Artificial Intelligence Review (2021) 54:4137–4167
https://doi.org/10.1007/s10462-020-09950-2

Probabilistic linguistic multiple attribute group decision making for location planning of electric vehicle charging stations based on the generalized Dice similarity measures

Guiwu Wei[1] · Cun Wei[2] · Jiang Wu[2] · Yanfeng Guo[3]

Published online: 16 January 2021

Abstract
The location of the electric vehicle charging station is deemed to be a multiple attribute group decision making (MAGDM) issue involving many experts and many conflicting attributes. In practical MAGDM issues, the information of uncertain and fuzzy cognitive decision is well-depicted by linguistic term sets (LTSs). These LTSs could be simply shifted into the probabilistic linguistic sets (PLTSs). In such paper, we design some novel probabilistic linguistic weighted Dice similarity measures (PLWDSM) and the probabilistic linguistic weighted generalized Dice similarity measures (PLWGDSM). Subsequently, the PLWGDSM-based MAGDM methods are presented under PLTSs. In the end, a practical case which concerns about the location planning of electric vehicle charging stations is offered to demonstrate the proposed PLWGDSM's applicability and advantages.

Keywords Multiple attribute group decision making · Probabilistic linguistic term set · Dice similarity measures · Generalized Dice similarity measures · Site selection · Electric vehicle charging stations

1 Introduction

In our everyday lives, decision-making issues are the regular behavior activities (Braglia et al. 2003; Liu et al. 2019a; Tian et al. 2017, 2018). It has been deemed that almost all assessing information is expressed with numerical values (Deng et al. 2000; Bourguignon and Massart 1994; Tsoulfas and Pappis 2008). Due to the human thinking's fuzziness and vagueness and the objective things' complexity (Chen et al. 2019a; Chen and Han 2019; Wei et al. 2020a, b; Wang et al. 2020), individuals are willing to express their

✉ Guiwu Wei
weiguiwu@163.com

1 School of Business, Sichuan Normal University, Chengdu 610101, People's Republic of China

2 School of Statistics, Southwestern University of Finance and Economics, Chengdu 611130, People's Republic of China

3 School of Finance, Southwestern University of Finance and Economics, Chengdu 610074, People's Republic of China

assessment information by linguistic term sets (LTSs) rather than the form of quantitative in more and more vague decision-making issues. For instance, the DMs may utilize the LTSs like "bad", "medium" and "good" when a family car's satisficing degree is assessed. Therefore, more and more studies on diverse linguistic models have brought about extensive attention by more and more scholars. In order to handle these qualitative assessment information easily, Herrera and Martinez (2000a) designed the linguistic term sets (LTSs) for calculating with given words. Herrera and Martinez (2000b) used 2-tuple fuzzy linguistic to combine the linguistic information and numerical. Herrera and Martinez (2001) designed the multi-granular hierarchical linguistic to handle linguistic 2-tuples. Geng et al. (2017) proposed the extension of 2-tuple linguistic DEA for tackling MAGDM issues which considered the attributes' effect relationships. Furthermore, Rodriguez et al. (2012) brought up the hesitant fuzzy LTSs (HFLTSs) with the aid of HFSs (Torra 2010) and LTSs (Zadeh 1975) which allowed DMs to provide some possible LTSs. However, in most living studies on HFLTSs, all possible values are handled by the DMs by using same weight or importance. Visibly, it is not consistent with our real life. In living situations, the DMs may assign possible linguistic terms so that the furnished information may have different probability distributions. Thus, Pang et al. (2016) raised the probabilistic linguistic term sets (PLTSs) to surmount this defect. Bai et al. (2017) built a comparison method which is more appropriate and proposed a tool which is more effective to manage PLTSs. Zhang et al. (2016) introduced the PLTSs to express the DMs' preferences information and discussed additive consistency of PLPR from preference relation graph. Lin et al. (2019) put up with the ELECTRE II method to manage PLTSs for edge computing. Liao et al. (2019) raised the novel operations of PLTSs to work out the probabilistic linguistic ELECTRE III method. Liang et al. (2018) developed the probabilistic linguistic grey relational analysis (PL-GRA) for MAGDM based on geometric Bonferroni mean (Wang et al. 2018; Wei et al. 2019; Zhu and Xu 2013). Liao et al. (2017) designed a linear programming algorithm to settle the MADM issues with PLTSs. Chen et al. (2019b) extended MULTIMOORA to the probabilistic linguistic for cloud-based ERP system selection. Feng et al. (Feng et al. 2019) set up the probabilistic linguistic QUALIFLEX method for MAGDM issue. Lu et al. (2019) designed TOPSIS method with entropy weight under the probabilistic linguistic environment for MAGDM issues to select the most appropriable supplier with new agricultural machinery products. Kobina et al. (2017) planned some power operators for MAGDM with PLTSs and classical power aggregation operators (Wei 2019a; Yager 2001; Xu and Yager 2010).

As a major and effective tool, the similarity measure is utilized to depict similarity's degree between objects (Wang et al. 2019a; Wei 2017, 2018; Wei and Wei 2018). Actually, similarity or dissimilarity's degree between objects plays a significant role under the currently researches (Li and Cheng 2002; Li 2004; Chen et al. 2016; Peng and Garg 2018; Sharaf 2018; Peng and Li 2019; Xian et al. 2019; Zhang et al. 2019). The measures of Jaccard, cosine and Dice similarity are frequently utilized in different domains (Dice 1945; Jaccard 1901; Salton and McGill 1987). However, Ye (2012a) designed the measures of Jaccard, cosine and Dice similarity between TIFNs for MAGDM issues. Ye (2012b) worked out the MADM approaches by employing the measure of Dice similarity between the expected ITFNs. Ye (2014) designed the Dice measures to tackle the simplified neutrosophic sets. Ye (2016) put up with the generalized Dice measures for MADM issues under simplified neutrosophic setting. Tang et al. (2017) came up with the GDSM to deal with MAGDM issues with intuitionistic information. Mahmood et al. (2016) designed three measures of similarity between simplified neutrosophic HFSs. Mandal and Basu (2016) defined two novel measures of similarity to conquer several limitations of existing all kinds of similarity measures. Wei (2019b) developed the GDSM for MADM issues with HFLTSs. Wang et al. (2019b) designed some new DSM of PFSs and the GDSM of PFSs to tackle MAGDM issues for choosing the most appropriable ERP system. Wei and Gao (2018) worked out some new DSM of picture fuzzy sets for building material recognition.

However, these DSM do not directly tackle the measures of similarity for PLTSs. Hence, extending the Dice measure to PLTSs to tackling MAGDM issues is essential which can fulfill DMs preference. To do so, the main aims of such paper are: (1) two DSM' forms within PLTSs are designed; (2) the GDSM and weighted GDSM with PLTSs are proposed; (3) the weighted GDSM are employed to work out the MAGDM issues under PLTSs; (4) within the process of MAGDM, the developed methods' major merit is more flexible and useful compared with the existing MAGDM issues with PLTSs.

To do so, this essay's remainder is given in the following. In Sect. 2, some fundamental theories which concerns about PLTSs is proposed. In Sect. 3, several measures of DSM and weighted DSM between PLTSs are put forward. In Sect. 4, the weighted GDSM are used to work out MAGDM issues with PLTSs. A practical case study for site selection of EVCS is offered to validate the developed weighted Dice similarity measure in Sect. 5. The paper with some remarks is concluded in the final section.

2 Preliminaries

Firstly, Xu (2005)worked out the additive linguistic scale and Gou et al. (2017) put up with the corresponding transformation function between the linguistic terms and [0, 1].

Definition 1 (Xu 2005; Gou et al. 2017) Let $L = \left\{ l_\alpha | \alpha = -\theta, \dots, -2, -1, 0, 1, 2, \dots \theta \right\}$ be an LTS (Xu 2005), the linguistic terms l_α can express the same information to β which is expressed with transformation mathematical function g (Gou et al. 2017):

$$g : \left[l_{-\theta}, l_\theta \right] \to [0, 1],\ g\left(l_\alpha \right) = \frac{\alpha + \theta}{2\theta} = \beta \tag{1}$$

β can also be represented the equal information by linguistic terms l_α which is denoted with the transformation function g^{-1}:

$$g^{-1} : [0, 1] \to \left[l_{-\theta}, l_\theta \right],\ g^{-1}(\beta) = l_{(2\beta-1)\theta} = l_\alpha \tag{2}$$

Definition 2 (Pang et al. 2016) Given the LTS $L = \left\{ l_\alpha | \alpha = -\theta, \dots, -2, -1, 0, 1, 2, \dots \theta \right\}$, a PLTS could be designed as:

$$PL(p) = \left\{ l^{(\phi)}\left(p^{(\phi)}\right) \middle| l^{(\phi)} \in L,\quad p^{(\phi)} \geq 0,\quad \phi = 1, 2, \dots,\quad \#PL(p), \sum_{\phi=1}^{\#PL(p)} p^{(\phi)} \leq 1 \right\} \tag{3}$$

where $l^{(\phi)}\left(p^{(\phi)}\right)$ is the ϕth linguistic term $l^{(\phi)}$ connected with the corresponding probability $p^{(\phi)}$, and $\#PL(p)$ is the length of $PL(p)$. The linguistic term $l^{(\phi)}$ in $PL(p)$ are listed by ascending order.

In order to easy computation, Pang et al. (Pang et al. 2016) normalized the PLTS $PL(p)$

a s

$$NPL(p)=\left\{l^{(\phi)}\left(\tilde{p}^{(\phi)}\right)\middle| l^{(\phi)}\in L,\quad \tilde{p}^{(\phi)}\geq 0,\quad \phi=1,2,\ldots,\#NPL(\tilde{p}),\quad \sum_{\phi=1}^{\#NPL(\tilde{p})}\tilde{p}^{(\phi)}=1\right\},$$

where $\tilde{p}^{(\phi)}=p^{(\phi)}\Big/\sum_{\phi=1}^{\#NPL(p)}p^{(\phi)}$ for all $\phi=1,2,\ldots,\#L(\tilde{p})$.

Definition 3 (Pang et al. 2016) Let $L=\left\{l_\alpha|\alpha=-\theta,\ldots,-1,0,1,\ldots\theta\right\}$ be an LTS, $NPL_1(p)=\left\{l_1^{(\phi)}\left(p_1^{(\phi)}\right)|\ \phi=1,2,\ldots,\#NPL_1(p)\right\}$ and $NPL_2(p)=\left\{l_2^{(\phi)}\left(p_2^{(\phi)}\right)|\ \phi=1,2,\ldots,\#NPL_2(p)\right\}$ be two PLTSs, where $\#NPL_1(p)$ and $\#NPL_2(p)$ are the length of $NPL_1(p)$ and $NPL_2(p)$, respectively. If $\#NPL_1(p)>\#NPL_2(p)$, then add $\#NPL_1(p)-\#NPL_2(p)$ linguistic terms to $NPL_2(p)$. In addition, the newly added linguistic terms should be the smallest linguistic term in $NPL_2(p)$ and the corresponding probabilities of newly added linguistic terms should be zero.

Definition 4 (Pang et al. 2016) For a PLTS $NPL(p)=\left\{l^{(\phi)}\left(p^{(\phi)}\right)|\ \phi=1,2,\ldots,\#NPL(p)\right\}$, the expected value $EV(NPL(p))$ and standard deviation $SD(NPL(p))$ of $NPL(p)$ is designed in the following:

$$E(NPL(p))=\sum_{\phi=1}^{\#NPL(p)}g(NPL(p))p^{(\phi)}\Big/\sum_{\phi=1}^{\#NPL(p)}p^{(\phi)} \tag{4}$$

$$SD(NPL(p))=\sqrt{\sum_{\phi=1}^{\#NPL(\tilde{p})}\left(g(NPL(p))p^{(\phi)}-EV(NPL(p))\right)^2}\Big/\sum_{\phi=1}^{\#NPL(p)}p^{(\phi)} \tag{5}$$

By using the Eqs. (4)–(5), the order relation between two PLTSs is distinguished as: (1) if $EV\left(NPL_1(p)\right)>EV\left(NPL_2(p)\right)$, then $NPL_1(p)>NPL_2(p)$; (2) if $EV\left(NPL_1(p)\right)=EV\left(NPL_2(p)\right)$, then if $SD\left(NPL_1(p)\right)=SD\left(NPL_2(p)\right)$, then $NPL_1(p)=NPL_2(p)$; if $SD\left(NPL_1(p)\right)<SD\left(NPL_2(p)\right)$, then, $NPL_1(p)>NPL_2(p)$.

Definition 5 (Lin and Xu 2018) Let $L=\left\{l_\alpha|\alpha=-\theta,\ldots,-1,0,1,\cdots\theta\right\}$ be a LTS. And let $NPL_1(p)=\left\{l_1^{(\phi)}\left(p_1^{(\phi)}\right)|\ \phi=1,2,\ldots,\#NPL_1(p)\right\}$ and $NPL_2(p)=\left\{l_2^{(\phi)}\left(p_2^{(\phi)}\right)|\ \phi=1,2,\ldots,\#NPL_2(p)\right\}$ be two PLTSs with $\#NPL_1(p)=\#NPL_2(p)=NPL(p)$, then Hamming distance $HD\left(NPL_1(p),NPL_2(p)\right)$ between $NPL_1(p)$ and $NPL_2(p)$ is derived:

$$HD\left(NPL_1(p),NPL_2(p)\right)=\frac{\sum_{\phi=1}^{\#NPL(p)}\left|p_1^{(\phi)}g\left(l_1^{(\phi)}\right)-p_2^{(\phi)}g\left(l_2^{(\phi)}\right)\right|}{\#NPL(p)} \tag{6}$$

3 Some Dice similarity measure for PLTSs

When one vector is zero, the DSM (Dice 1945) can't induce this undefined setting which conquers the cosine similarity measure's limitation. Hence, the DSM's concept is designed in the chapter (Dice 1945).

Definition 6 (Dice 1945) Let $A=\left(a_1,a_2,\dots,a_n\right)$ and $B=\left(b_1,b_2,\dots,b_n\right)$ be two set of positive real numbers. Then the DSM is defined in the following:

$$DSM(A,B)=\frac{2A\cdot B}{\|A\|_2^2+\|B\|_2^2}=\frac{2\sum_{j=1}^{n}a_jb_j}{\sum_{j=1}^{n}\left(a_j\right)^2+\sum_{j=1}^{n}\left(b_j\right)^2} \tag{7}$$

where $A\cdot B=\sum_{j=1}^{n}a_jb_j$ is the inner product between A and B and $\|A\|_2=\sqrt{\sum_{j=1}^{n}\left(a_j\right)^2}$ and $\|B\|_2=\sqrt{\sum_{j=1}^{n}\left(b_j\right)^2}$ are the Euclidean norms of A and B.

The value of DSM belongs to the interval $[0,1]$. Thus, if $a_j=b_j=0(j=1,2,\dots,n)$, then $DSM(A,B)$=0.

3.1 Dice similarity measure for PLTSs

In such section, some DSM and some weighted DSM (WDSM) between PLTs are designed on the basis of the concept of the DSM.

Definition 7 Let $L=\left\{l_\alpha|\alpha=-\theta,\dots,-1,0,1,\cdots\theta\right\}$ be an LTS, $NPL_1(p)=\left\{NPL_{1j}(p)\right\}(j=1,2,\dots,n)=\left\{l_{1j}^{(\phi)}\left(p_{1j}^{(\phi)}\right)\middle|\phi=1,2,\dots,\#NPL_{1j}(p)\right\}(j=1,2,\dots,n)$ and $NPL_2(p)=\left\{NPL_{2j}(p)\right\}(j=1,2,\dots,n)=\left\{l_{2j}^{(\phi)}\left(p_{2j}^{(\phi)}\right)\middle|\phi=1,2,\dots,\#NPL_{2j}(p)\right\}(j=1,2,\dots,n)$ be two sets of PLTSs, where $\#NPL_{1j}(p)$ and $\#NPL_{2j}(p)$ are the numbers of PLTS $NPL_{1j}(p)$ and $NPL_{2j}(p)$, $\#NPL_{1j}(p)=\#NPL_{2j}(p)=\#NPL_j(p)$ respectively, the probabilistic linguistic DSM (PLDSM) between $NPL_{1j}(p)$ and $NPL_{2j}(p)$ is designed as follows:

$$\begin{aligned}&PLDSM_{PLTSs}^{1}\left(NPL_1(p),NPL_2(p)\right)\\&=\frac{1}{n}\sum_{j=1}^{n}\frac{2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\end{aligned} \tag{8}$$

Example 1 Let $NPL_1(p)=\left[\left[\left\{l_2(0.4),l_3(0.6)\right\},\left\{l_1(0.2),l_2(0.8)\right\},\left\{l_{-1}(0.2),l_1(0.8)\right\}\right]\right]$ and $NPL_2(p)=\left[\left[\left\{l_{-3}(0.8),l_{-1}(0.2)\right\},\left\{l_2(0.6),l_3(0.4)\right\},\left\{l_{-2}(0.7),l_{-1}(0.3)\right\}\right]\right]$ be two sets of normalized PLTSs, then in terms of the Eq. (8), we can acquire:

$$\begin{aligned}
&PLDSM^1_{PLTSs}\left(NPL_1(p),NPL_2(p)\right)\\
&=\frac{1}{n}\sum_{j=1}^{n}\frac{2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&=\frac{1}{3}\times\left(\begin{array}{l}
\frac{2\times\left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right)\times\left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)}{\left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right)^2+\left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)}\\
+\frac{2\times\left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right)\times\left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)}{\left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right)^2+\left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)}\\
+\frac{2\times\left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right)\times\left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)}{\left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right)^2+\left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)}
\end{array}\right)\\
&=0.5914
\end{aligned}$$

The DSM between $NPL_1(p)$ and $NPL_2(p)$ also fulfills the subsequently properties:

(1) $0\le PLDSM^1_{PLTSs}\left(NPL_1(p),NPL_2(p)\right)\le 1$;
(2) $PLDSM^1_{PLTSs}\left(NPL_1(p),NPL_2(p)\right)=PLDSM^1_{PLTSs}\left(NPL_2(p),NPL_1(p)\right)$;
(3) $PLDSM^1_{PLTSs}\left(NPL_1(p),NPL_2(p)\right)=1,\quad \text{if}\quad \sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}=\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}$, $j=1,2,\ldots,n.$

If we take into account the weights ω_j of $NPL_{kj}(p)(k=1,2)$, then, a probabilistic linguistic weighted DSM (PLWDSM) between $NPL_1(p)$ and $NPL_2(p)$ is designed in the following:

$$\begin{aligned}
&PLWDSM^1_{PLTSs}\left(NPL_1(p),NPL_2(p)\right)\\
&=\sum_{j=1}^{n}\omega_j\frac{2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}
\end{aligned}\tag{9}$$

where $\omega=\left(\omega_1,\omega_2,\ldots,\omega_n\right)^T$ is the weight of $NPL_{kj}(p)(k=1,2)$, with $\omega_j\in[0,1]$, $\sum_{j=1}^{n}\omega_j=1$. Particularly, if $\omega=(1/n,1/n,\ldots,1/n)^T$, then the PLWDSM reduces to the PLDSM. Then there is: $PLWDSM^1_{PLTSs}\left(NPL_1(p),NPL_2(p)\right)=PLDSM^1_{PLTSs}\left(NPL_1(p),NPL_2(p)\right)$.

Example 2 Let $NPL_1(p)=\left[\left[\left\{l_2(0.4),l_3(0.6)\right\},\left\{l_1(0.2),l_2(0.8)\right\},\left\{l_{-1}(0.2),l_1(0.8)\right\}\right]\right]$ and $NPL_2(p)=\left[\left[\left\{l_{-3}(0.8),l_{-1}(0.2)\right\},\left\{l_2(0.6),l_3(0.4)\right\},\left\{l_{-2}(0.7),l_{-1}(0.3)\right\}\right]\right]$ be two sets of normalized PLTSs, the weight values are: $\omega=(0.2,0.5,0.3)^T$, then in accordance with the Eq. (9), we can acquire:

$$PLWDSM^1_{PLTSs}\left(NPL_1(p), NPL_2(p)\right)$$
$$= \sum_{j=1}^{n} \omega_j \frac{2\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right) \cdot \left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2 + \left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}$$
$$= 0.2 \times \frac{2 \times \left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right) \times \left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)}{\left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right)^2 + \left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)}$$
$$+ 0.5 \times \frac{2 \times \left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right) \times \left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)}{\left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right)^2 + \left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)}$$
$$+ 0.3 \times \frac{2 \times \left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right) \times \left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)}{\left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right)^2 + \left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)}$$
$$= 0.7167$$

Obviously, the $PLWDSM^1_{PLTSs}\left(NPL_1(p), NPL_2(p)\right)$ also fulfills the subsequently properties:

(1) $0 \leq PLWDSM^1_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) \leq 1$;
(2) $PLWDSM^1_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) = PLWDSM^1_{PLTSs}\left(NPL_2(p), NPL_1(p)\right)$;
(3) $PLWDSM^1_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) = 1, \quad \text{if} \quad \sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)} = \sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}$, $j = 1, 2, \ldots, n$.

3.2 Another form of the DSM for PLTSs

In such chapter, Dice similarity measure' another form for PLTSs is designed below:

Definition 8 Let $L = \left\{l_\alpha | \alpha = -\theta, \ldots, -1, 0, 1, \ldots \theta\right\}$ be an LTS, $NPL_1(p) = \left\{NPL_{1j}(p)\right\}(j = 1, 2, \ldots, n) = \left\{ l_{1j}^{(\phi)}\left(p_{1j}^{(\phi)}\right) \middle| \phi = 1, 2, \ldots, \#NPL_{1j}(p) \right\}(j = 1, 2, \ldots, n)$ and $NPL_2(p) = \left\{NPL_{2j}(p)\right\}(j = 1, 2, \ldots, n) = \left\{ l_{2j}^{(\phi)}\left(p_{2j}^{(\phi)}\right) \middle| \phi = 1, 2, \ldots, \#NPL_{2j}(p) \right\}(j = 1, 2, \ldots, n)$ be two sets of PLTSs, where $\#NPL_{1j}(p)$ and $\#NPL_{2j}(p)$ are the numbers of PLTS $NPL_{1j}(p)$ and $NPL_{2j}(p)$, $\#NPL_{1j}(p) = \#NPL_{2j}(p) = \#NPL_j(p)$ respectively, another form of probabilistic linguistic DSM (PLDSM) between $NPL_{1j}(p)$ and $NPL_{2j}(p)$ is designed in the following:

$$PLDSM^2_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) = \frac{2\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\cdot\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\sum_{j=1}^{n}\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2\right)} \tag{10}$$

Example 3 Let $NPL_1(p) = [[\{l_2(0.4), l_3(0.6)\}, \{l_1(0.2), l_2(0.8)\}, \{l_{-1}(0.2), l_1(0.8)\}]]$ and $NPL_2(p) = [[\{l_{-3}(0.8), l_{-1}(0.2)\}, \{l_2(0.6), l_3(0.4)\}, \{l_{-2}(0.7), l_{-1}(0.3)\}]]$ be two sets of normalized PLTSs, then in accordance with the Eq. (10), we can acquire:

$$\begin{aligned} &PLDSM^2_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) \\ &= \frac{2\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\cdot\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\sum_{j=1}^{n}\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2\right)} \\ &= \frac{2\times\left(\begin{array}{l}\left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right)\times\left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)\\ +\left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right)\times\left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)\\ +\left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right)\times\left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)\end{array}\right)}{\left(\begin{array}{l}\left(\left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right)^2+\left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)\right)\\ +\left(\left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right)^2+\left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)\right)\\ +\left(\left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right)^2+\left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)\right)\end{array}\right)} \\ &= 0.6677 \end{aligned}$$

Another form of probabilistic linguistic DSM (PLDSM) between $NPL_{1j}(p)$ and $NPL_{2j}(p)$ also fulfills the subsequently properties:

(1) $0 \le PLDSM^2_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) \le 1$;
(2) $PLDSM^2_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) = PLDSM^2_{PLTSs}\left(NPL_1(p), NPL_2(p)\right)$;
(3) $PLDSM^2_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) = 1,\ if\ NPL_1(p) = NPL_2(p)$, i . e . $\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)} = \sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}, j = 1, 2, \dots, n.$

If we consider the weights ω_j of $\tilde{L}_{kj}(\tilde{p})$ $(k = 1, 2)$, then, another form of probabilistic linguistic weighted DSM (PLWDSM) between $NPL_{1j}(p)$ and $NPL_{2j}(p)$ is developed in the following:

$$PLWDSM_{_{PLTSs}}^{2}\left(NPL_1(p),NPL_2(p)\right)=\frac{2\sum_{j=1}^{n}\left(\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2\right)} \tag{11}$$

where $\omega=\left(\omega_1,\omega_2,\ldots,\omega_n\right)^T$ is the weight vector of $NPL_{kj}(p)$ $(k=1,2)$, with $\omega_j\in[0,1]$, $\sum_{j=1}^{n}\omega_j=1$. Particularly, if $\omega=(1/n,1/n,\ldots,1/n)^T$, then the PLWDSM reduces to the PLDSM. Then there is: $PLWDSM_{_{PLTSs}}^{2}\left(NPL_1(p),NPL_2(p)\right)=PLWDSM_{_{PLTSs}}^{2}\left(NPL_1(p),NPL_2(p)\right)$.

Example 4 Let $\tilde{L}_1(\tilde{p})=\left[\left[\left\{l_2(0.4),l_3(0.6)\right\},\left\{l_1(0.2),l_2(0.8)\right\},\left\{l_{-1}(0.2),l_1(0.8)\right\}\right]\right]$ and $\tilde{L}_2(\tilde{p})=\left[\left[\left\{l_{-3}(0.8),l_{-1}(0.2)\right\},\left\{l_2(0.6),l_3(0.4)\right\},\left\{l_{-2}(0.7),l_{-1}(0.3)\right\}\right]\right]$ be two sets of normalized PLTSs, the weight values are: $\omega=(0.2,0.5,0.3)^T$, then according to the Eq. (9), we can obtain:

$$\begin{aligned}&PLWDSM_{_{PLTSs}}^{2}\left(NPL_1(p),NPL_2(p)\right)\\&=\frac{2\sum_{j=1}^{n}\left(\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2\right)}\\&=\frac{2\times\left(\begin{array}{l}0.2^2\times\left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right)\times\left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)\\\quad+0.5^2\times\left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right)\times\left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)\\\quad+0.3^2\times\left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right)\times\left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)\end{array}\right)}{\left(\begin{array}{l}0.2^2\times\left(\left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right)^2+\left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)\right)\\+0.5^2\times\left(\left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right)^2+\left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)\right)\\+0.3^2\times\left(\left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right)^2+\left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)\right)\end{array}\right)}\\&=0.8946\end{aligned}$$

Obviously, the $PLWDSM_{_{PLTSs}}^{2}\left(NPL_1(p),NPL_2(p)\right)$ also meets the following properties:

(1) $0\le PLWDSM_{_{PLTSs}}^{2}\left(NPL_1(p),NPL_2(p)\right)\le1$;
(2) $PLWDSM_{_{PLTSs}}^{2}\left(NPL_1(p),NPL_2(p)\right)=PLWDSM_{_{PLTSs}}^{2}\left(NPL_1(p),NPL_2(p)\right)$;
(3) $PLWDSM_{_{PLTSs}}^{2}\left(NPL_1(p),NPL_2(p)\right)=1$, if $\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}=\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}$, $j=1,2,\ldots,n$.

3.3 The generalized Dice similarity measure for PLTSs

In such chapter, as the Dice similarity measure's generalization, the probabilistic linguistic generalized Dice similarity measure (PLGDSM) between $NPL_{1j}(p)$ and $NPL_{2j}(p)$ are designed below.

Definition 9 Let $L = \{l_\alpha | \alpha = -\theta, \ldots, -1, 0, 1, \ldots \theta\}$ be an LTS, $NPL_1(p) = \{NPL_{1j}(p)\}(j = 1, 2, \ldots, n) = \left\{ l_{1j}^{(\phi)}\left(p_{1j}^{(\phi)}\right) \middle| \phi = 1, 2, \ldots, \#NPL_{1j}(p) \right\}(j = 1, 2, \ldots, n)$ and $NPL_2(p) = \{NPL_{2j}(p)\}(j = 1, 2, \ldots, n) = \left\{ l_{2j}^{(\phi)}\left(p_{2j}^{(\phi)}\right) \middle| \phi = 1, 2, \ldots, \#NPL_{2j}(p) \right\}(j = 1, 2, \ldots, n)$ be two sets of PLTSs, where $\#NPL_{1j}(p)$ and $\#NPL_{2j}(p)$ are the numbers of PLTS $NPL_{1j}(p)$ and $NPL_{2j}(p)$, $\#NPL_{1j}(p) = \#NPL_{2j}(p) = \#NPL_j(p)$ respectively, the PLGDSM between $NPL_{1j}(p)$ and $NPL_{2j}(p)$ is proposed in the following:

$$PLGDSM_{PLTSs}^{1}\left(NPL_1(p), NPL_2(p)\right) = \frac{1}{n}\sum_{j=1}^{n} \frac{\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right) \cdot \left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\lambda\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2 + (1-\lambda)\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \tag{12}$$

$$PLGDSM_{PLTSs}^{2}\left(NPL_1(p), NPL_2(p)\right) = \frac{\sum_{j=1}^{n}\left(\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right) \cdot \left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\lambda \sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2 + (1-\lambda)\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \tag{13}$$

where λ is a positive parameter for $0 \le \lambda \le 1$.

Example 5 Let $NPL_1(p) = [[\{l_2(0.4), l_3(0.6)\}, \{l_1(0.2), l_2(0.8)\}, \{l_{-1}(0.2), l_1(0.8)\}]]$ and $NPL_2(p) = [[\{l_{-3}(0.8), l_{-1}(0.2)\}, \{l_2(0.6), l_3(0.4)\}, \{l_{-2}(0.7), l_{-1}(0.3)\}]]$ be two sets of normalized PLTSs, let $\lambda = 0.3$, then according to the Eqs. (12–13), we can obtain:

$$
\begin{aligned}
&PLGDSM^{1}_{PLTSs}\left(NPL_1(p),NPL_2(p)\right)\\
&=\frac{1}{n}\sum_{j=1}^{n}\frac{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\lambda\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&=\frac{1}{3}\times\left(\begin{array}{l}
\frac{2\times\left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right)\times\left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)}{0.3\times\left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right)^2+0.7\times\left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)}\\
+\frac{2\times\left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right)\times\left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)}{0.3\times\left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right)^2+0.7\times\left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)}\\
+\frac{2\times\left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right)\times\left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)}{0.3\times\left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right)^2+0.7\times\left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)}
\end{array}\right)\\
&=0.7023
\end{aligned}
$$

$$
\begin{aligned}
&PLGDSM^{2}_{PLTSs}\left(NPL_1(p),NPL_2(p)\right)\\
&=\frac{\sum_{j=1}^{n}\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\lambda\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&=\frac{2\times\left(\begin{array}{l}
\left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right)\times\left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)\\
+\left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right)\times\left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)\\
+\left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right)\times\left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)
\end{array}\right)}{\left(\begin{array}{l}
\left(0.3\times\left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right)^2+0.7\times\left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)\right)\\
+\left(0.3\times\left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right)^2+0.7\times\left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)\right)\\
+\left(0.3\times\left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right)^2+0.7\times\left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)\right)
\end{array}\right)}\\
&=0.7982
\end{aligned}
$$

Then, the PLGDSM involves some special situations by modifying the parameter value λ.

If $\lambda = 0.5$, the two PLGDSM (12) and (13) reduced to PLDSM (14) and (15):

$$PLGDSM^1_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) = \frac{1}{n}\sum_{j=1}^{n}\frac{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\lambda\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}$$
$$= \frac{1}{n}\sum_{j=1}^{n}\frac{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{0.5\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-0.5)\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \tag{14}$$
$$= \frac{1}{n}\sum_{j=1}^{n}\frac{2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}$$

$$PLGDSM^2_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) = \frac{\sum_{j=1}^{n}\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\lambda\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}$$
$$= \frac{\sum_{j=1}^{n}\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{0.5\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-0.5)\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \tag{15}$$
$$= \frac{2\sum_{j=1}^{n}\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}$$

If $\lambda = 0, 1$, the two PLGDSM reduced to the subsequently measures of asymmetric similarity respectively:

$$
\begin{aligned}
&PLGDSM^{1}_{PLTSs}\left(NPL_1(p), NPL_2(p)\right)\\
&= \frac{1}{n}\sum_{j=1}^{n} \frac{\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\lambda\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2 + (1-\lambda)\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&= \frac{1}{n}\sum_{j=1}^{n} \frac{\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{0\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2 + (1-0)\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&= \frac{1}{n}\sum_{j=1}^{n} \frac{\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}, \quad for \quad \lambda = 0.
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&PLGDSM^{1}_{PLTSs}\left(NPL_1(p), NPL_2(p)\right)\\
&= \frac{1}{n}\sum_{j=1}^{n} \frac{\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\lambda\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2 + (1-\lambda)\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&= \frac{1}{n}\sum_{j=1}^{n} \frac{\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{1\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2 + (1-1)\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&= \frac{1}{n}\sum_{j=1}^{n} \frac{\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{2j}^{(\phi)} g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\left(\sum_{\phi=1}^{\#NPL_j(p)} \frac{p_{1j}^{(\phi)} g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}, \quad for \quad \lambda = 1.
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
&PLGDSM^{2}_{PLTSs}\left(NPL_1(p),NPL_2(p)\right)\\
&=\frac{\sum_{j=1}^{n}\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\lambda\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&=\frac{\sum_{j=1}^{n}\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{0\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-0)\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&=\frac{\sum_{j=1}^{n}\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2},\quad for\quad \lambda=0
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&PLGDSM^{2}_{PLTSs}\left(NPL_1(p),NPL_2(p)\right)\\
&=\frac{\sum_{j=1}^{n}\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\lambda\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&=\frac{\sum_{j=1}^{n}\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{1\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-1)\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&=\frac{\sum_{j=1}^{n}\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\sum_{j=1}^{n}\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2},\quad for\quad \lambda=1.
\end{aligned}
\tag{19}
$$

In terms of the above analysis, it could be found that these four measures of asymmetric similarity are the corresponding extension of the relative projection measure of the PLTSs.

In various situations, the weight $\omega_j(j=1,2,\ldots,n)$ of the elements $\tilde{L}_{kj}(\tilde{p})(k=1,2)$ could be taken into consideration. For instance, in the process of MADM, there exist different importance for the considered attributes, thus different weights should be considered to assign. However, the subsequently two probabilistic linguistic weighted GDSM (PLWGDSM) for PLTSs are further to proposed, respectively, as follows:

$$PLWGDSM^1_{PLTSs}\left(\tilde{L}_1(\tilde{p}),\tilde{L}_2(\tilde{p})\right) = \sum_{j=1}^{n}\omega_j \frac{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\lambda\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \tag{20}$$

$$PLWGDSM^2_{PLTSs}\left(NPL_1(p),NPL_2(p)\right) = \frac{\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\lambda\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \tag{21}$$

where $\omega=\left(\omega_1,\omega_2,\ldots,\omega_n\right)^T$ is the weight of $NPL_{kj}(p)(k=1,2)$, with $\omega_j\in[0,1]$, $\sum_{j=1}^{n}\omega_j=1$. Particularly, if $\omega=(1/n,1/n,\ldots,1/n)^T$, then the PLWGDSM reduces to the PLGDSM. Then there is $PLWGDSM^k_{PLTSs}\left(\tilde{L}_1(\tilde{p}),\tilde{L}_2(\tilde{p})\right)=PLGDSM^k_{PLTSs}\left(\tilde{L}_1(\tilde{p}),\tilde{L}_2(\tilde{p})\right)(k=1,2)$.

Example 6 Let $NPL_1(p)=\left[\left[\left\{l_2(0.4),l_3(0.6)\right\},\left\{l_1(0.2),l_2(0.8)\right\},\left\{l_{-1}(0.2),l_1(0.8)\right\}\right]\right]$ and $NPL_2(p)=\left[\left[\left\{l_{-3}(0.8),l_{-1}(0.2)\right\},\left\{l_2(0.6),l_3(0.4)\right\},\left\{l_{-2}(0.7),l_{-1}(0.3)\right\}\right]\right]$ be two sets of normalized PLTSs, the weight values are: $\omega=(0.2,0.5,0.3)^T$, $\lambda=0.3$ then according to the Eq. (20–21), we can get:

$$PLWGDSM^1_{PLTSs}\left(NPL_1(p),NPL_2(p)\right) = \sum_{j=1}^{n}\omega_j \frac{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\lambda\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}$$

$$= 0.2\times\frac{2\times\left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right)\times\left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)}{0.3\times\left(\frac{(2+3)/6\times0.4+(3+3)/6\times0.6}{2}\right)^2+0.7\times\left(\frac{(-3+3)/6\times0.8+(-1+3)/6\times0.2}{2}\right)}$$

$$+0.5\times\frac{2\times\left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right)\times\left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)}{0.3\times\left(\frac{(1+3)/6\times0.2+(2+3)/6\times0.8}{2}\right)^2+0.7\times\left(\frac{(2+3)/6\times0.6+(3+3)/6\times0.4}{2}\right)}$$

$$+0.3\times\frac{2\times\left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right)\times\left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)}{0.3\times\left(\frac{(-1+3)/6\times0.2+(1+3)/6\times0.8}{2}\right)^2+0.7\times\left(\frac{(-2+3)/6\times0.7+(-1+3)/6\times0.3}{2}\right)}$$

$$= 0.7835$$

$$
\begin{aligned}
& PLWGDSM^2_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) \\
&= \frac{\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\lambda\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \\
&= \frac{2\times\left(\begin{array}{c}
0.2^2\times\left(\frac{(2+3)/6\times 0.4+(3+3)/6\times 0.6}{2}\right)\times\left(\frac{(-3+3)/6\times 0.8+(-1+3)/6\times 0.2}{2}\right)\\
+0.5^2\times\left(\frac{(1+3)/6\times 0.2+(2+3)/6\times 0.8}{2}\right)\times\left(\frac{(2+3)/6\times 0.6+(3+3)/6\times 0.4}{2}\right)\\
+0.3^2\times\left(\frac{(-1+3)/6\times 0.2+(1+3)/6\times 0.8}{2}\right)\times\left(\frac{(-2+3)/6\times 0.7+(-1+3)/6\times 0.3}{2}\right)
\end{array}\right)}{\left(\begin{array}{c}
0.2^2\times\left(0.3\times\left(\frac{(2+3)/6\times 0.4+(3+3)/6\times 0.6}{2}\right)^2+0.7\times\left(\frac{(-3+3)/6\times 0.8+(-1+3)/6\times 0.2}{2}\right)\right)\\
+0.5^2\times\left(0.3\times\left(\frac{(1+3)/6\times 0.2+(2+3)/6\times 0.8}{2}\right)^2+0.7\times\left(\frac{(2+3)/6\times 0.6+(3+3)/6\times 0.4}{2}\right)\right)\\
+0.3^2\times\left(0.3\times\left(\frac{(-1+3)/6\times 0.2+(1+3)/6\times 0.8}{2}\right)^2+0.7\times\left(\frac{(-2+3)/6\times 0.7+(-1+3)/6\times 0.3}{2}\right)\right)
\end{array}\right)} \\
&= 0.9117
\end{aligned}
$$

After that, the PLWGDSM involves some special cases by modifying the parameter value λ.

If $\lambda = 0.5$, the two weighted GDSM (20) and (21) reduced to weighted DSM (22) and (23):

$$
\begin{aligned}
& PLWGDSM^1_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) \\
&= \sum_{j=1}^{n}\omega_j\frac{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\lambda\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \\
&= \sum_{j=1}^{n}\omega_j\frac{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{0.5\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-0.5)\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \\
&= \sum_{j=1}^{n}\omega_j\frac{2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}
\end{aligned} \quad (22)
$$

$$
\begin{aligned}
&PLWGDSM^2_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) \\
&= \frac{\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\lambda\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \\
&= \frac{\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{0.5\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-0.5)\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \\
&= \frac{2\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}
\end{aligned}
\tag{23}
$$

If $\lambda = 0, 1$, the two PLWGDSM reduces to the subsequent asymmetric weighted DSM, respectively:

$$
\begin{aligned}
&PLWGDSM^1_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) \\
&= \sum_{j=1}^{n}\omega_j\frac{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\lambda\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \\
&= \sum_{j=1}^{n}\omega_j\frac{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{0\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-0)\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \\
&= \sum_{j=1}^{n}\omega_j\frac{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}, \quad for \quad \lambda = 0.
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
&PLWGDSM^{1}_{PLTSs}\left(NPL_1(p), NPL_2(p)\right)\\
&=\sum_{j=1}^{n}\omega_j\frac{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\lambda\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&=\sum_{j=1}^{n}\omega_j\frac{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{1\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-1)\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&=\sum_{j=1}^{n}\omega_j\frac{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)}{\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2},\quad for\ \ \lambda=1.
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
&PLWGDSM^{2}_{PLTSs}\left(NPL_1(p), NPL_2(p)\right)\\
&=\frac{\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\lambda\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&=\frac{\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{0\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-0)\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}\\
&=\frac{\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2},\quad for\ \ \lambda=0.
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
&PLWGDSM^{2}_{PLTSs}\left(NPL_1(p), NPL_2(p)\right) \\
&= \frac{\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\lambda\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-\lambda)\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \\
&= \frac{\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{1\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2+(1-1)\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2} \\
&= \frac{\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{2j}^{(\phi)}g\left(l_{2j}^{(\phi)}\right)}{\#NPL_j(p)}\right)\right)}{\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_j(p)}\frac{p_{1j}^{(\phi)}g\left(l_{1j}^{(\phi)}\right)}{\#NPL_j(p)}\right)^2}, \quad for \;\; \lambda = 1.
\end{aligned}
\tag{27}
$$

In terms of the above analysis, it could be found that these four measures of asymmetric weighted similarity are the corresponding extension of the relative weighted projection measure of PLTSs.

4 The weighted GDSM for probabilistic linguistic MAGDM with entropy weight

In this chapter, we put forward a novel probabilistic linguistic weighted GDSM (PLWGDSM) method for MAGDM issues with unknown weight. The subsequently mathematical notations are made use of solving the probabilistic linguistic MAGDM issues. Let $A = \{A_1, A_2, \dots, A_m\}$ be a discrete collection of alternatives, and $G = \{G_1, G_2, \dots, G_n\}$ with weight vector $w = (w_1, w_2, \dots, w_n)$, where $\omega_j \in [0, 1]$, $j = 1, 2, \dots, n$, $\sum_{j=1}^{n} w_j = 1$, and a collection of experts $E = \{E_1, E_2, \dots, E_q\}$. Suppose that there are n qualitative attribute $A = \{A_1, A_2, \dots, A_m\}$ and their values are evaluated by qualified experts and denoted as linguistic expressions information $l_{ij}^{k}(i = 1, 2, \dots, m,\ j = 1, 2, \dots, n,\ k = 1, 2, \dots, q)$.

Then, PLWGDSM method is designed to solve the MAGDM problems with entropy weight. The elaborated calculating procedures are given in the following:

Step 1 Shift cost attribute into beneficial attribute. If the cost attribute value is l_τ, then the corresponding beneficial attribute value is $l_{-\tau}(\tau = -3, -2, -1, 0, 1, 2, 3)$.

Step 2 Convert the linguistic information $l_{ij}^{k}(i = 1, 2, \dots, m,\ j = 1, 2, \dots, n,\ k = 1, 2, \dots, q)$ into PLTs $l_{ij}^{(\phi)}\left(p_{ij}^{(\phi)}\right)$, $\phi = 1, 2, \dots, \#L_{ij}(p)$ and construct the probabilistic linguistic assessing matrix $PL = \left(PL_{ij}(p)\right)_{m\times n}$, $PL_{ij}(p) = \left\{ l_{ij}^{(\phi)}\left(p_{ij}^{(\phi)}\right) \middle| \phi = 1, 2, \dots, \#L_{ij}(p) \right\}$ $(i = 1, 2, \dots, m, j = 1, 2, \dots, n)$.

Step 3 Derive the normalized probabilistic linguistic matrix $NPL = \left(NPL_{ij}(p)\right)_{m\times n}$, $NPL_{ij}(p) = \left\{ l_{ij}^{(\phi)}\left(p_{ij}^{(\phi)}\right) \middle| \ \phi = 1,2,\ldots,\#NPL_{ij}(p) \right\}(i = 1,2,\ldots,m,\ j = 1,2,\ldots,n)$. Thus, probabilistic linguistic information for given alternative $A_i \in A$ with regard to all the attribute G can be depicted as: $PLA_i = \left(l_{i1}^{(\phi)}\left(\tilde{p}_{i1}^{(\phi)}\right), l_{i2}^{(\phi)}\left(\tilde{p}_{i2}^{(\phi)}\right), \ldots, l_{in}^{(\phi)}\left(\tilde{p}_{in}^{(\phi)}\right)\right)$, $\phi = 1,2,\ldots,\#L_{ij}(\tilde{p})$.

Step 4 Compute the weight values with entropy.

The attributes' weight is very significant in decision making issues. Entropy (Shannon 1948) is a conventional term from information theory which is also used to determine weight of attributes. Firstly, the normalized decision matrix $NL_{ij}(p)$ is derived as follows:

$$NL_{ij}(p) = \frac{\sum_{\phi=1}^{\#NPL_{ij}(p)}\left(p_{ij}^{(\phi)}g\left(l_{ij}^{(\phi)}\right)\right)}{\sum_{i=1}^{m}\sum_{\phi=1}^{\#NPL_{ij}(p)}\left(p_{ij}^{(\phi)}g\left(l_{ij}^{(\phi)}\right)\right)}, \quad j = 1,2,\ldots,n, \tag{28}$$

Then, the information of Shannon entropy $E = \left(E_1, E_2, \ldots, E_n\right)$ is calculated in the following:

$$E_j = -\frac{1}{\ln m}\sum_{i=1}^{m} NL_{ij}(p)\ln NL_{ij}(p) \tag{29}$$

and $NL_{ij}(p)\ln NL_{ij}(p)$ is defined as 0, if $NL_{ij}(p) = 0$.

Finally, the attribute weights $w = \left(w_1, w_2, \ldots, w_n\right)$ is computed:

$$w_j = \frac{1 - E_j}{\sum_{j=1}^{n}\left(1 - E_j\right)}, \quad j = 1,2,\ldots,n. \tag{30}$$

Step 5 Decide the probabilistic linguistic positive ideal solution (PLPIS):

$$PLPIS = \left(PLPIS_1, PLPIS_2, \ldots, PLPIS_n\right) \tag{31}$$

$$PLPIS_j = \left\{ pl_j^{(\phi)}\left(p_j^{(\phi)}\right) \middle| \ \phi = 1,2,\ldots,\#NPL_{ij}(p) \right\},\ E\left(PLPIS_j\right) = \left\{ \max_i E\left(NPL_{ij}(p)\right) \right\} \tag{32}$$

Step 6 Calculate the PLWGDSM between $PLA_i(i = 1,2,\ldots,m)$ and $PLPIS$ as follows:

$$\begin{aligned} & PLWGDSM^1_{PLTSs}\left(PLA_i, PLPIS\right) \\ & = \sum_{j=1}^{n}\omega_j \frac{\left(\sum_{\phi=1}^{\#NPL_{ij}(p)}\frac{p_{ij}^{(\phi)}g\left(l_{ij}^{(\phi)}\right)}{\#NPL_{ij}(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_{ij}(p)}\frac{p_j^{(\phi)}g\left(pl_j^{(\phi)}\right)}{\#NPL_{ij}(p)}\right)}{\lambda\left(\sum_{\phi=1}^{\#NPL_{ij}(p)}\frac{p_{ij}^{(\phi)}g\left(l_{ij}^{(\phi)}\right)}{\#NPL_{ij}(p)}\right)^2 + (1-\lambda)\left(\sum_{\phi=1}^{\#NPL_{ij}(p)}\frac{p_j^{(\phi)}g\left(pl_j^{(\phi)}\right)}{\#NPL_{ij}(p)}\right)^2} \end{aligned} \tag{33}$$

or

$$PLWGDSM^2_{PLTSs}\left(PLA_i, PLPIS\right) = \frac{\sum_{j=1}^{n}\omega_j^2\left(\left(\sum_{\phi=1}^{\#NPL_{ij}(p)}\frac{p_{ij}^{(\phi)}g\left(l_{ij}^{(\phi)}\right)}{\#NPL_{ij}(p)}\right)\cdot\left(\sum_{\phi=1}^{\#NPL_{ij}(p)}\frac{p_j^{(\phi)}g\left(pl_j^{(\phi)}\right)}{\#NPL_{ij}(p)}\right)\right)}{\lambda\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_{ij}(p)}\frac{p_{ij}^{(\phi)}g\left(l_{ij}^{(\phi)}\right)}{\#NPL_{ij}(p)}\right)^2+(1-\lambda)\sum_{j=1}^{n}\omega_j^2\left(\sum_{\phi=1}^{\#NPL_{ij}(p)}\frac{p_j^{(\phi)}g\left(pl_j^{(\phi)}\right)}{\#NPL_{ij}(p)}\right)^2} \tag{34}$$

Step 7 All the given alternatives $A_i(i = 1, 2, \ldots, m)$ can be ranked and the optimal one(s) could be selected by $PLWGDSM^1_{PLTSs}\left(PLA_i, PLPIS\right)$ or $PLWGDSM^2_{PLTSs}\left(PLA_i, PLPIS\right)$ $(i = 1, 2, \ldots, m)$. If any alternative has the highestvalues $PLWGDSM^1_{PLTSs}\left(PLA_i, PLPIS\right)$ or $PLWGDSM^2_{PLTSs}\left(PLA_i, PLPIS\right)$, then, it is the optimal alternative.

5 A case study and comparative analysis

5.1 A case study for site selection of EVCS

With the rapid development of economy, resource shortage and environmental pollution become more and more serious, thus, people pay more and more attention to their health and living environment. At present, the huge car market is aggravating the cost of resources and adding more pressure to the urban environment. And electric vehicle because of its energy saving and environmental protection characteristics is becoming the main development direction of the automobile industry. Liu et al. (2019b) proposed the integrated MCDM method by a grey decision-making trial and evaluation laboratory (DEMATEL) and uncertain linguistic multi-objective optimization by ratio analysis plus full multiplicative form (UL-MULTIMOORA) for obtaining the most suitable EVCS site in terms of multiple interrelated criteria. Wu et al. (2017) defined the hesitant fuzzy integrated MCDM method for quality function deployment with a case study in electric vehicle. Liu et al. (2017) explored the critical factors influencing the diffusion of electric vehicles in China form the multi-stakeholder perspective. As a supporting infrastructure for electric vehicles, charging stations must be planned and constructed first. The site selection of EVCS is deemed as a kind of MAGDM issue (Wu et al. 2019a, b; Deng and Gao 2019; Li and Lu 2019; Lu and Wei 2019; Wang et al. 2019c). Thus, in this section a numerical case is designed for site selection of EVCS. There are five possible EVCS sites $A_i(i = 1, 2, 3, 4, 5)$ to be assessed. The invited experts select four attributes to assess five underlying EVCS sites: ③ G_1 the traffic convenience; ④ G_2 the service capability; ① $G_3$is waste discharge; ② G_4 is construction cost. The construction cost (G_4) is not beneficial attribute, others are beneficial attribute. The five underlying EVCS sites $A_i(i = 1, 2, 3, 4, 5)$ are to be assessed by utilizing the linguistic variables

$$\begin{aligned} L = \{&l_{-3} = extremely\ poor(EP), \quad l_{-2} = very\ poor(VP),\\ &l_{-1} = poor(P), \quad l_0 = medium(M), \quad l_1 = good(G),\\ &l_2 = very\ good(VG), \quad l_3 = extremely\ good(EG)\}\end{aligned}$$

by five DMs, as listed in the Tables 1, 2, 3, 4 and 5.

Whereafter, we employ the PLWGDSM method developed for selecting the optimal EVCS sites.

Step 1 Shift cost attribute G_4 into beneficial attribute. If the cost attribute value is l_τ, then the corresponding beneficial attribute value is $l_{-\tau}(\tau = -3, -2, -1, 0, 1, 2, 3)$ (See Tables 6, 7, 8, 9 and 10)

Step 2 Shift the linguistic information into probabilistic linguistic assessing matrix (Table 11).

Step 3 Calculate the normalized assessing matrix with PTSs (Table 12).

Step 4 Compute the weight values for attributes from Eqs. (28)–(30): $w_1 = 0.1416, w_2 = 0.4892, w_3 = 0.1021, w_4 = 0.2671$.

Step 5 Determine the PLPIS by Eqs. (31)–(32) (Table 13):

Step 6 Calculating the PLWGDSM between $PLA_i(i = 1, 2, \ldots, 5)$ and $PLPIS$ (Tables 14, 15):

Step 7 All the given alternatives $A_i(i = 1, 2, \ldots, m)$ can be ranked and the optimal one(s) can be selected by $PLWGDSM^1_{PLTSs}\left(PLA_i, PLPIS\right)$ or $PLWGDSM^2_{PLTSs}\left(PLA_i, PLPIS\right)$ $(i = 1, 2, \ldots, 5)$ (Tables 16, 17).

From the Tables 16 and 17, taking different λ and different PLWGDSM, the ranking orders can be different. Then A_1, A_5 and A_3 should be the optimal EVCS sites in accordance with the principle of the maximum PLWGDSM.

Furthermore, for the two PLWGDSM's special situations, we acquire the subsequent results:

Table 1 Linguistic assessing matrix by DM_1

Alternatives	G_1	G_2	G_3	G_4
A_1	VG	EG	G	G
A_2	P	VP	EP	P
A_3	EP	EP	G	VG
A_4	G	EP	VG	EP
A_5	EG	M	P	G

Table 2 Linguistic assessing matrix by DM_2

Alternatives	G_1	G_2	G_3	G_4
A_1	VG	VG	G	VG
A_2	M	VP	P	VP
A_3	P	VP	VG	VG
A_4	G	EP	EG	VP
A_5	EG	M	P	G

Table 3 Linguistic assessing matrix by DM_3

Alternatives	G_1	G_2	G_3	G_4
A_1	VG	VG	G	EG
A_2	M	VP	P	EP
A_3	EP	VP	VG	VG
A_4	G	EP	VG	EP
A_5	VG	M	P	EG

Table 4 Linguistic assessing matrix by DM_4

Alternatives	G_1	G_2	G_3	G_4
A_1	VG	VG	G	EG
A_2	M	VP	P	P
A_3	EP	VP	VG	VG
A_4	G	P	VG	EP
A_5	VG	M	EP	VG

Table 5 Linguistic assessing matrix by DM_5

Alternatives	G_1	G_2	G_3	G_4
A_1	VG	VG	G	EG
A_2	M	VP	P	EP
A_3	EP	P	EG	VG
A_4	G	VP	EG	EP
A_5	EG	M	VP	VG

Table 6 Linguistic assessing matrix by DM_1

Alternatives	G_1	G_2	G_3	G_4
A_1	VG	EG	G	P
A_2	P	VP	EP	G
A_3	EP	EP	G	VP
A_4	G	EP	VG	EG
A_5	EG	M	P	P

Table 7 Linguistic assessing matrix by DM_2

Alternatives	G_1	G_2	G_3	G_4
A_1	VG	VG	G	VP
A_2	M	VP	P	VG
A_3	P	VP	VG	VP
A_4	G	EP	EG	VG
A_5	EG	M	P	P

- When $\lambda = 0$, the two PLWGDSM reduced to the corresponding weighted projection measures of A_i $(i = 1, 2, 3, 4, 5)$ on PLPIS. Thus, A_1 should be the optimal EVCS site in accordance with the maximum PLWGDSM. In such situation, we could obtain the same optimal EVCS site as the above mentioned three methods. Thus our proposed method is effective.
- When $\lambda = 0.5$, the two PLWGDSM reduced to the PLWDSM on PLPIS. Thus, A_1 should be the optimal EVCS site in accordance with maximum PLWDSM between

Table 8 Linguistic assessing matrix by DM_3

Alternatives	G_1	G_2	G_3	G_4
A_1	VG	VG	G	EP
A_2	M	VP	P	EG
A_3	EP	VP	VG	VP
A_4	G	EP	VG	EG
A_5	VG	M	P	EP

Table 9 Linguistic assessing matrix by DM_4

Alternatives	G_1	G_2	G_3	G_4
A_1	VG	VG	G	EP
A_2	M	VP	P	G
A_3	EP	VP	VG	VP
A_4	G	P	VG	EG
A_5	VG	M	EP	VP

Table 10 Linguistic assessing matrix by DM_5

Alternatives	G_1	G_2	G_3	G_4
A_1	VG	VG	G	EP
A_2	M	VP	P	EG
A_3	EP	P	EG	VP
A_4	G	VP	EG	EG
A_5	EG	M	VP	VP

Table 11 Probabilistic linguistic assessing matrix

Alternatives	G_1	G_2
A_1	$\{l_2(1)\}$	$\{l_2(0.8),\ l_3(0.2)\}$
A_2	$\{l_{-1}(0.2),\ l_0(0.8)\}$	$\{l_{-2}(1)\}$
A_3	$\{l_{-3}(0.8),\ l_{-1}(0.2)\}$	$\{l_{-3}(0.2),\ l_{-2}(0.6),\ l_{-1}(0.2)\}$
A_4	$\{l_1(1)\}$	$\{l_{-3}(0.6),\ l_{-2}(0.2),\ l_{-1}(0.2)\}$
A_5	$\{l_2(0.4),\ l_3(0.6)\}$	$\{l_0(1)\}$
Alternatives	**G_3**	**G_4**
A_1	$\{l_1(1)\}$	$\{l_{-3}(0.6),\ l_{-2}(0.2),\ l_{-1}(0.2)\}$
A_2	$\{l_{-3}(0.2), l_{-1}(0.8)\}$	$\{l_1(0.4),\ l_2(0.2),\ l_3(0.4)\}$
A_3	$\{l_1(0.2),\ l_2(0.6),\ l_3(0.2)\}$	$\{l_{-2}(1)\}$
A_4	$\{l_2(0.6),\ l_3(0.4)\}$	$\{l_2(0.2),\ l_3(0.8)\}$
A_5	$\{l_{-3}(0.2),\ l_{-2}(0.2),\ l_{-1}(0.6)\}$	$\{l_{-3}(0.2),\ l_{-2}(0.4),\ l_{-1}(0.4)\}$

Table 12 The assessing matrix with Normalized PTSs

Alternatives	G_1	G_2
A_1	$\{l_2(0),\ l_2(0),\ l_2(1)\}$	$\{l_2(0),\ l_2(0.8),\ l_3(0.2)\}$
A_2	$\{l_{-1}(0),\ l_{-1}(0.2),\ l_0(0.8)\}$	$\{l_{-2}(0),\ l_{-2}(0),\ l_{-2}(1)\}$
A_3	$\{l_{-3}(0),\ l_{-3}(0.8),\ l_{-1}(0.2)\}$	$\{l_{-3}(0.2),\ l_{-2}(0.6),\ l_{-1}(0.2)\}$
A_4	$\{l_1(0),\ l_1(0),\ l_1(1)\}$	$\{l_{-3}(0.6), l_{-2}(0.2), l_{-1}(0.2)\}$
A_5	$\{l_2(0), l_2(0.4), l_3(0.6)\}$	$\{l_0(0), l_0(0), l_0(1)\}$
Alternatives	G_3	G_4
A_1	$\{l_1(0), l_1(0), l_1(1)\}$	$\{l_{-3}(0.6), l_{-2}(0.2), l_{-1}(0.2)\}$
A_2	$\{l_{-3}(0), l_{-3}(0.2), l_{-1}(0.8)\}$	$\{l_1(0.4), l_2(0.2), l_3(0.4)\}$
A_3	$\{l_1(0.2), l_2(0.6), l_3(0.2)\}$	$\{l_{-2}(0), l_{-2}(0), l_{-2}(1)\}$
A_4	$\{l_2(0), l_2(0.6), l_3(0.4)\}$	$\{l_2(0), l_2(0.2), l_3(0.8)\}$
A_5	$\{l_{-3}(0.2), l_{-2}(0.2), l_{-1}(0.6)\}$	$\{l_{-3}(0.2), l_{-2}(0.4), l_{-1}(0.4)\}$

Table 13 PLPIS

	G_1	G_2
PLPIS	$\{l_2(0), l_2(0.4), l_3(0.6)\}$	$\{l_2(0), l_2(0.8), l_3(0.2)\}$
	G_3	G_4
PLPIS	$\{l_2(0), l_2(0.6), l_3(0.4)\}$	$\{l_2(0), l_2(0.2), l_3(0.8)\}$

Table 14 The $PLWGDSM^1_{PLTSs}(PLA_i, PLPIS)$

λ	$(PLA_1, PLPIS)$	$(PLA_2, PLPIS)$	$(PLA_3, PLPIS)$	$(PLA_4, PLPIS)$	$(PLA_5, PLPIS)$
0	0.7189	0.4254	0.1883	0.5268	0.5056
0.1	0.7281	0.4502	0.1999	0.5381	0.5343
0.2	0.7386	0.4795	0.2140	0.5516	0.5682
0.3	0.7506	0.5148	0.2318	0.5679	0.6087
0.4	0.7648	0.5584	0.2548	0.5882	0.6583
0.5	0.7823	0.6145	0.2862	0.6145	0.7207
0.6	0.8049	0.6906	0.3319	0.6508	0.8022
0.7	0.8370	0.8021	0.4056	0.7056	0.9149
0.8	0.8910	0.9885	0.5459	0.8035	1.0871
0.9	1.0233	1.3922	0.9257	1.0541	1.4118
1.0	3.3674	3.4816	10.0016	4.8074	2.6743

PLTSs. For such case, we could also obtain the same optimal EVCS site as the above mentioned three methods. Thus our proposed method is effective.

Table 15 The $PLWGDSM^{2}_{PLTSs}(PLA_i, PLPIS)$

λ	$(PLA_1, PLPIS)$	$(PLA_2, PLPIS)$	$(PLA_3, PLPIS)$	$(PLA_4, PLPIS)$	$(PLA_5, PLPIS)$
0	0.7657	0.3792	0.1263	0.3977	0.5036
0.1	0.7868	0.4110	0.1397	0.4269	0.5418
0.2	0.8092	0.4487	0.1563	0.4606	0.5861
0.3	0.8329	0.4940	0.1775	0.5002	0.6384
0.4	0.8580	0.5495	0.2052	0.5471	0.7009
0.5	0.8847	0.6191	0.2433	0.6039	0.7770
0.6	0.9131	0.7087	0.2986	0.6737	0.8716
0.7	0.9434	0.8288	0.3865	0.7618	0.9924
0.8	0.9757	0.9977	0.5479	0.8764	1.1522
0.9	1.0104	1.2533	0.9403	1.0316	1.3732
1.0	1.0476	1.6849	3.3150	1.2536	1.6992

Table 16 The $PLWGDSM^{1}_{PLTSs}(PLA_i, PLPIS)$ and ranking orders

λ	Ranking orders	The worst alternative	The optimal alternative
0	$A_1 > A_4 > A_5 > A_2 > A_3$	A_1	A_3
0.1	$A_1 > A_4 > A_5 > A_2 > A_3$	A_1	A_3
0.2	$A_1 > A_5 > A_4 > A_2 > A_3$	A_1	A_3
0.3	$A_1 > A_5 > A_4 > A_2 > A_3$	A_1	A_3
0.4	$A_1 > A_5 > A_4 > A_2 > A_3$	A_1	A_3
0.5	$A_1 > A_5 > A_2 = A_4 > A_3$	A_1	A_3
0.6	$A_1 > A_5 > A_2 > A_4 > A_3$	A_1	A_3
0.7	$A_5 > A_1 > A_2 > A_4 > A_3$	A_5	A_3
0.8	$A_5 > A_2 > A_4 > A_4 > A_3$	A_5	A_3
0.9	$A_5 > A_2 > A_4 > A_1 > A_3$	A_5	A_3
1.0	$A_3 > A_4 > A_2 > A_1 > A_5$	A_3	A_5

Table 17 The $PLWGDSM^{2}_{PLTSs}(PLA_i, PLPIS)$ and ranking orders

λ	Ranking orders	The worst alternative	The optimal alternative
0	$A_1 > A_5 > A_4 > A_2 > A_3$	A_1	A_3
0.1	$A_1 > A_5 > A_4 > A_2 > A_3$	A_1	A_3
0.2	$A_1 > A_5 > A_4 > A_2 > A_3$	A_1	A_3
0.3	$A_1 > A_5 > A_4 > A_2 > A_3$	A_1	A_3
0.4	$A_1 > A_5 > A_2 > A_4 > A_3$	A_1	A_3
0.5	$A_1 > A_5 > A_2 > A_4 > A_3$	A_1	A_3
0.6	$A_1 > A_5 > A_2 > A_4 > A_3$	A_1	A_3
0.7	$A_5 > A_1 > A_2 > A_4 > A_3$	A_5	A_3
0.8	$A_5 > A_2 > A_1 > A_4 > A_3$	A_5	A_3
0.9	$A_5 > A_2 > A_4 > A_1 > A_3$	A_5	A_3
1.0	$A_3 > A_5 > A_2 > A_4 > A_1$	A_3	A_1

- When $\lambda = 1$, the two PLWGDSM reduced to the corresponding weighted projection measures of PLPIS on A_i $(i = 1, 2, 3, 4, 5)$. Thus, A_3 should be the optimal EVCS site in accordance with the principle of the maximum degree of PLWDSM between PLTSs.

However, in accordance with different PLWGDSM and different λ, there exists slightly different with ranking orders. Therefore some value of λ and some measure can assign to the presented PLWGDSM methods to fulfill the requirements of DMs preference and flexible decision making issues.

5.2 Comparative analysis

Firstly, PL-GRA method (Liang et al. 2018) (let $\rho = 0.5$) is used to compare with our proposed PLWGDSM method, then we can get the calculating results: $\varepsilon_1^+ = 0.6865, \varepsilon_2^+ = 0.4094, \varepsilon_3^+ = 0.3848, \varepsilon_4^+ = 0.6220, \varepsilon_5^+ = 0.4451$. Furthermore, we can derive the ranking order: $A_1 > A_4 > A_5 > A_2 > A_3$. Thus, we also have the same optimal EVCS site A_1.

Secondly, probabilistic linguistic weighted average (PLWA) operator (Pang et al. 2016) is used to compare with our proposed PLWGDSM method. If the attribute weights are completely known, the calculating results is: $E\big(Z_1(w)\big) = s_{0.2735}$, $E\big(Z_2(w)\big) = s_{-0.2052}$, $E\big(Z_3(w)\big) = s_{-0.6567}$, $E\big(Z_4(w)\big) = s_{-0.0132}$, $E\big(Z_5(w)\big) = s_{-0.0920}$ and we can obtain the sorting order: $A_1 > A_4 > A_5 > A_2 > A_3$, thus, we have the same optimal EVCS site A_1.

Finally, PL-TOPSIS method (Pang et al. 2016) is employed to compare with PLWGDSM method, then we can acquire the calculating results and sorting results (Table 18).

In terms of the above analysis, it can be found that these above mentioned methods have the same optimal EVCS site A_1, and there are slightly different in the three methods' ranking results from our presented PLWGDSM methods, which can confirm the PLWGDSM methods we presented are more flexible and fulfill the requirements of DMs' preference. All these methods have their good advantages: (1) PL-GRA method emphasis the shape similarity degree from the positive ideal solution; (2) PLWA operator emphasis group influences; (3) PL-TOPSIS method emphasis the distance similarity degree from the positive and negative ideal solution with incomplete weight information. (4) Some value of λ and some measure can assign to the presented PLWGDSM methods to fulfill the requirements of DMs' preference and flexible decision making. Evidently, on the basis of the Dice measures and the projection measures, the MAGDM methods are the special situations of the presented MAGDM methods based on PLWGDSM. Thus, in the process of MAGDM, the MAGDM methods put forward in such paper are more useful and more flexible compared with existing MAGDM issues under PLTSs.

6 Conclusion

In this paper, we design some novel DSM of PLTSs and the GDSM of PLTSs and indicate that the DSM and PLTSs' asymmetric measures are special situations of the PLWGDSM with different parameter values. Then, we propose the PLWGDSM-based MAGDM methods with PLTSs. In the end, a demonstrative case study for location planning of electric vehicle charging stations is offered to illustrate the PLWGDSM's efficiency. Thus, the main

Table 18 The calculating results and sorting results by using PL-TOPSIS method

TOPSIS method	calculating results and sorting results
The distances of each alternative from PLPIS	$d_1^+ = 0.6421,\ d_2^+ = 1.3843,\ d_3^+ = 2.0281,\ d_4^+ = 0.8854,\ d_5^+ = 1.1077$
The distances of each alternative from PLNIS	$d_1^- = 1.2300,\ d_2^- = 1.0939,\ d_3^- = 0.7477,\ d_4^- = 1.4381,\ d_5^- = 0.8015$
The relative closeness degree of each alternative from PLNIS	$d_1 = -0.1447,\ d_2 = -1.3952,\ d_3 = -2.6386,\ d_4 = -0.3789,\ d_5 = -1.1678$
Ordering	$A_1 > A_4 > A_5 > A_2 > A_3$

contributions of such paper are: (1) two DSM' forms within PLTSs are designed; (2) the GDSM and weighted GDSM with PLTSs are defined; (3) the weighted GDSM are utilized to tackle the MAGDM issues under PLTSs; (4) within the process of MAGDM, the developed methods' major merit is more flexible and useful compared with the existing MAGDM issues with PLTSs. In the future, the proposed PLWGDSM of PLTSs can be widely applied and investigated in dynamic and intricate MADM or MAGDM issues and various unpredictable environments. The designed methods could also tackle other issues, such as environmental sustainability competency analysis, intelligent sustainable supplier selection and comprehensive assessment for water pollution.

References

Bai CZ, Zhang R, Qian LX, Wu YN (2017) Comparisons of probabilistic linguistic term sets for multi-criteria decision making. Knowl Based Syst 119:284–291
Bourguignon B, Massart DL (1994) The oreste method for multicriteria decision-making in experimental chemistry. Chemometr Intell Lab Syst 22:241–256
Braglia M, Frosolini M, Montanari R (2003) Fuzzy TOPSIS approach for failure mode, effects and criticality analysis. Qual Reliab Eng Int 19:425–443
Chen SM, Han WH (2019) Multiattribute decision making based on nonlinear programming methodology, particle swarm optimization techniques and interval-valued intuitionistic fuzzy values. Inf Sci 471:252–268
Chen SM, Cheng SH, Lan TC (2016) A novel similarity measure between intuitionistic fuzzy sets based on the centroid points of transformed fuzzy numbers with applications to pattern recognition. Inf Sci 343:15–40
Chen ZS, Chin KS, Tsui KL (2019a) Constructing the geometric Bonferroni mean from the generalized Bonferroni mean with several extensions to linguistic 2-tuples for decision-making. Appl Soft Comput 78:595–613
Chen SX, Wang JQ, Wang TL (2019b) Cloud-based ERP system selection based on extended probabilistic linguistic MULTIMOORA method and Choquet integral operator. Comput Appl Math 38:88
Deng XM, Gao H (2019) TODIM method for multiple attribute decision making with 2-tuple linguistic Pythagorean fuzzy information. J Intell Fuzzy Syst 37:1769–1780
Deng H, Yeh C-H, Willis RJ (2000) Inter-company comparison using modified TOPSIS with objective weights. Comput Oper Res 27:963–973
Dice LR (1945) Measures of the amount of ecologic association between species. Ecology 26:297–302
Feng XQ, Liu Q, Wei CP (2019) Probabilistic linguistic QUALIFLEX approach with possibility degree comparison. J Intell Fuzzy Syst 36:719–730
Geng XL, Qiu HQ, Gong XM (2017) An extended 2-tuple linguistic DEA for solying MAGDM problems considering the influence relationships among attributes. Comput Ind Eng 112:135–146

Gou XJ, Xu ZS, Liao HC (2017) Multiple criteria decision making based on Bonferroni means with hesitant fuzzy linguistic information. Soft Comput 21:6515–6529
Herrera F, Martinez L (2000a) A 2-tuple fuzzy linguistic representation model for computing with words. IEEE Trans Fuzzy Syst 8:746–752
Herrera F, Martinez L (2000b) An approach for combining linguistic and numerical information based on the 2-tuple fuzzy linguistic representation model in decision-making. Int J Uncertain Fuzziness Knowl Based Syst 8:539–562
Herrera F, Martinez L (2001) A model based on linguistic 2-tuples for dealing with multigranular hierarchical linguistic contexts in multi-expert decision-making. IEEE Trans Syst Man Cybern Part B Cybern 31:227–234
Jaccard P (1901) Distribution de la flore alpine dans le Bassin des Drouces et dans quelques regions voisines. Bull Soc Vaudoise Sci Nat 37:241–272
Kobina A, Liang DC, He X (2017) Probabilistic linguistic power aggregation operators for multi-criteria group decision making. Symmetry-Basel 9:320
Li DF (2004) Some measures of dissimilarity in intuitionistic fuzzy structures. J Comput Syst Sci 68:115–122
Li DF, Cheng CT (2002) New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions. Pattern Recogn Lett 23:221–225
Li ZX, Lu M (2019) Some novel similarity and distance and measures of Pythagorean fuzzy sets and their applications. J Intell Fuzzy Syst 37:1781–1799
Liang DC, Kobina A, Quan W (2018) Grey relational analysis method for probabilistic linguistic multi-criteria group decision-making based on geometric Bonferroni mean. Int J Fuzzy Syst 20:2234–2244
Liao HC, Jiang LS, Xu ZH, Xu JP, Herrera F (2017) A linear programming method for multiple criteria decision making with probabilistic linguistic information. Inf Sci 415:341–355
Liao HC, Jiang LS, Lev B, Fujitac H (2019) Novel operations of PLTSs based on the disparity degrees of linguistic terms and their use in designing the probabilistic linguistic ELECTRE III method. Appl Soft Comput 80:450–464
Lin MW, Xu ZS (2018) Probabilistic linguistic distance measures and their applications in multi-criteria group decision making. In: Collan M, Kacprzyk J (eds) Soft computing applications for group decision-making and consensus modeling. Studies in fuzziness and soft computing. Springer, Cham, p 357
Lin MW, Chen ZY, Liao HC, Xu ZS (2019) ELECTRE II method to deal with probabilistic linguistic term sets and its application to edge computing. Nonlinear Dyn 96:2125–2143
Liu HC, You XY, Xue YX, Luan X (2017) Exploring critical factors influencing the diffusion of electric vehicles in China: a multi-stakeholder perspective. Res Transp Econ 66:46–58
Liu HC, Quan MY, Li ZW, Wang ZL (2019a) A new integrated MCDM model for sustainable supplier selection under interval-valued intuitionistic uncertain linguistic environment. Inf Sci 486:254–270
Liu HC, Yang MY, Zhou MC, Tian GD (2019b) An integrated multi-criteria decision making approach to location planning of electric vehicle charging stations. IEEE Trans Intell Transp Syst 20:362–373
Lu JP, Wei C (2019) TODIM method for performance appraisal on social-integration-based rural reconstruction with interval-valued intuitionistic fuzzy information. J Intell Fuzzy Syst 37:1731–1740
Lu JP, Wei C, Wu J, Wei GW (2019) TOPSIS method for probabilistic linguistic MAGDM with entropy weight and its application to supplier selection of new agricultural machinery products. Entropy 21:953
Mahmood T, Ye J, Khan Q (2016) Vector similarity measures for simplified neutrosophic hesitant fuzzy set and their applications. J Inequal Spec Funct 7:176–194
Mandal K, Basu K (2016) Improved similarity measure in neutrosophic environment and its application in finding minimum spanning tree. J Intell Fuzzy Syst 31:1721–1730
Pang Q, Wang H, Xu ZS (2016) Probabilistic linguistic term sets in multi-attribute group decision making. Inf Sci 369:128–143
Peng XD, Garg H (2018) Algorithms for interval-valued fuzzy soft sets in emergency decision making based on WDBA and CODAS with new information measure. Comput Ind Eng 119:439–452
Peng XD, Li WQ (2019) Algorithms for interval-valued pythagorean fuzzy sets in emergency decision making based on multiparametric similarity measures and WDBA. IEEE Access 7:7419–7441
Rodriguez RM, Martinez L, Herrera F (2012) Hesitant fuzzy linguistic term sets for decision making. IEEE Trans Fuzzy Syst 20:109–119
Salton G, McGill MJ (1987) Introduction to modern information retrieval. McGraw-Hill, New York
Shannon CE (1948) A mathematical theory of communication. Bell Syst Tech J 27:379–423
Sharaf IM (2018) TOPSIS with similarity measure for MADM applied to network selection. Comput Appl Math 37:4104–4121

Tang Y, Wen LL, Wei GW (2017) Approaches to multiple attribute group decision making based on the generalized Dice similarity measures with intuitionistic fuzzy information. Int J Knowl Based Intell Eng Syst 21:85–95

Tian GD, Zhang HH, Feng YX, Jia HF, Zhang CY, Jiang ZG, Li ZW, Li PG (2017) Operation patterns analysis of automotive components remanufacturing industry development in China. J Clean Prod 164:1363–1375

Tian GD, Zhang HH, Feng YX, Wang DQ, Peng Y, Jia HF (2018) Green decoration materials selection under interior environment characteristics: a grey-correlation based hybrid MCDM method. Renew Sustain Energy Rev 81:682–692

Torra V (2010) Hesitant fuzzy sets. Int J Intell Syst 25:529–539

Tsoulfas GT, Pappis CP (2008) A model for supply chains environmental performance analysis and decision making. J Clean Prod 16:1647–1657

Wang J, Wei GW, Wei Y (2018) Models for green supplier selection with some 2-tuple linguistic neutrosophic number Bonferroni mean operators. Symmetry-Basel 10:131

Wang P, Wang J, Wei GW, Wei C (2019a) Similarity measures of q-rung orthopair fuzzy sets based on cosine function and their applications. Mathematics 7:340

Wang J, Gao H, Wei GW (2019b) The generalized Dice similarity measures for Pythagorean fuzzy multiple attribute group decision making. Int J Intell Syst 34:1158–1183

Wang J, Gao H, Lu M (2019c) Approaches to strategic supplier selection under interval neutrosophic environment. J Intell Fuzzy Syst 37:1707–1730

Wang J, Wei GW, Wei C, Wu J (2020) Maximizing deviation method for multiple attribute decision making under q-rung orthopair fuzzy environment. Def Technol 16:1073–1087

Wei GW (2017) Some cosine similarity measures for picture fuzzy sets and their applications to strategic decision making. Informatica 28:547–564

Wei GW (2018) Some similarity measures for picture fuzzy sets and their applications. Iran J Fuzzy Syst 15:77–89

Wei GW (2019a) Pythagorean fuzzy Hamacher Power aggregation operators in multiple attribute decision making. Fundam Inform 166:57–85

Wei GW (2019b) The generalized dice similarity measures for multiple attribute decision making with hesitant fuzzy linguistic information. Econ Res Ekonomska Istrazivanja 32:1498–1520

Wei GW, Gao H (2018) The generalized dice similarity measures for picture fuzzy sets and their applications. Informatica 29:107–124

Wei GW, Wei Y (2018) Similarity measures of Pythagorean fuzzy sets based on the cosine function and their applications. Int J Intell Syst 33:634–652

Wei GW, Wang R, Wang J, Wei C, Zhang Y (2019) Methods for evaluating the technological innovation capability for the high-tech enterprises with generalized interval neutrosophic number Bonferroni mean operators. IEEE Access 7:86473–86492

Wei GW, Lu JP, Wei C, Wu J (2020a) Probabilistic linguistic GRA method for multiple attribute group decision making. J Intell Fuzzy Syst 38:4721–4732

Wei GW, Wang J, Lu JP, Wu J, Wei C, Alsaadi FE, Hayat T (2020b) VIKOR method for multiple criteria group decision making under 2-tuple linguistic neutrosophic environment. Econ Res Ekonomska Istrazivanja 33:3185–3208

Wu SM, Liu HC, Wang LE (2017) Hesitant fuzzy integrated MCDM approach for quality function deployment: a case study in electric vehicle. Int J Prod Res 55:4436–4449

Wu LP, Gao H, Wei C (2019a) VIKOR method for financing risk assessment of rural tourism projects under interval-valued intuitionistic fuzzy environment. J Intell Fuzzy Syst 37:2001–2008

Wu LP, Wang J, Gao H (2019b) Models for competiveness evaluation of tourist destination with some interval-valued intuitionistic fuzzy Hamy mean operators. J Intell Fuzzy Syst 36:5693–5709

Xian SD, Chai JH, Yin YB (2019) A visual comparison method and similarity measure for probabilistic linguistic term sets and their applications in multi-criteria decision making. Int J Fuzzy Syst 21:1154–1169

Xu ZS (2005) Deviation measures of linguistic preference relations in group decision making. Omega Int J Manag Sci 33:249–254

Xu ZS, Yager RR (2010) Power-geometric operators and their use in group decision making. IEEE Trans Fuzzy Syst 18:94–105

Yager RR (2001) The power average operator. IEEE Trans Syst Man Cybern Part A 31:724–731

Ye J (2012a) Multicriteria group decision-making method using vector similarity measures for trapezoidal intuitionistic fuzzy numbers. Group Decis Negot 21:519–530

Ye J (2012b) Multicriteria decision-making method using the Dice similarity measure between expected intervals of trapezoidal fuzzy numbers. J Decis Syst 21:307–317

Ye J (2014) Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making. Int J Fuzzy Syst 16:204–211
Ye J (2016) The generalized Dice measures for multiple attribute decision making under simplified neutrosophic environments. J Intell Fuzzy Syst 31:663–671
Zadeh LA (1975) The concept of a linguistic variable and its application to approximate reasoning. Inf Sci 8:301–357
Zhang YX, Xu ZS, Wang H, Liao HC (2016) Consistency-based risk assessment with probabilistic linguistic preference relation. Appl Soft Comput 49:817–833
Zhang XF, Xu ZS, Ren PJ (2019) A novel hybrid correlation measure for probabilistic linguistic term sets and crisp numbers and its application in customer relationship management. Int J Inf Technol Decis Mak 18:673–694
Zhu B, Xu ZS (2013) Hesitant fuzzy Bonferroni means for multi-criteria decision making. J Oper Res Soc 64:1831–1840