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Intuitionistic fuzzy three-way decision method based on data envelopment analysis

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Abstract
As a powerful mathematical tool for tackling uncertain decision problems, three-way decision has garnered substantial attention since its inception. However, real-world decision problems are inherently complex, and decision-makers often exhibit characteristics of incomplete rationality. Traditional three-way decision models, which rely on functions or relationships, face challenges when confronted with multi-output problems. In response to this challenge, this paper introduces an intuitionistic fuzzy three-way decision model grounded in data envelopment analysis (DEA). Initially, we propose an input–output correlation degree that integrates hesitancy information and serves as a procedural indicator for benefit scores. Subsequently, the traditional DEA is extended to accommodate the intuitionistic fuzzy environment and utilized to construct a comprehensive loss function. Furthermore, a novel intuitionistic fuzzy three-way decision model is developed, incorporating three dimensions: optimism, neutrality, and pessimism, and corresponding decision rules and algorithms are provided. Finally, the effectiveness of the proposed model is rigorously validated through a series of experiments and comparative analyses. The model offers a pioneering approach to address uncertain multi-input–output decision problems, effectively integrating decision-maker's risk preferences within an intuitionistic fuzzy environment.

Keywords Three-way decision · DEA · Intuitionistic fuzzy set · Input–output connection degree · Loss function

1 Introduction

The decision environment in real life is complex and changeable, and the decision-maker's preferences are subject to uncertainty. This makes the limitations of the traditional two-way decision method more and more prominent. In this context, Yao proposed a three-way decision theory (Yao 2009) based on Pawlak rough sets (Pawlak 1982) and decision rough sets (Yao and Wong 1992). It introduces a delay option based on traditional two-way decisions to reduce decision risks. In practice, the three-way decision divides the domain of discussion into three disjoint regions: the positive region (POS), the boundary region (BND), and the Negative Region (NEG). These regions respectively correspond to the

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Table 1 Examples of multiple input–output decision problems

U	e_1	e_2	e_3	y_1	y_1
x_1	1	2	3	3	4
x_2	5	3	5	2	3
x_3	4	6	2	1	3
x_4	7	3	5	2	2

acceptance decision, delay decision, and reject decision. The core of the three-way decision involves the determination of the loss function and conditional probability. It does not regard maintaining the precise consistency of decision knowledge and data as the only goal but rather focuses more on the losses or risks caused by different decisions. This makes the model more cost-sensitive to the misclassification of decisions. Due to its strong theoretical foundation and excellent interpretability, the three-way decision has attracted much attention and has been successfully applied in many fields, such as shadow sets (Yang and Yao 2021), information systems (Huang et al. 2020), cost-sensitive learning (Qian et al. 2022), transfer learning (Xin et al. 2023), cognitive concept learning (Yan et al. 2021), recommendation system (Ye and Liu 2022), medical diagnosis (Chu et al. 2023) and clustering integration (Wu et al. 2022), etc.

In the face of the continuously complex decision-making environment, new decision-making challenges persistently arise. For instance, Table 1 represents a multi-input–output decision information system, with $x_i(i = 1, 2, 3, 4)$ denoting various objects, $e_i(i = 1, 2, 3)$ representing inputs, and $y_i(i = 1, 2)$ representing outputs. Traditional information systems predominantly cater to single-output (decision or classification) issues and are ill-suited for addressing multi-input–output problems, as exemplified by the scenario in Example 1. The DEA method is an efficient mathematical planning tool in management, economics, and operations research, and can evaluate decision-making units (DMUs) with multiple inputs and outputs. Additionally, a notable characteristic of DEA is its independence from considering the functional relationship between input and output variables or estimating parameters. This independence effectively prevents the influence of subjective factors on the evaluation results.

Currently, DEA-related research mainly focuses on the following three aspects. (1) Uncertain decision-making. For example, a novel rough-set decision method derived from the DEA model was introduced by Dun et al. (2010) to address decision problems with multiple decision attributes. Liu and Liang (2017) introduced three-way decisions into the field of DEA for the first time and proposed a three-way decision model based on DEA. Bagherikahvarin and De Smet (2016) developed an integrated DEA multi-criteria decision auxiliary model, aiming to limit the weight value of DEA

to improve the discriminative ability of the DEA model. (2) Evaluation method integrating decision-maker preference information. For example, Yang et al. (2013) combined the DEA model with the evidential reasoning method, providing a new method for the DEA model to reflect the decision maker's preferences or value judgments. Omrani et al. (2020) proposed a combined DEA-group best-worst method to evaluate road safety, incorporating the decision-maker's preferences into the decision process and overcoming the shortcomings of flexible weights in the DEA model. (3) Sorting strategy. For example, An et al. (2018) combined the DEA model and the analytic hierarchy process to construct an interval multiplicative preference relationship to derive the ranking of DMUs. Namazi and Mohammadi (2018) used DEA based on preference ranking techniques that approximate ideal solutions to explore the efficiency of national innovation systems. Rakhshan (2017) studied the effective DMU ranking problem in DEA and proposed a new combined ranking method. In summary, the integration of DEA expands the application scope of traditional decision methods while simultaneously offering a feasible and effective mathematical tool for addressing multi-output decision or ranking problems.

It is worth noting that most current DEA problems assume that inputs and outputs are precise values. Due to the complexity of practical problems, the input and output data of DMU are usually imprecise, such as interval values, ordinal values, probability values, or fuzzy values. Therefore, the effectiveness evaluation of DMU under an uncertain environment is a research hotspot. Despotis and Smirlis (2002) deal with imprecise data through an imprecise DEA model. Subsequently, Entani et al. (2002); Puri and Yadav (2015) constructed various DEA models, such as interval efficiency and fuzzy efficiency to measure the effectiveness of each DMU. In addition, Ref. Puri and Yadav (2015) extended fuzzy DEA to intuitionistic fuzzy DEA for the first time and analyzed DEA of optimistic and pessimistic efficiency and intuitionistic fuzzy input–output data. Chen et al. (2022) proposed a three-way decision method based on the interval data DEA model to deal with interval-type fuzzy data. Combining DEA with uncertainty theory or methods proves effective in enhancing its adaptability to the environment. In a similar vein, the integration of three-way decisions into DEA offers a top-down decision framework, facilitating the inclusion of risk cost measurement. Their combination will provide a new idea and decision framework for processing and analyzing uncertain multiple input–output decision problems. However, there are currently few studies on the three-way decision model combined with DEA, and existing research ignores the interactive impact of uncertain information on input–output. Due to the notable reliability of DEA, various decision methods have been integrated with it to explore more general and interpretable intelligent decision-making models. In general, the research on decision or ranking methods combined with DEA can be categorized into three types:

1. **Combining multi-purpose and multi-attribute decision-making methods:** Examples include preference ranking organization method for enrichment evaluations (PROMETHEE II), TOPSIS, and analytic hierarchy process (AHP). By integrating DEA, the scope of application of existing decision methods is expanded, improving the accuracy and comprehensiveness of decisions. For instance, combining PROMETHEE II and DEA achieves more precise ranking and selection in complex multi-attribute decision problems.
2. **Based on various uncertainty measures:** Methods such as distance entropy and acceptability. Introducing uncertainty measurement into the decision process establishes a new decision or ranking model driven by both "data and knowledge." This approach relies on data quantity while considering data quality and knowledge background, leading to more robust decisions in uncertain environments.
3. **Granular computing approach:** Methods like three-way decision and fuzzy sets, when combined with DEA, enhance the application of granular computing methods and effectively tackle complex and fuzzy decision problems. For example, integrating three-way decision with DEA provides more interpretable decision results under uncertainty and ambiguity.

To clearly describe the differences between our method and existing ones, we selected fifteen representative models from the literature and elaborated on them from two perspectives: data processing and decision method.

- **Data processing level:** Reference (Bagherikahvarin and De Smet 2016), References (Omrani et al. 2020; An et al. 2018; Namazi and Mohammadi 2018; Rakhshan 2017), Reference (Liu and Chen 2022) and References (Ebrahimi et al. 2020; Liu et al. 2022) focus on decision problems with precise information, while References (Chen et al. 2022; Wang et al. 2005, 2016; Jie et al. 2013; Yul et al. 2019) and Reference (Wang et al. 2021) address decision problems with uncertain information. Our method processes data as intuitionistic fuzzy numbers, considering the impact of uncertain information on decision results. We propose the input–output connection degree to apply the influence of uncertain information in the model-solving process, effectively addressing multiple input–output decision-making problems in an intuitionistic fuzzy environment and expanding the scope of DEA.
- **Decision method level:** Reference (Bagherikahvarin and De Smet 2016) obtains the ranking results via the net flow fraction of PROMETHEE II. Reference (An et al. 2018) combines the AHP with interval efficiency of DMUs for comprehensive ranking. Reference (Namazi and Mohammadi 2018) ranks DMUs based on their distance from positive/negative ideals. Reference (Wang et al. 2021) obtains the ranking by introducing the group best-worst method to solve the benefit score of DMUs. Reference (Chen et al. 2022) uses three-way decision to rank DMUs. Reference (Wang et al. 2005) uses the minimum-maximum regret method to rank DMU. Reference (Wang et al. 2016) ranks DMU according to the distance from the positive ideal cross-efficiency. References (Rakhshan 2017) and (Jie et al. 2013) use TOPSIS to rank DMU. Reference (Yul et al. 2019) ranks DMU by stochastic multi-criteria acceptability analysis 2 (SMAA2). Reference (Liu and Chen 2022) classifies DMUs by input–output slack and benefit score. Reference (Ebrahimi et al. 2020) finds efficient DMU by considering decision maker preferences. Reference (Liu et al. 2022) uses a multi-attribute group decision-making (MAGDM) method based on trust relationship to rank DMUs. Reference (Wu et al. 2021) ranks DMU by calculating the Shannon entropy of efficiency score and Reference (Omrani et al. 2020) uses the fuzzy weighted aggregate sum-product assessment method to rank DMUs.

Comparison of different models as shown in Table 2, where columns represent specific features of the proposed method: fuzzy information (a_1), attribute connection (a_2), loss function (a_3), multiple strategies (a_4), psychological cognition (a_5), conditional probability (a_6), classification (a_7), and ranking (a_8).

At the same time, how to weaken the impact and result deviation brought by their subjective loss functions during the combination process is also a key issue. Their combination will provide a new idea and decision framework for processing and analyzing uncertain multiple input–output decision problems. Inspired by the above ideas, this paper constructs two DEA models (Intuitionistic Fuzzy Sets-CCR, IFS-CCR) and (Intuitionistic Fuzzy Sets-BCC, IFS-BCC) in an intuitionistic fuzzy environment and proposes an intuitionistic fuzzy three-way decision model based on DEA. Table 2 shows the characteristics of the existing classical methods and the proposed methods. The main contributions are as follows:

Table 2 Comparison of different models

Methods	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	Work and contribution
Smet et al. (2016)	–	–	–	–	–	–	–	✓	A ranking method based on DEA and PROMETHEE II is proposed
An et al. (2018)	–	–	–	–	–	–	–	✓	Based on the DEA model of the TOPSIS method, a multi-criteria decision-making method is proposed
Namazi et al. (2018)	–	–	–	–	–	–	–	✓	A comprehensive ranking method for DMUs is developed by leveraging the advantages of DEA and AHP models
Rakhshan et al. (2017)	–	–	–	–	–	–	–	✓	A DEA model based on the TOPSIS method is proposed to avoid the subjective interference of decision makers in the decision process
Wang et al. (2021)	✓	–	–	–	–	–	–	✓	A two-phase framework of multi-criteria decision making was used to rank the DMUs
Liu et al. (2022)	✓	–	–	✓	–	✓	✓	✓	Classification and ranking of DMUs with interval efficiency using three-way decision
Wang et al. (2005)	✓	–	–	–	–	–	–	✓	Compare and sort the efficiency intervals of DMUs according to the maximum and minimum regret method
Wang et al. (2016)	✓	–	–	–	–	–	–	✓	The DMUs are evaluated and ranked according to their distance from the positive ideal cross-efficiency
Wu et al. (2013)	✓	–	–	–	–	–	–	✓	A new TOPSIS method is proposed to rank DMUs with imprecise information
Yu et al. (2019)	✓	–	–	–	–	–	–	✓	Solving the full sequence problem of interval efficiency using SMAA 2 method
Liu and Chen (2022)	–	–	–	✓	–	–	✓	–	Divide DMU into three orderly areas and develop effective forecasting strategies to act on these three areas to achieve better results
Toloo et al. (2020)	–	–	–	–	✓	–	–	✓	A new mixed binary linear DEA model is proposed to find the most efficient DMU by considering the decision-makers' preferences
Liu et al. (2022)	–	–	–	–	✓	–	–	✓	Based on the DEA regret cross-efficiency method, a trust propagation and trust aggregation model is constructed to solve the MAGDM problem, and all solutions are ranked according to the aggregation value
Wu et al. (2021)	–	–	–	–	–	–	–	✓	A combined DEA-group best-worst method is proposed to evaluate road safety, which incorporates the decision maker's preferences into the decision-making process
Amini et al. (2020)	–	–	–	–	✓	–	–	✓	A combined DEA-group best-worst method is proposed to evaluate road safety, which incorporates the decision maker's preferences into the decision-making process and overcomes the disadvantage of flexible weights in the DEA model
Our method	✓	✓	✓	✓	✓	✓	✓	✓	An intuitionistic fuzzy three-way decision model based on DEA is proposed to quantitatively analyze the impact of uncertain information on the input–output correlation in the decision process, classify and sort the DMUs based on three strategies, and propose a loss function that integrates the input–output distance

1. A method for measuring the correlation between input and output is proposed, and the optimal efficiency score is determined by using fuzzy measures as constraints.
2. Four IFS-CCR optimization models for different scenarios are proposed, and the corresponding algorithms are given.
3. The losses of DMUs under three strategies are given and an intuitionistic fuzzy three-way decision model based on DEA is proposed.

The rest of this paper is organized as follows: Sect. 2 introduces the basic knowledge of intuitionistic fuzzy set theory, three-way decisions, and DEA. Section 3 explains the calculation method of input–output connection, gives four IFS-CCR optimization models, and induces intuitionistic fuzzy three-way decision rules under three strategies of optimism, neutrality, and pessimism. Section 4 verifies the effectiveness of the proposed model through a series of experiments and comparative analysis, and finally summarizes this article.

2 Preliminaries

This section introduces some relevant basic concepts of the decision rough set (Pawlak 1982), intuitionistic fuzzy set (Atanassov and Stoeva 1986; Szmidt and Kacprzyk 2001), three-way decisions (Yao and Wong 1992), as well as the CCR model and BCC model (Charnes et al. 1978).

2.1 Three-way decision

Traditional decision theory regards people as rational decision-makers who follow the principle of maximizing economic interests in the decision-making process. Affected by the uncertainty of information and the cognition of decision-makers, it is difficult to be completely rational even if you fully understand and master the information and intelligence of the decision-making environment. This makes the limitations of the conventional binary decision method increasingly prominent. In response, Yao developed the three-way decision theory, which introduces delayed decisions to the conventional binary decision framework to mitigate decision risks. Given a decision information system $DS = (U, AT, V, f)$, $\forall X \subseteq U$, $A \subseteq AT$, the upper and lower approximations of X based on the equivalence relation R_A is as follows:

$$\underline{R}(X) = \{x \in U | [x]_A \subseteq X\},$$

$$\bar{R}(X) = \{x \in U | [x]_A \cap X \neq \emptyset\}.$$

The upper and lower approximations can be equivalently converted into POS, BND and NEG to describe the target concept X as follows:

$$POS(X) = \underline{R}(X),$$

$$BND(X) = \bar{R}(X) - \underline{R}(X),$$

$$NEG(X) = U - \bar{R}(X).$$

Among them, *POS* means accepting that object x belongs to X, *NEG* means refusing to accept that x belongs to X, and *BND* means that x may belong to X. Since the equivalence relationship is too strict, in order to reduce boundary redundant information, an improved three-way decision method based on probabilistic rough sets is introduced below. Given a decision information system $DS = (U, AT, V, f)$, $\forall X \subseteq U, A \subseteq AT$, assume that (α, β) is a pair of probability thresholds, meet the conditions $0 \le \beta < \alpha \le 1$. Then the conditional probability function is as follows:

$$P(X|[x]) = \frac{|X \cap [x]|}{|[x]|}.$$

The upper and lower approximations of R_A can be rewritten as follows:

$$\underline{R}_{(\alpha,\beta)}(X) = \{x \in U | P(X|[x]) \geqslant \alpha\},$$
$$\bar{R}_{(\alpha,\beta)}(X) = \{x \in U | P(X|[x]) > \beta\}.$$

Correspondingly, we can obtain the decision rules as follows:

$$POS(X) = \{x \in U | P(X|[x]) \geqslant \alpha\},$$
$$BND(X) = \{x \in U | \beta < P(X|[x]) < \alpha\},$$
$$NEG(X) = \{x \in U | P(X|[x]) \leqslant \beta\}.$$

2.2 Decision-theoretic rough sets

Yao and Wong (1992) introduced the Bayesian minimum risk decision based on the traditional Pawlak rough set proposed the decision rough set theory and then proposed a three-way decision model. Let $U = \{x_1, x_2, ..., x_n\}$ be a non-empty finite domain, R be the equivalence relation on U, $R \in U \times U$, and $U/R = \{[x]_R | x \in U\}$ be the equivalence class of x with respect to R. If the state set $\Omega = \{C, \neg C\}$ is given which represents $x \in C$ or $x \notin C$, and the action set $\Gamma = \{a_P, a_B, a_N\}$ which represents the three decision actions of acceptance, deferment and rejection. Namely, the object x is divided into the *POS* domain, *BND* domain, and *NEG* domain, the decision cost matrix of each action in different states As shown in Table 3.

Among them, when $x \in C$, the decision losses correspond to taking the three actions a_P, a_B and a_N are λ_{PP}, λ_{BP} and λ_{NP}. Similarly, when $x \notin C$, the decision losses corresponding to taking the above three actions are λ_{PN}, λ_{BN} and λ_{NN}. Therefore, the expected losses from taking the three actions a_P, a_B and a_N can be expressed as:

$$\begin{aligned} R(a_P|[x]_R) &= \lambda_{PP}P(C|[x]_R) + \lambda_{PN}P(\neg C|[x]_R), \\ R(a_B|[x]_R) &= \lambda_{BP}P(C|[x]_R) + \lambda_{BN}P(\neg C|[x]_R), \\ R(a_N|[x]_R) &= \lambda_{NP}P(C|[x]_R) + \lambda_{NN}P(\neg C|[x]_R). \end{aligned} \tag{1}$$

Table 3 Decision cost matrix

	C	$\neg C$
a_P	λ_{PP}	λ_{PN}
a_B	λ_{BP}	λ_{BN}
a_N	λ_{NP}	λ_{NN}

Among them, $P(C|[x]_R) = \left|C \bigcap [x]_R\right| / \left|[x]_R\right|$ represents the conditional probability that the equivalence class $[x]_R$ is in state C. Because of $P(C|[x]_R) + P(\neg C|[x]_R) = 1$, the above loss is only related to the classification conditional probability and the loss function $\lambda_{\bullet\bullet}(\bullet \in \{P, B, N\})$. Furthermore, consider that the loss of accepting the right thing is no greater than the loss of delaying acceptance of the right thing, and both are less than the loss of rejecting the right thing. Similarly, the loss of rejecting the wrong thing is no greater than the loss of delaying rejecting the wrong thing, and both are less than the loss of rejecting the wrong thing. Therefore, the size relationship between the loss functions satisfies: $\lambda_{PP} \le \lambda_{BP} < \lambda_{NP}$ and $\lambda_{PN} > \lambda_{BN} \ge \lambda_{NN}$.

According to the Bayesian decision criterion, the action set with the smallest expected loss is selected as the best action plan. The following three decision rules can be obtained as follows:

If $R(a_P|[x]_R) \le R(a_B|[x]_R) \wedge R(a_P|[x]_R) \le R(a_N|[x]_R)$, decide $x \in POS(C)$,
If $R(a_B|[x]_R) \le R(a_P|[x]_R) \wedge R(a_B|[x]_R) \le R(a_N|[x]_R)$, decide $x \in BND(C)$,
If $R(a_N|[x]_R) \le R(a_P|[x]_R) \wedge R(a_N|[x]_R) \le R(a_B|[x]_R)$, decide $x \in NEG(C)$.

2.3 Intuitionistic fuzzy sets

During the decision process, people usually make uncertain judgments about vague and complex objective things. For this reason, Zadeh proposed the concept of fuzzy sets to represent uncertain information. To overcome the shortcomings of a single membership degree, Atanassov proposed the theory of Intuitionistic Fuzzy Sets (IFS), which is described by three scalars: membership degree, non-membership degree, and hesitation degree to describe uncertain information.

Definition 1 Atanassov and Stoeva (1986) Let $U = \{x_1, x_2, ..., x_n\}$ be a non-empty finite domain, $\forall T \subseteq U$, $u \in T$, call $T = \{< \mu_T(u), v_T(u) > |u \in U\}$ an intuitionistic fuzzy set on U, where $\mu_T(u) : U \to [0, 1]$ represents the membership degree of u to T, $v_T(u) : U \to [0, 1]$ represents the membership degree of u that does not belong to T (i.e., non-membership degree), and satisfying conditions $0 \le \mu_T(u), v_T(u) \le 1$ and $0 \le \mu_T(u) + v_T(u) \le 1$. $\pi_T(u) = 1 - \mu_T(u) - v_T(u)$ represents the degree of hesitation that u belongs to T. In addition, if $\forall u \in U$ has $\pi_T(u) = 0$, then the intuitionistic fuzzy set T degenerates into a fuzzy set. For the convenience of expression, the sequence pair $(\mu_T(u), v_T(u))$ is called an intuitionistic fuzzy number, and $\alpha = (\mu_a, v_a)$ is usually used to represent the intuitionistic fuzzy number α.

Definition 2 Xu (2007) Let $\alpha = (\mu_\alpha, v_\alpha)$ and $\beta = (\mu_\beta, v_\beta)$ be two intuitionistic fuzzy numbers, then there are the following operations:

(1) $\alpha \oplus \beta = (\mu_\alpha + \mu_\beta - \mu_\alpha \mu_\beta, v_\alpha v_\beta)$,
(2) $\alpha \otimes \beta = (\mu_\alpha \mu_\beta, v_\alpha + v_\beta - v_\alpha v_\beta)$,
(3) $\omega\alpha = (1 - (1 - \mu_\alpha)^\omega, v_\alpha^\omega), \omega > 0$,
(4) $\alpha^\omega = (\mu_\alpha^\omega, 1 - (1 - v_\alpha)^\omega), \omega > 0$.

Definition 3 Xu (2007) Let $\alpha = (\mu_a, v_a)$ and $\beta = (\mu_b, v_b)$ be two intuitionistic fuzzy numbers, then we have

$$S(\alpha) = \mu_\alpha - \nu_\alpha \, Z(\alpha) = \mu_a + \nu_a. \tag{2}$$

Among them, $S(\alpha)$ is the score function, $S(\alpha) \in [-1, 1]$ and $Z(\alpha)$ is the exact function, $Z(\alpha) \in [0, 1]$ satisfy the following relationships:

(1) If $S(\alpha) < S(\beta)$, decide α less than β, recorded as $\alpha < \beta$;
(2) If $S(\alpha) = S(\beta)$, decide If $Z(\alpha) < Z(\beta)$, decide α less than β, recorded as $\alpha < \beta$; If $Z(\alpha) = Z(\beta)$, decide α equal β, recorded as $\alpha = \beta$.

2.4 Choquet integral

Choquet integral as nonlinear integration, is a method of aggregating or integrating functions in the context of multi-criteria decisions. It is appropriate for addressing non-additive measurement or decision problems. The method aggregates input–output values based on the set function L, substituting the additivity of probability measures with monotonicity.

Definition 4 Grabisch and Labreuche (2016) Let $(X, P(X), L)$ be a fuzzy measure space, where $X = \{e_1, e_2, ..., e_n\}$ is a multidimensional data set, $P(X)$ is the power set of X, L is the set function, and the measurable function is $f : X \to (-\infty, +\infty)$. If the set function $L : P(X) \to [0, +\infty)$ satisfies $L(\emptyset) = 0$; $E, F \in P(X)$, $E \subset F$ and $L(E) \le L(F)$, it is called the benefit measure on X. Then the choquet integral of $f(e)$ with respect to the benefit measure L is as follows:

$$\int fdL = \int_{-\infty}^{0} [L(F_\xi) - L(X)]d\xi + \int_{0}^{+\infty} L(F_\xi)d\xi. \tag{3}$$

Among them, $F_\xi = \{e|f(e) \ge \xi, e \in X\}$, $\xi \in [0, +\infty)$, when f is a non-negative function, $\int fdL = \int_0^{+\infty} L(F_\xi)d\xi$. When $X = \{e_1, e_2, ..., e_m\}$ is a finite set, the function value $f(e_1), f(e_2), ..., f(e_m)$ of f can be arranged in increasing order $f(e_1{}') \le f(e_2{}') \le ... \le f(e_m{}')$; where the set $\{e_1, e_2, ..., e_m\}$ is rearranged from the subsets in the set $\{e_1{}', e_2{}', ..., e_m{}'\}$ from small to large. Therefore, the Choquet integral can be obtained as follows:

$$\int fdL = \sum_{i=1}^{n} [f(e_i{}') - f(e_{i-1}{}')]L(A_i); f(e_0{}') = 0, A_i = \{e_i{}', e_{i+1}{}', ..., e_m{}'\}. \tag{4}$$

Taking input variables as an example, an equivalent integral corresponding to the Choquet integral (Wang and Guo 2003) is as follows:

$$\int fdL = \sum_{j=1}^{2^m-1} z_j L(A_j), \tag{5}$$

where $z_j = \max\{\min_{i \in A_j} f(e_i) - \max_{i \notin A_j} f(e_i), (0, 1)\}, A_1 = \{1\}, A_2 = \{2\}, ..., A_{2^m-1} = \{1, 2, ..., 2^m - 1\}$, the output variable ρ_j is the same.

2.5 DEA

DEA can evaluate the efficiency of a DMU by considering multiple inputs and outputs. It assesses whether the DMU achieves a balance between input and output and further evaluates the relative effectiveness of efficiency.

Definition 5 Charnes et al. (1978) Let $M = \{u_1, u_2, ..., u_n\}$ represents n DMUs, denoted as $u_j(j = 1, 2, ..., n)$. each DMU has m input variables and s output variables. The p-th input of the j-th DMU is $e_{pj}(p = 1, 2, ..., m; j = 1, 2, ..., n)$, and the corresponding weight is o. The q-th output is $y_{qj}(q = 1, 2, ..., s; j = 1, 2, ..., n)$, and the corresponding weight is i, then the CCR model is as follows:

$$\max \psi_0 = \sum_{q=1}^{s} o_q y_{0q} / \sum_{p=1}^{m} i_p e_{0p},$$
$$s.t. \begin{cases} \sum_{q=1}^{s} o_q y_{qj} / \sum_{p=1}^{m} i_p e_{pj} \leq 1, j = 1, 2, \ldots n, \\ o_q \geq 0, q = 1, 2, ..., s, \\ i_p \geq 0, p = 1, 2, ..., m. \end{cases} \tag{6}$$

The above planning model is a fractional planning, which can be obtained by changing the Charnes-Cooper as follows:

$$\max \psi_0 = \sum_{q=1}^{s} O_q y_{0q},$$
$$s.t. \begin{cases} \sum_{q=1}^{s} O_q y_{qj} - \sum_{p=1}^{m} I_p e_{pj} \leq 0, j = 1, 2, \ldots n, \\ \sum_{p=1}^{m} I_p e_{0p} = 1, \\ I_p \geq 0, O_q \geq 0, \end{cases} \tag{7}$$

where I_p and O_q are the input and output weights used by each DMU to construct the best practice boundary, ψ_0 is the efficiency score of u_0, and u_0 is considered effective when $\psi_0 = 1$. The essence of the CCR model becomes apparent as the evaluated DMU endeavors to discover its weight vector. This pursuit aims to maximize its weighted output, subject to the constraints that its weighted input remains fixed at unity, and its weighted output does not exceed the weighted input of all DMUs.

Since the traditional CCR model assumes Constant Returns to Scale (CRS), the type of returns to scale of DMU cannot be estimated. In order to make up for this shortcoming, Banker et al. developed a BCC model that considers Variable Returns to Scale (VRS) based on the CCR model. The BCC model is described as follows:

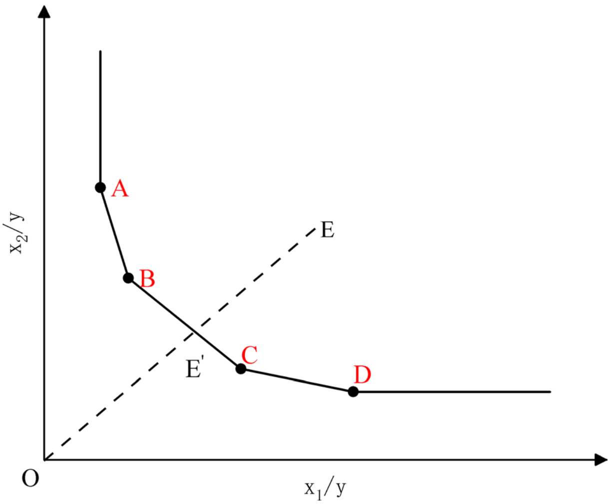


Fig. 1 DMU production frontier

$$\min \psi_0,\quad s.t.\begin{cases}\sum_{j=1}^{n}\lambda_j y_{qj} \geqslant y_{qo}, q = 1, 2, \ldots s,\\ \sum_{j=1}^{n}\lambda_j x_{pj} \leqslant \psi_0 x_{p0}, p = 1, 2, \ldots m,\\ \sum_{j=1}^{n}\lambda_j = 1,\\ \lambda_j \geqslant 0.\end{cases} \tag{8}$$

where ψ_0 is the best efficiency score of the DMU calculated based on the BCC model.

Among the above models, if the efficiency score of a DMU achieves DEA effectiveness if its efficiency score equals unity and it resides on the production frontier. Conversely, if the efficiency score deviates from unity, the DMU is deemed DEA ineffective. The corresponding DEA production frontier is shown in Fig. 1. Figure 1 shows five DMUs, each DMU has two input variables e_1, e_2 and one output variable y. The polyline in Fig. 1 is DEA's production frontier and DMU's relative efficiency value on the production frontier. Observing the figure above reveals the validity of DEA for A, B, C, and D, whereas E is identified as invalid. By mapping E to E' on the production frontier, DEA becomes valid at E', with the efficiency at E calculated as $OE'/OE < 1$.

3 Intuitionistic fuzzy three-way decision model based on DEA

To quantitatively analyze the impact of hesitancy on input–output interaction, this section proposes the input–output connection degree based on the set pair analysis theory and constructs the IFS-CCR model. Furthermore, the input and output of each DMU are integrated into the measurement process of the loss function. Finally, according to the difference in

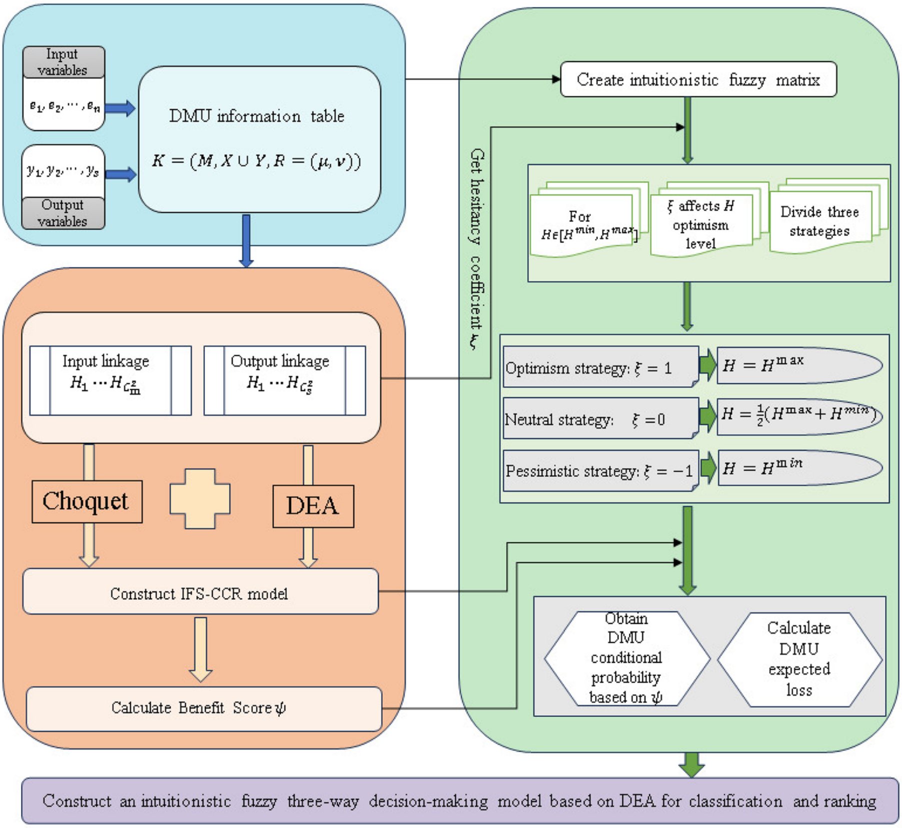


Fig. 2 Intuitionistic fuzzy three-way decision model framework based on DEA

the value of the hesitation degree in the input–output connection degree, the corresponding intuitionistic fuzzy three-way decision model is constructed from the three dimensions of optimism, neutrality, and pessimism. At the same time, a multi-strategy ranking method using the DEA benefit score as the conditional probability is proposed. The overall framework is shown in Fig. 2.

3.1 Intuitionistic fuzzy DEA model construction

The DEA model is an important tool for efficiency evaluation. Since the objective data in the real world are full of uncertainties, the traditional CCR model is limited to processing accurate data. To this end, this section also proposes an intuitionistic fuzzy DEA model that integrates the input–output connection degree to improve the model's ability to handle uncertain and complex problems.

Definition 6 Let $U = \{x_1, x_2, ..., x_n\}$ is a non-empty finite domain, T is an intuitionistic fuzzy set on U, $\alpha = (\mu_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \nu_\beta)$ are two intuitionistic fuzzy inputs on T, and their weights are ω_α and ω_β respectively, then

$$H = \mu_H + \pi_H \xi - \nu_H. \tag{9}$$

Among them, H is the connection degree between two input variables, $\xi \in [-1, 1]$ is the hesitancy value coefficient of H, $\mu_H = (\omega_\alpha \mu_\alpha + \omega_\beta \mu_\beta)$, $\pi_H = (\omega_\alpha \pi_\alpha + \omega_\beta \pi_\beta)$, and $\nu_H = (\omega_\alpha \nu_\alpha + \omega_\beta \nu_\beta)$ are the connection components between α and β respectively. Namely, the degree of identity, the degree of difference, and the degree of opposition. Decision makers can use H to measure the degree of connection between two input variables. Furthermore, the integration process reflects the impact of the input–output connection degree on the DMU benefit score based on the degree of optimism of the hesitancy value coefficient.

Theorem 1 *The input–output connection H satisfies the following properties*:

(1) *When* $\xi \in [-1, 1]$, *the input–output connection* $H \in [-1, 1]$;
(2) *When* $\xi \in [-1, 1]$, *input–output connection degree* $\pi_H \xi$ *and* $-\pi_H \xi$ *have the same value range*.

Proof

(1) In the input–output connection H, there is $\mu_H : T \to [0, 1]\ \nu_H : T \to [0, 1]$. For any $u \in T$, there is $0 \le \mu_H + \nu_H \le 1$, $\xi \in [-1, 1]$, so $H \in [-1, 1]$ is established.
(2) When $\xi \in [-1, 1]$, $\pi_H \xi \in [\pi_H \xi - \pi_H \xi]$ is the same as $-\pi_H \xi \in [\pi_H \xi - \pi_H \xi]$, so its value range is the same.

□

Definition 7 Let there are n DMUs, denoted as $u_j (j = 1, 2, ..., n)$, each DMU has m input variables denoted as $X = \{e_1, e_2, ..., e_p\}$, $(p = 1, 2, ..., m)$, and s output variables denoted as $Y = \{y_1, y_2, ..., y_q\}$, $(q = 1, 2, ..., s)$. Considering the interaction between multiple input and output variables, use $o(\{y_q\})$ to represent the benefit measure of the output variable, $i(\{e_p\})$ to represent the benefit measure of the input variables, $g_j(y_q)$ to represent the data information of the output variables of the j-th DMU, and $f_j(\{e_p\})$ to represent the j-th DMU's output variables data information. The efficiency score of DMU is expressed by the maximum ratio of total output to total input. The IFS-CCR I model can be obtained as follows:

$$\max \psi_0 = \frac{\int g_{q0} do_q}{\int f_{p0} di_p},$$
$$s.t. \begin{cases} \frac{\int g_{qj} do_q}{\int f_{pj} di_p} \le (1, 0), j = 1, 2, \dots n, \\ 0 \le i(A) \le i(B); A \subseteq B, A, B \in P(X), \\ 0 \le o(C) \le o(D); C \subseteq D, C, D \in P(Y). \end{cases} \tag{10}$$

Perform the following conversion on IFS-CCR I:

Let $t_j = (1,0)/\int f_j di[t_j > (0,1)]$, $O_q = t_j o_q$, $I_p = t_j i_p$, get the IFS-CCR II model as follows:

$$\max \psi_0 = \int g_{q0} dO_q$$
$$s.t. \begin{cases} \int g_{qj} dO_q - \int f_{pj} dI_p \le (0,1), j = 1,2,\dots n, \\ \int f_{p0} dI_p = (1,0), \\ 0 \le I(A) \le I(B); A \subseteq B, A, B \in P(X), \\ 0 \le O(C) \le O(D); C \subseteq D, C, D \in P(Y). \end{cases} \tag{11}$$

In IFS-CCR II, if there are fuzzy measures O and I such that the DMU's efficiency score is equal to unity, the DMU is deemed to be effective; if the efficiency of the DMU is less than unity, the DMU is deemed to be invalid. The nature of the IFS-CCR II model becomes apparent as it presents itself as a linear programming problem. The objective function only considers the output data of DMU. The entire model is only related to the benefit measure $i(\{e_p\})$ of the input variable and the benefit measure $o(\{y_q\})$ of the output variable. To adapt the classic equivalent formula to the intuitionistic fuzzy set environment, presented below are the intuitionistic fuzzy subtraction operation and the multi-constraint intuitionistic fuzzy division operation, which integrate the input–output connection degree.

Definition 8 Let $\alpha = (\mu_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \nu_\beta)$ be two intuitionistic fuzzy numbers, then we have

$$\alpha - \beta = ((\mu_\alpha - \mu_\beta + \mu_\alpha \mu_\beta)|H|, \tfrac{\nu_\alpha}{\nu_\beta})$$
$$s.t. \begin{cases} \mu_\alpha - \mu_\beta + \mu_\alpha \mu_\beta \geqslant 0 \\ \dfrac{\nu_\alpha}{\nu_\beta} \geqslant 0 \\ (\mu_\alpha - \mu_\beta + \mu_\alpha \mu_\beta)|H| + \nu_\alpha / \nu_\beta \leqslant 1 \end{cases} \tag{12}$$

$$\alpha \div \beta = (\tfrac{\mu_\alpha}{\mu_\beta}, \nu_\alpha - \nu_\beta + \tfrac{\nu_\alpha - \nu_\beta}{1 - \nu_{\min}}),$$
$$s.t. \begin{cases} \frac{\mu_\alpha}{\mu_\beta} \ge 0, \\ \nu_\alpha - \nu_\beta + \frac{\nu_\alpha - \nu_\beta}{1 - \nu_{\min}} \ge 0, \\ \frac{\mu_\alpha}{\mu_\beta} + \nu_\alpha - \nu_\beta + \frac{\nu_\alpha - \nu_\beta}{1 - \nu_{\min}} \le 1. \end{cases} \tag{13}$$

In Eq. 12, the closer the connection between input and output, the bigger the impact of membership after integration, and the overall positive connection appears. If Eq. 12 does not satisfy its constraints, let $\alpha - \beta = (0, 1)$. Similarly, if Eq. 13 does not satisfy its constraints, let $\alpha \div \beta = (0, 1)$. Therefore, according to Definition 4 and Eqs. 12 and 13, the IFS-CCR III model is as follows:

$$\max \psi_0 = \sum_{q=1}^{2^s-1} \rho_{q0} O_q,$$
$$s.t. \begin{cases} \sum_{q=1}^{2^s-1} \rho_{qj} O_{qj} - \sum_{p=1}^{2^m-1} z_{pj} I_{pj} \leq (0,1), j = 1,2,\dots n \\ \sum_{p=1}^{2^m-1} z_{pj} I_{pj} = (1,0), \\ 0 \leq I(A) \leq I(B); A \subseteq B, A, B \in P(X), \\ 0 \leq O(C) \leq O(D); C \subseteq D, C, D \in P(Y). \end{cases} \tag{14}$$

IFS-CCR III uses the intuitionistic fuzzy integration operator (Xu 2007) to integrate the input–output variables z_{pj} and ρ_{qj} in the model to obtain the optimal benefit score. The intuitionistic fuzzy weighted average operator of $g_k(y_r)$ regarding the fuzzy measure is:

$$IFWA(g_j) = \sum_{q=1}^{2^s-1} \rho_{qj} O_{qj} = (1 - \prod_{q=1}^{2^s-1} (1-\mu_{qj})^{O_{qj}}, \prod_{q=1}^{2^s-1} v_{qj}^{O_{qj}}). \tag{15}$$

In the same way, $f_k(e_i)$'s intuitionistic fuzzy weighted average operator regarding fuzzy measures is:

$$IFWA(f_j) = \sum_{p=1}^{2^m-1} z_{pj} I_{pj} = (1 - \prod_{p=1}^{2^m-1} (1-\mu_{pj})^{I_{pj}}, \prod_{p=1}^{2^m-1} v_{pj}^{I_{pj}}). \tag{16}$$

Applying Eqs. 15 and 16 to IFS-CCR III, IFS-CCR IV based on the intuitionistic fuzzy weighted average operator can be obtained:

$$\max \psi_0 = IFWA(g_0)$$
$$s.t. \begin{cases} IFWA(g_j) - IFWA(f_j) \leq (1,0), j = 1,2,\dots n, \\ IFWA(f_j) = (1,0), \\ 0 \leq I(A) \leq I(B); A \subseteq B, A, B \in P(X), \\ 0 \leq O(C) \leq O(D); C \subseteq D, C, D \in P(Y). \end{cases} \tag{17}$$

In the same way, BCC use the intuitionistic fuzzy integration operator [30] to integrate the input–output variables x_{pj} and y_{qj} in the model to obtain the optimal benefit score. The intuitionistic fuzzy weighted average operator of $g_k(y_r)$ regarding the weight λ:

$$IFWA(g_j) = \sum_{q=1}^{2^s-1} y_{qj} \lambda_j = (1 - \prod_{q=1}^{2^s-1} (1-\mu_{qj})^{\lambda_j}, \prod_{q=1}^{2^s-1} v_{qj}^{\lambda_j}). \tag{18}$$

The intuitionistic fuzzy weighted average operator of $f_k(e_i)$ regarding the weight λ:

$$IFWA(f_j) = \sum_{p=1}^{2^m-1} x_{pj} \lambda_j = (1 - \prod_{p=1}^{2^m-1} (1-\mu_{pj})^{\lambda_j}, \prod_{p=1}^{2^m-1} v_{pj}^{\lambda_j}). \tag{19}$$

Applying eqs. (18–19) to BCC model, IFS-BCC model based on the intuitionistic fuzzy weighted average operator can be obtained:

$$\min \psi_0$$
$$s.t.\begin{cases} \sum_{j=1}^{n} IFWA(g_j) \geqslant IFWA(g_0), j = 1, 2, \ldots n, \\ \sum_{j=1}^{n} IFWA(f_j) \leqslant \psi_0 IFWA(f_0), \\ \sum_{j=1}^{n} \lambda_j = 1, \\ \lambda_j \geqslant 0. \end{cases} \tag{20}$$

In the process of solving the benefit score, IFS-CCR IV integrates intuitionistic fuzzy data of input and output using the intuitionistic fuzzy weighted average operator, while also incorporating the input–output connection degree.

Different from the traditional CCR model, IFS-CCR IV introduces the benefit measure as an optimization parameter. This approach enhances the objectivity of the benefit measure, facilitating the derivation of the benefit score under optimal conditions. Consequently, it addresses the calculation problem associated with the DMU benefit score in an intuitionistic fuzzy environment. The key steps of the IFS-CCR IV model are outlined below, with corresponding pseudocode provided in Algorithm 1.

Step 1 Calculate the input–output connection H between each variable according to Eq. 9.
Step 2 Expand the input and output variables in the DMU information table according to Definition 4.
Step 3 Calculate the difference between the expanded variables according to Eq. 12.
Step 4 Obtain IFS-CCR IV based on Eqs. 15 and 16 and the results in step 3.
Step 5 Solve the benefit score based on IFS-CCR IV.

Algorithm 1 DMU benefit score solving algorithm

Input: (1) DMU information table $K = (M, X \cup Y, V = (\mu, \nu))$; (2) Weight ω.
Output: Benefit score ψ.

begin
  for *each* $u_j \in M$ **do**
    Calculate the input-output connection degree according to Eq. 9 H
    compute According to Definition 4, the input and output variables in the DMU information table are expanded, and the rules are as follows: $A_1 = \{1\}, A_2 = \{2\}, A_3 = \{1, 2\}, A_4 = \{3\}, ..., A_{2^{n-1}} = \{1, 2, ..., 2^m - 1\}$
  end
  Generator The matrix of input-output connection.
  Establishing computational rules grounded in input-output connection, according to Eq. 12
  for *Get the data* $R = (\mu, \nu)$ *and weight* ω *of all* $u_j \in M$ *from the DMU information table* K **do**
    Substitute the choquet integrals for the inputs and outputs of each DMU with the equivalent formula provided in Definition 4.
    if $\min_{p \in A_j} f(e_p) - \max_{p \notin A_j} f(e_p) < (0, 1)$ **then**
      $z_j = (0, 1);$
    end
    else
      $z_j = \min_{p \in A_j} f(e_p) - \max_{p \notin A_j} f(e_p);$
    end
    for *Build model: IFS-CCR* **do**
      Set objective function: $\max \psi_0 = \sum_{q=1}^{2^s-1} \rho_{qj} O_{qj}$

      Set constraints:
$$\begin{aligned} &\sum_{q=1}^{2^s-1} \rho_{qj} O_{qj} - \sum_{p=1}^{2^m-1} z_{pj} I_{pj} \leq (0, 1), j = 1, 2, \ldots n \\ &\sum_{p=1}^{2^m-1} z_{pj} I_{pj} = (1, 0) \\ &0 \leq I(A) \leq I(B); A \subseteq B, A, B \in P(X) \\ &0 \leq O(C) \leq O(D); C \subseteq D, C, D \in P(Y) \end{aligned}$$
    end
    Utilize Eqs. 15 and 16 to integrate the intuitionistic fuzzy information from z_j and ρ_j resulting in the IFS-CCR IV model.
  end
  Obtain efficiency scores for DMUs using the IFS-CCR IV model ψ_j
end

Table 4 Loss function integrating input–output distance

	C	$\neg C$
a_P	$\lambda^j_{PP} = 0$	$\lambda^j_{PN} = d_{\max} - \lvert \frac{\max \cos_Y\theta - \min \cos_X\theta}{\max \cos_Y\theta + \max \cos_X\theta} \rvert$
a_B	$\lambda^j_{BP} = \eta(\lvert \frac{\max \cos_X\theta - \min \cos_Y\theta}{\max \cos_Y\theta + \max \cos_X\theta} \rvert - d_{\min})$	$\lambda^j_{BN} = \eta(d_{\max} - \lvert \frac{\max \cos_Y\theta - \min \cos_X\theta}{\max \cos_Y\theta + \max \cos_X\theta} \rvert)$
a_N	$\lambda^j_{NP} = \lvert \frac{\max \cos_X\theta - \min \cos_Y\theta}{\max \cos_Y\theta + \max \cos_X\theta} \rvert - d_{\min}$	$\lambda^j_{NN} = 0$

3.2 Loss function integrating input–output impact

The loss function of the traditional three-way decision model is generally given by experts, which means that the actions $\Gamma = \{a_P, a_B, a_N\}$ taken by different plans in state $\Omega = \{C, \neg C\}$ have the same loss value. This phenomenon contradicts reality, since different schemes may lead to different losses depending on their characteristics. To incorporate the impact of intuitionistic fuzzy input–output on decision loss, this section proposes a loss-cost function that fuses cosine distance.

Definition 9 Let $K = (M, X \bigcup Y, V = (\mu, \nu))$ represent a DMU information table, where $M = \{u_1, u_2, ..., u_n\}$ is a DMU set, $X = \{e_1, e_2, ..., e_p\}$ is an input variable set, $Y = \{y_1, y_2, ..., y_q\}$ is an output variable set, and $V = (\mu, \nu)$ is an intuitionistic fuzzy input–output variable value set. The intuitionistic fuzzy mean of the input variable is as follows:

$$\overline{\mu_p} = 1 - \overline{\pi_p} - \overline{\nu_p}, \overline{\nu_p} = \prod_{j=1}^{n} \nu_{pj}^{\frac{1}{n}}, \overline{\pi_p} = \frac{1}{n} \sum_{j=1}^{n} \pi_{pj}, j = 1, 2, ..., n. \tag{21}$$

Among them, $\overline{\mu_p}$ is the average membership degree of the input variable e_p, $\overline{\nu_p}$ is the average non-membership degree of input variable e_p, $\overline{\pi_p}$ is the average hesitation degree of the input variable e_p, and meets the conditions $0 \le \overline{\mu_p}, \overline{\nu_p} \le 1, 0 \le \overline{\mu_p} + \overline{\nu_p} \le 1$ and $\overline{\pi_p} = 1 - \overline{\mu_p} - \overline{\nu_p}$.

Definition 10 In the DMU information table $K = (M, X \bigcup Y, V = (\mu, \nu))$, u_j is in state $\Omega = \{C, \neg C\}$, and the loss function for taking action $\Gamma = \{a_P, a_B, a_N\}$ is shown in Table 4.

Among them, $\eta(\eta \in (0, 1))$ is the risk aversion coefficient, $d_{\min} = 0$, $d_{\max} = 1$. This loss function is constructed using the difference between the input distance and the output distance of the variables in u_j from the average in the most favorable situation, that is when the input–output is closest to the average. Based on Definition 10, we know that $\lambda^j_{PP} < \lambda^j_{BP} < \lambda^j_{NP}, \lambda^j_{NN} < \lambda^j_{BN} < \lambda^j_{PN}$, the loss function exhibits the following characteristics.

(1) When the decision unit u_j is in state C, the loss incurred by the DMU when making an acceptance decision is $\lambda^j_{PP} = 0$. If it chooses to reject the decision, the loss incurred by the DMU at this time is $\lambda^j_{NP} = \lvert \frac{\max \cos_X\theta - \min \cos_Y\theta}{\max \cos_Y\theta + \max \cos_X\theta} \rvert - d_{\min}$. Namely, as the output-input ratio increases, the expected Decision losses increase. In addition, considering people's risk aversion in the real world, when making delayed decisions, the loss value should be smaller than the loss of rejecting the decision, so the delayed decision loss function can be expressed as $\lambda^j_{BP} = \eta(\lvert \frac{\max \cos_X\theta - \min \cos_Y\theta}{\max \cos_Y\theta + \max \cos_X\theta} \rvert - d_{\min})$.

(2) When the decision unit u_j is in state $\neg C$, the loss incurred by the DMU when making a rejection decision is $\lambda_{NN}^j = 0$. If it chooses to accept the decision, the loss incurred by the DMU at this time is $\lambda_{PN}^j = d_{\max} - |\frac{\max\cos_Y\theta - \min\cos_X\theta}{\max\cos_Y\theta + \max\cos_X\theta}|$. Namely, as the output-input ratio decreases, expected decision losses increase. In addition, considering people's risk aversion in the real world, when making a delayed decision, the loss value should be smaller than the loss of accepting the decision, so the loss function of the delayed decision can be expressed as $\lambda_{BN}^j = \eta(d_{\max} - |\frac{\max\cos_Y\theta - \min\cos_X\theta}{\max\cos_Y\theta + \max\cos_X\theta}|)$.

Among them, $\eta(\eta \in (0,1))$ is the risk aversion coefficient. Referring to Definition 6, it becomes evident that the input–output relationship incorporates the hesitancy coefficient ξ. Considering the unique personality and attitude variations among DMUs, the hesitation coefficient ξ acts as a parameter, reflecting the decision-maker's optimism in managing the input–output connection degree within the designated number of intervals.

Definition 11 Assume that the input–output connection degree is H, $H \in [H^{\min}, H^{\max}]$, and the hesitation degree coefficient is ξ, $\xi \in [-1, 1]$. There are three situations as follows:

Case 1 When $\xi = 1$, according to Definition 8 and IFS-CCR IV, at this time the membership degree is the biggest and the benefit score is the largest. Therefore, the current DMU is completely optimistic, which is the Optimistic Strategy (Opt), with $H = H^{\max}$;

Case 2 When $\xi = -1$, it means that the current DMU is completely pessimistic, that is, it is the Pessimistic Strategy (Pes), with $H = H^{\min}$;

Case 3 When $\xi = 0$, it means that the current decision attitude of DMU is unknown, which is the Neutral Strategy (Neu), with $H = \frac{1}{2}(H^{\max} + H^{\min})$.

Therefore, based on the above three strategies, the decision rule in section 2.1 can be rewritten as follows:

If $R(a_P|u_j)^\Delta \leq R(a_B|u_j)^\Delta \wedge R(a_P|u_j)^\Delta \leq R(a_N|u_j)^\Delta$, decide $u_j \in POS(C)$;

If $R(a_B|u_j)^\Delta \leq R(a_P|u_j)^\Delta \wedge R(a_B|u_j)^\Delta \leq R(a_N|u_j)^\Delta$, decide $u_j \in BND(C)$;

If $R(a_N|u_j)^\Delta \leq R(a_P|u_j)^\Delta \wedge R(a_N|u_j)^\Delta \leq R(a_B|u_j)^\Delta$, decide $u_j \in NEG(C)$.

Among them, $\Delta = \{Opt, Pes, Neu\}$, $C \in \Omega$, $u_j \in M$.

3.3 Intuitionistic fuzzy three-way decision model based on DEA

This section gives the intuitionistic fuzzy three-way decision model algorithm based on DEA, and provides a detailed description of the decision process based on the IFS-CCR model. The classification and ranking results of each plan are obtained through the proposed model. The key steps are as follows:

Step 1 Calculate the benefit score ψ of each DMU according to Algorithm 1.

Step 2 Calculate the input–output intuitionistic fuzzy mean and distance.

Step 3 Calculate the loss function of each DMU according to Definition 10.

Step 4 Calculate the decision loss of each DMU based on the benefit score and the loss function and divide M into three domains: POS, BND, and NEG based on the size of the loss.

Step 5 Sort the DMUs in each domain according to the loss value. The corresponding pseudocode is as follows:

Algorithm 2 Intuitionistic fuzzy three-way decision model based on DEA

Input: (1)DMU information table $K = (M, X \cup Y, V = (\mu, \nu))$; (2)Benefit score ψ; (3)Degree of input-output connection H;
Output: Classify and sort.

```
begin
    for each e_p ∈ X, y_q ∈ Y do
        Calculate the intuitive fuzzy mean of each input-output according to Eq. 21
        (\overline{μ_p^{input}}, \overline{ν_p^{input}}) and (\overline{μ_q^{output}}, \overline{ν_q^{output}});
        Calculate the cosine distance between each data and the intuitionistic fuzzy
        mean according to Definition 10 d_{cos θ};
        Construct six loss functions based on cosine distance
        λ_PP, λ_PN, λ_BP, λ_BN, λ_NP, λ_NN;
        Calculate the decision loss by combining the benefit scores of Algorithm 1
        R(a_P|[u]_R)^Δ, R(a_B|[u]_R)^Δ, R(a_N|[u]_R)^Δ;
    end
    for each u_j ∈ M do
        if R(a_P|[u_j]_R)^Δ ≤ R(a_B|[u_j]_R)^Δ ∧ R(a_P|[u_j]_R)^Δ ≤ R(a_N|[u_j]_R)^Δ then
            u_j ∈ POS;
        end
        if R(a_B|[u_j]_R)^Δ ≤ R(a_P|[u_j]_R)^Δ ∧ R(a_B|[u_j]_R)^Δ ≤ R(a_N|[u_j]_R)^Δ then
            u_j ∈ BND;
        end
        if R(a_N|[u_j]_R)^Δ ≤ R(a_P|[u_j]_R)^Δ ∧ R(a_N|[u_j]_R)^Δ ≤ R(a_B|[u_j]_R)^Δ then
            u_j ∈ NEG;
        end
    end
    for arbitrary u_a, u_b, u_c ∈ M do
        if u_a, u_b ∈ POS ∨ u_a, u_b ∈ BND then
            R_a < R_b ⇒ u_a ≻ u_b;
            else if u_a, u_b ∈ NEG then
                R_a > R_b ⇒ u_a ≻ u_b;
                else if u_a ∈ POS, u_b ∈ BND, u_c ∈ NEG then
                    u_a ≻ u_b ≻ u_c;
                end
            end
        end
    end
    return: Classification and ranking results of the three strategies.
end
```

3.4 Complexity analysis

The efficiency score solution algorithm includes two processes: input–output variable expansion and model solution. This article assumes that n represents the number of DMUs, m represents the number of input variables, and s represents the number of output variables. In the input–output variable expansion stage, all DMUs are first traversed and the degree of connection is calculated. The time complexity is $O(n \times (m + s))$, then expand the input–output variables of each DMU respectively, and its time complexity is $O(n \times (2^m + 2^s))$. Therefore, the time complexity of the input–output variable expansion stage is $O(n \times (2^m + 2^s + m + s))$. In the model solving phase, the expanded variables are initialized with a time complexity of $O(n + 2^m + 2^s)$, and then the data structure of the DEA model is created, including input variables, output variables, etc., with a time complexity of $O(n \times (2^m + 2^s))$. In the model-solving phase, the time complexity is mainly determined by the model, then the time complexity of solving the model is $O(n^2 \times (2^m + 2^s) \times (\log(2^s + 2^m)))$. Therefore, $O(n \times (2^m + 2^s + m + s) + n^2 \times (2^m + 2^s) \times (\log(2^s + 2^m)))$ is the overall time complexity of the benefit score solving algorithm.

4 Experiment and comparative analysis

4.1 Numerical example

To better explain the loss function based on the input–output connection degree and the input–output distance, this section uses a numerical example to illustrate the calculation process of the proposed connection degree and loss function.

Digital education refers to an education model that uses advanced information and communication technology to digitize and make all aspects of the education process intelligent. In the international environment, with the development of cloud computing and big data technology and the rapid globalization trend, digital education has become an important direction for many countries to promote education reform. At the same time, most countries have also introduced policy incentives and regulations to effectively promote the innovation and development of digital education-related technical facilities. The reason lies in the indispensable importance of digital education, which plays a pivotal role in enhancing the quality of education, fostering innovation, and mitigating educational inequality. This is significant for improving national competitiveness, promoting social development, and coping with changes. In response to the national call to improve digital teaching capabilities, a school is preparing to introduce a digital education system. When evaluating the digital education systems to be selected, due to the differences in thinking between management and branches, relevant experts are invited to evaluate the 10 digital education systems to be selected ($M = \{u_1, u_2, ..., u_{10}\}$). The input evaluation indicators set by experts during the evaluation process include acquisition cost (e_{1j}), required personnel (e_{2j}), and system operability (e_{3j}). The output indicators include the level of integration of educational information resources (y_{1j}), and the intelligence of education platform facilities. Level (y_{2j}). The intuitionistic fuzzy evaluation information of the utility evaluation object is shown in Table 5. Now it is necessary to analyze the multi-input–output connection degree of the digital education system. The specific process is as follows:

Firstly, calculate the input–output connection degree based on Definition 6, as shown in Table 6. Among them, the value range of ξ is $\xi \in [-1, 1]$, which has the effect of converting uncertainty into certainty. Therefore, according to Definition 11, the input–output connection degree under different strategies can be obtained. According to the definition of input–output connection, when ξ equals unity, it means that all the uncertain

Table 5 DMU information table

DMU	X			Y	
	e_1	e_2	e_3	y_1	y_2
u_1	(0.8, 0.1)	(0.2, 0.4)	(0.6, 0.4)	(0.3, 0.5)	(0.7, 0.2)
u_2	(0.5, 0.5)	(0.2, 0.7)	(0.4, 0.4)	(0, 0.7)	(0.3, 0.5)
u_3	(0.5, 0.4)	(0.3, 0.7)	(0.7, 0.1)	(0.5, 0.3)	(0.3, 0.4)
u_4	(0.9, 0.1)	(0.2, 0.3)	(0.6, 0.3)	(0.3, 0.5)	(0.5, 0.5)
u_5	(0.3, 0.6)	(0, 0.8)	(0.1, 0.7)	(0, 0.1)	(0.2, 0.8)
u_6	(0.3, 0.7)	(0, 1)	(0.5, 0.4)	(0.2, 0.6)	(0.4, 0.4)
u_7	(0.4, 0.5)	(0.1, 0.7)	(0.3, 0.7)	(0.3, 0.6)	(0.3, 0.5)
u_8	(0.7, 0.3)	(0.1, 0.9)	(0.3, 0.6)	(0.4, 0.4)	(0.2, 0.6)
u_9	(0.3, 0.5)	(0, 0.9)	(0.5, 0.5)	(0.4, 0.5)	(0.1, 0.7)
u_{10}	(0.3, 0.4)	(0.1, 0.8)	(0.6, 0.4)	(0.4, 0.5)	(0.3, 0.6)
ω	1/3	1/3	1/3	1/2	1/2

Table 6 Input–output connection degree

DMU	Input variable connection			Output variable connection
	$H_{e_1e_2}$	$H_{e_1e_3}$	$H_{e_2e_3}$	$H_{y_1y_2}$
u_1	$0.17\xi + 0.17$	$0.03\xi + 0.3$	0.13ξ	$0.15\xi + 0.15$
u_2	$0.03\xi - 0.17$	0.07ξ	$0.1\xi - 0.17$	$0.25\xi - 0.45$
u_3	$0.03\xi - 0.1$	$0.1\xi + 0.23$	$0.07\xi + 0.07$	$0.3\xi + 0.05$
u_4	$0.17\xi + 0.23$	$0.03\xi + 0.37$	$0.2\xi + 0.07$	$0.1\xi - 0.1$
u_5	$0.1\xi - 0.37$	$0.1\xi - 0.3$	$0.13\xi - 0.47$	$0.45\xi - 0.35$
u_6	-0.47	$0.03\xi - 0.1$	$0.03\xi - 0.3$	$0.2\xi - 0.2$
u_7	$0.1\xi - 0.23$	$0.03\xi - 0.17$	$0.07\xi - 0.33$	$0.15\xi - 0.25$
u_8	-0.4	$0.03\xi + 0.03$	$0.03\xi - 0.37$	$0.2\xi - 0.2$
u_9	$0.1\xi - 0.37$	$0.07\xi - 0.07$	$0.03\xi - 0.3$	$0.15\xi - 0.35$
u_{10}	$0.13\xi - 0.27$	$0.1\xi + 0.03$	$0.03\xi - 0.17$	$0.1\xi - 0.2$

Table 7 The distance between each input–output and the average value

DMU	X			Y	
	e_1	e_2	e_3	y_1	y_2
u_1	0.911	0.855	0.987	0.989	0.792
u_2	0.964	0.995	0.974	0.827	0.992
u_3	0.992	0.960	0.860	0.935	0.956
u_4	0.896	0.715	0.979	0.989	0.959
u_5	0.853	0.975	0.730	0.500	0.899
u_6	0.810	0.949	1.000	0.944	0.984
u_7	0.943	0.995	0.872	0.942	0.992
u_8	0.980	0.968	0.908	0.988	0.954
u_9	0.876	0.968	0.984	0.960	0.890
u_{10}	0.860	0.990	0.987	0.960	0.982

Table 8 Loss function of each DMU

	λ^j_{PP}	λ^j_{BP}	λ^j_{NP}	λ^j_{PN}	λ^j_{BN}	λ^j_{NN}
u_1	0	0.020	0.099	0.962	0.192	0
u_2	0	0.009	0.044	0.986	0.197	0
u_3	0	0.004	0.018	0.949	0.190	0
u_4	0	0.002	0.010	0.910	0.182	0
u_5	0	0.030	0.150	0.909	0.182	0
u_6	0	0.007	0.037	0.825	0.165	0
u_7	0	0.000	0.001	0.938	0.188	0
u_8	0	0.003	0.013	0.945	0.189	0
u_9	0	0.010	0.048	0.946	0.189	0
u_{10}	0	0.001	0.003	0.938	0.188	0

parts of u_j hold a supportive attitude at this time. At this time, u_j is in line with the optimistic attitude in the real world and adopts an optimistic strategy; when ξ is equal to zero, indicating that the uncertain part of u_j at this time has an attitude of neither support nor opposition. At this time, u_j conforms to the neutral attitude in the real world and adopts a neutral strategy; when ξ is equal to negative unity, it indicates that the uncertain part of u_j are opposed and adopt a pessimistic strategy at this time.

Then, according to the DMU information table and Definition 9, the intuitionistic fuzzy mean of each input–output is obtained:

$(\overline{\mu_p^{input}}, \overline{v_p^{input}}) = \{(0.56, 0.35), (0.16, 0.68), (0.51, 0.4)\}$,

$(\overline{\mu_q^{output}}, \overline{v_q^{output}}) = \{(0.33, 0.42), (0.36, 0.49)\}$.

Furthermore, the cosine distance shown in Table 7 can be obtained based on the intuitionistic fuzzy mean and DMU information table. According to Table 7, the following results can be obtained:

(1) The cosine value of the angle between the input of each DMU and the average value in the optimal state is is as follows: 0.987, 0.995, 0.992, 0.979, 0.975, 1.000, 0.995, 0.980, 0.984, 0.990;

(2) The cosine value of the angle between the output of each DMU under optimal conditions and the average value is as follows: 0.989, 0.992, 0.956, 0.989, 0.899, 0.984, 0.992, 0.988, 0.960, 0.982;

(3) The cosine value of the angle between the input of each DMU in the worst state and the average value is as follows: 0.855, 0.964, 0.860, 0.715, 0.730, 0.810, 0.872, 0.908, 0.876, 0.860;

(4) The cosine value of the angle between the input of each DMU in the worst state and the average value is as follows: 0.792, 0.827, 0.935, 0.959, 0.500, 0.944, 0.942, 0.954, 0.890, 0.960. Finally, according to Definition 10 and Table 7, the loss functions of all DMUs can be calculated as shown in Table 8.

Taking u_1, u_2 and u_3 as an example, let the risk coefficient $\eta = 0.2$ when in state C, we can get $\lambda^3_{NP} < \lambda^2_{NP} < \lambda^1_{NP}$ and $\lambda^3_{BP} < \lambda^2_{BP} < \lambda^1_{BP}$, which means that no matter whether the expert takes the acceptance or rejection decision during the evaluation process, the loss function of u_3 is smaller than the plan u_1 and u_2. The loss function of the first scheme is greater than the loss function of the other schemes. In the same way, when in state $\neg C$, there are $\lambda_{BN}^3 < \lambda_{BN}^1 < \lambda_{BN}^2$ and $\lambda_{PN}^3 < \lambda_{PN}^1 < \lambda_{PN}^2$.

Observing Definitions 6 and 11 highlights the influence of the input–output connection on the benefit score. Consequently, based on the relationships outlined in Table 6, the following conclusions can be derived.

(1) Adopt an optimistic strategy: For input indicators, the largest connection degree between input indicators e_{1j} and e_{2j} of the system u_4 means that the system $u_4$can consume less input under the same output standard. The connection degree among the overall input indicators is the largest, which is higher than other systems. For output indicators, the largest connection degree between the output indicators y_{1j} and y_{2j} of system u_3 means that the system u_3 can obtain higher output under the same input standard. Among the overall output indicators, u_3 has the largest connection degree, which is better than other systems.
(2) Adopt a neutral strategy: System u_4 can consume less input under the same output standard. u_4 has the highest degree of connection among the overall input indicators and has higher benefits than other systems. System u_1 can obtain higher output under the same input standard. Among the overall output indicators, u_1 has the largest connection degree and has higher benefits than other systems.
(3) Adopt a pessimistic strategy: System u_4 can consume less input under the same output standard, u_4 has the highest degree of connection among the overall input indicators and has higher benefits than other systems. System u_1 can obtain higher output under the same input standard. Among the overall output indicators, u_1 has the highest degree of connection and has higher benefits than other systems.

4.2 Experimental analysis

In this section, experiments are conducted on the reservoir dam data in the literature (Chen et al. 2022), and the classification and ranking results are obtained for comparative analysis with other methods. Among them, there are a total of 19 reservoir dams, denoted as $M = \{u_1, u_2, ..., u_{19}\}$. To test their risk levels in earthquakes, they are evaluated by setting three input indicators and three output indicators. The evaluation inputs set during the evaluation process include the degree of earthquake risk. (e_{1j}), dam characteristics (e_{2j}) and pre-earthquake status (e_{3j}), and the output indicators include seismic cracks (y_{1j}), leakage (y_{2j}) and seismic deformation (y_{3j}). When conducting the evaluation, the intuitionistic fuzzy evaluation information of these dams is shown in Table 9.

Firstly, based on the division rules of the three strategies in Definition 11, the benefit scores of DMU under the three strategies are calculated according to IFS-CCR IV and IFS-BCC, as shown in Tables 10 and 11.

Secondly, if actions a_P, a_B and a_N are taken against a DMU that actually belongs to C, the losses are λ_{PP}, λ_{BP} and λ_{NP} respectively; On the contrary, if actions a_P, a_B and a_N are taken against a DMU that actually belongs to $\neg C$, the losses are λ_{PN}, λ_{BN} and λ_{NN} respectively. Therefore, based on the original loss function and according to Definition 10, the loss function of each DMU fusion input–output is calculated as shown in Table 12.

Finally, the benefit scores in Table 11 are used as conditional probabilities to calculate the expected decision losses of each DMU taking corresponding actions under

Table 9 DMU information table

DMU	X			Y		
	e_1	e_2	e_3	y_1	y_2	y_3
u_1	(0.877,0.062)	(0.344,0.482)	(0.572,0.214)	(0.656,0.117)	(0.219,0.634)	(0.096,0.788)
u_2	(0.873,0.063)	(0.236,0.623)	(0.291,0.556)	(0.505,0.303)	(0.765,0.006)	(0.492,0.318)
u_3	(0.874,0.063)	(0.116,0.759)	(0.304,0.526)	(0.369,0.447)	(0.220,0.629)	(0.724,0.036)
u_4	(0.852,0.074)	(0.313,0.525)	(0.426,0.39)	(0.437,0.378)	(0.683,0.093)	(0.273,0.574)
u_5	(0.847,0.076)	(0.292,0.537)	(0.161,0.7)	(0.560,0.219)	(0.260,0.576)	(0.369,0.444)
u_6	(0.851,0.075)	(0.422,0.393)	(0.43,0.384)	(0.656,0.122)	(0.396,0.424)	(0.150,0.723)
u_7	(0.949,0.025)	(0.167,0.671)	(0.099,0.759)	(0.027,0.855)	(0.246,0.571)	(0.301,0.503)
u_8	(0.85,0.075)	(0.204,0.64)	(0.048,0.84)	(0.232,0.605)	(0.751,0.124)	(0.205,0.639)
u_9	(0.861,0.069)	(0.172,0.69)	(0.048,0.846)	(0.656,0.114)	(0.683,0.083)	(0.219,0.631)
u_{10}	(0.862,0.069)	(0.122,0.739)	(0.205,0.633)	(0.587,0.174)	(0.178,0.667)	(0.137,0.719)
u_{11}	(0.796,0.102)	(1.0,0.0)	(0.304,0.539)	(0.519,0.286)	(0.410,0.413)	(0.096,0.795)
u_{12}	(0.865,0.067)	(0.16,0.71)	(0.521,0.277)	(0.082,0.808)	(0.738,0.028)	(0.519,0.279)
u_{13}	(0.936,0.032)	(0.391,0.417)	(0.24,0.601)	(0.301,0.526)	(0.601,0.171)	(0.000,0.908)
u_{14}	(0.813,0.093)	(0.257,0.578)	(0.378,0.431)	(0.396,0.409)	(0.328,0.491)	(0.232,0.609)
u_{15}	(0.947,0.026)	(0.332,0.469)	(0.275,0.539)	(0.000,0.896)	(0.123,0.732)	(0.232,0.593)
u_{16}	(0.842,0.079)	(0.209,0.637)	(0.572,0.203)	(0.109,0.764)	(0.464,0.329)	(0.219,0.625)
u_{17}	(0.829,0.085)	(0.555,0.253)	(0.378,0.459)	(0.642,0.153)	(0.383,0.453)	(0.683,0.106)
u_{18}	(0.925,0.038)	(0.306,0.506)	(0.235,0.593)	(0.096,0.771)	(0.437,0.348)	(0.041,0.843)
u_{19}	(0.977,0.012)	(0.078,0.802)	(0.369,0.439)	(0.246,0.590)	(0.383,0.422)	(0.287,0.539)

Table 10 DMU's benefit score for CRS

DMU	Benefit score		
	ψ_{Opt}	ψ_{Pes}	ψ_{Neu}
u_1	0.778	0.894	0.895
u_2	0.976	0.982	0.977
u_3	0.912	0.948	0.908
u_4	0.851	0.943	0.845
u_5	0.656	0.874	0.616
u_6	0.798	0.918	0.778
u_7	0.670	0.558	0.570
u_8	0.825	0.948	0.949
u_9	0.887	0.970	0.895
u_{10}	0.672	0.845	0.846
u_{11}	0.557	0.841	0.491
u_{12}	0.936	0.963	0.935
u_{13}	0.697	0.867	0.641
u_{14}	0.331	0.738	0.206
u_{15}	0.164	0.394	0.429
u_{16}	0.417	0.742	0.746
u_{17}	0.869	0.973	0.883
u_{18}	0.342	0.678	0.690
u_{19}	0.295	0.714	0.717

Table 11 DMU's benefit score for VRS

DMU	Benefit score		
	ψ_{Opt}	ψ_{Pes}	ψ_{Neu}
u_1	0.784	0.799	0.809
u_2	1.000	0.880	0.992
u_3	0.938	1.000	0.920
u_4	0.8641	0.876	0.888
u_5	0.679	0.973	0.722
u_6	0.809	0.877	0.847
u_7	0.342	0.937	0.388
u_8	0.826	1.000	1.000
u_9	0.864	1.000	0.876
u_{10}	0.717	1.000	0.738
u_{11}	0.667	1.000	0.743
u_{12}	0.962	0.955	0.955
u_{13}	0.639	0.663	0.683
u_{14}	0.514	1.000	0.580
u_{15}	0.293	0.621	0.338
u_{16}	0.559	0.976	0.617
u_{17}	0.884	0.938	0.927
u_{18}	0.466	0.705	0.518
u_{19}	0.378	1.000	0.439

Table 12 Loss function of each DMU

	λ_{PP}	λ_{BP}	λ_{NP}	λ_{PN}	λ_{BN}	λ_{NN}
u_1	5.25	5.282	8.156	10.194	7.169	3.95
u_2	0.25	4.758	9.809	7.191	5.138	4.75
u_3	0.50	4.032	8.406	6.594	4.869	4.50
u_4	5.85	6.004	6.784	10.266	5.703	2.55
u_5	5.50	5.774	7.750	10.200	4.80	4.15
u_6	1.75	1.961	9.347	8.553	4.531	5.00
u_7	5.75	5.885	8.011	9.639	4.128	2.15
u_8	1.00	1.264	8.572	6.928	5.486	4.35
u_9	0.00	5.011	10.354	5.396	4.079	4.00
u_{10}	1.95	2.086	9.135	8.115	5.623	5.35
u_{11}	5.45	5.515	6.052	10.248	5.350	3.95
u_{12}	1.25	1.481	10.107	7.643	4.379	5.50
u_{13}	5.35	5.672	6.299	9.851	7.020	5.15
u_{14}	4.85	5.714	6.931	10.719	7.144	4.25
u_{15}	4.75	4.894	5.536	10.814	8.663	6.85
u_{16}	4.65	5.977	6.774	10.462	6.585	5.00
u_{17}	0.75	1.675	8.505	8.745	4.549	4.25
u_{18}	4.50	5.029	6.218	10.286	7.906	6.00
u_{19}	5.35	5.462	8.686	9.314	6.363	4.00

different strategies. To display the expected losses of the three strategies more intuitively, an expected loss bar chart is drawn as shown in Fig. 3.

Among them, Fig. 3(*a-c*) shows the expected decision loss of DMU under the optimistic strategy, neutral strategy and pessimistic strategy based on CRS method, and Fig. 3(*d-f*) shows the expected decision loss of DMU under the optimistic strategy, neutral strategy and pessimistic strategy based on VRS method. The figure can be seen that when the DMU takes action $\Gamma = \{a_P, a_B, a_N\}$, it will be classified into the POS domain, BND domain or NEG domain. Therefore, according to Definition 11, the corresponding decision rules can be obtained as shown in Tables 13 and 14.

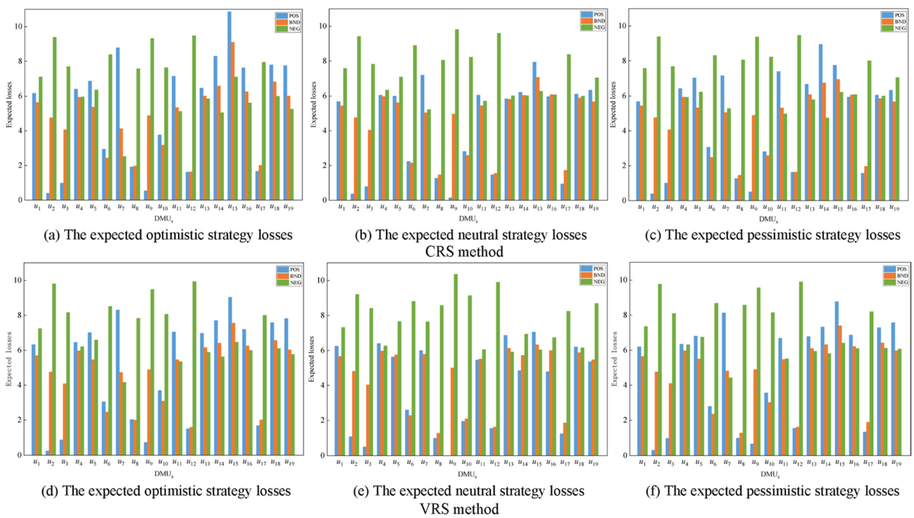
According to Tables 13 and 14, the decision domains under the three strategies can be drawn as shown in Fig. 4.

Finally, the schemes in each classification area are sorted according to the expected decision loss, and all schemes are sorted according to the rule $POS > BND > NEG$. The results are shown in Table 15.

Table 15 reveals that, in both the optimistic and pessimistic strategies, dam u_2 exhibits the highest risk level, followed by dam u_9. In the neutral strategy, the risk level of dam u_9 is the highest, followed by dam u_2, of which three under the strategy, the risk level of dam u_{15} is the lowest, indicating that dam u_{15} has better seismic performance in earthquakes.

4.3 Comparative analysis

This section compares the IFS-CCR model and IFS-BCC model with several existing DEA models to illustrate the effectiveness of the proposed IFS-CCR model and IFS-BCC model. These five methods are Wang's method (Wang et al. 2005), Wang's method (Wang et al. 2016), Wu's method (Jie et al. 2013), Yu's method (Yul et al. 2019) and Liu's method (Chen et al. 2022). The decision areas and ranking results of the five models and the model in this article are shown in Fig. 4 and Table 14. The outcomes presented in the table and



(a) The expected optimistic strategy losses (b) The expected neutral strategy losses (c) The expected pessimistic strategy losses
CRS method

(d) The expected optimistic strategy losses (e) The expected neutral strategy losses (f) The expected pessimistic strategy losses
VRS method

Fig. 3 Expected losses of DMU under three strategies

Table 13 Decision rules of DMU under three strategies based on CRS

DMU	*Opt*		*Neu*		*Pes*	
	decision rules	decision loss	decision rules	decision loss	decision rules	decision loss
u_1	*BND*	5.701	*BND*	5.482	*BND*	5.480
u_2	*POS*	0.417	*POS*	0.375	*POS*	0.410
u_3	*POS*	1.036	*POS*	0.817	*POS*	1.061
u_4	*BND*	5.959	*BND*	5.987	*BND*	5.957
u_5	*BND*	5.439	*BND*	5.651	*BND*	5.532
u_6	*BND*	2.480	*BND*	2.172	*BND*	2.532
u_7	*NEG*	2.543	*BND*	5.108	*BND*	5.129
u_8	*BND*	2.003	*BND*	1.308	*POS*	1.302
u_9	*POS*	0.610	*POS*	0.162	*POS*	0.567
u_{10}	*BND*	3.246	*BND*	2.634	*BND*	2.631
u_{11}	*NEG*	5.121	*BND*	5.489	*NEG*	4.982
u_{12}	*POS*	1.659	*POS*	1.487	*POS*	1.666
u_{13}	*NEG*	5.951	*BND*	5.851	*NEG*	5.887
u_{14}	*NEG*	5.137	*BND*	6.089	*NEG*	4.802
u_{15}	*NEG*	6.635	*NEG*	6.332	*NEG*	6.286
u_{16}	*NEG*	5.740	*BND*	5.134	*POS*	6.117
u_{17}	*POS*	1.797	*POS*	0.966	*POS*	1.685
u_{18}	*NEG*	6.075	*BND*	5.955	*BND*	5.921
u_{19}	*NEG*	5.382	*BND*	5.720	*BND*	5.717

figure make it apparent that Wang's model in Wang et al. (2005) is evidently incapable of achieving a comprehensive ranking for all DMUs. In the model (Wang et al. 2016), the sorting relationship between u_2 and u_9 cannot be obtained, and both u_2 and u_9 are regarded as optimal solutions. Although the sorting order of different models is not the same, the optimal DMU is the same, and the consistent results illustrate the rationality and effectiveness of the proposed model. In addition, the reason why there are certain differences in the ranking results of the three strategies is that the degree of optimism of the input–output connection is different. The ranking results of different methods are shown in Table 16.

Reviewing Table 16, it becomes apparent that the comparison model utilized closely aligns with the top-ranked DMU in the proposed method. This implies that all ranking methods employ identical criteria for evaluation. In addition, from the perspective of local ranking results, for the group of rankings where the optimal two DMUs are $u_9 \succ u_2$. Neutral strategy for this method, method (Chen et al. 2022) and method (Jie et al. 2013) of this article all consider $u_9 \succ u_2$, while the method (Wang et al. 2005) and method (Wang et al. 2016). Then consider both as the optimal solution, namely $u_9 \approx u_2$. In the method (Yul et al. 2019), the optimal DMU is u_9, and the one next to u_9 is not u_2.

Table 14 Decision rules of DMU under three strategies based on VRS

DMU	*Opt*		*Neu*		*Pes*	
	decision rules	decision loss	decision rules	decision loss	decision rules	decision loss
u_1	*BND*	5.691	*BND*	5.661	*BND*	5.643
u_2	*POS*	0.250	*POS*	1.083	*POS*	0.304
u_3	*POS*	0.876	*POS*	0.500	*POS*	0.986
u_4	*BND*	5.963	*BND*	5.967	*BND*	5.970
u_5	*BND*	5.461	*POS*	5.627	*BND*	5.503
u_6	*BND*	2.453	*POS*	2.277	*BND*	2.355
u_7	*NEG*	4.156	*BND*	5.774	*NEG*	4.426
u_8	*BND*	1.998	*POS*	1.000	*POS*	1.000
u_9	*POS*	0.736	*POS*	0.000	*POS*	0.670
u_{10}	*BND*	3.087	*POS*	1.950	*BND*	3.011
u_{11}	*NEG*	5.351	*POS*	5.450	*BND*	5.473
u_{12}	*POS*	1.493	*POS*	1.538	*POS*	1.538
u_{13}	*NEG*	5.884	*NEG*	5.912	*NEG*	5.935
u_{14}	*NEG*	5.627	*POS*	4.850	*NEG*	4.805
u_{15}	*NEG*	6.464	*NEG*	6.034	*NEG*	6.406
u_{16}	*NEG*	5.991	*POS*	4.789	*NEG*	6.094
u_{17}	*POS*	1.680	*POS*	1.246	*POS*	1.334
u_{18}	*NEG*	6.102	*BND*	5.878	*NEG*	6.113
u_{19}	*NEG*	5.772	*POS*	5.350	*BND*	5.967

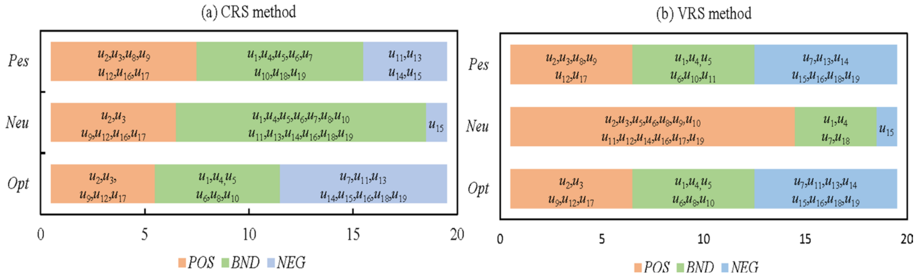


Fig. 4 Classification results of three strategies

In addition, because the method in this article considers the input–output connection, resulting in differences in local ranking results, the optimal DMU in the optimistic strategy and pessimistic strategy in this article is u_2. Based on several methods, the selection of the worst DMU is the same, and u_{15} is ranked as the worst DMU. Since different decision methods have differences in program selection, decision-makers need to clearly understand the background and positioning of each method to correctly select the most appropriate decision method.

Table 15 Ranking results of the three strategies

Methods	Strategies	Sort results
	Opt	$u_2 \succ u_9 \succ u_3 \succ u_{12} \succ u_{17} \succ u_8 \succ u_6 \succ u_{10} \succ u_5 \succ u_1 \succ u_4 \succ u_7 \succ u_{11} \succ u_{14} \succ u_{19} \succ u_{16} \succ u_{13} \succ u_{18} \succ u_{15}$
CRS	*Neu*	$u_9 \succ u_2 \succ u_3 \succ u_{17} \succ u_{12} \succ u_8 \succ u_6 \succ u_{10} \succ u_7 \succ u_1 \succ u_{11} \succ u_5 \succ u_{19} \succ u_{13} \succ u_{18} \succ u_4 \succ u_{14} \succ u_{16} \succ u_{15}$
	Pes	$u_2 \succ u_9 \succ u_3 \succ u_8 \succ u_{12} \succ u_{17} \succ u_{16} \succ u_6 \succ u_{10} \succ u_7 \succ u_5 \succ u_1 \succ u_{19} \succ u_{18} \succ u_4 \succ u_{14} \succ u_{11} \succ u_{13} \succ u_{15}$
	Opt	$u_2 \succ u_9 \succ u_3 \succ u_{12} \succ u_{17} \succ u_8 \succ u_6 \succ u_{10} \succ u_5 \succ u_1 \succ u_4 \succ u_7 \succ u_{11} \succ u_{14} \succ u_{19} \succ u_{13} \succ u_{16} \succ u_{18} \succ u_{15}$
VRS	*Neu*	$u_9 \succ u_3 \succ u_8 \succ u_2 \succ u_{17} \succ u_{12} \succ u_{10} \succ u_6 \succ u_{16} \succ u_{14} \succ u_{19} \succ u_{11} \succ u_5 \succ u_1 \succ u_7 \succ u_{18} \succ u_4 \succ u_{13} \succ u_{15}$
	Pes	$u_2 \succ u_9 \succ u_3 \succ u_8 \succ u_{17} \succ u_{12} \succ u_6 \succ u_{10} \succ u_{11} \succ u_5 \succ u_1 \succ u_{19} \succ u_4 \succ u_7 \succ u_{14} \succ u_{13} \succ u_{16} \succ u_{18} \succ u_{15}$

Table 16 Sidewaystable

Method	Sort results	Optimal
Wang et al. (2005)	$u_2 \approx u_3 \approx u_6 \approx u_8 \approx u_9 \approx u_{10} \approx u_{12} \approx u_{17} \succ u_1 \succ u_{19} \succ u_5 \succ u_4 \succ u_{11} \succ u_7 \succ u_{13} \succ u_{14} \succ u_{16} \succ u_{18} \succ u_{15}$	-
Wang et al. (2016)	$u_2 \approx u_9 \succ u_{17} \succ u_3 \succ u_{12} \succ u_8 \succ u_6 \succ u_{10} \succ u_4 \succ u_3 \succ u_1 \succ u_{19} \succ u_{11} \succ u_{13} \succ u_{14} \succ u_7 \succ u_{16} \succ u_{18} \succ u_{15}$	u_2, u_9
Wu et al. (2013)	$u_9 \succ u_2 \succ u_{17} \succ u_8 \succ u_4 \succ u_3 \succ u_{12} \succ u_5 \succ u_6 \succ u_{10} \succ u_{19} \succ u_{14} \succ u_{11} \succ u_{13} \succ u_1 \succ u_{16} \succ u_7 \succ u_{18} \succ u_{15}$	u_9
Yu et al. (2019)	$u_9 \succ u_3 \succ u_2 \succ u_8 \succ u_{17} \succ u_{12} \succ u_5 \succ u_4 \succ u_{10} \succ u_6 \succ u_{19} \succ u_7 \succ u_{14} \succ u_1 \succ u_{13} \succ u_{11} \succ u_{16} \succ u_{18} \succ u_{15}$	u_9
Liu et al.(Opt) Chen et al. (2022)	$u_9 \succ u_2 \succ u_3 \succ u_{17} \succ u_8 \succ u_{12} \succ u_6 \succ u_{10} \succ u_1 \succ u_{19} \succ u_{11} \succ u_5 \succ u_7 \succ u_{13} \succ u_4 \succ u_{14} \succ u_{18} \succ u_{16} \succ u_{15}$	u_9
Liu et al.(Pes) Chen et al. (2022)	$u_9 \succ u_2 \succ u_3 \succ u_{17} \succ u_8 \succ u_{12} \succ u_6 \succ u_{10} \succ u_5 \succ u_1 \succ u_{19} \succ u_{11} \succ u_7 \succ u_4 \succ u_{14} \succ u_{13} \succ u_{16} \succ u_{18} \succ u_{15}$	u_9
Liu et al.(Neu) Chen et al. (2022)	$u_9 \succ u_2 \succ u_3 \succ u_{17} \succ u_8 \succ u_{12} \succ u_6 \succ u_{10} \succ u_1 \succ u_{19} \succ u_5 \succ u_{11} \succ u_4 \succ u_7 \succ u_{13} \succ u_{14} \succ u_{16} \succ u_{18} \succ u_{15}$	u_9
Ours IFS-CCR (Opt)	$u_2 \succ u_9 \succ u_3 \succ u_{12} \succ u_{17} \succ u_8 \succ u_6 \succ u_{10} \succ u_5 \succ u_1 \succ u_4 \succ u_7 \succ u_{11} \succ u_{14} \succ u_{19} \succ u_{16} \succ u_{13} \succ u_{18} \succ u_{15}$	u_2
Ours IFS-CCR (Neu)	$u_9 \succ u_2 \succ u_3 \succ u_{17} \succ u_{12} \succ u_8 \succ u_6 \succ u_{10} \succ u_7 \succ u_1 \succ u_{11} \succ u_5 \succ u_{19} \succ u_{13} \succ u_{18} \succ u_4 \succ u_{14} \succ u_{16} \succ u_{15}$	u_9
Ours IFS-CCR (Pes)	$u_2 \succ u_9 \succ u_3 \succ u_8 \succ u_{12} \succ u_{17} \succ u_{16} \succ u_6 \succ u_{10} \succ u_7 \succ u_5 \succ u_1 \succ u_{19} \succ u_{18} \succ u_4 \succ u_{14} \succ u_{11} \succ u_{13} \succ u_{15}$	u_2
Ours IFS-BCC (Opt)	$u_2 \succ u_9 \succ u_3 \succ u_{12} \succ u_{17} \succ u_8 \succ u_6 \succ u_{10} \succ u_5 \succ u_1 \succ u_4 \succ u_7 \succ u_{11} \succ u_{14} \succ u_{19} \succ u_{13} \succ u_{16} \succ u_{18} \succ u_{15}$	u_2
Ours IFS-BCC (Neu)	$u_9 \succ u_3 \succ u_8 \succ u_2 \succ u_{17} \succ u_{12} \succ u_{10} \succ u_6 \succ u_{16} \succ u_{14} \succ u_{19} \succ u_{11} \succ u_5 \succ u_1 \succ u_7 \succ u_{18} \succ u_4 \succ u_{13} \succ u_{15}$	u_9
Ours IFS-BCC (Pes)	$u_2 \succ u_9 \succ u_3 \succ u_8 \succ u_{17} \succ u_{12} \succ u_6 \succ u_{10} \succ u_{11} \succ u_5 \succ u_1 \succ u_{19} \succ u_4 \succ u_7 \succ u_{14} \succ u_{13} \succ u_{16} \succ u_{18} \succ u_{15}$	u_2

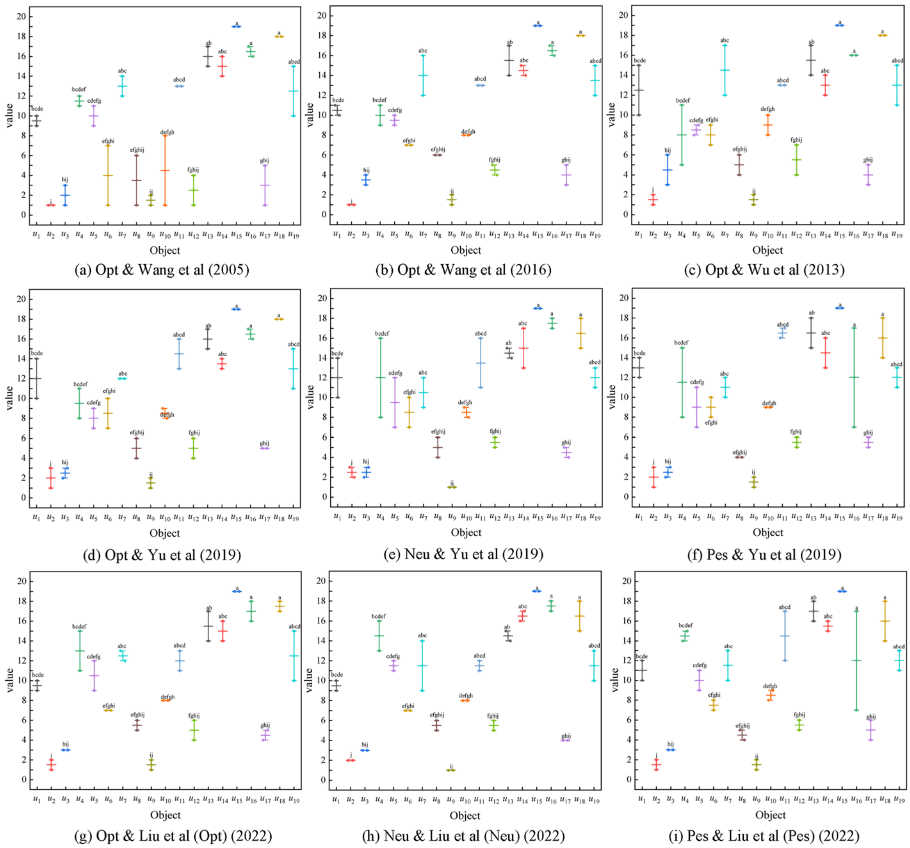


Fig. 5 Comparison of sorting results based on CRS

4.4 Relevance analysis

To further demonstrate the rationality and feasibility of this method compared to other methods, the significance analysis of the proposed model and each method is shown in Figs. 5 and 6 for the excellence of all DMUs. The letters represent whether the difference is significant. If those with the same marking letters are considered to have insignificant differences, and those with different marking letters are considered to have significant differences, among which $p \leq 0.01$.

Figure 5a illustrates a pronounced and significant difference between u_2 and u_9 when compared to u_{15}, u_{16}, and u_{18}. There is no significant difference between u_2 and u_9, and there is no significant difference between u_{15}, u_{16} and u_{18}. Observing Fig. 5b to i, a distinction is evident in the approach of this paper compared to the method Yul et al. (2019) when assessing the dissimilarity between u_2, u_9, and u_{15}, u_{16}, u_{18}. It is consistent with the judgment in this section, namely u_2 and u_9 are the optimal solutions. On the contrary, u_{15}, u_{16} and u_{18} are the worst solutions.

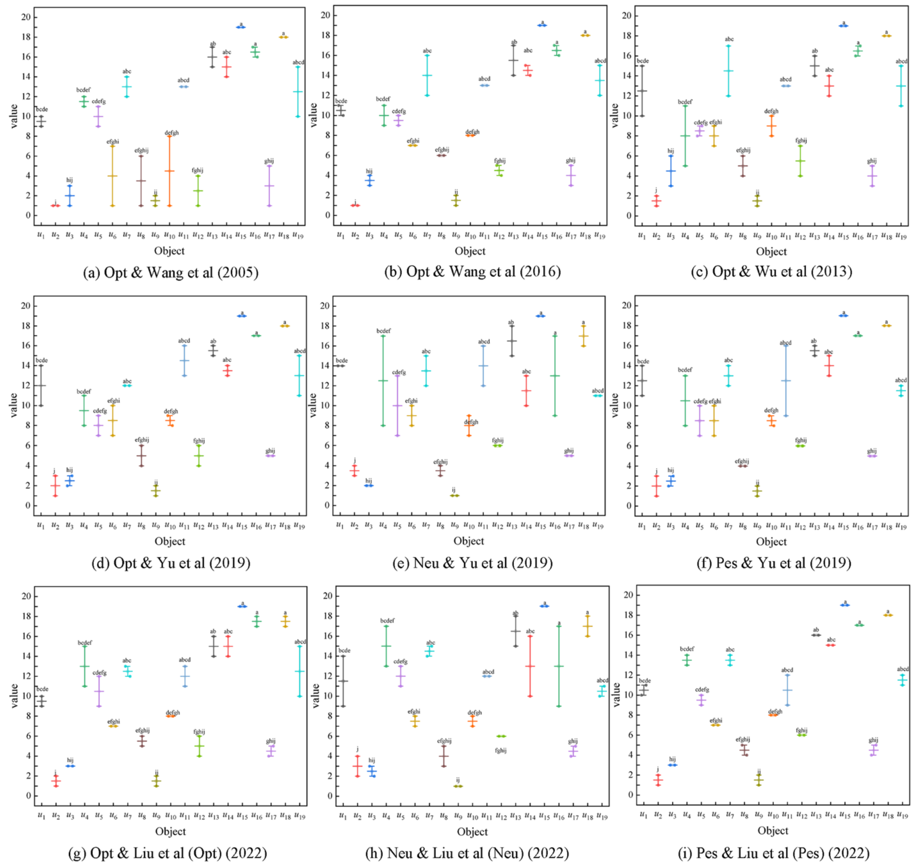


Fig. 6 Comparison of sorting results based on VRS

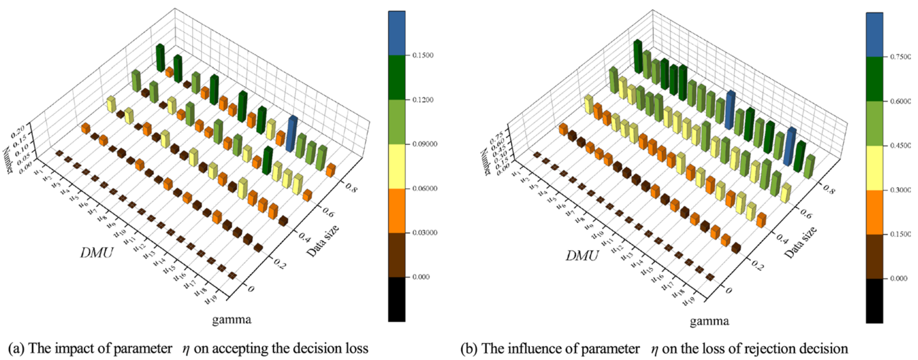


Fig. 7 The impact of η on the loss function

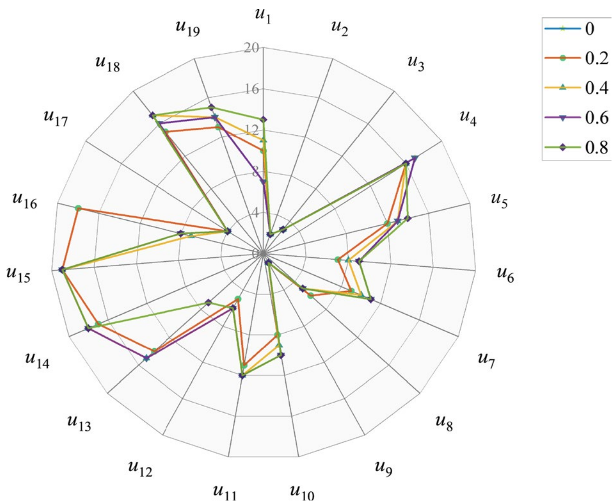


Fig. 8 The impact of η on DMU sorting

4.5 Parameter analysis

The risk coefficient analysis results of the loss function in the proposed intuitionistic fuzzy three-way decision model are shown in Fig. 7. When there is no risk coefficient, the loss function of each DMU is not affected. As the risk aversion coefficient increases, the overall loss function of each DMU shows an upward trend.

Due to the changes in the loss function, the expected decision loss of DMU also changes. According to the decision rules in Definition 11, different ranking results due to different risk coefficients can be obtained, as shown in Fig. 8. Observing the figure, it is evident that the ranking results of this program remain constant as the risk coefficient steadily increases. This stability implies that the impact of the risk coefficient on the results is relatively consistent. Additionally, it is notable that the risk coefficient significantly influences the three DMUs u_1, u_{13}, and $u16$, while the impact is minimal on the four DMUs u_2, u_3, u_9, and u_{15}. It is consistent with the decision result in Table 13, u_2, u_3 and u_9 are DMUs with excellent ranking results and u_{15} is the DMU with the worst ranking results.

5 Conclusions

In this paper, a novel intuitionistic fuzzy three-way decision model based on DEA is proposed to address decision-making and ranking problems involving multi-input–output intuitionistic fuzzy information. Several key advancements over traditional approaches are introduced. Firstly, by extending the DEA model to an intuitionistic fuzzy environment, a corresponding model is developed that derives the benefit scores of DMUs, thereby enhancing the model's capability to handle uncertainty. Additionally, the proposed loss function integrates the impacts of inputs and outputs, eliminating the subjectivity inherent in setting loss functions and improving the reliability of the results. Secondly, the model incorporates the input–output connection degree during the solving process, considering interactions between different inputs and outputs. This extension includes optimistic, neutral, and pessimistic strategies, accommodating decision-makers with varying risk preferences. Thirdly, an intuitionistic fuzzy three-way decision model is constructed from three dimensions, thus expanding the application scope of traditional DEA. Despite these advancements, the approach has some limitations. The preprocessing process for large-scale datasets may require significant computational resources. Additionally, the relationship between optimal decision-making units and decision rules has not been explored in depth from the perspective of logical implication.

The scientific contributions of the proposed method lie in its ability to combine DEA and intuitionistic fuzzy set theory, offering novel perspectives and tools for optimization and ordering in complex decision environments with multiple inputs and outputs. Practically, this approach provides a more nuanced and reliable way to rank decision-making units under uncertainty, which can be beneficial in various fields such as economics, management, and engineering. For future research, further investigation into the correlation between multiple input–output decision-making units and three-way decision rules from the perspectives of conflict analysis and logical implication is planned. Additionally, exploring methods to optimize the computational efficiency of the model will be a priority. Techniques such as adversarial generation networks and data block/layering methods could be employed to transform large-scale data, using a "divide and conquer" strategy to solve the DEA model for each block. By addressing these challenges, the goal is to enhance the practical applicability and generalizability of this approach, providing robust solutions for intelligent decision-making in environments characterized by uncertain information.

Given the uncertainty of data, future research could explore several avenues to enhance and expand the current work. This includes investigating robust optimization techniques to enhance the model's robustness against uncertainties in input data, and exploring stochastic programming methods to handle probabilistic uncertainty in decision-making processes. Additionally, developing hybrid models that combine intuitionistic fuzzy logic with other decision-making frameworks, such as multi-criteria decision analysis, could broaden the model's applicability and effectiveness. Applying the model to various real-world scenarios, including healthcare, finance, logistics, and environmental management, would help validate its practical effectiveness and versatility. Furthermore, creating more advanced algorithms to improve the computational efficiency and scalability of the model will make it suitable for large-scale problems. Extending the model to dynamic decision-making environments, where the decision-making process evolves over time with changing data and conditions, is another promising direction. Finally, integrating machine learning techniques could enhance the decision-making process by learning from large datasets and improving predictive accuracy. These potential directions offer valuable pathways for further exploration and development in this field.

Appendix A: table of acronyms

Abbreviations	Meaning	Abbreviations	Meaning
DEA	Data Envelopment Analysis	Neg	Negative
IFS	Intuitionistic Fuzzy	Bnd	Boundary
DMUs	Decision Making Units	Opt	Optimistic strategy
IFS-CCR	Intuitionistic Fuzzy-CCR	Pes	Pessimistic strategy
IFWA	Iterative Fuzzy Weighted Averaging	Neu	Neutral strategy
IFS-BCC	Intuitionistic Fuzzy-BCC	Pos	Positive
CRS	Constant Returns to Scale	VRS	Variable Returns to Scale
PROMETHEE II	Preference Ranking Organization Method for Enrichment Evaluations	AHP	Analytic Hierarchy Process
SMAA2	Stochastic Multi-criteria Acceptability Analysis 2	MAGDM	Multi-Attribute Group Decision-Making

Appendix B: symbol thumbnail table

Symbols	Meaning
U	Domain of discourse
$u \in M$	M is a finite non-empty DMU set, u is a DMU
T	Intuition fuzzy sets on U
$\alpha = (\mu_\alpha, \nu_\alpha)$	intuitionistic fuzzy numbers
$S(\alpha)$	Score function
L	Set function (benefit measure$L(A_j)$)
$E, F \in P(X)$	Power set
A	Assemble ($A_j = \{x_1, ..., x_n\}$)
j	Number of DMUs
m	Number of input variables
s	Number of output variables
X	Input variable set
Y	Output variable set
e	Input variables
y	Output variables
p	p-th input
q	q-th output
$o//O$	Output weights
$i//I$	Input weights
$I(\{x\})$	Benefit measurement of input variable x
$O(\{y\})$	Benefit measurement of output variable y
H	Input-output connection
$\eta(\eta \in (0, 1))$	Risk aversion coefficient
λ	Loss function

Symbols	Meaning
θ	vector angle
W	Set of weight
ω	Variable weights
ψ	Benefit score
ξ	Hesitation value coefficient
$\Gamma = \{a_P, a_B, a_N\}$	Action set
$\Omega = \{C, \neg C\}$	State set

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Declarations

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Research involving human participants or animals This article does not contain any studies with human participants or animals performed by any of the authors.

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