begin{cases} \int_{t=T}^{\infty} (Mp_{i}^{Win}(t) - c) f(t) dt & A_{i} \text{ is informed} \\ p(M \int_{t=-\infty}^{\infty} \hat{p}_{i}^{Win}(t) f(t) dt - c) & \text{otherwise} \end{cases}$$
(5)

For a non-informed agent  $A_i$  that does not participate in the contest the expected profit is  $B_i^{\neg P} = 0$ . Therefore the following must hold in equilibrium:

$$\begin{cases} B_i \ge 0, \quad p = 1 \\ B_i = 0, \quad 0 \le p < 1 \end{cases}$$
(6)

A solution (p, T) is thus in equilibrium if it satisfies both (4) and (6).

Since the equilibrium is defined both by p and T, it is possible that a given setting will have more than a single (symmetric) equilibrium solution (i.e., multi-equilibria).<sup>9</sup> This is illustrated in Fig. 1 which depicts the expected profit of the non-informed agents, when choosing to participate in the contest, as a function of the participation probability p used. The setting used includes four agents (n = 4), where two of them receive their exact performance from the organizer (k = 2). All agents are characterized by a uniform performance distribution function between 0 and 1. The prize is M = 0.38 and participation cost is c = 0.14. For each value p we calculate the corresponding threshold T to be used by the informed agents (according to (4)) and this threshold value is then used for calculating  $B_i^P(p)$ . Thus any pvalue for which  $B_i^P(p) = 0$  constitutes an equilibrium solution along with its corresponding T value. As can be seen from Fig. 1, in this setting there are two equilibria: p = 0.085(corresponds to T = 0.408), and p = 0.603 (corresponds to T = 0.62). With both equilibria the non-informed agents end up with zero expected profit. The informed agents, however, end up with a profit  $B_i^P = 0.067$  and  $B_i^P = 0.006$  in the first and second equilibrium, respectively.

For some classes of settings, we can prove somehow counter-intuitive properties of the equilibrium, in particular, equivalence in the thresholds used for settings differing in the number of informed agents and similar expected profits of informed and non-informed agents. These are summarized in Propositions 2 and 3. This is stated in the following Proposition.

**Proposition 2** In any two contests characterized by M, f(q) and  $c^{org}(A')$ , in which the equilibrium solution is of type (p = 1, T), the threshold used by the informed agents is the same, regardless of the number of informed agents  $1 < k \le n$ .

**Proof** The proof is based on showing that any such contest with 1 < k < n is equivalent to a contest with k = n informed agents. For k = n we obtain from (2) that  $p_i^{Win}(t) = F(t)^{n-1}$  for t > T. Similarly, substituting k < n and p = 1 in (2) we obtain  $p_i^{Win}(t) = F(t)^{n-1}$ 

<sup>&</sup>lt;sup>8</sup> In the extreme where k = n this can be simplified to  $B_i = \int_{t=T}^{\infty} (MF(t)^{n-1} - c)dt$ , while in the other extreme in which k = 0 it is  $B_i = p(M\sum_{j=0}^{n-1} {n-1 \choose j} \frac{p^j(1-p)^{n-j-1}}{1+j} - c)$ .

<sup>&</sup>lt;sup>9</sup> The question of which of those will be used is interesting, yet beyond the scope of the current paper.



**Fig. 1** The expected profit of a participating non-informed agent as a function of the participation probability p of the other non-informed agents. The setting used is  $n = 4, k = 2, M = 0.38, c = 0.14, c^{org}(\cdot) = 0$  and f is uniform over (0, 1)

for t > T. Meaning that the winning probability of any informed agent does not change for t > T—it only changes when  $t \le T$ , alas in those cases the agent will not participate in the first place. Consequently, the calculation of the equilibrium threshold T (based on (1)-(4)) remains unchanged.

Intuitively, the *certain* participation of the n - k non-informed agents will affect the informed agents only when their performance value turns to be greater than T. From the informed agents point of view, this is equivalent to having all non-informed agents participate according to a threshold T, hence the strategic situation is similar to the case where all agents are informed.

**Proposition 3** For any contest characterized by M, f(q),  $c^{org}(A')$ , k = n - 1 (i.e., only one of the agents remains non-informed) and equilibrium solution (1, T), all agents gain the exact same expected profit, regardless of being informed or not.

**Proof** Consider the non-informed agent that always participates. For any performance  $t \ge T$ , its winning probability is equal to the winning probability of an informed agent with the same performance measure (this was already established in the proof of Proposition 2). As for values t < T, the winning probability of the agent is  $F(T)^k$ , as all *k* informed agents will participate only if their performance is *T* or above. Recall that the threshold *T* was set such that the expected profit from participating when t = T is zero (see (4)). Therefore, for the non-informed agent, participating with a value t < T also results in a zero expected profit. To summarize, for any performance measure *t* the non-informed agent gains the same expected benefit as the informed agents, hence the agents' expected overall profit is the same.

**Corollary 1** For any participation cost c for which disclosing the information to n - 1 agents results in having the non-informed agent always participating, the organizer's choice of disclosing the information to all agents is strictly dominated by disclosing the information to n - 1 agents. The difference in the organizer's expected profit is  $c^{org}(A) - c^{org}(A')$ , where A' is a subset of size n - 1.

#### 4.3 Organizer's profit

Since the agents are a priori homogeneous the organizer has no preference of providing the information to a specific agent and so her strategy is captured by the number of agents she wants to purchase (and provide) information to, denoted k ( $0 \le k \le n$ ). Let  $\overline{F}(y)$  be the probability that the maximum performance obtained in a contest involving k informed agents is less than y:

$$\bar{F}(y) = \begin{cases} F(T)^k (pF(y) + (1-p))^{n-k} & y \le T \\ F(y)^k (pF(y) + (1-p))^{n-k} & y > T \end{cases}$$
(7)

Here we require that all k informed agents are associated with performance lower than T (i.e., ending up opting not to participate) when  $y \le T$  and performance lower than y when y > T. Similarly, we require that all n - k non-informed agents will either not participate or participate and realized later to be associated with a value lower than y. The corresponding probability distribution function, denoted  $\overline{f}(y)$  is, by definition, the first derivative of  $\overline{F}(y)$ . Consequently, the organizer's expected profit from obtaining and revealing information to subset A'(|A'| = k), denoted  $B^{org}(A')$ , is given by:

$$B^{org}(A') = \int_{y=-\infty}^{\infty} y \bar{f}(y) dy - M \left( 1 - F(T)^k (1-p)^{n-k} \right) - c^{org}(A')$$
(8)

where  $(1 - F(T)^k (1 - p)^{n-k})$  is the probability that at least one of the agents participates, hence the prize *M* is being awarded.

As mentioned earlier (below (3)), with the first extreme of the proposed model, in which the organizer obtains and discloses the performance of all agents (i.e., k = n), the organizer's expected profit, given in (8), can be simplified to:

$$B^{org}(A) = \int_{y=T}^{\infty} y \frac{d\bar{F}(y)}{dy} - M(1 - F(T)^n) - c^{org}(A)$$
(9)

where  $\bar{F}(y) = F(\max(y, T))^n$ . With the second extreme, where information is not obtained at all (i.e., k = 0), the organizer's expected profit can be simplified to:

$$B^{org}(\emptyset) = \int_{y=-\infty}^{\infty} y \frac{d\bar{F}(y)}{dy} dy - M(1 - (1-p)^n)$$
(10)

where  $\bar{F}(y) = (pF(y) + (1-p))^n$ .

Naturally, the organizer's decision on the set  $A' \subseteq A$  of agents for which information will be obtained depends on the cost function  $c^{org}(A')$  that determines the cost of obtaining the information. The greater the cost of obtaining the information, the smaller the benefit in obtaining it. Consequently, examples where the organizer prefers obtaining information for merely a subset of the agents are easy to construct. Interestingly, even when the information is costless, and the organizer needs not consume any resources in order to obtain it  $(c^{org}(A') = 0$  $\forall A' \subseteq A)$ , one cannot trivially assume that the information should be obtained and provided to all the agents. This is illustrated in Fig. 2. The figure depicts the organizer's expected profit as a function of the agents' participation cost, for different number of informed agents, while the information is costless (Graph (a)). It also provides the participation probability of the non-informed agents and the threshold used by the informed agents under each condition (Graphs (b) and (c), respectively). The optimal number of informed agents (k) as a function of the agents' participation cost is given in Graph (d)). The setting used includes four agents (n = 4), a prize M = 0.38 and a uniform performance distribution function between 0 and



**Fig. 2** The influence of the participation cost over the organizer's expected profit for different number of informed agents (Graph (a)); the strategies of the non-informed agents (Graph (b)); the strategies of the informed agents (Graph (c)); and the optimal number of informed agents (Graph (d)). The setting used is n = 4, M = 0.38,  $c^{org}(\cdot) = 0$  and f is uniform over (0, 1)

1. From the figure we observe that for low participation costs ( $c \leq 0.04$ ) the organizer's preference is k = 4 (i.e., all agents become informed). This is somehow surprising, as from Graph (b) we observe that even if not providing any information whatsoever, all four agents will choose to participate if participation cost is small. With four informed agents there is a chance that all four agents will realize their expected performance is below the threshold set and opt not to participate (Graph (c)). Meaning that from the contest outcome point of view (i.e., the contributed performance) the contest's organizer loses from not having any agent participating in the contest whenever all agents' performances are low. Still, in the latter case where none of the agents participate in the contest the organizer saves the prize that needs to be awarded. Since the threshold used by the informed agents is rather small, the saving from not awarding the prize is necessarily greater than the performance gained in such cases. For 0.04 < c < 0.1, the organizer benefits from having all the agents remain uninformed (i.e., k = 0). This may seem counter-intuitive, as in most realistic settings the common practice is to provide contestants as much information possible-providing agents information about their own performance will result in having only the most competent ones (i.e., those associated with the highest performances) taking part in the contest, whereas in the absence of such information, agents whose performances might turn to be the highest will potentially opt not to participate in the first place. Still, in this case, since the participation cost is quite small, the organizer benefits from the relatively high participation probability which compensates for cases where the agents, being fully informed, all end up not participating. For  $0.1 < c \le 0.12$  the organizer's best strategy is k = 2, for  $0.12 < c \le 0.18$  it is k = 4, for 0.18 < c < 0.32 it is k = 3 and for c > 0.32 it is k = 1.<sup>10</sup> The optimal number of informed agents k as a function of the participation  $\cot c$  for the above setting is illustrated in Fig. 2(d). As can be seen from the figure, the change in the optimal number of informed agents is not monotonic in the increase in the agents' participation cost: in the setting used the optimal number of informed agents drops from four to zero for a certain c value, and then increases and decreases, with sharp changes (e.g., from two to four agents and three to one agents).

The fact that for small participation costs it was found mostly beneficial to provide all agents with the information is not occasional. In fact, we can prove that regardless of the setting used, when information is costless there is always an interval of *c*-values for which the organizer is better off obtaining and disclosing the information to *all* agents, as given in the following proposition.

**Proposition 4** For any contest characterized by M, f(q) and  $c^{org}(A') = 0 \forall A' \subseteq A$  there is at least one interval of c values where obtaining and disclosing the information to all agents (k = n) dominates informing only a subset of the agents (k < n).

**Proof** Consider an arbitrarily small participation  $\cot c \rightarrow 0$ . With this cost, all the n - k non-informed agents participate, as the expected profit according to (5) is positive. Alas, the k informed agents participate only with performance greater than their optimal threshold  $T_{n-k}$  as calculated based on (4). We therefore consider two possible scenarios: (i) the agent associated with the highest performance among the n agents is a non-informed agent and its performance is lower than the equilibrium threshold  $T_n$  holding when all n agents are informed, and (ii) any other case. For the first case, we show that the organizer necessarily benefits if all non-informed agents become informed. The difference between the two cases is only when a winning non-informed participant opts not to participate once becoming informed, i.e., when the maximum performance is of a non-informed agent and it is below

<sup>&</sup>lt;sup>10</sup> The setting is ill-defined for c > 0.38 as the prize offered is smaller than the participation cost.

the threshold  $T_n$ . Since p = 1 for the non-informed agents in the case of k < n, the value  $T_n$  (following (2) and (4)) is necessarily smaller than  $T_{n-k}$ . Therefore, in such case there is no other winner (as all agents use  $T_n$  hence their performance dictates not participating)—the organizer saves herself awarding the prize M and loses the gain from the performance of the non-informed agent that once becoming informed chooses not to participate. The expected net profit over all such cases is  $M - E[\max(t_1, ..., t_{n-k})|t_i < T_n \forall 1 \le i \le n-k]$ . Since  $c \to 0, T$  is small enough to guarantee that the latter expression is positive. As for all other cases (second-type above scenario), the prize awarded and the expected best performance are the same both when the agents are informed and non-informed, hence there is no change in the expected profit of the organizer.

#### 4.4 Influence of information providing cost

We now turn to illustrate the effect of the information  $\cos t c^{org}$  on the optimal number of informed agents, from the contest's organizer point of view. Figure 3 depicts the organizer's expected profit for different number of agents becoming informed, when the cost of providing the information to k agents is  $c^{org} = \alpha \cdot k$ , and the horizontal axis represents the coefficient  $\alpha$ . The setting used is of four agents (n = 4), a prize M = 0.35, participation cost c = 0.26and a uniform performance distribution function between 0 and 1. As can be seen from the graph, when the cost of purchasing the information is relatively low ( $c^{org} < 0.01 \cdot k$ ) the organizer maximizes its profit with three informed agents (k = 3). When the cost of obtaining and providing the information is relatively high  $(c^{org} \ge 0.09 \cdot k)$  the information is never purchased. For  $0.01 \cdot k < c^{org} < 0.03 \cdot k$ , the organizer purchases the information for two agents (k = 2) while for  $0.03 \cdot k < c < 0.09 \cdot k$  the information to only one agent (k = 1). The intuition for this revealed pattern is that the gain in contributed performance when providing k agents with information does not depend on  $c^{org}$ . Therefore, when the cost of obtaining and providing information increases, the optimal number of informed agents cannot increase (as otherwise, that same number of agents should have been used also with a smaller value of  $c^{org}$ ).



**Fig. 3** The influence of the information-cost coefficient over the organizer's expected profit, for different number of informed agents. The setting used is n = 4, M = 0.35, c = 0.26 and f is uniform over (0, 1). The cost of obtaining and providing the information to k agents is  $c^{org} = \alpha \cdot k$ 

#### 4.5 Agents' profit

Much like with the organizer's preference of whether or not to obtain and provide the information to all agents, we demonstrate that, somehow counter-intuitively, the agents themselves are sometimes better off without having the information regarding their performance. Figure 4 depicts the agents' average expected profit (calculated cross all agents, whether informed or not), as a function of their participation cost c, in a four-agents contest, for different number of informed agents. The setting used is the same as in Fig. 2. The two bottom graphs depict the agents' expected profit based on their type: informed (Graph (b)) and non-informed (Graph (c)). Non-informed agents gain a positive expected profit as long as their equilibrium participation probability is 1 (Graph (c)). When their participation probability drops below 1, they are indifferent between participating to not participating, hence their expected profit is zero. The greater the number of informed agents (k), the lesser the number of non-informed agents, and consequently the greater the participation cost at which they become indifferent to participation (as they can benefit from cases where the informed agents choose not to participate). As for the informed agents, much like with the non-informed agents, here the general (and natural) pattern is of a decrease in expected profit as the cost of participation increases. The only exception is at the participation cost at which the non-informed agents become indifferent to participating in the contest (i.e., use a mixed strategy). Here, the expected profit of the informed agents increases with any further decrease in the participation probability of the non-informed agents. This continues until the increase in the cost of participation becomes more significant than the reduced competition. From that point and on, the expected profit of the informed agents decreases once again as the cost of participation increases.

The above dynamics lead to a complex behavior of the cross-agent average expected profit as reflected in Graph (a) of Fig. 4. From this graph we obtain that from the agents' perspective, the highest expected profit is obtained when only some of them become informed: for relatively low values of participation cost ( $c \le 0.14$ ), the agents gain the highest expected profit when k = 3, for  $0.15 < c \le 0.16$  whereas for high values of c (c > 0.17) k = 1dominates. More specifically, when k = 0 (see black line), for c < M/4 = 0.095 all (noninformed) agents participate and the expected profit decreases linearly as the participation cost c increases. For  $c \ge 0.095$  a mixed-strategy equilibrium holds and the expected profit of the agents is zero as they are indifferent between participating and not participating in the contest. When k = 4 (see blue line), agents' participation threshold increases in c and their expected profit decreases consequently. For  $k \in \{1, 2, 3\}$  we observe a temporal increase in the agents expected profit following an increase in the profit of the informed agents as explained above. When k = 1, for  $c \le 0.1$  all (non-informed) agents participate and so the expected profit decreases linearly as the participation cost c increases. For higher values of c, the non-informed agents use a decreasing mixed-strategy for their participation decision while the threshold used by the informed agents increases; up to c = 0.26 in which the latter becomes 1 (i.e., the non-informed agents do not participate). For  $0.1 < c \le 0.18$  the agents' expected profit increases due to the sharp increase in the informed-agents' expected profit, up to c = 0.19 where the threshold used by the informed agents is high enough and so their expected profit goes down again. Similar phenomenon is illustrated when k = 2 and k = 3, only for different c values.

We note that from the agent's profit point of view, regardless of the setting used, having at least some of the agents become informed always dominates keeping them all non-informed. This is because when the agents participate based on some participation probability p, as in the case of keeping all agents non-informed, their expected profit is zero (according to (6)), whereas with k > 0 informed agents, the profit of the non-informed agents remains zero



**Fig. 4** The influence of the participation cost *c* over the agents' average expected profit (Graph (a)), the informed-agents' strategies (Graph (b)) and the non-informed strategies (Graph (c)). The setting used is  $n = 4, M = 0.38, c^{org}(\cdot) = 0$  and *f* is uniform over (0, 1)

and the profit of the informed ones becomes positive. The only exception is when all agents initially use p = 1 when non-informed. Here, it is possible that the agents' profit when being non-informed is positive to begin with, and once some of them become informed the increase in the profit of the informed ones does not compensate for the decrease in the profit of the others. Still, from the organizer's point of view, offering a prize *M* that results in p = 1 and positive expected benefit to the agents is dominated by offering M' < M for which p = 1

and the agents' expected profit is zero. Therefore, the above exception is only theoretical and a rational organizer will never run into it.

#### 4.6 Social welfare

One interesting question that arises when analyzing the influence of partial information providing over the contest outcome is the change in social welfare. We stick with the traditional definition of social welfare as the sum of the expected profit of all agents (organizer and contestants). Since the organizer is rational, she will only obtain the information (for a partial or the full set of agents) if this will increase her expected profit. As for the agents' expected profit, we have already established above that unless the prize M is exceptionally high, in the sense that it pushes all agents to participate in the contest, informing a subset of the agents regarding their performance can only improve the agents' expected profit. Therefore, the social welfare measure can only increase in value when providing information to the agents. This does not necessarily mean that social welfare is maximized by making all nagents informed. This is illustrated in Fig. 5 that depicts the social welfare in a four-agents setting for different number of informed agents. The setting used is the same as the one used in Fig. 2. As can be seen from the graph, the social welfare decreases as agents' participation cost increases. Specifically, the social welfare is maximized when k = 4 for low values of c  $(c \le 0.19)$  and for high values of c (c > 0.29) when k = 1. For  $0.19 < c \le 0.29$  making only a subset of the agents informed (i.e., k = 2) maximizes the social welfare.

#### 5 Heterogeneous participants

Our model assumes all agents are a priori alike, in the sense that they all incur the same cost if choosing to participate in the contest and their performance derives from a common probability distribution function. This is a common practice in binary-contests literature [5–7]. Still, the analysis can trivially be extended to support the case where agents are heterogeneous in these two parameters. Assume each agent  $A_i \in A$  is characterized by a participation cost  $c_i$  and its performance derives from a probability distribution function  $f_i(x)$ . This difference



**Fig. 5** Social welfare for different number of informed agents. The setting used is n = 4, M = 0.38,  $c^{org}(\cdot) = 0$  and f is uniform over (0, 1)

in participation cost can be attributed to differences in the way agents value their time, or when they arrive from different locations hence their cost of travel and accommodation is different. The difference in the underlying performance probability distribution function can be attributed to inherent differences in the agents' capabilities.

#### 5.1 Equilibrium analysis

Consider an informed agent  $A_i \in A'$ . The probability that this agent will win the contest, if choosing to participate knowing that its performance will be *t*, given the strategies of all other agents  $(T_j, \forall A_{j \neq i} \in A' \text{ and } p_j \forall A_j \notin A')$ , can be calculated using the following modification of (2):

$$p_i^{Win}(t, A') = \prod_{A_j \in A' - A_i} F_j(\max(t, T_j)) \prod_{A_w \in A - A'} (p_w F_w(t) + (1 - p_w))$$
(11)

i.e., for  $A_i$  to win, all other informed agents need to have a value smaller than the threshold they use (in which case they opt not to participate) or below t (in which case they lose). The probability of this event is  $F_j(max(t, T_j))$  for any  $A_{j \neq i} \in A'$ . As for the remaining n - knon-informed agents, each such agent  $A_w \notin A'$  participates with probability  $p_w$  and needs to have a value smaller than t for  $A_i$  to win, and with probability  $1 - p_w$  does not participate. If not participating,  $A_i$  gains zero. The expected-profit-maximizing threshold  $T_i^*$  is therefore the one satisfying  $B_i^P(T_i^*) = 0$ .

Now consider a non-informed agent  $A_i$  which participates with probability  $p_i$ . The probability that the agent will win should it participate and its performance (at retrospect) is realized to be *t* can be expressed using the following modification of (3):

$$\bar{p}_i^{Win}(t, A') = \prod_{A_j \in A'} F_j(\max(t, T_j)) \prod_{A_w \in A - A' - A_i} (p_w F_w(t) + (1 - p_w))$$
(12)

Similarly, the expected a priori profit of any agent  $A_i$ , depending on whether or not it is about to receive information, is given by (5) except for using  $p_i^{Win}(t, A')$ ,  $\bar{p}_i^{Win}(t, A')$  and changing *c* and *f* to  $c_i$  and  $f_i$ . A solution ( $p_i(\forall A_i \in A - A')$ ,  $T_j(\forall A_j \in A')$  is an equilibrium if it satisfies the condition given for our a priori homogeneous model.

#### 5.2 Organizer's profit

We now turn to calculating the expected profit of the organizer. Let  $\overline{F}(y)$  be the probability that the maximum performance obtained in a contest involving a subset A' of informed agents is less than y:

$$\bar{F}(y) = \prod_{A_j \in A'} F_j(\max(y, T_j) \prod_{A_w \in A - A'} (p_w F_w(y) + (1 - p_w))$$
(13)

Here we require that every informed agent  $A_j$  is associated with performance lower than their threshold  $T_j$  (i.e., ending up opting not to participate) when  $y \leq T_j$  and performance lower than y when  $y > T_j$ . Similarly, we require that all non-informed agents will either not participate or participate and realized later to be associated with a value lower than y. The corresponding probability distribution function, denoted  $\overline{f}(y)$  is, by definition, the first derivative of  $\overline{F}(y)$ . Consequently, the organizer's expected profit from obtaining and revealing information to A', denoted  $B^{org}(A')$ , is given by:

$$B^{org}(A') = \int_{y=-\infty}^{\infty} y\bar{f}(y)dy - M\left(1 - \prod_{A_j \in A'} F_j(T_j) \prod_{A_w \in A - A'} (1 - p_w)\right) - c^{org}(A')$$
(14)

where  $1 - \prod_{A_j \in A'} F_j(T_j) \prod_{A_w \in A - A'} (1 - p_w)$  is the probability that at least one of the agents participates, hence the prize *M* is being awarded.

#### 5.3 Numerical illustration

Similar to the homogeneous case, we can find examples where even if the information can be obtained and provided for free, it is the best interest of the organizer to provide such information only to a subset of the agents. However, in the heterogeneous case, we can also illustrate some counter-intuitive organizer's preference related to the type of agents to be included in the contest. For example, we can show that for some settings the organizer will prefer that some of the agents will be associated with high participation cost rather than low ones. The common practice is that the smaller the participation cost of the agent, the stronger is the agent, as the prize awarded is primarily intended to cover the participation cost of the agents. Therefore the organizer is expected to prefer that the potential contestants will be associated with low participation cost.

We provide an example where having a weaker agent (in terms of its participation cost) improves the expected profit of both the agent and the organizer (compared to having a strong agent). The setting used includes five agents, where  $c_1 = c_2 = c_3 = 0.15$  (i.e., three "homogenous" agents) and  $c_4$  and  $c_5$  are the independent parameters (i.e., two "heterogeneous" agents) with only  $c_3$  being informed (k = 1). Meaning that  $A_1$ ,  $A_2$ ,  $A_4$  and  $A_5$ 's equilibrium strategies are in the form of participation probability, while  $A_3$ 's strategy is threshold-based. All five agents are characterized by a uniform performance distribution function between 0 and 1. The prize to be awarded to the winner is M = 0.42.

Figure 6 illustrates the agents' expected profit as a function of the participation cost  $c_4$  $(= c_5)$ . We note that since  $A_1$  and  $A_2$  are symmetric, the most natural (and fair) equilibrium is the one where they use the same strategy p (and gain the same profit). This holds also for agents  $A_4$  and  $A_5$  that use the same strategy  $p_h$  for their participation decision. As can be seen from the figure, up to  $c_4 = 0.09$ , we obtain an equilibrium (0, 0, 0.597, 1, 1)and so the expected profits of A<sub>4</sub> and A<sub>5</sub> decrease as their participation cost increases. For  $0.09 < c_4 < 0.108$ , the heterogeneous non-informed agents  $A_4$  and  $A_5$  find it nonbeneficial to participate (since their cost is relatively high), which decreases the threshold used by the single informed agent  $(A_3)$  to T = 0.5, and consequently increases its expected profit. For 0.108  $< c_4 \leq 0.178$ , we obtain an equilibrium  $(p, p, T, p_h, p_h)$ , i.e, all noninformed agents use a mixed-strategy solution (results in a zero expected profit), which causes a collapse in the informed-agent expected profit and an increase afterwards. When  $c \ge 0.179$  we obtain an equilibrium (0, 0, 0.5, 0, 0) and so A<sub>3</sub>'s expected profit increases to 0.135. Figure 7 complements the above figure by illustrating the organizer's expected profit as a function of  $c_4$  (=  $c_5$ ) under the same setting. Interestingly, in the transition that takes place at  $c_4 = 0.108$  we observe a counter-intuitive phenomenon according to which a decrease in the competence of  $A_4$  and  $A_5$  (i.e., an increase in their participation costs) results in an increase in the organizer's expected profit.



**Fig. 6** The agents' expected profit as a function of participation cost  $c_4$  of the heterogeneous agents. The setting used is n = 5, k = 1, M = 0.42,  $c^{org}(\cdot) = 0$ , c = 0.15 and f is uniform over (0, 1)

## 6 Discussion and conclusions

Obtaining and providing performance-related information to contestants in a contest offer many advantages to the contestants and the organizer. The contestants, once becoming more informed, can be more selective in their participation decisions, avoiding costly participation in cases their chance of winning is a priori poor. Similarly, the organizer will benefit from the improved selectiveness in contestants' participation decision, potentially saving herself paying the reward whenever all contestants are a priori poor. Still, as shown throughout the paper, having fully informed contestants, in terms of knowing their own performance measure if participating in the contest, is not always the best choice for the contest organizer. We show this holds even in settings where the organizer can obtain and provide contestants such information at no cost. Meaning that the benefits from partial information providing (as opposed to making all participants fully informed) derive not only from reduced cost of obtaining the information but also from the different equilibrium dynamics that hold in this more generalized contest model. Having some contestants deciding on their participation



**Fig. 7** The organizer's expected profit as a function of participation cost  $c_4$  of the heterogeneous agents. The setting used is n = 5, k = 1, M = 0.42,  $c^{org}(\cdot) = 0$ , c = 0.15 and f is uniform over (0, 1)

decision probabilistically and some based on a performance threshold often leads to better overall performance, as demonstrated in the paper. Partial information providing can be advantageous also for the agents and at times can even improve overall social welfare. While the benefit in partial information disclosure has been acknowledged in effort-based contests literature, the type of information considered and the governing mechanisms are different from those that hold in binary contests, as discussed throughout the first two sections of this paper.

While the model used assumes the organizer benefits from the best performance of agents participating in the contest, the analysis provided largely support also the case where the organizer's benefit is the sum of participants' performances [5, 40, 41]. In particular, the analysis of the subgame when the agents decide on participation remains unchanged and the only change required is in the calculation of the organizer's expected profit—taking the sum of the expected contribution of each non-informed agent and expected conditional contribution of each informed agent (based on the threshold used).

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# Declarations

**Competing Interests** The authors declare no competing interests.

**Compliance with Ethical Standards** NA. The authors have no relevant financial or non-financial interests to disclose.

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