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Query answering in circumscribed OWL2 profiles

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Abstract
This paper partially bridges a gap in the literature on Circumscription in Description Logics by investigating the tractability of conjunctive query answering in OWL2's profiles. It turns out that the data complexity of conjunctive query answering is coNP-hard in circumscribed $\mathcal{EL}$ and *DL-lite*, while in circumscribed OWL2-RL conjunctive queries retain their classical semantics. In an attempt to capture nonclassical inferences in OWL2-RL, we consider conjunctive queries with safe negation. They can detect some of the nonclassical consequences of circumscribed knowledge bases, but data complexity becomes coNP-hard. In circumscribed $\mathcal{EL}$, answering queries with safe negation is undecidable.

Keywords Low-complexity nonmonotonic description logics · Circumscription · Query answering

Mathematics Subject Classification (2010) 68T30 · 68Q17

1 Introduction

There is recurring evidence in the literature that adding nonmonotonic inferences to the standard ontology language OWL2 and Description Logics (DLs, for short) would greatly help in modeling biomedical knowledge, policies, and other important application domains for knowledge representation languages [9, 27, 29]. Many nonmonotonic extensions of DLs have been proposed to address these needs, for example [5, 6, 9, 11, 14, 16, 18, 19, 25], but in spite of all this work, OWL2 and its reasoners do not yet support nonmonotonic inferences. This is due to several semantic and computational issues that are summarized, for example, in [9]. Here we focus on a desideratum related to complexity: in order to be applicable to large volumes of data and knowledge, a nonmonotonic semantics should preserve the tractability of the low-complexity profiles of OWL2, that is, OWL2-EL, OWL2-QL, and OWL2-RL.[1] In particular, the question addressed in this paper is whether Circumscription satisfies this requirement.

[1] https://www.w3.org/TR/owl2-profiles/

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Circumscription is one of the major nonmonotonic semantics. It is based on a natural idea: in order to reason about what normally holds, the extensions of the predicates that represent abnormal situations (called *abnormality predicates*) are minimized, so that reasoning is based on maximally normal models only. If needed, the extension of some predicates may be fixed while abnormality predicates are minimized; in this way, such *fixed predicates* retain their classical semantics. The remaining predicates, called *varying predicates*, may be modified in order to make the extension of abnormality predicates smaller. When the minimization of a predicate is in conflict with the minimization of another predicate, it is possible to specify priorities that state which predicate should be minimized first. Circumscription has been already investigated as a semantics of DLs; the complexity of concept satisfiability, subsumption checking, and instance checking has been characterized for a set of DLs that ranges from $\mathcal{ALC}$ to relatively expressive DLs such as $\mathcal{ALCIO}$ and $\mathcal{ALCQO}$ [11]. There are also decidability results for $DL\text{-}lite^{R}_{bool}$ and $\mathcal{ALCFI}$ [8].

This paper contributes to the literature on circumscribed DLs by investigating the tractability of reasoning in the profiles of OWL2. The focus is on conjunctive query answering, since this may be regarded as the primary reasoning task for OWL2-QL and OWL2-RL. To the best of our knowledge, neither OWL2 profiles nor conjunctive query answering have been investigated in the context of Circumscribed DLs.[2] The results of the tractability analysis can be summarized as follows: either query answering is insensitive to Circumscription and preserves its classical semantics, or reasoning becomes intractable, and even undecidable in some case.

The next section provides the necessary preliminaries on DLs, first-order queries, and Circumscription. Section 3 first investigates the tractability of conjunctive queries in Circumscription. Since in many cases the answers to conjunctive queries are the same in Circumscription and classical logic, we consider also a more expressive language, namely, conjunctive queries with safe negation. They can detect some of the additional inferences supported by Circumscription but, unfortunately, tractability is not preserved in any of the cases considered here. The paper is concluded by a discussion of related work and interesting topics for further research.

2 Preliminaries

Here we report the basics about the Description Logics (DL) needed for our work and refer the reader to [4] for a more general treatment. The DL languages of our interest are built from countably infinite sets of concept names ($\mathsf{N_C}$), role names (NR), and individual names (NI). For brevity, individual names will be called *constants*, sometimes. *Concepts* are built from concept names and from the concept constructors listed in Table 1. Similarly, *roles* are built from role names and from the role constructors listed in Table 1. Unless stated otherwise, we will use metavariables A, B for concept names, P for role names, C, D for (possibly compound) concepts, R, S for roles, and a, b for individual names.

An *interpretation* $\mathcal{I}$ is a structure $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ where $\Delta^{\mathcal{I}}$ is a nonempty set, and the *interpretation function* $\cdot^{\mathcal{I}}$ is such that (i) $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ if $A \in \mathsf{N_C}$; (ii) $P^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ if $P \in \mathsf{N_R}$; (iii) $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ if $a \in \mathsf{N_I}$.

Table 1 Syntax and semantics of DL constructs and axioms

Name	Syntax	Semantics
Concept and role constructors (the latter recognizable by the word "role" in the name)		
inverse role	R^-	$\{(y,x) \mid (x,y) \in R^{\mathcal{I}}\} \quad (R \in \mathsf{N_R})$
role complement	$\neg R$	$(\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}) \setminus R^{\mathcal{I}}$
top	$\top$	$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$
bottom	$\bot$	$\bot^{\mathcal{I}} = \emptyset$
intersection	$C \sqcap D$	$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
complement	$\neg C$	$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
existential restriction	$\exists R.C$	$\{d \in \Delta^{\mathcal{I}} \mid \exists (d,e) \in R^{\mathcal{I}} : e \in C^{\mathcal{I}}\}$
universal restriction	$\forall R.C$	$\{d \in \Delta^{\mathcal{I}} \mid \forall (d,e) \in R^{\mathcal{I}} : e \in C^{\mathcal{I}}\}$
number restrictions	$(\bowtie n\ S.C)$	$\{x \in \Delta^{\mathcal{I}} \mid \#\{y \mid (x,y) \in S^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\} \bowtie n\} \quad (\bowtie = \leq, \geq)$
nominals	$\{a\}$	$\{a\}^{\mathcal{I}} = \{a^{\mathcal{I}}\} \quad (a \in \mathsf{N_I})$
Terminological axioms		$\mathcal{I}$ satisfies the axiom if:
GCI	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
equivalence	$C \equiv D$	$C^{\mathcal{I}} = D^{\mathcal{I}}$
role inclusions	$R \sqsubseteq S$	$R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$
irreflexivity	$\mathsf{irr}(R)$	$\forall x \in \Delta^{\mathcal{I}},\ \neg R(x,x)$
transitivity	$\mathsf{tran}(R)$	$\forall x,y,z \in \Delta^{\mathcal{I}},\ R(x,y) \wedge R(x,z) \rightarrow R(x,z)$
Concept and role assertion axioms		
conc. assrt.	$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}} (A \in \mathsf{N_C},\ a \in \mathsf{N_I})$
role assrt.	$R(a,b)$	$(a,b)^{\mathcal{I}} \in R^{\mathcal{I}} \quad (P \in \mathsf{N_R},\ a,b \in \mathsf{N_I})$

The third column of Table 1 shows how to extend the valuation $\cdot^{\mathcal{I}}$ of an interpretation $\mathcal{I}$ to compound DL expressions and axioms. GCI stands for "general concept inclusion". An interpretation $\mathcal{I}$ *satisfies* an axiom α (equivalently, $\mathcal{I}$ is a *model* of α) if $\mathcal{I}$ satisfies the corresponding semantic condition in Table 1. When $\mathcal{I}$ satisfies α we write $\mathcal{I} \models \alpha$.

A *knowledge base* $\mathcal{KB}$ is a finite set of DL axioms. Its *terminological part* (or *TBox*) is the set of *terminological axioms* in $\mathcal{KB}$, while its *ABox* is the set of its *assertion axioms* (these axiom categories are specified in Table 1).

An interpretation $\mathcal{I}$ is a *model* of $\mathcal{KB}$ (in symbols, $\mathcal{I} \models \mathcal{KB}$) if $\mathcal{I}$ satisfies all the axioms in $\mathcal{KB}$. We say that $\mathcal{KB}$ *entails* an axiom α (in symbols, $\mathcal{KB} \models \alpha$) if all the models of $\mathcal{KB}$ satisfy α.

[2] Apparently, [10] and related conference papers deal with some of the OWL2 profiles, however the *defeasible inclusions* adopted there are equivalent to inclusions of the form $C \sqcap \neg Ab \sqsubseteq D$ [10, Remark 3.5] that are not expressible in any profile of OWL2 unless $D = \bot$.

2.1 The description logics used in this paper

Our results concern different fragments of the constructors and axioms listed in Table 1. $\mathcal{ALC}$ supports $\top$, $\bot$, $\sqcap$, $\neg$, $\exists$, $\forall$, GCIs, and both types of assertions. $\mathcal{ALCIO}$ further supports inverse roles and nominals, while $\mathcal{ALCQO}$ extends $\mathcal{ALC}$ with number restrictions and nominals. Both logics enjoy the *finite model property*, that is, all consistent knowledge bases in these logics have a finite model.

The standard ontology language OWL2 has three tractable profiles: OWL2-EL, OWL2-QL, and OWL2-RL.

OWL2-EL is an extension of $\mathcal{EL}_\bot$, that is, the DL that supports only atomic concepts and roles, $\top$, $\bot$, $\sqcap$, and existential restrictions. Supported axioms are GCIs and assertions. $\mathcal{EL}$ is $\mathcal{EL}_\bot$ without $\bot$. The additional features of OWL2-EL will not be needed in this paper because query answering is already intractable in $\mathcal{EL}$.

OWL2-QL is an extension the logic *DL-lite core* (hereafter *DL-lite* , for short), that supports only inverse roles, unqualified existential restrictions of the form $\exists R$ (an abbreviation of $\exists R.\top$), GCIs and assertions. Moreover, complements ($\neg$) are allowed on the right-hand side of GCIs. The additional features of OWL2-QL will not be needed.

OWL2-RL is equivalent to the logic $\mathcal{RL}$ that supports inverse roles and the following kinds of axioms:[3] assertion axioms; role axioms of the form $R \sqsubseteq R'$, $R \sqsubseteq \neg R'$, $\mathsf{irr}(R)$, $\mathsf{tran}(R)$; inclusion axioms of the form $C_L \sqsubseteq C_R$, where C_L and C_R – called *left concepts* and *right concepts*, respectively[4] – are specified by the following grammar, where $n \in \{0, 1\}$:

$$C_L ::= A \mid C_L \sqcap C_L \mid C_L \sqcup C_L \mid \{a\} \mid \exists R.C_L \mid \exists R.\top$$
$$C_R ::= A \mid C_R \sqcap C_R \mid \bot \mid \neg C_L \mid \exists R.\{a\} \mid \forall R.C_R \mid \leq n\ R.C_L \mid \leq n\ R.\top\,.$$

2.2 Queries

An *atom* is an expression of the form $A(t)$ or $P(t, u)$, where $A \in \mathsf{N_C}$, $P \in \mathsf{N_R}$, and t, u are either constants in $\mathsf{N_I}$ or variables, taken from a countably infinite set of variable names $\mathsf{N_V}$. A *literal* is either an atom or a negated atom (called *positive* and *negative* literals, respectively).

A *conjunctive query with safe negation* (CQSN for short) is a first-order formula $q(\mathbf{x})$ of the form $\exists\mathbf{y}.\phi(\mathbf{x}, \mathbf{y})$, where $\mathbf{x}$ and $\mathbf{y}$ are tuples of mutually distinct variables, $\phi(\mathbf{x}, \mathbf{y})$ is a conjunction of literals whose variables occur in $\mathbf{x}$ or $\mathbf{y}$, and all the variables that occur in a negative literal occur also in a positive literal (safety property).

A *conjunctive query* (CQ for short) is a CQSN with no occurrences of negation. A query is $\exists$-*free* if it contains no occurrences of $\exists$.

The *query output tuple* (QOT) problem consists in deciding whether $\mathcal{KB} \models q(\mathbf{a})$ holds, given a query $q(\mathbf{x})$, a knowledge base $\mathcal{KB}$, and a tuple of constants $\mathbf{a}$ occurring in $\mathcal{KB}$, of the same length as $\mathbf{x}$.[5] The *data complexity* of the QOT problem is measured by fixing the TBox of $\mathcal{KB}$ and the query, so that only the ABox is allowed to vary.

[3]We omit some of the constructs listed in the specification, that can be expressed by means of the other constructs. For simplicity, we omit also the constructs related to datatypes, that play no role in our results.

[4]They are called *subclass expression* and *superclass expression* in the standard specification.

[5]If $\mathbf{x}$ is empty we write $q()$ and the QOT problem consists in checking whether $\mathcal{KB} \models \exists\mathbf{y}.\phi(\mathbf{y})$.

2.3 Circumscription

The semantics of a knowledge base in Circumscription is specified by a *circumscription pattern*, that is a tuple $\mathsf{CP} = \langle \prec, \mathsf{M}, \mathsf{V}, \mathsf{F} \rangle$, where M, V, and F constitute a partition of $\mathsf{N_C} \cup \mathsf{N_R}$, and $\prec$ is an irreflexive and transitive priority relation on M. The predicates in M, V, and F are called *minimized, variable*, and *fixed*, respectively. Two interpretations $\mathcal{I}$ and $\mathcal{J}$ are F-*equivalent*, in symbols $\mathcal{I} \sim_{\mathsf{F}} \mathcal{J}$, if

$$\begin{aligned} \Delta^{\mathcal{I}} &= \Delta^{\mathcal{J}} \\ a^{\mathcal{I}} &= a^{\mathcal{J}}, \quad \text{for all } a \in \mathsf{N_I} \\ P^{\mathcal{I}} &= P^{\mathcal{J}}, \quad \text{for all } P \in \mathsf{F}. \end{aligned}$$

Each circumscription pattern CP induces a partial order $\leq_{\mathsf{CP}}$ on interpretations, such that $\mathcal{I} \leq_{\mathsf{CP}} \mathcal{J}$ holds iff $\mathcal{I} \sim_{\mathsf{F}} \mathcal{J}$ and for all $X \in \mathsf{M}$, if $X^{\mathcal{I}} \not\subseteq X^{\mathcal{J}}$, then there exists $Y \prec X$ in M such that $Y^{\mathcal{I}} \subset Y^{\mathcal{J}}$.

We write $\mathcal{I} <_{\mathsf{CP}} \mathcal{J}$ iff $\mathcal{I} \leq_{\mathsf{CP}} \mathcal{J}$ and $\mathcal{J} \not\leq_{\mathsf{CP}} \mathcal{I}$.

Note that if $\prec$ is empty (i.e. there are no priorities on minimized predicates), then $\mathcal{I} \leq_{\mathsf{CP}} \mathcal{J}$ holds iff $\mathcal{I} \sim_{\mathsf{F}} \mathcal{J}$ and for all $X \in \mathsf{M}$, $X^{\mathcal{I}} \subseteq X^{\mathcal{J}}$.

A model $\mathcal{I}$ of $\mathcal{KB}$ is $\leq_{\mathsf{CP}}$-*minimal* if there exists no other model $\mathcal{J}$ of $\mathcal{KB}$ such that $\mathcal{J} <_{\mathsf{CP}} \mathcal{I}$.

If $\mathcal{DL}$ is a description logic, a *circumscribed* $\mathcal{DL}$ *knowledge base* is a pair $(\mathsf{CP}, \mathcal{KB})$, where CP is a circumscription pattern and $\mathcal{KB}$ is a knowledge base in $\mathcal{DL}$. A first-order sentence ϕ is *entailed* by a circumscribed knowledge base $(\mathsf{CP}, \mathcal{KB})$ iff ϕ is satisfied by all the $\leq_{\mathsf{CP}}$-minimal models of $\mathcal{KB}$. In this case we write $\mathcal{KB} \models_{\mathsf{CP}} \phi$.

If $\mathcal{Q}$ is a class of queries, then the *QOT problem for* $\mathcal{Q}$ *in circumscribed* $\mathcal{DL}$ consists in deciding whether $\mathcal{KB} \models_{\mathsf{CP}} q(\mathbf{a})$ holds, given a query $q(\mathbf{x}) \in \mathcal{Q}$, a circumscribed $\mathcal{DL}$ knowledge base $(\mathsf{CP}, \mathcal{KB})$, and a tuple $\mathbf{a}$ of constants occurring in $\mathcal{KB}$ (of the same lenght as $\mathbf{x}$). The data complexity of this problem is defined as in the classical case. The QOT problem for $\mathcal{Q}$ in circumscribed $\mathcal{DL}$ *without priorities/variable predicates* is the restriction of the QOT problem for $\mathcal{Q}$ in circumscribed $\mathcal{DL}$ to knowledge bases whose circumscription pattern satisfies the specified restriction.

2.3.1 Examples

We illustrate Circumscription by means of two examples adapted from [11, Sec. 2.1]. In the first example, the knowledge to be formalized is: *Mammals normally inhabitate land, but whales – that are mammals – do not live on land*. These statements can be encoded with three inclusions:

$$\begin{aligned} \mathsf{Mammal} \sqcap \neg \mathsf{Ab_{Mammal}} &\sqsubseteq \exists \mathsf{habitat.Land}, \\ \mathsf{Whale} &\sqsubseteq \mathsf{Mammal}, \\ \mathsf{Whale} &\sqsubseteq \neg \exists \mathsf{habitat.Land}. \end{aligned}$$

Concept $\mathsf{Ab_{Mammal}}$ is an abnormality predicate that represents the class of abnormal mammals. So, the first axiom says that normal mammals live on land (and, equivalently, that every mammal that does not live on land is an abnormal mammal). Additionally, with two assertions, we introduce two individuals: a whale and a mammal that is not a whale:

$$\begin{aligned} &\mathsf{Whale(mobydick)}, \\ &(\mathsf{Mammal} \sqcap \neg \mathsf{Whale})(\mathsf{pongo}). \end{aligned}$$

Let $\mathcal{KB}_1$ consist of the above five axioms.

The abnormality concept $\mathsf{Ab_{Mammal}}$ shall be minimized. A reasonable circumscription pattern for this example is

$$\mathsf{CP}_1 = \langle \emptyset, \{\mathsf{Ab_{Mammal}}\}, \{\mathsf{habitat}, \mathsf{Land}\}, \{\mathsf{Mammal}, \mathsf{Whale}\}\rangle$$

(where $\mathsf{Ab_{Mammal}}$ is minimized, habitat and Land can vary as $\mathsf{Ab_{Mammal}}$ is minimized, while Mammal and Whale retain their classical semantics).

In the following we are going to illustrate why CP_1 is appropriate for this example, and what would happen with different choices of variable and fixed predicates. For this purpose, we will make use of the following observations. Note that for all the classical models $\mathcal{I}$ of $\mathcal{KB}_1$, there exists a classical model $\mathcal{J} \leq_{\mathsf{CP}_1} \mathcal{I}$ of $\mathcal{KB}_1$ such that $\mathsf{pongo}^{\mathcal{J}} \notin \mathsf{Ab_{Mammal}}^{\mathcal{J}}$. Such $\mathcal{J}$ can be obtained from $\mathcal{I}$ by modifying $\mathsf{habitat}^{\mathcal{I}}$ and $\mathsf{Land}^{\mathcal{I}}$, if necessary, in order to insert pongo in $\exists\mathsf{habitat.Land}$ ($\mathcal{I}$ and $\mathcal{J}$ are equal in all other respects).[6] Then, in all CP_1-minimal models of $\mathcal{KB}_1$, pongo is a normal mammal; equivalently:

$$\mathcal{KB}_1 \models_{\mathsf{CP}_1} \neg\mathsf{Ab_{Mammal}}(\mathsf{pongo})\,. \tag{1}$$

Now consider the following query:

$$q_1(x) = \exists y.\mathsf{habitat}(x, y) \wedge \mathsf{Land}(y)\,,$$

This query should return the animals that live on land. We expect it to hold for $x = \mathsf{pongo}$, because it is not a whale, so the default property of living on land should apply. Indeed, (1) and the first axiom of $\mathcal{KB}_1$ imply that pongo lives on land.

This inference is possible only because habitat and Land are variable predicates. For example, there exists a classical model $\mathcal{I}$ of $\mathcal{KB}_1$ such that $\mathsf{Land}^{\mathcal{I}} = \emptyset$; so if Land could not be modified during the minimization of $\mathsf{Ab_{Mammal}}$, then $\mathsf{Land}^{\mathcal{J}}$ would be empty also in all the $\leq_{\mathsf{CP}}$-minimal models $\mathcal{J} \leq_{\mathsf{CP}} \mathcal{I}$, consequently pongo would not live on land (and would belong to $\mathsf{Ab_{Mammal}}$ even in some CP-minimal model). Similarly, if habitat were not allowed to vary, then there would be CP-minimal models where pongo has no habitat and is an abnormal mammal, so pongo would not be returned by q_1.

The same would happen if habitat and Land were minimized like $\mathsf{Ab_{Mammal}}$. In this case, in every minimally abnormal model $\mathcal{I}$, either pongo is a member of $\mathsf{Ab_{Mammal}}$, or there exists a pair $(\mathsf{pongo}^{\mathcal{I}}, d) \in \mathsf{habitat}^{\mathcal{I}}$ such that $d \in \mathsf{Land}^{\mathcal{I}}$. So there would be CP-minimal models where pongo is abnormal, which is compensated by smaller extensions for habitat or Land (or both).

Now consider concept Whale. Since it is fixed, its semantics is classical: we only know that it contains mobydick but not pongo, and that its instances are abnormal mammals that do not live on land. Clearly we do not want Whale to be minimized: as a result it would become a singleton containing only mobydick – which would not model correctly the notion of whale.

Interestingly, in this example, Whale would be a singleton also if it were a variable predicate. Since all whales are abnormal mammals, the minimization of $\mathsf{Ab_{Mammal}}$ would remove, as a side effect, all elements from Whale with the exception of mobydick (the only individual that is forced to be a member of Whale by an axiom).

The second example illustrates the purpose of priorities. Let us modify $\mathcal{KB}_1$ by introducing Willy, a whale that sings opera in theaters, so its habitat is on land[7]. The important

[6]Just pick any individual $d \in \Delta^{\mathcal{I}}$ and let $\mathsf{habitat}^{\mathcal{J}} = \mathsf{habitat}^{\mathcal{I}} \cup \{(\mathsf{pongo}^{\mathcal{I}}, d)\}$ and $\mathsf{Land}^{\mathcal{J}} = \mathsf{Land}^{\mathcal{I}} \cup \{d\}$.
[7]Willy is the main character of a classical cartoon produced by Walt Disney in 1946.

feature, in this example, is that there are multiple levels of exceptions: whales are exceptional mammals that live in the sea, and Willy is an exceptional whale whose habitat is on land. The new individual makes it necessary to replace the third axiom of $\mathcal{KB}_1$ with a weaker version that makes it possible to override the property of whales. Accordingly, obtain $\mathcal{KB}_2$ from $\mathcal{KB}_1$ by replacing the third axiom with

$$\mathsf{Whale} \sqcap \neg \mathsf{Ab}_{\mathsf{Whale}} \sqsubseteq \neg \exists \mathsf{habitat}.\mathsf{Land}$$

and by adding the assertions Whale(willy), habitat(willy, theater) and Land(theater). We expect ∃habitat.Land to contain pongo and willy but not mobydick, according to the principle of *specificity*, that is: the default properties of more specific classes should override those of more generic classes (in this example, the default habitat of whales should override that of generic mammals). In order to obtain these inferences, let

$$\mathsf{Ab}_{\mathsf{Whale}} \prec \mathsf{Ab}_{\mathsf{Mammal}},$$

and let the circumscription pattern be

$$\mathsf{CP}_2 = \langle \prec, \{\mathsf{Ab}_{\mathsf{Mammal}}, \mathsf{Ab}_{\mathsf{Whale}}\}, \{\mathsf{habitat}, \mathsf{Land}\}, \{\mathsf{Mammal}, \mathsf{Whale}\}\rangle .$$

The priority relation is essential for the above expected inferences. To see why, note that by classical inferences every whale (including mobydick) is either a member of $\mathsf{Ab}_{\mathsf{Mammal}}$ or a member of $\mathsf{Ab}_{\mathsf{Whale}}$. Without the priority relation, both choices are equally preferred (more precisely, the resulting models are incomparable in $\leq_{\mathsf{CP}_1}$). Then, with CP_1, Circumscription would not entail that mobydick does *not* live on land. On the contrary, due to the priority relation, in all $\leq_{\mathsf{CP}_2}$-minimal models $\mathsf{Ab}_{\mathsf{Whale}}$ contains only willy (that is an abnormal whale by classical inferences) while $\mathsf{Ab}_{\mathsf{Mammal}}$ contains all whales but willy.

In sufficiently expressive description logics, fixed predicates can be simulated with variable predicates. It suffices to add a fresh predicate X' for each fixed predicate X (a concept or a role). The new predicates are axiomatized simply by $X' \equiv \neg X$. Then all such X and X' are minimized (while the sets of variable predicates in the old and new circumscription patterns are the same). For instance, the first example would be transformed into the circumscribed knowledge base $(\mathsf{CP}'_1, \mathcal{KB}'_1)$ such that

$$\mathcal{KB}'_1 = \mathcal{KB}_1 \cup \{\mathsf{Mammal}' \equiv \neg\mathsf{Mammal}, \mathsf{Whale}' \equiv \neg\mathsf{Whale}\} ,$$
$$\mathsf{CP}'_1 = \langle \emptyset, \{\mathsf{Ab}_{\mathsf{Mammal}}, \mathsf{Mammal}, \mathsf{Mammal}', \mathsf{Whale}, \mathsf{Whale}'\}, \{\mathsf{habitat}, \mathsf{Land}\}, \emptyset\rangle .$$

The resulting circumscribed knowledge base is equivalent to the original (there is a one-to-one correspondence between their minimally abnormal models). The reason is simple: due to the definition of the predicates X', every attempt at reducing the extension of X' increases the extension of X and viceversa, so that eventually every extension of X is equally preferred, as if X were fixed.

The profiles of OWL2 investigated in the following sections cannot express the equivalences $X' \equiv \neg X$ (that are not Horn). Therefore we will not be able to exploit the above method for fixed predicate elimination.

Before leaving this section, let us go back to the first example and illustrate a mild limitation of Circumscription. Extend $\mathcal{KB}_1$ with $\mathsf{Mammal} \sqcap \mathsf{Land} \sqsubseteq \bot$. Since no axioms state that Land should be nonempty, there exists a model $\mathcal{I}$ of this knowledge base where $\mathsf{Mammal}^{\mathcal{I}} = \Delta^{\mathcal{I}}$ and – consequently – Land is empty (by the new axiom); in turn this implies (by the first axiom of $\mathcal{KB}_1$) that all individuals are in $\mathsf{Ab}_{\mathsf{Mammal}}$. The same holds for all the models $\mathcal{J}$ of the knowledge base such that $\mathcal{J} \leq_{\mathsf{CP}_1} \mathcal{I}$ (because $\Delta^{\mathcal{J}} = \Delta^{\mathcal{I}}$ and $\mathsf{Mammal}^{\mathcal{J}} = \mathsf{Mammal}^{\mathcal{I}}$). It follows that $\mathcal{I}$ and all such $\mathcal{J}$ are CP_1-minimal, despite their abnormality, because $\mathsf{Ab}_{\mathsf{Mammal}}$ has the same extension in all of them. Due to such interpretations, for no mammal (including pongo) it can be derived that it lives on land. Informally speaking, this example shows that Circumscription is not able to create new individuals (in this case a land) in order to make other individuals more normal. This limitation could be removed by making interpretations with different domains comparable; however such interpretation orderings have too many infinite descending chains, that make most knowledge bases inconsistent (CP_1-minimal models almost never exist). Fortunately, the above limitation of Circumscription can be handled easily, without changing semantics. If $\mathcal{KB}_1$ were not a toy example, then Land would probably be axiomatized in a way that makes it nonempty (e.g. by introducing geographic areas that are the natural habitat of specific species), and the above problem would not arise. Anyway, if needed, Land can be made nonempty simply through a class assertion, or by means of an inclusion $\top \sqsubseteq \exists R.\mathsf{Land}$, where R is a fresh auxiliary role.

2.3.2 Similar formalisms

A recent work, related to circumscribed knowledge bases, studies query answering in description logics with *closed predicates*. Similarly to database relations, closed predicates contain only the instances specified in the ABox [1]. Closed predicates are akin but not equivalent to minimized predicates. Consider a knowledge base $\mathcal{KB}$ with ABox $\{A(a), B(a)\}$ and TBox $\{A \sqsubseteq \exists P.(A \sqcap \neg B)\}$. If A is closed, then this knowledge base is inconsistent, because the extension of A contains only a, by definition. In Circumscription, instead, if A is minimized then A contains two individuals in every minimal model of the knowledge base. This example is an instance of a more general result, namely, if $\mathcal{KB}$ has a finite (classical) model, then its circumscribed versions are consistent, too, for all circumscription patterns:

Proposition 1 *Let* $(\mathsf{CP}, \mathcal{KB})$ *be a circumscribed knowledge base. If* $\mathcal{KB}$ *has a finite model, then there exists a* $\leq_{\mathsf{CP}}$*-minimal model of* $\mathcal{KB}$*.*

Proof Let $\mathcal{I}$ be a finite model of $\mathcal{KB}$. Since $\mathcal{I}$ is finite, there are no infinite descending $\leq_{\mathsf{CP}}$-chains. The proposition immediately follows. □

Corollary 2 *If* $\mathcal{KB}$ *has a finite model, then for all circumscribed knowledge bases* $(\mathsf{CP}, \mathcal{KB})$*,*

$$\mathcal{KB} \models_{\mathsf{CP}} \top \sqsubseteq \bot \ \textit{iff} \ \mathcal{KB} \models \top \sqsubseteq \bot\,.$$

Clearly, this is not true of knowledge bases with closed predicates. The above example shows a knowledge base that is classically consistent, while the version with closed predicates is not. It follows immediately that, in general, *closed predicates cannot be defined by means of circumscription patterns*, and in particular *this holds in the description logics considered in the following section*, that enjoy the finite model property.

Note that the differences between the two semantics are not confined to inconsistent knowledge bases only. Consider a slight variation of the above example:

$$\mathcal{KB}' = \left\{A \sqsubseteq \exists P.(A \sqcap \neg B),\ A(a),\ A(b),\ B(b)\right\}.$$

If A and B are closed, then $\mathcal{KB}'$ entails $P(a, a)$ and $P(b, a)$, because a is the only member of $A \sqcap \neg B$. On the contrary, with Circumscription, if A and B are minimized, then none of the above facts is entailed, because there exists a $\leq_{\mathsf{CP}}$-minimal model $\mathcal{I}$ such that $a^{\mathcal{I}} = b^{\mathcal{I}}$, $B^{\mathcal{I}} = \{a^{\mathcal{I}}\}$, $A^{\mathcal{I}} = \{a^{\mathcal{I}}, d\}$, and $P^{\mathcal{I}} = \{(a^{\mathcal{I}}, d)\}$, where d is an anonymous individual. Such $\mathcal{I}$ is $\leq_{\mathsf{CP}}$-minimal no matter how P is dealt with in CP (it might be mimimized, variable, or fixed).

An approach similar to closed predicates is *grounded circumscription* [15, 23], that restricts the extension of minimized predicates to denotable individuals only. The reader may easily verify that, in the above $\mathcal{KB}$, the minimization of A under grounded circumscription yields the same result as making A a closed predicate (i.e. the knowledge base is inconsistent), because the extension of A may only contain a, while the TBox implies that it should contain at least a second individual. Consequently, circumscription patterns are not able to define grounded circumscription, either.

3 Results

In classical logic, CQ answering is tractable in OWL2-EL and OWL2-QL [13, 26]. Circumscription raises the complexity of CQ answering at least at the first level of the polynomial hierarchy. We prove two versions of this claim: first we consider circumscription patterns with variable predicates and no priorities, then circumscription patterns with priorities and no variable predicates.

Theorem 3 *The data complexity of the QOT problem for CQ in circumscribed $\mathcal{EL}$ and DL-lite without priorities is coNP-hard.*

Proof We prove this theorem by reducing 3-coloring (an NP-complete problem) to the complement of CQ answering. The instances of 3-coloring consist of a graph $G = (V, E)$. A coloring is a function $c : V \to \{1, 2, 3\}$ such that for all edges $(i, j) \in E$, $c(i) \neq c(j)$. The answer to a 3-coloring instance is *yes* iff there exists a coloring.

We start with the reduction to CQ answering in $\mathcal{EL}$. The graph is encoded with the following ABox, where each a_i represents node $i \in V$, c_j represents a color ($j = 1, 2, 3$), N encodes the set of nodes, P encodes the set of edges, and C the set of colors.

$$\begin{array}{ll} N(a_i) & (i \in V) \\ P(a_i, a_j) & ((i, j) \in E) \\ C(c_k) & (k \in \{1, 2, 3\})\,. \end{array}$$

The TBox contains only $N \sqsubseteq \exists col.C$, where the role col represents the coloring. The circumscription pattern is $\mathsf{CP} = \langle \emptyset, \mathsf{M}, \mathsf{V}, \emptyset \rangle$ where $\mathsf{M} = \{N, C\}$ and $\mathsf{V} = \{P, col\}$.[8] The conjunctive query below checks whether the coloring encoded by col violates the constraint that adjacent nodes must be colored differently.

$$q() = \exists y_1.\exists y_2.\exists y_3.N(y_1) \wedge N(y_2) \wedge P(y_1, y_2) \wedge col(y_1, y_3) \wedge col(y_2, y_3)\,.$$

Clearly this reduction can be computed in polynomial time. Note that only the ABox depends on the problem instance, as required for data complexity. Now we have to prove that the graph has a coloring iff $\mathcal{KB} \not\models_{\mathsf{CP}} q()$, where $\mathcal{KB}$ is the above knowledge base.

[8] P can be alternatively placed in M. However, minimized roles usually increase complexity [11].

(Only if part) Assume that the graph has a coloring c. Construct an interpretation $\mathcal{I}$ as follows:

$$\begin{aligned} \Delta^{\mathcal{I}} &= \{a_i \mid i \in V\} \cup \{c_1, c_2, c_3\} \\ a^{\mathcal{I}} &= a \quad \text{for all } a \in \Delta^{\mathcal{I}} \\ N^{\mathcal{I}} &= \{a_i \mid i \in V\} \\ C^{\mathcal{I}} &= \{c_1, c_2, c_3\} \\ P^{\mathcal{I}} &= \{(a_i, a_j) \mid (i, j) \in E\} \\ col^{\mathcal{I}} &= \{(a_i, c_k) \mid c(i) = k\}\,. \end{aligned}$$

We have both $\mathcal{I} \models \mathcal{KB}$ and $\mathcal{I} \not\models q()$ by construction. Thus, in order to prove that $\mathcal{KB} \not\models_{\mathsf{CP}} q()$, we are left to prove that $\mathcal{I}$ is $\leq_{\mathsf{CP}}$-minimal. To this end, let $\mathcal{J}$ be any other model of $\mathcal{KB}$ comparable with $\mathcal{I}$ (i.e. with the same domain and the same interpretation of individual names). In order to satisfy the above ABox, $\mathcal{J}$ must satisfy $A^{\mathcal{J}} \supseteq A^{\mathcal{I}}$ for all $A \in \mathsf{M}$. It follows immediately that $\mathcal{I} \leq_{\mathsf{CP}} \mathcal{J}$, therefore $\mathcal{I}$ is $\leq_{\mathsf{CP}}$-minimal.

(If part) Assume that $\mathcal{KB} \not\models_{\mathsf{CP}} q()$. We are going to construct a coloring starting from a witness of this assumption, that is, a $\leq_{\mathsf{CP}}$-minimal model $\mathcal{I}$ of $\mathcal{KB}$ such that $\mathcal{I} \not\models q()$.

Claim 1: $C^{\mathcal{I}} = \{c_1^{\mathcal{I}}, c_2^{\mathcal{I}}, c_3^{\mathcal{I}}\}$.

To prove this claim, note that if $C^{\mathcal{I}}$ contained any elements besides $c_1^{\mathcal{I}}, c_2^{\mathcal{I}}, c_3^{\mathcal{I}}$, then we could find a model $\mathcal{J} <_{\mathsf{CP}} \mathcal{I}$ of $\mathcal{KB}$ by removing each additional element x from $C^{\mathcal{I}}$, and replacing all pairs $(y, x) \in col^{\mathcal{I}}$ with $(y, c_1^{\mathcal{I}})$.[9] This contradicts the assumption that $\mathcal{I}$ is $\leq_{\mathsf{CP}}$-minimal, so Claim 1 is proved.

By Claim 1, the inclusion $N \sqsubseteq \exists col.C$ in the TBox makes sure that for all individual names a_i there exists a c_k such that $(a_i^{\mathcal{I}}, c_k^{\mathcal{I}}) \in col^{\mathcal{I}}$ $(1 \leq k \leq 3)$. Choose any of such c_k and define $c(i) = k$. If two adjacent nodes i and j had the same color k, then the query $q()$ would be satisfied by $\mathcal{I}$, with the variable assignment $y_1 = a_i^{\mathcal{I}}$, $y_2 = a_j^{\mathcal{I}}$, and $y_3 = c_k^{\mathcal{I}}$ (a contradiction). Therefore c is a coloring. This completes the hardness proof for $\mathcal{EL}$. The proof for *DL-lite* is similar. It suffices to replace the above TBox with $\{N \sqsubseteq \exists col,\ \exists col^- \sqsubseteq C\}$.[10] □

Theorem 4 *The data complexity of the QOT problem for CQ in circumscribed $\mathcal{EL}$ and DL-lite without variable predicates is coNP-hard.*

Proof It suffices to replace pattern $\mathsf{CP} = \langle \emptyset, \mathsf{M}, \mathsf{V}, \emptyset \rangle$ in the proof of Theorem 3 with $\mathsf{CP}' = \langle \prec, \mathsf{M}', \emptyset, \emptyset \rangle$, where $\mathsf{M}' = \mathsf{M} \cup \mathsf{V}$ and $\prec$ is $\mathsf{M} \times \mathsf{V}$. Since $C \prec col$, the extension of col can be changed to minimize C, so the proof of Claim 1 is still valid. □

The next theorem shows that variable predicates and priorities are essential to achieve the above hardness results; however, without such features, Circumscription is significantly less interesting, because in a wide range of cases it collapses to classical logic. We will need the following relation between interpretations: Given two F-equivalent interpretations $\mathcal{I}$ and $\mathcal{J}$, we write $\mathcal{I} \subseteq \mathcal{J}$ if

$$P^{\mathcal{I}} \subseteq P^{\mathcal{J}}, \quad \text{for all } P \in \mathsf{N_C} \cup \mathsf{N_R}\,.$$

[9] Note that here it is essential that *col* be a variable predicate.

[10] Incidentally, the same reduction works for DL with closed predicates, see Lemma 4.1 of [17]. In general, however, minimized and closed predicates behave differently, as shown in Section 2.3.2.

Theorem 5 *Let $q(\mathbf{x})$ be an ∃-free CQ and $\mathcal{KB}$ be a knowledge base in $\mathcal{ALCIO}$ or $\mathcal{ALCQO}$. If $\prec$ and V are empty then, for all tuples of individual names* $\mathbf{a}$, $\mathcal{KB} \models q(\mathbf{a})$ *iff* $\mathcal{KB} \models_{\mathsf{CP}} q(\mathbf{a})$.

Proof By definition, the consequences of $\mathcal{KB}$ under Circumscription are valid in a subset of the classical models of $\mathcal{KB}$, so $\mathcal{KB} \models q(\mathbf{a})$ clearly implies $\mathcal{KB} \models_{\mathsf{CP}} q(\mathbf{a})$. We are left to prove the opposite implication, namely:

$$\mathcal{KB} \not\models q(\mathbf{a}) \text{ implies } \mathcal{KB} \not\models_{\mathsf{CP}} q(\mathbf{a})\,.$$

To this end, suppose that $\mathcal{KB} \not\models q(\mathbf{a})$, and let $\mathcal{J}$ be a model of $\mathcal{KB}$ such that

$$\mathcal{J} \not\models q(\mathbf{a})\,. \tag{2}$$

We may assume without loss of generality that $\mathcal{J}$ is finite.[11] Then the set of $\mathcal{J}' \leq_{\mathsf{CP}} \mathcal{J}$ is finite, too, so there exists a $\leq_{\mathsf{CP}}$-minimal model $\mathcal{I}$ of $\mathcal{KB}$ such that $\mathcal{I} \leq_{\mathsf{CP}} \mathcal{J}$. Note that when $\prec$ and V are empty, it holds that $\mathcal{I} \leq_{\mathsf{CP}} \mathcal{J}$ implies $\mathcal{I} \subseteq \mathcal{J}$. Then, since q is positive (hence monotonic in the interpretation), fact (2) and $\mathcal{I} \subseteq \mathcal{J}$ imply $\mathcal{I} \not\models q(\mathbf{a})$. Therefore $\mathcal{I}$ witnesses that $\mathcal{KB} \not\models_{\mathsf{CP}} q(\mathbf{a})$. □

The significance of ∃-free queries can be appreciated by noting that SPARQL queries are ∃-free. In OWL2-RL the above result can be extended to *all* conjunctive queries and circumscription patterns.

Theorem 6 *Let $q(\mathbf{x})$ be a CQ and $(\mathsf{CP}, \mathcal{KB})$ be a knowledge base in $\mathcal{RL}$. Then, for all tuples of individual names* $\mathbf{a}$ *of the appropriate length,* $\mathcal{KB} \models q(\mathbf{a})$ *iff* $\mathcal{KB} \models_{\mathsf{CP}} q(\mathbf{a})$.

In order to prove this theorem we need some auxiliary notation and a lemma. Given a nonempty set of F-equivalent interpretations $\mathcal{S}$, the interpretation $\bigcap \mathcal{S}$ is defined as follows:

$$\begin{aligned} \Delta^{\mathcal{I}} &= \Delta^{\mathcal{J}}, \quad \text{for all } \mathcal{J} \in \mathcal{S}\,. \\ a^{\mathcal{I}} &= a^{\mathcal{J}}, \quad \text{for all } \mathcal{J} \in \mathcal{S}\,. \\ P^{\mathcal{I}} &= \textstyle\bigcap_{\mathcal{J} \in \mathcal{S}} P^{\mathcal{J}}, \quad \text{for all } P \in \mathsf{N_C} \cup \mathsf{N_R}\,. \end{aligned}$$

Clearly $\bigcap \mathcal{S}$ is F-equivalent to all the members of $\mathcal{S}$ by definition. Now we can formulate and prove a lemma that extends a well-known property of logic programs and Herbrand models to $\mathcal{RL}$ and non-Herbrand domains. Roughly speaking, the models of $\mathcal{RL}$ knowledge bases are closed under intersection.

Lemma 7 *Let $\mathcal{KB}$ be a knowledge base in $\mathcal{RL}$, and let $\mathcal{S}$ be a nonempty set of mutually* F*-equivalent (classical) models of $\mathcal{KB}$. Then $\bigcap \mathcal{S} \models \mathcal{KB}$.*

Proof Let $\mathcal{I} = \bigcap \mathcal{S}$. Clearly $\mathcal{I} \subseteq \mathcal{J}$ for all $\mathcal{J} \in \mathcal{S}$. Moreover, $\mathcal{I} \subseteq \mathcal{J}$ implies that for all left concepts C, $C^{\mathcal{I}} \subseteq C^{\mathcal{J}}$ (this can be proved by a simple structural induction; the details are left to the reader). It follows that:

Claim 1: for all $\mathcal{J} \in \mathcal{S}$, and for all left concepts C, $C^{\mathcal{I}} \subseteq C^{\mathcal{J}}$.

Using Claim 1, it is easy to prove:

[11] Here we leverage the finite model property: Every counterexample to $\mathcal{KB} \not\models q(\mathbf{a})$, where $\mathcal{KB}$ is in $\mathcal{ALCIO}$ or $\mathcal{ALCQO}$, can be transformed into such a $\mathcal{J}$ by standard filtration techniques. It is essential that q be ∃-free, otherwise all counterexamples might be infinite.

Claim 2: for all right concepts D and all $x \in \Delta^{\mathcal{I}}$, if $x \notin D^{\mathcal{I}}$, then there exists $\mathcal{J} \in \mathcal{S}$ such that $x \notin D^{\mathcal{J}}$.

Also this claim can be proved with a straightforward structural induction, omitted here. Now we have to show that $\mathcal{I} \models \mathcal{KB}$. Let α be any axiom in $\mathcal{KB}$. The possible cases are:

a) α is an assertion $A(a)$. By definition of $\mathcal{I}$, and since all $\mathcal{J} \in \mathcal{S}$ satisfy α by hypothesis, $a^{\mathcal{I}} = a^{\mathcal{J}} \in \mathcal{A}^{\mathcal{J}}$ for all such $\mathcal{J}$, hence $a^{\mathcal{I}} \in \bigcap_{\mathcal{J} \in \mathcal{S}} \mathcal{A}^{\mathcal{J}} = A^{\mathcal{I}}$, that is, $\mathcal{I} \models \alpha$.

b) α is an assertion $P(a, b)$. Similarly to case a), we have $\mathcal{I} \models \alpha$.

c) α is a role inclusion $R_1 \sqsubseteq R_2$. Let (x, y) be any member of $R_1^{\mathcal{I}}$. By definition of $\mathcal{I}$, and since all $\mathcal{J} \in \mathcal{S}$ satisfy α by hypothesis, $(x, y) \in R_1^{\mathcal{I}} \subseteq R_1^{\mathcal{J}} \subseteq R_2^{\mathcal{J}}$ for all such $\mathcal{J}$. Then $(x, y) \in \bigcap_{\mathcal{J} \in \mathcal{S}} R_2^{\mathcal{J}} = R_2^{\mathcal{I}}$, which proves that $\mathcal{I} \models \alpha$.

d) α is a role inclusion $R_1 \sqsubseteq \neg R_2$. By analogy with case c), for all $\mathcal{J} \in \mathcal{S}$ we have:

$$(x, y) \in R_1^{\mathcal{I}} \subseteq R_1^{\mathcal{J}} \subseteq \Delta^{J} \setminus R_2^{\mathcal{J}} .$$

Then $(x, y) \notin \bigcap_{\mathcal{J} \in \mathcal{S}} R_2^{\mathcal{J}} = R_2^{\mathcal{I}}$, that is, $(x, y) \in (\neg R_2)^{\mathcal{I}}$, which proves that $\mathcal{I} \models \alpha$.

e) α is of the form $\mathsf{irr}(R)$ or $\mathsf{tran}(R)$. In this case, since all $\mathcal{J} \in \mathcal{S}$ must satisfy α by hypothesis, we have that $R^{\mathcal{J}}$ is irreflexive or transitive, respectively, for all such $\mathcal{J}$. Recall that $R^{\mathcal{I}} = \bigcap_{\mathcal{J} \in \mathcal{S}} R^{\mathcal{J}}$ and that the intersection of irreflexive or transitive relations is irreflexive or transitive, respectively. It follows immediately that $\mathcal{I} \models \alpha$.

f) α is of the form $C \sqsubseteq D$ where C is a left concept and D is a right concept. Assume that $\mathcal{I} \not\models \alpha$; we shall derive a contradiction. By assumption, there exists $x \in \Delta^{\mathcal{I}}$ such that: (i) $x \in C^{\mathcal{I}}$, and (ii) $x \notin D^{\mathcal{I}}$. By (i) and Claim 1, $x \in C^{\mathcal{J}}$ for all $\mathcal{J} \in \mathcal{S}$. By (ii) and Claim 2, for some $\mathcal{J}_0 \in \mathcal{S}$, $x \notin D^{\mathcal{J}_0}$. But then $\mathcal{J}_0 \not\models \alpha$, which contradicts the hypothesis that all members of $\mathcal{S}$ are models of $\mathcal{KB}$.

Since $\mathcal{I}$ satisfies α in all cases, the Lemma is proved. □

We are finally ready to prove the theorem.

Proof of Theorem 6 The left-to-right implication follows as in Theorem 5. Now we have to prove the opposite implication, namely:

$$\mathcal{KB} \not\models q(\mathbf{a}) \text{ implies } \mathcal{KB} \not\models_{\mathsf{CP}} q(\mathbf{a}) .$$

To this end, suppose that $\mathcal{KB} \not\models q(\mathbf{a})$. Then there exists a model $\mathcal{J}_0$ of $\mathcal{KB}$ such that

$$\mathcal{J}_0 \not\models q(\mathbf{a}) . \tag{3}$$

Let $\mathcal{S}$ be the set of all models of $\mathcal{KB}$ that are F-equivalent to $\mathcal{J}_0$, and let $\mathcal{I} = \bigcap \mathcal{S}$. Since q is positive (hence monotonic in the interpretation) and $\mathcal{I} \subseteq \mathcal{J}_0$, fact (3) implies $\mathcal{I} \not\models q(\mathbf{a})$. Therefore, in order to prove that $\mathcal{KB} \not\models_{\mathsf{CP}} q(\mathbf{a})$, it suffices to show that $\mathcal{I}$ is a model of the circumscription of $\mathcal{KB}$. By Lemma 7, $\mathcal{I}$ is a classical model of $\mathcal{KB}$. So we are only left to prove that $\mathcal{I}$ is $\leq_{\mathsf{CP}}$-minimal. For this purpose, let $\mathcal{J}$ be any model of $\mathcal{KB}$ such that $\mathcal{J} \leq_{\mathsf{CP}} \mathcal{I}$; we shall prove that $\mathcal{I} \leq_{\mathsf{CP}} \mathcal{J}$. Since $\mathcal{J}$ is comparable to $\mathcal{I}$ by assumption, $\mathcal{J}$ must be F-equivalent to $\mathcal{I}$. Then $\mathcal{J} \in \mathcal{S}$. It follows, by definition of $\mathcal{I}$, that $\mathcal{I} \subseteq \mathcal{J}$. This fact, together with F-equivalence, easily implies that $\mathcal{I} \leq_{\mathsf{CP}} \mathcal{J}$. □

When query answering in circumscribed knowledge bases collapses to classical query answering, as in the above two results, it is interesting to consider queries with negation. The rationale for this choice is that predicate minimization introduces nonclassical negative consequences, and negative queries may capture such additional inferences. Unfortunately, the increased expressiveness of CQSN leads to intractability in most cases. Rosati [28] proved that answering CQSN in classical *DL-lite* is coNP-hard by using a knowledge base that belongs to $\mathcal{RL}$, too. This knowledge base contains negation ($\neg$) and inverse roles. In Circumscription, the same lower complexity bound can be obtained without inverse roles, and in the absence of $\neg$ and $\bot$ in the knowledge base. To see this, let $\mathcal{RL}_0$ be the fragment of $\mathcal{RL}$ where axioms are restricted to role assertions and inclusions of the form

$$A_1 \sqcap A_2 \sqsubseteq A_3 \text{ and } A_1 \sqcap \exists P.A_2 \sqsubseteq A_3$$

(where $A_i \in \mathsf{N_C}$ and $P \in \mathsf{N_R}$).

Theorem 8 *The data complexity of the* QOT *problem for* CQSN *in circumscribed* $\mathcal{RL}_0$ *without priorities is coNP-hard.*

Proof We prove the theorem by reducing 3-coloring to the complement of CQSN answering. The instances of 3-coloring (that are graphs $G = (V, E)$, cf. Theorem 3) are encoded as a knowledge base $\mathcal{KB}$ as follows. Each node $i \in V$ is encoded by a distinguished individual a_i. The ABox represents the edges with a role e, by means of the assertions:

$$e(a_i, a_j) \quad \text{such that } (i, j) \in E\,.$$

The concept names C_k ($k = 1, 2, 3$) represent the classes of nodes whose color is k. The TBox makes the concepts C_k disjoint, and enforces the constraint on colorings (no adjacent nodes should have the same color):

$$\begin{array}{ll} C_i \sqcap C_j \sqsubseteq \mathit{Empty} & (1 \le i < j \le 3) \\ C_k \sqcap \exists e.C_k \sqsubseteq \mathit{Empty} & (1 \le k \le 3)\,. \end{array}$$

The following query checks whether the coloring uses more than three colors:

$$q() = \exists y.\neg C_1(y) \wedge \neg C_2(y) \wedge \neg C_3(y)\,.$$

Finally, $\mathsf{CP} = \langle \emptyset, \mathsf{M}, \mathsf{V}, \emptyset \rangle$, where $\mathsf{M} = \{\mathit{Empty}\}$ and $\mathsf{V} = \{e\} \cup \{C_i \mid 1 \le i \le 3\}$. Note that only the ABox depends on the instance of 3-coloring, as required for data complexity. Clearly the above reduction can be computed in polynomial time. We are left to prove its correctness, that is:

G has a coloring iff $\mathcal{KB} \not\models_{\mathsf{CP}} q()$.

First suppose that G has a coloring c. Define $\mathcal{I}$ as follows:

$$\begin{aligned} \Delta^{\mathcal{I}} &= \{a_i \mid i \in V\} \\ a_i^{\mathcal{I}} &= a_i \quad \text{for all } i \in V \\ e^{\mathcal{I}} &= \{(a_i, a_j) \mid (i, j) \in E\} \\ C_k^{\mathcal{I}} &= \{a_i \mid c(i) = k\} \quad (1 \le i \le 3) \\ \mathit{Empty}^{\mathcal{I}} &= \emptyset\,. \end{aligned}$$

Clearly, by construction, $\mathcal{I}$ satisfies $\mathcal{KB}$ and $\mathcal{I} \not\models q()$, which proves that $\mathcal{KB} \not\models q()$. Moreover, $\mathcal{I}$ is $\le_{\mathsf{CP}}$-minimal, since $\mathit{Empty}^{\mathcal{I}} = \emptyset$. Consequently, $\mathcal{KB} \not\models_{\mathsf{CP}} q()$.

Conversely, suppose that $\mathcal{KB} \not\models_{\mathsf{CP}} q()$. Then there exists a $\le_{\mathsf{CP}}$-minimal model $\mathcal{I}$ of $\mathcal{KB}$ such that $\mathcal{I} \models \neg q()$.

Claim: Empty $^{\mathcal{I}} = \emptyset$.

The Claim follows by noting that the interpretation $\mathcal{J}$ obtained from $\mathcal{I}$ by setting *Empty* $^{\mathcal{J}} = \emptyset$ and $C_i^{\mathcal{J}} = \emptyset$ for $i = 1, 2, 3$, is trivially a model of $\mathcal{KB}$. Therefore, if *Empty* $^{\mathcal{I}} \neq \emptyset$, then $\mathcal{J} <_{\mathsf{CP}} \mathcal{I}$ (that is a contradiction, since $\mathcal{I}$ is assumed to be $\le_{\mathsf{CP}}$-minimal). This proves the claim.

Since $\neg q()$ is satisfied, all the members of $\Delta^{\mathcal{I}}$ (including all $a_i^{\mathcal{I}}$) belong to $C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}} \cup C_3^{\mathcal{I}}$. Then, for all $i \in V$, we can define:

$$c(i) = k \quad \text{if } a_i^{\mathcal{I}} \in C_k^{\mathcal{I}} \ (1 \leq k \leq 3)\,.$$

Note that c is well-defined since the concepts C_k are mutually disjoint, by the first TBox axiom and the Claim. The second TBox axiom, the Claim, and the ABox make sure that for all $(i, j) \in E$, $c(i) \neq c(j)$. Then c is a coloring. □

Remark 9 In this case, we cannot prove a similar result without variable predicates by simulating them with priorities, as in Theorem 4. If we applied the same transformation in Theorem 8, then the minimization of the concepts C_i would make them empty in all minimal models of the knowledge base, so the query would always be entailed.

The last two theorems of this paper prove that negation in the queries makes query answering undecidable in circumscribed $\mathcal{EL}$ and $\mathcal{EL}_\bot$. Only the result for $\mathcal{EL}$ needs priorities. Note that answering CQSN in classical $\mathcal{EL}_\bot$ is known to be coNP-hard [22].

Theorem 10 *The QOT problem for* CQSN *in circumscribed $\mathcal{EL}_\bot$ without priorities is undecidable.*

Proof We prove this theorem by reducing the domino problem [7] to CQSN answering. Domino problems are structures $\langle T, H, V \rangle$ where T is a finite set of *tile types*, $H \subseteq T \times T$, and $V \subseteq T \times T$. A *tiling* is a function $\tau : \mathbb{N}^2 \to T$ such that for all $(i, j) \in \mathbb{N}^2$, $(\tau(i, j), \tau(i+1, j)) \in H$ and $(\tau(i, j), \tau(i, j+1)) \in V$. Deciding whether a given instance of the domino problem has a tiling is undecidable (co-re), even if we assume that for each tile $u \in T$ there exist w and z such that $(u, w) \in H$ and $(u, z) \in V$. The domino problems that satisfy this property can be reduced to query answering in circumscribed $\mathcal{EL}_\bot$ as follows. Informally speaking, we use a role t to encode the tiling, and roles h and v to connect adjacent elements of $\mathbb{N}^2$ (horizontally and vertically, respectively). The set $\mathbb{N}^2$ is represented by concept name N. Of course $\mathcal{EL}_\bot$ cannot force h and v to form a grid isomorphic to $\mathbb{N}^2$; when they fail to do so, the following SCQ is satisfied

$$\begin{aligned} q() = \ & \exists y_1 \exists y_2 \exists y_3 \exists y_4 . N(y_1) \wedge N(y_2) \wedge N(y_3) \wedge N(y_4) \\ & \wedge h(y_1, y_2) \wedge v(y_2, y_3) \wedge v(y_1, y_4) \wedge \neg h(y_4, y_3)\,. \end{aligned}$$

Additionally, the knowledge base $\mathcal{KB}$ makes use of a concept name T_i for each tile type $i \in T$, and a concept $\hat{T}$ that subsumes all T_i. The ABox of $\mathcal{KB}$ uses individuals a_i, b_i, for all $i \in T$, and contains the axioms

$$T_i(a_i) \quad (i \in T)\,, \tag{4}$$

in order to prevent predicate minimization from emptying T_i. The ABox contains also the assertions

$$t(b_i, a_i) \quad (i \in T)\,, \tag{5}$$

in order to have at least one individual b_i labeled with T_i, for each $i \in T$.[12] Finally, the ABox contains the assertion $N(c)$, where c is a fresh individual that represents (1, 1). The TBox of $\mathcal{KB}$ contains the following inclusions:

$$T_i \sqsubseteq \hat{T} \quad \text{for } i \in T \tag{6}$$

$$N \sqsubseteq \exists t.\hat{T} \quad \text{(every grid node is labelled with a tile)} \tag{7}$$

$$N \sqsubseteq \exists h.N \quad \text{(every node has neighbors)} \tag{8}$$

$$N \sqsubseteq \exists v.N \tag{9}$$

$$\exists t.T_i \sqcap \exists t.T_j \sqsubseteq \bot \quad \text{for } i \neq j \text{ (the tiling must be well-defined)} \tag{10}$$

$$\exists t.T_i \sqcap \exists h.\exists t.T_j \sqsubseteq \bot \quad \text{for } (i, j) \notin H \text{ (constraints should be satisfied)} \tag{11}$$

$$\exists t.T_i \sqcap \exists v.\exists t.T_j \sqsubseteq \bot \quad \text{for } (i, j) \notin V\,. \tag{12}$$

The circumscription pattern is $\mathsf{CP} = \langle \emptyset, \mathsf{M}, \mathsf{V}, \emptyset \rangle$, where M contains $\hat{T}$ and all T_i ($i \in T$), and V contains all role names and N. We have to prove that

the domino instance has a tiling iff $\mathcal{KB} \not\models_{\mathsf{CP}} q()$.

First suppose that a tiling τ exists. Define an interpretation $\mathcal{I}$ as follows:

$$\begin{aligned}
\Delta^{\mathcal{I}} &= \mathbb{N}^2 \cup \{a_i \mid i \in T\} \cup \{b_i \mid i \in T\} \\
a^{\mathcal{I}} &= a \text{ for all individuals } a \neq c \\
c^{\mathcal{I}} &= (1, 1) \\
N^{\mathcal{I}} &= \mathbb{N}^2 \\
T_i^{\mathcal{I}} &= \{a_i\} \\
\hat{T}^{\mathcal{I}} &= \textstyle\bigcup_{i \in T} T_i^{\mathcal{I}} \\
h^{\mathcal{I}} &= \{((i, j), (i + 1, j)) \mid (i, j) \in \mathbb{N}^2\} \\
v^{\mathcal{I}} &= \{((i, j), (i, j + 1)) \mid (i, j) \in \mathbb{N}^2\} \\
t^{\mathcal{I}} &= \{((i, j), a_k) \mid \tau(i, j) = k\} \cup \{(b_i, a_i) \mid i \in T\}\,.
\end{aligned}$$

Clearly, by construction, $\mathcal{I} \models \mathcal{KB}$ and $\mathcal{I} \not\models q()$. We are left to prove that $\mathcal{I}$ is $\leq_{\mathsf{CP}}$-minimal. It suffices to note that each $T_i^{\mathcal{I}}$ is the least set needed to satisfy (4), and $\hat{T}^{\mathcal{I}}$ is the least set needed to satisfy (6). This completes the "only if" part of the equivalence.

Conversely, suppose that $\mathcal{KB} \not\models_{\mathsf{CP}} q()$. Then there exists a $\leq_{\mathsf{CP}}$-minimal $\mathcal{I}$ such that $\mathcal{I} \models \mathcal{KB}$ and $\mathcal{I} \not\models q()$. □

Claim 1 $\mathcal{T}^{\mathcal{I}} = \bigcup_{i \in T} T_i^{\mathcal{I}}$.

Clearly $\mathcal{T}^{\mathcal{I}} \supseteq \bigcup_{i \in T} T_i^{\mathcal{I}}$ holds by (6). Suppose the opposite inclusion does not hold. Then the $\leq_{\mathsf{CP}}$-minimality of $\mathcal{I}$ is contradicted, because from $\mathcal{I}$ we can construct a model of $\mathcal{KB}$ $\mathcal{J} <_{\mathsf{CP}} \mathcal{I}$ as follows:

a) let $N^{\mathcal{J}} = \{b_i^{\mathcal{I}} \mid i \in T\}$ and $t^{\mathcal{J}} = \{(b_i^{\mathcal{I}}, a_i^{\mathcal{I}}) \mid i \in T\}$;
b) let $T_i^{\mathcal{J}} = \{a_i^{\mathcal{I}}\}$, and $\mathcal{T}^{\mathcal{J}} = \{a_i^{\mathcal{I}} \mid i \in T\}$;
c) choose, for each tile type $i \in T$, a tile type j_i such that $(i, j_i) \in H$, and let

$$h^{\mathcal{I}} = \{(b_i^{\mathcal{I}}, b_{j_i}^{\mathcal{I}}) \mid i \in T\}.$$

[12] The individuals b_i are not necessarily grid nodes. They guarantee that the TBox (6)–(12) can always be satisfied with a suitable choice of extensions for h and v, no matter how grid nodes are labelled (of course h and v do not necessarily form a grid). This property will be needed in the proof of Claim 1 below, see points c) and d).

d) choose, for each tile type $i \in T$, a tile type k_i such that $(i, k_i) \in V$, and let

$$v^{\mathcal{I}} = \{(b_i^{\mathcal{I}}, b_{k_i}^{\mathcal{I}}) \mid i \in T\}.$$

It is easy to verify that $\mathcal{J} \models \mathcal{KB}$ and $\mathcal{J} <_{\mathsf{CP}} \mathcal{I}$; the details are left to the reader.

Claim 2 There is a mapping $\mu : \mathbb{N}^2 \to N^{\mathcal{I}}$ such that:

i) $\mu(1, 1) = c^{\mathcal{I}}$;
ii) $(\mu(i, j), \mu(i + 1, j)) \in h^{\mathcal{I}}$;
iii) $(\mu(i, j), \mu(i, j + 1)) \in v^{\mathcal{I}}$.

Such a mapping must exist because the query $q()$ is false in $\mathcal{I}$, so it is possible to pick elements from $N^{\mathcal{I}}$ in such a way that $h^{\mathcal{I}}$ and $v^{\mathcal{I}}$ commute. It can be proved with two nested inductions on i and j. The details are long but straightforward, and are omitted here.

We are finally ready to define a tiling from $\mathcal{I}$. Let

$$\tau(i, j) = k \text{ if } \mu(i, j) \in (\exists t.T_k)^{\mathcal{I}}.$$

This function is total, by (7) and Claim 1. It is well-defined, too, since $\mathcal{I}$ satisfies (10). Moreover, since $\mathcal{I}$ satisfies (11) and (12), it holds that for all $(i, j) \in \mathbb{N}^2$,

$(\tau(i, j), \tau(i + 1, j)) \in H$ and $(\tau(i, j), \tau(i, j + 1)) \in V$.

Then τ is a tiling.

Theorem 11 *The QOT problem for CQSN in circumscribed $\mathcal{EL}$ without variable predicates is undecidable.*

Proof Similar to the proof of Theorem 10; it suffices to:

i) replace $\bot$ with a new top-priority, minimized concept name *Empty*;
ii) simulate variable predicates with priorities, as in Theorem 4.

More precisely, pattern $\mathsf{CP} = \langle \emptyset, \mathsf{M}, \mathsf{V}, \emptyset \rangle$ is replaced with $\mathsf{CP}' = \langle \prec, \mathsf{M}', \emptyset, \emptyset \rangle$, where $\mathsf{M}' = \{Empty\} \cup \mathsf{M} \cup \mathsf{V}$ and $\prec$ is $(\{Empty\} \times (\mathsf{M} \cup \mathsf{V})) \cup (\mathsf{M} \times \mathsf{V})$.

Note that $Empty \prec X$ for all other predicates X in M'. In each $\leq_{\mathsf{CP}}$-minimal model $\mathcal{I}$ of the resulting $\mathcal{KB}$, *Empty* behaves exactly like $\bot$, i.e. $Empty^{\mathcal{I}} = \emptyset$. To see this, obtain an interpretation $\mathcal{J}$ from $\mathcal{I}$ by setting $Empty^{\mathcal{J}} = \emptyset$, and specifying the extension of the other predicates as in points a)–d) of the proof of Theorem 10. By construction, $\mathcal{J}$ is a model of $\mathcal{KB}$. Clearly, if $Empty^{\mathcal{I}} \neq \emptyset$, then $Empty^{\mathcal{J}} \subset Empty^{\mathcal{I}}$, hence $\mathcal{J} <_{\mathsf{CP}} \mathcal{I}$. This would contradict the minimality of $\mathcal{I}$. Since *Empty* is equivalent to $\bot$, and the predicates in V can be modified in order to minimize those in M (by definition of $\prec$), the rest of the proof can proceed exactly as in Theorem 10. □

4 Conclusions and related work

Table 2 collects our results and some of the known lower bounds for classical description logics. Unfortunately, we have to conclude that, in general, query answering in the circumscribed profiles of OWL2 is either intractable or uninteresting (because it collapses to classical query answering). In this paper we focussed on lower complexity bounds only, since the research question being investigated is whether Circumscription preserves tractability in some profile. Tight complexity characterizations are an interesting topic for

Table 2 Summary of old and new lower complexity bounds$^{(*)}$

	$\mathcal{EL}$	*DL-lite* core	$\mathcal{RL}$ (OWL2-RL)
	Circumscription patterns with variable predicates or priorities		
CQ	coNP-hard	coNP-hard	classical
CQSN	undecidable	coNP-hard$^{(1)}$	coNP-hard$^{(1)(2)(3)}$
	Circumscription patterns without variable predicates and priorities		
CQ	classical for ∃-free queries up to $\mathcal{ALCIO}$ and $\mathcal{ALCQO}^{(4)}$		classical
CQSN	coNP-hard in $\mathcal{EL}_{\bot}^{(1)}$	coNP-hard$^{(1)}$	coNP-hard$^{(1)(2)}$

$^{(*)}$All results hold even in the absence of fixed predicates

$^{(1)}$This lower bound holds in classical logic, too [22, 28]

$^{(2)}$It follows from the proof for classical *DL-lite* $_R$ [28], that uses a KB in $\mathcal{RL}$

$^{(3)}$This lower bound holds also for circumscribed $\mathcal{RL}_0$ without priorities

$^{(4)}$SPARQL queries are ∃-free

future work. Besides that, a few interesting questions are still open. It is not known whether CQSN answering is tractable in circumscribed $\mathcal{EL}$ without priorities and variable predicates; currently there is only a coNP-hardness result for classical $\mathcal{EL}_{\bot}$ [22]. Moreover, it is not yet known whether restricted classes of queries (e.g. tree-shaped, or guarded), that have better computational properties in classical logic, are tractable in Circumscription, too. Therefore, query answering might be tractable for some restricted classes of circumscription patterns and queries. Another open question is whether conjunctive queries with negation are decidable in *DL-lite* core.

Recall that description logics with *closed predicates* are akin to circumscribed knowledge bases, but closed predicates, in general, cannot be defined by means of circumscription patterns (cf. Section 2.3.2). An interesting subject for future work is the characterization of the cases under which closed predicates can be simulated with Circumscription, in order to extend the results of [1] to classes of circumscribed knowledge bases. Analogous considerations apply to *grounded circumscription* (see Section 2.3.2). Conjunctive queries have not yet been investigated in this formalism, so reductions of circumscription to grounded circumscription may extend our lower complexity bounds to the latter formalism, in particular cases.

Another relevant line of research concerns *Datalog*$^{\pm}$, that is, Datalog extended with existentially quantified variables in rule heads. Datalog$^{\pm}$ is closely related to Horn description logics, such as the profiles of OWL2. Several works study queries and Datalog$^{\pm}$ programs with negation, where ¬ is interpreted as negation as failure; see for example [2, 3, 12, 20, 21]. Conjunctive queries with negation over negation-free programs are particularly similar to the setting based on Circumscription investigated here. The semantics of Datalog$^{\pm}$ is based on the Herbrand models of the program's skolemization, with the exception of [2, 20, 24]. The available tractability results might depend precisely on the restriction to Herbrand domains; if this conjecture were true, then it would help in understanding the complexity sources of Circumscription (where interpretation domains are not restricted). This topic will be the subject of future work.

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