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Chaotic slime mould optimization algorithm for global optimization

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Abstract

Metaheuristic optimization methods; It is a well-known global optimization approach for large-scale search and optimization problems, commonly used to find the solution many different optimization problems. Slime mould optimization algorithm (SMA) is a recently presented metaheuristic technique that is inspired by the behavior of slime mould. Slow convergence speed is a fundamental problem in SMA as in other metaheuristic optimization methods. In order to improve the SMA method, 10 different chaotic maps have been applied for the first time in this article to generate chaotic values instead of random values in SMA. Using chaotic maps, it is aimed to increase the speed of SMA's global convergence and prevent it from getting stuck in its local solutions. The Chaotic SMA (CSMA) proposed for the first time in this study was applied to 62 different benchmark functions. These are unimodal, multimodal, fixed dimension, CEC2019, and CEC2017 test suite. The results of the application have been comparatively analyzed and statistical analysis performed with the well-known metaheuristic optimization methods, particle swarm optimization and differential evolution algorithm, and recently proposed grey wolf optimization (GWO) and whale optimization algorithm (WOA). In addition, in the CEC2017 test suite, the CSMA method has been compared with the SMA, WOA, GWO, harris hawk optimization, archimedes optimization algorithm and COOT algorithms that have been proposed in recent years, and statistical analyzes have been made. In addition, CSMA has been tested in 3 different real-world engineering design problems. According to the experimental results, it was observed that CSMA achieved relatively more successful results in 62 different benchmark functions and real-world engineering design problems compared to other compared methods and standard SMA.

Keywords Chaotic map · Metaheuristic optimization methods · Slime mould optimization

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1 Introduction

Optimization is the process of finding the most suitable solution among all available solutions of a particular problem under given conditions (Törn and Žilinskas 1989). It is difficult to create a mathematical model for complex systems. Even if the model is established, it is not preferred because it is very costly and takes a lot of time. The use of mathematical models created from natural phenomena in optimization problems has recently become very popular. Thus, many complex nonlinear optimization problems have been solved. In such cases, classical algorithms may not give the desired results and therefore alternative methods should be used. Metaheuristic optimization methods; It is a well-known global optimization approach that is widely used to solve many different optimization problems and produces near-optimum solutions in acceptable time for large-scale search and optimization problems where mathematical models cannot be created (Yang 2011). This approach; aims to find the most appropriate solution for the given problem by imitating the events, mechanisms, or social behavior of the species (Altay and Alatas 2019).

There is no single optimization algorithm that can solve all optimization problems at the desired level. Various optimization methods have been proposed in the literature that can be used for optimization problems. These methods have become quite popular not only in the computer science area but also in other research fields (Ewees et al. 2018). Especially in engineering optimization problems, metaheuristic methods have become quite common due to flexibility, gradient-free mechanism, and being based on uncomplicated simple concepts (Houssein et al. 2020a). When the literature is examined, some of them are; To solve topology optimization problems of nonlinear single-layer domes (Bigham and Gholizadeh 2020), seismic design optimization of steel moment frames based on discrete performance (Gholizadeh and Danesh 2020), discrete sizing optimization of steel skeletal structures (Gholizadeh and Milany 2018), seismic design optimization of steel frames (Gholizadeh and Baghchevan 2017), obtaining quantum cloning circuit parameters (Houssein et al. 2021b), feature selection (Hussain et al. 2021), to solve motif discovery problem (Hashim et al. 2020), to solve the optimal Economic Emission Dispatch problem (Hassan et al. 2021), drug design and discovery in chemoinformatics (Houssein et al. 2020a), wireless sensor networks (Houssein et al. 2020b), association rule mining problem (Altay and Alatas 2021). One of the suggested methods to solve such problems is slime mould optimization algorithm (SMA) (Li et al. 2020). SMA is a new stochastic optimization method proposed in 2020 and based on the oscillation mode of slime mould in nature (Li et al. 2020). Since SMA is a very new optimization method, there are very few studies. Lack of an appropriate balance between discovery and exploitation due to its stochastic nature is one of the main difficulties encountered in the development of meta-heuristic optimization methods (Mirjalili et al. 2016). In SMA, as in other metaheuristic methods, there is a balance problem between exploration and exploitation. The exploration phase helps the optimizer to globalize the search area by ensuring the widest possible coverage. On the contrary, the exploitation phase includes the improvement of promising solutions obtained from the exploration phase. Solving this problem and increasing the speed of convergence and the ability to obtain the global optimal solution in SMA and increasing the performance of SMA is an important area of research. Because SMA still suffers from reasons such as global convergence, increase to population diversity and, prevent SMA from getting stuck in local solutions.

There is no best optimization method that can give the best results in all optimization problems. Considering the no free lunch theorem, different versions of SMA will be proposed, helping to achieve better performance of SMA in different optimization problems. There are different versions of SMA in the literature in which its performance has been improved. SMA and its different versions have been successfully applied to optimization problems in different fields.

Izci proposed a new hybrid method by combining the SMA and Nelder-Mead simplex search method to increase the performance of SMA. The performance of the proposed method has been examined in 4 different benchmark functions (Izci 2021). Ekinci et al. carried out the optimal design of the power system balancer in a single-machine infinite bus power system using SMA. They defined the parameter setting of the power system stabilizer as an optimization problem. They compared their results with conventional power system balancer and grasshopper optimization (Ekinci et al. 2020). In order to demonstrate the performance of SMA in real-life engineering problems, Izci and Ekinci applied it to the problems of efficient Design of Proportional–Integral–Derivative Controller integral derivative controller. SMA has been shown to be an effective algorithm in these problems (Izci and Ekinci 2021).

A hybrid method was proposed by Zhao et al. by combining SMA with the Harris Hawk optimization method. As a result of the study, it has been shown that the proposed method achieves better performance by combining location updates (Zhao and Gao 2020). Levy flight was proposed by Zhao et al. to replace random numbers in SMA. However, the proposed method in the study gave equal or worse results than the standard SMA (Zhao et al. 2020). In the study by Naik et al., the exploration feature of SMA was developed using opposition-based learning. The study was tested in 29 different benchmark functions. As a result of the study, the Friedman test showed that the proposed method is in the first place (Naik et al. 2021). In the study by Gao et al., the performance of SMA was developed with cosine controlling parameters. The study has shown that the performance of SMA can be improved by using cosine controlling parameters (Gao et al. 2020). In the study by Houssein et al., a hybrid method was proposed using SMA and Adaptive Guided Differential Evolution Algorithm. The proposed method was tested in the CEC2017 test suite, three engineering design problems tension/compression spring, pressure vessel, and rolling element bearing and two combinatorial optimization problems bin packing and quadratic assignment. As a result of the study, the proposed method produced promising values (Houssein et al. 2021a).

With the development of nonlinear dynamics, chaos theory has been widely used in various applications (Kellert 1994). Chaos theory is concerned with the study of chaotic dynamical systems that are highly sensitive to initial conditions and involve infinite unstable periodic movements. Chaotic systems are deterministic and should not be confused with stochastic systems. The chaos in a system is not a random external effect but the internal dynamics of the system itself (Ozer 2010). Various metaheuristic algorithms have also improved performance using chaos theory. Thus, they had better convergence speed and not get stuck in local solutions. Previously chaos theory genetic algorithm (Yang and Chen 2002), particle swarm optimization (PSO) (Liu et al. 2005), bird swarm algorithm (BSA) (Altay and Alatas 2020), whale optimization algorithm (WOA) (Kaur and Arora 2018), grasshopper optimization algorithm (Arora and Anand 2019), krill herd optimization algorithm (Wang et al. 2014), memetic differential evolution algorithm (Jia et al. 2011), fruit fly optimization algorithm (Mitić et al. 2015), optics optimization algorithm (Bingol and Alatas 2020) have been successfully applied in various metaheuristic optimization methods. There are two different studies of SMA using chaotic maps. First, Rizk-Allah et al. use a single chaotic map and crossover-opposition strategy. The proposed method aims to obtain the optimum wind turbine design under high-altitude sites, a nonlinear and nonconvex model (Rizk-Allah et al. 2021). Second, Dhawale et al. used the sinusoidal chaotic function in the SMA method. The proposed method has been applied to 23 benchmark problems and 10 multidisciplinary design problems (Dhawale et al. 2021).

In this article, chaotic maps have been integrated into the SMA and ten new SMAs have been proposed to improve the performance of the standard SMA. In addition, the effect of each chaotic map on SMA was examined separately. Discrete-time chaotic systems were chosen while chaotic maps were selected. The biggest advantage of these chaotic maps is their high-performance thanks to their simple mathematical models. For the first time, ten different chaotic systems are integrated and chaotic slime mould (CSMA) algorithms are proposed instead of random number sequences to obtain SMA parameters. In this way, it is aimed to improve global convergence and prevent getting stuck on a local solution. The experimental results are thought to show that the application of deterministic chaotic maps instead of random values can be a possible strategy to improve the performance of SMA.

The remainder of the article is organized as follows. First of all, the SMA and chaotic maps that make up the background study of this article are described in Sect. 2 and Sect. 3, respectively. In Sect. 2, the SMA that inspired slime mould, the mathematical model, pseudo-code and rules, are introduced. The equations and parameters of the chaotic maps used in the article are shown in Sect. 3. The novel CSMAs, which form the motivation and basis of the article, is described in Sect. 4. In Sect. 5, the CSMAs have been applied to 62 different benchmark functions (including CEC2017 test suite and CEC2019 test suite) and 3 different real-world problems. The obtained results are compared with other known and recent artificial intelligence-based metaheuristic optimization methods and Friedman and Wilcoxon statistical analyzes are performed. Finally, Sect. 6 concludes the article and provides information about future works.

2 Slime mould swarm algorithm

Inspired by the optimization method, Slime mould was first named by Howard (1931). Slime mould is a eukaryote that survives in cold and humid places. Plasmodium, which is its active and dynamic phase, forms the basis of its nutrition. This stage is also the basis of the Slime mould swarm algorithm. Slime mould is looking for food with organic matter in it during this phase. After the slime mould completes the search process, it wraps around the food and secretes enzymes to digest the food. During the migration process, the front end extends into a fan-shaped mesh. It then extends into an interconnected venous network allowing it to flow in. Due to its unique patterns and characteristic structure, it can create a venous network for more than one food at the same time. Slime mould can grow over an area of more than 900 square centimeters if it finds enough nutrients in its environment. The mathematical model of slime mould has been applied in road networks and graph theory (Yu et al. 2018; Šešum-Čavić et al. 2016; Becker 2015). There are three different correlations between the morphological changes of the venous structure of slime mould and the mode of contraction of slime mould (Nakagaki et al. 2000).

- When contraction frequencies change from outside to inside, thick veins are formed roughly along a radius.
- Anisotropy starts to appear when the contraction mode is unsteady.
- The venous structure is absent when the contraction pattern of slime mould is not ordered according to time and place.

It has recently been shown in the literature that foraging regulations of slime mould are based on optimization theory (Nakagaki et al. 2000). The foraging structure of slime mould is dynamic. In other words, the search pattern changes according to the food quality. This change changes in parallel with the quality of food resources. If the density of the first food source is not at the desired level, the slime mould will leave its source to look for other food sources in the area. Thanks to the search strategy that can vary depending on the situation, it will be able to get the chance to search all the food in a region. The mathematical model of the slime mould algorithm is discussed in section three (Li et al. 2020). The symbols used in the mathematical model of the standard SMA are given in the Table 1 (Li et al. 2020).

2.1 Approach food

Equation (1) is given to model the rule of contraction mode in the approach behavior of slime mould as a mathematical Eq. (1) (Li et al. 2020):

$$\overrightarrow{X(t+1)} = \begin{cases} \overrightarrow{X_b(t)} + \overrightarrow{vb} \cdot \left(\vec{W} \cdot \overrightarrow{X_A(t)} - \overrightarrow{X_B(t)} \right), & r < p \\ \overrightarrow{vc} \cdot \overrightarrow{X(t)}, & r \geq p \end{cases}. \tag{1}$$

The formula of p is shown in Eq. (2):

$$p = \tanh |S(i) - DF|. \tag{2}$$

The formula of $\overrightarrow{vb}$ is shown in Eq. (3) and the formula of a is shown in Eq. (4):

$$\overrightarrow{vb} = [-a, a] \tag{3}$$

Table 1 Symbols used in the standard SMA and their definitions

Symbols	Definitions
$\overrightarrow{vb}$	Range of $[-a, a]$
$\overrightarrow{vc}$	Decreases linearly from one to zero
t	Current iteration
$\vec{X}$	Location of slime mould
$\overrightarrow{X_b}$	The individuals location with the highest odor concentration
$\overrightarrow{X_A}$ and $\overrightarrow{X_B}$	Two individuals randomly selected from the swarm
$\vec{W}$	Weight of slime mould
$S(i)$	The fitness of $\vec{X}$
DF	The best fitness
r	The random value interval of [0,1]
bF	Optimal fitness obtained in the current iterative process
wF	Worst fitness value obtained in the iterative process currently
LB and UB	The lower and upper boundaries of the search range

$$a = \operatorname{arctanh}\left(-\left(\frac{t}{\max\_t}\right) + 1\right). \tag{4}$$

The formula of $\vec{W}$ is listed in Eq. (5):

$$\overrightarrow{W(SmellIndex(i))} = \begin{cases} 1 + r \cdot \log\left(\frac{bF-S(i)}{bF-wF} + 1\right), & condition \\ 1 - r \cdot \log\left(\frac{bF-S(i)}{bF-wF} + 1\right), & others \end{cases} \tag{5}$$

$$SmellIndex = sort(S). \tag{6}$$

In the Eq. (6) represents the sequence of fitness values sorted (Li et al. 2020).

2.2 Wrap food

Wrap food mathematically simulates the contraction mode of the venous tissue structure of the slime mould during the search. The concentration of food in contact with the container is directly proportional to the wavelength produced by the bio-oscillator. Thus, the higher the wavelength, the faster the cytoplasm and the thicker it will be in the container. Weight changes according to food concentration. When the weight is low, the slime mould tends to explore other areas (Li et al. 2020).

The mathematical formula and equation for updating the location of slime mould are given in Eq. (7):

$$\overrightarrow{X^*} = \begin{cases} rand \cdot (UB - LB) + LB, & rand < z \\ \overrightarrow{X_b(t)} + \overrightarrow{vb} \cdot \left(W \cdot \overrightarrow{X_A(t)} - \overrightarrow{X_B(t)}\right), & r < p \\ \overrightarrow{vc} \cdot \overrightarrow{X(t)}, & r \geq p \end{cases}. \tag{7}$$

2.3 Grabble food (oscillation)

The search for nutrients of the slime mould changes the cytoplasmic flow in the vessels according to the spreading wave generated by the biological oscillator, thus enabling a better food concentration to be found (Li et al. 2020) (Table 2).

3 Chaotic maps

Numbers created using chaotic maps have been successfully applied in different application areas. In general, chaotic maps have three basic properties: ergodicity, initial conditions, and semi-stochastic properties. Chaotic behaviors attract the attention of researchers in different fields such as ecology, medicine, economics, and engineering applications. It is also widely used in improving the performance of optimization methods. Successful results have been obtained by applying chaotic maps in different studies to improve the stochastic structure of the optimization methods. In this study, 10 different chaotic maps were used to increase the global convergence speed of the standard SMA and not to get stuck in local solutions. Chaotic maps used are chebyshev map, gauss map, circle map, sine map, logistic

Table 2 Pseudo-code of SMA

```
Algorithm 1 Pseudo-code of CSMAs
Initialize the parameters pop size, Max_iteraition;
Initialize the positions of slime mould X_i (i = 1,2, ..., n);
While (t ≤ Max_iteration)
    Calculate the fitness of all slime mould;
    Update bestFitness, X_b
    Calculate the W by Eq. (5) Using with chaotic maps;
    For each search portion
       Update p, vb, vc;
       Update positions by Eq. (7);
    End For
    t = t + 1;
End While
Return bestFitness, X_b;
```

map, piecewise map, iterative map, singer map, tent map, and sinusoidal map. The parameters and equations related to these maps are explained below.

Chebyshev map Chebyshev equation is given in Eq. (8).

$$X_{n+1} = \cos\left(kcos^{-1}x_n\right) \tag{8}$$

Circle map Circle map equation and parameters are given in Eq. (9).

$$X_{n+1} = X_n + b - \left(\frac{a}{2\pi}\right)\sin\left(2\pi X_n\right)mod(1) \quad a = 0.5 \text{ and } b = 0.2 \text{ are taken.} \tag{9}$$

Gauss map Gauss map equation and parameters are given in Eq. (10).

$$X_{n+1} = \begin{cases} 0, & X_n = 0 \\ \frac{1}{X_n mod(1)}, & X_n \epsilon (0, 1) \end{cases} \tag{10}$$

$$\frac{1}{X_n mod(1)} = \frac{1}{X_n} - \left\lfloor \frac{1}{X_n} \right\rfloor .$$

Iterative map Iterative map equation and parameters are given in Eq. (11).

$$X_{n+1} = sin\left(\frac{a\pi}{x_n}\right) \quad a = 0.7. \tag{11}$$

Logistic map Logistic map equation is given in Eq. (12).

$$X_{n+1} = aX_n\left(1 - X_n\right) \quad a = 4. \tag{12}$$

Piecewise map Piecewise map equation and parameters are given in Eq. (13).

$$x_{n+1} = \begin{cases} \frac{x_n}{P}, & 0 \le x_n < P \\ \frac{x_n - P}{0.5 - P}, & P \le x_n < 0.5 \\ \frac{1 - P - x_n}{0.5 - P}, & 0.5 \le x_n < 1 - P \\ \frac{1 - x_n}{P}, & 1 - P \le x_n < 1 \end{cases}, \quad P = 0.4. \tag{13}$$

Sine map Sine map equation and parameters are given in Eq. (14). In this article a value is taken as 4.

$$X_{n+1} = \frac{a}{4} sin\left(\pi x_n\right) \quad \text{for } 0 < a \le 4. \tag{14}$$

Singer map Singer map equation and parameters are given in Eq. (15).

$$X_{n+1} = \mu\left(7.86x_n - 23.31x_n^2 + 28.75x_n^3 - 13.302875x_n^4\right), \quad \mu = 1.07. \tag{15}$$

Sinusoidal map Sinusoidal map equation and parameters are given in Eq. (16).

$$X_{n+1} = ax_n^2 \sin\left(\pi X_n\right) \tag{16}$$

when $a = 2.3$ and $X_0 = 0.7$ are selected, it can be simplified as Eq. (17).

$$X_{n+1} = \sin\left(\pi X_n\right). \tag{17}$$

Tent map Tent map equation and parameters are given in Eq. (18).

$$X_{n+1} = \begin{cases} X_n/0.7, & X_n < 0.7 \\ 10/3X_n(1 - X_n), & \text{otherwise} \end{cases}. \tag{18}$$

The graphics of the ten chaotic maps are shown in Fig. 1.

4 Proposed chaotic map slime mould optimization algorithm

In this section, the determination of the parameters of the proposed (novel) SMA method with chaotic maps is explained. The steps of the proposed CSMAs are described below. In the basic working principle of the SMA method, the region to be discovered according to the food concentration is expressed mathematically. The SMA method, like SM, scans the regions to be discovered according to the density of the food concentration. Randomization was used as shown in Eq. 5 to calculate the fitness weight of each slime mould for each slime mould (Li et al. 2020). The flowchart of the CSMAs is demonstrated in Fig. 2.

By using chaotic maps, the solution quality of optimization algorithms can be increased significantly. Although it has not been mathematically proven yet, it has been shown in different studies that metaheuristic optimization algorithms made by many researchers are improved by using chaotic maps. The results obtained in this study show that chaotic maps increase the performance of optimization methods. It may be more convenient to use chaotic maps instead of using random variables to search in SMA. Chaotic maps were used instead of random in Eq. 5 to increase convergence and overall working speed in SMA.*r*

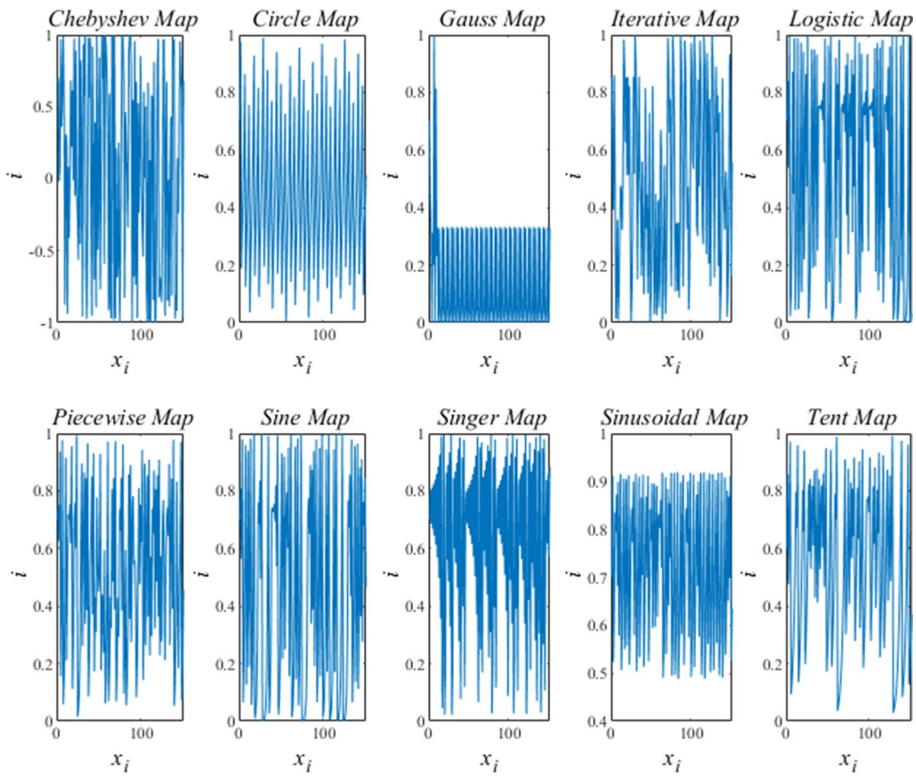

Fig. 1 Demonstration of chaotic maps

values of Eq. (5) with respect to the iterations are taken from the selected chaotic map and the update equation is replaced by Eq. (19):

$$\overrightarrow{W(SmellIndex(i))} = \begin{cases} 1 + c^{t+1} \cdot \log\left(\frac{bF - S(i)}{bF - wF} + 1\right), & condition \\ 1 - c^{t+1} \cdot \log\left(\frac{bF - S(i)}{bF - wF} + 1\right), & others \end{cases} \tag{19}$$

here t represents the current iteration value of the algorithm. The r value in Eq. (5) represents a randomly assigned sequence of numbers. In Eq. (19), c^{t+1} is used instead of this r. In c^{t+1} it shows the sequence of numbers created with chaotic maps. That is, it is expressed as $c^{t+1} = DC_{type}c^t$. Here DC represents discrete-time chaotic map function and type represents selected chaotic map. Proposed algorithms using 10 different chaotic; chebyshev map based SMA (CSMA-1), circle map based SMA (CSMA-2), gauss map based SMA (CSMA-3), iterative map based SMA (CSMA-4), logistic map based SMA (CSMA-5), piecewise map based SMA (CSMA-6), sine map based SMA (CSMA-7), singer map based SMA (CSMA-8), sinusoidal map based SMA (CSMA-9) and tent map based SMA (CSMA-10). The pseudo-code of CSMAs is given in Table 3.

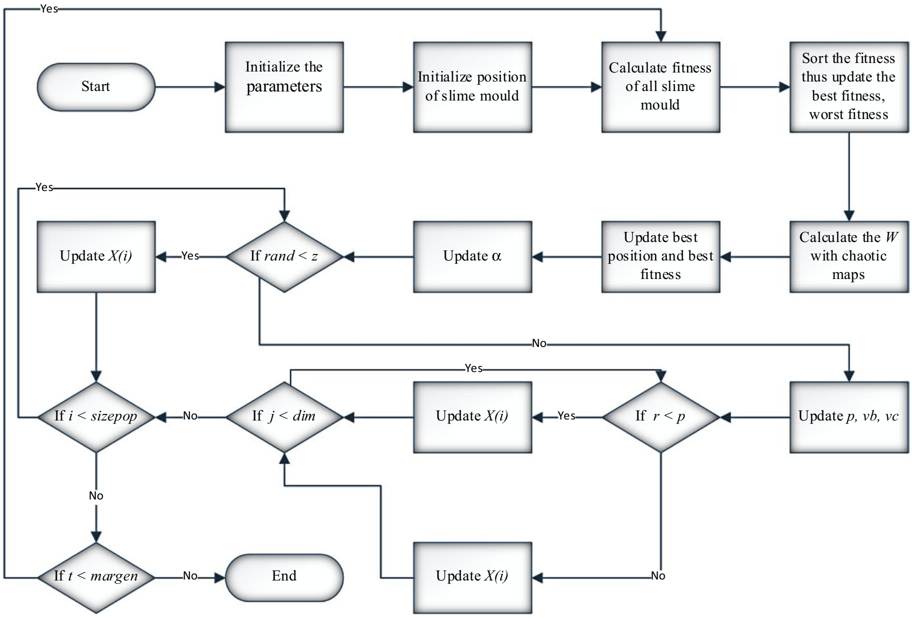

Fig. 2 Flowchart of CSMAs

Table 3 Pseudo-code of CSMAs

Algorithm 1 Pseudo-code of CSMAs
Initialize the parameters *pop size*, $Max\_iteraition$;
Initialize the positions of slime mould $X_i (i = 1,2, \ldots, n)$;
While $(t \leq Max\_iteration)$
Calculate the fitness of all slime mould;
$Update\ bestFitness,\ X_b$
Calculate the W by **Eq.** (5) $Using\ with\ chaotic\ maps$;
For $each\ search\ portion$
$Update\ p, vb, vc$;
$Update\ positions\ by$ **Eq.** (7);
End For
$t = t + 1$;
End While
Return $bestFitness,\ X_b$;

5 Experimental results and analyses

The CSMAs suggested in this section have been implemented in 62 benchmark functions and 3 real-engineering problems. Implementation results in CEC2019 test suite have been compared with the standard SMA, well-known DE (Storn and Price 1997), and PSO

(Kennedy and Eberhart 1995), and the recently proposed GWO (Mirjalili et al. 2014), and WOA (Mirjalili and Lewis 2016). In the CEC2017 test suite, the CSMA method has been compared with the standard SMA, WOA, GWO, HHO (Heiadari et al. 2019), AOA (Hashim et al. 2021), and COOT (Naruei and Keynia 2021) algorithms that have been proposed in recent years and statistical analyzes have been made. In addition, the application results in the real engineering problem were compared with the results of previous studies in the literature. The experimentations were run on the operating system of Windows 10 with 16 GB RAM and CPU of Intel (R) core i7-4790 k (4.00 GHz). The algorithms for comparison were coded by MATLAB R2018b.

5.1 Basic benchmark functions and CEC2019

33 different benchmark functions were used in this study. These are F1–F7 unimodal, F8–f13 multi-modal, f14–f23 fixed dimension multi-modal, and f24–f33 CEC2019 (Abdullah et al. 2019). These benchmark functions equations, dimension, range, and optimal values of functions are given in Tables 4, 5, 6 and 7. Dim indicates the size of the function; Range shows the domain of the function, f_{min} shows the optimum value of the function.

All algorithms have been run under the same conditions so that a fair evaluation can be made. The number of search agents has been taken as 30, dimension 30, the maximum number of iterations 1000, and repeated time 20 due to the stochastic nature of the algorithms. All optimization methods used in the comparison were run 20 times in order to minimize the effects of random factors in the algorithms' own structures on the results. In order to measure the experimental results, mean (AVG), Standard deviation (STD), maximum (MAX), and minimum (MIN) were used to evaluate the results. Parameter settings of SMA, DE, PSO, GWO, and WOA where CSMAs are compared are given in Table 8. The Parameter selections are based on commonly used parameters. Such as used by the original author in the article or parameters commonly used by various researchers.

In the study, it is aimed to increase the performance of SMA by using 10 different chaotic maps. Therefore, the first proposed CSMAs were compared with the standard SMA, and the superiority of the proposed CSMAs were proved by performing the Friedman test. Then, CSMA was compared with SMA, GWO, which are the current methods, and DE and PSO, which are the most used methods, and statistical analyzes were made with CSMA-SMA, CSMA-GWO, CSMA-DE, and CSMA-PSO by performing the Wilcoxon test.

Table 4 Unimodal benchmark functions

Unimodal functions	Dim	Range	f_{min}
$f_1(x) = \sum_{i=1}^{n} x_i^2$	n	[− 100, 100]	0
$f_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	n	[− 10, 10]	0
$f_3(x) = \sum_{i=1}^{n} \left(\sum_{j-1}^{i} x_j \right)^2$	n	[− 100, 100]	0
$f_4(x) = \max_i \left\{ \lvert x_i \rvert, 1 \le i \le n \right\}$	n	[− 100, 100]	0
$f_5(x) = \sum_{i=1}^{n-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + \left(x_i - 1\right)^2 \right]$	n	[− 30, 30]	0
$f_6(x) = \sum_{i=1}^{n} \left(\left[x_i + 0.5 \right] \right)^2$	n	[− 100, 100]	0
$f_7(x) = \sum_{i=1}^{n} ix_i^4 + random[0, 1]$	n	[− 128, 128]	0

Table 5 Multimodal benchmark functions

Multimodal functions	Dim	Range	f_{min}
$f_8(x) = \sum_{i=1}^{n} -x_i sin\left(\sqrt{\lvert x_i \rvert}\right)$	n	[− 500, 500]	− 418.9829*n
$f_9(x) = \sum_{i=1}^{n} \left[x_i^2 - 10\cos\left(2\pi x_i\right) + 10\right]$	n	[− 5.12, 5.12]	0
$f_{10}(x) = -20\exp\left(-0.2\left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right)^{0.5}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos\left(2\pi x_i\right)\right) + 20 + e$	n	[− 32, 32]	0
$f_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	n	[− 600, 600]	0
$f_{12}(x) = \frac{\pi}{n}\left\{10sin\left(\pi y_1\right) + \sum_{i=1}^{n-1}\left(y_i - 1\right)^2\left[1 + 10sin^2\left(\pi y_{i+1}\right)\right] + \left(y_n - 1\right)^2\right\}$ $+ \sum_{i=1}^{n} u\left(x_i, 10, 100, 4\right), y_i = 1 + \frac{x_i + 1}{4}$	n	[− 50, 50]	0
$u\left(x_i, a, k, m\right) = \begin{cases} k\left(x_i - a\right)^m x_i > a \\ 0 - a < x_i < a \\ k\left(-x_i - a\right)^m x_i < a \end{cases}$			
$f_{13}(x) = 0.1\left\{sin^2\left(3\pi x_1\right) + \sum_{i=1}^{n}\left(x_i - 1\right)^2\left[1 + sin^2\left(3\pi x_i + 1\right)\right]\right.$ $\left.+\left(x_n - 1\right)^2\left[1 + sin^2\left(2\pi x_n\right)\right]\right\} + \sum_{i=1}^{n} u\left(x_i, 5, 100, 4\right)$	n	[− 50, 50]	0

Table 6 Fixed-dimension multimodal benchmark functions

Fixed-dimension multimodal functions	Dim	Range	f_{min}
$f_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25}\frac{1}{j + \sum_{i=1}^{2}\left(x_i - a_{ij}\right)^6}\right)^{-1}$	2	[− 65, 65]	1
$f_{15}(x) = \sum_{i=1}^{11}\left[a_i - \frac{x_1\left(b_i^2 + b_i x_2\right)}{b_i^2 + b_i x_3 + x_4}\right]^2$	4	[− 5, 5]	0.00030
$f_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$	2	[− 5, 5]	− 1.0316
$f_{17}(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)cosx_1 + 10$	2	[− 5, 5]	0.398
$f_{18}(x) = \left[1 + \left(x_1 + x_2 + 1\right)^2\left(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2\right)\right]$ $\times\left[30 + \left(2x_1 - 3x_2\right)^2 \times \left(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2\right)\right]$	2	[− 2, 2]	3
$f_{19}(x) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{j=1}^{3} a_{ij}\left(x_j - p_{ij}\right)^2\right)$	3	[1, 3]	− 3.86
$f_{20}(x) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{j=1}^{6} a_{ij}\left(x_j - p_{ij}\right)^2\right)$	6	[0, 1]	− 3.32
$f_{21}(x) = -\sum_{i=1}^{5}\left[\left(X - a_i\right)\left(X - a_i\right)^T + c_i\right]^{-1}$	4	[0, 10]	− 10.1532
$f_{22}(x) = -\sum_{i=1}^{7}\left[\left(X - a_i\right)\left(X - a_i\right)^T + c_i\right]^{-1}$	4	[0, 10]	− 10.4028
$f_{23}(x) = -\sum_{i=1}^{10}\left[\left(X - a_i\right)\left(X - a_i\right)^T + c_i\right]^{-1}$	4	[0, 10]	− 105,363

Table 7 CEC-C06 2019 benchmark functions

Functions	Dim	Range	f_{min}
F24 Storn's Chebyshev Polynomial Fitting Problem	9	[− 8192, 8192]	1
F25 Inverse Hilbert Matrix Problem	16	[− 16,384, 16,384]	1
F26 Lennard–Jones Minimum Energy Cluster	18	[− 4, 4]	1
F27 Shifted Rotated Rastrigin's Function	10	[− 100, 100]	1
F28 Shifted Rotated Grienwangk's Function	10	[− 100, 100]	1
F29 Shifted Rotated Weierstrass Function	10	[− 100, 100]	1
F30 Modified Schwefel's Function	10	[− 100, 100]	1
F31 Expanded Schaffer's Function	10	[− 100, 100]	1
F32 Shifted Rotated Happy Cat Function	10	[− 100, 100]	1
F33 Shifted Rotated Ackley Function	10	[− 100, 100]	1

Table 8 Parameter settings of other metaheuristic optimization algorithms

Algorithm	Parameter settings
SMA	$z = 0.03$
DE	*Lower bound of scaling factor* = 0.2; *Upper bound of scaling factor* = 0.2; *crossover probability* = 0.2
PSO	$C1 = 1.5$; $c2 = 2.0$; *inertia weight* = 1; *inertia damping ratio* = 0.99
GWO	$a = [2, 0]$
WOA	$A1 = [2, 0]; a2 = [-2, -1]; b = 1$

All benchmark function results obtained from CSMAs and standard SMA are given in the Table 9. The comparison of algorithms in benchmark functions is based on the average value. When Table 9 is examined; It has been observed that the proposed CSMAs are more successful than standard SMA in all unimodal functions except F1 and F3 functions. It is seen that F1 and F3 functions do not have any superiority over each other. The use of bold within the all tables shows the best result obtained.

In multimodal benchmark functions; all of CSMAs in F12 and F13 showed good performance compared to the standard SMA. It has been observed that the standard SMA and CSMAs show the same performances in the F8–F11.

In fixed dimension benchmark functions; Gauss map based SMA from CSMA has achieved the best performance in F15. All of the methods compared in F16-F19 showed the same performance. It has been observed that CSMAs in F14, F20-F23 show the same performance as the standard SMA.

In CEC2019 benchmark functions; it has been observed that sinusoidal map based SMA in F29 and gauss map based SMA in F30 and F33 give better results than all other algorithms. All methods used in F25 and F26 showed the same performance. Most of the CSMAs in F27, F28, F31, and F32 performed better than the standard SMA. When we examine the benchmark functions of CEC2019 in general, the proposed chaotic based SMAs in all functions except F25 and F26 produced better results than the standard SMA.

Table 9 Comparison results on benchmark functions

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
	F1					F2				
CSMA-1	0	0	0	0	6.00	5.11E−225	0	1.02E−223	**0**	5.95
CSMA-2	0	0	0	0	6.00	7.58E−210	0	1.52E−208	**0**	5.63
CSMA-3	0	0	0	0	6.00	**3.97E−246**	0	3.23E−245	**0**	2.60
CSMA-4	0	0	0	0	6.00	1.97E−236	0	3.94E−235	**0**	3.25
CSMA-5	0	0	0	0	6.00	3.13E−233	0	6.25E−232	**0**	3.93
CSMA-6	0	0	0	0	6.00	6.49E−228	0	1.26E−226	**0**	6.90
CSMA-7	0	0	0	0	6.00	1.29E−224	0	1.27E−223	8.02E−277	9.90
CSMA-8	0	0	0	0	6.00	6.75E−235	0	1.15E−233	**0**	7.03
CSMA-9	0	0	0	0	6.00	4.89E−226	0	9.77E−225	**0**	7.28
CSMA-10	0	0	0	0	6.00	3.68E−239	0	4.44E−238	**0**	4.20
SMA	0	0	0	0	6.00	4.78E−176	0	9.09E−175	1.7E−244	9.35
	F3					F4				
CSMA-1	0	0	0	0	6.00	2.15E−237	0	4.14E−236	0	8.70
CSMA-2	0	0	0	0	6.00	8.43E−236	0	1.57E−234	0	7.53
CSMA-3	0	0	0	0	6.00	3.61E−222	0	3.66E−221	0	5.98
CSMA-4	0	0	0	0	6.00	**1.64E−257**	0	1.95E−256	0	9.75
CSMA-5	0	0	0	0	6.00	2.32E−234	0	3.87E−233	0	2.53
CSMA-6	0	0	0	0	6.00	1.95E−245	0	3.90E−244	0	6.88
CSMA-7	0	0	0	0	6.00	3.73E−229	0	7.45E−228	0	1.93
CSMA-8	0	0	0	0	6.00	1.22E−237	0	2.45E−236	0	8.45
CSMA-9	0	0	0	0	6.00	2.31E−246	0	4.60E−245	0	5.03
CSMA-10	0	0	0	0	6.00	5.10E−241	0	5.78E−240	0	3.68
SMA	0	0	0	0	6.00	1.24E−197	0	2.36E−196	2.15E−264	5.58
	F5					F6				
CSMA-1	1.48E−01	1.19E−01	3.72E−01	4.11E−04	5.70	9.58E−04	3.07E−04	1.50E−03	3.60E−04	6.90
CSMA-2	1.17E−01	9.57E−02	2.96E−01	1.55E−05	3.20	**9.77E−04**	6.18E−04	2.54E−03	**9.11E−06**	5.65
CSMA-3	1.00E−01	7.96E−02	2.48E−01	1.21E−03	2.25	8.95E−04	4.07E−04	1.63E−03	3.08E−04	4.85
CSMA-4	1.78E−01	**9.70E−02**	3.69E−01	**3.68E−05**	7.60	9.57E−04	3.74E−04	1.57E−03	4.19E−04	7.05
CSMA-5	1.39E−01	9.21E−02	2.92E−01	1.74E−04	4.35	9.02E−04	4.45E−04	1.68E−03	7.33E−05	5.85

Table 9 (continued)

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
CSMA-6	**2.04E−01**	8.47E−02	3.14E−01	1.16E−02	9.55	9.59E−04	4.84E−04	1.87E−03	3.69E−04	6.05
CSMA-7	1.40E−01	8.60E−02	3.24E−01	1.75E−03	4.55	7.96E−04	2.83E−04	1.21E−03	1.31E−04	5.45
CSMA-8	1.93E−01	1.08E−01	4.02E−01	7.88E−03	9.35	7.64E−04	1.68E−04	1.04E−03	4.36E−04	4.45
CSMA-9	1.30E−01	8.68E−02	2.72E−01	3.40E−03	4.25	8.70E−04	2.74E−04	1.26E−03	3.80E−04	6.65
CSMA-10	1.72E−01	9.06E−02	3.61E−01	2.11E−02	7.80	7.99E−04	2.23E−04	1.19E−03	4.08E−04	5.25
SMA	1.63E+00	5.83E+00	2.70E+01	1.12E−01	7.40	1.25E−03	4.25E−04	2.43E−03	8.11E−04	**7.85**
	F7					F8				
CSMA-1	4.67E−05	3.32E−05	1.00E−04	5.20E−07	4.40	−1.26E+04	9.56E−02	−1.26E+04	−1.26E+04	6.30
CSMA-2	5.16E−05	2.69E−05	1.05E−04	4.72E−06	7.05	−1.26E+04	6.65E−02	−1.26E+04	−1.26E+04	5.60
CSMA-3	3.98E−05	2.83E−05	1.10E−04	9.68E−07	3.10	− 1.26E+04	1.69E−02	− 1.26E+04	− 1.26E+04	4.10
CSMA-4	5.17E−05	3.33E−05	1.33E−04	1.34E−05	6.80	− 1.26E+04	8.67E−02	− 1.26E+04	− 1.26E+04	7.40
CSMA-5	4.50E−05	3.43E−05	1.22E−04	3.18E−06	4.60	− 1.26E+04	5.53E−02	− 1.26E+04	− 1.26E+04	5.95
CSMA-6	5.53E−05	4.04E−05	1.16E−04	1.18E−06	6.45	− 1.26E+04	7.41E−02	− 1.26E+04	− 1.26E+04	6.70
CSMA-7	3.76E−05	2.05E−05	**7.21E−05**	3.46E−06	3.35	− 1.26E+04	9.72E−02	− 1.26E+04	− 1.26E+04	**7.80**
CSMA-8	**7.85E−05**	4.21E−05	1.56E−04	9.08E−06	10.35	− 1.26E+04	9.13E−02	− 1.26E+04	− 1.26E+04	6.05
CSMA-9	3.69E−05	1.98E−05	8.11E−05	3.55E−06	3.35	− 1.26E+04	1.26E−02	− 1.26E+04	− 1.26E+04	3.00
CSMA-10	5.62E−05	3.33E−05	1.25E−04	9.72E−06	8.10	− 1.26E+04	7.04E−02	− 1.26E+04	− 1.26E+04	6.70
SMA	1.21E−04	**7.71E−05**	3.95E−04	5.03E−05	8.45	− 1.26E+04	7.78E−02	− 1.26E+04	− 1.26E+04	6.40
	F9					F10				
CSMA-1	0	0	0	0	6.00	8.88E−16	0	8.88E−16	8.88E−16	6.00
CSMA-2	0	0	0	0	6.00	8.88E−16	0	8.88E−16	8.88E−16	6.00
CSMA-3	0	0	0	0	6.00	8.88E−16	0	8.88E−16	8.88E−16	6.00
CSMA-4	0	0	0	0	6.00	8.88E−16	0	8.88E−16	8.88E−16	6.00
CSMA-5	0	0	0	0	6.00	8.88E−16	0	8.88E−16	8.88E−16	6.00
CSMA-6	0	0	0	0	6.00	8.88E−16	0	8.88E−16	8.88E−16	6.00
CSMA-7	0	0	0	0	6.00	8.88E−16	0	8.88E−16	8.88E−16	6.00
CSMA-8	0	0	0	0	6.00	8.88E−16	0	8.88E−16	8.88E−16	6.00
CSMA-9	0	0	0	0	6.00	8.88E−16	0	8.88E−16	8.88E−16	6.00

Table 9 (continued)

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
CSMA-10	0	0	0	0	6.00	8.88E−16	0	8.88E−16	8.88E−16	6.00
SMA	0	0	0	0	6.00	8.88E−16	0	8.88E−16	8.88E−16	6.00
	F11					F12				
CSMA-1	0	0	0	0	6.00	3.19E−04	2.89E−04	**9.70E−04**	1.91E−06	6.75
CSMA-2	0	0	0	0	6.00	3.09E−04	2.30E−04	8.78E−04	4.76E−07	6.00
CSMA-3	0	0	0	0	6.00	2.54E−04	2.33E−04	7.08E−04	1.20E−06	2.75
CSMA-4	0	0	0	0	6.00	**4.21E−04**	3.67E−04	1.13E−03	**8.93E−09**	7.30
CSMA-5	0	0	0	0	6.00	2.73E−04	2.64E−04	8.26E−04	1.76E−06	3.75
CSMA-6	0	0	0	0	6.00	3.51E−04	4.05E−04	1.29E−03	3.89E−07	5.35
CSMA-7	0	0	0	0	6.00	2.58E−04	2.57E−04	7.11E−04	4.99E−07	3.55
CSMA-8	0	0	0	0	6.00	3.27E−04	2.57E−04	7.83E−04	2.39E−06	7.35
CSMA-9	0	0	0	0	6.00	3.13E−04	2.59E−04	8.68E−04	5.52E−06	6.70
CSMA-10	0	0	0	0	6.00	3.34E−04	2.65E−04	8.47E−04	1.57E−05	8.10
SMA	0	0	0	0	6.00	1.02E−03	**7.16E−04**	2.69E−03	2.36E−04	**8.40**
	F13					F14				
CSMA-1	6.16E−04	3.22E−04	1.12E−03	3.30E−05	7.40	9.98E−01	4.71E−14	9.98E−01	9.98E−01	6.00
CSMA-2	4.41E−04	2.42E−04	7.55E−04	1.46E−05	2.30	9.98E−01	1.26E−13	9.98E−01	9.98E−01	6.00
CSMA-3	4.72E−04	2.55E−04	9.26E−04	4.10E−05	3.10	9.98E−01	5.17E−15	9.98E−01	9.98E−01	6.00
CSMA-4	6.35E−04	2.25E−04	**9.94E−04**	2.58E−04	7.30	9.98E−01	1.75E−13	9.98E−01	9.98E−01	6.00
CSMA-5	**6.76E−04**	2.23E−04	9.79E−04	1.86E−04	**8.95**	9.98E−01	8.13E−14	9.98E−01	9.98E−01	6.00
CSMA-6	6.45E−04	**3.23E−04**	1.06E−03	2.95E−05	8.60	9.98E−01	1.29E−13	9.98E−01	9.98E−01	6.00
CSMA-7	4.89E−04	2.96E−04	9.17E−04	**3.12E−07**	2.90	9.98E−01	2.34E−13	9.98E−01	9.98E−01	6.00
CSMA-8	5.56E−04	2.65E−04	9.63E−04	3.31E−05	5.60	9.98E−01	3.92E−13	9.98E−01	9.98E−01	6.00
CSMA-9	5.67E−04	3.25E−04	1.05E−03	2.06E−06	6.10	9.98E−01	8.72E−15	9.98E−01	9.98E−01	6.00
CSMA-10	5.49E−04	3.13E−04	9.78E−04	8.35E−06	5.05	9.98E−01	2.57E−13	9.98E−01	9.98E−01	6.00
SMA	1.05E−03	2.41E−04	1.49E−03	6.42E−04	8.70	9.98E−01	1.50E−13	9.98E−01	9.98E−01	6.00
	F15					F16				
CSMA-1	3.33E−04	3.99E−05	4.54E−04	3.08E−04	2.15	− 1.03E+00	1.76E−11	− 1.03E+00	− 1.03E+00	6.00

Table 9 (continued)

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
CSMA-2	3.45E−04	3.72E−05	4.30E−04	3.09E−04	6.45	− 1.03E+00	6.59E−11	− 1.03E+00	− 1.03E+00	6.00
CSMA-3	**3.29E−04**	2.28E−05	3.85E−04	3.08E−04	3.25	− 1.03E+00	1.38E−11	− 1.03E+00	− 1.03E+00	6.00
CSMA-4	3.44E−04	2.73E−05	3.92E−04	3.08E−04	7.10	− 1.03E+00	1.80E−10	− 1.03E+00	− 1.03E+00	6.00
CSMA-5	3.53E−04	6.75E−05	5.98E−04	3.09E−04	6.80	− 1.03E+00	3.88E−11	− 1.03E+00	− 1.03E+00	6.00
CSMA-6	3.98E−04	1.08E−04	6.79E−04	3.08E−04	7.45	− 1.03E+00	8.30E−11	− 1.03E+00	− 1.03E+00	6.00
CSMA-7	3.43E−04	3.47E−05	4.19E−04	3.08E−04	4.90	− 1.03E+00	9.80E−11	− 1.03E+00	− 1.03E+00	6.00
CSMA-8	4.33E−04	1.41E−04	7.36E−04	3.08E−04	**9.30**	− 1.03E+00	5.29E−11	− 1.03E+00	− 1.03E+00	6.00
CSMA-9	3.42E−04	4.60E−05	5.04E−04	3.08E−04	4.55	− 1.03E+00	2.22E−11	− 1.03E+00	− 1.03E+00	6.00
CSMA-10	3.69E−04	9.26E−05	6.66E−04	3.08E−04	6.20	− 1.03E+00	7.35E−11	− 1.03E+00	− 1.03E+00	6.00
SMA	5.27E−04	2.28E−04	1.22E−03	3.38E−04	7.85	− 1.03E+00	1.09E−10	− 1.03E+00	− 1.03E+00	6.00
	F17					F18				
CSMA-1	3.98E−01	1.79E−08	3.98E−01	3.98E−01	6.00	3.00E+00	9.55E−12	3.00E+00	3.00E+00	6.00
CSMA-2	3.98E−01	1.37E−08	3.98E−01	3.98E−01	6.00	3.00E+00	1.17E−11	3.00E+00	3.00E+00	6.00
CSMA-3	3.98E−01	3.37E−10	3.98E−01	3.98E−01	6.00	3.00E+00	9.16E−12	3.00E+00	3.00E+00	6.00
CSMA-4	3.98E−01	3.97E−08	3.98E−01	3.98E−01	6.00	3.00E+00	2.32E−12	3.00E+00	3.00E+00	6.00
CSMA-5	3.98E−01	6.16E−09	3.98E−01	3.98E−01	6.00	3.00E+00	4.18E−12	3.00E+00	3.00E+00	6.00
CSMA-6	3.98E−01	1.71E−08	3.98E−01	3.98E−01	6.00	3.00E+00	2.89E−11	3.00E+00	3.00E+00	6.00
CSMA-7	3.98E−01	1.01E−08	3.98E−01	3.98E−01	6.00	3.00E+00	4.81E−12	3.00E+00	3.00E+00	6.00
CSMA-8	3.98E−01	4.21E−08	3.98E−01	3.98E−01	6.00	3.00E+00	5.74E−11	3.00E+00	3.00E+00	6.00
CSMA-9	3.98E−01	7.30E−10	3.98E−01	3.98E−01	6.00	3.00E+00	8.16E−12	3.00E+00	3.00E+00	6.00
CSMA-10	3.98E−01	1.73E−08	3.98E−01	3.98E−01	6.00	3.00E+00	6.09E−12	3.00E+00	3.00E+00	6.00
SMA	3.98E−01	1.23E−08	3.98E−01	3.98E−01	6.00	3.00E+00	4.34E−12	3.00E+00	3.00E+00	6.00
	F19					F20				
CSMA-1	− 3.86E+00	1.55E−08	− 3.86E+00	− 3.86E+00	6.00	− 3.23E+00	4.85E−02	− 3.20E+00	− 3.32E+00	6.00
CSMA-2	− 3.86E+00	3.04E−08	− 3.86E+00	− 3.86E+00	6.00	− 3.24E+00	5.70E−02	− 3.20E+00	− 3.32E+00	6.00
CSMA-3	− 3.86E+00	3.52E−09	− 3.86E+00	− 3.86E+00	6.00	**− 3.25E+00**	5.99E−02	− 3.20E+00	− 3.32E+00	6.00
CSMA-4	− 3.86E+00	3.75E−08	− 3.86E+00	− 3.86E+00	6.00	− 3.23E+00	5.35E−02	− 3.20E+00	− 3.32E+00	6.00
CSMA-5	− 3.86E+00	1.98E−08	− 3.86E+00	− 3.86E+00	6.00	**− 3.25E+00**	5.83E−02	− 3.20E+00	− 3.32E+00	6.00

Table 9 (continued)

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
CSMA-6	− 3.86E+00	5.66E−08	− 3.86E+00	− 3.86E+00	6.00	− 3.23E+00	5.35E−02	− 3.20E+00	− 3.32E+00	6.00
CSMA-7	− 3.86E+00	3.27E−08	− 3.86E+00	− 3.86E+00	6.00	− 3.25E+00	5.92E−02	− 3.20E+00	− 3.32E+00	6.00
CSMA-8	− 3.86E+00	5.09E−08	− 3.86E+00	− 3.86E+00	6.00	− 3.24E+00	5.72E−02	− 3.20E+00	− 3.32E+00	6.00
CSMA-9	− 3.86E+00	2.67E−09	− 3.86E+00	− 3.86E+00	6.00	− 3.23E+00	5.35E−02	− 3.20E+00	− 3.32E+00	6.00
CSMA-10	− 3.86E+00	1.77E−08	− 3.86E+00	− 3.86E+00	6.00	− 3.23E+00	5.35E−02	− 3.20E+00	− 3.32E+00	6.00
SMA	− 3.86E+00	2.93E−08	− 3.86E+00	− 3.86E+00	6.00	− 3.23E+00	5.35E−02	− 3.20E+00	− 3.32E+00	6.00
	F21					F22				
CSMA-1	− 1.02E+01	8.16E−05	− 1.02E+01	− 1.02E+01	6.00	− 1.04E+01	8.53E−05	− 1.04E+01	− 1.04E+01	6.00
CSMA-2	− 1.02E+01	1.15E−04	− 1.02E+01	− 1.02E+01	6.00	− 1.04E+01	5.15E−05	− 1.04E+01	− 1.04E+01	6.00
CSMA-3	− 1.02E+01	1.76E−06	− 1.02E+01	− 1.02E+01	6.00	− 1.04E+01	2.97E−06	− 1.04E+01	− 1.04E+01	6.00
CSMA-4	− 1.02E+01	8.20E−05	− 1.02E+01	− 1.02E+01	6.00	− 1.04E+01	8.54E−05	− 1.04E+01	− 1.04E+01	6.00
CSMA-5	− 1.02E+01	9.51E−05	− 1.02E+01	− 1.02E+01	6.00	− 1.04E+01	8.59E−05	− 1.04E+01	− 1.04E+01	6.00
CSMA-6	− 1.02E+01	6.70E−05	− 1.02E+01	− 1.02E+01	6.00	− 1.04E+01	5.92E−05	− 1.04E+01	− 1.04E+01	6.00
CSMA-7	− 1.02E+01	6.24E−05	− 1.02E+01	− 1.02E+01	6.00	− 1.04E+01	7.07E−05	− 1.04E+01	− 1.04E+01	6.00
CSMA-8	− 1.02E+01	1.07E−04	− 1.02E+01	− 1.02E+01	6.00	− 1.04E+01	9.82E−05	− 1.04E+01	− 1.04E+01	6.00
CSMA-9	− 1.02E+01	2.35E−06	− 1.02E+01	− 1.02E+01	6.00	− 1.04E+01	2.64E−06	− 1.04E+01	− 1.04E+01	6.00
CSMA-10	− 1.02E+01	1.16E−04	− 1.02E+01	− 1.02E+01	6.00	− 1.04E+01	8.55E−05	− 1.04E+01	− 1.04E+01	6.00
SMA	− 1.02E+01	9.63E−05	− 1.02E+01	− 1.02E+01	6.00	− 1.04E+01	9.74E−05	− 1.04E+01	− 1.04E+01	6.00
	F23					F24				
CSMA-1	− 1.05E+01	7.01E−05	− 1.05E+01	− 1.05E+01	6.00	4.53E+04	3.10E+03	5.17E+04	3.92E+04	5.20
CSMA-2	− 1.05E+01	6.47E−05	− 1.05E+01	− 1.05E+01	6.00	5.21E+04	1.13E+04	8.31E+04	4.05E+04	7.60
CSMA-3	− 1.05E+01	3.18E−06	− 1.05E+01	− 1.05E+01	6.00	4.61E+04	2.38E+03	5.05E+04	4.12E+04	6.75
CSMA-4	− 1.05E+01	7.20E−05	− 1.05E+01	− 1.05E+01	6.00	4.70E+04	2.55E+03	5.04E+04	4.15E+04	**8.65**
CSMA-5	− 1.05E+01	1.47E−04	− 1.05E+01	− 1.05E+01	6.00	4.65E+04	3.08E+03	5.21E+04	4.06E+04	8.05
CSMA-6	− 1.05E+01	5.91E−05	− 1.05E+01	− 1.05E+01	6.00	4.46E+04	1.89E+03	4.76E+04	4.05E+04	3.15
CSMA-7	− 1.05E+01	8.27E−05	− 1.05E+01	− 1.05E+01	6.00	**4.45E+04**	2.37E+03	4.84E+04	3.94E+04	3.20
CSMA-8	− 1.05E+01	1.05E−04	− 1.05E+01	− 1.05E+01	6.00	4.56E+04	4.35E+03	5.31E+04	3.79E+04	4.70
CSMA-9	− 1.05E+01	2.21E−06	− 1.05E+01	− 1.05E+01	6.00	4.46E+04	2.16E+03	4.87E+04	4.08E+04	3.25

Table 9 (continued)

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
CSMA-10	− 1.05E+01	9.28E−05	− 1.05E+01	− 1.05E+01	6.00	4.67E+04	2.66E+03	5.27E+04	4.14E+04	8.10
SMA	− 1.05E+01	6.72E−05	− 1.05E+01	− 1.05E+01	6.00	5.05E+04	7.25E+03	7.19E+04	4.53E+04	7.35
	F25					F26				
CSMA-1	1.73E+01	9.53E−05	1.73E+01	1.73E+01	6.00	1.27E+01	1.78E−04	1.27E+01	1.27E+01	6.00
CSMA-2	1.73E+01	1.23E−04	1.73E+01	1.73E+01	6.00	1.27E+01	1.16E−04	1.27E+01	1.27E+01	6.00
CSMA-3	1.73E+01	8.56E−05	1.73E+01	1.73E+01	6.00	1.27E+01	2.05E−05	1.27E+01	1.27E+01	6.00
CSMA-4	1.73E+01	1.21E−04	1.73E+01	1.73E+01	6.00	1.27E+01	2.30E−05	1.27E+01	1.27E+01	6.00
CSMA-5	1.73E+01	1.52E−04	1.73E+01	1.73E+01	6.00	1.27E+01	1.22E−04	1.27E+01	1.27E+01	6.00
CSMA-6	1.73E+01	1.52E−04	1.73E+01	1.73E+01	6.00	1.27E+01	1.53E−05	1.27E+01	1.27E+01	6.00
CSMA-7	1.73E+01	1.43E−04	1.73E+01	1.73E+01	6.00	1.27E+01	2.06E−04	1.27E+01	1.27E+01	6.00
CSMA-8	1.73E+01	1.23E−04	1.73E+01	1.73E+01	6.00	1.27E+01	2.24E−05	1.27E+01	1.27E+01	6.00
CSMA-9	1.73E+01	1.33E−04	1.73E+01	1.73E+01	6.00	1.27E+01	1.07E−04	1.27E+01	1.27E+01	6.00
CSMA-10	1.73E+01	1.42E−04	1.73E+01	1.73E+01	6.00	1.27E+01	6.98E−05	1.27E+01	1.27E+01	6.00
SMA	1.73E+01	1.43E−04	1.73E+01	1.73E+01	6.00	1.27E+01	1.83E−04	1.27E+01	1.27E+01	6.00
	F27					F28				
CSMA-1	**1.32E+01**	3.97E+00	1.89E+01	4.98E+00	1.15	1.17E+00	4.33E−02	1.24E+00	1.09E+00	3.70
CSMA-2	1.70E+01	3.59E+00	2.39E+01	9.95E+00	**9.30**	**1.16E+00**	3.64E−02	1.22E+00	1.09E+00	3.25
CSMA-3	1.79E+01	5.30E+00	2.49E+01	5.98E+00	7.30	1.18E+00	5.99E−02	1.28E+00	1.06E+00	5.70
CSMA-4	1.50E+01	4.06E+00	2.09E+01	4.00E+00	4.00	1.19E+00	6.47E−02	1.29E+00	1.06E+00	7.00
CSMA-5	1.47E+01	4.39E+00	2.09E+01	6.00E+00	3.40	1.22E+00	8.03E−02	1.35E+00	1.08E+00	**9.60**
CSMA-6	1.48E+01	3.54E+00	1.89E+01	7.96E+00	3.60	1.20E+00	4.68E−02	1.28E+00	1.11E+00	8.00
CSMA-7	1.70E+01	4.44E+00	2.49E+01	8.96E+00	8.20	**1.16E+00**	3.67E−02	1.22E+00	1.07E+00	3.05
CSMA-8	1.57E+01	4.23E+00	2.19E+01	5.97E+00	5.25	1.18E+00	5.03E−02	1.25E+00	1.05E+00	5.85
CSMA-9	1.71E+01	3.97E+00	2.19E+01	7.96E+00	8.10	1.18E+00	4.81E−02	1.26E+00	1.08E+00	5.90
CSMA-10	1.67E+01	4.48E+00	2.29E+01	6.97E+00	7.60	1.18E+00	5.72E−02	1.27E+00	1.09E+00	4.95
SMA	2.43E+01	7.44E+00	3.98E+01	1.69E+01	8.10	1.32E+00	9.28E−02	1.50E+00	1.20E+00	9.00
	F29					F30				
CSMA-1	5.81E+00	8.95E−01	6.95E+00	4.06E+00	8.90	3.23E+01	9.16E+01	2.24E+02	− 1.41E+02	6.75

Table 9 (continued)

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
CSMA-2	5.29E+00	7.37E−01	6.38E+00	4.00E+00	4.95	4.30E+01	1.03E+02	1.87E+02	− 1.94E+02	7.95
CSMA-3	3.58E+00	6.86E−01	4.66E+00	1.60E+00	1.65	2.45E+01	7.73E+01	1.36E+02	− 1.35E+02	5.55
CSMA-4	5.79E+00	9.79E−01	7.00E+00	3.83E+00	8.85	3.42E+01	1.16E+02	2.19E+02	− 2.70E+02	7.30
CSMA-5	5.71E+00	7.47E−01	6.70E+00	4.30E+00	8.00	− 2.87E+01	1.06E+02	1.29E+02	− 2.57E+02	2.25
CSMA-6	5.70E+00	9.12E−01	7.02E+00	3.47E+00	8.35	3.94E+01	1.34E+02	1.68E+02	− 3.55E+02	7.75
CSMA-7	5.19E+00	9.02E−01	6.69E+00	3.20E+00	4.50	3.06E+01	8.04E+01	1.48E+02	− 1.23E+02	6.55
CSMA-8	5.46E+00	8.95E−01	6.90E+00	3.01E+00	6.25	1.78E+01	1.00E+02	1.65E+02	− 1.63E+02	4.80
CSMA-9	**3.54E+00**	7.05E−01	4.50E+00	1.61E+00	1.35	**− 2.74E+01**	9.55E+01	1.34E+02	− 2.27E+02	2.10
CSMA-10	5.08E+00	7.61E−01	6.46E+00	3.65E+00	3.95	1.68E+01	1.07E+02	1.85E+02	− 1.28E+02	5.50
SMA	6.92E+00	8.65E−01	8.84E+00	5.51E+00	9.25	2.52E+02	1.21E+02	5.50E+02	5.08E+01	9.50
	F31					F32				
CSMA-1	4.86E+00	5.58E−01	5.57E+00	3.43E+00	4.00	2.41E+00	2.82E−02	2.48E+00	2.35E+00	6.00
CSMA-2	**4.73E+00**	7.29E−01	5.51E+00	2.92E+00	3.15	2.41E+00	4.47E−02	2.52E+00	2.35E+00	6.00
CSMA-3	4.85E+00	6.55E−01	5.55E+00	3.13E+00	4.75	2.41E+00	4.67E−02	2.52E+00	2.36E+00	6.00
CSMA-4	4.87E+00	7.34E−01	5.86E+00	3.12E+00	5.20	2.41E+00	4.02E−02	2.56E+00	2.36E+00	6.00
CSMA-5	4.78E+00	7.64E−01	5.72E+00	2.86E+00	3.95	2.42E+00	3.24E−02	2.49E+00	2.36E+00	6.00
CSMA-6	5.21E+00	5.27E−01	5.83E+00	3.56E+00	9.80	2.41E+00	3.90E−02	2.51E+00	2.36E+00	6.00
CSMA-7	4.92E+00	4.80E−01	5.65E+00	4.07E+00	4.50	2.41E+00	5.49E−02	2.63E+00	2.35E+00	6.00
CSMA-8	5.07E+00	6.54E−01	5.93E+00	3.39E+00	8.50	**2.40E+00**	3.82E−02	2.53E+00	2.35E+00	6.00
CSMA-9	4.95E+00	5.92E−01	5.74E+00	3.78E+00	5.75	2.41E+00	3.66E−02	2.52E+00	2.36E+00	6.00
CSMA-10	5.05E+00	4.68E−01	5.77E+00	4.16E+00	7.15	2.41E+00	4.30E−02	2.53E+00	2.36E+00	6.00
SMA	5.76E+00	2.53E−01	6.20E+00	5.37E+00	9.25	2.41E+00	3.04E−02	2.47E+00	2.36E+00	6.00
	10-F33									
CSMA-1	2.01E+01	6.26E−02	2.01E+01	1.99E+01	6.30					
CSMA-2	1.91E+01	4.36E+00	2.02E+01	8.32E−02	8.80					
CSMA-3	**1.70E+01**	7.15E+00	2.01E+01	7.12E−03	1.95					
CSMA-4	2.01E+01	5.90E−02	2.02E+01	1.99E+01	7.20					
CSMA-5	1.91E+01	4.36E+00	2.01E+01	4.80E−02	3.85					

Table 9 (continued)

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
CSMA-6	2.01E+01	3.63E−02	2.01E+01	2.00E+01	4.75					
CSMA-7	1.81E+01	5.99E+00	2.01E+01	5.47E−02	5.75					
CSMA-8	1.91E+01	4.37E+00	2.02E+01	7.48E−02	9.40					
CSMA-9	1.80E+01	6.01E+00	2.00E+01	5.37E−03	1.50					
CSMA-10	2.01E+01	5.98E−02	2.02E+01	1.99E+01	5.85					
SMA	2.02E+01	5.81E−02	2.04E+01	2.01E+01	10.65					

Convergence graphs of some benchmark functions where CSMAs converge better than standard SMA are shown in the Fig. 3.

The obtained results from Table 9 were analyzed using the non-parametric Friedman test to determine if there was a significant difference between the standard SMA and CSMAs with different chaotic maps. The results were shown in Table 10. If $X_R^2 > X_{table}^2$, H_0 is rejected. For the Friedman test, the k-1 degree-of-freedom Chi-square distribution is applied. That is why for the table α = 0.05 and X_{table}^2 is 18.307. When looking at the results obtained from Table 9, in F2, F4–F8, F12, F13, F15, F24, F27-F31, F33 functions, H_0 is rejected due to $X_R^2 > X_{table}^2$ inequality. According to the results of the Friedman test, we can conclude that the difference between the results obtained from SMA and CSMAs is statistically significant. Since all the results are the same in the remaining functions, we can conclude that there is no statistical difference.

CSMA was compared with SMA, GWO, which are the current methods, and DE and PSO, which are the most used methods. These comparison results are shown in Table 11. Derrac et al. (2011) recommended that in order to evaluate the performance of algorithms, statistical tests should be done. This means a statistical test is mandatory in order to verify that the proposed algorithm shows a noteworthy enhancement in comparison with other algorithms. Therefore, a nonparametric statistical test, Wilcoxon's rank-sum test, is conducted at a 95% confidence interval (Wilcoxon et al. 1963). The *p* values calculated by the Wilcoxon's test are presented in the results as well. The *p* values reported using the Wilcoxon's test also prove that this superiority is statistically significant. Also; It has been shown that CSMA is superior to other methods by using the Wilcoxon's test.

When Table 11 is examined; according to the average results, the chaotic-based CSMA method proposed in 27 of 33 different functions finds the best result. If we express it as a percentage, CSMA produced better results in 81.82% of 33 different optimization processes. While the proposed CSMA produced the same result as SMA in 7 of the 33 functions, all methods found the same result in 6 functions. These results are also the optimum values of the function, that is, they produced the best result.

5.2 CEC2017 benchmark functions

The CEC2017 test suite (Awad et al. 2016) contain global optimization problems (Houssein et al. 2021a). First of all, the CSMAs we recommend were compared with the standard SMA using the CEC2017 test suite. Here, mean rank values of CSMAs were calculated by the Friedman test. Based on these mean rank values, the best CSMA was selected. The selected CSMA CEC2017 test suite have been compared with the standard SMA, WOA, GWO, HHO, AOA, and COOT optimization algorithms that have been proposed and popular in recent years. The CEC2017 test suite includes 29 different benchmark functions. These functions are divided into 4 different categories as unimodal shifted and rotated functions, multimodal shifted and rotated functions, hybrid functions, and composition functions. Parameters for algorithms in the CEC2017 test suite are given in Table 12. Except for the specific parameters of the algorithms, all the common parameters such as the number of iterations, search agent, and run are taken at the same values as the benchmark functions in 5.1. The lower and upper bounds of all functions included in the CEC 2017 test suite are between − 100 and 100. In order to evaluate the performances of the algorithms, the average (AVG), standard deviation (SD), maximum (MAX) and minimum (MIN) values for each method were calculated separately. CSMAs produced better solutions in all functions than the standard SMA. When the results in the Table 13 are

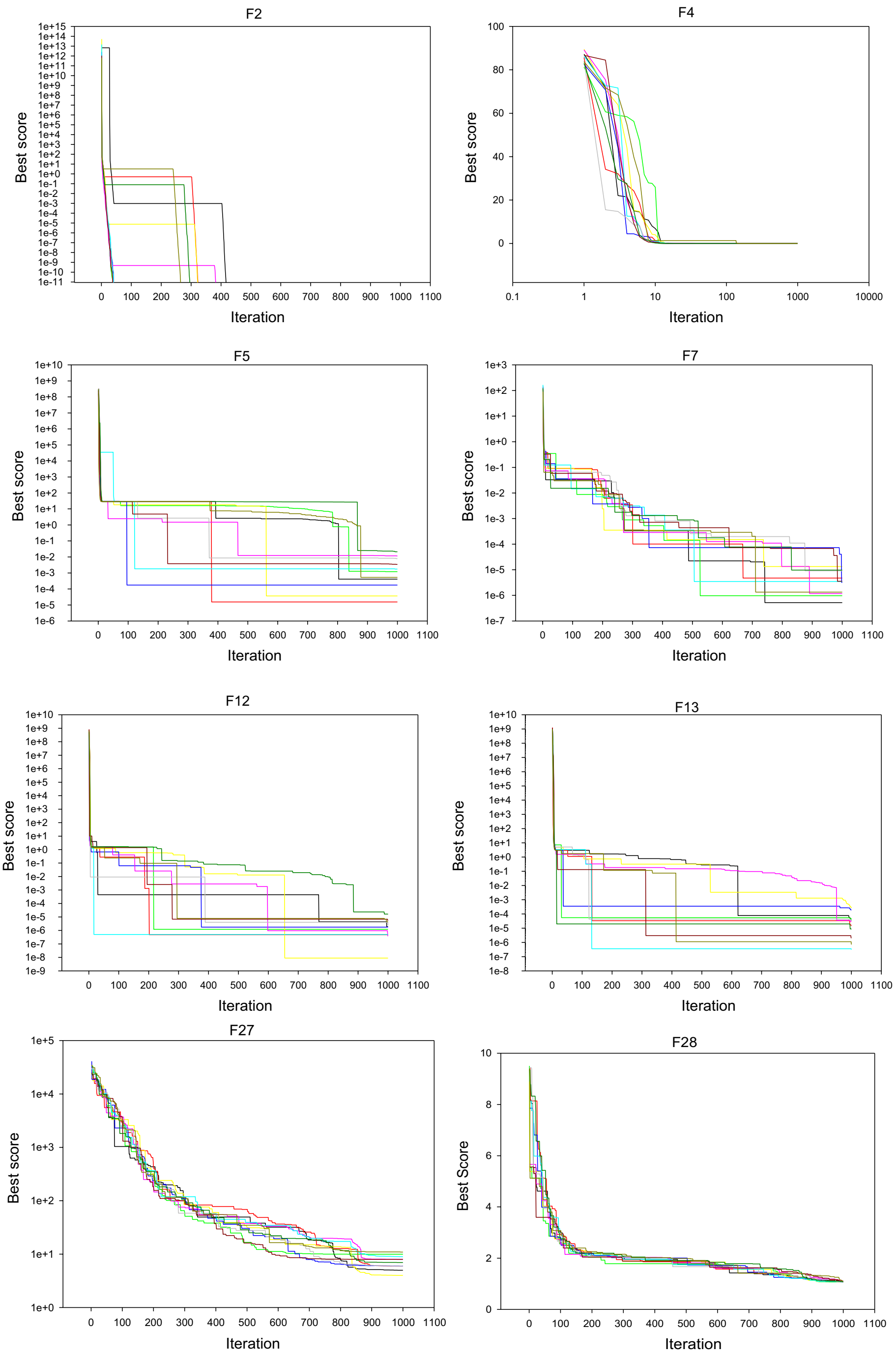

Fig. 3 Convergence curve of CSMAs and standard SMA

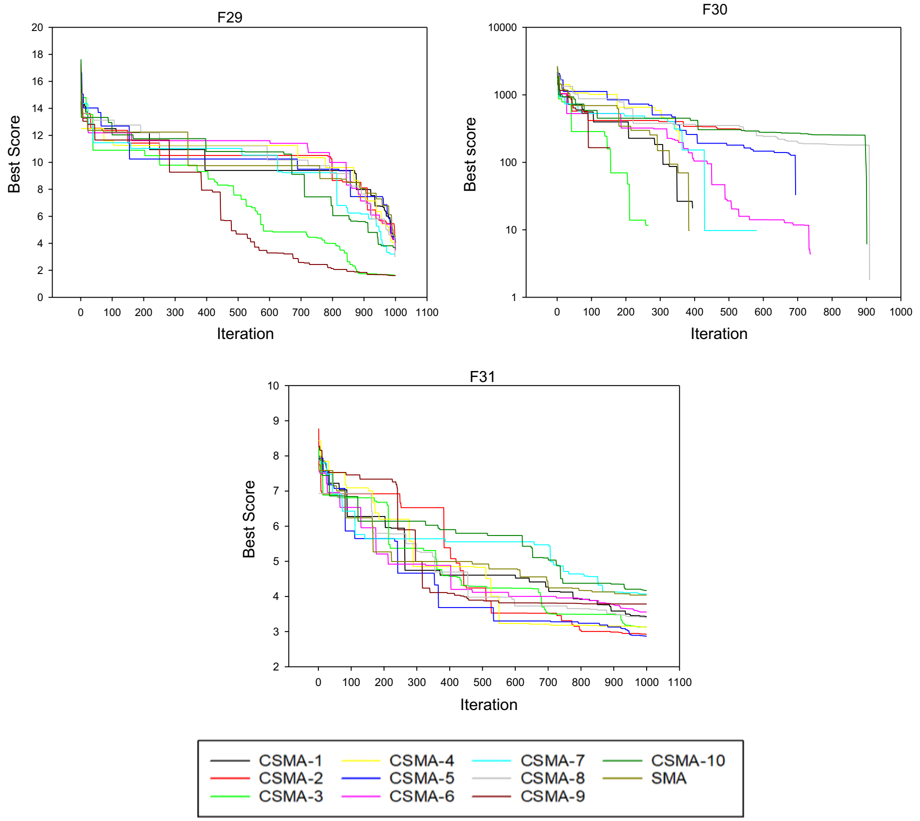

Fig. 3 (continued)

examined in detail, CSMA-3 produced the best solution in unimodal shifted and rotated functions (f1). It is seen that the exploitation ability of CSMA-3 is better than SMA in such problems. In multimodal shifted and rotated functions (f3–f10), 3 times CSMA-8, 2 times CSMA-10, 2 times CSMA-1 and 1 time CSMA-6 obtained the best values as seen in the table. These problems are used to evaluate the escape capabilities of optimization algorithms from two or more local optimum regions. These problems allow to evaluation of the local optima capability and discovery capabilities of optimization algorithms. The hybrid functions and composition functions categories have multiple local optimums to evaluate their avoidance of local minimum regions. For the algorithm to avoid the local minimum, it needs to balance exploration and exploitation. CSMAs have generally performed better than SMA in such problems. Considering all of the CEC2017 test suite, when the average of the mean rank values in the Table 13 is calculated, CSMA-1 from the CSMAs achieved the best result in the average of the Friedman average ranks. CSMA-1 was followed by CSMA-8 and CSMA-10. The obtained results from Table 13 were analyzed using the non-parametric Friedman test to determine if there was a significant difference between the standard SMA and CSMAs with different chaotic maps. The results were shown in Table 14.

Table 10 Friedman test results

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15	F16
18.307	–	108.419	–	127.753	111.755	19.082	105.918	34.918	–	–	–	66.882	107.609	–	80.373	–
H_0		Reject	–	Reject	Reject	Reject	Reject	Reject	–	–	–	Reject	Reject	–	Reject	–
	F17	F18	F19	F20	F21	F22	F23	F24	F25	F26	F27	F28	F29	F30	F31	F33
18.307	–	–	–	–	–	–	–	84.427	–	–	126.191	90.436	154.145	96.064	97.118	155.41
H_0	–	–	–	–	–	–	–	Reject	–	–	Reject	Reject	Reject	Reject	Reject	Reject

Table 11 Results obtained by comparing CSMA with other methods

Algorithm	AVG	STD	MAX	MIN	*p* values	AVG	STD	MAX	MIN	*p* values
	F1					F2				
CSMA	**0**	0	0	0	0.00	**3.97E−246**	0	3.23E−245	0	0.00
SMA	**0**	0	0	0	1.00	4.78E−176	0	9.09E−175	1.7E−244	1.00
DE	3.14E−12	1.86E−12	8.16E−12	6.82E−13	8.90E−05	3.44E−08	1.25E−08	8.09E−08	1.80E−08	8.90E−05
PSO	1.45E−08	2.04E−07	1.10E−06	1.05E−17	8.90E−05	3.73E−02	4.73E−02	1.95E−01	1.42E−03	8.90E−05
GWO	6.68E−59	1.08E−58	4.42E−58	5.06E−61	8.90E−05	8.25E−35	7.87E−35	2.99E−34	5.88E−36	8.90E−05
WOA	3.69E−147	1.97E−146	1.08E−145	2.66E−169	8.90E−05	4.89E−104	2.32E−103	1.27E−102	3.07E−115	8.90E−05
	F3					F4				
CSMA	**0**	0	0	0	0.00	**1.64E−257**	0	1.95E−256	0	0.00
SMA	**0**	0	0	0	1.00	1.24E−197	0	2.36E−196	2.15E−264	1.00
DE	2.50E+04	4.65E+03	3.51E+04	1.53E+04	8.90E−05	1.99E+00	4.28E−01	3.04E+00	1.19E+00	8.90E−05
PSO	7.29E+00	1.60E+01	7.81E+01	3.23E−01	8.90E−05	5.94E−01	3.61E−01	1.56E+00	8.53E−02	8.90E−05
GWO	1.07E−14	3.56E−14	1.92E−13	1.12E−19	8.90E−05	1.18E−14	1.66E−14	8.05E−14	9.58E−17	8.90E−05
WOA	1.86E+04	1.18E+04	4.20E+04	2.21E+03	8.90E−05	3.73E+01	3.13E+01	8.66E+01	2.44E−02	8.90E−05
	F5					F6				
CSMA	**2.04E−01**	8.47E−02	3.14E−01	1.16E−02	0.00	9.77E−04	6.18E−04	2.54E−03	9.11E−06	0.00
SMA	1.63E+00	5.83E+00	2.70E+01	1.12E−01	0.191	1.25E−03	4.25E−04	2.43E−03	8.11E−04	0.117
DE	5.08E+01	2.82E+01	9.61E+01	2.55E+01	8.90E−05	**3.57E−12**	1.97E−12	9.24E−12	1.30E−12	8.90E−05
PSO	4.07E+01	2.67E+01	1.02E+02	1.57E+01	8.90E−05	3.97E−05	2.16E−04	1.18E−03	1.80E−14	8.90E−05
GWO	2.71E+01	7.91E−01	2.87E+01	2.62E+01	8.90E−05	6.32E−01	3.38E−01	1.51E+00	2.24E−05	1.03E−04
WOA	2.73E+01	6.84E−01	2.88E+01	2.64E+01	8.90E−05	9.46E−02	1.28E−01	4.40E−01	8.26E−03	8.90E−05
	F7					F8				
CSMA	**7.85E−05**	4.21E−05	1.56E−04	9.08E−06	0.00	**− 1.26E+04**	9.56E−02	− 1.26E+04	− 1.26E+04	0.00
SMA	1.21E−04	7.71E−05	3.95E−04	5.03E−05	0.232	**− 1.26E+04**	7.78E−02	− 1.26E+04	− 1.26E+04	0.01
DE	2.81E−02	5.72E−03	4.02E−02	1.73E−02	8.90E−05	− 1.25E+04	1.09E+02	− 1.22E+04	− 1.26E+04	2.93E−04
PSO	1.59E−02	7.71E−03	3.50E−02	7.77E−03	8.90E−05	− 6.22E+03	8.83E+03	− 5.17E+03	− 8.58E+03	8.90E−05
GWO	9.55E−04	6.42E−04	2.97E−03	2.55E−04	8.90E−05	− 5.87E+03	8.93E+02	− 3.11E+03	− 7.16E+03	8.90E−05
WOA	1.80E−03	1.64E−03	5.42E−03	1.29E−04	1.03E−04	− 1.15E+04	1.38E+03	− 8.39E+03	− 1.26E+04	8.90E−05
	F9					F10				
CSMA	**0**	0	0	0	0.00	**8.88E−16**	0	8.88E−16	8.88E−16	0.00

Table 11 (continued)

Algorithm	AVG	STD	MAX	MIN	*p* values	AVG	STD	MAX	MIN	*p* values
SMA	**0**	0	0	0	1.00	**8.88E−16**	0	8.88E−16	8.88E−16	1.00
DE	5.87E+01	7.65E+00	7.40E+01	3.87E+01	8.90E−05	4.06E−07	1.11E−07	7.00E−07	1.96E−07	8.90E−05
PSO	5.30E+01	1.85E+01	1.14E+02	2.49E+01	8.90E−05	1.63E+00	9.08E−01	3.09E+00	1.21E−06	8.90E−05
GWO	1.57E−01	8.58E−01	4.70E+00	0.00E+00	8.90E−05	1.64E−14	3.55E−15	2.58E−14	7.99E−15	5.00E−05
WOA	**0**	0	0	0	1.00	4.44E−15	2.29E−15	7.99E−15	8.88E−16	3.12E−04
	F11					F12				
CSMA	**0**	0	0	0	0.00	4.21E−04	3.67E−04	1.13E−03	8.93E−09	0.00
SMA	**0**	0	0	0	1.00	1.02E−03	7.16E−04	2.69E−03	2.36E−04	0.040
DE	4.30E−10	1.55E−09	8.37E−09	6.38E−12	8.90E−05	**4.09E−13**	3.65E−13	1.50E−12	1.17E−13	8.90E−05
PSO	3.08E−02	3.81E−02	1.42E−01	1.16E−11	8.90E−05	9.86E−02	2.53E−01	1.14E+00	7.66E−17	0.313
GWO	9.50E−04	3.66E−03	1.64E−02	0.00E+00	8.90E−05	4.01E−02	2.26E−02	1.06E−01	1.96E−02	8.90E−05
WOA	4.61E−03	1.45E−02	5.92E−02	0.00E+00	8.90E−05	1.24E−02	1.66E−02	7.27E−02	1.12E−03	8.90E−05
	F13					F14				
CSMA	6.76E−04	2.23E−04	9.79E−04	1.86E−04	0.00	**9.98E−01**	8.72E−15	9.98E−01	9.98E−01	0.00
SMA	1.05E−03	2.41E−04	1.49E−03	6.42E−04	0.05	**9.98E−01**	1.50E−13	9.98E−01	9.98E−01	1.00
DE	**1.72E−12**	1.12E−12	4.35E−12	6.65E−13	8.90E−05	**9.98E−01**	0.00E+00	9.98E−01	9.98E−01	1.00
PSO	1.47E−01	3.00E−01	1.08E+00	6.73E−14	0.001	4.47E+00	3.32E+00	1.27E+01	9.98E−01	8.90E−05
GWO	4.48E−01	2.19E−01	1.03E+00	1.07E−01	8.90E−05	3.36E+00	3.29E+00	1.27E+01	9.98E−01	8.90E−05
WOA	1.94E−01	1.23E−01	4.76E−01	4.16E−02	8.90E−05	2.18E+00	2.48E+00	1.08E+01	9.98E−01	8.90E−05
	F15					F16				
CSMA	**4.33E−04**	1.41E−04	7.36E−04	3.08E−04	0.00	**− 1.03E+00**	8.30E−11	− 1.03E+00	− 1.03E+00	0.00
SMA	5.27E−04	2.28E−04	1.22E−03	3.38E−04	0.279	**− 1.03E+00**	1.09E−10	− 1.03E+00	− 1.03E+00	1.00
DE	6.72E−04	9.84E−05	8.13E−04	4.71E−04	0.001	**− 1.03E+00**	6.78E−16	− 1.03E+00	− 1.03E+00	1.00
PSO	1.31E−03	4.48E−03	2.04E−02	3.07E−04	0.002	**− 1.03E+00**	6.78E−16	− 1.03E+00	− 1.03E+00	1.00
GWO	3.38E−03	7.32E−03	2.04E−02	3.07E−04	0.411	**− 1.03E+00**	5.79E−09	− 1.03E+00	− 1.03E+00	1.00
WOA	5.54E−04	3.10E−04	1.49E−03	3.08E−04	0.263	**− 1.03E+00**	4.88E−11	− 1.03E+00	− 1.03E+00	1.00
	F17					F18				
CSMA	**3.98E−01**	7.30E−10	3.98E−01	3.98E−01		**3.00E+00**	9.16E−12	3.00E+00	3.00E+00	

Table 11 (continued)

Algorithm	AVG	STD	MAX	MIN	p values	AVG	STD	MAX	MIN	p values
SMA	**3.98E−01**	1.23E−08	3.98E−01	3.98E−01	1.00	**3.00E+00**	4.34E−12	3.00E+00	3.00E+00	1.00
DE	**3.98E−01**	0.00E+00	3.98E−01	3.98E−01	1.00	**3.00E+00**	1.30E−15	3.00E+00	3.00E+00	1.00
PSO	**3.98E−01**	0.00E+00	3.98E−01	3.98E−01	1.00	**3.00E+00**	6.65E−16	3.00E+00	3.00E+00	1.00
GWO	**3.98E−01**	5.70E−07	3.98E−01	3.98E−01	1.00	**3.00E+00**	8.36E−06	3.00E+00	3.00E+00	1.00
WOA	**3.98E−01**	1.16E−06	3.98E−01	3.98E−01	1.00	**3.00E+00**	2.09E−05	3.00E+00	3.00E+00	1.00
	F19					F20				
CSMA	**− 3.86E+00**	2.67E−09	− 3.86E+00	− 3.86E+00	0.00	− 3.25E+00	5.99E−02	− 3.20E+00	− 3.32E+00	0.00
SMA	**− 3.86E+00**	2.93E−08	− 3.86E+00	− 3.86E+00	1.00	− 3.23E+00	5.35E−02	− 3.20E+00	− 3.32E+00	1.00
DE	**− 3.86E+00**	2.71E−15	− 3.86E+00	− 3.86E+00	1.00	**− 3.31E+00**	2.60E−02	− 3.20E+00	− 3.32E+00	1.00
PSO	**− 3.86E+00**	2.65E−15	− 3.86E+00	− 3.86E+00	1.00	− 3.29E+00	5.11E−02	− 3.20E+00	− 3.32E+00	1.00
GWO	**− 3.86E+00**	2.23E−03	− 3.85E+00	− 3.86E+00	1.00	− 3.24E+00	7.74E−02	− 3.11E+00	− 3.32E+00	1.00
WOA	**− 3.86E+00**	3.33E−03	− 3.85E+00	− 3.86E+00	1.00	− 3.23E+00	1.15E−01	− 2.84E+00	− 3.32E+00	1.00
	F21					F22				
CSMA	**− 1.02E+01**	1.76E−06	− 1.02E+01	− 1.02E+01	0.00	**− 1.04E+01**	2.97E−06	− 1.04E+01	− 1.04E+01	0.00
SMA	**− 1.02E+01**	9.63E−05	− 1.02E+01	− 1.02E+01	0.117	**− 1.04E+01**	9.74E−05	− 1.04E+01	− 1.04E+01	0.654
DE	**− 1.02E+01**	3.69E−03	− 1.01E+00	− 1.02E+01	0.014	**− 1.04E+01**	1.34E−04	− 1.04E+01	− 1.04E+01	0.002
PSO	− 4.77E+00	3.27E+00	− 2.63E+00	− 1.02E+01	0.001	− 7.10E+00	3.48E+00	− 2.75E+00	− 1.04E+01	0.062
GWO	− 9.27E+00	2.20E+00	− 2.63E+00	− 1.02E+01	1.63E−04	**− 1.04E+01**	3.06E−04	− 1.04E+01	− 1.04E+01	0.001
WOA	− 9.64E+00	1.57E+00	− 5.06E+00	− 1.02E+01	8.90E−05	− 9.19E+00	2.47E+00	− 3.72E+00	− 1.04E+01	8.90E−05
	F23						F24			
CSMA	**− 1.05E+01**	3.18E−06	− 1.05E+01	− 1.05E+01	0.00	**4.45E+04**	2.37E+03	4.84E+04	3.94E+04	0.00
SMA	**− 1.05E+01**	6.72E−05	− 1.05E+01	− 1.05E+01	0.794	5.05E+04	7.25E+03	7.19E+04	4.53E+04	0.008
DE	**− 1.05E+01**	6.96E−14	− 1.05E+01	− 1.05E+01	8.90E−05	1.04E+10	5.98E+09	2.58E+10	1.53E+09	8.90E−05
PSO	− 7.10E+00	3.93E+00	− 1.68E+00	− 1.05E+01	0.145	1.90E+08	1.76E+08	6.62E+08	2.49E+07	8.90E−05
GWO	**− 1.05E+01**	2.44E−04	− 1.05E+01	− 1.05E+01	0.001	8.77E+07	9.17E+07	3.23E+08	7.22E+05	8.90E−05
WOA	− 7.60E+00	3.75E+00	− 1.68E+00	− 1.05E+01	8.90E−05	1.56E+10	2.53E+10	9.42E+10	1.27E+06	8.90E−05
	F25					F26				
CSMA-1	**1.73E+01**	9.53E−05	1.73E+01	1.73E+01	0.00	**1.27E+01**	2.05E−05	1.27E+01	1.27E+01	0.00

Table 11 (continued)

Algorithm	AVG	STD	MAX	MIN	p values	AVG	STD	MAX	MIN	p values
SMA	**1.73E+01**	1.43E−04	1.73E+01	1.73E+01	1.00	**1.27E+01**	1.83E−04	1.27E+01	1.27E+01	1.00
DE	**1.73E+01**	7.23E−15	17.34286	17.34286	1.00	**1.27E+01**	8.43E−07	1.27E+01	1.27E+01	1.00
PSO	**1.73E+01**	7.14E−15	1.73E+01	1.73E+01	1.00	**1.27E+01**	3.61E−15	1.27E+01	1.27E+01	1.00
GWO	**1.73E+01**	1.37E−04	1.73E+01	1.73E+01	1.00	**1.27E+01**	1.22E−06	1.27E+01	1.27E+01	1.00
WOA	**1.73E+01**	1.34E−02	1.74E+01	1.73E+01	1.00	**1.27E+01**	3.40E−08	1.27E+01	1.27E+01	1.00
	F27						F28			
CSMA	**1.32E+01**	3.97E+00	1.89E+01	4.98E+00	0.00	1.16E+00	3.73E−02	1.22E+00	1.09E+00	0.00
SMA	2.43E+01	7.44E+00	3.98E+01	1.69E+01	2.93E−04	1.32E+00	9.52E−02	1.50E+00	1.20E+00	1.63E−04
DE	1.48E+01	2.93E+00	1.88E+01	9.04E+00	0.279	**1.09E+00**	3.76E−02	1.17E+00	1.04E+00	3.38E−04
PSO	2.05E+01	6.86E+00	3.18E+01	5.97E+00	0.002	1.11E+00	3.58E−02	1.18E+00	1.06E+00	0.001
GWO	4.72E+01	2.50E+01	1.06E+02	1.71E+01	8.90E−05	1.28E+00	1.93E−01	2.74E+00	1.09E+00	0.012
WOA	2.55E+02	1.54E+02	6.88E+02	7.85E+01	8.90E−05	1.69E+00	3.54E−01	2.72E+00	1.24E+00	8.90E−05
	F29						F30			
CSMA	**3.54E+00**	7.05E−01	4.50E+00	1.61E+00	0.00	**− 2.74E+01**	9.55E+01	1.34E+02	− 2.27E+02	0.00
SMA	6.92E+00	8.65E−01	8.84E+00	5.51E+00	8.90E−05	2.52E+02	1.24E+02	5.50E+02	5.08E+01	2.19E−04
DE	7.25E+00	7.14E−01	8.30E+00	5.68E+00	8.90E−05	9.79E+01	9.88E+01	2.86E+02	− 1.58E+02	0.002
PSO	6.83E+00	1.48E+00	9.98E+00	4.64E+00	8.90E−05	1.65E+02	1.41E+02	4.26E+02	− 1.78E+02	4.49E−04
GWO	1.06E+01	5.32E−01	1.14E+01	9.79E+00	8.90E−05	3.62E+02	3.04E+02	1.19E+03	3.50E+01	1.03E−04
WOA	8.82E+00	9.26E−01	1.10E+01	6.79E+00	8.90E−05	5.40E+02	2.86E+02	1.01E+03	3.28E+01	8.90E−05
	F31						F32			
CSMA	**4.73E+00**	7.29E−01	5.51E+00	2.92E+00	0.00	2.41E+00	2.82E−02	2.48E+00	2.35E+00	0.00
SMA	5.76E+00	2.53E−01	6.20E+00	5.37E+00	2.19E−04	2.41E+00	3.04E−02	2.47E+00	2.36E+00	0.126
DE	5.19E+00	4.45E−01	5.74E+00	3.89E+00	0.019	2.41E+00	2.29E−02	2.45E+00	2.37E+00	0.823
PSO	4.98E+00	6.91E−01	6.08E+00	3.77E+00	0.351	**2.36E+00**	1.69E−02	2.42E+00	2.34E+00	0.001
GWO	5.04E+00	9.48E−01	6.36E+00	3.15E+00	0.247	4.37E+00	7.31E−01	5.79E+00	3.29E+00	8.90E−05
WOA	5.77E+00	4.37E−01	6.42E+00	4.78E+00	2.54E−04	4.27E+00	8.19E−01	6.20E+00	3.23E+00	8.90E−05
	F33									
CSMA	**1.70E+01**	7.15E+00	2.01E+01	7.12E−03	0.00					

Table 11 (continued)

Algorithm	AVG	STD	MAX	MIN	p values	AVG	STD	MAX	MIN	p values
SMA	2.02E+01	5.81E−02	2.04E+01	2.01E+01	8.90E−05					
DE	2.01E+01	3.18E−02	2.02E+01	2.01E+01	8.90E−05					
PSO	1.90E+01	4.48E+00	2.03E+01	4.44E−15	0.681					
GWO	2.04E+01	6.95E−02	2.06E+01	2.03E+01	8.90E−05					
WOA	2.02E+01	1.23E−01	2.05E+01	2.00E+01	8.90E−05					

The results obtained as a result of comparing the best of the CSMAs with the methods that have emerged in recent years standard SMA, WOA, GWO, HHO, AOA, and COOT are given in Table 15. Table 15 includes the average (AVG), standard deviation (SD), minimum (MIN), maximum (MAX), Wilcoxon test results (*p* value), and Friedman mean rank (R) results of the methods. In the table, it is underlined that Wilcoxon values are insignificant. Wilcoxon's rank-sum test, is conducted at a 95% confidence interval. The *p* values calculated by Wilcoxon's test are presented in the results as well. The p values reported using Wilcoxon's test also prove that this superiority is statistically significant. Also; It has been shown that CSMA is superior to other methods by using Wilcoxon's test. When the Table 16 is evaluated, CSMA is followed by SMA and COOT algorithms in unimodal benchmark functions. In multi-modal benchmark functions, CSMA is followed by SMA and GWO algorithms. In hybrid benchmark functions, CSMA is followed by COOT and AOA algorithms, while in composition benchmark functions, which are more difficult problems, CSMA is followed by SMA and COOT algorithms. In the light of all the results obtained, it is seen that the CSMA method provides better results in different problems by using different chaotic maps, not with a single chaotic map. Table 16 shows the average of average Friedman rank in unimodal, multi-modal, hybrid, composition functions in CEC2017 with CSMA, standard SMA, WOA, GWO, HHO, AOA, and COOT methods. Convergence graphs of CEC2017 results are shown in the Fig. 4.

5.3 Parameter sensitivity analysis

In this section, parameter sensitivity analysis was performed to evaluate the effect of the number of iterations, the number of search agents, and the z value of CSMA on the algorithm. During the analysis, CSMA-1 was chosen from CSMAs. The reason for this is that it gave the best performance in the Friedman test for the selected problems. The parameters of CSMA were kept constant and applied to the first 11 CEC2017 benchmark functions. First, the effect of the change in the number of iterations was examined. The number of iterations was evaluated by choosing the values of 100, 500, and 1000. When Table 17 is examined, it is observed that the increase in the number of iterations increases the performance of CSMA in all functions.

Table 18 shows how the increase in the number of search agents affects the performance of CSMA. Here, the number of search agents is taken as 10, 30, and 100. When Table y is examined, it is observed that the increase in the number of search agents increases the performance of CSMA.

In Table 19, the effect of the z value of CSMA's own parameter was examined. The parameter range of the Z value should be between 0 and 0.1. In the experiment, the z value

Table 12 Parameter settings of other metaheuristic optimization algorithms

Algorithm	Parameter settings
SMA	$z = 0.03$
WOA	$A1 = [2, 0]; a2 = [-2, -1]; b = 1$
GWO	$a = [2, 0]$
HHO	$EO \in [-1, 1]; \beta = 1.5$
AOA	$C1 = 2; C2 = 6; u = 0.9; l = 0.1$
COOT	–

Table 13 Results obtained by comparing CSMA with SMA on CEC2017 test suite

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
	F1					F3				
CSMA-1	8.26E+03	3.07E+03	1.65E+04	2.07E+03	5.40	**2.93E+03**	1.09E+03	4.77E+03	9.36E+02	**1.90**
CSMA-2	8.05E+03	2.63E+03	1.21E+04	2.09E+03	5.15	4.06E+03	1.53E+03	6.68E+03	1.78E+03	5.50
CSMA-3	**4.49E+03**	2.78E+03	9.63E+03	9.91E+02	**1.55**	5.44E+03	2.62E+03	9.40E+03	8.46E+02	9.15
CSMA-4	9.28E+03	4.27E+03	1.74E+04	3.45E+03	7.15	3.82E+03	1.51E+03	6.55E+03	1.05E+03	4.60
CSMA-5	1.09E+04	4.55E+03	2.05E+04	4.29E+03	9.65	4.43E+03	2.17E+03	8.09E+03	1.21E+03	6.90
CSMA-6	9.41E+03	4.48E+03	2.00E+04	3.52E+03	6.90	4.92E+03	1.87E+03	8.47E+03	1.33E+03	9.00
CSMA-7	8.87E+03	2.57E+03	1.25E+04	3.87E+03	7.15	3.97E+03	1.32E+03	5.76E+03	1.38E+03	5.70
CSMA-8	8.60E+03	2.26E+03	1.25E+04	5.19E+03	6.10	3.37E+03	1.80E+03	5.96E+03	7.66E+02	2.50
CSMA-9	5.26E+03	2.30E+03	1.08E+04	1.82E+03	2.00	4.63E+03	2.09E+03	8.22E+03	7.62E+02	7.60
CSMA-10	8.71E+03	3.30E+03	1.26E+04	2.87E+03	7.20	3.97E+03	2.05E+03	7.13E+03	6.55E+02	4.90
SMA	1.66E+04	8.85E+03	3.53E+04	6.33E+03	7.75	8.32E+03	4.51E+03	1.84E+04	3.36E+03	8.25
	F4					F5				
CSMA-1	4.91E+02	7.62E+00	5.00E+02	4.64E+02	4.60	5.98E+02	1.46E+01	6.22E+02	5.68E+02	3.80
CSMA-2	4.93E+02	1.15E+01	5.03E+02	4.47E+02	9.70	6.03E+02	1.17E+01	6.20E+02	5.82E+02	5.75
CSMA-3	4.92E+02	3.15E+00	4.97E+02	4.84E+02	5.25	6.09E+02	1.37E+01	6.35E+02	5.92E+02	8.30
CSMA-4	4.91E+02	6.78E+00	5.00E+02	4.74E+02	4.20	5.95E+02	2.19E+01	6.26E+02	5.57E+02	3.25
CSMA-5	4.91E+02	1.03E+01	5.07E+02	4.59E+02	4.95	6.06E+02	2.15E+01	6.33E+02	5.68E+02	7.50
CSMA-6	4.89E+02	2.06E+01	5.01E+02	4.03E+02	7.15	**5.92E+02**	1.35E+01	6.13E+02	5.69E+02	**2.00**
CSMA-7	4.97E+02	8.90E+00	5.16E+02	4.88E+02	9.00	6.10E+02	1.52E+01	6.34E+02	5.80E+02	8.70
CSMA-8	4.89E+02	1.69E+01	4.98E+02	4.19E+02	5.05	6.02E+02	1.94E+01	6.37E+02	5.65E+02	5.35
CSMA-9	4.92E+02	5.23E+00	4.99E+02	4.77E+02	5.35	6.14E+02	2.69E+01	6.47E+02	5.56E+02	9.05
CSMA-10	**4.88E+02**	1.25E+01	4.98E+02	4.43E+02	**2.35**	5.98E+02	1.94E+01	6.20E+02	5.51E+02	4.40
SMA	5.07E+02	1.55E+01	5.38E+02	4.92E+02	8.40	6.29E+02	2.49E+01	6.94E+02	5.96E+02	7.90
	F6					F7				
CSMA-1	6.11E+02	8.16E+00	6.35E+02	6.03E+02	6.00	8.41E+02	2.05E+01	8.70E+02	8.04E+02	2.50
CSMA-2	6.10E+02	4.98E+00	6.20E+02	6.02E+02	6.45	8.51E+02	3.02E+01	8.89E+02	7.77E+02	7.05
CSMA-3	6.13E+02	6.85E+00	6.26E+02	6.02E+02	9.15	8.56E+02	1.59E+01	8.84E+02	8.27E+02	7.60
CSMA-4	6.11E+02	6.62E+00	6.31E+02	6.02E+02	7.60	8.53E+02	2.90E+01	9.00E+02	8.01E+02	6.85
CSMA-5	6.06E+02	2.88E+00	6.11E+02	6.02E+02	4.40	8.47E+02	2.01E+01	8.75E+02	7.91E+02	4.85

Table 13 (continued)

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
CSMA-6	6.09E+02	3.36E+00	6.14E+02	6.03E+02	8.05	8.56E+02	1.38E+01	8.84E+02	8.20E+02	7.55
CSMA-7	6.06E+02	2.03E+00	6.10E+02	6.02E+02	3.35	8.48E+02	2.14E+01	8.73E+02	8.08E+02	5.25
CSMA-8	**6.06E+02**	2.20E+00	6.10E+02	6.02E+02	**2.75**	8.46E+02	2.77E+01	8.90E+02	8.00E+02	4.65
CSMA-9	6.07E+02	2.56E+00	6.12E+02	6.02E+02	5.30	8.59E+02	1.78E+01	8.92E+02	8.27E+02	9.05
CSMA-10	6.06E+02	2.20E+00	6.10E+02	6.02E+02	4.40	**8.39E+02**	1.91E+01	8.70E+02	8.10E+02	**2.10**
SMA	6.16E+02	6.40E+00	6.30E+02	6.07E+02	8.55	8.89E+02	2.58E+01	9.36E+02	8.55E+02	8.55
	F8					F9				
CSMA-1	**9.00E+02**	2.04E+01	9.31E+02	8.59E+02	**2.65**	3.13E+03	9.92E+02	4.72E+03	1.42E+03	**4.05**
CSMA-2	9.04E+02	2.43E+01	9.42E+02	8.60E+02	4.65	3.39E+03	8.41E+02	4.73E+03	2.08E+03	7.30
CSMA-3	9.20E+02	1.66E+01	9.38E+02	8.69E+02	9.45	3.32E+03	1.16E+03	5.17E+03	1.18E+03	5.85
CSMA-4	9.10E+02	1.90E+01	9.37E+02	8.62E+02	6.25	3.26E+03	9.75E+02	4.84E+03	1.33E+03	6.25
CSMA-5	9.03E+02	3.07E+01	9.39E+02	8.44E+02	5.60	3.22E+03	8.29E+02	4.61E+03	1.24E+03	5.55
CSMA-6	9.15E+02	1.56E+01	9.40E+02	8.83E+02	8.20	3.13E+03	8.83E+02	4.48E+03	1.36E+03	4.10
CSMA-7	9.12E+02	1.95E+01	9.40E+02	8.71E+02	7.50	3.30E+03	6.58E+02	4.40E+03	1.90E+03	6.45
CSMA-8	9.02E+02	**1.19E+01**	9.20E+02	8.84E+02	3.65	**3.12E+03**	6.17E+02	4.20E+03	1.68E+03	4.35
CSMA-9	9.12E+02	1.58E+01	9.36E+02	8.88E+02	7.20	3.44E+03	1.07E+03	4.99E+03	1.57E+03	7.95
CSMA-10	9.00E+02	1.49E+01	9.30E+02	8.76E+02	2.75	3.40E+03	1.19E+03	5.12E+03	1.51E+03	6.80
SMA	9.37E+02	2.64E+01	9.95E+02	9.05E+02	8.10	4.24E+03	8.42E+02	6.14E+03	3.50E+03	7.35
	F10					F11				
CSMA-1	4.38E+03	5.08E+02	4.88E+03	3.13E+03	6.40	1.24E+03	3.89E+01	1.29E+03	1.18E+03	3.10
CSMA-2	4.34E+03	5.45E+02	5.01E+03	3.13E+03	6.10	1.25E+03	3.60E+01	1.30E+03	1.18E+03	7.90
CSMA-3	4.36E+03	3.40E+02	4.81E+03	3.68E+03	5.30	**1.23E+03**	3.06E+01	1.28E+03	1.15E+03	**2.50**
CSMA-4	4.36E+03	4.36E+02	4.87E+03	3.35E+03	6.00	1.25E+03	3.85E+01	1.29E+03	1.16E+03	6.45
CSMA-5	4.45E+03	4.41E+02	4.92E+03	3.37E+03	8.85	1.25E+03	3.39E+01	1.29E+03	1.17E+03	6.80
CSMA-6	4.46E+03	3.79E+02	4.99E+03	3.79E+03	8.25	**1.23E+03**	3.57E+01	1.29E+03	1.16E+03	3.25
CSMA-7	4.40E+03	4.57E+02	4.89E+03	3.03E+03	7.40	1.24E+03	3.83E+01	1.29E+03	1.15E+03	5.95
CSMA-8	**4.23E+03**	5.31E+02	4.85E+03	2.86E+03	3.15	1.25E+03	3.74E+01	1.30E+03	1.18E+03	7.60
CSMA-9	4.24E+03	4.16E+02	4.73E+03	3.35E+03	**2.65**	1.24E+03	3.95E+01	1.30E+03	1.17E+03	4.15

Table 13 (continued)

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
CSMA-10	4.25E+03	3.58E+02	4.71E+03	3.45E+03	2.85	1.26E+03	3.91E+01	1.30E+03	1.17E+03	9.70
SMA	5.12E+03	4.05E+02	5.98E+03	4.61E+03	9.05	1.32E+03	4.37E+01	1.41E+03	1.26E+03	8.60
	F12					F13				
CSMA-1	2.12E+06	1.42E+06	5.72E+06	1.49E+05	4.80	2.56E+04	1.17E+04	4.80E+04	7.06E+03	7.00
CSMA-2	**1.82E+06**	1.17E+06	3.74E+06	1.78E+05	**2.50**	2.07E+04	6.37E+03	3.35E+04	1.15E+04	5.55
CSMA-3	3.32E+06	1.67E+06	6.04E+06	5.86E+05	10.00	**1.15E+04**	1.67E+03	1.35E+04	6.62E+03	**1.35**
CSMA-4	2.06E+06	1.22E+06	4.38E+06	2.13E+05	4.85	2.06E+04	8.05E+03	3.60E+04	6.49E+03	4.80
CSMA-5	2.07E+06	1.02E+06	4.32E+06	3.90E+05	5.80	1.99E+04	7.64E+03	3.60E+04	1.04E+04	3.95
CSMA-6	3.28E+06	1.81E+06	5.79E+06	3.81E+05	9.45	2.10E+04	9.79E+03	5.00E+04	1.02E+04	4.80
CSMA-7	1.97E+06	1.05E+06	3.71E+06	4.99E+05	4.65	3.45E+04	1.70E+04	6.86E+04	1.47E+04	9.85
CSMA-8	1.92E+06	1.27E+06	4.24E+06	3.12E+05	3.65	3.41E+04	2.25E+04	7.20E+04	9.91E+03	8.35
CSMA-9	2.39E+06	1.35E+06	4.61E+06	3.36E+05	7.75	2.19E+04	2.04E+04	6.52E+04	7.86E+03	3.25
CSMA-10	2.15E+06	1.50E+06	5.02E+06	4.07E+05	4.55	2.56E+04	9.52E+03	4.86E+04	1.18E+04	8.10
SMA	4.25E+06	1.91E+06	8.09E+06	1.77E+06	8.00	5.48E+04	2.20E+04	8.28E+04	2.64E+04	9.00
	F14					F15				
CSMA-1	7.28E+04	3.92E+04	1.22E+05	7.56E+03	5.65	1.41E+04	1.00E+04	3.09E+04	2.19E+03	5.00
CSMA-2	7.57E+04	3.66E+04	1.17E+05	2.40E+03	6.25	1.70E+04	1.18E+04	3.18E+04	2.67E+03	8.60
CSMA-3	6.90E+04	4.14E+04	1.34E+05	3.19E+03	4.55	1.55E+04	1.11E+04	3.15E+04	2.15E+03	6.30
CSMA-4	6.96E+04	4.08E+04	1.45E+05	1.30E+04	4.55	1.70E+04	1.17E+04	3.15E+04	2.48E+03	7.55
CSMA-5	7.57E+04	4.23E+04	1.38E+05	1.96E+04	6.40	2.24E+04	1.27E+04	3.65E+04	2.44E+03	9.95
CSMA-6	6.77E+04	4.49E+04	1.52E+05	6.69E+03	3.80	1.24E+04	1.05E+04	2.88E+04	1.98E+03	2.75
CSMA-7	**5.07E+04**	3.03E+04	1.00E+05	9.08E+03	**1.95**	1.49E+04	1.26E+04	3.23E+04	2.56E+03	6.05
CSMA-8	9.43E+04	4.01E+04	1.56E+05	1.35E+04	9.75	1.10E+04	1.05E+04	3.02E+04	2.00E+03	2.75
CSMA-9	8.89E+04	4.58E+04	1.63E+05	2.33E+04	9.05	1.71E+04	1.25E+04	3.16E+04	1.99E+03	6.65
CSMA-10	7.05E+04	4.24E+04	1.50E+05	8.61E+03	5.60	**8.17E+03**	7.77E+03	2.79E+04	1.88E+03	**1.85**
SMA	1.29E+05	3.33E+04	1.87E+05	7.94E+04	8.45	3.44E+04	7.77E+03	4.44E+04	1.94E+04	8.55
	F16					F17				
CSMA-1	2.43E+03	2.04E+02	2.76E+03	2.02E+03	4.45	**2.15E+03**	1.42E+02	2.34E+03	1.86E+03	**2.90**

Table 13 (continued)

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
CSMA-2	**2.39E+03**	2.49E+02	2.70E+03	1.88E+03	**2.95**	2.17E+03	1.17E+02	2.35E+03	1.95E+03	4.05
CSMA-3	2.51E+03	2.54E+02	2.84E+03	1.99E+03	8.05	2.24E+03	1.48E+02	2.47E+03	1.91E+03	8.10
CSMA-4	2.43E+03	2.11E+02	2.67E+03	1.90E+03	4.15	2.25E+03	1.27E+02	2.39E+03	1.98E+03	8.15
CSMA-5	2.45E+03	1.31E+02	2.69E+03	2.19E+03	4.75	**2.15E+03**	1.47E+02	2.40E+03	1.80E+03	3.40
CSMA-6	2.45E+03	2.07E+02	2.69E+03	1.93E+03	5.30	2.30E+03	1.38E+02	2.48E+03	2.07E+03	10.25
CSMA-7	2.53E+03	2.12E+02	2.78E+03	2.03E+03	9.40	2.23E+03	1.76E+02	2.44E+03	1.83E+03	7.65
CSMA-8	**2.39E+03**	2.48E+02	2.76E+03	1.96E+03	3.65	2.16E+03	1.22E+02	2.33E+03	1.88E+03	3.40
CSMA-9	2.52E+03	2.14E+02	2.84E+03	2.05E+03	8.35	2.20E+03	1.58E+02	2.41E+03	1.81E+03	5.55
CSMA-10	2.49E+03	1.83E+02	2.73E+03	2.19E+03	6.55	2.18E+03	1.73E+02	2.47E+03	1.81E+03	4.65
SMA	2.75E+03	1.95E+02	3.15E+03	2.46E+03	8.40	2.39E+03	1.76E+02	2.78E+03	2.17E+03	7.90
	F18					F19				
CSMA-1	**5.81E+05**	3.10E+05	1.36E+06	2.22E+05	**2.55**	2.58E+04	2.15E+04	5.66E+04	2.58E+03	6.60
CSMA-2	8.29E+05	6.43E+05	2.17E+06	1.50E+05	5.80	2.59E+04	2.34E+04	5.65E+04	2.43E+03	6.30
CSMA-3	7.55E+05	4.19E+05	1.86E+06	2.31E+05	7.45	3.04E+04	2.23E+04	5.66E+04	2.74E+03	8.25
CSMA-4	7.87E+05	3.82E+05	1.48E+06	2.89E+04	8.15	**1.06E+04**	1.07E+04	4.54E+04	2.15E+03	**3.35**
CSMA-5	8.40E+05	4.08E+05	1.42E+06	3.29E+05	7.55	1.24E+04	1.32E+04	4.98E+04	2.05E+03	4.00
CSMA-6	8.59E+05	5.03E+05	1.84E+06	2.48E+05	8.50	1.59E+04	1.29E+04	4.88E+04	3.09E+03	6.45
CSMA-7	7.17E+05	4.44E+05	1.84E+06	2.70E+05	5.90	1.08E+04	1.01E+04	3.47E+04	2.08E+03	4.05
CSMA-8	5.93E+05	3.08E+05	1.37E+06	1.15E+05	3.45	1.16E+04	5.73E+03	1.79E+04	2.47E+03	5.15
CSMA-9	6.46E+05	4.10E+05	1.70E+06	1.48E+05	3.45	1.62E+04	1.10E+04	4.26E+04	1.96E+03	7.10
CSMA-10	7.04E+05	3.97E+05	1.67E+06	2.07E+05	6.10	2.22E+04	1.93E+04	5.01E+04	2.07E+03	6.65
SMA	1.92E+06	2.15E+06	7.78E+06	5.29E+05	7.10	3.99E+04	1.85E+04	5.67E+04	1.09E+04	8.10
	F20					F21				
CSMA-1	2.47E+03	1.25E+02	2.64E+03	2.16E+03	4.50	**2.39E+03**	1.48E+01	2.42E+03	2.36E+03	**1.30**
CSMA-2	2.46E+03	1.51E+02	2.67E+03	2.12E+03	3.90	2.41E+03	1.54E+01	2.43E+03	2.38E+03	4.95
CSMA-3	2.54E+03	1.37E+02	2.72E+03	2.31E+03	9.45	2.42E+03	1.32E+01	2.43E+03	2.38E+03	8.80
CSMA-4	2.49E+03	1.50E+02	2.71E+03	2.25E+03	6.25	2.41E+03	1.97E+01	2.43E+03	2.37E+03	6.75
CSMA-5	2.52E+03	1.41E+02	2.70E+03	2.23E+03	7.85	2.40E+03	1.84E+01	2.43E+03	2.35E+03	3.40

Table 13 (continued)

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
CSMA-6	2.47E+03	1.67E+02	2.67E+03	2.13E+03	5.05	2.41E+03	1.94E+01	2.44E+03	2.37E+03	6.45
CSMA-7	**2.38E+03**	1.39E+02	2.59E+03	2.17E+03	**1.45**	2.41E+03	1.14E+01	2.42E+03	2.38E+03	4.70
CSMA-8	2.45E+03	1.59E+02	2.65E+03	2.14E+03	3.50	2.40E+03	1.79E+01	2.43E+03	2.37E+03	4.85
CSMA-9	2.51E+03	1.52E+02	2.73E+03	2.23E+03	7.25	2.43E+03	1.69E+01	2.45E+03	2.39E+03	10.40
CSMA-10	2.53E+03	1.37E+02	2.73E+03	2.33E+03	8.80	2.41E+03	1.57E+01	2.43E+03	2.36E+03	5.50
SMA	2.67E+03	1.41E+02	3.09E+03	2.49E+03	8.00	2.44E+03	2.91E+01	2.52E+03	2.41E+03	8.90
	F22					F23				
CSMA-1	5.56E+03	1.17E+03	6.46E+03	2.30E+03	8.15	2.75E+03	1.41E+01	2.76E+03	2.72E+03	5.20
CSMA-2	5.84E+03	9.11E+02	6.64E+03	2.31E+03	9.85	2.75E+03	1.43E+01	2.77E+03	2.72E+03	7.70
CSMA-3	5.14E+03	1.50E+03	6.44E+03	2.30E+03	4.25	2.76E+03	1.34E+01	2.78E+03	2.73E+03	9.55
CSMA-4	5.19E+03	1.30E+03	6.28E+03	2.30E+03	3.15	**2.74E+03**	1.14E+01	2.76E+03	2.72E+03	2.85
CSMA-5	5.35E+03	1.17E+03	6.65E+03	2.30E+03	4.90	**2.74E+03**	1.74E+01	2.76E+03	2.71E+03	**2.25**
CSMA-6	5.49E+03	1.39E+03	6.50E+03	2.30E+03	8.25	**2.74E+03**	2.01E+01	2.77E+03	2.70E+03	3.95
CSMA-7	5.12E+03	1.48E+03	6.26E+03	2.30E+03	4.55	2.75E+03	1.71E+01	2.77E+03	2.71E+03	5.80
CSMA-8	**4.76E+03**	1.49E+03	6.09E+03	2.30E+03	**1.45**	**2.74E+03**	1.33E+01	2.76E+03	2.72E+03	4.00
CSMA-9	5.49E+03	1.22E+03	6.65E+03	2.30E+03	6.85	2.76E+03	1.51E+01	2.78E+03	2.72E+03	9.90
CSMA-10	5.45E+03	1.17E+03	6.37E+03	2.30E+03	6.40	2.75E+03	1.40E+01	2.77E+03	2.72E+03	5.40
SMA	6.49E+03	4.84E+02	7.39E+03	5.68E+03	8.20	2.78E+03	1.44E+01	2.81E+03	2.76E+03	9.40
	F24					F25				
CSMA-1	2.92E+03	2.17E+01	2.96E+03	2.89E+03	3.85	**2.89E+03**	1.24E+00	2.89E+03	2.88E+03	5.75
CSMA-2	2.92E+03	1.43E+01	2.94E+03	2.89E+03	5.45	**2.89E+03**	1.88E+00	2.89E+03	2.88E+03	4.25
CSMA-3	2.94E+03	1.99E+01	2.97E+03	2.90E+03	9.60	**2.89E+03**	2.60E+00	2.90E+03	2.88E+03	6.75
CSMA-4	2.92E+03	1.49E+01	2.95E+03	2.90E+03	4.60	**2.89E+03**	1.97E+00	2.89E+03	2.88E+03	7.35
CSMA-5	**2.91E+03**	1.82E+01	2.93E+03	2.86E+03	2.55	**2.89E+03**	1.17E+00	2.89E+03	2.88E+03	9.20
CSMA-6	2.92E+03	2.20E+01	2.95E+03	2.87E+03	6.40	**2.89E+03**	1.41E+00	2.89E+03	2.88E+03	4.90
CSMA-7	2.92E+03	2.15E+01	2.96E+03	2.88E+03	6.60	**2.89E+03**	1.81E+00	2.89E+03	2.88E+03	**1.45**
CSMA-8	2.92E+03	2.22E+01	2.95E+03	2.88E+03	6.10	**2.89E+03**	1.94E+00	2.89E+03	2.88E+03	3.30
CSMA-9	2.94E+03	1.59E+01	2.97E+03	2.92E+03	10.30	**2.89E+03**	1.84E+00	2.89E+03	2.88E+03	8.10

Table 13 (continued)

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
CSMA-10	2.91E+03	1.76E+01	2.94E+03	2.87E+03	3.10	**2.89E+03**	1.51E+00	2.89E+03	2.88E+03	5.75
SMA	2.95E+03	2.78E+01	3.02E+03	2.92E+03	7.45	2.90E+03	1.71E+01	2.94E+03	2.89E+03	9.20
	F26					F27				
CSMA-1	4.69E+03	1.94E+02	4.93E+03	4.31E+03	5.40	3.22E+03	1.03E+01	3.23E+03	3.19E+03	2.60
CSMA-2	**4.56E+03**	1.47E+02	4.80E+03	4.27E+03	**1.65**	3.22E+03	8.83E+00	3.24E+03	3.21E+03	6.10
CSMA-3	4.88E+03	1.75E+02	5.14E+03	4.37E+03	9.55	3.22E+03	9.79E+00	3.23E+03	3.20E+03	6.95
CSMA-4	4.68E+03	4.50E+02	5.00E+03	2.90E+03	7.20	3.22E+03	7.31E+00	3.23E+03	3.21E+03	5.10
CSMA-5	4.66E+03	1.69E+02	4.93E+03	4.31E+03	4.45	3.22E+03	7.79E+00	3.23E+03	3.20E+03	7.15
CSMA-6	4.71E+03	2.18E+02	5.03E+03	4.25E+03	6.40	**3.21E+03**	6.57E+00	3.22E+03	3.20E+03	**1.95**
CSMA-7	4.55E+03	4.43E+02	4.93E+03	2.90E+03	2.90	3.23E+03	9.37E+00	3.24E+03	3.21E+03	10.15
CSMA-8	4.65E+03	1.56E+02	4.96E+03	4.37E+03	4.20	3.22E+03	1.17E+01	3.24E+03	3.20E+03	5.95
CSMA-9	4.90E+03	1.96E+02	5.13E+03	4.44E+03	10.05	3.22E+03	1.11E+01	3.24E+03	3.20E+03	4.35
CSMA-10	4.65E+03	4.92E+02	5.19E+03	2.90E+03	6.25	3.22E+03	9.00E+00	3.23E+03	3.20E+03	7.50
SMA	4.98E+03	1.94E+02	5.39E+03	4.72E+03	7.95	3.24E+03	1.46E+01	3.26E+03	3.22E+03	8.20
	F28					F29				
CSMA-1	3.24E+03	2.27E+01	3.28E+03	3.21E+03	9.00	3.86E+03	1.12E+02	4.04E+03	3.62E+03	6.80
CSMA-2	3.24E+03	2.01E+01	3.27E+03	3.21E+03	7.95	3.88E+03	1.11E+02	4.06E+03	3.60E+03	8.15
CSMA-3	3.24E+03	1.99E+01	3.27E+03	3.22E+03	8.85	3.84E+03	1.63E+02	4.08E+03	3.50E+03	5.50
CSMA-4	**3.23E+03**	2.32E+01	3.27E+03	3.20E+03	4.55	3.81E+03	1.60E+02	4.03E+03	3.55E+03	4.20
CSMA-5	3.24E+03	1.47E+01	3.26E+03	3.22E+03	8.15	**3.79E+03**	1.22E+02	3.97E+03	3.53E+03	**2.55**
CSMA-6	**3.23E+03**	2.09E+01	3.28E+03	3.21E+03	4.65	3.89E+03	1.49E+02	4.10E+03	3.57E+03	9.35
CSMA-7	**3.23E+03**	1.51E+01	3.25E+03	3.20E+03	**1.95**	3.86E+03	8.79E+01	3.99E+03	3.67E+03	6.30
CSMA-8	**3.23E+03**	1.67E+01	3.26E+03	3.20E+03	5.25	3.80E+03	1.76E+02	4.02E+03	3.44E+03	4.15
CSMA-9	**3.23E+03**	1.56E+01	3.26E+03	3.21E+03	2.85	3.81E+03	1.48E+02	4.10E+03	3.54E+03	4.25
CSMA-10	3.24E+03	1.83E+01	3.27E+03	3.21E+03	5.85	3.85E+03	1.40E+02	4.02E+03	3.56E+03	7.05
SMA	3.26E+03	3.25E+01	3.35E+03	3.23E+03	6.95	4.02E+03	1.79E+02	4.45E+03	3.83E+03	7.70
	F30									
CSMA-1	3.02E+04	9.54E+03	4.62E+04	1.31E+04	5.35					

Table 13 (continued)

Algorithm	AVG	STD	MAX	MIN	Mean rank	AVG	STD	MAX	MIN	Mean rank
CSMA-2	3.40E+04	1.18E+04	5.38E+04	1.72E+04	7.15					
CSMA-3	**2.42E+04**	6.87E+03	3.60E+04	1.32E+04	**1.60**					
CSMA-4	3.67E+04	1.73E+04	6.74E+04	1.30E+04	7.20					
CSMA-5	3.94E+04	1.49E+04	7.01E+04	2.25E+04	9.25					
CSMA-6	3.02E+04	7.78E+03	4.24E+04	1.76E+04	5.10					
CSMA-7	3.23E+04	1.24E+04	5.27E+04	1.40E+04	6.10					
CSMA-8	3.95E+04	1.63E+04	6.91E+04	1.90E+04	9.25					
CSMA-9	2.55E+04	1.02E+04	4.36E+04	1.34E+04	2.15					
CSMA-10	2.82E+04	9.93E+03	4.46E+04	8.29E+03	3.60					
SMA	7.29E+04	3.30E+04	1.52E+05	3.55E+04	9.25					

was taken as 0.01, 0.03, 0.006, and 0.1. When Table z is examined, the best performance of CSMA is seen when the z value is 0.003.

In general, it is seen that the increase in the number of search agents increases the performance of CSMA. The reason for this is that the increase in the number of search agents increases the search efficiency of the algorithm and the increase in the number of iterations increases the search time and increases the accuracy of subsequent searches. Because the result values do not increase proportionally, in different problems, researchers can use different iteration and search agent values in accordance with the nature of the problem.

5.4 Experiments on engineering design problems

In this section, suggested CSMAs using 10 different chaotic maps and standard SMA are applied to welded beam design, pressure vessel design, and tension/compression spring design constraint real-world engineering design optimization problems. In the following sections, the tests conducted on these three design problems and their comparison with the results obtained from different articles are given.

5.4.1 Welded beam design problem

The aim of the Welded beam design problem is to be able to design the beam with minimum cost under certain constraints. Welded beam design problem consists of four design variables. These four variables are shown as variables; h $\left(x_1\right)$, l $\left(x_2\right)$, t $\left(x_3\right)$ ve b $\left(x_4\right)$. Among these values, h and b are defined between 0.1 and 2, and l and t values between 0.1 and 10. The objective function and constraints of the welded beam design problem are shown in the Eq. (20–35). Welded beam design problem is demonstrated in Fig. 5.

$$\text{Minimize}: f(x) = \left(1 + C_1\right)x_1^2 x_2 + C_2 x_3 x_4 \left(L + x_2\right) \tag{20}$$

Subject to:

$$g_1(x) = \tau(\mathrm{x}) - \tau_d \leq 0 \tag{21}$$

$$\tau(x) = \sqrt{(\tau')^2 + (2\tau'\tau'')\frac{x_2}{2R} + (\tau'')^2} \tag{22}$$

$$\tau' = \frac{6000}{\sqrt{2}x_1 x_2} \tag{23}$$

$$\tau'' = \frac{MR}{J} \tag{24}$$

$$M = 6000\left(14 + \frac{x_2}{2}\right) \tag{25}$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2} \tag{26}$$

$$J = \left\{ x_1 x_2 \sqrt{2} \left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2 \right] \right\} \tag{27}$$

$$g_2(x) = \sigma(x) - \sigma_d \leq 0 \tag{28}$$

$$\sigma(x) = \frac{504000}{x_4 x_3^2} \tag{29}$$

$$g_3(x) = x_1 - x_4 \leq 0 \tag{30}$$

$$g_4(x) = C_1 x_1^2 + C_2 x_3 x_4 \left(L + x_2\right) - 5 \leq 0 \tag{31}$$

$$g_5(x) = 0.125 - x_1 \leq 0$$

$$g_6(x) = \delta(x) - \delta_d \leq 0 \tag{32}$$

$$\delta(x) = \frac{2.1952}{x_4 x_3^3} \tag{33}$$

$$g_7(x) = P - P_c(x) \leq 0 \tag{34}$$

$$P_c(x) = \frac{4.013E\sqrt{\frac{x_3^2 x_4^6}{36}}}{196}\left(1 - \frac{\sqrt[x_3]{\frac{E}{4G}}}{28}\right). \tag{35}$$

Constant values of specifications have been taken $C_1 = 0.10471$, $C_2 = 0.04811$, $\sigma_d = 30000$, $\tau_D = 13600$, $E = 30 \times 106$, $\delta_d = 0.25$, $P = 6000$, $G = 12 \times 106$, and $L = 14$. These values taken respectively represent cost per volume of the welded material, cost per volume of the bar stock, design normal stress of the bar material, design shear stress of the welded material, young's modulus of bar stock, design bar end deflection, loading condition, shear modulus of bar stock and overhang length of the beam.

SMA and CSMAs have been tested by applying the welded beam design problem. The population size was 30, the number of iterations was 1000, and the algorithms were run 30 times. In addition, it has been compared with the results of the studies obtained from the literature. These are WOA (Mirjalili et al. 2016), firefly algorithm (FA) (Erdal 2017), competitive Bird Swarm Algorithm (COBSA) (Wang et al. 2018), chaotic grey wolf optimization algorithm (CGWO) (Kohli et al. 2018), differential big bang church algorithm (DBCA) (Prayogo et al. 2018), sonar inspired optimization (SIO) (Tzanetos and Dounias 2020), uses adaptive inertia weight technique PSO (CBPPSO) (Agrawal and Tripathi

Table 14 Friedman test results

	F1	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15	F16
18.307	105.773	108.955	93.245	107.618	84.700	97.745	98.536	33.782	98.864	107.473	108.118	129.500	100.582	129.327	91.000
H_0	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
	F17	F18	F19	F20	F21	F22	F23	F24	F25	F26	F27	F28	F29	F30	
18.307	113.427		49.645	115.100	126.527	118.964	137.255	111.418	108.482	125.464	105.536	104.155	77.173	137.518	
H_0	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	

Table 15 Results obtained by comparing CSMA with other methods on CEC2017 test suite

Algorithm	AVG	STD	MIN	MAX	*p* values	R	AVG	STD	MIN	MAX	*p* values	R
	F1						F3					
CSMA	**4.49E+03**	2.78E+03	9.91E+02	9.63E+03	0.00E+00	**1.20**	**2.93E+03**	1.09E+03	9.36E+02	4.77E+03	0.00E+00	**1.10**
SMA	1.66E+04	8.85E+03	6.33E+03	3.53E+04	6.81E−04	1.80	8.32E+03	4.51E+03	3.36E+03	1.84E+04	2.54E−04	1.95
WOA	1.59E+09	7.00E+08	4.13E+08	3.03E+09	8.86E−05	6.60	2.57E+05	4.71E+04	1.96E+05	3.67E+05	8.86E−05	7.00
GWO	1.28E+09	9.43E+08	1.07E+08	4.04E+09	8.86E−05	6.15	5.48E+04	1.21E+04	3.40E+04	7.49E+04	8.86E−05	5.80
HHO	2.91E+07	6.82E+06	1.76E+07	4.84E+07	8.86E−05	4.10	4.13E+04	8.93E+03	2.19E+04	5.72E+04	8.86E−05	5.10
AOA	4.40E+08	4.07E+08	1.92E+07	1.18E+09	8.86E−05	5.15	2.59E+04	1.03E+04	1.43E+04	6.22E+04	8.86E−05	3.80
COOT	3.36E+05	2.13E+05	6.36E+04	8.80E+05	8.86E−05	3.00	1.74E+04	6.09E+03	6.83E+03	3.08E+04	8.86E−05	3.25
	F4						F5					
CSMA	**4.88E+02**	1.25E+01	4.43E+02	4.98E+02	0.00E+00	**1.30**	**5.92E+02**	1.35E+01	5.69E+02	6.13E+02	0.00E+00	**1.35**
SMA	5.07E+02	1.55E+01	4.92E+02	5.38E+02	1.02E−03	2.45	6.29E+02	2.49E+01	5.96E+02	6.94E+02	5.93E−04	2.85
WOA	8.13E+02	1.47E+02	6.33E+02	1.18E+03	8.86E−05	6.95	8.59E+02	3.68E+01	8.03E+02	9.33E+02	8.86E−05	7.00
GWO	6.00E+02	6.35E+01	5.01E+02	7.26E+02	8.86E−05	4.95	6.20E+02	1.56E+01	5.99E+02	6.61E+02	5.93E−04	2.25
HHO	5.78E+02	4.33E+01	5.24E+02	6.85E+02	8.86E−05	4.90	7.72E+02	3.11E+01	7.37E+02	8.47E+02	8.86E−05	5.85
AOA	5.95E+02	1.75E+02	4.86E+02	1.29E+03	1.63E−04	4.20	7.12E+02	3.21E+01	6.33E+02	7.61E+02	8.86E−05	5.05
COOT	5.27E+02	2.81E+01	4.88E+02	5.86E+02	1.20E−04	3.25	6.51E+02	2.16E+01	5.97E+02	6.86E+02	1.03E−04	3.65
	F6						F7					
CSMA	**6.06E+02**	2.20E+00	6.02E+02	6.10E+02	0.00E+00	**1.20**	**8.39E+02**	1.91E+01	8.10E+02	8.70E+02	0.00E+00	**1.35**
SMA	6.16E+02	6.40E+00	6.07E+02	6.30E+02	2.19E−04	2.75	8.89E+02	2.58E+01	8.55E+02	9.36E+02	3.38E−04	2.45
WOA	6.86E+02	8.16E+00	6.74E+02	7.04E+02	8.86E−05	7.00	1.32E+03	6.00E+01	1.25E+03	1.48E+03	8.86E−05	6.25
GWO	6.15E+02	5.07E+00	6.09E+02	6.25E+02	1.63E−04	2.25	9.09E+02	5.72E+01	8.57E+02	1.08E+03	2.54E−04	3.30
HHO	6.70E+02	4.17E+00	6.64E+02	6.77E+02	8.86E−05	6.00	1.33E+03	3.40E+01	1.27E+03	1.43E+03	8.86E−05	6.70
AOA	6.40E+02	1.35E+01	6.10E+02	6.54E+02	8.86E−05	4.30	1.05E+03	1.39E+02	8.85E+02	1.30E+03	8.86E−05	4.90
COOT	6.37E+02	1.15E+01	6.16E+02	6.60E+02	8.86E−05	4.50	9.04E+02	4.91E+01	8.34E+02	1.06E+03	1.40E−04	3.05
	F8						F9					
CSMA	**9.00E+02**	2.04E+01	8.59E+02	9.31E+02	0.00E+00	**2.15**	3.13E+03	9.92E+02	1.42E+03	4.72E+03	0.00E+00	2.70
SMA	9.37E+02	2.64E+01	9.05E+02	9.95E+02	3.19E−03	3.95	4.24E+03	8.42E+02	3.50E+03	6.14E+03	2.76E−02	3.65
WOA	1.05E+03	3.06E+01	1.01E+03	1.11E+03	8.86E−05	7.00	1.31E+04	3.29E+03	9.26E+03	2.11E+04	8.86E−05	7.00
GWO	9.08E+02	2.63E+01	8.86E+02	9.98E+02	6.54E−01	2.25	**2.34E+03**	4.90E+02	1.81E+03	3.77E+03	3.04E−02	**1.50**
HHO	9.91E+02	1.63E+01	9.71E+02	1.03E+03	8.86E−05	5.95	8.93E+03	6.88E+02	7.81E+03	1.04E+04	8.86E−05	6.00

Table 15 (continued)

Algorithm	AVG	STD	MIN	MAX	*p* values	R	AVG	STD	MIN	MAX	*p* values	R
AOA	9.49E+02	2.57E+01	9.12E+02	9.93E+02	1.03E−04	4.30	5.01E+03	1.32E+03	2.52E+03	6.86E+03	2.93E−04	4.55
COOT	9.11E+02	2.63E+01	8.65E+02	9.65E+02	1.45E−01	2.40	3.23E+03	9.87E+02	1.78E+03	5.09E+03	9.40E−01	2.60
	F10						F11					
CSMA	**4.24E+03**	4.16E+02	3.35E+03	4.73E+03	0.00E+00	**1.60**	**1.23E+03**	3.06E+01	1.15E+03	1.28E+03	0.00E+00	**1.45**
SMA	5.12E+03	4.05E+02	4.61E+03	5.98E+03	3.38E−04	3.40	1.32E+03	4.37E+01	1.26E+03	1.41E+03	1.89E−04	3.40
WOA	7.37E+03	5.46E+02	6.67E+03	8.82E+03	8.86E−05	6.85	1.03E+04	2.71E+03	6.83E+03	1.59E+04	8.86E−05	7.00
GWO	5.26E+03	1.34E+03	4.23E+03	9.43E+03	7.19E−03	2.85	2.64E+03	7.95E+02	1.60E+03	4.35E+03	8.86E−05	6.00
HHO	6.13E+03	5.93E+02	5.43E+03	7.63E+03	8.86E−05	5.55	1.33E+03	3.81E+01	1.28E+03	1.41E+03	8.86E−05	4.55
AOA	5.52E+03	8.11E+02	4.25E+03	7.86E+03	8.86E−05	4.40	1.32E+03	6.96E+01	1.23E+03	1.53E+03	3.90E−04	3.25
COOT	5.12E+03	8.75E+02	3.66E+03	6.88E+03	2.00E−03	3.35	1.27E+03	5.49E+01	1.16E+03	1.39E+03	3.30E−02	2.35
	F12						F13					
CSMA	**1.82E+06**	1.17E+06	1.78E+05	3.74E+06	0.00E+00	**1.40**	**1.15E+04**	1.67E+03	6.62E+03	1.35E+04	0.00E+00	**1.05**
SMA	4.25E+06	1.91E+06	1.77E+06	8.09E+06	4.55E−03	2.25	5.48E+04	2.20E+04	2.64E+04	8.28E+04	8.86E−05	2.65
WOA	3.78E+08	1.87E+08	1.53E+08	7.83E+08	8.86E−05	7.00	1.91E+06	1.09E+06	6.18E+05	4.52E+06	8.86E−05	6.85
GWO	1.38E+08	1.24E+08	3.74E+07	4.57E+08	8.86E−05	5.85	3.04E+06	8.21E+06	8.51E+04	2.94E+07	8.86E−05	5.05
HHO	3.93E+07	2.16E+07	1.68E+07	9.65E+07	8.86E−05	4.80	1.29E+06	1.85E+06	5.51E+05	8.69E+06	8.86E−05	5.85
AOA	2.33E+07	4.10E+07	1.37E+06	1.64E+08	1.89E−04	3.60	1.08E+05	5.76E+04	1.89E+04	2.62E+05	8.86E−05	3.75
COOT	8.45E+06	5.97E+06	8.60E+05	2.61E+07	1.89E−04	3.10	6.42E+04	5.17E+04	1.09E+04	2.40E+05	1.03E−04	2.80
	F14						F15					
CSMA	5.07E+04	3.03E+04	9.08E+03	1.00E+05	0.00E+00	2.60	**8.17E+03**	7.77E+03	1.88E+03	2.79E+04	0.00E+00	**1.50**
SMA	1.29E+05	3.33E+04	7.94E+04	1.87E+05	3.90E−04	3.90	3.44E+04	7.77E+03	1.94E+04	4.44E+04	1.63E−04	3.60
WOA	2.96E+06	2.64E+06	8.94E+05	1.01E+07	8.86E−05	7.00	2.28E+06	2.02E+06	3.98E+05	7.49E+06	8.86E−05	7.00
GWO	6.59E+05	7.20E+05	6.65E+04	2.68E+06	3.38E−04	4.80	1.38E+06	1.38E+06	6.51E+04	4.44E+06	8.86E−05	5.80
HHO	9.94E+05	4.46E+05	4.23E+05	2.08E+06	8.86E−05	5.85	1.21E+05	4.55E+04	7.66E+04	2.43E+05	8.86E−05	5.20
AOA	5.47E+04	6.63E+04	5.17E+03	2.78E+05	6.01E−01	2.00	9.20E+03	4.66E+03	2.57E+03	1.68E+04	1.79E−01	1.80
COOT	**3.23E+04**	2.74E+04	5.52E+03	1.08E+05	6.74E−02	**1.85**	2.50E+04	2.05E+04	7.31E+03	9.24E+04	1.71E−03	3.10
	F16						F17					
CSMA	**2.39E+03**	2.49E+02	1.88E+03	2.70E+03	0.00E+00	**1.90**	**2.15E+03**	1.42E+02	1.86E+03	2.34E+03	0.00E+00	2.40

Table 15 (continued)

Algorithm	AVG	STD	MIN	MAX	p values	R	AVG	STD	MIN	MAX	p values	R
SMA	2.75E+03	1.95E+02	2.46E+03	3.15E+03	4.05E−03	3.20	2.39E+03	1.76E+02	2.17E+03	2.78E+03	5.11E−03	4.10
WOA	4.14E+03	3.23E+02	3.66E+03	4.66E+03	8.86E−05	7.00	2.89E+03	2.01E+02	2.50E+03	3.30E+03	8.86E−05	6.65
GWO	2.72E+03	2.85E+02	2.39E+03	3.49E+03	2.28E−02	2.65	2.18E+03	5.48E+01	2.10E+03	2.31E+03	6.27E−01	**2.10**
HHO	3.64E+03	3.62E+02	3.23E+03	4.58E+03	8.86E−05	5.95	2.84E+03	2.63E+02	2.55E+03	3.40E+03	8.86E−05	6.35
AOA	2.81E+03	3.12E+02	2.18E+03	3.52E+03	1.16E−03	3.45	2.26E+03	2.02E+02	1.92E+03	2.61E+03	1.17E−01	3.25
COOT	2.90E+03	3.11E+02	2.29E+03	3.39E+03	2.93E−04	3.85	2.23E+03	2.32E+02	1.76E+03	2.48E+03	1.91E−01	3.15
	F18						F19					
CSMA	5.81E+05	3.10E+05	2.22E+05	1.36E+06	0.00E+00	3.15	**1.06E+04**	1.07E+04	2.15E+03	4.54E+04	0.00E+00	**2.00**
SMA	1.92E+06	2.15E+06	5.29E+05	7.78E+06	2.06E−02	3.90	3.99E+04	1.85E+04	1.09E+04	5.67E+04	1.94E−03	3.30
WOA	1.58E+07	1.21E+07	6.67E+06	5.93E+07	8.86E−05	7.00	1.60E+07	1.74E+07	3.51E+06	8.40E+07	8.86E−05	6.95
GWO	1.93E+06	1.75E+06	3.84E+05	5.98E+06	6.42E−03	4.20	5.54E+06	1.18E+07	2.46E+05	3.94E+07	8.86E−05	5.40
HHO	3.77E+06	3.70E+06	6.08E+05	1.43E+07	1.02E−03	5.65	1.03E+06	4.29E+05	5.29E+05	1.96E+06	8.86E−05	5.65
AOA	**2.35E+05**	2.38E+05	3.38E+04	8.82E+05	5.73E−03	**1.55**	2.00E+04	2.96E+04	2.62E+03	1.21E+05	2.63E−01	2.15
COOT	6.77E+05	5.27E+05	5.18E+04	1.70E+06	6.81E−01	2.55	2.06E+04	1.98E+04	2.99E+03	6.31E+04	5.22E−02	2.55
	F20						F21					
CSMA	2.38E+03	1.39E+02	2.17E+03	2.59E+03	0.00E+00	2.00	2.39E+03	1.48E+01	2.36E+03	2.42E+03	0.00E+00	**1.60**
SMA	2.67E+03	1.41E+02	2.49E+03	3.09E+03	7.80E−04	4.05	2.44E+03	2.91E+01	2.41E+03	2.52E+03	1.63E−04	3.65
WOA	2.97E+03	1.63E+02	2.68E+03	3.27E+03	8.86E−05	6.85	2.66E+03	4.04E+01	2.61E+03	2.75E+03	8.86E−05	7.00
GWO	2.53E+03	1.81E+02	2.32E+03	3.16E+03	6.74E−02	2.80	2.42E+03	1.26E+01	2.39E+03	2.44E+03	3.59E−03	2.15
HHO	2.91E+03	1.25E+02	2.75E+03	3.12E+03	8.86E−05	5.95	2.60E+03	2.99E+01	2.57E+03	2.68E+03	8.86E−05	6.00
AOA	2.52E+03	1.72E+02	2.29E+03	2.84E+03	1.69E−02	3.20	2.48E+03	3.58E+01	2.40E+03	2.54E+03	1.03E−04	4.65
COOT	2.53E+03	1.34E+02	2.23E+03	2.86E+03	1.71E−03	3.15	2.43E+03	3.20E+01	2.38E+03	2.49E+03	3.90E−04	2.95
	F22						F23					
CSMA	4.76E+03	1.49E+03	2.30E+03	6.09E+03	0.00E+00	2.35	**2.74E+03**	1.74E+01	2.71E+03	2.76E+03	0.00E+00	**1.30**
SMA	6.49E+03	4.84E+02	5.68E+03	7.39E+03	3.38E−04	4.00	2.78E+03	1.44E+01	2.76E+03	2.81E+03	2.19E−04	2.00
WOA	8.55E+03	6.42E+02	7.33E+03	9.63E+03	8.86E−05	6.55	3.18E+03	6.28E+01	3.09E+03	3.33E+03	8.86E−05	5.95
GWO	5.34E+03	1.55E+03	2.65E+03	7.38E+03	3.51E−01	2.85	2.80E+03	4.60E+01	2.75E+03	2.93E+03	1.40E−04	3.30
HHO	7.54E+03	5.25E+02	6.70E+03	8.57E+03	8.86E−05	5.40	3.34E+03	9.62E+01	3.23E+03	3.63E+03	8.86E−05	7.00

Table 15 (continued)

Algorithm	AVG	STD	MIN	MAX	*p* values	R	AVG	STD	MIN	MAX	*p* values	R
AOA	7.26E+03	2.70E+03	2.33E+03	1.11E+04	3.19E−03	5.40	2.96E+03	1.21E+02	2.79E+03	3.26E+03	8.86E−05	4.80
COOT	**2.75E+03**	1.37E+03	2.30E+03	6.76E+03	<u>8.97E−03</u>	**1.45**	2.83E+03	5.67E+01	2.73E+03	2.97E+03	1.63E−04	3.65
	F24						F25					
CSMA	**2.91E+03**	1.82E+01	2.86E+03	2.93E+03	0.00E+00	**1.40**	**2.89E+03**	1.81E+00	2.88E+03	2.89E+03	0.00E+00	**1.15**
SMA	2.95E+03	2.78E+01	2.92E+03	3.02E+03	<u>5.73E−03</u>	2.20	2.90E+03	1.71E+01	2.89E+03	2.94E+03	1.63E−04	2.00
WOA	3.32E+03	6.23E+01	3.23E+03	3.42E+03	8.86E−05	5.80	3.13E+03	2.01E+01	3.10E+03	3.17E+03	8.86E−05	6.90
GWO	2.97E+03	5.30E+01	2.93E+03	3.09E+03	1.40E−04	3.35	3.08E+03	1.07E+02	3.01E+03	3.47E+03	8.86E−05	6.05
HHO	3.55E+03	1.00E+02	3.40E+03	3.76E+03	8.86E−05	6.85	2.96E+03	2.46E+01	2.93E+03	3.02E+03	8.86E−05	4.45
AOA	3.18E+03	2.13E+02	2.94E+03	3.67E+03	8.86E−05	5.10	2.96E+03	3.69E+01	2.91E+03	3.04E+03	8.86E−05	4.20
COOT	2.97E+03	4.26E+01	2.91E+03	3.06E+03	8.86E−05	3.30	2.93E+03	1.93E+01	2.89E+03	2.96E+03	1.03E−04	3.25
	F26						F27					
CSMA	**4.56E+03**	1.47E+02	4.27E+03	4.80E+03	0.00E+00	**1.85**	3.21E+03	6.57E+00	3.20E+03	3.22E+03	0.00E+00	2.20
SMA	4.98E+03	1.94E+02	4.72E+03	5.39E+03	2.19E−04	3.30	3.24E+03	1.46E+01	3.22E+03	3.26E+03	4.49E−04	3.00
WOA	8.72E+03	7.72E+02	7.43E+03	1.04E+04	8.86E−05	6.80	3.57E+03	1.83E+02	3.38E+03	4.08E+03	8.86E−05	6.35
GWO	4.94E+03	3.34E+02	4.57E+03	5.70E+03	2.50E−03	2.85	3.28E+03	2.44E+01	3.26E+03	3.35E+03	8.86E−05	4.75
HHO	8.44E+03	5.58E+02	7.42E+03	9.70E+03	8.86E−05	6.00	3.56E+03	1.71E+02	3.40E+03	4.18E+03	8.86E−05	6.65
AOA	5.84E+03	1.94E+03	3.40E+03	8.55E+03	<u>1.00E−02</u>	3.65	**3.20E+03**	2.91E−04	3.20E+03	3.20E+03	8.86E−05	**1.00**
COOT	5.30E+03	1.27E+03	2.81E+03	7.25E+03	<u>2.51E−02</u>	3.55	3.27E+03	3.13E+01	3.22E+03	3.34E+03	1.03E−04	4.05
	F28						F29					
CSMA	**3.23E+03**	1.51E+01	3.20E+03	3.25E+03	0.00E+00	**1.35**	**3.79E+03**	1.22E+02	3.53E+03	3.97E+03	0.00E+00	**1.85**
SMA	3.26E+03	3.25E+01	3.23E+03	3.35E+03	<u>1.11E−02</u>	2.15	4.02E+03	1.79E+02	3.83E+03	4.45E+03	4.05E−03	3.50
WOA	3.63E+03	9.38E+01	3.53E+03	3.84E+03	8.86E−05	6.95	5.65E+03	4.11E+02	4.91E+03	6.51E+03	8.86E−05	7.00
GWO	3.51E+03	1.21E+02	3.42E+03	3.94E+03	8.86E−05	6.00	3.96E+03	1.46E+02	3.76E+03	4.27E+03	<u>1.37E−02</u>	2.55
HHO	3.35E+03	2.69E+01	3.33E+03	3.43E+03	8.86E−05	4.70	4.96E+03	4.00E+02	4.57E+03	6.04E+03	8.86E−05	6.00
AOA	3.34E+03	6.41E+01	3.27E+03	3.49E+03	8.86E−05	4.20	4.06E+03	2.96E+02	3.59E+03	4.55E+03	<u>5.11E−03</u>	3.10
COOT	3.27E+03	2.89E+01	3.21E+03	3.31E+03	1.20E−04	2.65	4.15E+03	2.15E+02	3.76E+03	4.47E+03	3.38E−04	4.00
	F30											
CSMA	**2.42E+04**	6.87E+03	1.32E+04	3.60E+04	0.00E+00	**1.60**						

Table 15 (continued)

Algorithm	AVG	STD	MIN	MAX	p values	R	AVG	STD	MIN	MAX	p values	R
SMA	7.29E+04	3.30E+04	3.55E+04	1.52E+05	1.03E−04	2.65						
WOA	6.89E+07	4.94E+07	3.48E+07	2.29E+08	8.86E−05	7.00						
GWO	1.40E+07	8.32E+06	5.34E+06	3.43E+07	8.86E−05	6.00						
HHO	5.89E+06	1.97E+06	3.51E+06	1.13E+07	8.86E−05	5.00						
AOA	2.97E+05	6.05E+05	4.76E+03	2.03E+06	2.63E−01	1.90						
COOT	6.88E+05	5.45E+05	6.50E+04	2.44E+06	8.86E−05	3.85						

Table 16 Average of Friedman mean rank

	CSMA	SMA	WOA	GWO	HHO	AOA	COOT
Unimodal shifted and rotated functions	**1.2**	1.8	6.6	6.15	4.1	5.15	3
Multimodal shifted and rotated functions	1.59	2.86	6.89	3.19	5.79	4.44	3.24
Hybrid functions	1.91	3.37	6.93	4.47	5.54	2.92	2.87
Composition functions	1.70	2.95	6.65	3.88	5.81	3.75	3.26

2018), new hybrid GA-ACO-PSO (Tam et al. 2019), BSA (Varol Altay and Alatas 2020) and bird swarm algorithm with chaotic map (CMBSA) (Varol Altay and Alatas 2020). The results of the comparisons are given in the Table 20. As seen in the table, in the welded beam design problem, sinusoidal map based SMA has achieved more minimum values compared to other algorithms. Furthermore; the performance of the proposed CSMAs on this problem has also been compared with the studies in the literature, and its graphic is given in Fig. 6.

5.4.2 Pressure vessel design

There are 4 variables in the pressure vessel design problem. These variables are shown in the Fig. 7, namely Ts (x_1), Th $\left(x_2\right)$, R $\left(x_3\right)$ ve L $\left(x_4\right)$. The ranges of R and L values from these four variables should be between 10 and 200. The range for $\mathrm{T_s}$ and $\mathrm{T_h}$ values are $1 \times 0.0625 \leq T_s, T_h \leq 99 \times 0.0625$. The objective function and constraints are shown in the Eqs. (36–40).

$$\text{Minimize}: \ f(x) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3. \tag{36}$$

Subject to:

$$g_1(x) = -x_1 + 0.0193x_3 \leq 0 \tag{37}$$

$$g_2(x) = -x_2 + 0.00954x_3 \leq 0 \tag{38}$$

$$g_3(x) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \leq 0 \tag{39}$$

$$g_4(x) = x_4 - 240 \leq 0 \tag{40}$$

SMA and CSMAs have been tested by applying pressure to the vessel design problem. The population size was 30, the number of iterations was 1000, and the algorithms were run 30 times. In addition, it has been compared with the results of the studies obtained from the literature. These are WOA (Mirjalili et al. 2016), AAO (Czerniak et al. 2017), COBSA (Wang et al. 2018), CBPPSO (Agrawal and Tripathi 2018), GA-ACO-PSO (Tam et al. 2019), an adaptive reinforcement learning-based bat algorithm (ARLBAT) (Meng et al. 2019), BSA (Varol Altay and Alatas 2020) and CMBSA (Varol Altay and Alatas 2020). The results of the comparisons are given in the Table 21. As can be seen from the table, it has obtained a better optimum cost value for CSMA than other algorithms.

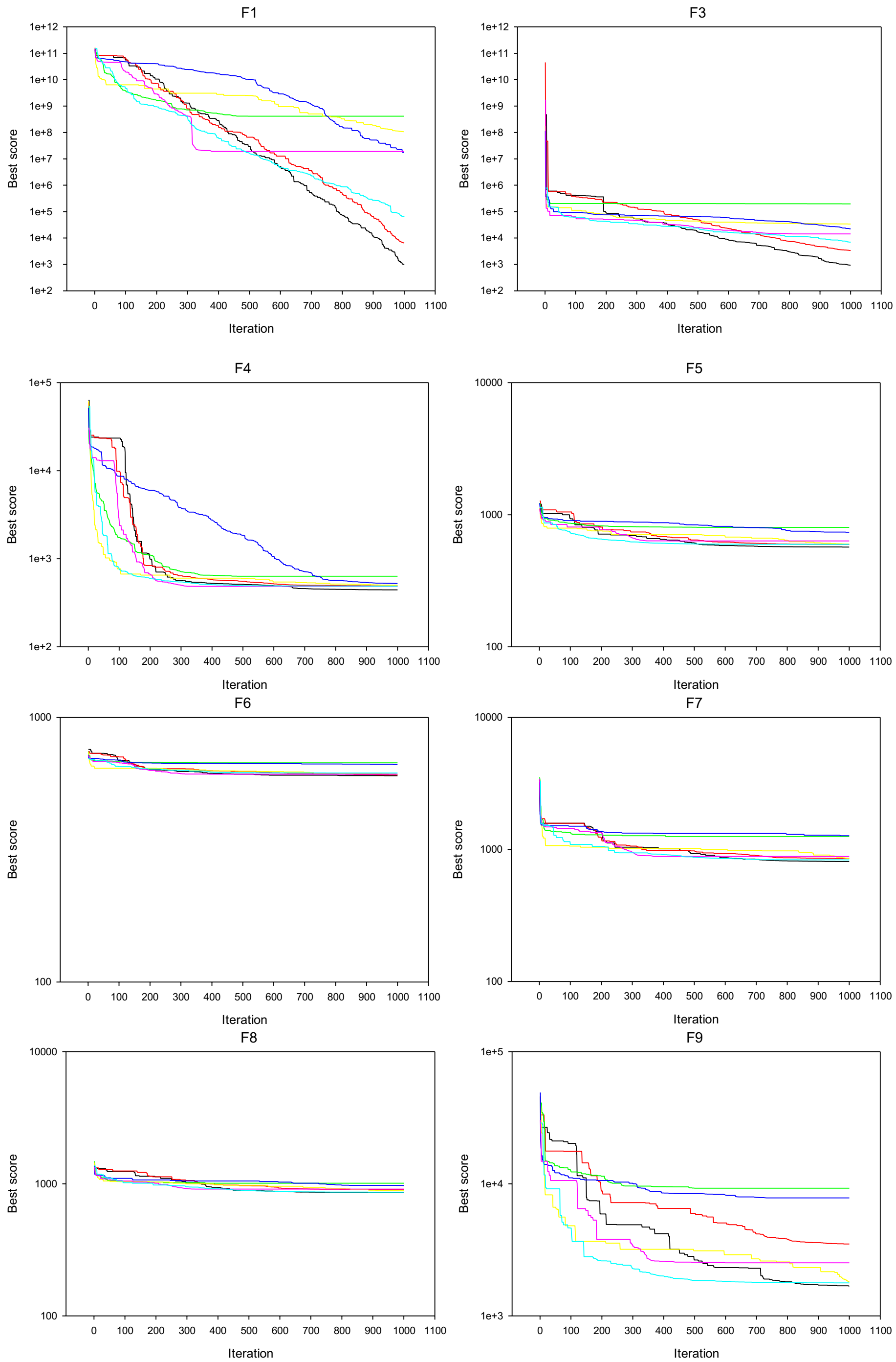

Fig. 4 Convergence curve of CSMAs and other methods on the CEC2017 test suite

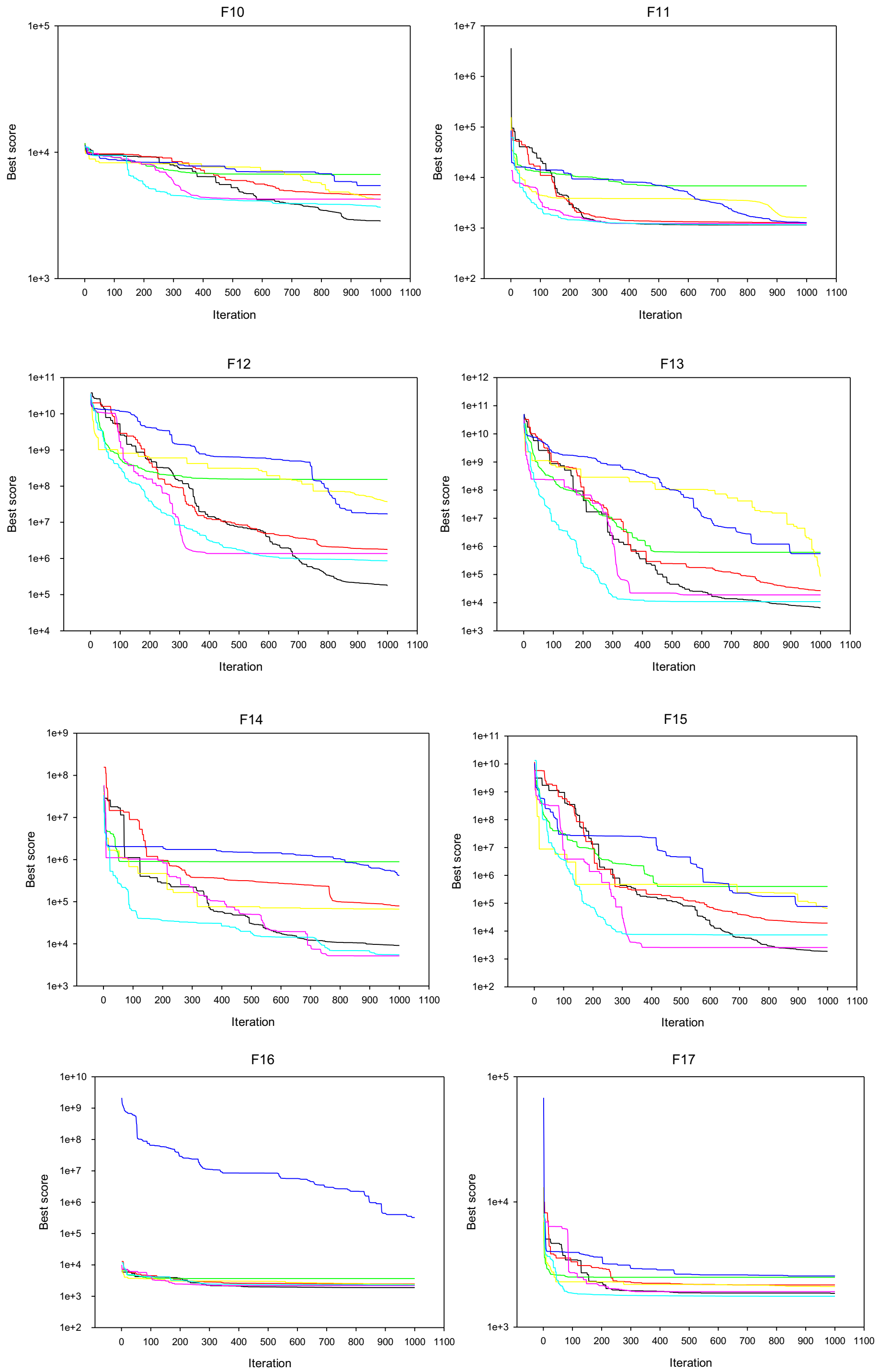

Fig. 4 (continued)

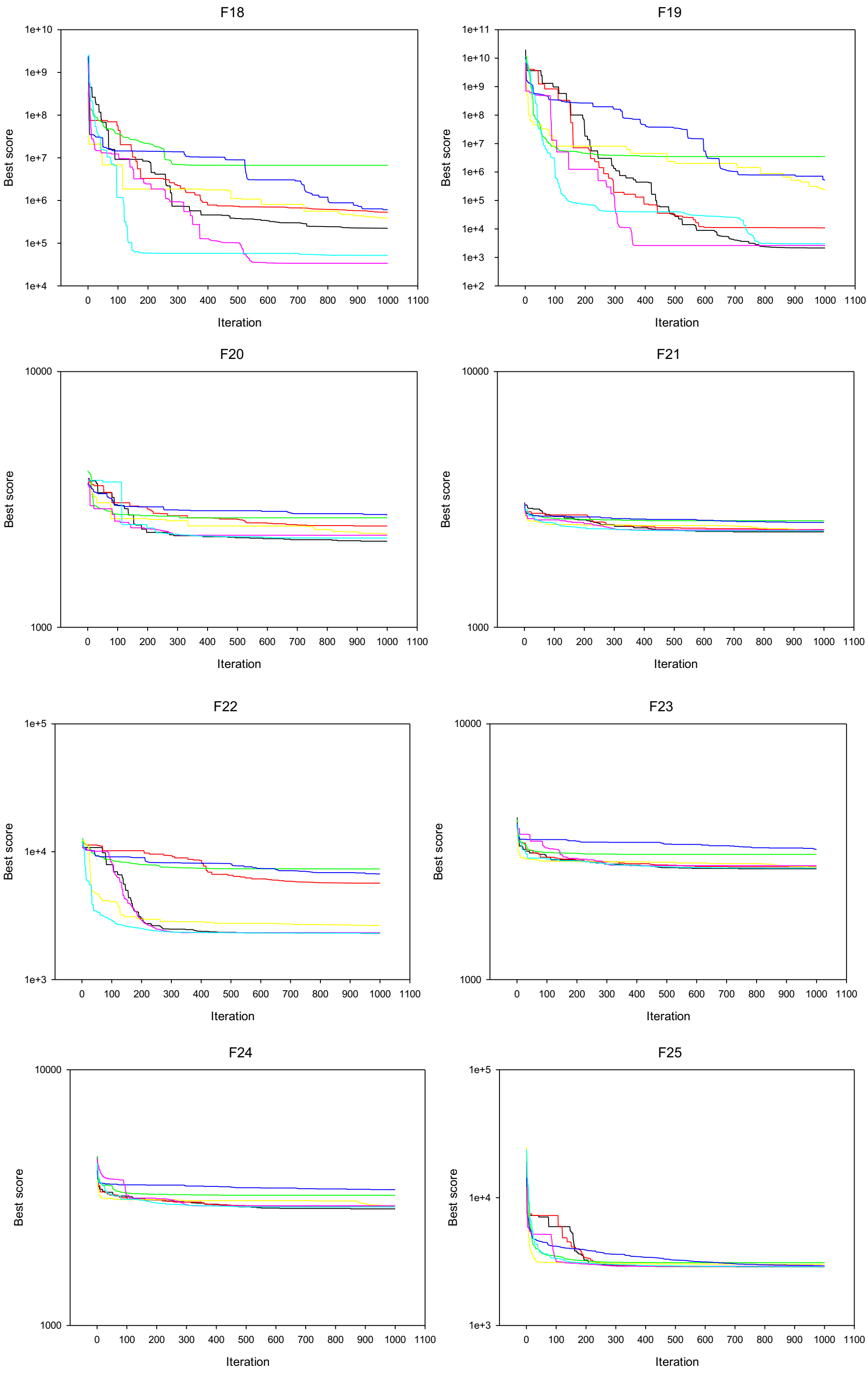

Fig. 4 (continued)

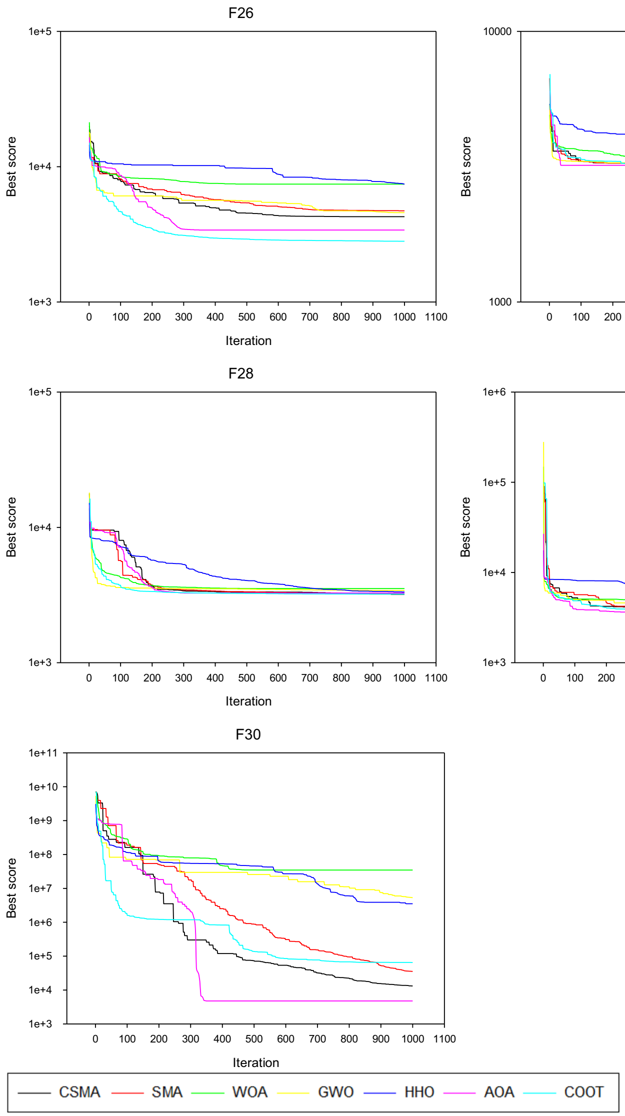

Fig. 4 (continued)

Furthermore; the performance of the proposed CSMAs on this problem has also been compared with the studies in the literature, and its graphic is given in Fig. 8.

5.4.3 Tension/compression spring design

Tension/compression spring design problem aims to create a spring design with a minimum weight level. It consists of 3 design variables and 4 constraints. Variable wire diameter d (x_1), the mean coil diameter D (x_2) ve number of active coil N (x_3) are shown in

Table 17 Results with varied values of the number of iterations

	f1	f3	f4	f5	f6	f7	f8	f9	f10	f11
Iterations = 100	1.01E+08	1.40E+05	5.76E+02	6.90E+02	6.43E+02	9.86E+02	9.73E+02	6.96E+03	5.62E+03	1.57E+03
Iterations = 500	1.10E+05	4.06E+04	5.05E+02	6.33E+02	6.19E+02	8.82E+02	9.32E+02	5.24E+03	4.67E+03	1.30E+03
Iterations = 1000	8.26E+03	2.93E+03	4.91E+02	5.98E+02	6.11E+02	8.41E+02	9.00E+02	3.13E+03	4.38E+03	1.24E+03

Table 18 Results with varied values of search agent

	f1	f3	f4	f5	f6	f7	f8	f9	f10	f11
Search agent = 10	1.44E+06	5.01E+04	5.15E+02	6.54E+02	6.28E+02	9.60E+02	9.58E+02	5.36E+03	5.19E+03	1.31E+03
Search agent = 30	8.26E+03	2.93E+03	4.91E+02	5.98E+02	6.11E+02	8.41E+02	9.00E+02	3.13E+03	4.38E+03	1.24E+03
Search agent = 100	6.36E+03	3.17E+02	4.90E+02	5.97E+02	6.02E+02	8.40E+02	8.94E+02	3.06E+03	4.34E+03	1.23E+03

Table 19 Results with varied values of parameter z

	f1	f3	f4	f5	f6	f7	f8	f9	f10	f11
z = 0.01	1.11E+04	1.33E+04	5.00E+02	6.14E+02	6.12E+02	8.73E+02	9.21E+02	4.46E+03	4.89E+03	1.30E+03
z = 0.03	**8.26E+03**	**2.93E+03**	**4.91E+02**	**5.98E+02**	6.11E+02	**8.41E+02**	**9.00E+02**	**3.13E+03**	**4.38E+03**	**1.24E+03**
z = 0.06	8.59E+04	4.83E+03	5.08E+02	6.14E+02	**6.08E+02**	8.69E+02	9.14E+02	3.74E+03	4.73E+03	1.27E+03
z = 0.1	5.40E+05	5.38E+03	5.02E+02	6.09E+02	6.10E+02	8.68E+02	9.12E+02	3.37E+03	4.68E+03	1.27E+03

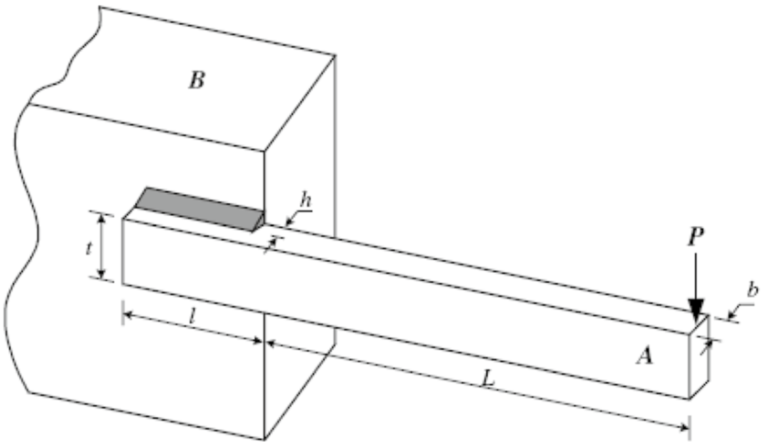

Fig. 5 Welded beam design problem

Fig. 9. The constraints of these values are $0.05 \le x_1 \le 2$, $0.25 \le x_2 \le 1.3$ and $2 \le x_3 \le 15$, respectively. The objective function and constraints are shown in the Eq. (41–45).

$$\text{Minimize}: \ f(x) = \left(x_3 + 2\right)x_1^2x_2 \tag{41}$$

Subject to:

$$g_1(x) = 1 - \frac{x_2^3x_3}{71785x_1^4} \le 0 \tag{42}$$

$$g_2(x) = \frac{4x_2^2 - x_1x_2}{12566\left(x_2x_{1\_}^3x_1^4\right)} + \frac{1}{5108x_1^2} \le 0 \tag{43}$$

$$g_3(x) = 1 - \frac{140.45x_1}{x_2^3x_3} \le 0 \tag{44}$$

$$g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \le 0 \tag{45}$$

SMA and CSMAs have been tested by applying the tension–compression string design problem. The population size was 30, the number of iterations was 1000, and the algorithms were run 30 times. In addition, it has been compared with the results of the studies obtained from the literature. These are (WOA) (Mirjalili et al. 2016), AAO (Czerniak et al. 2017), CGWO (Kohli et al. 2017), COBSA (Wang et al. 2018), CBPPSO (Agrawal and Tripathi 2018), BSA (Varol Altay and Alatas 2020) and CMBSA (Varol Altay and Alatas 2020). The results of the comparisons are given in the Table 22. As seen in the Table 22, in the tension–compression string design problem, the sine map based SMA has achieved relatively minimum values compared to other algorithms. Furthermore; the performance

Table 20 Results of welded beam design problem compared with other algorithms

Algorithms	Optimum values for variables				Optimum cost
	h	*l*	*t*	*b*	
CSMA-1	0.2057	3.4710	9.0366	0.2057	1.7246
CSMA-2	0.2057	3.4772	9.0362	0.2057	1.7249
CSMA-3	0.2057	3.4709	9.0369	0.2057	1.7247
CSMA-4	0.2057	3.4711	9.0367	0.2057	1.7247
CSMA-5	0.2057	3.4710	9.0365	0.2057	1.7246
CSMA-6	0.2057	3.4706	9.0375	0.2057	1.7247
CSMA-7	0.2057	3.4713	9.0366	0.2057	1.7247
CSMA-8	0.2057	3.4713	9.0366	0.2057	1.7247
CSMA-9	0.2056	3.4710	9.0366	0.2057	**1.7245**
CSMA-10	0.2056	3.4723	9.0370	0.2057	1.7247
SMA	0.2056	3.4723	9.0366	0.2057	1.7247
WOA (Mirjalili et al. 2016)	0.2054	3.4843	9.0374	0.2063	1.7305
FA (Erdal 2017)	0.2015	3.5620	9.0414	0.2057	1.7312
COBSA (Wang et al. 2018)	0.2057	3.4705	9.0366	0.2057	1.7249
CGWO (Kohli et al. 2018)	0.3439	1.8836	9.0313	0.2121	1.7255
DBCA (Prayogo et al. 2018)	0.2057	3.4705	9.0366	0.2057	1.7249
SIO (Tzanetos and Dounias 2020)	0.3314	2.0174	9.0459	0.2088	1.7621
CBPPSO (Agrawal and Tripathi 2018)	0.2057	3.4704	9.0366	0.2057	1.7249
GA-ACO-PSO (Tam et al. 2019)	0.2057	3.4705	9.0366	0.2057	1.7249
BSA (Varol Altay and Alatas 2020)	0.2057	3.4706	9.0366	0.2057	1.7249
CMBSA (Varol Altay and Alatas 2020)	0.2057	3.4702	9.0377	0.2057	1.7249

of the proposed CSMAs on this problem has also been compared with the studies in the literature, and its graphic is given in Fig. 10.

6 Conclusions

In this study, chaotic maps have been integrated into SMA. It has been developed with 10 different chaotic maps with spreading spectrum, unpredictable, irregular, ergodic, and stochastic features to achieve faster convergence and higher accuracy than standard SMA. Chaotic map based SMA has been tested and statistical analysis performed with a total of 62 different benchmark functions, including unimodal, multimodal, fixed dimension, CEC2019, and CEC2017. CSMAs have been compared under equal conditions with the commonly used metaheuristic optimization algorithms (DE and PSO) and the newly proposed metaheuristic optimization algorithms (standard SMA, WOA, and GWO) in the unimodal, multimodal, fixed dimension, and CEC2019 test suite. In addition, CSMAs have been compared with the SMA, WOA, GWO, HHO, AOA and COOT algorithms proposed in recent years in the CEC2017 test suite. Chaotic map-based SMA methods have generally performed better than the standard SMA method. In addition, CSMAs has been applied to three different real-world engineering design problems. According to the experimental results, it has been seen that CSMAs give better results than other metaheuristic

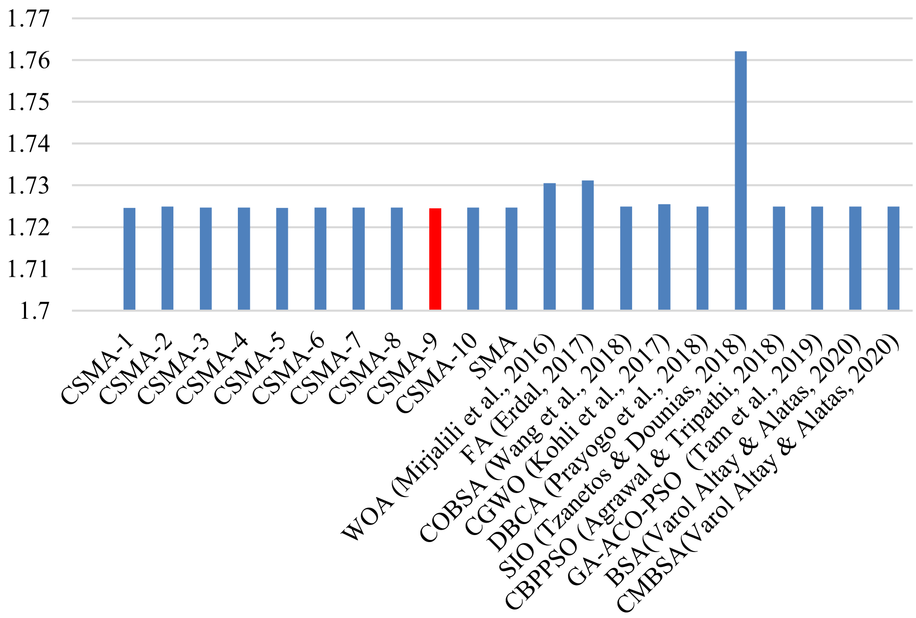

Fig. 6 The results of the cost values obtained by comparing the proposed CSMAs on the welded beam design problem with the literature

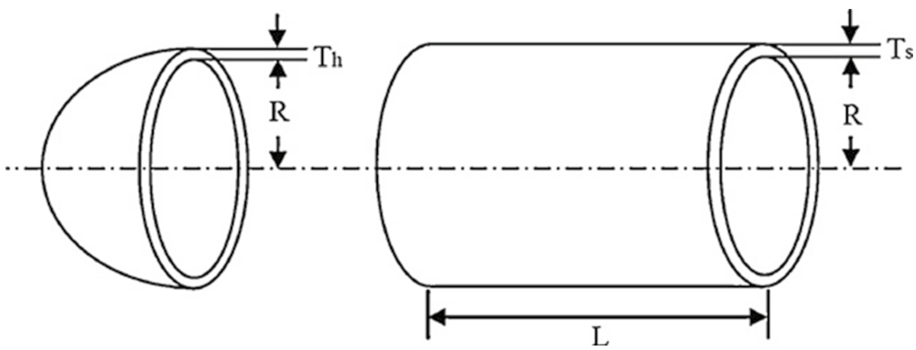

Fig. 7 Pressure vessel design problem

optimization algorithms compared. The proposed CSMA methods generally increased the solution quality and speed of convergence. Thus, the global search capability of the standard SMA has been developed by avoiding local solutions. Since SMA has been proposed more recently, multidimensional chaotic maps, continuously chaotic systems, and fraction-based chaotic systems, and hybrid multidimensional maps can be used to improve SMA's performance. Thus, the proposed methods can be used more effectively than SMA such as in machine learning methods (support vector machine and artificial neural network models, etc.), power system problems, mechanical problems, industrial chemical processes, process synthesis, and design problems, and mechanical engineering problems and power

Table 21 Results of pressure vessel design problem compared with other algorithms

Algorithms	Optimum values for variables				Optimum cost
	x_1	x_2	x_3	x_4	
CSMA-1	0.7778	0.3845	40.3196	200.0	5882.2262
CSMA-2	0.7778	0.3845	40.3196	200.0	5882.2262
CSMA-3	0.7778	0.3845	40.3196	200.0	5882.2262
CSMA-4	0.7779	0.3845	40.3221	199.9649	5882.6085
CSMA-5	0.7778	0.3845	40.3196	200.0	5882.2262
CSMA-6	0.7778	0.3845	40.3196	199.9998	5882.2219
CSMA-7	0.7778	0.3845	40.3207	199.9850	**5882.0851**
CSMA-8	0.7778	0.3845	40.3196	200.0	5882.2262
CSMA-9	0.7804	0.3858	40.4513	198.1744	5887.1324
CSMA-10	0.7778	0.3845	40.3196	200.0	5882.2262
SMA	0.8260	0.4083	42.8155	167.9488	5970.0507
WOA (Mirjalili et al. 2016)	0.8125	0.4375	42.0982	176.6389	6059.7410
AAO (Czerniak et al. 2017)	0.8125	0.4375	42.0985	176.6366	6059.7140
COBSA (Wang et al. 2018)	0.8115	0.4375	42.0984	176.6366	6059.7143
CBPPSO (Agrawal and Tripathi 2018)	1.125	0.6250	62.9866	20.00000	6952.7200
ARLBAT (Meng et al. 2019)	0.8125	0.4375	42.0984	176.6366	6059.7143
GA-ACO-PSO (Tam et al. 2019)	0.8125	0.4375	42.0984	176.6366	6059.7143
BSA (Varol Altay and Alatas 2020)	0.7788	0.3846	40.3542	199.5188	5886.8000
CMBSA (Varol Altay and Alatas 2020)	0.7780	0.3850	40.3200	200.0000	5883.8610

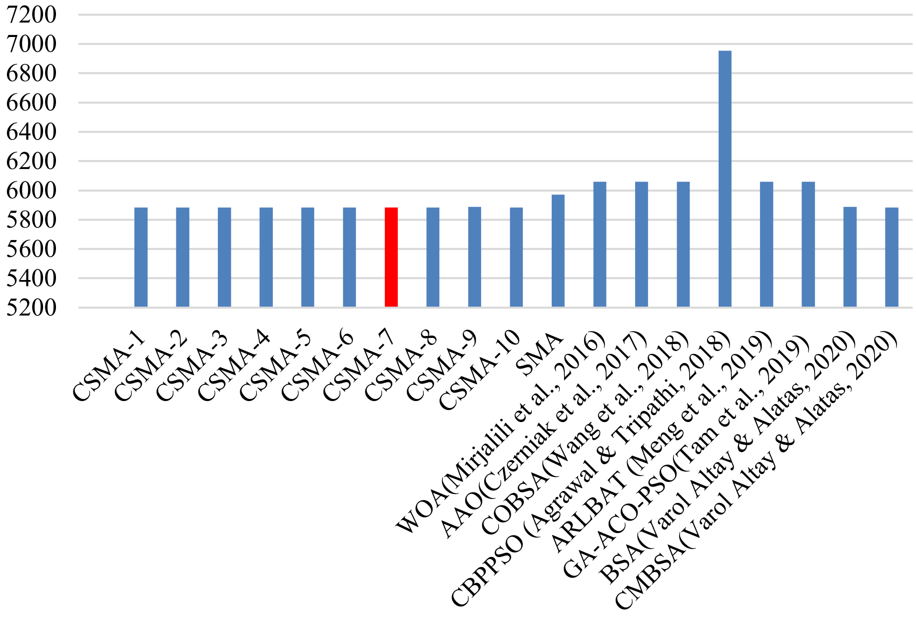

Fig. 8 The results of the cost values obtained by comparing the proposed CSMAs on the pressure vessel design problem with the literature

Fig. 9 Tension–compression spring design problem

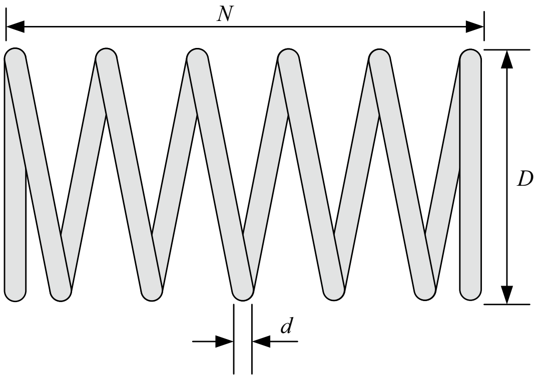

Table 22 Results of tension–compression spring design compared with other algorithms

Algorithms	Optimum values for variables			Optimum cost
	x_1	x_2	x_3	
CSMA-1	0.7939	0.6506	6.9594	0.0127
CSMA-2	0.05	0.3174	14.0279	0.0127
CSMA-3	0.05	0.3174	14.0278	0.0127
CSMA-4	0.05	0.3174	14.0280	0.0127
CSMA-5	0.05	0.3174	14.0279	0.0127
CSMA-6	0.05	0.3174	14.0291	0.0127
CSMA-7	0.0511	0.3427	12.1605	0.0126
CSMA-8	0.05	0.3175	14.0195	0.0127
CSMA-9	0.05	0.3174	14.0282	0.0127
CSMA-10	0.05	0.3174	14.0278	0.0127
SMA	0.05	0.3174	14.0276	0.0127
WOA (Mirjalili et al. 2016)	0.0512	0.3452	12.004	0.0127
AAO (Czerniak et al. 2017)	0.0517	0.3581	11.2015	0.0127
CGWO (Kohli et al. 2018)	0.0528	0.8044	2.000	**0.0120**
COBSA (Wang et al. 2018)	0.0516	0.3566	11.2918	0.0127
CBPPSO (Agrawal and Tripathi 2018)	0.5126	0.3465	11.9097	0.0127
BSA (Varol Altay and Alatas 2020)	0.0528	0.3835	9.8751	0.0127
CMBSA8 (Varol Altay and Alatas 2020)	0.0519	0.3618	11.000	0.0127

electronic problems. In addition, distributed, multi-objective, and parallel versions of SMA can be brought to the literature and used efficaciously for metaheuristic optimization problems.

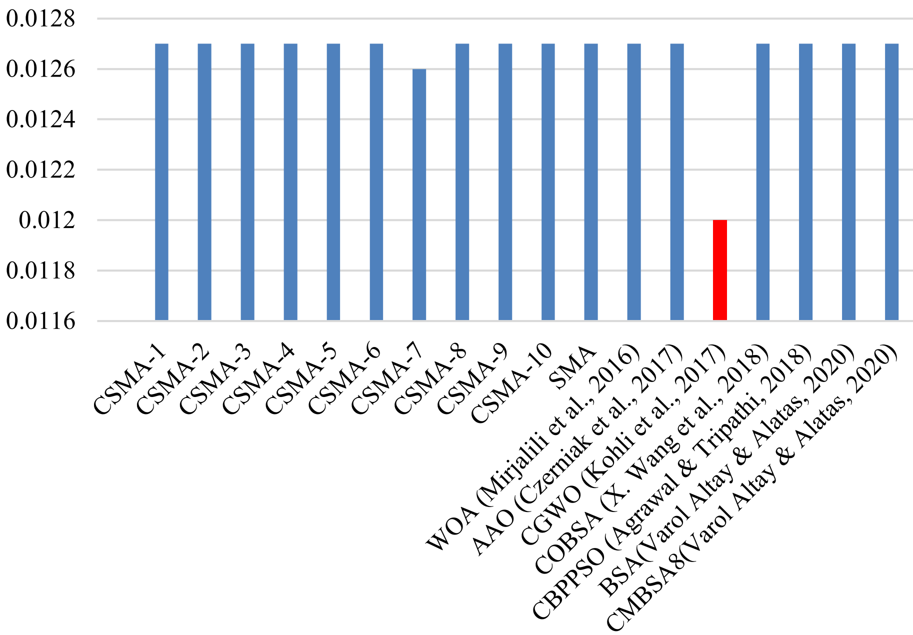

Fig. 10 The results of the cost values obtained by comparing the proposed CSMAs on the tension–compression string design problem with the literature

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