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A CLASS OF PERIODIC GROUPS

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We deal with periodic groups saturated with dihedral groups. In particular, it is proved that periodic groups of bounded period, and also periodic Shunkov groups, saturated with dihedral groups, are locally finite.

1. DEFINITIONS AND MAIN RESULTS

We say that a group G *is saturated* with groups from a set $\mathfrak{R}$ if every finite subgroup K of G is contained in a subgroup isomorphic to some group of $\mathfrak{R}$ (see [1]). If G is saturated with groups from a set $\mathfrak{R}$, and for any $X \in \mathfrak{R}$, G contains a subgroup $L \simeq X$, then we say that G *is saturated with the set* $\mathfrak{R}$ *of groups*, and we call $\mathfrak{R}$ a *saturating set of groups* for G.

Recall that a group generated by two involutions is called a *dihedral group*, or *dihedron*. And if such a group is finite, then we refer to it as a *finite dihedron*. A *locally finite dihedron* is a group that is a union of an infinite ascending chain of finite dihedrons. A *Shunkov group* is one in which every pair of conjugate elements of prime order generate a finite group, with this property preserved over finite sections.

In the present paper we prove the following theorems.

THEOREM 1. A periodic Shunkov group saturated with dihedral groups is locally finite.

THEOREM 2. A periodic group of bounded period saturated with dihedral groups is finite.

THEOREM 3. If G is a periodic group saturated with dihedral groups and S is its Sylow 2-subgroup, then either S is a group of order 2 and G is a (locally) finite dihedron, or $G = ABC = ACB = BCA = CBA$ where A is the centralizer of some involution z in the center of S, $B = O(C_G(v))$ where v is an arbitrary involution in S, distinct from z, and $C = O(C_G(zv))$. Moreover, A is a (locally) finite dihedron and B, C are (locally) cyclic groups.

2. PRELIMINARIES

LEMMA 1. A locally finite dihedron G is generated by involutions.

Proof. Indeed, G is generated by finite dihedral groups, which are generated by involutions; so, G is generated by involutions. The lemma is proved.

LEMMA 2. Let G be a locally finite dihedron. Then $G = H\lambda\langle i\rangle$, where H is a locally cyclic group, i is an involution, and for any element $h \in H$, $h^i = h^{-1}$. In particular, every (locally) cyclic subgroup of G,

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whose order is greater than 4, belongs to H and is normal in G, and each non-(locally)cyclic subgroup of G contains its centralizer in G.

Proof. By the definition of a locally finite dihedron, $G = \bigcup_{k=1}^{\infty} D_k$, where $D_1 < \ldots < D_k < \ldots$ is an infinite ascending chain of finite dihedrons, $D_k = \langle h_k \rangle \lambda \langle i_k \rangle$, and i_k is an involution inverting every element of $\langle h_k \rangle$. Clearly, $\langle h_k \rangle \subseteq \langle h_{k+1} \rangle$ and every involution i of D_k that is non-central in D_{k+1} inverts all elements of $\langle h_{k+1} \rangle$, in which case $D_{k+1} = \langle h_{k+1} \rangle \lambda \langle i \rangle$. Put $H = \bigcup_{k=1}^{\infty} \langle h_k \rangle$ and let i be a non-central involution, for instance, in D_2. Then $G = H\lambda\langle i \rangle$. The lemma is proved.

The next three lemmas are obvious.

LEMMA 3. Let G be a (locally) finite dihedron and H be its non-Abelian subgroup. Then H is a (locally) finite dihedron.

LEMMA 4. Let G be a (locally) finite dihedron and H be its Abelian subgroup. Then either H is elementary Abelian of order 4, or H is a (locally) cyclic group.

LEMMA 5. If G is a finite or locally finite dihedron, and M, K are its finite cyclic subgroups, with $|H| = |K| > 2$, then $M = K$.

In what follows, the expression "$H\lambda\langle x \rangle$ is a locally finite dihedron" will imply that x is an involution, H is a locally cyclic group, and $h^{-1} = h^x$ for any $h \in H$.

LEMMA 6. Let $G = H\lambda\langle x \rangle$ be a locally finite dihedron. Then G contains one, two, or three conjugacy classes of involutions, and namely,

class $K_1 = \{x^g \mid g \in G\}$;

class $K_2 = \{z\}$ where $z \in H$ is a central involution in G, if it exists;

class $K_3 = \{(bx)^g \mid g \in G\}$ where b is an element of H that is not a square, if it exists.

Proof. Let y be any involution in G. If $y \in H$ then $y = z \in K_2$. Let $y = hx$ for some $h \in H$. If h is a square then $y = hx = h_1^2 = h_1 x h_1^{-1} = x^{h^{-1}}$ and $y \in K_1$. If we cannot take a square root of h then the squares of all elements of H constitute in H a subgroup B of index 2, and so $h = bh_1^2$ for some $h_1 \in H$ and some fixed element $b \in H \setminus B$. In this case $y = hx = bh_1^2 x = (bx)^{h_1^{-1}} \in K_3$. Thus $y \in K_1 \cup K_2 \cup K_3$. The lemma is proved.

LEMMA 7 [2]. A periodic group containing an involution whose centralizer is finite is locally finite and almost locally soluble.

LEMMA 8 [3]. Let G be a 2-group and K be its proper finite subgroup. Then there exists a finite subgroup $M \leq G$ such that $K < M$. In particular, the normalizer of K in G is distinct from K.

LEMMA 9 [3]. An arbitrary 2-group with a sole involution either is (locally) cyclic or is a (generalized) quaternion group, finite or infinite.

LEMMA 10 [4]. If some Sylow 2-subgroup of a periodic group G is finite then all Sylow 2-subgroups of G are finite and conjugate.

3. PROPERTIES OF PERIODIC GROUPS SATURATED WITH DIHEDRAL GROUPS

Throughout this section, G is a periodic group saturated with dihedral groups.

LEMMA 11. If G is locally finite then it is a locally finite dihedron.

Proof. Since G is locally finite, any one of its dihedral subgroups is finite. We choose in G two elements, x and y, satisfying $|x| > 2$ and $|y| > 2$, and we consider a finite group $\langle x, y\rangle$. By the saturation condition, $\langle x, y\rangle \leq D < G$, where $D = \langle a\rangle\lambda\langle i\rangle$ is a finite dihedral group. Hence $x \in \langle a\rangle$, $y \in \langle a\rangle$, and $xy = yx$; consequently, all elements of G whose order is greater than 2 generate in G a normal locally cyclic subgroup H. Therefore the set $G \setminus H$ is non-empty and consists of involutions only. Let t be a fixed involution in $G \setminus H$, and let x be an arbitrary involution in $G \setminus H$, with $1 \neq h \in H$ and $|h| > 2$. Consider a finite group $\langle h, x, t\rangle$. By saturation, $\langle h, x, t\rangle \subset D = \langle h_1\rangle\lambda\langle t\rangle$ is a finite dihedral group. The definition of H implies that $h_1 \in H$. Consequently, all involutions of G sit in $H\lambda\langle t\rangle$, and $G = H\lambda\langle t\rangle$. The lemma is proved.

LEMMA 12. Let $b \in G$, $|b| > 2$. Then $C_G(b)$ contains at most one involution.

Proof. Assume, to the contrary, that $C_G(b)$ contains two distinct involutions — x and y. By saturation, the finite group $\langle x, y, b\rangle$ is contained in some finite dihedron of G. Then $x = y$, and we are led to a contradiction. The lemma is proved.

LEMMA 13. G contains an involution i such that $x^2 = i$ has no solution in G.

Proof. Assume, to the contrary, that i is an involution in G and g is an element of G for which $g^2 = i$. By saturation, G contains an involution j such that $g^j = g^{-1}$, j centralizes i, and g normalizes a Klein subgroup $T = \langle i, j\rangle$. Let h be an element of G for which $h^2 = j$. A subgroup $D = \langle i, h\rangle$ is finite since its factor group w.r.t. a subgroup $\langle j\rangle$ is generated by two involutions. By saturation, the group D embeds in a finite dihedron, and so is itself a dihedron of order 8. Consequently, $h \in N_G(T)$ and the subgroup T is normal in $M = \langle g, h\rangle$. The factor group M/T is generated by two involutions, so M is finite. By saturation, M is a subgroup of a finite dihedron in G. It follows that $\langle h\rangle = \langle g\rangle$ and $j = h^2 = g^2 = i$; the latter is impossible. The lemma is proved.

LEMMA 14. If G is a 2-group then G is a (locally) finite dihedron.

Proof. By Lemma 11, it suffices to prove that G is locally finite. Assume the contrary, letting G be a counterexample and letting z be an involution in G. By Lemma 7, the subgroup $C = C_G(z)$ is infinite. We take in C a finite Abelian subgroup K of order greater than 2, containing z. By saturation, $K < D = \langle d\rangle\lambda\langle i\rangle$ is a finite dihedron, and by Lemma 4, $z \in \langle d\rangle$. Thus C contains an involution $i \neq z$. Consider $C_1 = C_C(i)$. If C_1 is an infinite group then it contains an element c such that $M = \langle c, i, z\rangle$ is a non-cyclic Abelian group of order at least 8. On the other hand, M lies in some finite dihedral group of G, and by Lemma 4, $|M| = 4$, a contradiction.

Thus C_1 is a finite group. By Lemma 7, C is locally finite, and by Lemma 5, $C = H\lambda\langle i\rangle$ is a locally finite dihedron, with $z \in H$. Similarly, $C(i) = C_G(i) = R\lambda\langle z\rangle$ is a locally finite dihedron, with $i \in R$. Let $h \in H$, $r \in R$, and $|h| = |r| = 4$. Since h and r belong to the normalizer of a Klein group $T = \langle i, z\rangle$, the group $\langle h, r\rangle$ is finite. By saturation, $\langle h, r\rangle$ is a subgroup of a finite dihedron in G. Therefore, by Lemma 5, $\langle h\rangle = \langle r\rangle$ and $i = r^2 = h^2 = z$; the latter is impossible since i and z are distinct involutions. Thus G is locally finite. The lemma is proved.

LEMMA 15. For any involution $t \in G$, the centralizer $C_G(t)$ is a (locally) finite dihedron.

Proof. By Lemmas 7 and 11, we may assume that $C = C_G(t)$ is an infinite group, and so there exists a finite Abelian subgroup $K < C$, containing an involution t for which $|K| > 4$. By saturation, $K < K_1$, where K_1 is a finite dihedron in G. Clearly, $K_1 < C$, and the latter contains involutions other than t. Denote by z one of such involutions and consider $C_C(z)$. It is not hard to verify that the group in question is finite, and by Lemma 7, C is locally finite. Since C is non-Abelian, it is saturated with dihedral groups, and is a locally finite dihedron by Lemma 11. The lemma is proved.

LEMMA 16. Let S be a Sylow 2-subgroup of G. Then either S is a locally finite dihedron, or S is a group of order 2 and G is a locally finite dihedron.

Proof. If S is of order 2 then $C_G(S) = S$ (saturation). By Lemma 7, G is locally finite, and is a locally finite dihedron by Lemma 11. Now, let the order of S be other than 2. In view of Lemma 14, it suffices to prove that S is saturated with dihedral groups. Suppose K is a finite subgroup of S and K is not a dihedron. By Lemmas 3 and 4, K is then a cyclic group with a sole involution k. If k lies in the center of S then S belongs to the centralizer of k, which is a (locally) finite dihedron by Lemma 15; hence, S is a (locally) finite dihedron.

If $k \notin Z(S)$ then there is a $g \in S$ such that $k \neq k^g$. The subgroup $\langle k, k^g \rangle$ is a dihedral group containing an involution i, which is distinct from k and centralizes k. Hence $D = \langle i, k \rangle$ lies in the centralizer of k, which is a locally finite group by Lemma 15. This implies that D is a finite non-cyclic 2-group, and consequently D is a dihedron. The lemma is proved.

LEMMA 17. Let H be a proper normal subgroup of G containing an involution. Then $G = HC_G(z)$ for some involution $z \in H$.

Proof. Let x, t be involutions such that $x \in H$, $t \in G \setminus H$. (Obviously, $G \setminus H$ contains involutions.) The finite dihedral group

$$D = \langle x, t \rangle = \langle d \rangle \lambda \langle x \rangle = \langle d \rangle \lambda \langle t \rangle$$

does not lie in H and contains a normal subgroup $H_1 = H \cap D = \langle d_1 \rangle \lambda \langle x \rangle$ of index 2, where $d_1 = (tx)^2$ (since $(tx)^2 = (tx)(tx) = (txt)x \in H$ in virtue of the fact that H is normal in G). Thus $\langle d \rangle$ has an involution z. Evidently, $z \in C_G(x)$. If $z \notin H$ then $D \subseteq H \cup zH$ and $t = zh$, for some $h \in H$, that is, $t \in HC_G(x)$.

Now, let $z \in H$. Then $\langle d \rangle$ contains an element v of order 4 for which $v^2 = z$. Hence the involution x can be chosen so that it lies in some cyclic group $\langle w \rangle$ of order 4 (e.g, as x we can take z). The group $\langle v, w \rangle$ is finite (in fact, $v, w \in N_G(\langle z \rangle \times \langle x \rangle)$, the latter group is periodic, the factor group $\bar{N} = N_G(\langle z \rangle \times \langle x \rangle)/(\langle z \rangle \times \langle x \rangle)$ is as well, $\bar{v}, \bar{w}$ are involutions in $\bar{N}$, and the preimage of a finite group $\langle \bar{v}, \bar{w} \rangle$ in N obviously coincides with $\langle v, w \rangle$), and is a subgroup of a finite dihedron in G. Then $\langle v \rangle = \langle w \rangle$ by Lemma 5, which is impossible since $v^2 = z \neq x = w^2$. Thus H contains an involution x such that any involution of $G \setminus H$ lies in $C_G(x)H$, and since G is generated by involutions (Lemma 1), we have $G = C_G(x)H$. The lemma is proved.

LEMMA 18. All Sylow 2-subgroups of G are conjugate.

Proof. Denote by S some Sylow 2-subgroup of G. If S is finite then we need only appeal to Lemma 10. Let S be an infinite group. Then $S = L\lambda\langle t \rangle$ is a locally finite dihedron by Lemma 16. If the subgroup L is normal in G then the conclusion of the lemma holds true. Moreover, the group G, in this case, is itself a locally finite dihedron by Lemma 16.

Let $R = H\lambda\langle i \rangle$ be another Sylow 2-subgroup of G, and let z and y be involutions in L and H, respectively, with $d = zy$. If the order of an element d is odd then z is conjugate to y, and so the subgroups S and R are conjugate. If the order of d is even, then by v we denote an involution in $\langle d \rangle$. Clearly, $v \in C_G(y) \cap C_G(z)$. Consider groups $M_1 = L\lambda\langle v \rangle$ and $M_2 = H\lambda\langle v \rangle$. Evidently, M_1 and M_2 are Sylow 2-subgroups that are conjugate to S in $N_G(L) = C_G(z)$ and to R in $N_G(H) = C_G(y)$, respectively. The subgroups $\langle v \rangle \times \langle z \rangle$ and $\langle v \rangle \times \langle y \rangle$, being Sylow 2-subgroups, are conjugate in $C_G(v)$; so, $\langle v \rangle \times \langle z \rangle < S \cap R^c$ for some $c \in C_G(v)$. Let $S \neq R^c$. Using the normalizer condition in S and in R^c (Lemma 8), we are led to a contradiction with G being saturated with dihedral subgroups. Thus $S = R^c$. The lemma is proved.

LEMMA 19. Let G contain a locally finite, non-trivial, normal subgroup H. Then G is locally finite.

Proof. If $G = H$, there is nothing to prove. Suppose $G \neq H$ and let G not be locally finite. Then Lemmas 15, 17 and the Shmidt theorem will imply that $2 \notin \pi(H)$. If G contains no Klein subgroup, then a

Sylow 2-subgroup of G has order 2, and by Lemma 16, G is locally finite. Consequently, G contains a Klein subgroup $K = \langle z \rangle \times \langle t \rangle, z^2 = t^2 = 1$. Lemmas 11 and 12 imply that $H\lambda K$ is a locally finite dihedron, and without loss of generality, we may assume that $z \in C_G(H)$. By the normality of H, $z^g \in C_G(H)$ holds, for any $g \in G$. By Lemma 12, the centralizer of H cannot contain more than one involution. It follows that $z^g = z$ for any $g \in G$, and by Lemma 15, G is a locally finite group. The lemma is proved.

LEMMA 20. For every $1 \neq b \in G$, the normalizer $N_G\langle b \rangle$ is a (locally) finite dihedron.

Proof. If $\langle b \rangle$ contains an involution then we need only appeal to Lemma 15. If $N_G\langle b \rangle$ is a finite group then the result will follow from the saturation condition. Let $N_G\langle b \rangle$ be infinite. Then $N_G\langle b \rangle$ is a periodic group saturated with dihedral groups and possessing a finite normal subgroup $\langle b \rangle$. And the conclusion then follows from Lemma 19. The lemma is proved.

LEMMA 21. Let H be an infinite locally finite subgroup of G. Then $H \leq L \leq G$, where L is a locally finite dihedron in G.

Proof. If G is locally finite then as L we can take G. Let G not be locally finite. Denote by M a normal subgroup of H generated by all elements in H whose order is greater than 2. The group H is locally finite, and since G is saturated with dihedral groups, it follows that M is locally cyclic. We claim that $C_G(M)$ is saturated with dihedral groups. Indeed, let K be a finite subgroup of $C_G(M)$. Take in M a cyclic group $\langle m \rangle$ such that $|m| > 2$. By saturation, the finite group $\langle m, K \rangle$ embeds in a finite dihedral group D, which can be easily shown to belong to $C_G(M)$. By Lemma 19, $C_G(M)$ is locally finite. Since $C_G(M)$ is non-Abelian (it contains D), $C_G(M)$ is a locally finite dihedron. The lemma is proved.

LEMMA 22. Let a Sylow 2-subgroup S of G be infinite. Then G contains exactly two conjugacy classes of involutions.

Proof. By Lemma 16, S is a locally finite dihedron. Take an involution $z \in Z(S)$ and consider $C_G(z) = L\lambda\langle t \rangle$, which is a locally finite dihedron by Lemma 15. We call z a long involution, and we call t a short involution (since t does not lie in a cyclic 2-group of order greater than 2). By Lemma 16, $C_G(z)$ contains two conjugacy classes of involutions — $I_t = \{z\}$ and $I_t = \{t^G\}$. By Lemma 18, the Sylow 2-subgroups of G are conjugate, and so all long involutions are as well. Denote the conjugacy class of long involutions by I_1. Let w, v be two short involutions. If w and v lie in $C_G(z)$ then they are conjugate. Let one of these, for instance, w, not belong to $C_G(z)$. Then $w \in S^g$ for some $g \in G$. Consequently, $w^{g^{-1}} \in C_G(z)$. We may therefore assume that v and w belong to $C_G(z)$, up to conjugation, and are conjugate in it. Denote by I_2 the set of short involutions. The argument above implies that every involution of G lies in $I_1 \cup I_2$, where $I_1 = \{z^G\}$, $I_2 = \{t^G\}$, and $I_1 \cap I_2 = \varnothing$. The lemma is proved.

LEMMA 23. If S is a Sylow 2-subgroup of G then $N_G(S) = S$.

Proof. If S is finite, and $n \in N_G(S)$, then $\langle S, n \rangle$ is contained in a finite dihedron, and $n \in SC_G(S) = S$. If S is infinite, then it contains a sole involution z, and so $N_G(S)$ is contained in $C_G(z)$, which, by Lemma 15, is a (locally) finite dihedron satisfying the conclusion of the present lemma. The lemma is proved.

LEMMA 24. Let V be a finite non-cyclic subgroup of G. Then $|N_G(V) : V| \leqslant 2$. If $|N_G(V) : V| = 2$ then V is a 2-group.

Proof. If $x \in C_G(V)$ then $\langle x, V \rangle$ is a finite subgroup lying in a finite dihedron of G, so $x \in V$, and hence $C_G(V) \leq V$. Since $|N_G(V)/C_G(V)| \leqslant |\mathrm{Aut}\, V| < \infty$, $N_G(V)$ is a finite subgroup lying in a finite dihedron. Now, the conclusion of the lemma follows from the structure of dihedrons.

LEMMA 25. Intersection of any two different Sylow 2-subgroups of G is a cyclic group.

Proof. Assume, on the contrary, that S and S_1 are two different Sylow 2-subgroups, but the subgroup

$V = S \cap S_1$ is non-cyclic. Then $N_S(V) \neq V \neq N_{S_1}(V)$. By Lemma 24, $N_S(V) = N_G(V) = N_{S_1}(V)$ and $V \neq N_G(V) \leq S \cap S_1 = V$, a contradiction. The lemma is proved.

LEMMA 26. Let S be a Sylow 2-subgroup of G. Then any two involutions of S conjugate in G are conjugate in S.

Proof. Let x, x^g be two involutions in S. We can assume that $x \neq x^g$. Evidently, $x \in S^{g^{-1}} = S_1$, and $C_S(x)$, $C_{S_1}(x)$ are non-cyclic groups belonging to $C_G(x)$. There exists a $c \in C_G(x)$ such that $\langle C_S(x), C^c_{S_1}(x)\rangle$ is a 2-group lying in some Sylow 2-subgroup S_2 of G. Since the groups $S \cap S_2$ and $S_2 \cap {S_1}^c$ are not cyclic, we have $S = S_2 = {S_1}^c = S^{g^{-1}c}$. It follows that $g = cn$, where $n \in N_G(S) = SC_G(S)$ by Lemma 23, and $x^g = x^n$. The lemma is proved.

4. PROVING THE MAIN RESULTS

Proof of Theorem 1. By the hypotheses of the theorem, every two conjugate elements of odd prime order in G generate a finite and (hence) cyclic subgroup. This implies that for every odd prime $p \in \pi(G)$, only one subgroup P of G is a group of order p. Therefore all elements of odd order generate in G a locally cyclic subgroup, the factor group with respect to which is a locally finite 2-group by Lemma 16. By the Shmidt theorem, G is locally finite. The proof is complete.

Proof of Theorem 2. Let G be infinite, and let z be an involution in G. If $C_G(z)$ is a finite group, then G is locally finite by Lemma 7, and $G = H\lambda\langle z\rangle$ is a locally finite dihedron by Lemma 11. The group H is infinite and locally cyclic; so, for any given natural n, H will contain an element h with $|h| > n$. This clashes with the assumption that G has bounded period. Consequently, $C_G(z)$ is infinite. By Lemma 15, $C_G(z) = L\lambda t$ is a locally finite dihedron and L is an infinite locally cyclic group; the latter contradicts the assumption that G has bounded period. The proof is complete.

Proof of Theorem 3. Let $g \in G$. Put $w = vz$. By Lemmas 23 and 26, v^g is not conjugate to z, and hence $D = \langle v^g, z\rangle$ is a finite dihedral group, whose order is divisible by 4. Consequently, $D \leq C_G(i)$, for some involution i. By Lemmas 23 and 26, no involution in $C_G(z) \setminus \langle z\rangle$ is conjugate to z. The involution v^g commutes with i, and so i is conjugate in G to an involution u of $V = \langle v, z\rangle$. Since i and u are in $C_G(z)$, by Lemmas 15 and 26, there exists an $a \in A$ such that $i^a = u$ and $v^{ga} \in C_G(u)$. If $u = v$ then v^{ga} and v are two distinct commuting, conjugate involutions. By Lemma 26, S is non-Abelian, and hence there exists an $a_1 \in S$ such that $u^{a_1} = w$. Replacing, if necessary, a with aa_1, we can obtain $i^a = w = zv$. Thus $v, v^{ga} \in C_G(w)$. Again by Lemmas 15 and 26, there exists a $c \in C$ such that $v = v^{gac}$, whence $gac = x \in C(v)$, that is, $g = xc^{-1}a^{-1}$. In view of $C(v) = BV$ and the fact that V normalizes C, we obtain $x = by$, for $b \in B$ and $y \in V$, and $g = bvc^{-1}a^{-1} = bc_1a_1$ for $a_1 = va^{-1} \in A$ and $c_1 = vc^{-1}v^{-1} \in C$. It follows that $G = BCA$. If in place of v we treat w then we can obtain $G = CBA$ similarly. The two remaining equalities of the theorem follow from the fact that $G = \{g^{-1} \mid g \in G\}$. The structure of groups A, B, C is determined by Lemma 15. The proof is complete.

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