

# **Brain works principle followed by neural information processing: a review of novel brain theory**

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## **Abstract**

The way the brain work and its principle of work has long been a big scientifc question that scientists have dreamed of solving. However, as is known to all, the brain works at diferent levels, and the operation at diferent levels is interactional and mutually coupled. Unfortunately, until now, we still do not know how the nervous system at diferent levels is interacting and coupling with each other. This review provides some preliminary discussions on how to address these scientifc questions, for which we propose a novel theory of the brain called neural energy. Such a theoretical and research approach can couple neural information with neural energy to address the interactions of the nervous system at various levels. Therefore, this review systematically summarizes the neural energy theories and methods proposed by our research in the feld of brain science, as well as the internal relationship between mechanics and neural energy theory. Focuses on how to construct a Wang–Zhang (W–Z) neuron model equivalent to Hodgkin–Huxley (H–H) model by using the idea of analytical dynamics. Then, based on this model, we proposed a large-scale neural model and a theoretical framework of global neural coding of the brain in the feld of neuroscience. It includes information processing of multiple sensory and perceptual nervous systems such as visual perception, neural mechanism of coupling between default mode network and functional network of brain, memory switching and brain state switching, brain navigation, prediction of new working mechanism of neurons, and interpretation of experimental phenomena that are difficult to be explained by neuroscience. It is proved that the new W–Z neuron model and neural energy theory have unique functions and advantages in neural modeling, neural information processing and methodology. The idea of large-scale neuroscience research with neural energy as the core will provide a potentially powerful research method for promoting the fusion of experimental neuroscience and theoretical neuroscience in the future, and propose a widely accepted brain theory system between experimental neuroscience and theoretical neuroscience. It is of great scientifc signifcance to abandon the shortcomings of reductive and holism research methods in the feld of neuroscience, and efectively integrate their respective advantages in methodology.

**Keywords** Brain theory · Large-scale neuroscience · Global neural coding · Neural energy models · Unifcation of reductionism and holism

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## **1 Introduction**

More than 20 years ago, a molecular neurobiologist at the University of Berkeley Professor Walter Freeman put forward the concept of neurodynamics (Freeman [2000\)](#page-59-0). It has become a new research feld to study the activities of cognition and nervous system with the theory and method of dynamics (Pouget and Latham [2002;](#page-62-0) Basar [1998;](#page-58-0) Çelik et al. [2021](#page-58-1); Brydges et al. [2020](#page-58-2); Yu et al. [2020](#page-64-0); Wouapi et al. [2021](#page-64-1); Wang et al. [2019a](#page-64-2); Iribarren and Moro [2009,](#page-60-0) [2008](#page-62-1); Memmesheimer and Timme [2006](#page-61-0); Navarro-López et al. [2021;](#page-61-1) Buxton [2012;](#page-58-3) Churchland et al.  $2002$ ; Hipp et al.  $2011a$ ; Ermentrout et al.  $2007$ ; Lakatos et al. [2008;](#page-60-2) Rabinovich and Huerta [2006;](#page-62-2) Sandrini et al. [2015;](#page-62-3) Hopfeld [2010;](#page-60-3) Hu et al. [2021](#page-60-4)), many of scientifc research achievements has been showed in this research area. Neurodynamics is more commonly known as computational neuroscience in European and American countries, while it is known as neuromechanics in Japan (Takeda [1999\)](#page-63-0). Particularly, the Dynamic Brain Group, originally organized by Japanese scientists, developed various kinds of collaborative researches such as the organization of Dynamic Brain Forum (DBF) in the world, where many researchers in the world, who have been engaged in neurosci-ence from the aspect of "dynamic brain" (Tsuda et al. [1987](#page-63-1); Tsuda [1991](#page-63-2), [1992](#page-63-3)). However, in the feld of experimental neuroscience, scientists prefer to use neuroinformatics to describe the basic laws of neural information processing qualitatively or quantitatively. But no matter what name we used with it, it doesn't change the fact that neuroscientists and artifcial intelligence scientists have come to realized that the development of cognitive neuroscience is increasingly dependent not only on advances in experimental techniques and rigorous experimental data, but also to understand the principle of brain network signal processing and transmission and insight into the internal mechanism of neural coding distribution mode with quantitative methods from a theoretical height, so as to discover the law and nature behind the vast experimental data. In order to better understand and master the operation of the brain and deal with various brain diseases, dynamic theory is used to accurately predict the potential patients with degenerative brain diseases at an early stage (Navarro-López et al. [2021;](#page-61-1) Ebrahimzadeh et al. [2021;](#page-59-2) Yang et al. [2021a;](#page-64-3) Jiang et al. [2020;](#page-60-5) Sharma and Acharya [2021\)](#page-62-4).

For a long time, cognitive neuroscience, which takes experiments as its basic research method, has focused on the improvement of experimental phenomena, experimental data and experimental techniques while ignoring the importance of theory. Because of this result, the brain science with hundreds of years of history has not had a systematic and complete theoretical system of its own. This abnormal phenomenon has made brain science so far developed to today is still an immature discipline. Although theoretical neuroscience was born 20 years ago, it is not widely accepted by the academic community to have only one theory. Although theoretical neuroscientists have made a series of excellent research achievements today (Wouapi et al. [2021;](#page-64-1) Wang et al. [2006](#page-63-4), [2021a;](#page-64-4) Clancy et al. [2017](#page-58-5); Videbech [2010](#page-63-5); Zhang et al. [2019](#page-65-0), [2020](#page-65-1); Yuan et al. [2021,](#page-65-2) [2022;](#page-65-3) Yao and Wang [2019](#page-64-5); Maltba et al. [2022](#page-61-2); Zhou et al. [2020](#page-65-4); Li et al. [2020](#page-61-3); Kim and Lim [2020;](#page-60-6) Yang et al. [2021b](#page-64-6)), it is still difficult to widely and effectively cooperate with experimental neuroscientists to promote each other and integrate development. One of the eforts for the integration of development has been performed is to publish a textbook of comprehensive neuroscience such as "Neuroscience in the 21st Century", 3rd Edition (Pfaff and Volkow [2022\)](#page-62-5). Among the chapters in that textbook, the following is devoted to the dynamics in neural systems: Ichiro Tsuda, Dynamics in neural systems: a dynamical systems viewpoint. The view of history, dynamic theory for neural information processing have been

proposed in several aspects. Minoru Tsukada frst treated the neural information processing as dynamic Markov channels from dynamics brain (Tsukada et al. [1975](#page-63-6)). Ichiro Tsuda proposed several dynamic aspects of the brain as a typical complex system (Kaneko and Tsuda [2001](#page-60-7)). Tsuda proposed Hermeneutics of the brain from the aspect that the brain interprets the external world to recognize internal images of external signals via action and sensation (Tsuda [1984\)](#page-63-7). Tsuda frst proposed a dynamic associative memory model in non-equilibrium neural networks (Tsuda et al. [1987\)](#page-63-1), in contrast to a typical static model of memory capacity such as Hopfeld model. Kazuyuki Aihara followed Tsuda's model and confrmed the presence of dynamic associative memory, by using his chaotic neural networks (Adachi and Aihara [1997](#page-58-6); Aihara et al. [1990](#page-58-7)). In this kind of study, Tsuda found complex dynamic transitions in his nonequilibrium neural networks, whose transition was proposed to be called "Chaotic Itinerancy" (Tsuda [1991,](#page-63-2) [1992,](#page-63-3) [2001,](#page-63-8) [2013,](#page-63-9) [2015](#page-63-10); Nara and Davis [1992\)](#page-61-4). In early 1990s, the Japanese Dynamic Brain Group was organized by Minoru Tsukada, Hiroshi Fujii, Shigetoshi Nara, Ichiro Tsuda, and Kazuyuki Aihara, and J-DBG developed various kinds of collaborative researches such as the organization of DBF, where many researchers in the world, who have been engaged in neuroscience from the aspect of "dynamic brain", gathered together. This activity of forum was led to the later organization of ICCN (International Conference on Cognitive Neurodynamics) and the publication of Cognitive Neurodynamics by Springer. M. Tsukada invited Masamich Sakagami in Tamagawa University to do the collaborative research on the dynamic mechanism of thoughts. An important aspect was whether or not the thoughts process can be discriminated with associative memories. M. Sakagami succeeded to make an experimental system, which can discriminate these two cognitive phases. X. Pan were enthusiastic to perform such an experiment and succeeded to fnd specifc neurons. Finally, they published important papers (Pan et al. [2008](#page-62-6), [2014](#page-62-7)). As another important works of DBG were for Cantor coding (Tsuda and Kuroda [2001](#page-63-11); Fukushima et al. [2007;](#page-59-3) Kuroda et al. [2009](#page-60-8); Yamaguti et al. [2011;](#page-64-7) Ryeu et al. [2001](#page-62-8)), gap junction-coupled neural network models (Fujii and Tsuda [2004](#page-59-4); Tsuda et al. [2004](#page-63-12); Tadokoro et al. [2011\)](#page-63-13) and complex visual hallucinations and so on (Collerton et al. [2016;](#page-58-8) Tsukada et al. [2015\)](#page-63-14).

Nevertheless, there still seems to be an invisible and unbridgeable chasm between the achievements of neuroscience at all levels. These conditions prevent the mutual use, infuence, and difusion of their research fndings, as well as major breakthroughs in cognitive neuroscience. As a result, the research feld of cognitive neuroscience has been unable to get out of the dilemma of "blind man touching the elephant". Especially in consciousness, thinking, creativity generation mechanism, emotion, the nature of intelligence, prediction, visual perception generation mechanism, memory storage and call, global brain function and many other aspects of the research progress is very slow, some even no progress. Moreover, a growing number of scientists in other felds are intrigued by the multitude of unanswered scientifc questions in the feld of brain research and their complexity.

The basic ideas of neurodynamics have been increasingly infltrated and embodied in many aspects, such as neuroscience, artifcial intelligence, brain-like computing, bioinformation, medical diagnosis, image processing, control science, complex network and engineering applications (Bullmore and Sporns [2009;](#page-58-9) Ullman [2019;](#page-63-15) Roy et al. [2019;](#page-62-9) Zeng et al. [2019;](#page-65-5) Wang and Zhu [2016;](#page-63-16) Deco et al. [2015;](#page-59-5) Kanwisher [2010a](#page-60-9)). Brain science is a largescale science, which not only involves the three-dimensional (3D) intersection of many disciplines, but also poses many unprecedented challenges to many mature disciplines. For example, does the weak magnetic feld inside our brains contribute to the transmission of nerve signals? If so, where is the experimental evidence? If there is no contribution, how to explain the negative power component of neurons (Wang et al. [2015a;](#page-63-17) Wang and Wang

[2018a\)](#page-63-18), and how to explain the equivalence between Wang–Zhang's (W–Z) neuron model and Hodgkin–Huxley's (H–H) model (Wang and Wang [2018b\)](#page-63-19). A further question is whether neurons in the brain, or even without synaptic connections and nerve fiber connections between brain regions, can still transmit neural signals in the case of electromagnetic feld coupling to achieve communication between neurons and various brain regions (Yang et al. [2022;](#page-64-8) Ma and Tang [2017](#page-61-5)).

As an unstable dynamic system, the brain has no controversy in the academic world. A large number of experimental data and results reveal that our brain has specifc functional characteristics at any level, and its activities are highly nonlinear and complex. The highly nonlinear and complex brain dynamics as well as their various functional expressions are not only related to gene and functional genomics, biology and biochemistry, but also to solid mechanics (McIntyre et al. [2001](#page-61-6)), fuid mechanics (Moore and Cao [2008\)](#page-61-7), dynamics and control (Lu et al. [2008a,](#page-61-8) [b](#page-61-9)). Our research shows that some experimental phenomena in cognitive neuroscience can be reproduced and repeated by mechanical models (Wang et al. [2015a;](#page-63-17) Li et al. [2022a](#page-61-10)), and some experimental data that cannot be explained by neurosci-ence can also be scientifically explained by our mechanical models (Peng and Wang [2021](#page-62-10)). We also can use mechanical models to predict new experimental phenomena and new neu-ral mechanisms not found in neuroscience (Wang and Wang [2018b](#page-63-19)). These results fully demonstrate the power of mechanical science in the feld of brain science research.

A comprehensive review article about "Neurodynamics and Mechanics" profoundly elaborated the internal connection between neurodynamics and mechanics (Lu [2020](#page-61-11)), explained the transformation from classical mechanics to generalized mechanics, and the one-to-one correspondence between generalized mechanics and neurodynamics. The review pointed that since the twentieth century, dynamical system theory and methods have been further developed and successfully used in various mechanical systems, even nonlinear diferential equations described in general system has universal theoretical signifcance and important application value. This indicates that modern mechanics research has broken through the traditional category of classical mechanical system and opened up a new category of "generalized" mechanical system. The research object of mechanics has expanded from "particle or particle system" to the general "dynamic system", the concept of "force" has expanded from "mechanical force" to the general "interaction", and the concept of "motion" has also expanded from "confguration change" in geometric space to "state evolution" in state space. These ideas are important and instructive for the modeling and calculation of mechanical science in cognitive neuroscience and the construction of the theoretical system of brain-like intelligence.

At present, there is no widely accepted theory in the feld of neuroscience, thus, theoretical neuroscience and experimental neuroscience cannot efectively integrate and promote each other. This has seriously hindered the development of various felds of cognitive neuroscience, so that we cannot efectively interpret experimental data, and reveal data behind the nature and regularities form the basis of scientifc predictions and explanations. In order to make a great breakthrough in neuroscience and establish a systematic and complete theoretical system of cognitive neuroscience, it is necessary to perform researches on brain theory. The core scientifc question in the study of brain theory is whether human intelligent behaviors depend on the activity of a single or a few neurons or whether they are realized by the interactions at molecular to systemic levels? The answer of this core scientifc question is now beyond dispute among neuroscientists. However, academic community has not yet come up with an efective solution to solve this core scientifc challenge. To this end, we proposed a defnition of large-scale neuroscience. The cornerstone of this defnition is that a large-scale neuroscience model is on the basis of the neural energy

model (Wang and Zhu [2016;](#page-63-16) Wang et al. [2015a](#page-63-17), [2008](#page-63-20); Wang and Wang [2018b](#page-63-19)), and the neural energy model arises from the theory and method of analytical dynamics (Wang and Pan [2021](#page-63-21)). The present study aimed to quantitatively obtain the global information of neural activity by fnding the relationship between neural energy and membrane potential, feld potential and fring rate of network. As the global information of the neural activity can be transformed into energy for research and analysis, neural energy coding constitutes the cornerstone of large-scale neuroscience models. The main contents include the following items: (a) a new research method that can unify reductionism and holism in theory. The new method could theoretically reproduce not only the electrophysiological recordings, but also the global information of the functional neural activity using functional magnetic resonance imaging, fMRI (Yuan et al. [2021;](#page-65-2) Wang et al. [2015a;](#page-63-17) Peng and Wang [2021;](#page-62-10) Cheng et al. [2020\)](#page-58-10); (b) the global functional model of brain is established computationally, which can be used to construct, analyze, and describe the experimental models of neuroscience at various levels (Wang and Zhu [2016;](#page-63-16) Wang et al. [2020\)](#page-64-9). Thus, the computational results at all levels are no longer mutually inapplicable, contradictory and irrelevant; (c) if a global functional model of brain cannot explain the function and energy consumption of the default mode network (DMN) and the resting state network, and how it transitions from the default model to the cognitive network and the corresponding energy transformation under task-induced condition, then, such model is not a global neural model (Yuan et al. [2021\)](#page-65-2). The adjustable parameters of the global functional model of brain are necessarily limited and simple; (d) this type of models can be used for the analysis of experimental data hidden behind the nature and regularity. As our neural energy model meets all of the above-mentioned requirements of the global functional model of brain, and it has already accumulated and published a series of original and innovative scientifc research results (Wang et al. [2008](#page-63-20), [2015a](#page-63-17), [2017a](#page-63-22), [2018a](#page-64-10), [b,](#page-64-11) [2019b](#page-64-12), [2021a](#page-64-4); Yuan et al. [2021;](#page-65-2) Wang and Zhu [2016;](#page-63-16) Wang and Wang [2018a](#page-63-18), [b](#page-63-19); Li et al. [2022a;](#page-61-10) Peng and Wang [2021;](#page-62-10) Lu [2020](#page-61-11); Cheng et al. [2020](#page-58-10)), this has laid a frm foundation for the creation of new brain theory that can stand the testing of experimental data. The main research directions of brain theory are refected in the following four aspects.

#### **1.1 Brain theory: exploration of working mechanism of the brain**

One of the frst scientifc questions that needs to be answered in the discussion of how the brain works is why neuroscience cannot explain the mechanism of about 95% of the brain's energy consumption (Fox and Raichle [2007a](#page-59-6); Balasubramanian [2021](#page-58-11); Raichle [2010](#page-62-11))? Although brain only accounts for about 2% of body weight, it consumes about 20% of the body's energy. The neural energy expenditure caused by task stimuli typically accounts for only 5% of the resting brain energy consumption. Much of our understanding of the brain comes from study of the 5% of brain activity (Raichle and Mintun [2006\)](#page-62-12). From these data, it is clear that the energy loss in the brain is almost independent of task stimuli. The following questions should be answered:

- (a) What is the nature of the persistent intrinsic activity that causes the great depletion of brain energy?
- (b) What is the relationship between the huge depletion of brain energy and cognitive function?

(c) Is it possible that the current mainstream view in cognitive neuroscience may mislead scholars to ignore the possibility that neuroscience experiments and cognitive psychology experiments reveal only some parts of the brain activity?

The important question is that if the brain is refex in nature, why is the energy consumption of the brain under stimulated conditions almost the same as the energy consumption of the brain under resting conditions? If brain is refex, task-induced brain energy consumption should increase signifcantly, why is it less than a 5% increase? In this research area, we have published a series of research results (Wang et al. [2021a](#page-64-4), [2018a;](#page-64-10) Yuan et al. [2021](#page-65-2)). Using the large-scale neural energy theory to reveal the neural mechanism of the hemodynamic phenomenon of the brain, it was revealed that neural energy is an important marker of the activity of the nervous system, and energy features contain information about external stimuli and neural responses (Yuan et al. [2021;](#page-65-2) Peng and Wang [2021\)](#page-62-10); using the neural energy theory to attain the biophysical mechanism of the mutual coupling and antagonism between the brain DMN and the working memory network, it can be demonstrated that neural energy can efectively fuse DMN s and cognitive networks to interpret and analyze the spatial information and encoded content contained in complex neural activities (Yuan et al. [2021](#page-65-2)). The computational simulation results are in complete agreement with the experimental data (Piccoli et al. [2015](#page-62-13); Compte [2000](#page-58-12); Wei et al. [2012;](#page-64-13) Hsieh and Ranganath [2014;](#page-60-10) Karlsgodt et al. [2005\)](#page-60-11). The neural energy theory can also efficiently express the neural coding of the cognitive system in a 3D space (Hsieh and Ranganath [2014](#page-60-10)); the nervous system can realize the maximization of information coding under the condition of energy constraints (Wang et al.  $2018a$ , [b,](#page-64-11)  $2019b$ ), and the neural energy coding can maximize the efficiency of intellectual exploration (Wang et al. [2017a](#page-63-22)). Our study also demonstrated a stimulus-related increase in energy expenditure of less than 5% compared with energy expenditure under a spontaneous activity, which is consistent with brain imaging results reported by Raichle et al. (Wang et al. [2021a\)](#page-64-4), and spontaneous activity consumes most of the energy compared with task state (Fox and Raichle [2007a;](#page-59-6) Balasubramanian [2021;](#page-58-11) Raichle [2010](#page-62-11); Raichle and Mintun [2006\)](#page-62-12). Therefore, the neural energy can be used to express the neural activity of the cerebral cortical network (Wang and Zhu [2016\)](#page-63-16).

Another view is that there is experimental evidence that the neural activity of the brain exhibits quasi-critical characteristics (Williams-García et al. [2014\)](#page-64-14). The biological cerebral cortex generally operates near the quasi-critical point. An article published in the Physical Review Letters revealed that external input forces the neural network of the brain away from a tipping point and operates in a non-equilibrium state. Under diferent conditions, the brain is in a "quasi-critical state" that satisfes the scaling relationship (Fosque et al. [2021](#page-59-7)). In particular, several recent articles have scientifcally concentrated on how the brain works, and have obtained experimental data for verifcation. They demonstrated that diverse cognitive processes set diferent demands on locally segregated and globally integrated brain activity. With emphasizing the multilevel, hierarchical modular structure of the functional connectivity of the brain to derive eigenmodebased measures, Wang et al. (Wang et al. [2021b\)](#page-64-15) showed that in healthy adults (range of age 22–36 years old), the healthy brain is characterized by a balance between functional segregation and integration. Crucially, a stronger integration is associated with a better cognitive ability, and a stronger segregation fosters crystallized intelligence and processing speed, and an individual's tendency toward balance supports a better memory. Thus, the segregation–integration balance empowers the brain to support diverse

cognitive abilities. This association between balance and cognitive abilities is not only consistent with the recently proposed Network Neuroscience Theory (NNT) of human intelligence (Barbey [2018a\)](#page-58-13), but also provides more contents to the NNT. In fact, the balance between segregation and integration requires the diversity from weak to strong functional connectivity in dynamic patterns. Using the eigenmode analysis, Wang et al. (Wang et al. xxxx) also found that the diverse functional interaction is generated by hierarchically activating and recruiting structural modes, which are inherent to the hierarchical modular organization of the structural connectome. The critical state can best explore the hierarchical modular organization, optimize the combination of intrinsic structural modes, and maximize the functional diversity.

We can conclude that whether it is the hypothesis that the operation of the brain is quasicritical, or the hypothesis that the cognitive ability of the brain conforms to the principle of separation–integration balance, under the condition that the experimental evidence is still lacking, we need to further analyze from the perspective of energy. Examine whether the energy in quasi-critical states, and the energy of state-space switching, is consistent with known features of complex, adaptive brain network dynamics in the presence of quasi-critical states. As for the separation–integration balance principle of the brain, it is necessary to quantitatively describe how the DMN transitions to the cognitive neural network from the perspective of neural energy. The establishment of the global neural model of the brain must conform to two basic principles supported by experimental data as follows: (1) costefectiveness—the activity of the neural network in the resting state and cognitive activity conforms to the principle of energy minimization;  $(2)$  high efficiency—the transmission efficiency of neural network signals in the cerebral cortex conforms to the principle of maximizing energy utilization (Laughlin and Sejnowski [2003a](#page-60-12); Zheng et al. [2022\)](#page-65-6).

### **1.2 Modeling of cerebral neural network and dynamic analysis of cognitive function**

Panayiota Poirazi and Athanasia Papoutsi presented modeling methods at three levels of abstraction, from "single neuron" to "microcircuit" and then to "large-scale network model", in a recent review on the relationship between dendrites and cerebral function. This study systematically summarizes the important role of dendrites in computational modeling, and expounds the great contribution of neurodynamics theory and computational neuroscience to the important progress of neuroscience by enumerating the successful complementarity or interaction between modeling and experimental neuroscience (Poirazi and Papoutsi [2020](#page-62-14)). In particular, a recent review of the structure, function and control of cerebral networks published in the Nature Reviews Physics comprehensively explains the possible complex operational mechanisms of the brain from the perspective of statistical physics, and analyzes the processes of cerebral cognition, creativity and consciousness from the perspective of complex network dynamics. From the perspective of computational network biodynamics, the changes of functional networks during the processes of brain diseases and improvement were summarized (Lynn and Bassett [2019\)](#page-61-12).

Our fndings showed that large-scale neuroscientifc models can not only analyze and explain the local neural activities of the brain, but also the global neural activities of the brain (Yuan et al. [2021](#page-65-2); Peng and Wang [2021\)](#page-62-10). This large-scale cerebral functional model is also a robust method to solve the conversion relationship between electroencephalography (EEG) and electrocorticography (ECoG) (Hipp et al. [2011b](#page-60-13)). It can also be used to describe the interaction between various cerebral regions (Yuan et al. [2021;](#page-65-2) Peng and Wang

[2021\)](#page-62-10), which can also explain the dependence between blood oxygen signals and states of consciousness through insight and analysis of the nature and laws behind the experimental data (Raichle et al. [2018;](#page-62-15) Stender and Mortensen [2016\)](#page-62-16), and the essence of intelligence (Barbey [2018b](#page-58-14); Kruegera et al. [2009\)](#page-60-14), the source of creativity (Kanwisher [2010b\)](#page-60-15), the laws of encoding and decoding of the perceptual neural system (Stelnmetz et al. [2019](#page-62-17); Esterman et al. [2009\)](#page-59-8), the neurophysiological mechanism of the hemodynamic phenomenon of the brain, the description and the content of brain waves, etc. (Cohen [2017\)](#page-58-15). Hence, neural energy models are the only option for large-scale neuroscience models until more efficient methods are found (Wang and Zhu [2016\)](#page-63-16).

#### **1.3 Research of dynamic coding based on classifcation**

#### **1.3.1 Receptor coding**

The essence of cognition is the process of information processing, and the essence of information processing is expressed through neural coding. Therefore, selection of a neural coding mode plays an extremely important role in determining the cerebral functions. To date, researches on neural coding have mainly concentrated on the fring of various membrane potentials to measure and characterize various stimulus properties, including light intensity, sound intensity, temperature, pressure, and motion. As the neural activity and applicable scope revealed by diferent neural coding patterns are very limited, we need to explore the coding characteristics at diferent levels, which are of great importance to reveal the cognitive activity corresponding to various types of coding patterns. Especially, when we combine coding patterns at diferent levels to explore the cognitive properties of diferent functions, it greatly increases the complexity of neural coding research and makes it more difficult to explore the working mechanism of the brain (Wang and Zhu [2016](#page-63-16)).

In the feld of neuroinformatics, several neural coding models have been proposed (Breakspear [2017](#page-58-16)). They can be mainly summarized as receptor coding, including spike count code, spike timing code, and temporal correlation code, respectively, corresponding to diferent assumptions of information units (Johnson and Ray [2004](#page-60-16); Nirenberg and Latham [2003](#page-61-13); Victor [1999\)](#page-63-23). Receptor encoding is the premise of realizing perceptual behavior, thus, in the process of realizing vision, hearing, and smell, the dynamic encoding patterns on photoreceptors, auditory receptors, and olfactory receptors are neurophysiological responses to various stimulus information. The characteristics of these neurophysiological responses strongly depend on the mechanism of the neural dynamics of each functional circuit. It has been found that photoreceptors and auditory receptors primarily encode the fring rate and fring time of spikes on neurons (Victor [1999](#page-63-23); Wang and Wang [2018c,](#page-63-24) [2020](#page-63-25); Jacobs et al. [2009;](#page-60-17) Malnic et al. [1999](#page-61-14)), while olfactory receptors simultaneously encode combinations of stimulus-sensitive neurons (Miyamichi and Luo [2009](#page-61-15); Xu et al. [2022a](#page-64-16)). In the recent decade, fring rate code has become the standard for describing the properties of various types of perceptual and cortical neurons. Humans detect the surface of objects through the fne movements of their fngers and recognize surface textures. The simplest action in the process is a fngertip rubbing across the surface. During tactile detection, material properties are converted into neural signals from the somatosensory system. Aiming at the tactile sensing and coding problems involved in softness, Wang et al. simulated the touch evaluation process and compared it with the psychophysical response of the softness of fabric materials. The study found that the average fring rate of action potentials evoked by all tactile receptors in the contact area between the fngertip and the object refects the bending stifness of fabric-like fexible materials, and there is a linear relationship between the average fring rate and softness (Yao and Wang [2019](#page-64-5); Hu and Wang [2013](#page-60-18); Hu et al. [2012\)](#page-60-19). Therefore, the encoding form in receptors is closely associated with stimulus characteristics. These fndings may enable us to further study the physiological mechanisms that receptors rely on, in order to process stimulus information.

#### **1.3.2 Coding of navigational information**

In the coding of navigational information, a recent article on how odor cues are recognized as location information by recording hippocampal CA1 neuron activity is of great signifcance. Scholars found that using smell as a landmark in the coding of navigational information, iteratively interacting with the route integration and the smell landmark in turn, can form a long-distance cognitive space map. The location information of place cells represented by odor cues greatly improves the spatial cognition and navigation abilities of rats (Fischler-Ruiz et al. [2021](#page-59-9)).

An important premise in the coding of navigational information is how the nervous system orders various information in order to encode the most important information, so as to achieve advantages and avoid disadvantages and the highest efficient navigation. Researchers found that paraventricular nucleus of the thalamus (PVT) plays an important role in tracking the salience of external stimuli. The brain uses this information to learn how to measure external stimuli and to ignore or avoid certain stimuli. The results showed that the importance of PVT in selecting external stimuli is not only determined by the physical characteristics of the stimulus itself, but also is related to the internal physiological state of animals and external environment (Zhu et al. [2018a](#page-65-7)). This provides a very important physiological basis for the coding of navigational information.

It is essential to emphasize that we biologically use the concept of neural energy to construct a computational model for the exploration of intelligence. The theory of neural energy coding is used to solve the path search problem: the model constructs a neural energy feld by the power of the place cell cluster, and calculates the gradient of the neural energy feld, so as to use the gradient vector to study the problem of intellectual exploration. The fndings demonstrate that our proposed new model of intellectual exploration based on the neural energy not only finds optimal paths more efficiently, but also presents a biophysically meaningful learning process. This new idea verifes the importance of hippocampal place cells and synapses for spatial memory and the efectiveness of energy encoding, and provides an important theoretical basis for understanding the neural dynamics of spatial memory (Wang et al. [2017b\)](#page-64-17).

#### **1.3.3 Coding of cortical information**

The coding of cortical information is a very signifcant research direction in the feld of neural information processing. It mainly involves time coding, frst-spike latency coding, population coding, and phase coding. Compared with time coding, fring rate coding has certain advantages in signal acquisition and energy efficiency. However, the firing of a single neuron in temporal coding can contain more information (Optican and Richmond [1987;](#page-61-16) Thorpe et al. [2001](#page-63-26)). It can express the characteristics of spike activity that cannot be expressed in the fring rate coding through the rank order code of the spike and the frstspike latency coding of the neuron. Experimental data showed that in the study of visual, auditory, and somatosensory cortex, more efficient and reliable neural signals could be

encoded based on the rank order code of the spike and the frst-spike latency coding of the neuron (Heil [2004](#page-60-20); Chase and Young [2007;](#page-58-17) Zhong and Wang [2021a,](#page-65-8) [b](#page-65-9), [c](#page-65-10); Xin et al. [2019\)](#page-64-18).

Studies revealed that both the encoding of receptors, the encoding of navigational information, and the encoding related to various cognitive information are not determined by the activity of a single or a few neurons, but also by the joint neural response of large-scale neural populations (Insel et al. [2004;](#page-60-21) Hipp et al. [2011c\)](#page-60-22). The population coding is produced to express such type of associative neural response. In particular, synchronized oscillation of neural populations in the cortex can be analyzed as units of information in population encoding (Hipp et al.  $2011c$ ), thereby facilitating the study of the intrinsic relationships between coupled networks in the cerebral cortex. The research on the phenomenon of synchronous oscillation is mainly based on the theory of phase synchronization and the theory of binding. Based on the binding theory (Feldman [2012](#page-59-10)), we can not only study how the codes in diferent cerebral regions are related to produce perception, decision-making, planning, and behavior (Churchland et al. [2012\)](#page-58-18), but also express the neural activity of the cortical network with diferent rhythms through the phase synchronization theory (Wang et al. [2008,](#page-63-20) [2009](#page-63-27); Wang and Zhang [2011;](#page-63-28) Rubin et al. [2012\)](#page-62-18). Therefore, phase synchronization theory is widely recognized as an important mechanism to support the study of binding phenomena (Panzeri et al. [2015\)](#page-62-19), facilitating the study of phase encoding on how synchronized oscillation depends greatly on enrichment of the population encoding. The most important disadvantage of population encoding is that it cannot handle the problem of high-dimensional nonlinear coupling (Wang and Zhu [2016\)](#page-63-16), followed by how the macroscopic properties of population encoding can be efectively integrated with encoding at the microscale, and some important mechanisms at the microscale, such as interactions between neurons, roles of diferent functional neurons in the macroscopic expression of population codes, and how the sparsity of coding and combinatorial selection of local encodes cause their intrinsic connections in the encoding of large-scale neuronal populations. These problems severely limit the application of population coding in the feld of complex brain dynamics research (Panzeri et al. [2015\)](#page-62-19). To address and explore the limitations of various coding theories, we proposed a theory of neural energy coding (Wang and Zhu [2016](#page-63-16)).

#### **1.3.4 Behavioral coding**

How do diferent neuronal populations, which are widely distributed in the brain, and communicate with each other to accomplish certain complex tasks? Are there certain working principles for these widely distributed neuronal populations when performing these complex tasks? One example is that in order to complete a perceptual decision, the brain needs to process sensory information and select behaviors that lead to rewarding efects. Neurons in multiple cerebral regions mediate some aspects of these processes, and it is not clear which cerebral region processes which information and whether the processing of this information depends on similar or diferent neural circuits. Therefore, the study of behavioral coding has noticeably attracted neuroscientists' attention in recent years to solve these scientifc problems.

In an article entitled "Spontaneous behaviors drive multidimensional, brainwide activity", Stringer et al. used two-photon calcium imaging combined with neuropixel electrode recording to monitor the activity of about 10,000 neurons in the visual cortex of awake mice. The primary visual cortex was found to encode visual and motor information related to facial movements (Stringer et al. [2019](#page-63-29)). In another article entitled "Thirst regulates motivated behavior through modulation of brainwide neural population dynamics" published in the Science by Allen et al. from Stanford University, the same neuropixel electrodes were used to record nearly 24,000 neurons in 34 cerebral regions of mice in a thirsty state. It was found that the state of drinking motivation could determine the activity of neuronal population throughout the brain to convert sensory information into behavioral efects (Allen et al. [2019](#page-58-19)). In the third article (Gründemann, et al. [2019\)](#page-59-11), entitled "Amygdala ensembles encode behavioral states" from the research group of Andreas Lüthi, University of Basel, Switzerland, the GRIN prism was used to study the encoding behavioral states of the basal amygdala under diferent behavioral paradigms, and found that in exploration during the non-exploratory and non-exploratory phases, two non-overlapping functional neuronal populations could encode opposite behavioral states. These results revealed the working state of neuronal population in diferent cerebral regions under complex tasks from a large-scale perspective, so as to better understand the working principle of the brain. This unprecedented large-scale recording of neuronal activity is due to the latest breakthrough in animal calcium imaging under the condition of awake activity with neuropixel electrodes, enabling us to explore scientifc problems that have never been imagined more boldly and freely.

#### **1.4 Neural energy coding**

Neural energy coding is a novel coding theory based on the corresponding relationship between neural energy consumption and neural activity.

#### **1.4.1 Studying the scientifc signifcance of neural energy encoding**

- (a) According to the fMRI fndings, when a part of the body is stimulated, according to the blood oxygenation level dependent (BOLD) imaging, it was found that there was a 31% increase in blood fow in the contralateral cerebral hemisphere, which was corresponded to only 6% increase in oxygen consumption (consistent with the 5:1 result obtained by the spectroscopy). Why is the increase in blood fow not accompanied by a substantial increase in oxygen consumption? Only a proportional increase in the rate can be consistent with an increase in energy expenditure continuously provided in the bloodstream. More importantly, is the large amount of remaining neural energy loss only used for the brain's physiological metabolism or is it related to cognitive activity? If these remaining energies are relevant to cognition, in what form are they involved in cognition? At present, neuroscience cannot explain these physiological phenomena, and neural energy coding has a strong potential to scientifcally explain and probe these physiological phenomena (Yuan et al. [2021;](#page-65-2) Peng and Wang [2021](#page-62-10); Cheng et al. [2020](#page-58-10)).
- (b) For the large number of unsolved neuroscience problems mentioned above, it is not enough to rely solely on neuroscience experiments and new neuroscience technologies, because experiments can only observe phenomena and new neuroscience techniques can only discover previously unobserved phenomena. For instance, the experimental data cannot reveal the working mechanism and the distribution pattern of dark energy of the DMN in the brain, or they cannot reveal the coupling relationship between the DMN and the functional neural network and their quantitative relationship with the biological energy provided by the brain blood fow.

(c) Completion of cognitive tasks is always associated with synchronized oscillation and synaptic transmission of neural activity in the network, and the neural energy consumption is generally considered to be only a small fraction (around 5%) of the overall energy expenditure. Thus, is there any other way of neural information transmission, which is one of the necessary conditions for the completion of cognitive tasks, as well as synaptic transmission? Professor Dominique Durand, an American scientist, recently published an article in the Journal of Neuroscience (Qiu et al. [2015\)](#page-62-20), pointing out that neural information can be efectively transmitted only using the brain's electrical feld to achieve cognitive tasks without the need for synaptic transmission, indicating that neural information may be involved in spontaneous neural activity. Therefore, studies based on spontaneous neural activity are likely to reveal how major energy consumption occurs in the brain and the internal relationship between the neural mechanism of energy consumption and cognitive function. Neural energy coding can unify spontaneous neural activity and task-induced neural activity in a group of neural models (Yuan et al. [2021;](#page-65-2) Peng and Wang [2021](#page-62-10)).

#### **1.4.2 Key features of neural energy coding**

As the neuronal membrane potential has a unique correspondence with neural energy (Wang et al. [2019a\)](#page-64-2), the time-varying energy fow can refect the conduction of information fow in the network, that is to say, neural information coding can be expressed by neural energy coding. This is certain that no matter how kaleidoscopic fring patterns of neurons are, how rich and varied are the synchronized oscillation of diferent frequencies of networks in the nervous system, or the changes in local feld potentials, the patterns of energy changes that constrain all these neural activities will always occur with the everchanging membrane potential or the oscillation mode of the network (Wang et al. [2015b](#page-63-30); Wang and Wang [2014\)](#page-63-31).

#### **1.4.3 Advantages of neural energy coding compared with the existing coding theories**

- (a) Global functional models of the brain based on energy can be used to analyze and describe experimental phenomenological neuroscience at various levels. It makes the computational results at various levels no longer mutually inapplicable, contradictory, and irrelevant. That is to say, neural information can be expressed in energy at various levels of molecules, neurons, networks, cognition, and behavior, as well as at the combination of each level, and a neural energy model can be used to unify the interactions between various levels (Yuan et al. [2021;](#page-65-2) Wang and Zhu [2016\)](#page-63-16), which is impossible to do this with any traditional neural coding theory.
- (b) It is difcult to simultaneously obtain recordings from multiple cerebral regions. Although EEG and magnetoencephalography (MEG) can obtain neuronal activities from various cerebral regions, estimating cortical interactions based on these extracranial signals is a clinical challenge. The main obstacle is the lack of a theoretical tool capable of efficiently analyzing interactions among each cortical area in highdimensional spaces. Additionally, a conversion relationship between scalp EEG and cortical potential was not found. The neural energy provides an efective solution to the above-mentioned problems.
- (c) As energy is a scalar quantity, whether it is a single neuron or a neuronal population, a network or a behavioral, linear or nonlinear neural model, their dynamic response

can be used to describe the mode of neural coding by the method of neural energy superposition (Yuan et al. [2021](#page-65-2); Peng and Wang [2021](#page-62-10); Wang et al. [2015b;](#page-63-30) Wang and Wang [2014\)](#page-63-31). This provides global information about functional neural activity in the brain, which is impossible to achieve with other traditional coding theories.

(d) The mode of network coupling oscillation can be ever-changing, and the coupling oscillation of neural network has a unique relationship with the oscillation of network energy. When the large-scale of the neural network modeling and numerical analysis becomes impossible to handle due to the extremely complexity of high-dimensional nonlinear coupling, it is suggested to use the neural energy coding to study the neural information processing. Thus, complex neural processing can be simply and easily performed without losing information (Zheng et al. [2022](#page-65-6)).

Our proposed model of neuronal energy could also accurately predict the presence of an unknown magnetic substance in the brain (Wang and Zhang [2006](#page-63-32)). After 10 years, an important academic article was published in the Nature Materials, which experimentally proved the existence of a magnetic protein called MagR in the brain, which can be used for direction and navigation in path exploration (Qin and Xie [2016](#page-62-21)).

As neural energy can describe the interactions at various levels from the global perspective of cerebral activity, neural energy coding can also be used to express various existing neural coding patterns. That is to say, all types of coding patterns based on membrane potential in the past can be expressed by neural energy coding patterns, which are special cases of neural energy coding.

## **2 Application of analysis dynamics in neuron modeling**

#### **2.1 Problem raising**

Observed from the Angle of mechanics, the growth of brain nerve system of connections between neurons function circuit formation and regeneration of nerve tissue degeneration after degradation ability, and the dynamics of the growth cone structure and their motions, and the change of the growth cone motion state and its movement trend depends on the interaction forces. Although the mechanics of neuron activity are well understood at the molecular level, all functional neural activity in the brain is based on the activity of 100 billion neurons. During the development of the nervous system, once neurons fnd their position in the brain, they are both the basic structural unit and the functional unit of the whole nervous system. In order to understand the complex multi-level interaction system of the brain and explore the law and essence behind the various functions of the system, according to the role and contribution of various forces of neuronal axons and dendrites in the signal transmission process. On the basis of previous studies on neurobiology and cell biology, we need to further investigate the efect of neuronal electromagnetic induction on membrane potential changes and corresponding energy metabolism. Some pioneering work has been done by Ma Jun of Lanzhou University of Technology. He pointed out that electromagnetic feld efects generated by ion transport inside and outside nerve cells should be considered during electrophysiological activities to further explain the dynamic mechanism of synaptic plasticity from a physical perspective (Wu et al. [2016\)](#page-64-19). Based on the principle of electromagnetic induction, magnetic fux was introduced into the neuron model, and induced current was used to express the electromagnetic feld efect and

external electromagnetic radiation efect generated by ion transport in the cell, which could explain the multi-mode oscillation of neuron electrical activity and the coupling synchronization process of neuron network internal feld.

In 1952, Alan Hodgkin and Andrew Huxley gave the frst quantitative description of neuron membrane potential (H–H equation), and proposed the concept of ion channel, thus revealing the veil of neuron excitability. Because this research, they won the Nobel Prize in Physiology and Medicine in 1963 and is the only two scientists who have won the Nobel Prize so far for constructing mathematical models. However, they propose that the H–H neuron model is a high-dimensional complex nonlinear equation. Due to the many parameters, although they can refect many nonlinear properties of nerve cells, they are suitable for the study at the subcellular level. But a large number of neurons can cause considerable computational difculty, or even impossibility. Therefore, it is not suitable for large-scale neural network and large-scale neural computing research. In order to solve these difficulties, some scholars proposed some simplifed neuron models before and after the publication of H–H precise models. Typical model is Hindermarsh–Rose nonlinear model (HR), which can be used as the basic unit of large-scale networks for the study of action potential emission characteristics of single neurons, due to its characteristic and applicable scope of much less computation and parameters than H–H model. The main disadvantage of HR model is that the rich nonlinear dynamic properties obtained under the condition of single neuron or several neurons cannot be directly extended to high-dimensional nonlinear system and neural network system, and there are many limitations in practical application. The integral-and-fre model (IF) can also be used for dynamic research of a large number of neuron clusters due to its simple calculation. However, this model is defective because it cannot record the integrity of neuron membrane potential changes. Compared with H–H model, FithzHuge–Nagumo nonlinear model (FHN) has much less computation and can only be used to study the action potential properties of single neurons. The Chay model is a simplifed version of H–H model, and the computational workload is relatively reduced. Its main shortcoming is that it is not suitable for large-scale neural network and large-scale neural computation. In fact, these situations not only seriously hinder the complete description of the nervous system in neurodynamics and computational neuroscience, but also seriously hinder the construction of the theoretical system of brain science. This research makes it impossible for us to understand the laws and nature behind big data in neuroscience from a theoretical height. As a result, there will continue to be a lack of a common language between experimental and theoretical neuroscience, leaving a hidden and unbridgeable gap in brain research that is supposed to understand, collaborate and develop together.

#### **2.2 Biophysical mechanism of W–Z neuron model**

In conclusion, it is necessary to fnd a simple and reliable neuron computing model that can perform large-scale combinatorial computation at all levels without losing the main information. The characteristics of this new neuron model are as follows: (1) It must be a neuron model equivalent to H–H model; (2) It is not only applicable to the calculation of the interaction between brain regions, but also applicable to the calculation of all levels of brain and the combination of all levels of brain; (3) At the neural nucleus, mesoscopic, complex network, macro cognitive and behavioral levels, the new neural model can ignore the secondary information but can't lose the main information; (4) The calculation is simple and reliable.

After more than 10 years of exploration, we have preliminarily found such a new neuron model called W–Z that basically meets the above requirements (Wang et al. [2015a;](#page-63-17) Wang and Wang [2018a,](#page-63-18) [b\)](#page-63-19). When we used analytical mechanics to construct this original neuron model at the cellular level, we accidentally discovered a new working mechanism of neurons. Although H–H neuron model and W–Z neuron model are two neuron models constructed at completely diferent levels, they are actually equivalent (Wang and Wang [2018b\)](#page-63-19). The biggest diference between the two neuron models is that inductor elements are introduced into the W–Z model. The biological basis for the inductance element was the discovery of a long-ignored phenomenon in neuroscience experiments. These experimental phenomena provide intracellular records of isolated brain and spinal cord sections of diferent types of mammals, indicating that central neurons can generate various complex action potential emission patterns and spontaneously and continuously generate action potentials by internal ionic mechanisms when there is no synaptic connection. Even if there is no stimulus input, the mechanism of neuron discharge mode is strongly intrinsically related to the activity of ionic current (Byrne and Roberts [2009\)](#page-58-20). Although Ma Jun and other authors did not see the above experimental phenomenon, they started from the perspective of physics and based on W–Z neuron model (Wang and Wang [2018b](#page-63-19)) and the working principle of memristor (Wu et al. [2016;](#page-64-19) Lv et al. [2016](#page-61-17)). They keenly feel that the electromagnetic feld efect generated by induced current in the process of ion transport in cells and the external electromagnetic radiation efect are keenly felt to contribute to the conduction of neural signals (Lv et al. [2016](#page-61-17)). They applied this method to study the electrical activity behavior of cardiac tissue under electromagnetic radiation and predicted the mechanism of sudden cardiac death and shock induced by electromagnetic radiation (Lv et al. [2016](#page-61-17); Ma et al. [2017](#page-61-18)). Thus, the physical mechanism of synaptic plasticity of neurons is scientifcally explained. The process of task synaptic current triggering is accompanied by the generation of electromagnetic felds. Therefore, the combination of capacitors and induction coils can be used to simulate the function of hybrid synapses (Ma et al. [2019](#page-61-19)).

Alan Hodgkin and Andrew Huxley show that ionic current can be described as the product of conductance and driving force (the diference between the membrane voltage and the ionic Nernst potential) in the case of constant membrane permeability. Conductance refects the permeability of cell membrane to ions, while the driving force refects the trend of ionic current movement of charged particles in the fuid inside and outside the cell under the dual efects of electric feld gradient and concentration gradient. Lots of experimental data (Byrne and Roberts [2009](#page-58-20)) shows that since the resting potential of the neuron is not at the equilibrium potential level of any particular ion, various ions continue to difuse along the concentration diference, which is evident in the generation of action potentials and synaptic potentials. Therefore, cells need to recover through active transport by the sodium–potassium pump, which is thought to complete transport through allosteric efects of protein phosphorylation and dephosphorylation. When the membrane potential exceeds the threshold for an action potential, it initiates a large, transient inward current (sodium current) followed by a persistent outward current (potassium current). They showed that when the membrane potential was depolarized, the sodium current (sodium conductance) was rapidly activated and then inactivated. The potassium current (potassium conductance) is activated after a delay, and a high activation level remains as long as depolarization is maintained. As the inactivation of sodium channel is slower than activation, when the sodium channel is activated but not inactivated, a large amount of sodium ions fow in, leading to the increase of sodium current, forming a positive feed. It then switches to repolarization due to sodium channel inactivation and potassium channel activation to prepare for the next action potential. All kinds of ion currents in H–H model, no matter sodium

current or potassium current, are very weak in each ion channel, so it is difficult to construct their own self-induction efects at the level of ion channel. Although the magnetic field intensity in the nervous system is very weak (Liu [2002](#page-61-20)), the electromagnetic induction efect exists objectively. Given this objective fact, our neuronal model is built directly on the level of nerve cells, the total efect of self-induction of all types of ion channel currents can be expressed in terms of an inductance. This inductance can in fact be used to express the total efect of the magnetic feld on the movement of all the ionic currents represented in the H–H model. This is the biological explanation for why inductance is introduced in the W–Z model, and the underlying reason why H–H and W–Z models are the equivalent neuron models.

### **2.3 Analytical dynamics reveals new working principles of neuronal activity**

Based on the experimental data provided by electrophysiology and the working characteristics of neurons, we present the biophysical model of coupled neuron activity as shown in Fig. [1](#page-15-0). This model refects the interaction of a single neuron with all other neurons connected to it. The mutual coupling between neurons is realized by the current formed by the input of the frst neuron to the frst neuron in the cerebral cortex.

According to Fig. [1](#page-15-0), we get the following circuit equation:

<span id="page-15-2"></span><span id="page-15-1"></span>
$$
P_m = U_m I_{0m} + U_{im} I_m,
$$
\n(1)

$$
U_{im} = C_m r_{3m} \dot{U}_{0m} + U_{0m}, \qquad (2)
$$

$$
U_m = r_{0m}I_{0m} + r_{1m}I_{1m} + L_m \dot{I}_{1m},
$$
\n(3)

$$
I_{0m} = I_{1m} - I_m + \frac{U_{im}}{r_m} + C_m \dot{U}_{0m}.
$$
\n(4)

Among them, membrane capacitance represents the accumulation of positive and negative ions inside and outside the cell membrane. It is the heat loss caused by the collision of the inner and outer membranes during ion exchange. As mentioned above, in this new



<span id="page-15-0"></span>**Fig. 1** Schematic description of W–Z model (Wang et al. [2015a](#page-63-17))

neuron model, we have introduced an inductance element that is not in the H–H model, which means that each ion channel on the cell membrane produces a self-induction efect caused by a loop current fowing through multiple ions.

It contains the transport process of a large number of charged ions in nerve cells such as sodium, potassium and calcium ions in ion channels, which can trigger uniform or nonuniform electromagnetic felds inside and outside the cell, which in turn will afect the transport of charged ions. That is to say, the change of the charge distribution density in the cell and the electromagnetic feld efect produced during the charge transport process will cause the induced current in the cell, which is exactly what the traditional neuron model does not consider (Ma et al. [2019](#page-61-19)). Represents the heat consumed by ions colliding with each other under the condition of current generation, which can be equivalent to resistance. m can be expressed as the chemical gradient inside the neuron, which will drive the fow of ions, and can be simulated by a voltage source or a current source in the electrical model. It is the loss caused by the unsatisfactory voltage source. And means that in addition to the chemical gradient, the neuron also accepts the action of other neurons, and at the same time maintains the resting potential of the neuron at rest. In order to express this function, set the input current as

$$
I_m = i_{1m} + \sum_{j=1}^{n} \left[ i_{0m(j-1)} \sin \left( \omega_{m(j-1)}(t_j - t_{j-1}) \right) \right] + i_{0m(n)} \sin \left( \omega_{m(n)}(t - t_n) \right). \tag{5}
$$

Among them is to maintain the resting potential, and the rest represents the input of the surrounding N neurons to the mth neuron.  $m$  is the frequency of the action potential. It is the resistance across and the loss caused by the imperfect current source. It is stimulated by peripheral neurons when it is subliminal. When the action potential is issued, it is not afected by external infuences, and is afected by the internal mechanism of neurons. The efect is that the voltage source and the current source are not produced at the same site. The voltage source mainly provides a small loop current of the ion channel, and the current source mainly accepts the stimulation of peripheral neurons, and they are almost closed to each other, but there is an internal connection between them, which can be represented by resistance. The observable physical quantities in this physical model are membrane potential and membrane current respectively.

The total power of N neurons can be expressed as

$$
P = \sum_{m=1}^{N} P_m.
$$
\n<sup>(6)</sup>

According to the circuit diagram, the power of the neuron is given by the following formula:

$$
P_m = d_{1m}\dot{U}_{0m}^2 + d_{2m}\dot{U}_{0m} + d_{3m}\dot{U}_{0m}U_{0m} + d_{4m}U_{0m}^2 + d_{5m}U_{0m} + d_{6m}.
$$
 (7)

The parameter  $d_{im}$  ( $i = 1, 2, ..., 6$ ) can be found in literature (Wang et al. [2015a\)](#page-63-17).

Since the expression of the voltage source  $U_m$  cannot be obtained from the circuit model in Fig. [1,](#page-15-0) generally the membrane potential cannot be determined. However, the interaction between coupled neurons in the cerebral cortex is orderly and follows the law of selforganization (Gu and Liang [2007;](#page-59-12) Haken [1996](#page-60-23)), and convincing evidence is that the data provided by neuroscientists at Yale University has confrmed that the activity of neurons in the cerebral cortex requires consume energy (Raichle and Gusnard [2002;](#page-62-22) Maandag et al.

[2007;](#page-61-21) Lin et al. [2010\)](#page-61-22). Their research work shows that compared with the resting state, the energy consumption of the brain under stimulation is mainly used for the propagation of action potentials, and neurotransmitter stimulation of receptors causes the restoration of post-synaptic ion current (Lin et al. [2010](#page-61-22)).

The neuron cluster model established by us with m neurons under coupling conditions can describe the basic characteristics of the subthreshold and threshold electrical activity of neuron cluster through its current coupling relationship with surrounding neurons (Wang et al. [2008](#page-63-20), [2009](#page-63-27)). However, the membrane potential cannot be obtained directly from the model. If we can fnd the functional form of neuron energy consumption and fnd the constraint conditions of neuron motion equation controlled by the energy function, we can get the solution of membrane potential. Due to spontaneous electrical activity of neuron clusters obey the law of self-organization (Haken [1996](#page-60-23)), at the same time, according to Yale university neuroscientist on nerve signal transmission and energy dissipation of tightly coupled with the experimental results, we determine the constraint condition of the given the new neuron model is likely to be the energy function in the circuit system. In mechanical analysis, we know that for a known dynamic system, we can write down the kinetic energy and potential energy of the system, thus obtaining its Lagrange function. But in our neuron electrical model, we assume that potential energy is equal to a constant, (and power is the average energy). It can be assumed that the power consumed on the circuit model can be regarded as the energy function of the dynamic system, thus leading to the Lagrange function as the constraint condition of the circuit model, which plays a key role in describing the neuron model completely. Whether such an idea is reasonable depends on whether the results obtained by such an idea agree with the results of neuroelectrophysiological experiments. The genius of this idea is to extend the modeling of dynamical systems from classical mechanical systems to nervous systems.

According to the above ideas, suppose that the Lagrange function in the model is related to the total power of the circuit model, and its dynamics equation is given by the following equation

$$
\frac{d}{dt}\left(\frac{\partial P_m}{\partial \dot{U}_{0m}}\right) - \frac{\partial P_m}{\partial U_{0m}} = 0 \quad (m = 1, 2, \dots, N). \tag{8}
$$

The solution to the above equation is

$$
U_{0m} = -\frac{\hat{g}_1}{\lambda_m^2} - \frac{\hat{g}_2 e^{-a(t-t_n)}}{\lambda_m^2 - a^2} - \frac{1}{\lambda_m^2 + \omega_m^2} (\hat{g}_3 \sin \omega_m(n)(t - t_n) + \hat{g}_4 \cos \omega_m(n)(t - t_n) + (U_{0m}(t_n) + \frac{\hat{g}_1}{\lambda_m^2} + \frac{\hat{g}_2}{\lambda_m^2 - a^2} + \frac{\hat{g}_4}{\lambda_m^2 + \omega_m^2(n)}) e^{-\lambda_m(t - t_n)} t_n < t < t_{n+1}, \quad n = 0, 1, 2, ..., t_0 = 0.
$$
\n(9)

When  $i_{0m}(n)$  is strongly stimulated and the membrane potential reaches the threshold level, we obtain the neuron membrane potential  $U_{0m}$  and the corresponding energy function  $P_m$ , as shown in Fig. [2](#page-18-0).

According to the calculated results, the action potential waveform is completely consistent with the experimental data, thus confrming our previous judgment is correct. It is important to note that the results of the analytical mechanics calculations show for the first time a previously undiscovered phenomenon that when neurons fire an action potential, the corresponding energy expenditure does not correspond to the conventional view of neuroscience, which is that neurons only use energy. It's about absorbing energy and then expending energy before expending energy. In fact, during the production of an action



<span id="page-18-0"></span>**Fig. 2** Action potential (**a**) and corresponding energy function (**b**) (Wang et al. [2015a\)](#page-63-17)

potential, the energy change of the neuron is composed of two parts. One part is the negative energy of the oxygenated hemoglobin obtained from the blood stream, which is used for energy storage. On the other hand, deoxygenated hemoglobin presents positive energy for energy consumption (Wang et al. [2015a;](#page-63-17) Wang and Wang [2018a,](#page-63-18) [b](#page-63-19)). For this novel energy calculation result, Zheng et al. (Zheng et al. [2014,](#page-65-11) [2016](#page-65-12)), combined with molecular biology knowledge and existing experimental data, provided a qualitative explanation for the regulation process of ion channel opening and closing, glutamate circulation and glucose by neurons and related glial cells during action potential generation. It is pointed out that the negative energy in action potential is a process of energy storage, that is, the amount of glucose and oxygen absorbed from blood stream is greater than the amount needed to consume. In other words, stimulated neurons will lead to increased cerebral blood fow, but there is a demand for oxygen consumption during depolarization (there is no oxygen consumption at the moment), mainly in the form of energy absorption. In the repolarization stage of neurons, the energy storage has been consumed, and at this time the oxygen consumption of neurons increases signifcantly, which is manifested as energy consumption. In short, the neuron is not only a energy-consuming device but also an energystoring device. From the perspective of an action potential, a neuron absorbs energy from the bloodstream and then uses it up again and again, reaching a dynamic equilibrium, suggesting that the energy storage capacity of a single neuron is limited. From the perspective of glucose and oxygen supply in blood stream, when the glucose and oxygen supply in blood stream is sufficient and the energy storage of neurons is not up to the upper limit,

neurons will spontaneously reserve energy at the initial stage of action potential depolarization. This may also explain why the oxygen metabolic rate,  $CMRO<sub>2</sub>$ , changes more rapidly than blood fow CBF, refecting the fact that neurons release stored oxygen in response to stimuli and then consume oxygen and glucose from the bloodstream. We have explained the neural mechanism of cerebral hemodynamic phenomena from a quantitative perspective (Peng and Wang [2021](#page-62-10)).

It emphasized that the existence of negative power component in the energy function corresponding to the fring of action potential by neurons is an extremely important new discovery of the working mechanism of neurons (Wang et al. [2015a\)](#page-63-17). This new mechanism reveals two previously undiscovered patterns of neuronal activity. The frst is that there is a corresponding relationship between membrane potential discharge of neurons and nerve energy. The second is that neurons consume energy when they are active below the threshold value, while absorb and consume energy when they are active above the threshold value. The frst law reveals a unique correspondence between functional acquisition of neuron membrane potential and energy function, which has been strongly confrmed by H–H models (Wang and Wang [2018b](#page-63-19)). The second rule verifes an experimental phenomenon currently unexplained by neuroscience, namely, activation of brain regions increases blood fow by 31% while oxygen consumption increases by only 6% (Zheng et al. [2016\)](#page-65-12), a relationship approximately equal to 5:1. Our calculation shows that the area of the positive and negative interval in the energy curve in Fig. [2](#page-18-0) is approximately equal to 5:1 (Adachi and Aihara [1997](#page-58-6)). It should be emphasized here that the positive and negative areas in this power curve have profound neurobiological signifcance: the positive and negative areas well correspond to the experimental result that blood fow increases about 31% during stimulus-induced neuronal activity, while the associated oxygen consumption increases only about 6% (Zheng et al. [2016](#page-65-12)). The use of the negative power component can also explain the hemodynamic phenomenon of the brain, that is, the signifcant increase in blood fow after the activation of the designated cortical area is delayed by 7–8 s from the activation moment. It also explains why tactile perception is synchronized with the emergence of consciousness and so on. The new neuron model based on experimental data enables us to propose the concept, theory and method of energy coding in an original way (Wang et al. [2008](#page-63-20), [2009,](#page-63-27) [2015a,](#page-63-17) [b](#page-63-30), [2017a](#page-63-22), [b,](#page-64-17) [2018a,](#page-64-10) [b](#page-64-11), [2019b,](#page-64-12) [2021a;](#page-64-4) Yuan et al. [2021;](#page-65-2) Wang and Zhu [2016;](#page-63-16) Wang and Wang [2018b](#page-63-19), [2020](#page-63-25), [2014](#page-63-31); Li et al. [2022a;](#page-61-10) Peng and Wang [2021;](#page-62-10) Zheng et al. [2014,](#page-65-11) [2016](#page-65-12), [2022](#page-65-6); Wang and Zhang [2006](#page-63-32)). This new concept and coding theory can not only explain some experimental phenomena that cannot be explained by neuroscience so far and quantitatively reveal the laws behind some experimental data, but also predict some phenomena that cannot be discovered by experimental neuroscience. When we fully understand and master the above nature of neuronal activity, we will have a new understanding of the rules of neural information processing and the principle of neural coding in the cerebral cortex. This fully refects the infuence and role of mechanics in promoting the progress of neuroscience and the infuence and role of mechanics in the feld of neuroscience and life science.

According to the discovery of the new working mechanism of neurons, the neuron model provided by us can also be used to quantitatively prove that the operating mode of brain works in accordance with the following criteria (Zheng et al. [2022;](#page-65-6) Laughlin and Sejnowski [2003b\)](#page-60-24): (1) Economy—The neural network activity conforms to the energy minimization principle when the brain is at rest and participating in cognitive activities (Zheng et al.  $2022$ ); (2) High efficiency—the transmission efficiency of cortical neural network signals conforms to the principle of maximum energy utilization (Zheng et al. [2022\)](#page-65-6); (3) Self-organizing neural computations—the relationship between membrane potential

and energy refects the coupling relationship between neural information and cerebral blood fow (Moore and Cao [2008](#page-61-7); Fox and Raichle [2007a;](#page-59-6) Raichle and Gusnard [2002](#page-62-22); Lin et al. [2010\)](#page-61-22). In addition, using this new neuron model, not only the action potential and the corresponding energy consumption of neurons can be simulated, but also the waveforms of presynaptic excitatory potential (EPSP) and postsynaptic inhibitory potential (IPSP) and the corresponding energy of EPSP and IPSP can be simulated, and the simulation results are in complete agreement with experimental records (Wang et al. [2015a\)](#page-63-17). This original neuronal model has been supported by a large number of neuroelectrophysiological experiments. The W–Z neuron model constructed by us and the coding theory of neural energy make it possible to transform a variety of complex, coupled and highly nonlinear membrane potential fring modes into energy fring modes for coding studies (Wang et al. [2008](#page-63-20), [2009,](#page-63-27) [2015a,](#page-63-17) [b](#page-63-30), [2017a](#page-63-22), [b,](#page-64-17) [2018a](#page-64-10), [b](#page-64-11), [2019b,](#page-64-12) [2021a,](#page-64-4); Yuan et al. [2021](#page-65-2); Wang and Zhu [2016;](#page-63-16) Wang and Wang [2014](#page-63-31), [2018b,](#page-63-19) [2020](#page-63-25); Li et al. [2022a;](#page-61-10) Peng and Wang [2021;](#page-62-10) Zheng et al. [2014,](#page-65-11) [2016,](#page-65-12) [2022;](#page-65-6) Wang and Zhang [2006](#page-63-32)). The theory holds that the encoding of neural information is closely related to the metabolism of neural energy, and the mechanism of encoding of neural information can be understood and revealed by using the energy method. What is particularly interesting is that the brain power calculated by W–Z neuron model is about 45 W, while the brain power provided by experimental data is 20 W (Wang et al. [2015a\)](#page-63-17). This is also the frst time that the power consumption of our own brains has been quantifed.

Thus, the discovery of this new and important working mechanism of neurons not only depends on the creative application of the theory of analytical mechanics in neuroscience, but also can perfectly bind neural information and neural energy together, laying a solid foundation for the research framework of global neural coding of the brain.

Experimental data show that the brain consumes only about 5% more energy in the task state than in the resting state (Fox and Raichle [2007a](#page-59-6)). The maximum energy consumption in the resting state comes from the DMN, which consumes more than 95% of the brain energy consumption in the resting state (Peppiatt and Attwell [2004\)](#page-62-23). Much of our understanding of the brain used to come from studying the activity of this 5%. Because the structure of different brain regions and their individual neural activity patterns are diferent, neuroscientists often use dynamic BOLD signals measured by fMRI to look at brain activity as a whole in order to obtain global information about brain activity. However, the average distribution of blood fow in the brain when the brain is activated and the nonlinear coupling relationship between blood flow and oxygen consumption make it difficult for us to obtain a quantitative and accurate understanding of the neural activity of the brain in various states, and we cannot understand the interaction between neurons in the activated brain using this technology (Clancy et al. [2017](#page-58-5)). At present, there is no new experimental technology in the feld of neuroscience that can perfectly integrate the accuracy of neural electrophysiological recording with fMRI technology that refects the global information of brain functional activity, including optogenetics technology which is only locally observable. If neuroscience cannot achieve such a new technique for a long time in the future, can we theoretically propose a new research method that combines the reductionism of the activity of individual neurons and the holism of the macroscopic efects of the brain, and take it as the main basis for studying the global neural activity of brain function. This new research method can not only accurately reproduce neural electrophysiology record, also can use fMRI data provided by the large repeat reproduce the global information of brain functional activity, and it also can be estimated in theory we haven't found the new phenomenon of neural activity, like found that neurons in the energy consumption negative power component so to do that involves understanding the nature of neuronal activity in the brain and to do that we need to compare the H–H neuronal model to

the W–Z neuronal model, right through the analysis and research of these two types of diferent neuron models, we can explain what factors dominate and control the various releasing modes of complex membrane potential of neurons, so as to understand and master the nature and change rules of neural information processing and signal transduction.

## **2.4 Equivalence between W–Z neuron model and H–H model and its molecular biological basis**

In order to verify the validity of W–Z neuron model established by analytical mechanics method, we used H–H model to calculate the energy characteristics of action potential and membrane potential (Wang and Wang [2018b\)](#page-63-19). That's because almost all previous work has been based on how neural activity causes changes in nerve energy. Under the same stimulation conditions, whether the neurons can transition from sub-threshold activity to supra-threshold activity is depended on whether the neurons can be fully energized. The two states of energy supply (both sub-threshold and supra-threshold) determine whether the ion pump can provide stable Nernst potential for sodium ions. That is, from the perspective of energy, the Nernst potential of sodium ion closely connects the sub-threshold and the supra-threshold activity. The inverse of this question is whether changes in nerve energy cause changes in neuronal activity, under insufficient energy supply, neurons will only manifest as the sub-threshold activity when stimulated, but cannot fre action potentials. In general, we are usually concerned about the imbalance distribution of neural energy caused by neural activities under stimulation. However, neural activities are also modulated and constrained by neural energy. The mechanism is that ion pump that lacks energy supply cannot provide constant Nernst potential for sodium ions, and in the early stage of the change of cell membrane permeability, it cannot provide continuous fow of sodium ions to reach the threshold potential (Hopfeld [2010](#page-60-3); Hu et al. [2021](#page-60-4)). Thus, the electrophysiological activity of neurons is strictly constrained by energy levels in the brain. When energy supply of ion pump of the sodium ion channels is insufficient, the response of neurons is sub-threshold activity to any form and intensity of stimulation. But only when the maximum power of sodium ion channels is not constrained, the neurons will be sup-threshold fring.

So it is necessary through calculating model to understand why when ion pump when there is no guarantee that a constant potential, namely when neurons system energy shortage, energy is how to regulate neuronal activity Its scientifc signifcance is when compared to the resting state network energy consumption, the task of related part of the increase of energy consumption is small (less than or equal to 5%). So most of what we know and understand about brain function so far comes from this tiny fraction of brain activity and if we want to get a complete picture of how the brain works, we have to think about the part that consumes most of the energy which is the innate spontaneous neural activity, right. Therefore, we need to further investigate the H–H neuron model established on the level of ion channel, and explore the nature of neuron activity through the comparative study of these two types of neuron models.

The circuit model of H–H equation is shown in Fig. [3.](#page-22-0)

Its diferential equation is described as

<span id="page-21-0"></span>
$$
C_m \frac{dV_m}{dt} = g_l (E_l - V_m) + g_{Na} m^3 h (E_{Na} - V_m) + g_K n^4 (E_K - V_m) + I,
$$
 (10)

where  $C_m$  is the membrane capacitance of neuron membrane,  $V_m$  is the membrane potential,  $/_{ENa}$  and  $_{EK}$  are the Nernst potential of sodium ion and potassium ion respectively,  $_{EI}$  is the potential when the leakage current is zero.  $\hat{g}_{Na}$  and  $\hat{g}_{K}$  sodium channel and potassium

<span id="page-22-0"></span>

channels of variable conductance,  $\hat{g}_{\text{Na}} = g_{\text{Na}} m^3 h$ ,  $\hat{g}_{\text{K}} = g_{\text{K}} m^4$ ,  $g_l$  for leakage conductance. The variable conductance of sodium and potassium channels is described by the following set of nonlinear diferential equations:

<span id="page-22-1"></span>
$$
\begin{cases}\n\frac{dn}{dt} = \alpha_n (1 - n) - \beta_n n, \\
\frac{dm}{dt} = \alpha_m (1 - m) - \beta_m m, \\
\frac{dh}{dt} = \alpha_h (1 - h) - \beta_h h.\n\end{cases}
$$
\n(11)

Each parameter in the above equation can be found in Tsuda [\(1991](#page-63-2)). In H–H equation of circuit model, the total energy can be table as follows:

$$
W_{all} = C \frac{dV_m}{dt} V_m + i_{\text{Na}} E_{\text{Na}} + i_{\text{K}} E_{\text{K}} + i_{l} E_{l},
$$
\n(12)

<span id="page-22-3"></span><span id="page-22-2"></span>
$$
C\frac{dV_m}{dt} = I - i_{\text{Na}} - i_{\text{K}} - i_l,\tag{13}
$$

$$
W_{all} = IV_m + i_{\text{Na}}(E_{\text{Na}} - V_m) + i_{\text{K}}(E_{\text{K}} - V_m) + i_l(E_l - V_m),\tag{14}
$$

$$
W_{all} = IV_m + (i_{\text{Na}}E_{\text{Na}} + i_{\text{K}}E_{\text{K}} + i_{l}E_{l}) - V_m + (i_{\text{Na}} + i_{\text{K}} + i_{l}).
$$
\n(15)

Including  $IV_m$  to provide energy to the outside world on the circuit system,  $(i_{Na}E_{Na} + i_KE_K + i_lE_l)$  to represent the Nernst potential energy provided by the voltage source, While  $V_m(i_{\text{Na}} + i_K + i_l)$  is the energy in membrane potential difference inside and outside. In the above process of releasing action potential from neurons, if the energy consumed by the change of membrane permeability is not taken into account, the energy involved is the energy provided by oxygen and glucose carried by blood fow to neurons respectively (the energy provided by the outside world to the circuit system). The increase in glucose consumption due to brain stimulation is mainly due to activation of the

sodium–potassium ATP pump (Zheng et al. [2014,](#page-65-11) [2016](#page-65-12); Sokoloff [2008;](#page-62-24) Maandag [2007](#page-61-23)). Described in the frst two of them are under the threshold of neurons and the relationship between biological energy, and the ion concentration diference through ion channels to help spread does not consume energy From the perspective of dynamic observation, however, under the threshold of neurons into functional neurons in the process, the sum of the three types of energy and H–H model of circuit is equal to the total energy in the system. The former two types of energy (the outside world to provide energy and the energy of the voltage source) and the circuit of  $IV_m$  and  $V_m(i_{\text{Na}} + i_K + i_l)$  correspond, so the energy provided by the Nernst potential  $(i_{Na}E_{Na} + i_KE_K + i_lE_l)$  equals the biological energy consumption of ion pump. In fact, in this process, the sodium–potassium pump continuously transports ions against the ion concentration gradient, which directly consumes biological energy, that is, one ATP can pump three sodium ions and two potassium ions. This also confrms that due to the existence of the ion pump, a stable Nernst potential is provided by the continuous transport of ions, thereby providing energy for neural activity. Therefore, we can calculate the power consumed by the ion pump by using the power of the voltage source represented by the Nernst potential, that is, the nerve energy consumed by neuronal activity is:

<span id="page-23-0"></span>
$$
P = |i_{\rm K} E_{\rm K}| + |i_{\rm I} E_{\rm I}| - |i_{\rm Na} E_{\rm Na}.\tag{16}
$$

The negative sign of the third term in Eq.  $(16)$  refers to the fact that in the circuit shown in Fig. [1,](#page-15-0) the voltage source/and current/are in opposite directions to  $E_{\text{Na}}$ ,  $i_{\text{Na}}$  and  $E_{\text{K}}$  and  $E_{\text{I}}$ (the sodium current is inward, while the potassium current and leakage fow outward). For an action potential, we can use the above equation to calculate the neural energy expend.

The calculated parameter values in Fig. [4](#page-24-0) can be found in Wang and Wang [2018b](#page-63-19) It can be seen that although the action potential of H–H model and W–Z model has some errors in waveform (mainly because W–Z model is constructed at diferent levels as H–H model), the nerve energy of H–H model also has negative power component, and almost has the same dynamic characteristics as that of W–Z neuron model. This result shows that the neuron energy model proposed by us has a deep internal connection with the H–H model. From the perspective of computation, H–H neuron model needs to calculate the conductance and current of multiple ion channels, so if H–H neurons are used to construct a large number of neuron networks, it will cost a lot of computation cost. However, if W–Z neurons are used to construct the network hierarchical model with a large number of neurons, it has a greater advantage because of its greatly reduced computational complexity (see Fig. [4\)](#page-24-0).

We can make biological explanation for the negative power component of neuron energy at the initial stage of fring action potential as follows.

This is mainly due to local congestion caused by nerve activity. As blood vessels dilate and blood fow increases, the amount of arterial infow increases, leading to an increase in oxygenated hemoglobin into the blood vessels (Peppiatt and Attwell [2004](#page-62-23)). Neurons take oxygen mainly through oxygenated hemoglobin, but the consumption of  $O<sub>2</sub>$  does not increase proportionally with the increase in blood flow and  $O_2$  (Peppiatt and Attwell [2004\)](#page-62-23). Fox et al. observed in PET that the event-induced oxygen uptake coefficient (OEF) decreased from 40% at rest to 20%, meaning that 80% of the oxygen delivered during the event was not physiologically metabolized. This suggests that the energy requirements associated with neural activation (compared to resting state requirements) are small and that the hyperemic response of cerebral blood fow is infuenced by products of non-oxidative metabolism such as lactic acid (Lin et al. [2010](#page-61-22)). Functional congestion plays a direct -20

ЗO

31

32

33



<span id="page-24-0"></span>**Fig. 4** Neuron action potential and corresponding power consumption based on H–H model (Wang and Wang [2018b](#page-63-19))

35

Time / ms

36

37

38

39

40

34

role in neuronal information processing. Local blood vessels dilate due to increased blood fow, increased blood volume and increased local vascular pressure. The anatomy shows that neurons and glial cells are located near blood vessels, so the dilation of blood vessels causes the deformation of cell membranes. Membrane deformation caused by these mechanical force signals such as blood fow, blood volume, pressure and local dilation and contraction of blood vessels can regulate ion channels sensitive to mechanical force, thus altering neural activity (Moore and Cao [2008](#page-61-7); Lin et al. [2010](#page-61-22); Peppiatt and Attwell [2004\)](#page-62-23). For example, in the somatosensory cortex, sensory stimulation induced an increase in the diameter of the arterioles, a mean net increase 10 ∼ 15*𝜇m*, with some experimental data suggesting that the dilation was even greater  $15 \mu m$  (Moore and Cao [2008\)](#page-61-7). According to Poiseuille's theory, a 23% reduction in blood vessel diameter results in a threefold decrease in blood fow, while an increase in blood vessel diameter results in a four- or fvefold increase in blood fow. In addition, blood fow plays a dominant role in the brain's temperature. Local congestion lowers brain temperature and reduces the efect of heat due to neural activity (see Fig. [5](#page-25-0)) (Moore and Cao [2008](#page-61-7)).

At the molecular level, glial cells, the most abundant cell type in the brain, have long been thought to function only as support and nourishment for neurons. But glial cells have recently been shown to play a crucial role in neural activity. It not only afects the growth and development of neurons, but also may directly participate in the transmission process of nerve signals. Astrocytes are the most abundant in glial cells, and glycogen in the brain is mainly stored in astrocytes. Magistretti et al. Astrocyte lactate shuttle hypothesis, ANLSH (Pellerin and Magistretti [1994](#page-62-25)), indicating that astrocytes play a crucial role in nerve energy metabolism and hemodynamics. Currently, the role of brain glycogen is not completely clear, but a large number of studies (Pellerin and Magistretti [1994](#page-62-25); Brown [2004a,](#page-58-21) [b](#page-58-22); DiNuzzo et al. [2012](#page-59-13)) have shown that brain glycogen is a very important brain energy reserve and the material basis of brain activity.



<span id="page-25-0"></span>**Fig. 5** Cellular/molecular and haemodynamic changes caused by lactate products and calcium waves in astrocyte end feet (Zheng et al. [2014](#page-65-11))

The increased activity of  $Na^+$ ,  $K^+$  and  $Ca^{2+}$  channels increased ATP consumption and stimulates ATP production. As shown in, during glycolysis of glucose in glial cells, one molecule of glucose produces two molecules of lactic acid and two ATPs, which are just used for glutamate uptake and metabolism. A molecule of glutamate, along with three Na<sup>+</sup>, is taken into the astrocyte via the cotransporter, which activates the Na<sup>+</sup>/K<sup>+</sup> pump to restore the osmotic gradient. Glutamate enters astrocytes and is converted into substances such as glutamine, which is sent back to neighboring neurons. Moreover,  $Na<sup>+</sup>$  uptake is a passive process; activation of the  $Na<sup>+</sup>/K<sup>+</sup>$  pump and conversion of glutamate to glutamine are energy consuming processes (each consuming 1 ATP) (Figley and Stroman [2011](#page-59-14)). Lactate, the product of glycolysis, is transported out of the cell through the lactate shuttling protein on the cell membrane, which is then absorbed by neighboring neurons, producing 36 ATP after oxidative metabolism (Sokoloff [2008\)](#page-62-24). It can be seen that although increased blood fow is mainly due to increased concentration of lactic acid, a product of non-oxidative metabolism. Most (98%) of the energy requirement comes from oxidative metabolic pathways (Eikenberry and Marmarelis [2015\)](#page-59-15). So when the activity of brain tissue increases, the corresponding energy demand rises rapidly, but the brain blood fow does not change enough, resulting in a lack of blood sugar. At this point, brain glycogen is rapidly digested to meet the energy needs of brain tissue activity (Zheng et al. [2014\)](#page-65-11).



<span id="page-26-0"></span>**Fig. 6** Neuron activity under diferent energy supply states (Wang and Wang [2018b\)](#page-63-19)

The above are the molecular mechanisms of negative power components caused by local congestion in the brain and the mechanical mechanisms of increased blood fow caused by vascular dilation. Thus, the above on vascular smooth muscle relaxation caused by diferent kinds of nerve chemical reactions lead to increased blood fow to the brain, in turn, control the neural activity in the brain is a neurochemistry, neural signal transduction, cell rheology of nonlinear viscoelastic mechanics, blood, blood vessels, non-Newtonian fuid mechanics and damage mechanics of multidisciplinary crossover study, There are a lot of mechanics problems which need to be further explored and solved by mechanics workers (Zheng et al. [2016](#page-65-12)).

From the perspective of the circuit, the power emitted by the voltage source represented by Nernst potential is negative, indicating that the work done by other components in the circuit to the voltage source is mainly done by the surrounding glial cells and neurons through various mechanical forces. This stage of capacitor discharge releases the energy stored in the capacitor (brain glycogen in the brain). Capacitance in the H–H model corresponds to the cell membrane of the neuron, while sodium ions enter the neuron through ion channels under the action of potential gradient caused by the potential diference between inside and outside the membrane. This can be regarded as the storage of cell membrane provides energy for the inward fow of sodium ions, which exactly corresponds to the discharge of the capacitor. It can be seen from the above discussion that the movement of ions corresponds completely to the circuit model (Wang and Wang [2018b](#page-63-19)).

Fig. [6](#page-26-0) correspond to the enlargement of the upper and lower graphs on the right. It can be seen from Fig. [6](#page-26-0) that when the sodium ion pump cannot provide stable Nernst potential, the activity of subthreshold membrane potential is mainly energy consumption. When the DMN is coupled with the resting state network, this explains why about 95% of the energy expenditure is devoted to the brain's intrinsic, spontaneous activity, whereas the neural energy expenditure caused by task stimulation usually accounts for only 5% of the brain energy expenditure under the resting state condition.

In short, there are many coding theories in the feld of neural coding such as frequency coding, rhythm coding, time coding, phase coding and other coding methods. However,

they can only be applied to local, single or few neuron systems and isolated or closed neural networks, while the actual neural coding must be the global neural coding of a large range, various levels of coupling and the interaction of various related brain regions. Energy coding studies neural coding from the energy characteristics of neuron activity, which can reflect global, economic and high efficiency advantages (Yuan et al. [2021;](#page-65-2) Wang and Zhu [2016](#page-63-16); Peng and Wang [2021;](#page-62-10) Wang et al. [2009](#page-63-27), [2015b,](#page-63-30) [2017a](#page-63-22), [2018a](#page-64-10), [b,](#page-64-11) [2019b;](#page-64-12) Wang and Wang [2014](#page-63-31), [2020;](#page-63-25) Wang and Zhang [2011;](#page-63-28) Zhu et al. [2018b](#page-65-13)). Objectively, the inductance element presented in the W–Z model not only theoretically proves the contribution of electromagnetic feld efect to signal conduction and information encoding, but also provides a theoretical basis for predicting the existence of an unknown magnetic substance in the brain. In the 10 years since we frst proposed the embryonic neural energy model in 2006 (Wang and Zhang [2006](#page-63-32)), we have seen an important academic paper published in Nature Material that experimentally demonstrated the existence of a magnetic protein called MagR in the brain. Navigation for direction and orientation in path exploration (Qin and Xie [2016\)](#page-62-21).

In short, complex changes in action potential and subthreshold membrane potential can reveal rich dynamic properties of neuronal fring activity. Our study not only reveals the equivalence of two kinds of diferent neuron models, but also discovers the nature and law of neuron fring activity behind the rich dynamic properties and a large amount of experimental data. From a reductionist point of view, this is an important contribution to neuroscience.

#### **2.5 Neural energy mechanism of cluster release and W–Z neuron model**

Cluster fring is also one of the common fring patterns of neurons. However, does cluster release also have the above properties? So far, the cellular mechanism of cluster release and its biological signifcance remain unclear. Therefore, from the perspective of energy, we proposed a neural energy calculation method based on The Chay cluster release model, and analyzed the ion current and its energy consumption (power) per unit time under two conditions with and without stimulation. Studies have found that the power becomes negative during the depolarization process of cluster release, which is consistent with the research results of W–Z neural energy model (Zhu et al. [2019](#page-65-14)). Furthermore, it was found that the energy consumption of neurons in cluster fring mode was minimal, especially in the spontaneous state without stimulation. The total energy consumption of cluster fring for 30 s was equivalent to the biological energy consumed by a single action potential. These results suggest that low-energy cluster firing is an energy-efficient way of neural information transmission that follows a brain strategy that minimizes energy. The energy efficiency of neural information transmission is considered to be an important constraint of neural information processing. It is usually measured by the energy consumed per unit of information. Most previous studies have focused on the energy efficiency of individual action potentials. However, neural information is more likely to be encoded by a spike sequence rather than a single spike. So far, it is not clear how energy efficiency depends on the discharge pattern of spike sequences. We simulate high, medium, and low frequency emission patterns based on the Chay neuron model and examine their energy efficiency. The results show that medium frequency mode is more efective than high and low frequency mode. Sparse cluster scattering (SBF) mode is the most efective, because it consumes the least energy and transmits the same amount of neural information as the high-frequency mode which consumes more energy. The SBF model minimizes energy consumption by

balancing the potential energy stored in the depletion ion concentration gradient. In addition, the combination of sparse cluster release (SBF) with a single spike maximizes the neural information carried by the SBF model, thus improving the energy efficiency. In conclusion, the nervous system may prioritize limiting energy costs over maximizing informa-tion to achieve higher energy efficiency (Zhu et al. [2020](#page-65-15)).

## **3 Equivalence between H–H model and W–Z model in structural network**

It has been known that the neural activity of the brain and the operation of the brain are subject to the principle of minimization of energy and maximization of signal transmis-sion efficiency (Laughlin and Sejnowski [2003b](#page-60-24)). This rigorous working mode of the brain has been proved by a large number of experimental data. This working principle governs the activity of the entire brain, but its role and contribution to cognition need to be further understood. In order to fnd the intrinsic correlation and essential connection between cognitive behavior and energy information, it is necessary to construct a series of structural and functional neural networks, and under what conditions the transformation from structural network to functional network should be solved, which involves large-scale neuroscience modeling and analysis.

## **3.1 Derived from the defnition of large‑scale neuroscience models based on analytical dynamics**

Large-scale neuroscience model is based on neural energy model, and the construction of neural energy model is derived from the theory and method of analyzing dynamics. Its purpose is to quantitatively obtain the global information of brain neural activity through the correspondence between nerve energy and membrane potential, feld potential and network emission rate. Since the global information of the brain can be converted into energy for study and analysis, neural energy coding forms the cornerstone of large-scale neuroscience models. Its defnition is as follows:

(1) It can analyze and interpret both local and global neural activities of the brain. At the same time, it can also be used to construct, analyze and describe the experimental phenomena of neuroscience at various levels from molecular to behavioral, and can establish the global brain function model on the combination of various levels, so that the calculation results at various levels are no longer unusable, contradictory and irrelevant. (2) Global brain function model can be used to solve the scalp EEG and conversion relation between cortical potentials. It is still difficult to describe large-scale neuronal interactions throughout the brain. It is currently difficult to record damage in multiple brain regions simultaneously. Although EEG and MEG can sample neuronal activity from various regions of the brain, it is very difcult to estimate cortical interactions on the basis of these extracranial signals, the main obstacle being the lack of a theoretical tool that can efectively analyze cortical to cortical interactions in high dimensional space. In addition, there is a lack of a conversion relationship between scalp electroencephalogram and cortical potential. One promising approach to these extremely difficult problems is the neuro-energy theory. (3) Can be used for insight and analysis of experimental data behind the essence and regularity of problems (such as oxygen dependencies between signals and state of consciousness; the meaning and the content of the EEG of brain waves, etc.). (4) If a global brain activity

model can under the condition of the degraded to explain the function of the DMN and static state interest, energy consumption, and under the task induced can explain the formation of the transition from the default mode to the cognitive network and the corresponding energy conversion, such a model is a global neural model. And the tunable parameters of the global model of brain function must be few and simple.

### **3.2 Comparison of network calculation results based on class H–H neuron model**

At the level of individual neurons, we have proved the equivalence of H–H and Z–W neuron models, but we still need to prove whether H–H and Z–W neuron models are also equivalent at the level of networks. If equivalent, the W–Z neuron model can be used to study cognition and behavior. As described above, W–Z model is much simpler than H–H model in terms of calculation and has the advantage of not losing the main information. Therefore, when studying the macro behavior model related to cognition, details such as ion concentration and ion current of synaptic connection between neurons need not be considered. Their dynamic characteristics are focused on the macro expression of cognitive and behavioral coding patterns.

A simple structural neural network is constructed according to the connection mode of neurons in functional columns of cerebral cortex (Fig. [7](#page-29-0)).

In the fully connected neural network structure shown in Fig. [7](#page-29-0), each neuron is composed of H–H model, and the coding mode of various parameter conditions and the behavioral response of the network are simulated through two diferent index systems (Wang et al. [2015b\)](#page-63-30). The research objective is to explore the relationship between synchronous discharge activity and network parameters of structural neural network, and to investigate the equivalence of H–H model and W–Z model under the same network structure as shown in Fig. [7.](#page-29-0)

According to equivalent circuit Fig. [3](#page-22-0) of H–H model, its diferential equation and variable conductance of ion channel are expressed by Eqs.  $(10)$  $(10)$  $(10)$  and  $(11)$ .

In the circuit model of H–H equation, the total power of the fully connected network in Fig. [7](#page-29-0) can be obtained according to the total energy Eq.  $(12)$  $(12)$  $(12)$ – $(15)$  $(15)$ . Where  $IV<sub>m</sub>$  is the external

<span id="page-29-0"></span>**Fig. 7** Schematic diagram of a fully connected structured network (Wang and Wang [2014](#page-63-31))



energy provided to the circuit system,  $(i_{Na}E_{Na} + i_KE_K + i_jE_l)$  is the energy supplied by the voltage source represented by the Nernst potential,  $V_m(i_{Na} + i_K + i_l)$  is the energy in the potential diference between the inner and outer membrane. Distributed in these neurons action potential in the process, if they do not consider the change of cell membrane permeability of consumed energy, the energy involved respectively carries blood oxygen and glucose for energy provided by the neuron, inside and outside the cell membrane potential and ion pump in against the concentration of energy in the transport ion TiDuCha when consumed by biological energy (ATP), Increased glucose consumption due to brain stimulation is mainly caused by activation of the sodium–potassium ATP pump (Churchland et al. [2002](#page-58-4); Rabinovich and Huerta [2006;](#page-62-2) Jiang et al. [2020](#page-60-5)). The frst two describe the relationship between subthreshold neurons and biological energy, and the assisted difusion of ions through ion channels along the diference in ion concentration does not consume energy. However, from a dynamic perspective, the sum of these three types of energy is equal to the total energy in the circuit system of H–H model during the transformation of subthreshold neurons into functional neurons. The first two types of energy correspond to the  $IV_m$  and  $V_m(i_{\text{Na}}+i_{\text{K}}+i_l)$  in the circuit respectively, so the energy provided by the Nernst potential  $(i_{Na}E_{Na} + i_KE_K + i_jE_l)$  is equal to the biological energy consumed by the ion pump. In fact, in this process, the sodium–potassium pump is constantly transporting ions up and down the concentration gradient, which directly depletes biological energy so that one ATP can pump out three sodium ions and two potassium ions. This also confrms the existence of an ion pump, which provides a steady energy potential through the continuous transport of ions, thus providing energy for neural activity. Thus, we can calculate the power consumed by the ion pump by using the power of the voltage source represented by Nernst potential in the circuit shown in Fig. [7](#page-29-0), that is, the nerve energy consumed by neuron activity is shown in Eq. [\(16](#page-23-0)).

The third in the minus sign refers to the circuit shown in Fig. [7,](#page-29-0) the direction of the voltage source and current  $E_{\text{Na}}$  in contrast to the  $i_{\text{Na}}$ ,  $E_K$ ,  $E_l$  and  $i_K$ ,  $i_l$  potassium (sodium current towards inside cells, and electric current and leakage flow). For an action potential, we can use the above equation to calculate the amount of nerve energy expended. The calculated parameter values are determined by Zhu et al. [\(2018b\)](#page-65-13). In fully connected neural networks, the dynamic properties of each neuron are derived from the above H–H model, so the network structure is strictly defned on the basis of neurobiology. The anatomical structure of neuronal connections in the cerebral cortex indicates that the neural network inside any brain region is a fully connected structural neural network if functional connections are not considered, such as cortical functional columns (Gazzaniga et al. [2002](#page-59-16)). If the cortical functional column is regarded as a closed system, and a local region within the closed system is intercepted for simplicity, the network structure of this region can be expressed by a structural neural network consisting of 20 neurons as shown in Fig. [7](#page-29-0). To understand the energy coding mode of cortical neural network under diferent parameters, the connection of neural network is simplifed to some extent. The connections between neurons in the fgure indicate that they are coupled to each other, but the coupling strength between any two neurons is not the same, and the coupling strength between two neurons is not symmetrical. According to the principle of synaptic plasticity, statistical data from experiments show that the range of synaptic coupling strength between neurons follows a uniform distribution (Rubinov et al. [2011\)](#page-62-26). That is, the following matrix is satisfed:

$$
W = \begin{bmatrix} w_{1,1} & w_{1,2} & \dots & w_{1,n} \\ w_{2,1} & \ddots & & w_{2,n} \\ \vdots & & \ddots & \vdots \\ w_{n,1} & w_{n,2} & \dots & w_{n,n} \end{bmatrix} .
$$
 (17)

 $w_{i,j}$  represents the coupling intensity from the first neuron to the first neuron, and represents the number of neurons.

$$
lin(t) = W \times Q(t - \tau)',
$$
  
\n
$$
I(t) = lin(t) + lex(t).
$$
\n(18)

Put  $I(t)$  into Eq. [\(1\)](#page-15-1) to obtain the membrane potential  $V_{im}(t)$ , and calculate the power consumed  $P_i(t)$  by the neuron through Eq. ([3](#page-15-2)). Where  $I(t)$  represents the sum of current stimuli received by neurons at any time,  $I_{in}(t)$  represents the interaction between neurons, and  $I_{ext}(t)$ represents the infuence of external stimuli on neurons.

$$
Q(t - \tau) = [Q_1(t - \tau), Q_2(t - \tau), \dots, Q_j(t - \tau), \dots, Q_n(t - \tau)].
$$
\n(19)

Moment of Qi on behalf of each neuron's action potential distribution state, for the sake of simplicity it is simplifed to the pulse of 0 or 1, when in the resting potential is 0, action potential constantly to 1, which said a neurons after issuing an action potential to another neuron stimulation time interval, which is exciting transfer delay, the range of possible values is uniformly distributed.

We use the traditional maximum correlation coefficient of synchronization index and the novel negative energy ratio to measure the synchronization activity of the network (Zhu et al. [2018b\)](#page-65-13).

The average maximum correlation coefficient is defined as follows:

$$
\rho_{mean} = \frac{\sum_{i=1}^{N} \max(C_{i,1}, C_{i,2}, \dots, C_{i,j}, \dots, C_{i,n})}{N} \quad (i \neq j),
$$
\n(20)

where  $C_{i,j}$  is the Pearson correlation coefficient between the membrane potential of the first *j* neuron and the first neuron. If the Pearson correlation coefficient between any two neurons is closer to 1, it indicates that the synchronization between these two neurons is greater. Previous studies have found that two or more oscillating groups will appear in steady state if the network achieves synchronization under transient stimuli. It can be seen that when the index of maximum correlation coefficient is adopted, the closer its value is to 1, the stronger the synchronization of neurons within the oscillating group is, that is to say, the closer the network state is to the common synchronization phenomenon of multiple groups. When the value is closer to 0, it indicates that the synchronization of neurons within the oscillating group is weaker, that is, only a few neurons are synchronized.

Negative energy ratio is defned as follows: the ratio of the absolute value of negative energy consumed by the whole neural network from moment 0 to moment 0 to the sum of the absolute value of positive and negative energy.

$$
\alpha(t) = \frac{E_{negative}}{E_{positive} + E_{negative}} \times 100\%,\tag{21}
$$

$$
E_{negative} = \sum_{i=1}^{n} \int_{o}^{t} P_i(t) \cdot \text{sgn}(-P_i(t))dt,
$$
\n(22)

$$
E_{positive} = \sum_{i=1}^{n} \int_{o}^{t} P_i(t) \cdot \text{sgn}(P_i(t))dt,
$$
\n(23)

where  $P_i(t)$  represents the power consumed by the neuron at the moment, and the integration  $P_i(t)$  in [0, *t*] represents the energy consumed by the neuron during [0, *t*].  $sgn(x) = \begin{cases} 1, & x > 0 \\ 0, & x \le 0 \end{cases}$  is symbolic function,  $E_{negative}$  and  $E_{positive}$  respectively represents the negative energy and positive energy consumed by the whole neural network in [0, *t*].

The synchronicity of network activity is measured by the two indexes of mean maximum correlation coefficient and negative energy ratio. The larger the two indexes are, the stronger the synchronicity of network activity. Figure [8](#page-33-0) compares the equivalence of H–H model and W–Z model through the changing relationship between the number of neurons and nerve energy:

The fgure above shows that the greater the number of neurons in the network, the greater the demand for energy for synchronous oscillation of the network, that is, more energy reserves are required, and the negative energy ratio just refects the energy stored in network activities. Like the traditional correlation coefficient method, the negative energy ratio can also reveal the synchronization state of the network, and both models show a positive correlation between the number of neurons and the network synchronization and the negative energy ratio. On the other hand, the negative energy ratio does not saturate quickly with the increase of the number of neurons, so that the number of neurons in the network can be more efectively distinguished, which is also one of the advantages of energy coding. It should be especially emphasized that, through the comparison of the left and right fgures, it can be clearly seen that for an identical fully connected neuron network, the maximum correlation coefficient and negative energy ratio of H–H model and W–Z model are almost the same with the increasing number of neurons.

Figure [9](#page-34-0) compares the equivalence of H–H model and W–Z model through the changing relationship between neuron coupling intensity and nerve energy.

Due to the coupling strength between the neurons afecting their information interaction, and the process relies on energy is required to complete, the coupling strength is larger, the synchronicity activities of the network is more intense, the stronger the information interaction between neurons, the higher the demand for energy, so you need a high energy reserve, namely refection is higher than the negative energy. Both models show that the coupling strength is positively correlated with network synchronization and negative energy ratio. By comparing Fig. [9](#page-34-0) left, right, and W–Z and H–H model for a same full connection neural network with increasing coupling strength between neurons, their negative energy, though there are some error, but also are increasing and the trend of growth is nearly the same, and both maximum correlation coefficient is exactly the same.

Figure [10](#page-35-0) compares the equivalence of H–H model and W–Z model through the change relation between neuron excitation transmission delay and nerve energy.

The longer the delay time for presynaptic neurons to release excitatory neurotransmitters to postsynaptic neurons, the weaker the correlation between the activities of presynaptic and postsynaptic neurons, the weaker the synchronous activity of the whole network, and the lower the demand for energy, thus the less energy stored in network activities, and the lower the corresponding negative energy ratio. The results based on the two



<span id="page-33-0"></span>**Fig. 8** Model based on W–Z (**b**) and H–H model (**a**) (Zhu et al. [2018b\)](#page-65-13) of the negative energy ratio and all curves of maximum correlation coefficient on the number of neurons

diferent models show similar decreasing curves with the increase of excitatory transfer



<span id="page-34-0"></span>**Fig. 9** Model based on W–Z (**b**) and H–H model (**a**) (Zhu et al. [2018b\)](#page-65-13) of the negative energy ratio and both maximum correlation coefficient curves of coupling strength

delay. Through the comparison of the left and right fgures, it can be clearly seen that for an identical fully connected neural network, the maximum correlation coefficient and negative energy ratio of H–H model and W–Z model are almost the same with the increase of signal delay time.



<span id="page-35-0"></span>**Fig. 10** Model based on W–Z (**b**) and H–H model (**a**) (Zhu et al. [2018b](#page-65-13)) of the negative energy ratio and both maximum correlation coefficient about excited transfer delay change curve

In addition, in the two published papers (Cohen [2017](#page-58-15); Zhu et al. [2018b](#page-65-13)), we also carried out a large number of studies on the network neural coding in the above three cases respectively by H–H model and W–Z model, and found that the coding modes are almost the

same under diferent parameters. Combined with the above two diferent synchronization indexes, the dynamic characteristics of synchronization are identical under the three conditions of increasing number of neurons, coupling strength and signal delay time. This proves that H–H model and W–Z model are equivalent at the level of structural neural network.

The results show that H–H model is suitable for the modeling, analysis and calculation of simple and local neural networks with a small number of neurons, while W–Z model is suitable for the modeling, analysis and calculation of complex networks with a large number of neurons. In particular, due to the correspondence between neural information and neural energy, and the advantages of simple calculation and no loss of main information, the neural energy theory and Z–W neuron model are very potential research methods that can be used to construct large-scale neuroscience models (Wang and Zhu [2016\)](#page-63-16).

## **4 Application of W–Z model and neural energy method in functional neural networks**

In fact, using neural energy method to study functional neural networks is not only very efective, but also can give scientifc explanations for some experimental phenomena that neuroscience has not been able to make clear up to now from a quantitative perspective.

#### **4.1 Neural mechanisms of cerebral hemodynamic phenomena**

Neuroscientists have long been puzzled by the phenomenon of brain hemodynamics. The so-called hemodynamic phenomenon is that blood fow to the nervous system always increases signifcantly about 7–8 s after the cerebral cortex is stimulated (Fox and Raichle [2007a;](#page-59-6) Peppiatt and Attwell [2004](#page-62-23)). Provided according to the literature, the current neuroscience community does not give an efective theory to the phenomenon of neural mechanisms of scientifc and reasonable explanation (Moore and Cao [2008;](#page-61-7) Maandag [2007](#page-61-23)), also didn't see related from neural modeling to calculate these two aspects of computer simulation and the experimental phenomenon that the hemodynamic phenomenon research reports. In order to simulate the phenomenon of blood fow delay in fMRI, we constructed a multi-level neural network based on W–Z neuron model and used the method of energy coding to give the neural energy changes that produce hemodynamic phenomena. The hemodynamic phenomenon of a large increase in cerebral blood fow in fMRI with a lag neuron activation area of 7–8 s is reconstructed quantitatively. Since this study is based on the negative energy mechanism of neuronal activity that we have revealed (Wang et al. [2015a\)](#page-63-17), we predict that the nature of brain hemodynamic phenomena is the existence of negative energy mechanism in neural activity (Peng and Wang [2021](#page-62-10)). Recently, based on the anatomical structure of the visual nervous system, we constructed a large-scale neural network model consisting of various visual areas for visual information processing using neuron energy model, using which we successfully simulated the hemodynamic phenomenon of the visual system in fMRI (Peng and Wang [2021\)](#page-62-10).

The signifcance of the above study lies in that it can provide a new vision for exploring the dynamic mechanism of hemodynamic phenomena in the future, and thus provide important scientifc support for establishing the framework of brain theoretical research in the future.

#### **4.2 Application of neural energy coding in brain navigation**

Spatial cognition and representation are critical to animal's survival, such as navigating to fnd hidden food or to avoid danger. It is believed that animal can form a cognitive map in the brain to solve spatial tasks. The concept of cognitive map was hypothesized by Tolman (Tolman [1948](#page-63-33)). Until 1971, O'Keefe and Dostrovsky discovered the frst neural basis of cognitive map (O'Keefe and Dostrovsky [1971](#page-61-24)). They reported a type of neuron in the rodent hippocampus that emitted spikes whenever it ran through a specifc set of spatial locations. This type of neurons is now termed as "place cell," and the particular subset of locations in the arena to which the place cell respond is called a "place feld". Diferent place cells are corresponding to diferent place felds, which varies in size, shape, and center. Thus, the environment is represented by the population of place cell entities in the hippocampus to support the concept of cognitive map (Wilson and McNaughton [1993\)](#page-64-20). Animals typically recruit diferent place cell populations when navigating in diferent environments, suggesting a remapping of spatial representations in the hippocampus across environments (Alme et al. [2014\)](#page-58-23). Other than spatial properties, the place cell also showed a striking temporal feature which is called phase precession, meaning the spike timing is progressively advancing relative to the local theta oscillation phase in the hippocampus as the rat pass a typical place feld (O'Keefe and Recce [1993](#page-61-25)). This phenomenon extends the place cell code from a pure spatial domain to a spatial–temporal domain. Thus, the hippocampus plays an important role not only in spatial representation, but also in spatial memory. Place cells are thought to be important for spatial tasks such as path fnding, and can also act as route planners.

However, the place cell is only the frst piece of the spatial computation system in the brain. It is a component of a more general circuit to represent the spatial information in a dynamic way (Moser et al. [2008](#page-61-26)). The place cell has particular spatial selectivity. So it is natural to ask where the spatial information received by place cell come from. The medial entorhinal cortex (MEC) is an important upstream of hippocampus and has attracted much attention during recent years. A similar type of neuron has been found in MEC called grid cell, which also responds to the location of the animal like the place cell, but multiple fring felds appear in a periodic fashion, forming a triangular grid pattern covering the entire arena (Sargolini et al. [2006](#page-62-27); Hafting et al. [2005\)](#page-60-25). It is hypothesized that projections from the grid cells to the hippocampal place cells support the generation of the place felds. From dorsal to ventral MEC, the space between the vertices of grid pattern increases in a modular manner (Sargolini et al. [2006](#page-62-27); Hafting et al. [2005;](#page-60-25) Fyhn et al. [2004\)](#page-59-17), while the positions of the grid vertices vary randomly, but each grid maintains a stable spatial phase (locations on the horizontal plane). These spatial features of the grid cell population are believed to form a global spatial coordinate system in the brain, which may be informatively redundant and robust, suggesting that grid cells are playing a role in path integration (Hafting et al. [2005;](#page-60-25) Barry et al. [2007](#page-58-24)), meaning tracking the locomotion of the animal. Furthermore, there is evidence that the fring felds of grid cells persist in the absence of sensory input, suggesting that the perception of self-movement is the main driver of grid cell activity, further implying that animals can continuously track and update their self-location in the environment through the grid coordinate system (Hafting et al. [2005](#page-60-25); McNaughton et al. [2006](#page-61-27)). Grid cells together with place cells constitute a quantitative spatial–temporal representation system for representing locations, paths, distance and associated behavioral and episodic memories.

Most of the understanding of the brain's navigation system came from experiments or theoretical models in two-dimensional (2D) space. The studies often conduct on fat, horizontal planes, while the actual world is three dimensional (3D), and all animals, more or less, need to navigate in 3D space. However, little is known about how 3D space is encoded in the brain. Do the place feld and grid felds have 3D properties? The regularly distributed fring felds of grid cells on the 2D plane constitute a metric system for navigation, and it is a difcult question how this hexagonal pattern can be generalized to volumetric space. Similarly, conclusion has not been made about the 3D counterpart of the circular or elliptical place feld on the 2D plane. Evidence has suggested the grid cell fring patterns on the one-dimensional linear track can be treated as cutting the 2D grid lattices (Yoon et al. [2016\)](#page-64-21), implying the grid cell representations may be global and the dimensionality could be higher than that of the experimental setup (Finkelstein et al. [2016](#page-59-18)). However, recordings from rodents do not support this hypothesis (Hayman et al. [2011,](#page-60-26) [2015](#page-60-27)). In these experiments, grid cells were recorded when rats navigated in 3D space or at least a section of volumetric space, such as on a helix stairs (Hayman et al. [2011\)](#page-60-26), titling or even vertical walls (Hayman et al. [2015;](#page-60-27) Casali et al. [2019\)](#page-58-25). The results show that the grid field maintains its horizontal character but is vertically elongated on multi-layered helix and is almost indistinguishable from the grid feld on the tilting plane from that on the horizontal plane. Place cell recordings from free-fying bats (Yartsev and Ulanovsky [2013\)](#page-64-22) and grid cell recordings from crawling bats (Yartsev et al. [2011](#page-64-23)) have also been published, indicating the existence of volumetric place felds in 3D space and hexagonal lattice patterns in the horizontal plane in such mammals. One theoretical analysis suggests that a face-centered cubic lattice is optimal for maximizing 3D spatial resolution (Mathis et al. [2015\)](#page-61-28). Another fact which should be taken into account is that rodents and bats have diferent natural movement behaviors, so their spatial encoding strategies are not necessarily identical. And conclusions about spatial representations of diferent species should be drawn with caution.

The complexity of the spatial computation problem forces us to consider an alternative besides electrophysiological experiments—neurodynamical modeling, since neural systems reveal abundant dynamical properties, such as oscillation and attractor dynamics. Regarding this, two main categories of models have been developed to reveal the mechanisms that form the grid cell activity, such as periodic fring felds, phase precession, and invariance to velocity changes. One is the oscillatory interference (OI) model and the other is the attractor network model (Giocomo et al. [2011](#page-59-19)). The frst type consists of several oscillator pairs. Inside each pair, there is a baseline oscillator and a velocity-modulated oscillator. The frequency diference between two oscillators is infuenced by the speed and direction (velocity) of the animal's movement. And diferent pairs corresponding to diferent allocentric directions. Thus, each pair of oscillator continuously tracks the distance the animal travels in a fxed direction by the phase diference of the two oscillators. And multiple oscillator pairs are integrated to form a grid pattern (Burgess et al. [2007\)](#page-58-26). The attractor network model arranges the neurons on a sheet and each neuron excites its neighbors and inhibits proximal neurons. The network can generate attractor states to represent position. Combining specifc input cues and structured recurrent connections, the activity bumps of the grid cell layer move in response to the animal's motion, which guarantees the periodic pattern can be formed for each neuron (Burak and Fiete [2009\)](#page-58-27).

OI model is often used to simulate the grid cell activity in 2D space. Classic OI model is consisted of several pairs of oscillators. Each pair has a somatic oscillator and a dendritic oscillator. Frequency of somatic oscillator is determined by the background theta rhythm and frequency of dendrite oscillator is modulated by velocity input on the basis of theta, which has an increment proportionate to the projection of velocity to the preferred

direction, which is assigned to each oscillator pair. When the preferred directions corresponding to the oscillator pairs are separated by  $60^\circ$ , the hexagonal pattern of grid cell fring can be generated. Based on this idea, a gravity-modulated OI model has been proposed to generate grid cell activity in 3D space (Wang et al. [2021c](#page-64-24)). The fxed preferred directions in 2D space of the classic model are changeable in this new model, which can be rotated onto the local plane. The rotation operation is presumably achieved by receiving the head direction signal with reference to gravity. The rotation axis is the intersection line of horizontal plane and body plane. By this method, the OI model can be modifed to simulate the grid cell activity in 3D space for crawling animals such as rodents. Simple as it is, the results can account for the known experimental phenomena found in rats, and the model also simulate and make testable predictions of grid cell activity on novel surfaces in 3D space.

Figure [11](#page-39-0) shows the grid pattern on the multi-layer helix stairs, which was used to recording spatial neurons in experiment (Hayman et al. [2011\)](#page-60-26). In this fgure, three "stripe patterns" (a, Stripe pattern 1–3) is the direct results of three preferred directions and the grid pattern can be formed by the threshold product of the stripe activities (a, bottom right). Projection of the grid patterns on the ground (top view) suggests the grid cell seems not sensitive to vertical locations, and the pattern is similar on each layer of the helix track (b). The histogram of grid fring locations on every layer (from bottom to top: lowest coil



<span id="page-39-0"></span>**Fig. 11** Grid fring analysis on helix track

to highest coil of the helix) in c further confrm this observation. The fring locations are represented by angles on each layer. Note that this phenomenon is a key fnding in experiment (Hayman et al. [2011\)](#page-60-26). The model can also generate grid patterns with diferent orientations and spatial periodicity (d).

Grid patterns on other complex surfaces in 3D space were also simulated, and it turns out that the grid cell activity can be trajectory-dependent, i.e., diferent trajectories can result in diferent grid patterns (Wang et al. [2021d\)](#page-64-25). As shown in Fig. [12](#page-40-0), the navigating terrains are a multimodal surface, a saddle surface and a unimodal surface. The two upper rows of Fig. [12](#page-40-0) illustrate the stripe and grid (low right) patterns of random moving while the two lower rows of Fig. [12](#page-40-0) are the same patterns but generated by regular trajectories, which are zigzag paths for multimodal and saddle surface and a spiraled path from top to bottom for unimodal surface. The stripes are very fuzzy and the grids are almost vanishing for random moving while the patterns generated by regular moving are quite inerratic. Mathematical analysis indicates that the condition for trajectory-independent grid pattern is quite rigorous, which demands the rotation of preferred direction vectors defned on every location on the surface forming a conservative feld, and merely the horizontal and tilting plane can satisfy the condition.

In 2D space, it is generally believed that grid cells are involved in the formation of the place feld because hippocampus receive primary input from MEC. The place cells are likely integrating multiple inputs from grid cells. Studies have shown that hippocampal pyramidal neurons perform linear summation of synaptic inputs (Cash and Yuste [1999](#page-58-28)). An elegant mathematical model inspired by Fourier Transformation has been proposed to generate a Gaussian-type place feld by linearly summing dendritic inputs from several grid cells with designed synaptic weights (Solstad et al. [2006](#page-62-28)). However, this model applied a sinusoidal function defned directly on the 2D plane to represent grid felds which did not consider the actual trajectory of the animal, and it did not suitable for 3D navigation either. Followed the gravity-modulated rotation scheme of the preferred directions in grid cell model, the place cell activity can be similarly modeled.

<span id="page-40-0"></span>

**Fig. 12** The trajectory-dependency of grid felds on smooth surfaces in 3D space

Grid patterns with diferent orientations and wavelengths are frst generated on the surfaces in 3D space, then these patterns are summed by the similar designed synaptic weights (Xu et al. [2022b](#page-64-26)).

Examples of simulation results are showed in Fig. [13](#page-41-0) (a: helix, b: sphere, c: multimodal surface, d: saddle surface). Trajectories were also generated by random movement. Each sub-fgure contains four panels. The frst column shows the place feld on the manifold. The fring rates are color-coded and warmer colors indicate higher fring rates. In the upper left panel, the rate is calculated for each position along the animal's trajectory, while the lower left panel shows only the rates above the threshold (half of the maximum value) for better illustration. The upper right panel in each subplot shows the weight values of each the grid cell input, which are color-coded. The lower right panel also shows the projection of place feld on the horizontal plane. The feature that place cell fres at almost the same location in each layer as the rat navigates the helix is consistent with experimental recordings (Hayman et al. [2011\)](#page-60-26). Future experiments could be set up similar apparatus to verify the place feld patterns predicted by the model.



<span id="page-41-0"></span>**Fig. 13** Place felds on complex 2D manifolds in 3D space



<span id="page-42-0"></span>**Fig. 14** Place cell network model encoding 3D spatial information constructed by neural energy method (Wang et al. [2018b\)](#page-64-11)

The aforementioned neurodynamical models focused on rodents, which are not a volumetric moving animal. The 3D movement of rodents is depending on the environmental apparatus. Bat is a volumetric navigating mammal which has hippocampal formation. A network model of 3D place cell based on neural energy was also proposed (Wang et al. [2018b\)](#page-64-11). This is a concrete examples of the application of neural energy theory in brain science. Neural energy was used to defne place feld and place feld center, and the locating performance and energy consumption characteristics of the place cell system were analyzed. Figure [14a](#page-42-0) displays the activity patterns of 16 randomly selected cells. The scatter plots in 3D space represent the diferent locations of the fying bat in the random search trajectory, and the fring power (in nW) of place cells at the corresponding position was coded by color. Centralized 3D place feld can be generated by this energy-based model. The distribution and size of place felds as well as fring powers vary among diferent place cells. Maximum power is about 3000 nW among these 16 cells and larger place felds usually have higher maximal power. Then this model was used to perform locating function in 3D space. Figure [14b](#page-42-0) shows the average locating error with respect to the size of place

feld. The result suggests that the locating error was not simply monotonously increasing as the place feld enlarging. There always exists a minimum localization error when the place feld is of the medium size. So the place feld with a reasonable optimal size will most accurately preform the localization function. Notably, larger place feld usually corresponding to more energy consumption, so a moderate feld size (moderate energy consumption) is the optimal solution for locating by place cell network. It implies the trade-of between energy consumption and spatial coverage of the place cell. The study using energy coding method validates the principle of energy economy of the brain in encoding 3D spatial information.

It has been shown that the spatial computation could vary among diferent species, especially in diferent dimensions. Is there a universal principle behind these diferences? The solution of this question is another successful application of the novel neural energy method. A new perspective of understanding the place cell activity in diferent dimensional spaces can be provided by neural energy and information theory. Place cell fre spikes to transmit information about locations. However, neural activity such as spike is energy expensive. The neural system ought to make full use of every spike to represent the largest amount of spatial information. Inspired by this designing principle, a theoretical work tries to answer the spike allocation problem for place cell, i.e., at what location to fre a spike (and forming the place field) with finite amount of total energy can achieve the most efficient representation for spatial information (Wang et al. [2019b](#page-64-12))? This question is actually a functional optimization problem with constrains which can be solved by a mathematical technique named calculus of variations. The place feld of diferent species can be treated as a function defned in diferent dimensional spaces and the amount of information is a functional taken place feld as the input function. The fnite amount of neural energy is the major constraint for the place cell. The functional optimization problem with constraint can be constructed based on neural energy by these steps. The variational method gave the optimal shape of place feld. When the moving trajectory is uniformly distributed in 1D,



<span id="page-43-0"></span>**Fig. 15** Optimal place feld in diferent dimensional space constrained by neural energy (Wang et al. [2019b](#page-64-12))

2D or 3D space, the spike location arrangement is a Gaussian-shape place feld in every dimensional space. These results are shown in Fig. [15a](#page-43-0) (1D space), b (2D space) and c (3D space). Figure [15d](#page-43-0) exemplifes the maximum information per spike (vertical axis) in 2D space with respect to the spatial variance and area of the 2D space. When the trajectory distribution is diferent, the resulting place feld is also afected, implying the animal's natural habitat and moving statistics play important roles in determining the distribution of place feld. It indicates that bat fying in 3D space and rat climbing in 3D space may result in diferently shaped 3D place felds, which can reconcile the inconsistency of the place feld symmetry found in animal experiments. It is a potential evidence that the brain complies certain designing principles such as energy economy and information efficiency, which is a representative application of neural energy theory.

The neural energy has very rich behaviors no matter in single neuron level or network level. After separating and defning neural energy supply and consumption, calculating suggested that the energy properties of supra-threshold and sub-threshold, such as power synchronization of ion channels and energy utilization ratio, have signifcant diferences. Especially the energy utilization ratio, which can rise to above 100% during sub-threshold activity, revealing an overdraft property of energy use (Wang et al. [2017b](#page-64-17)). The neural energy method has also been used to study the transformation of diferent types of memory based on a neurodynamical model (Wang et al. [2019c](#page-64-27)). A method has been developed to measure the changes in energy input of diferent stimuli and the corresponding energy consumption of the memory system. The results provide a comprehensive understanding in the memory transformation by an energy coding approach and also reveal the energy-efficient principle of the neural system.

The neural energy is also a promising prospective to study cognitive function such as path-fnding. Based on activity pattern of hippocampal place cells, a novel model of neural energy feld gradient has been proposed. A mapping among discrete spatial locations, place cell population and neural energy was constructed to defne the neural energy feld (Wang et al. [2017a\)](#page-63-22). The distribution pattern of fring power among the neuron cluster was utilized to encode the metric and topological information of space. Then it suggested that the energy feld gradient can sever as a navigational vector. By the coupling efect of gradient and noise vectors, the model can perform an efficient and biological plausible mental exploration. It is an important example that neural energy is an efective tool to study cognitive functions of the brain.

In conclusion, energy-based large-scale neuroscience models can profoundly reveal the relationship between energy, information and spatial position in the nervous system. Its advantages are as follows:

- $(1)$  nervous energy can efficiently express the cognitive system in 3D space of neural coding (Wang et al. [2019b](#page-64-12));
- (2) the energy constraint conditions to maximize information coding (Wang et al.  $2018a$ , [2018b\)](#page-64-11);
- (3) the neural coding can maximize energy to improve the efficiency of intellectual discovery (Wang et al. [2017a](#page-63-22)).

#### **4.3 Neural energy characteristics of memory switching**

Neuroscience provides a qualitative explanation of how short-term memory is transitioned to long-term memory using data from neuroanatomical experiments. However, on the basis

of experimental data, from the perspective of quantitative analysis, it seems that there is no relevant research report on how short-term memory is transferred to long-term memory under diferent stimulus conditions. We explore the interaction between working memory and long-term memory from the perspective of energy encoding based on a bi-stable working memory model. Long-term memory was induced using the working memory model using theta cluster stimulation (TBS) and high frequency stimulation (HFS), which induced LTP in experiments (Zhu et al. [2016a](#page-65-16), [b\)](#page-65-17). Based on electrical stimulation of the physical nature of nervous system, we developed a quantitative method to determine the stimulus to the energy input of the nervous system and the corresponding energy consumption system, and at the same time, further studied the two diferent long-term memory inducing stimulation protocols, their minimum energy consumption, and defned the energy ratios to quantitatively describe the stimulus energy efficiency. The results show that both of these commonly used LTP-inducing stimuli can successfully stimulate long-term memory based on the bi-stable dynamic model. However, by analyzing the minimum energy consumption and energy ratio, TBS is found to be a more energy efficient stimulus mode than HFS, which is also consistent with experimental results (Wang et al. [2019c\)](#page-64-27). The reason may be that TBS can push up the system response rhythmically, gradually raising it to a high steady state. In this study, by combining neural energy and dynamics, the energy characteristics of dynamic switching in memory model were found by investigating the response characteristics of the memory system to the stimulus modes commonly used in the two experiments (Wang et al.  $2019c$ ). This provides a strong dynamic evidence for understanding how working memory is transformed into long-term memory, which refects the high efficiency of the nervous system energy utilization during the formation of long-term memory. This example is also a successful application of neural energy coding theory.

#### **4.4 Dynamics and energy characteristics in spontaneous brain networks**

Most current researches on the brain have concentrated on task-related brain activities, both experimentally and theoretically. However, it is of much necessary to take spontaneous activities into account when we consider how the brain works, since these spontaneous brain activities always consume most of the brain's energy. The up–down oscillations of membrane potentials, which usually characterized by bistable and bimodal distribution, accompanied by some spontaneous spikes in up states, is considered to be one of the signifcant spontaneous activities. How these spontaneous phenomena occur? How much is the energy consumption of this kind of activity? Whether it implies something or not? Our work on spontaneous up–down network have tried to answer these questions and to provide a theoretical complement to the study of spontaneous brain networks.

In this work, a network model of spontaneous up–down oscillation has been designed, and on this basis the causes and key elements which infuence the spontaneous spikes have been revealed, as well as energy characteristics of spontaneous bistable networks (Wang et al. [2019c,](#page-64-27) [2021e](#page-64-28)). The results of our theoretical study of up–down oscillations, as shown in Fig. [16](#page-46-0), specifcally focus on intrinsic ion channel kinetics and synaptic transmission process. In Fig. [16](#page-46-0)a, b, it is obvious to fnd that the fast sodium current is critical to the generation of spontaneous neural spikes, while the persistent sodium current plays a role in whole spontaneous fuctuation, with or without external noise. Both of them infuence spontaneous fring rates and synchronous up–down activities, which illustrated in Fig. [16](#page-46-0)c as the combined efect. In terms of synaptic transmission, the blocking of



<span id="page-46-0"></span>**Fig. 16** Infuence of intrinsic ion channel and synaptic transmission on up and down oscillation. (Color fgure online)

excitatory connection reduces neural spikes and meanwhile still reveals spontaneous fring, as the experimental results recorded (Sanchez-Vives and McCormick [2000](#page-62-29); Compte et al. [2003\)](#page-59-20) (also see Fig. [16d](#page-46-0)), which indicate that some neurons produce spikes spontaneously through intrinsic membrane mechanisms. Furthermore, energy consumption of spontaneous up–down network and its characteristics has been concerned and the result is demonstrated in Fig. [17.](#page-47-0) The energy consumption of neurons in spontaneous up–down network is calculated and bistable characteristic and bimodal distribution of energy consumption is shown in Fig. [17a](#page-47-0), which is just in accordance with the feature of membrane potentials. At the same time, temporal and spatial characteristics of energy consumption which mostly occurs during up states and concentrates within the neuron rather than in synaptic transmission process, are refected in Fig. [17](#page-47-0)c. Besides, we also compare the indicator of energy with other commonly used ones, like fring rate and synchronizing rate, and expound its efectiveness and robustness as a global indicator (see Fig. [17](#page-47-0)b). In Fig. [17](#page-47-0)d, the results show that the energy consumption of stimulus-related energy is much smaller than that of spontaneous activity, indicating that energy consumption was driven by internal spontaneous activity rather than external stimulus, which are consistent with the evidence from



<span id="page-47-0"></span>**Fig. 17** Energy characteristics in spontaneous up and down oscillation networks. (Color fgure online)

brain imaging results and point of view put forward by Raichle (Raichle and Mintun [2006;](#page-62-12) Fox and Raichle [2007b\)](#page-59-21).

Figure [16](#page-46-0)a. Spontaneous membrane potential oscillation of two sample neurons—an excitatory neuron (EN) and an inhibitory neuron (IN)—with or without fast sodium current. Sample neurons with fast sodium current exhibit spontaneous spiking in up and down oscillation. Figure [16b](#page-46-0). Membrane potential oscillation of sample neurons under diferent cases with or without noise input. (A) Membrane potential oscillation of a sample neuron under cases without noise. (B) Membrane potential oscillation of a sample neuron under cases with noise input. Figure [16](#page-46-0)c. Mean fring rate of neurons in the network controlled by persistent sodium conductance together with fast sodium conductance. (Blue) Mean fring rate of neurons in the network controlled by persistent sodium conductance. (Red) Mean fring rate of neurons in the network controlled by fast sodium conductance. Figure [16](#page-46-0)d. Experimental and simulated results show block of excitatory synaptic transmission decreases neural fring and reveals spontaneous fring. (A) Extracellular recordings from layer V pyramidal cells: (A1)–(A3) Extracellular recordings from three example neurons (Sanchez-Vives and McCormick [2000](#page-62-29); Compte et al. [2003](#page-59-20)). (B) Simulated results based on our network model before and after blocking excitatory synaptic transmission: (B1)–(B2) Simulated results from two example neurons.

Figure [17](#page-47-0)a. Power and membrane potential are always stable at two states and both show bimodal distribution, whenever in spontaneous activities or during continuous external stimulus  $(10-12 s)$ . (A) Membrane potential distribution of all the neurons in the network. (B) Power distribution of all the neurons in the network. Figure [17](#page-47-0)b. The network size-dependent change of three indicators. (A) Mean synchronization rate for excitatory and inhibitory neurons. (B) Mean fring rate for excitatory and inhibitory neurons. (C) Mean energy consumption for excitatory and inhibitory neurons. (D) Mean energy consumption for all neurons (the green dotted line) in the network. Figure [17](#page-47-0)c. Temporal and spatial characteristics of energy consumption. (A) Membrane potential versus energy consumption plane of a single neuron. (B) Mean ratio of synaptic to total energy consumption of all the neurons in the network. Figure [17d](#page-47-0). Spontaneous and stimulation related energy consumption of neurons in the network. (A) Mean energy consumption of neurons in the network during spontaneous and stimulated periods. (B) Stimulation related increases in energy consumption of neurons in the network.

Through the observation and analysis of the fndings, we believe that these results shed light on the role of intrinsic sodium current and synaptic transmission in spontaneous fring and up and down transitions, lay the foundation for further work on spontaneous cortex activity, and would promote the progress of the energy theory in the study of spontaneous brain activity.

## **4.5 Biophysical mechanism of interaction between default mode network and working memory network**

The question of particular interest to neuroscientists is what causes the enormous persistent expenditure of brain energy. Is it possible that the current mainstream view of cognitive neuroscience has misled researchers into ignoring or ignoring the possibility that experiments in neuroscience and cognitive psychology reveal only part of brain activity (Fox and Raichle [2007a\)](#page-59-6). The answer may lie in the brain's DMN and its resting state network. To this end, we frst explore the neural mechanism of antagonism between DMN and task positive network (TPN). The results show that the synaptic connection strength has opposite



<span id="page-48-0"></span>**Fig. 18** Firing rate curves of TPN (left) and TNN (right) with diferent synaptic conductance (Cheng et al. [2020\)](#page-58-10)



<span id="page-49-0"></span>**Fig. 19** Firing rate curves of TPN (left) and TNN (right) with diferent 180◦ (Yuan et al. [2021\)](#page-65-2)



<span id="page-49-1"></span>**Fig. 20** Firing rate curves of task-positive (left) and task-negative network (right) with diferent σ (Yuan et al. [2021](#page-65-2))



<span id="page-49-2"></span>**Fig. 21** Average fring rate of excitatory neurons in the working memory network in model 1 (Yuan et al. [2021\)](#page-65-2)



<span id="page-50-0"></span>**Fig. 22** Average fring rate of excitatory neurons in the default-mode network in model 2 under various stimuli during the whole process (Cheng et al. [2020](#page-58-10))

efects on the positive task network (TPN) and the negative task network (TNN), thus concluding that the neural mechanism of antagonism between the DMN and the positive task network is mutual inhibition at the synaptic level (Cheng et al. [2020\)](#page-58-10).

As shown in Figs. [18,](#page-48-0) [19](#page-49-0) and [20,](#page-49-1) three diferent parameters (NMDA conductance parameter K5, Gaussian parameters  $\sigma$  and J+of internal preference weight of inner pyramidal cell population) strongly controlled TPN and TNN release, especially synaptic connection strength J+had opposite role to TPN and TNN, Thus, the antagonistic mechanism between them is caused by mutual inhibition at the synaptic level.

We further investigate the relationship between DMN activity as the number of working memories increases. It was found that with the increase of the number of stimuli in working memory (Fig. [21\)](#page-49-2), the neural activity of the default network decreased more rapidly (Fig. [22](#page-50-0)), indicating that the task was more difcult. It indicates that the default network is indispensable in the process of working memory.

The calculated results are in good agreement with the experimental data provided in Hu et al. ([2013\)](#page-60-28).

The energy expression of DMN and working memory network under coupling condition is consistent with the conclusion of synaptic mutual inhibition, as shown in Fig. [23](#page-51-0).

We also studied the control and energy expression of NMDA neurotransmitter on the interaction between TPN and TNN. As shown in Fig. [24,](#page-52-0) the energy of TPN and TNN under the action of NMDA neurotransmitter switch in TPN–TNN model was found. Furthermore, it is found that the NMDA conductance between TPN and TNN can be used as the switch of diferent stages of working memory and has good robustness.

The particularly interesting is that we coupled TNN1, the DMN of the posterior cingulate gyrus (PCC) and TNN2, the DMN of the inferior parietal lobe (IPL), with the working memory network to study and reconstruct three stages of working memory: encoding, storage and retrieval. What needs to be emphasized is that in the information extraction stage, the information in the coding stage can be fully refected. It was found that the antagonism between DMN and working memory network was not only negatively correlated in the traditional sense, but showed complex negative correlation and positive correlation successively in diferent brain regions.



<span id="page-51-0"></span>**Fig. 23** Firing results of TPN–TNN network after the introduction of AMPA with the same order of magnitude as NMDA. **a** Firing rate curves of excitatory population in TPN and TNN. Gaussian weight parameters: preferred direction: 180°. Red dotted line is the baseline of TNN fring rate after stimulus withdrawal. The baseline value is 32.21 Hz. Bright blue solid line is the baseline of TPN fring rate after stimulus withdrawal. The baseline value is 21.32 Hz. **b** Scatter plot of TPN with density color temperature. **c** Scatter plot of TNN with density color temperature. **d** Contained energy in TPN and TNN. Red curve is the contained energy of TPN, and the blue one is the contained energy of TNN (Yuan et al. [2021\)](#page-65-2). (Color fgure online)

Fig. [25](#page-53-0)a is the scatter color temperature diagram of the specifc emission rate of the three networks in the whole 9000 ms. Fig. [25b](#page-53-0) is the energy curve of the three networks in the coding, maintenance and extraction phases respectively. Task negative network 2 is the last blue dotted line, task negative network 1 is the middle magenta dotted line, and task positive network is the frst green solid line.

TPN is the positive network of task, TNN1 is the default network of posterior cingulate gyrus (PCC), TNN2 is the default network of IPL. Figure [25a](#page-53-0): Scatter color temperature diagram of TPN emission rate on the top, and emission rate of TNN1 and TNN2 in the presence of working memory in the middle and below (strong negative activation of the left and right ends and inhibition of the middle part). Figure [25b](#page-53-0) (energy graph): Above is the code. In the middle is the maintenance phase. Note that the green line in the maintenance phase is the result of processing the coded stimulus with the past information. The picture below is the extraction (recall stage). It can be seen that the information of the coding stage is fully refected in the recall stage.



<span id="page-52-0"></span>**Fig. 24** Contained energy of TPN and TNN with NMDA switch I. The whole network got stimulation in 750–1000 ms. NMDA channels between TPN and TNN was switched off in 3000–7000 ms (right below) and switched on in the rest of simulation time. Preference direction: 180°, Gaussian weight parameter:  $\sigma = 13.25$ ,  $J^+ = 3.62$ ,  $k5 = 95$  (Yuan et al. [2021](#page-65-2)). (Color figure online)

## **4.6 Neural energy as a new view to explain the mechanisms of neuropsychiatric disorders**

Some evidence suggests that neuropsychiatric disorders are related to energy metabolisms. At the molecular level, quantities of upstream genes associated with energy metabolisms are found to be signifcantly changed in animal models and human cerebrospinal fuid of major depressive disorder (Abdallah et al. [2014;](#page-58-29) Ågren and Niklasson [1988](#page-58-30); Głombik et al. [2020;](#page-59-22) Gu et al. [2021;](#page-59-23) Zuccoli et al. [2017\)](#page-65-18), schizophrenia (Zuccoli et al. [2017;](#page-65-18) Chase et al. [2015;](#page-58-31) Duarte and Xin [2019](#page-59-24); Martins-de-Souza et al. [2011;](#page-61-29) Pruett and Meador-Woodruf [2020;](#page-62-30) Konradi et al. [2004\)](#page-60-29) and bipolar disorder (Zuccoli et al. [2017](#page-65-18); Konradi et al. [2004](#page-60-29)). At the cellular level, some studies show that abnormal glial cell activities are also the potential pathological reasons of neuropsychiatric disorders (Cui et al. [2018](#page-59-25); Dietz et al. [2020\)](#page-59-26), while glial cells are responsible for the energy supply of neurons. In addition, a large number of fMRI results have also shown that diferent blood fow velocities and BOLD efect exist in brains with neuropsychiatric disorder (Chen et al. [2011](#page-58-32); Forbes et al. [2006;](#page-59-27) Gur et al. [2002](#page-60-30); Jaworska et al. [2015](#page-60-31); Zhou et al. [2007\)](#page-65-19), which lead to diferent neural energy consumption results. However, it is unable to tell how molecular/cellular/wholebrain levels changes lead to neuropsychiatric disorders only by these experiments, because brain is highly coupled by these non-independent components. Since neural energy theory



<span id="page-53-0"></span>**Fig. 25** Simulation results of the whole process of working memory (Yuan et al. [2021](#page-65-2)). (Color fgure online)

plays a signifcant role in encoding cognitive activities, could it also become a new explanation for neuropsychiatric disorders?

This is possible and valuable, but few researchers have been working on it. Although neuropsychiatric disorders are usually accompanied by much more complex intracellular activities (e.g., overexpression and underexpression of key proteins, interactions of different neurotransmitters and ion channels) than those in simple cognitive activities, these changes will fnally act on the membrane potentials and neural population activities. With some biophysical models like H–H model (Hodgkin and Huxley [1952](#page-60-32)), Rall's compartment model (Rall [1962](#page-62-31)) and neurotransmitter receptor binding models (Destexhe et al. [1995\)](#page-59-28), researchers can simulate the membrane potentials with ion currents and neurotransmitters data from the available electrophysiological experiments of neuropsychiatric disorders. After that, it is easy to calculate the neuronal energy consumption of ion channels, synaptic activities and neuronal activities as well as the neuronal population's behaviors.

Li's study in the feld of major depressive disorder has made an exploration of the application of neural energy theory to neuropsychiatric disorder (Li et al. [2022b](#page-61-30)). Li et al. chose medium spiny neurons (MSNs) of nucleus accumbens (NAc) as the research objective,



<span id="page-54-0"></span>**Fig. 26** Membrane potential, neuronal power and energy in single MSN model. **b** and **c** showed the 'positive' and 'negative' components (Li et al. [2022a](#page-61-10))



<span id="page-54-1"></span>**Fig. 27** The abnormalities of MSN under energy model in MDD group (Li et al. [2022a](#page-61-10)). **a** The total energy results. **b** The Negative-to-Positive energy ratio results. **c** The lag time (between power peak and the potential peak) results (Zhu et al. [2018b\)](#page-65-13)

which is a key neuronal type in dopaminergic pathway and closely related to major depressive disorder. Using H–H model, Li et al. frst successfully established the membrane potential computational model of single MSN in depression and normal group by adjusting several ion channel properties. Then, using energy model (Zhu et al. [2018b](#page-65-13)), Li et al. calcu-lated the neuronal power and energy consumption (Fig. [26\)](#page-54-0).

Further analyses showed diferences in energy encoding patterns between the depression and normal group (Fig. [27\)](#page-54-1): (1) the energy cost of MSN in MDD group was lower than that in control group; (2) the negative-to-total energy ratio of MSN in MDD group was higher than that in control group; and (3) the delay time of the power peak and the potential peak in MDD group were shorter than that in control group. These results are consistent with some behaviors and can be easily calculated by theoretical models, while are hardly to get only by biological experiments. In brief, it demonstrates that neural energy should be considered as an important part of decoding the mechanism of major depressive disorder, and it gives a new thought for the research of other neuropsychiatric disorders.

### **5 Conclusion remarks**

This review article systematically summarizes how neural energy combines various levels of molecules, cells, networks, and behavior, thus systematically interpreting the so-called large-scale neuroscience theory is actually neural energy theory. Only neural energy theory makes it possible to systematically construct models of global neural activity in the brain, and to unify their respective advantages in reductionism and holism in neuroscience within a research framework. Only the neural energy theory is possible to study interactions among the microscopic, mesoscopic and macroscopic neural activities in a theoretical system from the perspective of global neural coding. Therefore, it is able to draw out the global information of how the brain works through comprehensive research and comparison of experimental data obtained at various levels (Wouapi et al. [2021](#page-64-1); Navarro-López et al. [2021;](#page-61-1) Churchland et al. [2002](#page-58-4), [2012](#page-58-18); Tsuda et al. [1987](#page-63-1), [2004](#page-63-12); Tsuda [1991](#page-63-2), [1992,](#page-63-3) [1984,](#page-63-7) [2001](#page-63-8), [2013](#page-63-9), [2015;](#page-63-10) Ebrahimzadeh et al. [2021](#page-59-2); Yang et al. [2021a](#page-64-3), [b,](#page-64-6) [2022;](#page-64-8) Jiang et al. [2020](#page-60-5); Sharma and Acharya [2021;](#page-62-4) Wang et al. [2006,](#page-63-4) [2008,](#page-63-20) [2009,](#page-63-27) [2015a](#page-63-17), [b,](#page-63-30) [2017a](#page-63-22), [b](#page-64-17), [2018a,](#page-64-10) [b,](#page-64-11) [2019b](#page-64-12), [c,](#page-64-27) [2020,](#page-64-9) [2021a,](#page-64-4) [b](#page-64-15), [c](#page-64-24), [d,](#page-64-25) [e;](#page-64-28) Clancy et al. [2017;](#page-58-5) Videbech [2010](#page-63-5); Zhang et al. [2019,](#page-65-0) [2020](#page-65-1); Yuan et al. [2022,](#page-65-3) [2021;](#page-65-2) Yao and Wang [2019](#page-64-5); Maltba et al. [2022;](#page-61-2) Zhou et al. [2020,](#page-65-4) [2007;](#page-65-19) Li et al. [2020,](#page-61-3) [2022a](#page-61-10), [b;](#page-61-30) Kim and Lim [2020](#page-60-6); Pfaf and Volkow [2022;](#page-62-5) Tsukada et al. [1975](#page-63-6), [2015;](#page-63-14) Kaneko and Tsuda [2001;](#page-60-7) Adachi and Aihara [1997;](#page-58-6) Aihara et al. [1990;](#page-58-7) Nara and Davis [1992](#page-61-4); Pan et al. [2008,](#page-62-6) [2014;](#page-62-7) Tsuda and Kuroda [2001](#page-63-11); Fukushima et al. [2007;](#page-59-3) Kuroda et al. [2009](#page-60-8); Yamaguti et al. [2011](#page-64-7); Ryeu et al. [2001](#page-62-8); Fujii and Tsuda [2004;](#page-59-4) Tadokoro et al. [2011;](#page-63-13) Collerton et al. [2016](#page-58-8); Bullmore and Sporns [2009;](#page-58-9) Ullman [2019;](#page-63-15) Roy et al. [2019](#page-62-9); Zeng et al. [2019](#page-65-5); Wang and Zhu [2016](#page-63-16); Deco et al. [2015;](#page-59-5) Kanwisher [2010a](#page-60-9), [b;](#page-60-15) Wang and Wang [2018a](#page-63-18), [b](#page-63-19), [c](#page-63-24), [2020](#page-63-25), [2014;](#page-63-31) Ma and Tang [2017](#page-61-5); McIntyre et al. [2001](#page-61-6); Moore and Cao [2008;](#page-61-7) Moore and Cao [2008;](#page-61-7) Lu et al. [2008a](#page-61-8), [b](#page-61-9); Peng and Wang [2021;](#page-62-10) Lu [2020;](#page-61-11) Wang and Pan [2021](#page-63-21); Cheng et al. [2020](#page-58-10); Fox and Raichle [2007a,](#page-59-6) [b;](#page-59-21) Balasubramanian [2021;](#page-58-11) Raichle [2010;](#page-62-11) Raichle and Mintun [2006](#page-62-12); Piccoli et al. [2015;](#page-62-13) Compte [2000;](#page-58-12) Wei et al. [2012;](#page-64-13) Hsieh and Ranganath [2014](#page-60-10); Karlsgodt et al. [2005](#page-60-11); Williams-García et al. [2014;](#page-64-14) Fosque et al. [2021;](#page-59-7) Barbey [2018a,](#page-58-13) [b;](#page-58-14) Wang et al. xxxx; Laughlin and Sejnowski [2003a](#page-60-12), [2003b;](#page-60-24) Zheng et al. [2022,](#page-65-6) [2014](#page-65-11), [2016;](#page-65-12) Poirazi and Papoutsi [2020](#page-62-14); Lynn and Bassett [2019;](#page-61-12) Hipp et al. [2011b](#page-60-13); Hipp et al. [2011c;](#page-60-22) Raichle et al. [2018;](#page-62-15) Stender and Mortensen [2016](#page-62-16); Kruegera et al. [2009;](#page-60-14) Stelnmetz et al. [2019;](#page-62-17) Esterman et al. [2009](#page-59-8); Cohen [2017;](#page-58-15) Breakspear [2017;](#page-58-16) Johnson and Ray [2004;](#page-60-16) Nirenberg and Latham [2003;](#page-61-13) Victor [1999](#page-63-23); Jacobs et al. [2009;](#page-60-17) Malnic et al. [1999;](#page-61-14) Miyamichi and Luo [2009;](#page-61-15) Xu et al. [2022a](#page-64-16), [b;](#page-64-26) Hu and Wang [2013;](#page-60-18) Hu et al. [2012,](#page-60-19) [2013;](#page-60-28) Fischler-Ruiz et al. [2021;](#page-59-9) Zhu et al. [2016a,](#page-65-16) [b](#page-65-17), [2018a,](#page-65-7) [b](#page-65-13), [2019](#page-65-14), [2020](#page-65-15); Optican and Richmond [1987;](#page-61-16) Thorpe et al. [2001](#page-63-26); Heil [2004;](#page-60-20) Chase and Young [2007;](#page-58-17) Zhong and Wang [2021a](#page-65-8), [2021b](#page-65-9), [2021c](#page-65-10); Xin et al. [2019;](#page-64-18) Insel et al. [2004;](#page-60-21) Feldman [2012;](#page-59-10) Wang and Zhang [2011](#page-63-28), [2006;](#page-63-32) Rubin et al. [2012](#page-62-18); Panzeri etal. [2015;](#page-62-19) Stringer et al. [2019;](#page-63-29) Allen et al. [2019;](#page-58-19) Gründemann, et al. [2019;](#page-59-11) Qiu et al. [2015](#page-62-20); Qin and Xie [2016](#page-62-21); Wu et al. [2016;](#page-64-19) Byrne and Roberts [2009](#page-58-20); Lv et al. [2016](#page-61-17); Ma et al. [2017,](#page-61-18) [2019](#page-61-19); Liu [2002;](#page-61-20) Gu and Liang [2007;](#page-59-12) Haken [1996](#page-60-23); Raichle and Gusnard [2002;](#page-62-22) Maandag et al. [2007](#page-61-21); Lin et al. [2010;](#page-61-22) Peppiatt and Attwell [2004](#page-62-23); Eikenberry and Marmarelis [2015](#page-59-15); Sokolof [2008](#page-62-24); Maandag [2007;](#page-61-23)

Figley and Stroman [2011](#page-59-14); Pellerin and Magistretti [1994](#page-62-25); Brown [2004a](#page-58-21), [2004b](#page-58-22); DiNuzzo et al. [2012](#page-59-13); Rong et al. [2020](#page-62-32); Gazzaniga et al. [2002;](#page-59-16) Rubinov et al. [2011;](#page-62-26) Tolman [1948;](#page-63-33) O'Keefe and Dostrovsky [1971;](#page-61-24) Wilson and McNaughton [1993;](#page-64-20) Alme et al. [2014](#page-58-23); O'Keefe and Recce [1993;](#page-61-25) Moser et al. [2008](#page-61-26); Sargolini et al. [2006;](#page-62-27) Hafting et al. [2005](#page-60-25); Fyhn et al. [2004;](#page-59-17) Barry et al. [2007](#page-58-24); McNaughton et al. [2006;](#page-61-27) Yoon et al. [2016;](#page-64-21) Finkelstein et al. [2016;](#page-59-18) Hayman et al. [2011](#page-60-26), [2015;](#page-60-27) Casali et al. [2019;](#page-58-25) Yartsev and Ulanovsky [2013](#page-64-22); Yartsev et al. [2011;](#page-64-23) Mathis et al. [2015](#page-61-28); Giocomo et al. [2011;](#page-59-19) Burgess et al. [2007](#page-58-26); Burak and Fiete [2009;](#page-58-27) Cash and Yuste [1999](#page-58-28); Solstad et al. [2006](#page-62-28); Sanchez-Vives and McCormick [2000;](#page-62-29) Compte et al. [2003](#page-59-20); Abdallah et al. [2014](#page-58-29); Ågren and Niklasson [1988;](#page-58-30) Głombik et al. [2020](#page-59-22); Gu et al. [2021;](#page-59-23) Zuccoli et al. [2017](#page-65-18); Chase et al. [2015;](#page-58-31) Duarte and Xin [2019](#page-59-24); Martins-de-Souza et al. [2011;](#page-61-29) Pruett and Meador-Woodruf [2020](#page-62-30); Konradi et al. [2004;](#page-60-29) Cui et al. [2018](#page-59-25); Dietz et al. [2020;](#page-59-26) Chen et al. [2011,](#page-58-32) [2021;](#page-58-33) Forbes et al. [2006](#page-59-27); Gur et al. [2002](#page-60-30); Jaworska et al. [2015;](#page-60-31) Hodgkin and Huxley [1952](#page-60-32); Rall [1962;](#page-62-31) Destexhe et al. [1995;](#page-59-28) Déli and Kisvárday [2020](#page-59-29)). This is the only way out for us to walk out of the dilemma of blind men touching elephants in the feld of neuroscience research.

Finally, we want to emphasize that the modeling and analysis methods of neural energy theory are based on neuron energy models. All the other coupling factors of neuron and neural network activity, such as the regulation of blood fow phenomenon to networks and the regulation of glial cells to neuron, have been considered as a result of the dynamic changes of neuron or coupling network.

From the above introduction, we know that the neural energy method can encode not only diferent stimulus information, but also the fring of individual neurons and neural oscillations at diferent frequencies at the neural network level. This is because (1) as a global brain function model, the neural energy model can be used to analyze and describe the experimental phenomena of neuroscience at all levels, so that the calculation results at all levels are no longer unusable, contradictory and irrelevant. That is to say, neural information can be expressed with energy at various levels of molecules, neurons, networks, cognition and behavior and the combination of all levels. Energy can be used to unify the neural model among all levels (Yuan et al. [2021;](#page-65-2) Wang and Zhu [2016;](#page-63-16) Wang et al. [2008\)](#page-63-20); (2) Neural energy can be used together with the releasing pattern of membrane potential to interpret neural information processing (Chen et al. [2021\)](#page-58-33); (3) Neural energy can describe the interaction of large scale neurons throughout the brain (at the combination of molecular, neuronal and network levels) (Déli and Kisvárday xxxx), which is otherwise impossible to be achieved by any traditional neural coding theory;  $(4)$  It is currently difficult to record damage in multiple brain regions simultaneously. Although EEG and MEG can sample neuronal activity from various regions of the brain, it is very difficult to estimate cortical interactions on the basis of these extracranial signals. The main obstacle is the lack of a theoretical tool that can efectively analyze cortical to cortical interactions in a highdimensional space. In addition, there is no conversion relationship between scalp EEG and cortical potential. Nerve energy provides an efective solution to the above problems; (5) Since energy is a scalar quantity, whether it is a single neuron or a cluster, whether it is a network or a behavioral, linear or nonlinear neural model, their dynamic response can be described by the method of neural energy superposition. Thus, global information about functional neural activity in the brain can be obtained, which cannot be achieved by other traditional coding theories; (6) network coupling oscillation modes can be ever-changing, the coupling oscillation and the neural network and the network energy oscillation and there is a corresponding relationship, so when the large scale of the neural network modeling and numerical analysis for high dimensional nonlinear coupling is extremely complex and become impossible to deal with, you can use the nervous energy code to study the

neural information processing, this makes complex neuroinformatics research simple and easy to process without losing information. Electrophysiological experiments in neuroscience have revealed the relationship between spontaneous brain activity and behavior, but it is difcult to give the quantitative relationship between behavior and brain energy consumption through experiments. And the importance of studying the quantitative relationship between the two lies in the future in the calculation of brain agent (Yuan et al. [2021;](#page-65-2) Peng and Wang [2021](#page-62-10)). However, there are two potential limitations of the proposed theory: (1) the technique of directly measuring brain energy supply and consumption in diferent temporal and spatial scales need to be further developed to advance the neural energy method; (2) the energy feld model of the whole brain and its dynamic and interaction with diferent stimuli such as electric, magnetic or photic stimuli should be developed. These limitations are also promising future directions for research in this area. When new measuring techniques can determine the exact amount of energy consumed by a single neuron, a neural circuit and a brain area, deeper understanding about how the neural energy encodes stimulus, behavior and neural activity will be achieved. And a unifed energy feld model of the whole brain can provide the frst-principle perspective of how the human brain interacts with the physical world. Even so, if one can master the behavior of the agent and the relationship with energy consumption, one can design the intelligent body neural chip through energy constraints, and fnd the agent's behavior and optimize the relationship between network parameters. So nervous energy theory and the calculation method can not only help to deeply understand macro behavior and the dependencies between the brain activity, but also can through the analysis of the brain nerve energy consumption, master the information of the whole information dynamic changes in the brain, to provide parameters of the basis for the design of agent behavior.

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**Data availability** We promise that the data in the paper can be publicly available and shared.

## **Declarations**

**Confict of interest** All authors declare that they have no confict of interest.

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## **References**

- <span id="page-58-29"></span>Abdallah CG, Jiang L, De Feyter HM et al (2014) Glutamate metabolism in major depressive disorder. Am J Psychiatry 171(12):1320–1327
- <span id="page-58-6"></span>Adachi M, Aihara K (1997) Associative dynamics in chaotic neural networks. Neural Netw 10:83–98
- <span id="page-58-30"></span>Ågren H, Niklasson F (1988) Creatinine and creatine in CSF: indices of brain energy metabolism in depression. J Neural Transm 74(1):55–59
- <span id="page-58-7"></span>Aihara K, Takabe T, Toyoda M (1990) Chaotic neural networks. Phys Lett A 144:333–340
- <span id="page-58-19"></span>Allen WE et al (2019) Thirst regulates motivated behavior through modulation of brainwide neural population dynamics. Science 364(6437):253
- <span id="page-58-23"></span>Alme CB, Miao CL, Jezek K, Treves A, Moser EI, Moser MB (2014) Place cells in the hippocampus: eleven maps for eleven rooms. Proc Natl Acad Sci USA 111(52):18428–18435

<span id="page-58-11"></span>Balasubramanian V (2021) Brain power. Proc Natl Acad Sci USA 118(32):e2107022118

- <span id="page-58-13"></span>Barbey AK (2018a) Network neuroscience theory of human intelligence. Trends Cogn Sci 22(1):8–20
- <span id="page-58-14"></span>Barbey AK (2018b) Network neuroscience theory of human intelligence. Trends Cogn Sci 22:8–20
- <span id="page-58-24"></span>Barry C, Hayman R, Burgess N, Jefery K (2007) Experience-dependent rescaling of entorhinal grids. Nat Neurosci 10(6):682–684
- <span id="page-58-0"></span>Basar E (1998) Brain function and oscillations. Springer, Berlin
- <span id="page-58-16"></span>Breakspear M (2017) Dynamic models of large-scale brain activity. Nat Neurosci 20:340–352
- <span id="page-58-21"></span>Brown AM (2004a) Brain glycogen re-awakened. J Neurochem 89:537–552
- <span id="page-58-22"></span>Brown AM (2004b) Energy transfer from astrocytes to axons: the role of CNS glycogen. Neurochem Int 45:529–536
- <span id="page-58-2"></span>Brydges CR, Barceló F, Nguyen AT, Fox AM (2020) Fast fronto-parietal cortical dynamics of confict detection and context updating in a fanker task. Cogn Neurodyn 14:795–814
- <span id="page-58-9"></span>Bullmore E, Sporns O (2009) Complex brain networks: graph theoretical analysis of structural and functional systems. Nat Rev Neurosci 10:186–198
- <span id="page-58-27"></span>Burak Y, Fiete IR (2009) Accurate path integration in continuous attractor network models of grid cells. PLoS Comput Biol 5(2):e1000291
- <span id="page-58-26"></span>Burgess N, Barry C, O'Keefe J (2007) An oscillatory interference model of grid cell fring. Hippocampus 17(9):801–812
- <span id="page-58-3"></span>Buxton RB (2012) Dynamic models of BOLD contrast. Neuroimage 62(2):953–961
- <span id="page-58-20"></span>Byrne JH, Roberts JL (2009) From molecules to networks. Elsevier, Amsterdam
- <span id="page-58-25"></span>Casali G, Bush D, Jefery K (2019) Altered neural odometry in the vertical dimension. Proc Natl Acad Sci USA 116(10):4631–4636
- <span id="page-58-28"></span>Cash S, Yuste R (1999) Linear summation of excitatory inputs by CA1 pyramidal neurons. Neuron 22(2):383–394
- <span id="page-58-1"></span>Çelik S, Doğan RB, Parlatan CS, Güntekin B (2021) Distinct brain oscillatory responses for the perception and identifcation of one's own body from other's body. Cogn Neurodyn 15:609–620
- <span id="page-58-17"></span>Chase SM, Young ED (2007) First-spike latency information in single neurons increases when referenced to population onset. Proc Natl Acad Sci USA 104(12):5175–5180
- <span id="page-58-31"></span>Chase KA, Rosen C, Gin H et al (2015) Metabolic and infammatory genes in schizophrenia. Psychiatry Res 225(1–2):208–211
- <span id="page-58-32"></span>Chen CH, Suckling J, Lennox BR et al (2011) A quantitative meta-analysis of fMRI studies in bipolar disorder. Bipolar Disord 13(1):1–15
- <span id="page-58-33"></span>Chen H, Xie L, Wang Y, Zhang H (2021) Memory retention in pyramidal neurons: a unifed model of energy-based homo and heterosynaptic plasticity with homeostasis. Cogn Neurodyn 15:675–692
- <span id="page-58-10"></span>Cheng X, Yuan Y, Wang R (2020) Neural antagonistic mechanism between default-mode and task-positive networks. Neurocomputing 417:74–85
- <span id="page-58-4"></span>Churchland MM, Cunningham JP, Kaufman MT, Foster JD, Nuyujukian P, Ryu SI, Shenoy KV (2002) Neural population dynamics during reaching. Nature 487:51–56
- <span id="page-58-18"></span>Churchland MM, Cunningham JP, Kaufman MT, Foster JD, Nuyujukian P, Ryu SI, Shenoy KV (2012) Neural population dynamics during reaching. Nature 487:51–56
- <span id="page-58-5"></span>Clancy K, Ding M, Bernat E et al (2017) Restless 'rest': intrinsic sensory hyperactivity and disinhibition in post-traumatic stress disorder. Brain 140(7):2041–2050
- <span id="page-58-15"></span>Cohen MX (2017) Where does EEG come from and what does it mean? Trends Neurosci 40(4):208–218
- <span id="page-58-8"></span>Collerton D, Taylor J-P, Tsuda I, Fujii H, Nara S, Aihara K, Katori Y (2016) How can we see things that are not there? Current insights into complex visual hallucinations. J Conscious Stud 23(7–8):195–227
- <span id="page-58-12"></span>Compte A (2000) Synaptic mechanisms and network dynamics underlying spatial working memory in a cortical network model. Cereb Cortex 10(9):910–923
- <span id="page-59-20"></span>Compte A, Sanchez-Vives MV, Mccormick DA, Wang XJ (2003) Cellular and network mechanisms of slow oscillatory activity (< 1 Hz) and wave propagations in a cortical network model. J Neurophysiol 89(5):2707–2725
- <span id="page-59-25"></span>Cui Y, Yang Y, Ni Z et al (2018) Astroglial Kir4.1 in the lateral habenula drives neuronal bursts in depression. Nature 554(7692):323–327
- <span id="page-59-5"></span>Deco G et al (2015) Rethinking segregation and integration: contributions of whole-brain modelling. Nat Rev Neurosci 16(7):430–439
- <span id="page-59-29"></span>Déli E, Kisvárday Z (2020) The thermodynamic brain and the evolution of intellect: the role of mental energy. Cogn Neurodyn 14:743–756
- <span id="page-59-28"></span>Destexhe A, Mainen ZF, Sejnowski TJ (1995) Fast kinetic models for simulating AMPA, NMDA, GABA A and GABA B receptors. In: The neurobiology of computation, Springer, Berlin, pp 9–14
- <span id="page-59-26"></span>Dietz AG, Goldman SA, Nedergaard M (2020) Glial cells in schizophrenia: a unifed hypothesis. Lancet Psychiatry 7(3):272–281
- <span id="page-59-13"></span>DiNuzzo M, Mangia S, Maraviglia B, Giove F (2012) The role of astrocytic glycogen in supporting the energetics of neuronal activity. Neurochem Res 37:2432–2438
- <span id="page-59-24"></span>Duarte J, Xin L (2019) Magnetic resonance spectroscopy in schizophrenia: evidence for glutamatergic dysfunction and impaired energy metabolism. Neurochem Res 44(1):102–116
- <span id="page-59-2"></span>Ebrahimzadeh E, Shams M, Jounghani AR, Fayaz F, Mirbagheri M, Hakimi N, Rajabion L, Soltanian-Zadeh H (2021) Localizing confned epileptic foci in patients with an unclear focus or presumed multifocality using a component-based EEG–fMRI method. Cogn Neurodyn 15:207–222
- <span id="page-59-15"></span>Eikenberry SE, Marmarelis VZ (2015) Principal dynamic mode analysis of the Hodgkin-Huxley equations. Int J Neural Syst 25(2):1550001
- <span id="page-59-1"></span>Ermentrout GB, Galán RF, Urban NN (2007) Relating neural dynamics to neural coding. Phys Rev Lett 99:248103
- <span id="page-59-8"></span>Esterman M, Chiu Y-C, Tamber-Rosenau BJ (2009) Decoding cognitive control in human parietal cortex. Proc Natl Acad Sci USA 106(42):17974–17979
- <span id="page-59-10"></span>Feldman J (2012) The neural binding problem(s). Cogn Neurodyn 6(5):409–419
- <span id="page-59-14"></span>Figley CR, Stroman PW (2011) The role(s) of astrocytes and astrocyte activity in neurometabolism, neurovascular coupling, and the production of functional neuroimaging signals. Eur J Neurosci 33:577–588
- <span id="page-59-18"></span>Finkelstein A, Las L, Ulanovsky N (2016) 3-D maps and compasses in the brain. Annu Rev Neurosci 39:171–196
- <span id="page-59-9"></span>Fischler-Ruiz W, Clark DG, Joshi N, Devi-Chou V, Kitch L, Schnitzer M, Abbott LF, Axel R (2021) Olfactory landmarks and path integration converge to form a cognitive spatial map. Neuron 109(24):4036–4049
- <span id="page-59-27"></span>Forbes EE, Christopher May J, Siegle GJ et al (2006) Reward-related decision-making in pediatric major depressive disorder: an fMRI study. J Child Psychol Psychiatry 47(10):1031–1040
- <span id="page-59-7"></span>Fosque LJ, Williams-García RV, Beggs JM, Ortiz G (2021) Evidence for quasicritical brain dynamics. Phys Rev Lett 126:098101
- <span id="page-59-21"></span>Fox MD, Raichle ME (2007b) Spontaneous fuctuations in brain activity observed with functional magnetic resonance imaging. Nat Rev Neurosci 8(9):700–711
- <span id="page-59-6"></span>Fox MD, Raichle ME (2007a) Spontaneous fuctuations in brain activity observed with functional magnetic resonance imaging. Nature 8:710–711
- <span id="page-59-0"></span>Freeman WJ (2000) Neurodynamics. Springer, Berlin
- <span id="page-59-4"></span>Fujii H, Tsuda I (2004) Itinerant dynamics of class I neurons coupled by gap junctions. Lect Notes Comput Sci 3146:140–160
- <span id="page-59-3"></span>Fukushima Y, Tsukada M, Tsuda I, Yamaguti Y, Kuroda S (2007) Spatial clustering property and its self-similarity in membrane potentials of hippocampal CA1 pyramidal neurons for a spatio-temporal input sequence. Cogn Neurodyn 1:305–316
- <span id="page-59-17"></span>Fyhn M, Molden S, Moser EI, Moser MB (2004) Spatial representation in the entorhinal cortex. Science 305(5688):1258–1264
- <span id="page-59-16"></span>Gazzaniga MS, Ivry RB, Mangun GR (2002) Cognitive neuroscience. W.W. Norton & Company, London
- <span id="page-59-19"></span>Giocomo LM, Moser MB, Moser EI (2011) Computational models of grid cells. Neuron 71(4):589–603
- <span id="page-59-22"></span>Głombik K, Detka J, Kurek A et al (2020) Impaired brain energy metabolism: involvement in depression and hypothyroidism. Front Neurosci 14:586939
- <span id="page-59-11"></span>Gründemann J et al (2019) Amygdala ensembles encode behavioral states. Science 364(6347):eaav8736
- <span id="page-59-12"></span>Gu F, Liang P (2007) Neural information processing. Beijing University of Technology Press, Beijing (**in Chinese**)
- <span id="page-59-23"></span>Gu X, Ke S, Wang Q et al (2021) Energy metabolism in major depressive disorder: recent advances from omics technologies and imaging. Biomed Pharmacother 141:111869
- <span id="page-60-30"></span>Gur RE, McGrath C, Chan RM et al (2002) An fMRI study of facial emotion processing in patients with schizophrenia. Am J Psychiatry 159(12):1992–1999
- <span id="page-60-25"></span>Hafting T, Fyhn M, Molden S, Moser MB, Moser EI (2005) Microstructure of a spatial map in the entorhinal cortex. Nature 436(7052):801–806
- <span id="page-60-23"></span>Haken H (1996) Principles of brain functioning. Springer, Berlin
- <span id="page-60-26"></span>Hayman R, Verriotis MA, Jovalekic A, Fenton AA, Jefery KJ (2011) Anisotropic encoding of threedimensional space by place cells and grid cells. Nat Neurosci 14(9):1182–1188
- <span id="page-60-27"></span>Hayman RM, Casali G, Wilson JJ, Jefery KJ (2015) Grid cells on steeply sloping terrain: evidence for planar rather than volumetric encoding. Front Psychol 6:925
- <span id="page-60-20"></span>Heil P (2004) First-spike latency of auditory neurons revisited. Curr Opin Neurobiol 14:461–467
- <span id="page-60-1"></span>Hipp JF, Engel AK, Siegel M (2011a) Oscillatory synchronization in large-scale cortical networks predicts perception. Neuron 69:387–396
- <span id="page-60-13"></span>Hipp JF, Enge AK, Siege M (2011b) Oscillatory synchronization in large-scale cortical networks predicts perception. Neuron 69:387–396
- <span id="page-60-22"></span>Hipp JF et al (2011c) Oscillatory synchronization in large-scale cortical networks predicts perception. Neuron 69:387–396
- <span id="page-60-32"></span>Hodgkin AL, Huxley AF (1952) A quantitative description of membrane current and its application to conduction and excitation in nerve. J Physiol 117(4):500
- <span id="page-60-3"></span>Hopfeld JJ (2010) Neurodynamics of mental exploration. Proc Natl Acad Sci USA 107(4):1648–1653
- <span id="page-60-10"></span>Hsieh LT, Ranganath C (2014) Frontal midline theta oscillations during working memory maintenance and episodic encoding and retrieval. Neuroimage 85:721–729
- <span id="page-60-18"></span>Hu J, Wang R (2013) Responses of cutaneous mechanoreceptors within fngerpad to stimulus information for tactile softness sensation of materials. Cogn Neurodyn 7(5):441–447
- <span id="page-60-19"></span>Hu J, Yang X, Ding X, Wang R (2012) Probability of prickliness detection in a model of populations of fber ends prickling human skin. Fibers Polym 13(1):79–86
- <span id="page-60-28"></span>Hu Y, Chen X, Gu H, Yang Y (2013) Resting-state glutamate and GABA concentrations predict taskinduced deactivation in the default mode network. J Neurosci 33(47):18566–18573
- <span id="page-60-4"></span>Hu B, Xu M, Wang Z, Jiang D, Wang D, Zhang D (2021) The theoretical mechanism of Parkinson's oscillation frequency bands: a computational model study. Cogn Neurodyn 15:721–731
- <span id="page-60-21"></span>Insel TR, Landis ND, Li T-K, Sieving J, Sieving P (2004) Limits to growth: why neuroscience needs large-scale science. Nat Neurosci 7(5):426–427
- <span id="page-60-0"></span>Iribarren JL, Moro E (2009) Impact of human activity patterns on the dynamics of information difusion. Phys Rev Lett 103:038702
- <span id="page-60-17"></span>Jacobs AL et al (2009) Ruling out and ruling in neural codes. Proc Natl Acad Sci USA 106(14):5936–5941
- <span id="page-60-31"></span>Jaworska N, Yang X-R, Knott V et al (2015) A review of fMRI studies during visual emotive processing in major depressive disorder. World J Biol Psychiatry 16(7):448–471
- <span id="page-60-5"></span>Jiang P, Yang X, Sun Z (2020) Dynamics analysis of the hippocampal neuronal model subjected to cholinergic action related with Alzheimer's disease. Cogn Neurodyn 14:483–500
- <span id="page-60-16"></span>Johnson DH, Ray W (2004) Optimal stimulus coding by neural populations using rate codes. J Comput Neurosci 16:129–138
- <span id="page-60-7"></span>Kaneko K, Tsuda I (2001) Complex systems: chaos and beyond. Springer, Berlin
- <span id="page-60-9"></span>Kanwisher N (2010a) Functional specifcity in the human brain: a window into the functional architecture of the mind. Proc Natl Acad Sci USA 107(25):11163–11170
- <span id="page-60-15"></span>Kanwisher N (2010b) Functional specifcity in the human brain: a window into the functional architecture of the mind. Proc Natl Acad Sci USA 107(25):11163–11170
- <span id="page-60-11"></span>Karlsgodt KH et al (2005) Hippocampal activations during encoding and retrieval in a verbal working memory paradigm. Neuroimage 25(4):1224–1231
- <span id="page-60-6"></span>Kim S-Y, Lim W (2020) Cluster burst synchronization in a scale-free network of inhibitory bursting neurons. Cogn Neurodyn 14:69–94
- <span id="page-60-29"></span>Konradi C, Eaton M, MacDonald ML et al (2004) Molecular evidence for mitochondrial dysfunction in bipolar disorder. Arch Gen Psychiatry 61(3):300–308
- <span id="page-60-14"></span>Kruegera F, Barbey AK, McCabe K, Strenziok M, Zamboni G, Solomon J, Raymont V, Grafman J (2009) The neural bases of key competencies of emotional intelligence. Proc Natl Acad Sci USA 106(52):22486–22491
- <span id="page-60-8"></span>Kuroda S, Fukushima Y, Yamaguti Y, Tsukada M, Tsuda I (2009) Iterated function systems in the hippocampal CA1. Cogn Neurodyn 3(3):205–222
- <span id="page-60-2"></span>Lakatos P, Karmos G, Mehta AD, Ulbert I, Schroeder CE (2008) Entrainment of neuronal oscillations as a mechanism of attentional selection. Science 320:110–113
- <span id="page-60-12"></span>Laughlin SB, Sejnowski TJ (2003a) Communication in neuronal networks. Science 301:1870–1874
- <span id="page-60-24"></span>Laughlin SB, Sejnowski TJ (2003b) Communication in neural networks. Science 301:1870
- <span id="page-61-3"></span>Li X, Luo S, Xue F (2020) Efects of synaptic integration on the dynamics and computational performance of spiking neural network. Cogn Neurodyn 14:347–357
- <span id="page-61-10"></span>Li Y, Wang R, Zhang T (2022a) Two coding models of single neuron to predict the clinical symptoms in major depressive disorder. Nonlinear Dyn 107:3847–3862
- <span id="page-61-30"></span>Li Y, Wang R, Zhang T (2022b) Nonlinear computational models of dynamical coding patterns in depression and normal rats: from electrophysiology to energy consumption. Nonlinear Dyn 107:1–16
- <span id="page-61-22"></span>Lin AL, Fox PT, Hardies J, Duong TQ, Gao JH (2010) Nonlinear coupling between cerebral blood fow, oxygen consumption, and ATP production in human visual cortex. Proc Natl Acad Sci USA 107(18):8446–8451
- <span id="page-61-20"></span>Liu Y (2002) Electromagnetic biological effects. Beijing University of Posts and Telecommunications Press, Beijing (**in Chinese**)
- <span id="page-61-11"></span>Lu Q (2020) Neurodynamics and mechanics. J Dyn Control 18(1):6–10 (**in Chinese**)
- <span id="page-61-8"></span>Lu QS, Gu HG, Yang ZQ et al (2008a) Dynamics of fring patterns, synchronization and resonances in neuronal electrical activities: experiments and analysis. Acta Mech Sin 24(6):593–628
- <span id="page-61-9"></span>Lu Q, Liu S, Liu F et al (2008b) Research on dynamics and function of biological neural network systems. Adv Mech 38:366–393 (**in Chinese**)
- <span id="page-61-17"></span>Lv M, Wang C, Ren G, Ma J, Song X (2016) Model of electrical activity in a neuron under magnetic fow efect. Nonlinear Dyn 85:1479–1490
- <span id="page-61-12"></span>Lynn CW, Bassett DS (2019) The physics of brain network structure, function and control. Nat Rev Phys 1:318–332
- <span id="page-61-5"></span>Ma J, Tang J (2017) A review for dynamics in neuron and neuronal network. Nonlinear Dyn 89:1569–1578
- <span id="page-61-18"></span>Ma J, Wu F, Hayat T et al (2017) Electromagnetic induction and radiation-induced abnormality of wave propagation in excitable media. Physica A 486:508–516
- <span id="page-61-19"></span>Ma J, Yang Z, Yang L et al (2019) A physical view of computational neurodynamics. J Zhejiang Univ Sci A 20(9):639–657
- <span id="page-61-23"></span>Maandag NJG (2007) Energetics of neuronal signaling and fMRI activity. Proc Natl Acad Sci USA 104(51):20546–20551
- <span id="page-61-21"></span>Maandag JG, Coman D, Sanganahalli BG et al (2007) Energetics of neuronal signaling and fMRI activity. Proc Natl Acad Sci USA 104(51):20546–20551
- <span id="page-61-14"></span>Malnic B, Hirono J, Sato T, Buck LB (1999) Combinatorial receptor codes for odors. Cell 96:713–723
- <span id="page-61-2"></span>Maltba TE, Zhao H, Tartakovsky DM (2022) Autonomous learning of nonlocal stochastic neuron dynamics. Cogn Neurodyn 16:683–705
- <span id="page-61-29"></span>Martins-de-Souza D, Harris LW, Guest PC et al (2011) The role of energy metabolism dysfunction and oxidative stress in schizophrenia revealed by proteomics. Antioxid Redox Signal 15(7):2067–2079
- <span id="page-61-28"></span>Mathis A, Stemmler MB, Herz AV (2015) Probable nature of higher-dimensional symmetries underlying mammalian grid-cell activity patterns. Elife 4:e05979
- <span id="page-61-6"></span>McIntyre J, Zago M, Berthoz A, Lacquaniti F (2001) Does the brain model Newton's laws? Nat Neurosci 4:693–694
- <span id="page-61-27"></span>McNaughton BL, Battaglia FP, Jensen O, Moser EI, Moser MB (2006) Path integration and the neural basis of the 'cognitive map.' Nat Rev Neurosci 7(8):663–678
- <span id="page-61-0"></span>Memmesheimer R-M, Timme M (2006) Designing the dynamics of spiking neural networks. Phys Rev Lett 97:188101
- <span id="page-61-15"></span>Miyamichi K, Luo L (2009) Brain wiring by presorting axons. Science 325:44–45
- <span id="page-61-7"></span>Moore CI, Cao R (2008) The hemo-neural hypothesis: on the role of blood fow in information processing. J Neurophysiol 99:2035–2047
- <span id="page-61-26"></span>Moser EI, Kropff E, Moser MB (2008) Place cells, grid cells, and the brain's spatial representation system. Annu Rev Neurosci 31:69–89
- <span id="page-61-4"></span>Nara S, Davis P (1992) Chaotic wandering and search in a cycle-memory neural network. Prog Theor Phys 88:845–855
- <span id="page-61-1"></span>Navarro-López EM, Çelikok U, Şengör NS (2021) A dynamical model for the basal ganglia-thalamo-cortical oscillatory activity and its implications in Parkinson's disease. Cogn Neurodyn 15:693–720
- <span id="page-61-13"></span>Nirenberg S, Latham PE (2003) Decoding neuronal spike trains: how important are correlations? Proc Natl Acad Sci USA 100:7348–7353
- <span id="page-61-24"></span>O'Keefe J, Dostrovsky J (1971) The hippocampus as a spatial map. Preliminary evidence from unit activity in the freely-moving rat. Brain Res 34(1):171–175
- <span id="page-61-25"></span>O'Keefe J, Recce ML (1993) Phase relationship between hippocampal place units and the EEG theta rhythm. Hippocampus 3(3):317–330
- <span id="page-61-16"></span>Optican LM, Richmond BJ (1987) Temporal encoding of two-dimensional patterns by single units in primate inferior temporal cortex. III. Information theoretic analysis. J Neurophysiol 57(1):162–178
- <span id="page-62-6"></span>Pan X, Sawa K, Tsuda I, Tsukada M, Sakagami M (2008) Reward prediction based on stimulus categorization in primate lateral prefrontal cortex. Nat Neurosci 11:703–712
- <span id="page-62-7"></span>Pan X, Fan H, Sawa K, Tsuda I, Tsukada M, Sakagami M (2014) Reward inference by primate prefrontal and striatal neurons. J Neurosci 34(4):1380–1396
- <span id="page-62-19"></span>Panzeri S, Macke JH, Gross J, Kayser C (2015) Neural population coding: combining insights from microscopic and mass signals. Trends Cogn Sci 19(3):162–172
- <span id="page-62-25"></span>Pellerin L, Magistretti PJ (1994) Glutamate uptake into astrocytes stimulates aerobic glycolysis: a mechanism coupling neuronal activity to glucose utilization. Proc Natl Acad Sci USA 91:10625–10629
- <span id="page-62-10"></span>Peng J, Wang R (2021) Mechanism of global neural coding in fMRI hemodynamics. Nonlinear Dyn 103:883–895
- <span id="page-62-23"></span>Peppiatt C, Attwell D (2004) Feeding the brain. Nature 431:137–138
- <span id="page-62-5"></span>Pfaf DW, Volkow ND (2022) Neuroscience in the 21st century. Springer, Berlin
- <span id="page-62-13"></span>Piccoli T et al (2015) The default mode network and the working memory network are not anti-correlated during all phases of a working memory task. PLoS ONE 10(4):1–16
- <span id="page-62-14"></span>Poirazi P, Papoutsi A (2020) Illuminating dendritic function with computational models. Nat Rev Neurosci 21:303–321
- <span id="page-62-0"></span>Pouget A, Latham P (2002) Digitized neural networks: long-term stability from forgetful neurons. Nat Neurosci 5(8):709–710
- <span id="page-62-30"></span>Pruett BS, Meador-Woodruf JH (2020) Evidence for altered energy metabolism, increased lactate, and decreased pH in schizophrenia brain: a focused review and meta-analysis of human postmortem and magnetic resonance spectroscopy studies. Schizophr Res 223:29–42
- <span id="page-62-21"></span>Qin S, Xie C et al (2016) A magnetic protein biocompass. Nat Mater 15:217–226
- <span id="page-62-20"></span>Qiu C, Shivacharan RS, Zhang M, Durand DM (2015) Can neural activity propagate by endogenous electrical feld? J Neurosci 35(48):15800–15811
- <span id="page-62-2"></span>Rabinovich MI, Huerta R (2006) Dynamics of sequential decision making. Phys Rev Lett 97:188103
- <span id="page-62-11"></span>Raichle ME (2010) Two views of brain function. Trends Cogn Sci 14(4):180–190
- <span id="page-62-22"></span>Raichle ME, Gusnard DA (2002) Appraising the brain's energy budget. Proc Natl Acad Sci USA 99(16):10237–10239
- <span id="page-62-12"></span>Raichle ME, Mintun MA (2006) Brain work and brain imaging. Annu Rev Neurosci 29:449–476
- <span id="page-62-15"></span>Raichle ME et al (2018) Spontaneous infra-slow brain activity has unique spatiotemporal dynamics and laminar structure. Neuron 98(2):297–305
- <span id="page-62-31"></span>Rall W (1962) Electrophysiology of a dendritic neuron model. Biophys J 2(2Pt2):145
- <span id="page-62-1"></span>Rangan AV, Cai D, McLaughlin DW (2008) Quantifying neuronal network dynamics through coarsegrained event trees. Proc Natl Acad Sci USA 105(31):10990–10995
- <span id="page-62-32"></span>Rong W, Wang R, Zhang J, Kong W (2020) Neurodynamics analysis of cochlear hair cell activity. Theor Appl Mech Lett 10(1):8–15
- <span id="page-62-9"></span>Roy K, Jaiswal A, Panda P (2019) Towards spike-based machine intelligence with neuromorphic computing. Nature 575:607–617
- <span id="page-62-18"></span>Rubin W, Zhikang Z, Tse CK, Qu J, Cao J (2012) Neural coding in networks of multi-populations of neural oscillators. Math Comput Simul 86:52–66
- <span id="page-62-26"></span>Rubinov M, Sporns O, Thivierge J-P, Breakspear M (2011) Neurobiologically realistic determinants of selforganized criticality in networks of spiking neurons. PLoS Comput Biol 7(6):e1002038
- <span id="page-62-8"></span>Ryeu JK, Aihara K, Tsuda I (2001) Fractal encoding in a chaotic neural network. Phys Rev E 64:1–6
- <span id="page-62-29"></span>Sanchez-Vives MV, McCormick DA (2000) Cellular and network mechanisms of rhythmic recurrent activity in neocortex. Nat Neurosci 3(10):1027–1034
- <span id="page-62-3"></span>Sandrini M, Cohen LG, Censor N (2015) Modulating reconsolidation: a link to causal systems-level dynamics of human memories. Trends Cogn Sci 19(8):475–482
- <span id="page-62-27"></span>Sargolini F, Fyhn M, Hafting T, McNaughton BL, Witter MP, Moser MB et al (2006) Conjunctive representation of position, direction, and velocity in entorhinal cortex. Science 312(5774):758–762
- <span id="page-62-4"></span>Sharma M, Acharya UR (2021) Automated detection of schizophrenia using optimal wavelet-based L1 norm features extracted from single-channel EEG. Cogn Neurodyn 15:661–674
- <span id="page-62-24"></span>Sokolof L (2008) The physiological and biochemical bases of functional brain imaging. Cogn Neurodyn  $2:1-5$
- <span id="page-62-28"></span>Solstad T, Moser EI, Einevoll GT (2006) From grid cells to place cells: a mathematical model. Hippocampus 16(12):1026–1031
- <span id="page-62-17"></span>Stelnmetz NA, Zatka-Haas P, Carandlnl M, Harrls KD (2019) Distributed coding of choice, action and engagement across the mouse brain. Nature 576:266–273
- <span id="page-62-16"></span>Stender J, Mortensen KN et al (2016) The minimal energetic requirement of sustained awareness after brain injury. Curr Biol 26:1494–1499
- <span id="page-63-29"></span>Stringer C et al (2019) Spontaneous behaviors drive multidimensional, brainwide activity. Science 364(6437):255
- <span id="page-63-13"></span>Tadokoro S, Yamaguti Y, Fujii H, Tsuda I (2011) Transitory behaviors in difusively coupled nonlinear oscillators. Cogn Neurodyn 5(1):1–12
- <span id="page-63-0"></span>Takeda A (1999) Brain and physics. Shobo Co., Ltd., Tokyo (**in Japanese**)
- <span id="page-63-26"></span>Thorpe S, Delorme A, Van Rullen R (2001) Spike-based strategies for rapid processing. Neural Netw 14:715–725
- <span id="page-63-33"></span>Tolman EC (1948) Cognitive maps in rats and men. Psychol Rev 55(4):189–208
- <span id="page-63-7"></span>Tsuda I (1984) A hermeneutic process of the brain. Prog Theor Phys 79:241–259
- <span id="page-63-2"></span>Tsuda I (1991) Chaotic itinerancy as a dynamical basis of Hermeneutics in brain and mind. World Futures 32:167–184
- <span id="page-63-3"></span>Tsuda I (1992) Dynamic link of memory—chaotic memory map in nonequilibrium neural networks. Neural Netw 5:313–326
- <span id="page-63-8"></span>Tsuda I (2001) Toward an interpretation of dynamic neural activity in terms of chaotic dynamical systems. Behav Brain Sci 24(5):793–847
- <span id="page-63-9"></span>Tsuda I (2013) Chaotic itinerancy. Scholarpedia 8(1):4459
- <span id="page-63-10"></span>Tsuda I (2015) Chaotic itinerancy and its roles in cognitive neurodynamics. Curr Opin Neurobiol 31:67–71
- <span id="page-63-11"></span>Tsuda I, Kuroda S (2001) Cantor coding in the hippocampus. Jpn J Ind Appl Math 18(2):29–258
- <span id="page-63-1"></span>Tsuda I, Koerner E, Shimizu H (1987) Memory dynamics in asynchronous neural networks. Prog Theor Phys 78:51–71
- <span id="page-63-12"></span>Tsuda I, Fujii H, Tadokoro S, Yasuoka T, Yamaguti Y (2004) Chaotic itinerancy as a mechanism of irregular changes between synchronization and desynchronization in a neural network. J Integr Neurosci 3:159–182
- <span id="page-63-6"></span>Tsukada M, Ishii N, Sato R (1975) Temporal pattern discrimination of impulse sequences in the computersimulated nerve cells. Biol Cybern 17:19–28
- <span id="page-63-14"></span>Tsukada H, Fujii H, Aihara K, Tsuda I (2015) Computational model of visual hallucination in dementia with Lewy bodies. Neural Netw 62:73–82
- <span id="page-63-15"></span>Ullman S (2019) Using neuroscience to develop artifcial intelligence. Science 363(6428):692–693
- <span id="page-63-23"></span>Victor JD (1999) Temporal aspects of neural coding in the retina and lateral geniculate. Netw Comput Neural Syst 10:R1-66
- <span id="page-63-5"></span>Videbech P (2010) PET measurements of brain glucose metabolism and blood fow in major depressive disorder: a critical review. Acta Psychiatr Scand 101(1):11–20
- <span id="page-63-21"></span>Wang R, Pan X (2021) Research progress of neurodynamics in China. Cogn Neurodyn 15:1–2
- <span id="page-63-31"></span>Wang Z, Wang R (2014) Energy distribution property and energy coding of a structural neural network. Front Comput Neurosci.<https://doi.org/10.3389/fncom.2014.00014>
- <span id="page-63-18"></span>Wang Y, Wang R (2018a) An improved neuronal energy model that better captures of dynamic property of neuronal activity. Nonlinear Dyn 91(1):319–327
- <span id="page-63-19"></span>Wang R, Wang Z (2018b) The essence of neuronal activity from the consistency of two diferent neuron models. Nonlinear Dyn 92:973–982
- <span id="page-63-24"></span>Wang G, Wang R (2018c) Simulation of retinal ganglion cell response using fast independent component analysis. Cogn Neurodyn 12(6):615–624
- <span id="page-63-25"></span>Wang G, Wang R (2020) The relationship between sparseness and energy consumption of neural networks. Neural Plast 2020:8848901
- <span id="page-63-32"></span>Wang R, Zhang Z (2006) Mechanism on brain information processing: energy coding. Appl Phys Lett 89:123903
- <span id="page-63-28"></span>Wang R, Zhang Z (2011) Phase synchronization motion and neural coding in dynamic transmission of neural information. IEEE Trans Neural Netw 22(7):1097–1106
- <span id="page-63-16"></span>Wang R, Zhu Y (2016) Can the activities of the large-scale cortical network be expressed by neural energy? A brief review. Cogn Neurodyn 10:1–5
- <span id="page-63-4"></span>Wang Z, Kai L, Day M et al (2006) Dopaminergic control of corticostriatal long-term synaptic depression in medium spiny neurons is mediated by cholinergic interneurons. Neuron 50(3):443–452
- <span id="page-63-20"></span>Wang R, Zhang Z, Chen G (2008) Energy function and energy evolution on neural population. IEEE Trans Neural Netw 19(3):535–538
- <span id="page-63-27"></span>Wang R, Zhang Z, Chen G (2009) Energy coding and energy functions for local activities of brain. Neurocomputing 73(1–3):139–150
- <span id="page-63-17"></span>Wang R, Tsuda I, Zhang Z (2015a) A new work mechanism on neuronal activity. Int J Neural Syst 25(03):1450037
- <span id="page-63-30"></span>Wang Z, Wang R, Fang R (2015b) Energy coding in neural network with inhibitory neurons. Cogn Neurodyn 9(2):129–144
- <span id="page-63-22"></span>Wang Y, Wang R, Zhu Y (2017a) Optimal path-fnding through mental exploration based on neural energy feld gradients. Cogn Neurodyn 11(1):99–111
- <span id="page-64-17"></span>Wang Y, Wang R, Xu X (2017b) Neural energy supply–consumption properties based on Hodgkin-Huxley model. Neural Plast 2017:6207141
- <span id="page-64-10"></span>Wang Y, Xu X, Wang R (2018a) Intrinsic sodium currents and excitatory synaptic transmission infuence spontaneous fring in up and down activities. Neural Netw 98:42–50
- <span id="page-64-11"></span>Wang Y, Xu X, Wang R (2018b) An energy model of place cell network in three dimensional space. Front Neurosci 12:264
- <span id="page-64-2"></span>Wang C, Tang J, Ma J (2019a) Minireview on signal exchange between nonlinear circuits and neurons via feld coupling. Eur Phys J Spec Top 228(10):1907–1924
- <span id="page-64-12"></span>Wang Y, Xu X, Wang R (2019b) The place cell activity is information-efficient constrained by energy. Neural Netw 116:110–118
- <span id="page-64-27"></span>Wang YH, Xu XY, Zhu YT, Wang RB (2019c) Neural energy mechanism and neurodynamics of memory transformation. Nonlinear Dyn 97(1):697–714
- Wang R, Lin P, Liu M et al (2019) Hierarchical connectome modes and critical state jointly maximize human brain functional diversity. Phys Rev Lett 123:038301
- <span id="page-64-9"></span>Wang R et al (2020) Advances in neurodynamic research. J Dyn Control 18(1):1–5 (**in Chinese**)
- <span id="page-64-4"></span>Wang Y, Xu X, Wang R (2021a) Energy features in spontaneous up and down oscillations. Cogn Neurodyn 15:65–75
- <span id="page-64-15"></span>Wang R, Liu M, Cheng X et al (2021b) Segregation, integration, and balance of large-scale resting brain networks confgure diferent cognitive abilities. Proc Natl Acad Sci USA 118(23):e2022288118
- <span id="page-64-24"></span>Wang YH, Xu XY, Wang RB (2021c) Modeling the grid cell activity on non-horizontal surfaces based on oscillatory interference modulated by gravity. Neural Netw 141:199–210
- <span id="page-64-25"></span>Wang YH, Xu XY, Pan XC, Wang RB (2021d) Grid cell activity and path integration on 2-D manifolds in 3-D space. Nonlinear Dyn 104(2):1767–1780
- <span id="page-64-28"></span>Wang YH, Xu XY, Wang RB (2021e) Energy features in spontaneous up and down oscillations. Cogn Neurodyn 15(1):65–75
- <span id="page-64-13"></span>Wei Z et al (2012) From distributed resources to limited slots in multiple-item working memory: a spiking network model with normalization. J Neurosci 32(33):11228–11240
- <span id="page-64-14"></span>Williams-García RV, Moore M, Beggs JM, Ortiz G (2014) Quasicritical brain dynamics on a nonequilibrium Widom line. Phys Rev E 90:062714
- <span id="page-64-20"></span>Wilson MA, McNaughton BL (1993) Dynamics of the hippocampal ensemble code for space. Science 261(5124):1055–1058
- <span id="page-64-1"></span>Wouapi MK, Fotsin BH, Ngouonkadi EBM, Kemwoue FF, Njitacke ZT (2021) Complex bifurcation analysis and synchronization optimal control for Hindmarsh-Rose neuron model under magnetic fow efect. Cogn Neurodyn 15:315–347
- <span id="page-64-19"></span>Wu F, Wang C, Xu Y, Ma J (2016) Model of electrical activity in cardiac tissue under electromagnetic induction. Sci Rep 6(1):28
- <span id="page-64-18"></span>Xin Y, Zhong L, Zhang Y, Zhou T, Pan J, Xu N-L (2019) Sensory-to-category transformation via dynamic reorganization of ensemble structures in mouse auditory cortex. Neuron 103(5):909-921.e6
- <span id="page-64-16"></span>Xu X, Zhu Z, Wang Y, Wang R (2022a) Odor pattern recognition of a novel bio-inspired olfactory neural network based on kernel clustering. Commun Nonlinear Sci Numer Simul 109:106274
- <span id="page-64-26"></span>Xu XY, Wang YH, Wang RB (2022b) The place cell activity in three-dimensional space generated by multiple grid cell inputs. Nonlinear Dyn 108(2):1719–1731
- <span id="page-64-7"></span>Yamaguti Y, Kuroda S, Fukushima Y, Tsukada M, Tsuda I (2011) A mathematical model for cantor coding in the hippocampus. Neural Netw 24:43–53
- <span id="page-64-3"></span>Yang C, Liu Z, Wang Q, Luan G, Zhai F (2021a) Epileptic seizures in a heterogeneous excitatory network with short-term plasticity. Cogn Neurodyn 15:43–51
- <span id="page-64-6"></span>Yang Y, Ma J, Xu Y, Jia Y (2021b) Energy dependence on discharge mode of Izhikevich neuron driven by external stimulus under electromagnetic induction. Cogn Neurodyn 15:265–277
- <span id="page-64-8"></span>Yang H, Xu G, Wang H (2022) Efects of magnetic felds on stochastic resonance in Hodgkin-Huxley neuronal network driven by Gaussian noise and non-Gaussian noise. Cogn Neurodyn 16:707–717
- <span id="page-64-5"></span>Yao M, Wang R (2019) Neurodynamic analysis of Merkel cell–neurite complex transduction mechanism during tactile sensing. Cogn Neurodyn 13:293–302
- <span id="page-64-22"></span>Yartsev MM, Ulanovsky N (2013) Representation of three-dimensional space in the hippocampus of fying bats. Science 340(6130):367–372
- <span id="page-64-23"></span>Yartsev MM, Witter MP, Ulanovsky N (2011) Grid cells without theta oscillations in the entorhinal cortex of bats. Nature 479(7371):103–107
- <span id="page-64-21"></span>Yoon KJ, Lewallen S, Kinkhabwala AA, Tank DW, Fiete IR (2016) Grid cell responses in 1D environments assessed as slices through a 2D lattice. Neuron 89(5):1086–1099
- <span id="page-64-0"></span>Yu Y, Wang X, Wang Q, Wang Q (2020) A review of computational modeling and deep brain stimulation: applications to Parkinson's disease. Appl Math Mech 41:1747–1768
- <span id="page-65-3"></span>Yuan Z, Feng P, Fan Y, Yu Y, Wu Y (2022) Astrocytic modulation on neuronal electric mode selection induced by magnetic field effect. Cogn Neurodyn 16:183-194
- <span id="page-65-2"></span>Yuan Y, Pan X, Wang R (2021) Biophysical mechanism of the interaction between default mode network and working memory network. Cogn Neurodyn 15:1101–1124
- <span id="page-65-5"></span>Zeng G, Chen Y, Cui B, Yu S (2019) Continual learning of context-dependent processing in neural networks. Nat Mach Intell 1:364–372
- <span id="page-65-0"></span>Zhang T, Pan X, Xu X, Wang R (2019) A cortical model with multi-layers to study visual attentional modulation of neurons at the synaptic level. Cogn Neurodyn 13:579–599
- <span id="page-65-1"></span>Zhang W, Guo L, Liu D, Xu G (2020) The dynamic properties of a brain network during working memory based on the algorithm of cross-frequency coupling. Cogn Neurodyn 14:215–228
- <span id="page-65-6"></span>Zheng J, Wang R, Kong W, Zhang J (2022) A new patterns of self-organization activity of brain: neural energy coding. Inf Sci 608:1747–1755
- <span id="page-65-11"></span>Zheng H, Wang R, Qiao L, Du Y (2014) The molecular dynamics of neural metabolism during the action potential. Sci China Technol Sci 57(5):857–863
- <span id="page-65-12"></span>Zheng H, Wang R, Qu J (2016) Effect of different glucose supply conditions on neuronal energy metabolism. Cogn Neurodyn 10(6):563–571
- <span id="page-65-8"></span>Zhong H, Wang R (2021a) A new discovery on visual information dynamic changes from V1 to V2: corner encoding. Nonlinear Dyn 105:3551–3570
- <span id="page-65-9"></span>Zhong H, Wang R (2021b) A visual-degradation-inspired model with HSV color-encoding for contour detection. J Neurosci Methods 369:109423
- <span id="page-65-10"></span>Zhong H, Wang R (2021c) Neural mechanism of visual information degradation from retina to V1. Cogn Neurodyn 15:299–313
- <span id="page-65-19"></span>Zhou Y, Liang M, Tian L et al (2007) Functional disintegration in paranoid schizophrenia using resting-state fMRI. Schizophr Res 97(1–3):194–205
- <span id="page-65-4"></span>Zhou X, Xu Y, Wang G, Jia Y (2020) Ionic channel blockage in stochastic Hodgkin-Huxley neuronal model driven by multiple oscillatory signals. Cogn Neurodyn 14:569–578
- <span id="page-65-16"></span>Zhu Y, Wang R, Wang Y (2016a) A comparative study of the impact of theta-burst and high-frequency stimulation on memory performance. Front Hum Neurosci 10:19
- <span id="page-65-17"></span>Zhu Y, Wang R, Wang Y (2016b) The impact of theta-burst stimulation on memory mechanism: a modeling study. Appl Math Mech 37(3):395–402
- <span id="page-65-7"></span>Zhu Y, Nachtrab G, Keyes PC et al (2018a) Dynamic salience processing in paraventricular thalamus gates associative learning. Science 362(6413):423–429
- <span id="page-65-13"></span>Zhu Z, Wang R, Zhu F (2018b) The energy coding of a structural neural network based on the Hodgkin-Huxley model. Front Neurosci 12:122
- <span id="page-65-14"></span>Zhu F, Wang R, Pan X, Zhu Z (2019) Energy expenditure computation of a single bursting neuron. Cogn Neurodyn 13:75–87
- <span id="page-65-15"></span>Zhu F, Wang R, Aihara K, Pan X (2020) Energy-efficient firing patterns with sparse bursts in the Chay neuron model. Nonlinear Dyn 100:2657–2672
- <span id="page-65-18"></span>Zuccoli GS, Saia-Cereda VM, Nascimento JM et al (2017) The energy metabolism dysfunction in psychiatric disorders postmortem brains: focus on proteomic evidence. Front Neurosci 11:493

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