Artificial Intelligence Review (2021) 54:3141–3171
https://doi.org/10.1007/s10462-020-09923-5

Certain competition graphs based on picture fuzzy environment with applications

Sankar Das[1,2] · Ganesh Ghorai[1] · Madhumangal Pal[1]

Published online: 3 November 2020

Abstract
In this paper, the notion of picture fuzzy competition graph along with its few generalizations such as *m*-step picture fuzzy competition graphs, picture fuzzy economic competition graphs and picture fuzzy competition hypergraphs are introduced. Some related picture fuzzy graphs including picture fuzzy *m*-step neighborhood graph, picture fuzzy *m*-step economic competition graph and picture fuzzy *k*-competition hypergraphs are introduced. Some properties of these graphs have been investigated. Finally, applications of *m*-step picture fuzzy competition graphs and picture fuzzy competition hypergraphs are presented in several fields such as in education system, ecosystem, business market and job competition.

Keywords Picture fuzzy competition graphs · *m*-step picture fuzzy competition graphs · Picture fuzzy economic competition graphs · Picture fuzzy hypergraphs · Picture fuzzy competition hypergraphs

1 Introduction

1.1 Research background

The notion of competition graph (CG) is formally introduced by Cohen (1968). In ecology, there are some problems of competition between species of food cycles those are modeled by the digraph $\overrightarrow{G} = (V, \overrightarrow{B})$. These models are suitable to specify well defined nature of objects and especially species-victim relations. Nowadays, besides of ecosystem, CGs have many applications in other fields, such as coding and energy system, channel assignments,

✉ Ganesh Ghorai
math.ganesh@mail.vidyasagar.ac.in

Sankar Das
sankarkgp22@gmail.com

Madhumangal Pal
mmpalvu@gmail.com

[1] Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore 721 102, India

[2] Department of Mathematics, Kharagpur College, Kharagpur 721 305, India

social interactions, modeling of complex economic, communications over a noisy channel etc. Let $\overrightarrow{G} = (V, \overrightarrow{B})$ be a digraph, which corresponds to a food cycle, a vertex $r \in V(\overrightarrow{G})$ represent a species and arc $(r, q) \in \overrightarrow{B}(\overrightarrow{G})$ indicate that r kills the species q. If two species r and s have a common victim q then they will compete for q. The CG $C(\overrightarrow{G})$ of $\overrightarrow{G}$ is an undirected graph with V as vertex set and having an edge (r, s) in $C(\overrightarrow{G})$ iff there is a vertex $q \in V$ such that $(r, q), (s, q) \in \overrightarrow{B}(\overrightarrow{G})$ for any $r, s \in V$, $(r \neq s)$.

The notion of fuzzy set (FS) was first introduced by Zadeh (1965) to discuss the uncertainty in several real life problems. It was found that one component is not sufficient to describe some special types of information. In this situations, a component namely non-membership value is invited to illustrate the information properly and in addition to this new component (Atanassov 1986) defined intuitionistic fuzzy set (IFS). Later on, in some cases, another component namely 'neutrality' is needed to represent some information completely. To recover these scenarios, Cuong (2014) and Cuong and Hai (2015) presented the idea of picture fuzzy set (PFS) as a generalization of IFS, by incorporating the notion of truth, abstinence and false membership degree of an element in the set with sum of these three degrees less than or equal to 1. The structure of fuzzy graph (FG) was introduced by Rosenfeld (1975), whose first definition was given by Kauffman (1973) and intuitionistic fuzzy graph (IFG) were discussed by Shannon and Atanassov (1994). Poulik and Ghorai (2018, 2020a, b) introduced several new concepts of bipolar fuzzy graphs with their applications. After that, Al-Hawary et al. (2018) provided the new concept of picture fuzzy graph (PFG) and discussed some operations on it. Recently, Das and Ghorai (2020a) introduced the notion of picture fuzzy planar graphs and applied it to construct road map designs. After Cohen's introduction of CG, its several variations are found in literature, such as p-CGs of digraphs (Kim et al. 1995), tolerance CGs (Brigham et al. 1995), m-step CGs of digraphs (Cho et al. 2000), etc. The p-competition indicates that there is a competition between two species if they have at least p common victims. Samanta and Pal (2013) and Samanta et al. (2014) first utilized FGs in competition in ecosystems. Later, Samanta et al. (2015) introduced another generalization as m-step fuzzy CGs of digraphs. Sahoo and Pal (2015) defined intuitionistic fuzzy CGs and its novel properties studied by Nasir et al. (2017). The study of fuzzy ϕ-tolerance CGs and interval-valued fuzzy ϕ-tolerance CGs was presented by Pramanik et al. (2016a, b). Al-Shehrie and Akram (2015) introduced the concept of bipolar fuzzy CGs. On the other hand, some novel concepts of CGs and a decision making approach based on CGs ware discussed in bipolar fuzzy environment by Sarwar and Akram (2017) and Sarwar et al. (2018b). Akram and Nasir (2017) and Akram and Sarwar (2018) studied certain CGs in intuitionistic neutrosophic environment and m-polar fuzzy CGs with their applications. Many other works on CGs are found in Borzooei et al. (2016), Das et al. (2020), Habib et al. (2019), and Samanta and Sarkar (2018). A very recent work, applying the idea of PFSs to CGs, Das and Ghorai (2020b) introduced the notion of picture fuzzy competition graphs (PFCGs) and applied this idea in medical science.

Hypergraph theory originally developed by Berge (1973) in 1960, as a generalization of graph theory. The notion of fuzzy hypergraphs (FHs) was first discussed by Kaufmann (1975) and then this concept redefined and extended by Lee-Kwang and Lee (1995). After that, FHs theory increased in different branches, such as interval-valued fuzzy hypergraphs (Chen 1997), intuitionistic fuzzy hypergraphs (Parvathi et al. 2009), bipolar fuzzy hypergraphs (Samanta and Pal 2012), etc. In 2004, Sonnatag and Teichert (2004) first gave the idea of competition hypergraphs (CHs). Sarwar et al. (2018a) presented the notion of fuzzy CHs to generalize the concept of CHs and fuzzy CGs.

1.2 Research challenges and gaps

- Some problems in literature can not be modeled by using PFCG but m-step picture fuzzy competition graphs (m-SPFCG) are used successfully for these problems.
- The crisp CGs do not measure the strength of competitions between common victim and related species due to uncertainty.
- The crisp CGs are not sufficient to show the degree of dependence about the common victim and related species due to uncertainty.
- The crisp hypergraphs do not describe all the competitions of real world problems that contain uncertainty and fuzzy in nature.
- PFCH give clear representation of predator-prey relations than CGs.

1.3 Motivation and contribution of this study

In ecological problem, species may be of several types like lenten, non-lenten, strong, weak etc. Similarly, victims may be tasteful, digestive, injurious, etc. These terms have no proper meaning. They are fuzzy in nature. So the species and victims may be assumed as PFSs and inter-relation between them may be designed with a PFG. Due to uncertainty in description of species and victims, and to find more than 1-step relationships between them, it is necessary to design m-SPFCG model. As crisp hypergraphs do not demonstrate properly all the competitions of such problems, therefore the contribution of this article is not only restricted to m-SPFCGs but also we have applied the idea of PFS to CHs to handle the real problems having non-linear uncertainties or haziness.

Here, we have generalized the notion of picture fuzzy hypergraph by assuming picture fuzzy vertex instead of crisp vertex set and an interrelation between picture fuzzy vertex and edges. Also, various new concepts including m-SPFCGs, picture fuzzy economic competition graphs (PFECGs), m-step picture fuzzy neighborhood graphs (m-SPFNGs), picture fuzzy competition hypergraphs (PFCH), picture fuzzy k-competition hypergraphs (PFkCH) and picture fuzzy neighborhood hypergraphs (PFNH) are presented with some of their interesting properties. An applications of m-SPFCG in education system is presented.

The PFCG or m-SPFCG model, usually give only pair-wise competition between objects. But, when we are interested to find group-wise competition among three or more objects, then existing models are not fruitful. In such situations, PFCH models plays a important role to overcome this issue.

1.4 Framework of this study

This work is composed as follows: In Sect. 2, several basic definitions related to PFCGs are provided. In Sect. 3, the notion of m-SPFCG is presented and studied several properties of it. In Sect. 4, the notion of PFECG is presented. In Sect. 5, PFCH is introduced. In Sect. 6, the concept of PFNHs is presented and established the relations between PFkCHs and picture fuzzy k-neighborhood graphs (PFkNHs). In Sect. 7, an application of m-SPFCG in education system is given. Finally, a conclusion is drawn in Sect. 8.

2 Preliminaries

In this section, several basic definitions related to PFCGs are provided. Meantime, we introduce cardinality, support and height of PFSs that will be used in later sections.

A digraph is usually used to model the relationship between a given set of objects.

Definition 2.1 (Jenson and Gutin 2009) A digraph $\overrightarrow{G}$ consists of a non-empty finite vertex set V and a finite set $\overrightarrow{B}$ of edges that are ordered pairs of distinct members of V. Let $r_i \in V$, the out-neighborhood and in-neighborhood of r_i are the sets $\aleph^+(r_i) = \{r_j \in V - r_i : (r_i, r_j) \in \overrightarrow{B}\}$ and $\aleph^-(r_i) = \{r_j \in V - r_i : (r_j, r_i) \in \overrightarrow{B}\}$, respectively. Also the set $\aleph^+(r_i) \cup \aleph^-(r_i)$ is the open neighborhood of r_i in $\overrightarrow{G}$. A directed walk from a vertex r_i to r_j in $\overrightarrow{G}$ is an alternating sequence of vertices and edges begin with r_i and end with r_j such that each edge is incident with the vertices preceding and following it. No edge appears more than once but vertex can. A walk is closed if $r_i = r_j$. If all vertices in a walk are distinct, then it is known as path.

The open and closed neighborhood of vertices help to model neighborhood graphs.

Definition 2.2 (Achary and Vartak 1973) The open neighborhood $\aleph(r)$ of r in an undirected graph G is the set of all vertices adjoining to r and the closed neighborhood of r is $\aleph[r] = \aleph(r) \cup \{r\}$. The open-neighborhood graph $\aleph(G)$ and closed-neighborhood graph $\aleph[G]$ of G are the graphs with V as vertex set and having an edge (r, s) in $\aleph(G)$ and $\aleph[G]$ iff $\aleph(r) \cap \aleph(s) \neq \emptyset$ and $\aleph[r] \cap \aleph[s] \neq \emptyset$, respectively in G.

PFSs, superior to FSs and IFSs, amplify the space of uncertain information.

Definition 2.3 (Cuong 2014) Let X be the universe. Then a PFS A is defined on X as $A = \{r, (\mu_A(r), \eta_A(r), \nu_A(r)) : r \in X\}$, where $\mu_A(r), \eta_A(r), \nu_A(r) \in [0, 1]$ denote the degree of truth membership (DTMS), degree of abstinence membership (DAMS), degree of false membership (DFMS) of $r \in A$, respectively with $0 \leq \mu_A(r) + \eta_A(r) + \nu_A(r) \leq 1 \ \forall r \in X$. Also $\forall r \in X, D_A(r) = 1 - (\mu_A(r) + \eta_A(r) + \nu_A(r))$ represent denial degree of $r \in A$. Here, $\mu_A(r), \eta_A(r), \nu_A(r)$ all are independent.

Definition 2.4 (Das and Ghorai 2020b) Let $A = (r, \mu_A, \eta_A, \nu_A)$ be a PFS. The cardinality of A is defined as $|A| = \left(|A|_\mu, |A|_\eta, |A|_\nu\right)$, where $|A|_\mu, |A|_\eta$ and $|A|_\nu$ represent the sum of DTMS, DAMS and DFMS, respectively of all elements of A. The support of A is $supp(A)=\{r \in V\!: \ \mu_A(r) \neq 0, \eta_A(r) \neq 0$ and $\nu_A(r) \neq 0\}$ and the height of A is $h(A) = \left(\sup_{r \in V} \mu_A(r), \sup_{r \in V} \eta_A(r), \inf_{r \in V} \nu_A(r)\right) = \left(h_\mu(A), h_\eta(A), h_\nu(A)\right)$.

Picture fuzzy models provide more legibility, flexibility and suitability to the system as compared with the models in other fields.

Definition 2.5 (Al-Hawary et al. 2018) A PFG is $G = (V, A, B)$ where $A = (\mu_A, \eta_A, \nu_A)$, $B = (\mu_B, \eta_B, \nu_B)$ and

(i) $V = \{r_1, r_2, \ldots, r_n\}$ such that $\mu_A, \eta_A, \nu_A : V \to [0,1]$ denote the DTMS, DAMS and DFMS of $r_i \in V$, respectively with $0 \le \mu_A(r_i) + \eta_A(r_i) + \nu_A(r_i) \le 1 \;\; \forall r_i \in V$, $(i = 1, 2, \ldots, n)$.

(ii) $\mu_B, \eta_B, \nu_B : V \times V \to [0,1]$ denote the DTMS, DAMS and DFMS of edge (r_i, r_j), respectively such that $\mu_B(r_i, r_j) \le min\{\mu_A(r_i), \mu_A(r_j)\}$, $\eta_B(r_i, r_j) \le min\{\eta_A(r_i), \eta_A(r_j)\}$ and $\nu_B(r_i, r_j) \le max\{\nu_A(r_i), \nu_A(r_j)\}$ with $0 \le \mu_B(r_i, r_j) + \eta_B(r_i, r_j) + \nu_B(r_i, r_j) \le 1$ for every (r_i, r_j), $(i, j = 1, 2, \ldots, n)$.

Like as fuzzy digraph, picture fuzzy digraph (PFD) has the following definition.

Definition 2.6 (Das and Ghorai 2020b) A PFD is of the form $\overrightarrow{G} = (V, A, \overrightarrow{B})$ where $A = (\mu_A, \eta_A, \nu_A)$, $\overrightarrow{B} = (\overrightarrow{\mu}_B, \overrightarrow{\eta}_B, \overrightarrow{\nu}_B)$ and (i) $V = \{r_1, r_2, \ldots, r_n\}$ such that $\mu_A, \eta_A, \nu_A : V \to [0,1]$ denote the DTMS, DAMS and DFMS of $r_i \in V$, respectively with $0 \le \mu_A(r_i) + \eta_A(r_i) + \nu_A(r_i) \le 1 \; \forall r_i \in V$, $(i = 1, 2, \ldots, n)$.

(ii) $\overrightarrow{\mu}_B, \overrightarrow{\eta}_B, \overrightarrow{\nu}_B : V \times V \to [0,1]$ denote the DTMS, DAMS and DFMS of edge (r_i, r_j), respectively such that $\overrightarrow{\mu}_B(r_i, r_j) \le min\{\mu_A(r_i), \mu_A(r_j)\}$, $\overrightarrow{\eta}_B(r_i, r_j) \le min\{\eta_A(r_i), \eta_A(r_j)\}$ and $\overrightarrow{\nu}_B(r_i, r_j) \le max\{\nu_A(r_i), \nu_A(r_j)\}$ with $0 \le \overrightarrow{\mu}_B(r_i, r_j) + \overrightarrow{\eta}_B(r_i, r_j) + \overrightarrow{\nu}_B(r_i, r_j) \le 1$ for every (r_i, r_j), $(i, j = 1, 2, \ldots, n)$.

We illustrate it by giving an example.

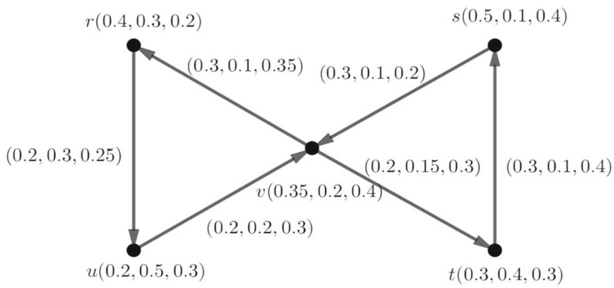
Example 2.7 We consider the PFD $\overrightarrow{G}$, as showing in Fig. 1.

In a PFD, strength of edges characterize the competitions between common feed and related species. This shows how much the species depend on the common feed.

Definition 2.8 (Mohamedlsmayil and AshaBosely 2019) A PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ is complete if $\overrightarrow{\mu}_B(r, s) = \mu_A(r) \wedge \mu_A(s)$, $\overrightarrow{\eta}_B(r, s) = \eta_A(r) \wedge \eta_A(s)$ and $\overrightarrow{\nu}_B(r, s) = \nu_A(r) \vee \nu_A(s)$ $\forall \;\; r, s \in V$. An edge (r, s) is independent strong if $\frac{1}{2} min\{\mu_A(r), \mu_A(s)\} < \overrightarrow{\mu}_B(r, s)$, $\frac{1}{2} min\{\eta_A(r), \eta_A(s)\} > \overrightarrow{\eta}_B(r, s)$ and $\frac{1}{2} max\{\nu_A(r), \nu_A(s)\} > \overrightarrow{\nu}_B(r, s)$. Otherwise, it is weak edge. Strength of the edge (r, s) is given by $\left(\frac{\overrightarrow{\mu}_B(r,s)}{\mu_A(r) \wedge \mu_A(s)}, \frac{\overrightarrow{\eta}_B(r,s)}{\eta_A(r) \wedge \eta_A(s)}, \frac{\overrightarrow{\nu}_B(r,s)}{\nu_A(r) \vee \nu_A(s)} \right)$.

Now, to construct PFCGs it is necessary to define PFON and PFIN of a vertex in the PFD.

Fig. 1 Example of a PFD



Definition 2.9 (Das and Ghorai 2020b) PFON of a vertex r in $\overrightarrow{G}$ is $\aleph^+(r) = (X_r^+, (\mu_r^+, \eta_r^+, \nu_r^+))$, where $X_r^+ = \{s : \overrightarrow{\mu}_B(r,s) > 0, \overrightarrow{\eta}_B(r,s) > 0$ and $\overrightarrow{\nu}_B(r,s) > 0\}$ and $\mu_r^+, \eta_r^+, \nu_r^+ : X_r^+ \to [0,1]$ are defined as $\mu_r^+(s) = \overrightarrow{\mu}_B(r,s), \eta_r^+(s) = \overrightarrow{\eta}_B(r,s)$ and $\nu_r^+(s) = \overrightarrow{\nu}_B(r,s)$.

PFIN of a vertex r of $\overrightarrow{G}$ is $\aleph^-(r) = (X_r^-, (\mu_r^-, \eta_r^-, \nu_r^-))$, where $X_r^- = \{s : \overrightarrow{\mu}_B(s,r) > 0, \overrightarrow{\eta}_B(s,r) > 0$ and $\overrightarrow{\nu}_B(s,r) > 0\}$ and $\mu_r^-, \eta_r^-, \nu_r^- : X_r^- \to [0,1]$ are defined as $\mu_r^-(s) = \overrightarrow{\mu}_B(s,r), \eta_r^-(s) = \overrightarrow{\eta}_B(s,r)$ and $\nu_r^-(s) = \overrightarrow{\nu}_B(s,r)$.

To cover all the competitions in real world, adding more uncertainty to intuitionistic fuzzy CGs, PFCGs are introduced.

Definition 2.10 (Das and Ghorai 2020b) The PFCG $C(\overrightarrow{G})$ of a PFD $\overrightarrow{G}$ is an undirected graph with V as vertex set and having an edge (r, s) in $C(\overrightarrow{G})$ iff $\aleph^+(r) \cap \aleph^+(s) \neq \emptyset$ in $\overrightarrow{G}$. The DTMS, DAMS and DFMS of (r, s) in $C(\overrightarrow{G})$ are respectively $\mu_B(r,s) = [\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph^+(r) \cap \aleph^+(s))$, $\eta_B(r,s) = [\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph^+(r) \cap \aleph^+(s))$ and $\nu_B(r,s) = [\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph^+(r) \cap \aleph^+(s))$.

3 *m*-step picture fuzzy competition graph (*m*-SPFCG)

In this section, we will introduce m-step picture fuzzy digraph and one of the generalization of PFCG is considered known as m-SPFCG. The following notations are used in this work:

$P^m_{(r,s)}$: A picture fuzzy path (PFP) between r and s of length m.
$\overrightarrow{P}^m_{(r,s)}$: A directed PFP of length m between r and s.
$\aleph_m^+(r)$: picture fuzzy m-step out neighborhood (PFmSON) of r.
$\aleph_m^-(r)$: picture fuzzy m-step in neighborhood (PFmSIN) of r.
$\aleph_m(r)$: picture fuzzy m-step neighborhood (PFmSN) of r.
$\aleph_m(G)$: picture fuzzy m-step neighborhood graph (PFmSNG)of the PFG G.
$C_m(\overrightarrow{G})$: m-step picture fuzzy competition graph (m-SPFCG) of the PFDG $\overrightarrow{G}$.

Definition 3.1 The m-SPFD of a PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ is denoted by $\overrightarrow{G_m} = (V, A, \overrightarrow{B'})$ with V as vertex set and having an edge (r, s) in $\overrightarrow{G_m}$ if there is a directed PFP $\overrightarrow{P}^m_{(r,s)}$ in $\overrightarrow{G}$.

In PFDs if a species r directly attacks a feed s, then their connection is showed by $(\overrightarrow{r,s})$ edge. But, if such connection made indirectly with the help of m mediators, this can be showed by a directed PFP of length m. So m-step feed (species) in a PFD is represented by a vertex which is the PFmSON (or PFmSIN) of some species (feed). Both of them help to construct m-SPFNGs and m-SPFCGs models. Now, PFmSON and PFmSIN of vertices in a PFD are defined below.

Definition 3.2 The PFmSON of a vertex r of $\overrightarrow{G}$ is the PFS $\aleph_m^+(r) = (X_r^+, (\mu_r^+, \eta_r^+, \nu_r^+))$, where $X_r^+ = \{s$: there is a directed PFP $\overrightarrow{P}^m_{(r,s)}$ of length m from r to $s\}$, $\mu_r^+, \eta_r^+, \nu_r^+ : X_r^+ \to$

$[0,1]$ are defined as $\mu_r^+(s) = min\{\overrightarrow{\mu}_B(a,b) : (a,b)$ is an edge of $\overrightarrow{P}^m_{(r,s)}\}$, $\eta_r^+(s) = min\{\overrightarrow{\eta}_B(a,b) : (a,b)$ is an edge of $\overrightarrow{P}^m_{(r,s)}\}$ and $\nu_r^+(s) = max\{\overrightarrow{\nu}_B(a,b) : (a,b)$ is an edge of $\overrightarrow{P}^m_{(r,s)}\}$.

The PFmSIN of a vertex r of $\overrightarrow{G}$ is the PFS $\aleph_m^-(r) = (X_r^-, (\mu_r^-, \eta_r^-, \nu_r^-))$, where $X_r^- = \{s$: there is a directed PFP $\overrightarrow{P}^m_{(s,r)}$ of length m from s to $r\}$, $\mu_A^-, \eta_A^-, \nu_A^- : X_r^- \rightarrow [0,1]$ are defined as $\mu_r^-(s) = min\{\overrightarrow{\mu}_B(a,b) : (a,b)$ is an edge of $\overrightarrow{P}^m_{(s,r)}\}$, $\eta_r^-(s) = min\{\overrightarrow{\eta}_B(a,b) : (a,b)$ is an edge of $\overrightarrow{P}^m_{(s,r)}\}$ and $\nu_r^-(s) = max\{\overrightarrow{\nu}_B(a,b) : (a,b)$ is an edge of $\overrightarrow{P}^m_{(s,r)}\}$.

There are some problems in literature, where the species and feed can not be connected directly. These problems of feed-species can not be properly modeled by using PFCGs. In such cases m-SPFCGs are effectively used. The m-SPFCG is the generalization of PFCG which is defined below.

Definition 3.3 Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD. The m-SPFCG of $\overrightarrow{G}$ is denoted by $C_m(\overrightarrow{G}) = (V, A, B')$ with V as vertex set and having an edge (r, s) in $C_m(\overrightarrow{G})$ iff $\aleph_m^+(r) \cap \aleph_m^+(s) \neq \emptyset$ in $\overrightarrow{G}$. The DTMS, DAMS and DFMS of an edge (r, s) are given by $\mu_{B'}(r,s) = [\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph_m^+(r) \cap \aleph_m^+(s))$, $\eta_{B'}(r,s) = [\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph_m^+(r) \cap \aleph_m^+(s))$ and $\nu_{B'}(r,s) = [\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph_m^+(r) \cap \aleph_m^+(s))$, respectively.

The following example illustrates 2-SPFCG.

Example 3.4 Consider a PFD $\overrightarrow{G}$ (see in Fig.2a). Here, $\aleph_2^+(r) = \{(y, (0.4, 0.1, 0.2)), (w, (0.7, 0.1, 0.2))\}$ and $\aleph_2^+(s) = \{(y, (0.4, 0.1, 0.3)), (w, (0.4, 0.2, 0.3))\}$. Therefore, $\aleph_2^+(r) \cap \aleph_2^+(s) = \{(y, (0.4, 0.1, 0.3)), (w, (0.4, 0.1, 0.3))\} \neq \emptyset$. Then there is an edge (r, s) in $C_2(\overrightarrow{G})$ with DTMS, DAMS and DFMS are respectively 0.2, 0.01 and 0.06 shown in Fig. 2b.

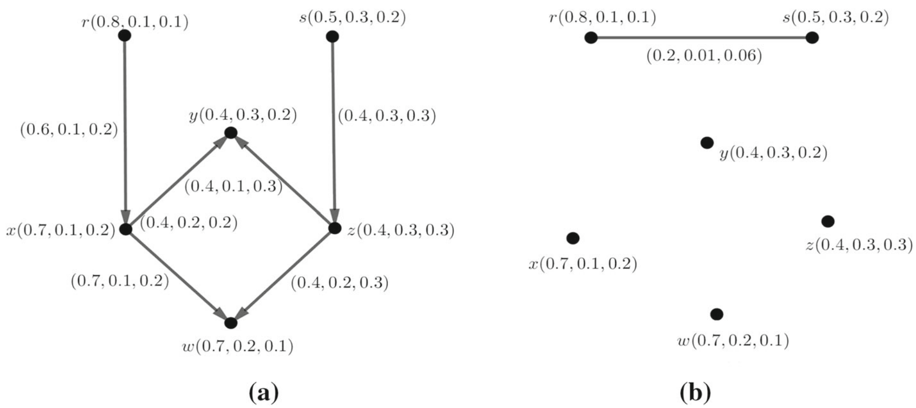


Fig. 2 Example of **a** PFD and **b** 2-SPFCG

Definition 3.5 The PFmSN of a vertex r of PFG $G=(V,A,B)$ is a PFS $\aleph_m(r)=(X_r,(\mu_r,\eta_r,\nu_r))$, where $X_r=\{s$: there exits a PFP $P^m_{(r,s)}$ of length m from r to $s\}$, $\mu_r,\eta_r,\nu_r:X_r\to[0,1]$ are defined by $\mu_r(s)=min\{\mu_B(a,b):(a,b)$ is an edge of $P^m_{(r,s)}\}$, $\eta_r(s)=min\{\eta_B(a,b):(a,b)$ is an edge of $P^m_{(r,s)}\}$ and $\nu_r(s)=max\{\nu_B(a,b):(a,b)$ is an edge of $P^m_{(r,s)}\}$.

Next, we define m-step picture fuzzy neighborhood graph (m-SPFNG). In m-SPFNG the relation among neighborhoods of any species are modeled.

Definition 3.6 Let $G=(V,A,B)$ be a PFG. The m-SPFNG of G is denoted by $\aleph_m(G)=(V,A,B)$, where $A=(\mu_A,\eta_A,\nu_A)$, $B=(\mu_B,\eta_B,\nu_B)$ and $\mu_A,\eta_A,\nu_A:V\to[0,1]$ and $\mu_B,\eta_B,\nu_B:V\times V\to[0,1]$ are such that $\mu_B(r,s)=[\mu_A(r)\wedge\mu_A(s)]h_\mu(\aleph_m(r)\cap\aleph_m(s))$, $\eta_B(r,s)=[\eta_A(r)\wedge\eta_A(s)]\ h_\eta(\aleph_m(r)\cap\aleph_m(s))$ and $\nu_B(r,s)=[\nu_A(r)\vee\nu_A(s)]h_\nu(\aleph_m(r)\cap\aleph_m(s))$.

The following example illustrates 2-SPFNG.

Example 3.7 Consider a PFG G given in Fig. 3a. Here, $\aleph_2(r)=\{(w,(0.5,0.1,0.2))\}$, $\aleph_2(s)=\{(u,(0.4,0.1,0.3)),(w,(0.4,0.2,0.3))\}$, $\aleph_2(t)=\{(v,(0.4,0.1,0.3))\}$, $\aleph_2(u)=\{(s,(0.4,0.1,0.3)),(w,(0.4,0.1,0.3))\}$, $\aleph_2(v)=\{(t,(0.4,0.1,0.3))\}$, $\aleph_2(w)=\{(r,(0.5,0.1,0.2)),(s,(0.4,0.2,0.3)),(u,(0.4,0.1,0.3))\}$. Also, $\aleph_2(r)\cap\aleph_2(s)=\{(w,(0.4,0.1,0.3))\}$, $\aleph_2(r)\cap\aleph_2(u)=\{(w,(0.4,0.1,0.3))\}$, $\aleph_2(s)\cap\aleph_2(u)=\{(w,(0.4,0.1,0.3))\}$, $\aleph_2(s)\cap\aleph_2(w)=\{(u,(0.4,0.1,0.3))\}$ and $\aleph_2(u)\cap\aleph_2(w)=\{(s,(0.4,0.1,0.3))\}$. So, ($r$, s), (r, u), (s, u), (s, w) and (u, w) are the edges of $\aleph_2(G)$ with DTMS, DAMS and DFMS are respectively (0.2, 0.01, 0.06), (0.16, 0.01, 0.06), (0.16, 0.03, 0.06), (0.2, 0.02, 0.06) and (0.16, 0.02, 0.06) shown in Fig. 3b.

Theorem 3.8 *Let $\overrightarrow{G}=(V,A,\overrightarrow{B})$ be a PFD. If $m>|V|$ then $C_m(\overrightarrow{G})$ is a PFG with no edges.*

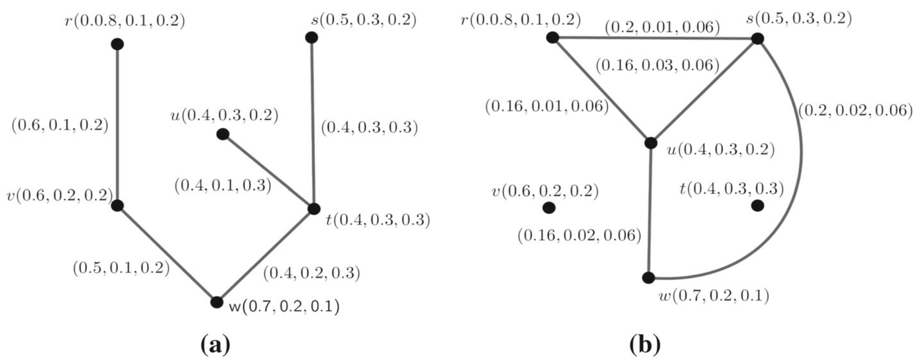


Fig. 3 Example of **a** PFG and **b** 2-SPFNG

Proof Let $C_m(\overrightarrow{G}) = (V, A, B')$ be the m-SPFCG corresponding to a PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$, where, $\mu_{B'}(r,s) = [\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph_m^+(r) \cap \aleph_m^+(s))$, $\eta_{B'}(r,s) = [\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph_m^+(r) \cap \aleph_m^+(s))$ and $\nu_{B'}(r,s) = [\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph_m^+(r) \cap \aleph_m^+(s))$.

If $m > |V|$, there does not exists any directed PFP $\overrightarrow{P}^m_{(r,s)}$ of length m in $\overrightarrow{G}$.

So, $\aleph_m^+(r) \cap \aleph_m^+(s) = \emptyset \; \forall \; r, s$ in $\overrightarrow{G}$. Hence, $C_m(\overrightarrow{G})$ has no edge. □

Example 3.9 Any PFD with n vertices has at most a directed PFP of length $(n-1)$ between its two vertices. In Fig. 2a, $|V(\overrightarrow{G})| = 6$. If we take $m = 7$, then there does not exists any directed PFP of length 7 in $\overrightarrow{G}$. So, $\aleph_7^+(r) \cap \aleph_7^+(s) = \emptyset \; \forall \; r, s$ in $\overrightarrow{G}$ and hence edge set of $C_7(\overrightarrow{G})$ is an empty set.

Here, the underlying PFG of a PFD is defined below.

Definition 3.10 Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD. The underlying PFG of $\overrightarrow{G}$ is the PFG $G = (V, A, B)$, where, $\mu_B(r,s) = min\{\overrightarrow{\mu}_B(r,s), \overrightarrow{\mu}_B(s,r)\}$, $\eta_B(r,s) = min\{\overrightarrow{\eta}_B(r,s), \overrightarrow{\eta}_B(s,r)\}$ and $\nu_B(r,s) = max\{\overrightarrow{\nu}_B(r,s), \overrightarrow{\nu}_B(s,r)\} \; \forall \; r, s \in V$.

Next, we established a relation between m-SPFCG and m-SPFNG.

Theorem 3.11 *If a PFD $\overrightarrow{G}$ does not contain any parallel edge, then $C_m(\overrightarrow{G}) = \aleph_m(G)$ for $m > 1$, where G is the underlying PFG of $\overrightarrow{G}$.*

Proof Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD having no parallel edges. Let $G = (V, A, B)$ be the underlying PFG of $\overrightarrow{G}$. Then $\mu_B(r,s) = min\{\overrightarrow{\mu}_B(r,s), \overrightarrow{\mu}_B(s,r)\}$, $\eta_B(r,s) = min\{\overrightarrow{\eta}_B(r,s), \overrightarrow{\eta}_B(s,r)\}$, $\nu_B(r,s) = max\{\overrightarrow{\nu}_B(r,s), \overrightarrow{\nu}_B(s,r)\} \; \forall \; r, s \in V$.

Since $\overrightarrow{G}$ contains no parallel edges, then $\mu_B(r,s) = \overrightarrow{\mu}_B(r,s)$, $\eta_B(r,s) = \overrightarrow{\eta}_B(r,s)$ and $\nu_B(r,s) = \overrightarrow{\nu}_B(r,s)$, $\forall \; r, s \in V$. Also, let $C_m(\overrightarrow{G}) = (V, A, B')$ be the m-SPFCG of $\overrightarrow{G}$ and $\aleph_m(G) = (V, A, B'')$ be the m-SPFNG of G. The vertex set of $C_m(\overrightarrow{G})$ and $\aleph_m(G)$ are same. We have to prove that the edges sets of them are also same. Here, $h_\mu(\aleph_m^+(r) \cap \aleph_m^+(s)) = h_\mu(\aleph_m(r) \cap \aleph_m(s))$, $h_\eta(\aleph_m^+(r) \cap \aleph_m^+(s)) = h_\eta(\aleph_m(r) \cap \aleph_m(s))$ and $h_\nu(\aleph_m^+(r) \cap \aleph_m^+(s)) = h_\nu(\aleph_m(r) \cap \aleph_m(s))$, $\forall r, s \in V$. Therefore, $\mu_{B'}(r,s) = \mu_{B''}(r,s)$, $\eta_{B'}(r,s) = \eta_{B''}(r,s)$ and $\nu_{B'}(r,s) = \nu_{B''}(r,s)$. Thus the edges of $C_m(\overrightarrow{G})$ and $\aleph_m(G)$ are same. Hence, $C_m(\overrightarrow{G}) = \aleph_m(G)$. □

Now, the strength of the feed in a PFD is defined below.

Definition 3.12 Let $\overrightarrow{G}$ be a PFD. Let q be a common vertex of m-step out neighborhood of the vertices $r_1, r_2, \ldots, r_k$. Also, let $\mu_B(b_i, c_i), \eta_B(b_i, c_i)$and $\nu_B(b_i, c_i)$ are the respective minimum DTMS, minimum DAMS and maximum DFMS of edges of the paths $\overrightarrow{P}^m_{(r_i,q)}, i = 1, 2, \ldots, k$. The m-step feed $q \in V$ is independent strong if $\mu_B(b_i, c_i) > 0.5, \eta_B(b_i, c_i) < 0.5$ and $\nu_B(b_i, c_i) < 0.5, i = 1, 2, \ldots, k$.

The strength of the feed q is defined by $(S_1(q), S_2(q), S_3(q))$, where $S_1, S_2, S_3 : V \rightarrow$

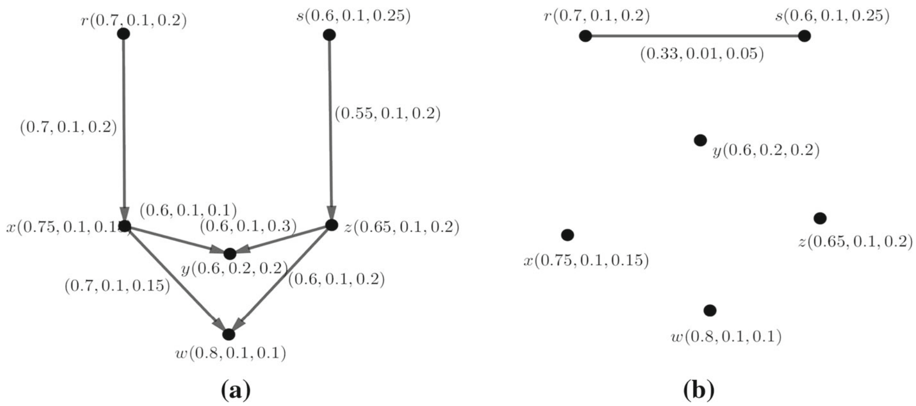


Fig. 4 Example of **a** PFD and **b** 2-SPFCG

[0, 1] are such that $S_1(q) = \frac{1}{k}\sum_1^k \overrightarrow{\mu}_B(b_i, c_i), S_2(q) = \frac{1}{k}\sum_1^k \overrightarrow{\eta}_B(b_i, c_i),\ S_3(q) = \frac{1}{k}\sum_1^k \overrightarrow{\nu}_B(b_i, c_i)$

Theorem 3.13 *If each feed of a PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ is independent strong, then each edge of $C_m(\overrightarrow{G})$ is independent strong.*

Proof Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD with strong feeds and $C_m(\overrightarrow{G}) = (V, A, B')$ be the corresponding m-SPFCG.

Case I: When $\aleph_m^+(r) \cap \aleph_m^+(s) = \emptyset$, then there is nothing to prove, as edges set of $C_m(\overrightarrow{G})$ is an empty set.

Case II: When $\aleph_m^+(r) \cap \aleph_m^+(s) \neq \emptyset$, then clearly $h_\mu(\aleph_m^+(r) \cap \aleph_m^+(s)) > 0.5$, $h_\eta(\aleph_m^+(r) \cap \aleph_m^+(s)) < 0.5$ and $h_\nu(\aleph_m^+(r) \cap \aleph_m^+(s)) < 0.5$ in $\overrightarrow{G}$ as each feed is independent strong.

Now, $\mu_{B'}(r, s) = [\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph_m^+(r) \cap \aleph_m^+(s))$ or, $\frac{\mu_{B'}(r,s)}{\mu_A(r) \wedge \mu_A(s)} > 0.5$.

Similarly, $\frac{\eta_{B'}(r,s)}{\eta_A(r) \wedge \eta_A(s)} < 0.5$ and $\frac{\nu_{B'}(r,s)}{\nu_A(r) \vee \nu_A(s)} < 0.5$.

Then (*r*, *s*) is an independent strong edge in $C_m(\overrightarrow{G})$. But, (*r*, *s*) is an arbitrary edge of $C_m(\overrightarrow{G})$. Thus each edge of $C_m(\overrightarrow{G})$ is independent strong. □

Example 3.14 Consider a PFD $\overrightarrow{G}$ (see Fig. 4a). Here, *y* and *w* both are independent strong 2-step feeds. We have, $\aleph_2^+(r) = \{(y, (0.6, 0.1, 0.2)), (w, (0.7, 0.1, 0.2))\}$ and $\aleph_2^+(s) = \{(y, (0.55, 0.1, 0.2)), (w, (0.55, 0.1, 0.2))\}$. Therefore, $\aleph_2^+(r) \cap \aleph_2^+(s) = \{(y, (0.55, 0.1, 0.2)), (w, (0.55, 0.1, 0.2))\} \neq \emptyset$. Then there exist an edge (*r*, *s*) in $C_2(\overrightarrow{G})$, which is independent strong with DTMS, DAMS and DFMS are respectively 0.33, 0.01 and 0.05, shown in Fig. 4b.

Theorem 3.15 *If a feed q of $\overrightarrow{G}$ is independent strong, then strength of q, $S_1(q) > 0.5, S_2(q) < 0.5$ and $S_3(q) < 0.5$.*

Proof Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD. Let q be a common vertex of PFmSON of the vertices $r_1, r_2, \ldots, r_k$. Also, let $\mu_B(b_i, c_i), \eta_B(b_i, c_i)$ and $\nu_B(b_i, c_i)$ are the respective minimum DTMS, minimum DAMS and maximum DFMS of edges of the paths $\overrightarrow{P}^m_{(r_i,q)}$, $i = 1, 2, \ldots, k$. If q is independent strong, then each edge (b_i, c_i), $i = 1, 2, \ldots, k$ is independent strong. So, $\mu_B(b_i, c_i) > 0.5$, $\eta_B(b_i, c_i) < 0.5$ and $\nu_B(b_i, c_i) < 0.5$, $i = 1, 2, \ldots, k$ and also, $S_1(q) > 0.5, S_2(q) < 0.5$ and $S_3(q) < 0.5$. □

Remark 3.16 The converse of the above theorem is not true. i.e., if $S_1(q) > 0.5, S_2(q) < 0.5$ and $S_3(q) < 0.5$, then feed q may not be independent strong. This can be explain as:

Let $S_1(q) > 0.5, S_2(q) < 0.5$ and $S_3(q) < 0.5$ for a feed q in $\overrightarrow{G}$. So, $\frac{1}{k}\sum_1^k \overrightarrow{\mu}_B(b_i, c_i) > 0.5$, $\frac{1}{k}\sum_1^k \overrightarrow{\eta}_B(b_i, c_i) < 0.5$ and $\frac{1}{k}\sum_1^k \overrightarrow{\nu}_B(b_i, c_i) < 0.5$. This result does not necessarily implies that $\overrightarrow{\mu}_B(b_i, c_i) > 0.5$, $\overrightarrow{\eta}_B(b_i, c_i) < 0.5$ and $\overrightarrow{\nu}_B(b_i, c_i) < 0.5\ \forall\ i = 1, 2, \ldots, k$ and hence, each edge of each directed PFP $\overrightarrow{P}^m_{(r_i,q)}$, $i = 1, 2, \ldots, k$ may not be independent strong. So, q may not be independent strong feed.

Example 3.17 In Fig.2(a)), strength of the feed w is $\big(S_1(w), S_2(w), S_3(w)\big) = (\frac{0.7+0.4}{2}, \frac{0.1+0.2}{2}, \frac{0.2+0.3}{2}) = (0.55, 0.15, 0.25)$. But w is not strong 2-step feed as edges of the PFP $\overrightarrow{P}^2_{(s,w)}$ are not independent strong.

Here, we establish a relation between m-SPFCG of a PFD and PFCG of m-SPFD.

Theorem 3.18 *If $\overrightarrow{G}$ be a PFD and $\overrightarrow{G}_m$ be its m -SPFD, then $C(\overrightarrow{G}_m) = C_m(\overrightarrow{G})$.*

Proof Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD and $\overrightarrow{G}_m = (V, A, B')$ is the m-SPFD of $\overrightarrow{G}$. Then vertex set of $C(\overrightarrow{G})$ and $C_m(\overrightarrow{G})$ are equal. we have to prove that the edges set of them are also equal.

Let (r, s) be an edge in $\overrightarrow{G}_m$. Then there exists an edges $(r, r_i), (s, r_i)$ in $\overrightarrow{G}_m, i = 1, 2, \ldots, k$.

In $\overrightarrow{G}_m$ we have $\aleph^+(r) \cap \aleph^+(s) = \{(r_i, \mu_i, \eta_i, \nu_i) | i = 1, 2, \ldots, k\}$, where $\mu_i = \overrightarrow{\mu}_{B'}(r, r_i) \wedge \overrightarrow{\mu}_{B'}(s, r_i), \eta_i = \overrightarrow{\eta}_{B'}(r, r_i) \wedge \overrightarrow{\eta}_{B'}(s, r_i), \nu_i = \overrightarrow{\nu}_{B'}(r, r_i) \vee \overrightarrow{\nu}_{B'}(s, r_i)$.

Let $P = max\{\mu_i | i = 1, 2, \ldots, k\}, Q = min\{\eta_i | i = 1, 2, \ldots, k\}$ and $R = min\{\nu_i | i = 1, 2, \ldots, k\}$. Then $\mu_{B'}(r, s) = [\mu_A(r) \wedge \mu_A(s)] h_\mu\big(\aleph^+(r) \cap \aleph^+(s)\big) = [\mu_A(r) \wedge \mu_A(s)] \times P$, $\eta_{B'}(r, s) = [\eta_A(r) \wedge \eta_A(s)] \times Q$ and $\nu_{B'}(r, s) = [\nu_A(r) \vee \nu_A(s)] \times R$. An edge (r, r_i) exists in $\overrightarrow{G}_m$ implies that there is a directed path from r to r_i of length m, $\overrightarrow{P}^m_{(r,r_i)}$ in $\overrightarrow{G}$ and $\overrightarrow{\mu}_{B'}(r, r_i) = min\{\overrightarrow{\mu}_B(x, y) : (x, y)$ is an edge in $\overrightarrow{P}^m_{(r,r_i)}\}$, $\overrightarrow{\eta}_{B'}(r, r_i) = min\{\overrightarrow{\eta}_B(x, y) : (x, y)$ is an edge in $\overrightarrow{P}^m_{(r,r_i)}\}$, $\overrightarrow{\nu}_{B'}(r, r_i) = max\{\overrightarrow{\nu}_B(x, y) : (x, y)$ is an edge in $\overrightarrow{P}^m_{(r,r_i)}\}$. Thus ($r$, s) is also an edge of $C_m(\overrightarrow{G})$.

Let $h_\mu(\aleph_m^+(r) \cap \aleph_m^+(s)) = P, h_\eta(\aleph_m^+(r) \cap \aleph_m^+(s)) = Q$ and $h_\mu(\aleph_m^+(r) \cap \aleph_m^+(s)) = R$ in $\overrightarrow{G}$. Therefore, $\mu_B(r,s) = [\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph_m^+(r) \cap \aleph_m^+(s)) = [\mu_A(r) \wedge \mu_A(s)] \times P$, $\eta_B(r,s) = [\eta_A(r) \wedge \eta_A(s)] \times Q$ and $\nu_B(r,s) = [\nu_A(r) \vee \nu_A(s)] \times R$. This shows that there is an edge in $C_m(\overrightarrow{G})$ for each edge in $C(\overrightarrow{G}_m)$. Similarly, for each edge in $C_m(\overrightarrow{G})$ there is an edge in $C(\overrightarrow{G}_m)$. Hence, $C(\overrightarrow{G}_m) = C_m(\overrightarrow{G})$. □

4 Picture fuzzy economic competition graphs (PFECG)

In this section, the definition of PFECG and *m*-step picture fuzzy economic competition graphs (*m*-SPFECG) are given and studied several properties.

Definition 4.1 The PFECG of a PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ is an undirected graph $E(\overrightarrow{G}) = (V, A, B)$ with V as vertex set and having an edge (r, s) in $E(\overrightarrow{G})$ iff $\aleph^-(r) \cap \aleph^-(s) \neq \emptyset$ in $\overrightarrow{G}$. The DTMS, DAMS and DFMS of (r, s) are respectively $\mu_B(r,s) = [\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph^-(r) \cap \aleph^-(s))$, $\eta_B(r,s) = [\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph^-(r) \cap \aleph^-(s))$ and $\nu_B(r,s) = [\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph^-(r) \cap \aleph^-(s))$.

Definition 4.2 The *m*-SPFECG $E_m(\overrightarrow{G})$ of a PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ is an undirected graph $E_m(\overrightarrow{G}) = (V, A, B^*)$ with V as vertex set and having an edge (r, s) in $E_m(\overrightarrow{G})$ iff $\aleph_m^-(r) \cap \aleph_m^-(s) \neq \emptyset$ in $\overrightarrow{G}$. The DTMS, DAMS and DFMS of (r, s) are respectively $\mu_{B^*}(r,s) = [\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph_m^-(r) \cap \aleph_m^-(s))$, $\eta_{B^*}(r,s) = [\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph_m^-(r) \cap \aleph_m^-(s))$ and $\nu_{B^*}(r,s) = [\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph_m^-(r) \cap \aleph_m^-(s))\ \forall\ r, s \in V$.

The following example illustrates PFECG and *m*-SPFECG.

Example 4.3 Consider a PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ (see in Fig. 5a). Here, $\aleph^-(s) \cap \aleph^-(u) = \{(t, (0.4, 0.1, 0.2))\}$ and $\aleph^-(t) \cap \aleph^-(u) = \{(v, (0.4, 0.1, 0.3))\}$. So, (s, u) and (t, u) are the edges of $E(\overrightarrow{G})$ with DTMS, DAMS and DFMS are (0.2, 0.01, 0.06) and (0.16, 0.01, 0.09), respectively shown in Fig. 5b. Also, $\aleph_2^-(r) \cap \aleph_2^-(s) = \{(v, (0.4, 0.1, 0.3))\}$, $\aleph_2^-(r) \cap \aleph_2^-(u) = \{(r, (0.4, 0.1, 0.3))\}$, $\aleph_2^-(r) \cap \aleph_2^-(v) = \{(t, (0.4, 0.1, 0.3))\}$, $\aleph_2^-(s) \cap \aleph_2^-(u) = \{(v, (0.4, 0.1, 0.2))\}$ and $\aleph_2^-(t) \cap \aleph_2^-(u) = \{(s, (0.4, 0.1, 0.3))\}$. So, (r, s), (r, u), (r, v), (s, u) and (t, u) are the edges of $E_2(\overrightarrow{G})$ with DTMS, DAMS and DFMS are respectively (0.2, 0.01, 0.03), (0.24, 0.01, 0.09), (0.2, 0.01, 0.09), (0.2, 0.01, 0.06) and (0.16, 0.01, 0.09) shown in Fig. 6 (Table 1).

Theorem 4.4 *The PFCGs and PFECGs of any complete PFD are same.*

Proof Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD and $C(\overrightarrow{G}) = (V, A, B)$ be the corresponding PFCG. Also, corresponding PFECG is $E_m(\overrightarrow{G}) = (V, A, B^*)$.

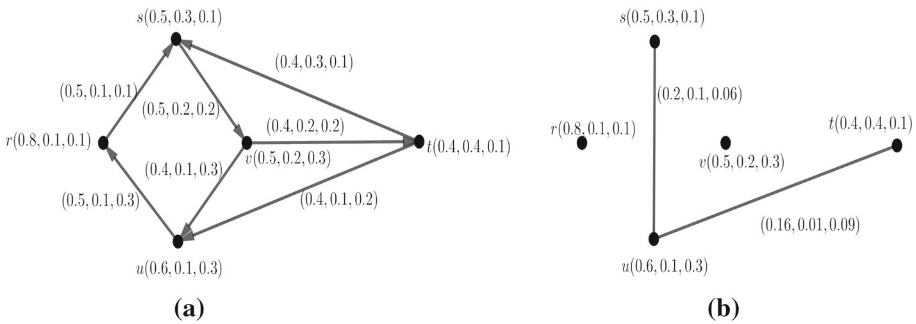


Fig. 5 Example of **a** PFD and **b** PFECG

Fig. 6 Example of m-SPFECG

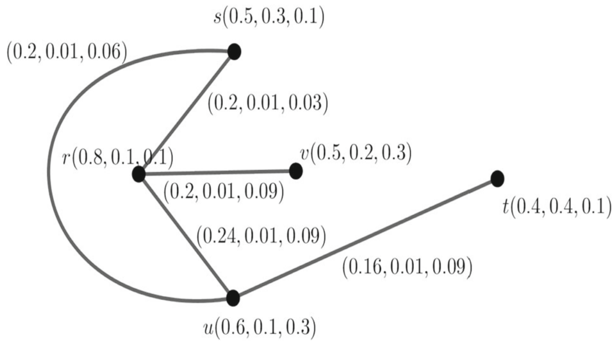


Table 1 PFON and 2-step PFON

$x \in V$	$\aleph^{-}(x)$	$\aleph_2^{-}(x)$
r	$\{(u,(0.5,0.1,0.3))\}$	$\{(v,(0.4,0.1,0.3)), (t,(0.4,0.1,0.3))\}$
s	$\{(r,(0.5,0.1,0.1)), (t,(0.4,0.3,0.1))\}$	$\{(u,(0.5,0.1,0.3)), (v,(0.4,0.2,0.2))\}$
t	$\{(v,(0.4,0.2,0.2))\}$	$\{(s,(0.4,0.2,0.2))\}$
u	$\{(t,(0.4,0.1,0.2)), (v,(0.4,0.1,0.3))\}$	$\{(t,(0.4,0.1,0.2)), (s,(0.4,0.1,0.3))\}$
v	$\{(s,(0.5,0.3,0.1))\}$	$\{(r,(0.5,0.1,0.2)), (t,(0.4,0.3,0.2))\}$

The vertex sets of $C(\overrightarrow{G})$ and $E_m(\overrightarrow{G})$ are same with the vertex set of $\overrightarrow{G}$. We have to prove that $\mu_B(r,s) = \mu_{B^*}(r,s)$, $\eta_B(r,s) = \eta_{B^*}(r,s)$, $\nu_B(r,s) = \nu_{B^*}(r,s)$, $\forall\ r,s \in V$. The DTMS, DAMS and DFMS of the edge (r, s) in $C(\overrightarrow{G})$ are respectively $\mu_B(r,s) = [\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph^+(r) \cap \aleph^+(s))$, $\eta_B(r,s) = [\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph^+(r) \cap \aleph^+(s))$ and $\nu_B(r,s) = [\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph^+(r) \cap \aleph^+(s))$. Also, the DTMS, DAMS and DFMS of the edge (r, s) in $E_m(\overrightarrow{G})$ are respectively $\mu_{B^*}(r,s) = [\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph_m^-(r) \cap \aleph_m^-(s))$, $\eta_{B^*}(r,s) = [\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph_m^-(r) \cap \aleph_m^-(s))$ and $\nu_{B^*}(r,s) = [\nu_A(r) \vee \nu_A(s)]\ \ h_\nu(\aleph_m^-(r) \cap$

$\aleph_m^-(s)\big)$. Since $\overrightarrow{G}$ is complete PFD, then $\aleph^+(r) \cap \aleph^+(s) = \aleph_m^-(r) \cap \aleph_m^-(s)$. Hence, PFCGs and PFECGs of any complete PFD are same. □

Theorem 4.5 *If $\overrightarrow{G_1}$ be the picture fuzzy sub-digraph of a PFD $\overrightarrow{G}$, then (i) $C_m(\overrightarrow{G_1}) \subset C_m(\overrightarrow{G})$ (ii) $E_m(\overrightarrow{G_1}) \subset E_m(\overrightarrow{G})$ (ii) $\aleph_m(\overrightarrow{G_1}) \subset \aleph_m(\overrightarrow{G})$.*

Proof Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ and $\overrightarrow{G_1} = (V_1, A_1, \overrightarrow{B_1})$, where $V_1 \subset V$ and $\mu_{A_1}(r) \leq \mu_A(r)$, $\eta_{A_1}(r) \leq \eta_A(r)$, $\nu_{A_1}(r) \geq \nu_A(r)\ \forall\ r \in V_1$. Also, $\overrightarrow{\mu}_{B_1}(r,s) \leq \overrightarrow{\mu}_B(r,s)$, $\overrightarrow{\eta}_{B_1}(r,s) \leq \overrightarrow{\eta}_B(r,s)$, $\overrightarrow{\nu}_{B_1}(r,s) \geq \overrightarrow{\nu}_B(r,s)\ \forall\ r,s \in V_1$. (i) Since, $V_1 \subset V$, the vertex set of $C_m(\overrightarrow{G_1})$ is a subset of $C_m(\overrightarrow{G})$. Also, for any edge (*r*, *s*) in $C_m(\overrightarrow{G_1})$, $\aleph_m^+(r) \cap \aleph_m^+(s)$ is picture fuzzy subset of the same in $C_m(\overrightarrow{G})$. Then $\overrightarrow{\mu}_{B_1}(r,s) \leq \overrightarrow{\mu}_B(r,s)$, $\overrightarrow{\eta}_{B_1}(r,s) \leq \overrightarrow{\eta}_B(r,s)$, $\overrightarrow{\nu}_{B_1}(r,s) \geq \overrightarrow{\nu}_B(r,s)$ $\forall r,s \in V_1$. This proves that $C_m(\overrightarrow{G_1}) \subset C_m(\overrightarrow{G})$. The proofs of (ii) and (iii) are similar to (i) □

5 Picture fuzzy competition hypergraphs (PFCHs)

Hypergraph theory is the most blooming tool for demonstrating several practical problems in different domains of science and technology. Moreover, crisp hypergraphs do not describe all the competitions of real world problems. Here, we introduce definitions and terminologies of picture fuzzy hypergraph (PFH) and PFCH.

Definition 5.1 The picture fuzzy hypergraph (PFH) is of the form $H = (V, A, B)$, V is the vertex set and B is the family of picture fuzzy hyperedges of H, where

(i) $V = \{a_1, a_2, \ldots, a_n\}$ is a finite set,
(ii) $A = \{\big(a_i, (\mu_A(a_i), \eta_A(a_i), \nu_A(a_i))\big) : i = 1, 2, \ldots, n\}$,
(iii) $B = \{E_1, E_2, \ldots, E_m\}$ is a family of picture fuzzy subsets of V,
(iv) $E_j = \{\big(a_i, (\mu_j(a_i), \eta_j(a_i), \nu_j(a_i))\big) : \mu_j(a_i), \eta_j(a_i), \nu_j(a_i) \geq 0$ and $0 \leq \mu_j(a_i) + \eta_j(a_i) + \nu_j(a_i) \leq 1\}$, $j = 1, 2, \ldots, m$,
(v) $E_j \neq \emptyset$, $j = 1, 2, \ldots, m$,
(vi) $\bigcup_{j=1}^m supp(E_j) = V$. The hyperedges E_j are PFSs of vertices, $\mu_j(a_i), \eta_j(a_i)$ and $\nu_j(a_i)$ are respectively the DTMS, DAMS and DFMS of a_i corresponding to E_j.

We illustrate it by giving an example.

Example 5.2 Consider a PFH $G = (V, A, B)$ such that $V = \{a_1, a_2, a_3\}$ and $B = \{E_1, E_2, E_3\}$ as shown in Fig. 7. Here, $E_1 = \{\big(a_1, (0.5, 0.3, 0.15)\big), \big(a_2, (0.6, 0.35, 0.1)\big)\}$, $E_2 = \{\big(a_2, (0.6, 0.25, 0.1)\big), \big(a_3, (0.3, 0.2, 0.35)\big)\}$ and $E_3 = \{\big(a_1, (0.5, 0.3, 0.15)\big), \big(a_3, (0.3, 0.2, 0.35)\big)\}$.

Now, we define picture fuzzy competition hypergraph (PFCH) and picture fuzzy double competition hypergraph (PFDCH) as follows:

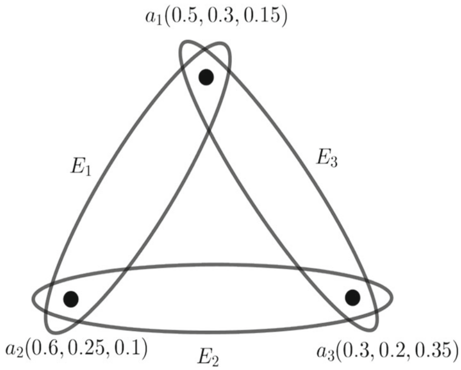


Fig. 7 Example of PFH

Definition 5.3 The PFCH $C_H(\overrightarrow{G}) = (V, A, B_c)$ of a PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ is an undirected graph with V as vertex set and has a hyperedge E consisting of vertices $a_1, a_2, \ldots, a_r$ if $\aleph^+(a_1) \cap \aleph^+(a_2) \cap \cdots \cap \aleph^+(a_r) \neq \emptyset$. The DTMS, DAMS and DFMS of the hyperedge $E = \{a_1, a_2, \ldots, a_r\}$ are define as $\mu_{B_c}(E) = [\mu_A(a_1) \wedge \mu_A(a_2) \wedge \cdots \wedge \mu_A(a_r)] h_\mu\big(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \cdots \cap \aleph^+(a_r)\big)$, $\eta_{B_c}(E) = [\eta_A(a_1) \wedge \eta_A(a_2) \wedge \cdots \wedge \eta_A(a_r)] h_\eta\big(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \cdots \cap \aleph^+(a_r)\big)$ and $\nu_{B_c}(E) = [\nu_A(a_1) \vee \nu_A(a_2) \vee \cdots \vee \nu_A(a_r)] h_\nu\big(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \cdots \cap \aleph^+(a_r)\big)$, respectively.

Definition 5.4 Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD. The PFDCH $C_{DH} = (V, A, B_d)$ is an undirected graph having same vertex set as in $\overrightarrow{G}$ and there is a hyperedge E consisting of vertices $a_1, a_2, \ldots, a_r$ if $\aleph^+(a_1) \cap \aleph^+(a_2) \cap \cdots \cap \aleph^+(a_r) \neq \emptyset$ and $\aleph^-(a_1) \cap \aleph^-(a_2) \cap \cdots \cap \aleph^-(a_r) \neq \emptyset$. The DTMS, DAMS and DFMS of the hyperedge $E = \{a_1, a_2, \ldots, a_r\}$ are defined as $\mu_{B_d}(E) = [\mu_A(a_1) \wedge \mu_A(a_2) \wedge \cdots \wedge \mu_A(a_r)] h_\mu\big(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \cdots \cap \aleph^+(a_r)) \vee h_\mu(\aleph^-(a_1) \cap \aleph^-(a_2) \cap \cdots \cap \aleph^-(a_r)\big)$, $\eta_{B_d}(E) = [\eta_A(a_1) \wedge \eta_A(a_2) \wedge \cdots \wedge \eta_A(a_r)] h_\eta\big(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \cdots \cap \aleph^+(a_r)) \vee h_\eta(\aleph^-(a_1) \cap \aleph^-(a_2) \cap \cdots \cap \aleph^-(a_r)\big)$ and $\nu_{B_d}(E) = [\nu_A(a_1) \vee \nu_A(a_2) \vee \cdots \vee \nu_A(a_r)] h_\mu\big(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \cdots \cap N^+(a_r)) \wedge h_\nu(\aleph^-(a_1) \cap \aleph^-(a_2) \cap \cdots \cap \aleph^-(a_r)\big)$, respectively.

The following example illustrates PFCH and PFDCH.

Example 5.5 Consider a PFD $\overrightarrow{G}$ shown in Fig. 8a. Here, $\aleph^+(a_1) \cap \aleph^+(a_6) = \{(a_2, (0.3, 0.2, 0.3))\}$, $\aleph^+(a_2) \cap \aleph^+(a_5) = \{(a_3, (0.4, 0.2, 0.3))\}$ and $\aleph^+(a_3) \cap \aleph^+(a_5) = \{(a_4, (0.4, 0.2, 0.1))\}$. Therefore, hyperedges of the PFCH are $E_2 = \{a_1, a_6\}$, $E_3 = \{a_2, a_5\}$ and $E_4 = \{a_3, a_5\}$. The DTMS, DAMS and DFMS of E_2, E_3 and E_4 are respectively (0.09, 0.06, 0.12), (0.16, 0.04, 0.09) and (0.16, 0.04, 0.02) (see Fig. 8b; Table 2).

Again, $\aleph^+(a_2) \cap \aleph^+(a_5) = \{(a_3, (0.4, 0.2, 0.3))\} \neq \emptyset$ and $\aleph^-(a_2) \cap \aleph^-(a_5) = \{(a_6, (0.3, 0.2, 0.3))\} \neq \emptyset$. So, there is only one hyperedge $E = \{a_2, a_5\}$ in PFDCH with DTMS, DAMS and DFMS are (0.16, 0.04, 0.09) shown in Fig. 8(c).

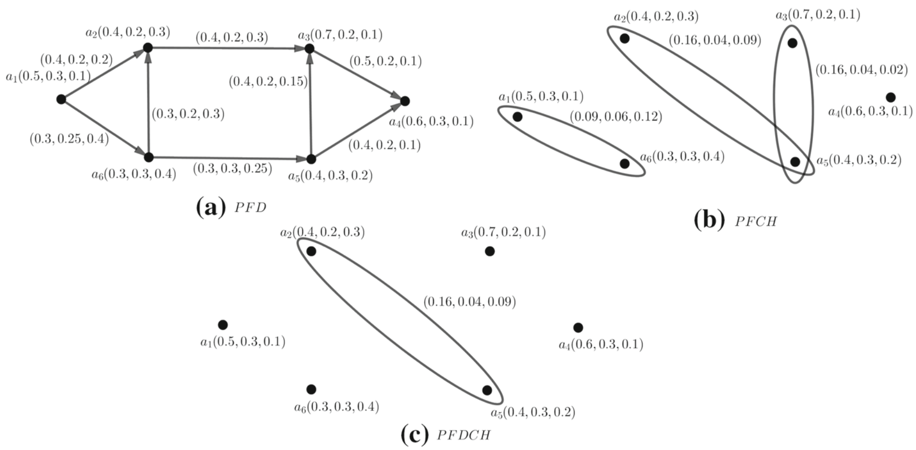


Fig. 8 Example of PFD and corresponding competition hypergraphs

Table 2 PFON and PFIN of the vertices

$a \in V$	$\aleph^+(a)$	$\aleph^-(a)$
a_1	$\{(a_2,(0.4,0.2,0.2)),(a_6,(0.3,0.25,0.4))\}$	$\emptyset$
a_2	$\{(a_3,(0.4,0.2,0.3))\}$	$\{(a_1,(0.4,0.2,0.2)),(a_5,(0.4,0.2,0.3)),(a_6,(0.3,0.2,0.3))\}$
a_3	$\{(a_4,(0.5,0.2,0.1))\}$	$\{(a_5,(0.4,0.2,0.15))\}$
a_4	$\emptyset$	$\{(a_3,(0.5,0.2,0.1)),(a_5,(0.4,0.2,0.1))\}$
a_5	$\{(a_3,(0.4,0.2,0.15)),(a_4,(0.4,0.2,0.1))\}$	$\{(a_6,(0.3,0.3,0.25))\}$
a_6	$\{(a_2,(0.3,0.2,0.3)),(a_5,(0.3,0.3,0.25))\}$	$\{(a_1,(0.3,0.25,0.4))\}$

Definition 5.6 Let $G=(V,A,B)$ be a PFH. A hyperedge $E_i=\{a_1,a_2,\ldots,a_r\}\subseteq V$ is independent strong if $\frac{1}{2}min\{\mu_A(a_1),\mu_A(a_2),\ldots,\mu_A(a_r)\}<\mu_B(E_i)$, $\frac{1}{2}min\{\eta_A(a_1),\eta_A(a_2),\ldots,\eta_A(a_r)\}>\eta_B(E_i)$ and $\frac{1}{2}max\{\nu_A(a_1),\nu_A(a_2),\ldots,\nu_A(a_r)\}>\nu_B(E_i)$. Otherwise, it is called weak edge.

Theorem 5.7 *Let $\overrightarrow{G}=(V,A,\overrightarrow{B})$ be a PFD. If $\aleph^+(a_1)\cap\aleph^+(a_2)\cap\ldots\cap\aleph^+(a_r)$ contains only one vertex of $\overrightarrow{G}$. Then the hyperedge $\{a_1,a_2,\ldots,a_r\}$ of $C_H(\overrightarrow{G})$ is independent strong iff $|\aleph^+(a_1)\cap\aleph^+(a_2)\cap\ldots\cap\aleph^+(a_r)|_\mu>0.5$, $|\aleph^+(a_1)\cap\aleph^+(a_2)\cap\ldots\cap\aleph^+(a_r)|_\eta<0.5$, and $|\aleph^+(a_1)\cap\aleph^+(a_2)\cap\ldots\cap\aleph^+(a_r)|_\nu<0.5$.*

Proof Let $\overrightarrow{G}$ be a PFD. If $\aleph^+(a_1)\cap\aleph^+(a_2)\cap\ldots\cap\aleph^+(a_r)=\{(x,(\theta,\phi,\psi))\}$, where θ,ϕ and ψ are the DTMS, DAMS and DFMS of either edge (a_1,x) or (a_2,x) or (a_r,x). Here,

$$|\aleph^+(a_1)\cap\aleph^+(a_2)\cap\ldots\cap\aleph^+(a_r)|_\mu=\theta=h_\mu(\aleph^+(a_1)\cap\aleph^+(a_2)\cap\ldots\cap\aleph^+(a_r)),$$
$$|\aleph^+(a_1)\cap\aleph^+(a_2)\cap\ldots\cap\aleph^+(a_r)|_\eta=\phi=h_\eta(\aleph^+(a_1)\cap\aleph^+(a_2)\cap\ldots\cap\aleph^+(a_r)),$$
$$|\aleph^+(a_1)\cap\aleph^+(a_2)\cap\ldots\cap\aleph^+(a_r)|_\nu=\psi=h_\nu(\aleph^+(a_1)\cap\aleph^+(a_2)\cap\ldots\cap\aleph^+(a_r)).$$

So that, $\mu_B(\{a_1,a_2,\ldots,a_r\})=[\mu_A(a_1)\wedge\mu_A(a_2)\wedge\cdots\wedge\mu_A(a_r)]\times\theta,\eta_B(\{a_1,a_2,\ldots,$

$a_r\}) = [\eta_A(a_1) \wedge \eta_A(a_2) \wedge \cdots \wedge \eta_A(a_r)] \times \phi, \nu_B(\{a_1, a_2, \ldots, a_r\}) = [\nu_A(a_1) \vee \nu_A(a_2) \vee \cdots \vee \nu_A(a_r)] \times \psi$.

Therefore, the hyperedge $\{a_1, a_2, \ldots, a_r\}$ in $C_H(\overrightarrow{G})$ is independent strong iff $\theta > 0.5, \phi < 0.5$ and $\psi < 0.5$, i.e., iff $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\mu > 0.5$, $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\eta < 0.5$ and $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\nu < 0.5$. □

Next, we define one of the extension of PFCH known as picture fuzzy k- competition hypergraph (PFkCH), k is a non-negative real number.

Definition 5.8 Let $k \geq 0$ be a real number. The PFkCH $C_{H_k}(\overrightarrow{G})$ of a PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ is a PFH $C_{H_k}(\overrightarrow{G}) = (V, A, B_k)$ whose vertex set is same as in $\overrightarrow{G}$ and there is a hyperedge $E = \{a_1, a_2, \ldots, a_r\}$ in $C_{H_k}(\overrightarrow{G})$ if $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\mu > k$, $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\eta > k$ and $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\nu > k$. Then the DTMS, DAMS and DFMS of E are respectively $\mu_{B_k}(E) = \frac{k_1 - k}{k_1}[\mu_A(a_1) \wedge \mu_A(a_2) \wedge \cdots \wedge \mu_A(a_r)] h_\mu(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)), \eta_B(E) = \frac{k_2 - k}{k_2}[\eta_A(a_1) \wedge \eta_A(a_2) \wedge \cdots \wedge \eta_A(a_r)] h_\eta(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r))$ and $\nu_B(E) = \frac{k_3 - k}{k_3}[\nu_A(a_1) \vee \nu_A(a_2) \vee \cdots \vee \nu_A(a_r)] h_\nu (\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r))$,

where $k_1 = |\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\mu$, $k_2 = |\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\eta$ and $k_3 = |\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\nu$. A PF$k$CH is simply a PFCH if $k = 0$.

Here, we provide the following example of a PF.1CH

Example 5.9 In Example 5.5, for the hyperedge E_2; $k_1 = 0.3, k_2 = 0.2, k_3 = 0.2$. For E_3; $k_1 = 0.4, k_2 = 0.2$, $k_3 = 0.15$. For E_4; $k_1 = 0.4, k_2 = 0.2, k_3 = 0.1$. If we choose $k = 0.1$, there exists hyperedges E_2 and E_3 in $C_{H_{0.1}}(\overrightarrow{G})$ with DTMS, DAMS and DFMS are (0.06, 0.03, 0.08) and (0.12, 0.02, 0.06), respectively as shown in Fig. 9.

Theorem 5.10 *Let $\overrightarrow{G}$ be a PFD. If $h_\mu(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)) = 1, h_\eta(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)) = 1, h_\nu(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)) = 1$ and if $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\mu > 2k, |\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\eta < 2k, |\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\nu < 2k$, then the hyperedge $E = \{a_1, a_2, \ldots, a_r\}$ is independent strong in $C_{H_k}(\overrightarrow{G})$.*

Proof Let $C_{H_k}(\overrightarrow{G}) = (V, A, B_k)$ be a PFkCH of a PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$. If $h_\mu(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)) = 1$ and $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\mu > 2k$, then $k_1 > 2k$.

Therefore, $\mu_{B_k}(E) = \frac{k_1 - k}{k_1}[\mu_A(a_1) \wedge \mu_A(a_2) \wedge \cdots \wedge \mu_A(a_r)] h_\mu(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r))$

or, $\frac{\mu_{B_k}(E)}{\bigwedge_1^r \mu_A(a_i)} = \frac{k_1 - k}{k_1} > 0.5$.

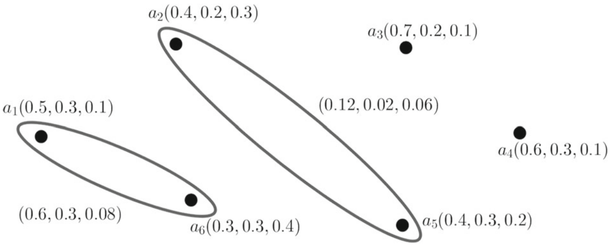


Fig. 9 Example of PF.1CH

Similarly, $\frac{\eta_{B_k}(E)}{\bigwedge_1^r \eta_A(a_i)} = \frac{k_1-k}{k_1} < 0.5$ and $\frac{\nu_{B_k}(E)}{\bigvee_1^r \nu_A(a_i)} = \frac{k_1-k}{k_1} < 0.5$.

Then E is independent strong in $C_{H_k}(\overrightarrow{G})$. □

6 Picture fuzzy neighborhood hypergraphs (PFNHs)

To design different types of PFCHs from a PFG the concepts of picture fuzzy open neighborhood (PFON) and picture fuzzy closed neighborhood (PFCN) are given below. The picture fuzzy neighborhoods (PFNs) of any species with their relations are describe in the PFNHs.

Definition 6.1 The PFON of a vertex r of a PFG $G = (V, A, B)$ is $\aleph(r) = (X_r, (\mu_r, \eta_r, \nu_r))$, where $X_r = \{s : \mu_B(r,s) > 0, \eta_B(r,s) > 0$ and $\nu_B(r,s) > 0\}$, $\mu_r, \eta_r, \nu_r : X_r \to [0,1]$ are defined as $\mu_r(s) = \mu_B(r,s), \eta_r(s) = \eta_B(r,s)$ and $\nu_r(s) = \nu_B(r,s)$. The PFCN of a vertex r is $\aleph[r] = \aleph(r) \cup \{(r, (\mu(r), \eta(r), \nu(r)))\}$.

Now, we define picture fuzzy open neighborhood hypergraphs (PFONH) and picture fuzzy closed neighborhood graphs (PFCNH).

Definition 6.2 Let $G = (V, A, B)$ be a PFG. The PFONH of G is $N(G) = (V, A, B')$, with V as vertex set and there is a hyperedge $E = \{a_1, a_2, \ldots, a_r\}$ in $N(G)$ if $\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r) \neq \emptyset$ in G. The DTMS, DAMS and DFMS of the hyperedge E are respectively $\mu_{B'}(E) = [\mu_A(a_1) \wedge \mu_A(a_2) \wedge \cdots \wedge \mu_A(a_r)] h_\mu(\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r))$, $\eta_{B'}(E) = [\eta_A(a_1) \wedge \eta_A(a_2) \wedge \cdots \wedge \eta_A(a_r)] h_\eta(\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r))$ and $\nu_{B'}(E) = [\nu_A(a_1) \vee \nu_A(a_2) \vee \cdots \vee \nu_A(a_r)] h_\nu(\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r))$.

Definition 6.3 Let $G = (V, A, B)$ be a PFG. The PFCNH of G is $N[G] = (V, A, B'')$, with V as vertex set and there is a hyperedge $E = \{a_1, a_2, \ldots, a_r\}$ in $\aleph[G]$ if $\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r] \neq \emptyset$ in G. The DTMS, DAMS and DFMS of the hyperedge E in $\aleph[G]$ are respectively $\mu_{B''}(E) = [\mu_A(a_1) \wedge \mu_A(a_2) \wedge \cdots \wedge \mu_A(a_r)] h_\mu(\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r])$, $\eta_{B''}(E) = [\eta_A(a_1) \wedge \eta_A(a_2) \wedge \cdots \wedge \eta_A(a_r)] h_\eta(\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r])$ and $\nu_{B''}(E) = [\nu_A(a_1) \vee \nu_A(a_2) \vee \cdots \vee \nu_A(a_r)] h_\nu(\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r])$.

To illustrate the preceding definitions, we provide the following example.

Example 6.4 Consider a PFG G given in Fig. 10a. Here, $\aleph(a_1) \cap \aleph(a_2) \cap \aleph(a_4) = \{(a_3, (0.3, 0.1, 0.3))\}$, $\aleph(a_2) \cap \aleph(a_3) = \{(a_4, (0.3, 0.1, 0.2))\}$ and $\aleph(a_3) \cap \aleph(a_4) = \{(a_2, (0.3, 0.1, 0.3))\}$. Therefore, $E_2 = \{a_3, a_4\}$, $E_3 = \{a_1, a_2, a_4\}$, $E_4 = \{a_2, a_3\}$ are hyperedges of $N(G)$ with DTMS, DAMS and DFMS are (0.12, 0.01, 0.06), (0.09, 0.02, 0.12) and (0.09, 0.01, 0.08), respectively shown in Fig. 10b. Also, $\aleph[a_1] \cap \aleph[a_2] \cap \aleph[a_3] \cap \aleph[a_4] = \{(a_3, (0.3, 0.1, 0.3))\}$. Therefore, $E = \{a_1, a_2, a_3, a_4\}$ is a hyperedge of $N[G]$ with DTMS, DAMS and DFMS (0.09, 0.01, 0.12) shown in Fig. 10c (Table 3).

Some other types of PFHs such as open picture fuzzy k-competition hypergraph (OPFkCH) and closed picture fuzzy k-competition hypergraph (CPFkCH) are defined here, using different types of PFN of the vertices.

Definition 6.5 Let $k \geq 0$ be a real number. The OPFkCH of a PFG G is $\aleph_{H_k}(G) = (V, A, B'_k)$, with V as vertex set and there is a hyperedge $E = \{a_1, a_2, \ldots, a_r\}$ in $N_{H_k}(G)$ if $|\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)|_\mu > k$, $|\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)|_\eta > k$ and $|\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)|_\nu > k$ in G.

Then the DTMS, DAMS and DFMS of the hyperedge E are respectively

$\mu_{B'_k}(E) = \frac{k'_1 - k}{k'_1}[\mu_A(a_1) \wedge \mu_A(a_2) \wedge \cdots \wedge \mu_A(a_r)] h_\mu(N(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r))$,

$\eta_{B'_k}(E) = \frac{k'_2 - k}{k'_2}[\eta_A(a_1) \wedge \eta_A(a_2) \wedge \cdots \wedge \eta_A(a_r)] h_\eta(\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r))$ and

$\nu_{B'_k}(E) = \frac{k'_3 - k}{k'_3}[\nu_A(a_1) \vee \nu_A(a_2) \vee \cdots \vee \nu_A(a_r)] h_\nu(\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r))$,

where $k'_1 = |\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)|_\mu$, $k'_2 = |\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)|_\eta$ and $k'_3 = |\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)|_\nu$.

Definition 6.6 Let $k \geq 0$ be a real number. The CPFkCH of a PFG G is $\aleph_{H_k}[G] = (V, A, B''_k)$, with V as vertex set and there is a hyperedge $E = \{a_1, a_2, \ldots, a_r\}$ in $\aleph_{H_k}[G]$ if $|\aleph[a_1] \cap \aleph[a_2])] \cap \ldots \cap \aleph[a_r]|_\mu > k$, $|\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r]|_\eta > k$ and $|\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r]|_\nu > k$ in G.

Then the DTMS, DAMS and DFMS of the hyperedge E are respectively

$\mu_{B''_k}(E) = \frac{k''_1 - k}{k''_1}[\mu_A(a_1) \wedge \mu_A(a_2) \wedge \cdots \wedge \mu_A(a_r)] h_\mu(\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r])$,

$\eta_{B''_k}(E) = \frac{k''_2 - k}{k''_2}[\eta_A(a_1) \wedge \eta_A(a_2) \wedge \cdots \wedge \eta_A(a_r)] h_\eta(\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r])$ and

$\nu_{B''_k}(E) = \frac{k''_3 - k}{k''_3}[\nu_A(a_1) \vee \nu_A(a_2) \vee \cdots \vee \nu_A(a_r)] h_\nu(\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r])$, where $k''_1 = |\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r]|_\mu$, $k''_2 = |\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r]|_\eta$ and $k''_3 = |\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r]|_\nu$.

Next, we established the relations between PFCHs and PFNHs in the following theorems.

Theorem 6.7 *Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a symmetric PFD without any loop. Then $C_{H_k}(\overrightarrow{G}) = N_{H_k}(U(\overrightarrow{G}))$, where $U(\overrightarrow{G})$ is the underlying PFG of $\overrightarrow{G}$.*

Proof Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD and the corresponding underlying PFG $U(G) = (V, A, B)$.

Let $C_{H_k}(\overrightarrow{G}) = (V, A, B')$ and $\aleph_{H_k}(U(G)) = (V, A, B'')$. The vertex sets of $C_{H_k}(\overrightarrow{G})$ and

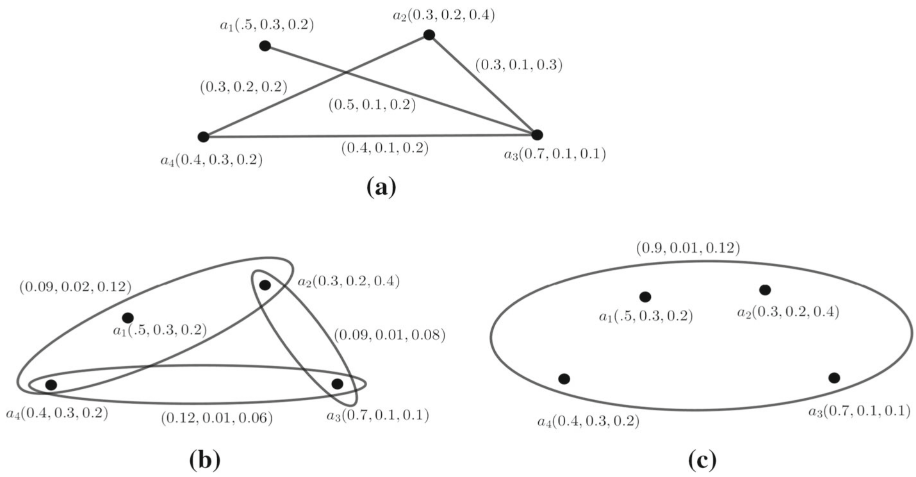


Fig. 10 Example of **a** PFG, **b** PFONH and **c** PFCNH

$\aleph_{H_k}(U(G))$ are same with the vertex set of $\overrightarrow{G}$. We have to prove that $\mu_{B'}(\{a_1, a_2, \ldots, a_r\}) = \mu_{B''}(\{a_1, a_2, \ldots, a_r\})$, $\eta_{B'}(\{a_1, a_2, \ldots, a_r\}) = \eta_{B''}(\{a_1, a_2, \ldots, a_r\})$, $\nu_{B'}(\{a_1, a_2, \ldots, a_r\}) = \nu_{B''}(\{a_1, a_2, \ldots, a_r\})$, $\forall\ a_1, a_2, \ldots, a_r \in V$.

Case I: If $\mu_{B'}(\{a_1, a_2, \ldots, a_r\}) = 0$, $\eta_{B'}(\{a_1, a_2, \ldots, a_r\}) = 0$, $\nu_{B'}(\{a_1, a_2, \ldots, a_r\}) = 0$ in $C_{H_k}(\overrightarrow{G})$, then there is no hyperedge in $C_{H_k}(\overrightarrow{G})$. So, $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\mu \leq k$, $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\eta \leq k$, $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\nu \leq k$.

Since $\overrightarrow{G}$ is symmetric, $|\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)|_\mu \leq k$, $|\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)|_\eta \leq k$, $|\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)|_\nu \leq k$ in $U(G)$. Hence, $\mu_{B''}(\{a_1, a_2, \ldots, a_r\}) = 0$, $\eta_{B''}(\{a_1, a_2, \ldots, a_r\}) = 0$, $\nu_{B''}(\{a_1, a_2, \ldots, a_r\}) = 0$ in $N_{H_k}(U(\overrightarrow{G}))$.

Case II: If $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\mu > k$, $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\eta > k$, $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\nu > k$. Then $\mu_{B'}(\{a_1, a_2, \ldots, a_r\}) > 0$, $\eta_{B'}(\{a_1, a_2, \ldots, a_r\}) > 0$, $\nu_{B'}(\{a_1, a_2, \ldots, a_r\}) > 0$ in $C_{H_k}(\overrightarrow{G})$ and there is an hyperedge $E = \{a_1, a_2, \ldots, a_r\}$ in $C_{H_k}(\overrightarrow{G})$ with DTMS, DAMS and DFMS are respectively

$\mu_{B'}(E) = \frac{k'-k}{k'}[\mu_A(a_1) \wedge \mu_A(a_2) \wedge \cdots \wedge \mu_A(a_r)]h_\mu\big(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)\big)$,
$\eta_{B'}(E) = \frac{k'-k}{k'}[\eta_A(a_1) \wedge \eta_A(a_2) \wedge \cdots \wedge \eta_A(a_r)]h_\eta\big(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)\big)$ and
$\nu_{B'}(E) = \frac{k'-k}{k'}[\nu_A(a_1) \vee \nu_A(a_2) \vee \cdots \vee \nu_A(a_r)]h_\nu\big(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)\big)$,
where $k' = |\big(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)\big)|$.

Since $\overrightarrow{G}$ is symmetric $|\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)|_\mu > k$, $|\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)|_\eta > k$, $|\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)|_\nu > k$ in $U(G)$. So,

$\mu_{B''}(E) = \frac{k''-k}{k''}[\mu_A(a_1) \wedge \mu_A(a_2) \wedge \cdots \wedge \mu_A(a_r)]h_\mu\big(\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)\big)$,
$\eta_{B''}(E) = \frac{k''-k}{k''}[\eta_A(a_1) \wedge \eta_A(a_2) \wedge \cdots \wedge \eta_A(a_r)]h_\eta\big(\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)\big)$ and
$\nu_{B''}(E) = \frac{k''-k}{k''}[\nu_A(a_1) \vee \nu_A(a_2) \vee \cdots \vee \nu_A(a_r)]h_\nu\big(\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)\big)$,
where $k'' = |\big(\aleph(a_1) \cap \aleph(a_2) \cap \ldots \cap \aleph(a_r)\big)|$. Here, $k' = k''$ as $\overrightarrow{G}$ is symmetric PFD.

Table 3 PFON and PFIN of the vertices

$x \in V$	$\aleph(x)$	$\aleph[x]$
a_1	$\{(a_3,(0.5,0.1,0.2))\}$	$\{(a_1,(0.5,0.3,0.2)),(a_3,(0.5,0.1,0.2))\}$
a_2	$\{(a_3,(0.3,0.1,0.3)),(a_4,(0.3,0.2,0.2))\}$	$\{(a_2,(0.3,0.2,0.4)),(a_3,(0.3,0.1,0.3)),(a_4,(0.3,0.2,0.2))\}$
a_3	$\{(a_1,(0.5,0.1,0.2)),(a_2,(0.3,0.1,0.3)),$ $(a_4,(0.4,0.1,0.2))\}$	$\{(a_1,(0.5,0.1,0.2)),(a_2,(0.3,0.1,0.3)),(a_3,(0.7,0.1,0.1)),$ $(a_4,(0.4,0.1,0.2))\}$
a_4	$\{(a_2,(0.3,0.2,0.2)),(a_3,(0.4,0.1,0.2))\}$	$\{(a_2,(0.3,0.2,0.2)),(a_3,(0.4,0.1,0.2)),(a_4,(0.4,0.3,0.2))\}$

Thus, $\mu_{B'}(\{a_1,a_2,\ldots,a_r\}) = \mu_{B''}(\{a_1,a_2,\ldots,a_r\})$, $\eta_{B'}(\{a_1,a_2,\ldots,a_r\}) = \eta_{B''}(\{a_1,a_2,\ldots,a_r\})$ and $\nu_{B'}(\{a_1,a_2,\ldots,a_r\}) = \nu_{B''}(\{a_1,a_2,\ldots,a_r\})$ $\forall$ $a_1,a_2,\ldots,a_r \in V$. □

Theorem 6.8 *Let $\overrightarrow{G} = (V,A,\overrightarrow{B})$ be a PFD having loops at every vertex. Then $C_{H_k}(\overrightarrow{G}) = \aleph_{H_k}[U(\overrightarrow{G})]$, where $U(\overrightarrow{G})$ is the loop less underlying PFG of $\overrightarrow{G}$.*

Proof Let $U(G) = (V,A,B)$ be an underlying loop less PFG corresponding to a PFD $\overrightarrow{G} = (V,A,\overrightarrow{B})$. Let $C_{H_k}(\overrightarrow{G}) = (V,A,B')$ and $\aleph_{H_k}[U(G)] = (V,A,B'')$. The vertex sets of $C_{H_k}(\overrightarrow{G})$ and $\aleph_{H_k}[U(G)]$ are same with the vertex set of $\overrightarrow{G}$. We have to prove that $\mu_{B'}(\{a_1,a_2,\ldots,a_r\}) = \mu_{B''}(\{a_1,a_2,\ldots,a_r\})$, $\eta_{B'}(\{a_1,a_2,\ldots,a_r\}) = \eta_{B''}(\{a_1,a_2,\ldots,a_r\})$, $\nu_{B'}(\{a_1,a_2,\ldots,a_r\}) = \nu_{B''}(\{a_1,a_2,\ldots,a_r\})$, $\forall$ $a_1,a_2,\ldots,a_r \in V$. Since $\overrightarrow{G}$ has a loop at each vertex, the PFON of every vertex contains the vertex itself.

Case I: If $\mu_{B'}(\{a_1,a_2,\ldots,a_r\}) = 0$, $\eta_{B'}(\{a_1,a_2,\ldots,a_r\}) = 0$, $\nu_{B'}(\{a_1,a_2,\ldots,a_r\}) = 0$ in $C_{H_k}(\overrightarrow{G})$, then there is no hyperedge in $C_{H_k}(\overrightarrow{G})$. So, $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\mu \le k$, $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\eta \le k$, $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\nu \le k$.

Since $\overrightarrow{G}$ is symmetric, $|\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r]|_\mu \le k$, $|\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r]|_\eta \le k$, $|\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r]|_\nu \le k$ in $U(G)$. Hence, $\mu_{B''}(\{a_1,a_2,\ldots,a_r\}) = 0$, $\eta_{B''}(\{a_1,a_2,\ldots,a_r\}) = 0$, $\nu_{B''}(\{a_1,a_2,\ldots,a_r\}) = 0$ in $N_{H_k}[U(\overrightarrow{G})]$.

Case II: If $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\mu > k$, $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\eta > k$, $|\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r)|_\nu > k$ in $U(G)$. Then $\mu_{B'}(\{a_1,a_2,\ldots,a_r\}) > 0$, $\eta_{B'}(\{a_1,a_2,\ldots,a_r\}) > 0$, $\nu_{B'}(\{a_1,a_2,\ldots,a_r\}) > 0$ in $C_{H_k}(\overrightarrow{G})$ and there is an hyperedge $E = \{a_1,a_2,\ldots,a_r\}$ in $C_{H_k}(\overrightarrow{G})$ with DTMS, DAMS and DFMS are respectively

$\mu_{B'}(E) = \frac{k'-k}{k'}[\mu_A(a_1) \wedge \mu_A(a_2) \wedge \cdots \wedge \mu_A(a_r)]h_\mu(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r))$,

$\eta_{B'}(E) = \frac{k'-k}{k'}[\eta_A(a_1) \wedge \eta_A(a_2) \wedge \cdots \wedge \eta_A(a_r)]h_\eta(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r))$ and

$\nu_{B'}(E) = \frac{k'-k}{k'}[\nu_A(a_1) \vee \nu_A(a_2) \vee \cdots \vee \nu_A(a_r)]h_\nu(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r))$,

where $k' = |(\aleph^+(a_1) \cap \aleph^+(a_2) \cap \ldots \cap \aleph^+(a_r))|$.

Since $\overrightarrow{G}$ is symmetric and has a loop at every vertex, then $|\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r]|_\mu > k$, $|\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r]|_\eta > k$, $|\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r]|_\nu > k$ in $U(G)$. So,

$\mu_{B''}(E) = \frac{k''-k}{k''}[\mu_A(a_1) \wedge \mu_A(a_2) \wedge \cdots \wedge \mu_A(a_r)]h_\mu(\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r])$,

$\eta_{B''}(E) = \frac{k''-k}{k''}[\eta_A(a_1) \wedge \eta_A(a_2) \wedge \cdots \wedge \eta_A(a_r)]h_\eta(\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r])$ and

$\nu_{B''}(E) = \frac{k''-k}{k''}[\nu_A(a_1) \vee \nu_A(a_2) \vee \cdots \vee \nu_A(a_r)]h_\nu(\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r])$,

where $k'' = |(\aleph[a_1] \cap \aleph[a_2] \cap \ldots \cap \aleph[a_r])|$. Here, $k' = k''$ as $\overrightarrow{G}$ is symmetric PFD.

Thus, $\mu_{B'}(\{a_1,a_2,\ldots,a_r\}) = \mu_{B''}(\{a_1,a_2,\ldots,a_r\})$, $\eta_{B'}(\{a_1,a_2,\ldots,a_r\}) = \eta_{B''}(\{a_1,a_2,\ldots,a_r\})$ and $\nu_{B'}(\{a_1,a_2,\ldots,a_r\}) = \nu_{B''}(\{a_1,a_2,\ldots,a_r\})$ $\forall$ $a_1,a_2,\ldots,a_r \in V$. □

7 Applications

7.1 An application of *m*-SPFCG in education system

7.1.1 Construction of model

The application of *m*-SPFCG is very useful in our real life. One of the application is in our education system. Consider a PFD shown in Fig. 11 representing the competition between government and non-government Primary Schools and also between Bengali Medium and English Medium high Schools in India. Let us consider the academic institutions as vertices of the digraph. Suppose in the Nursery Schools (NS), the degree of good infrastructure is 50 percent, indeterminacy of infrastructure is 10 percent and inadequate of infrastructure is 25 percent, i.e., the DTMS, DAMS and DFMS of the infrastructure of Nursery Schools is (0.5, 0.1, 0.25) and similarly for the other institutions. Initially, the students started their education in Nursery Schools and they complete their child education from either govt.(GPS) or non-govt. Primary Schools (NGPS). Then they clear school education from either Bengali Medium (BMHS) or English Medium high Schools (EMHS). After that they either admitted into General degree Colleges (GDC) or Engineering Colleges (EC) or Medical Colleges (MC) and lastly admitted into their respective Universities Institutions. The directed edges between institutions represents the rate of selection of institutions by the students. Suppose the degree of choosing govt. Primary School is 50 percent, indeterminacy of choosing is 15 percent and not choosing is 20 percent, i.e., the DTMS, DAMS and DFMS of choosing of the govt. Primary Schools is (0.5, 0.15, 0.2) and similarly for the other institutions as shown in Fig. 11. It is seen that if primary (or high) schools are removed from the education system, then higher education will be highly effected and also nursery students will be deprived for their next educations. Here, we evaluate the competition between institutions with the help of 2-SPFCG.

7.1.2 Decision making

We have, $\aleph_2^+(\text{GPS}) \cap \aleph_2^+(\text{NGPS}) = \{(GDC, (0.4, 0.05, 0.2)), (EC, (0.3, 0.05, 0.2)), (MC, (0.2, 0.05, 0.2))\}$, $\aleph_2^+(\text{BMHS}) \cap \aleph_2^+(\text{EMHS}) = \{(Univ, (0.3, 0.1, 0.25)), (WBUHS, (0.2, 0.05, 0.2))\}$ (see Table 4). Thus there is an edge between GPS and NGPS; BMHS and EMHS in the 2-SPFCG, which indicates the 2-step competition in PFCG. The DTMS, DAMS and DFMS of this edges are respectively (0.26, 0.0075, 0.05) and (0.18, 0.01, 0.04) in Fig. 12. Hence there is a 2-step competition between GPS and NGPS, BMHS and EMHS on the basis of infrastructure in our education system.

7.2 An application of *m*-SPFCG in ecosystem

7.2.1 Construction of model

The *m*-SPFCG is also applicable in our ecosystem. We consider a ecosystem with eight species, all of these are taken as vertices of the digraph as shown in Fig. 13a. Here fox eats goat and bird, goat eats grain and grass, owl eats bird and mouse, bird eats grain and grasshopper, mouse eats grain, grasshopper eats grain and grass. Suppose the degree of existence in the environment of the species fox is 40 percent, indeterminacy of existence is

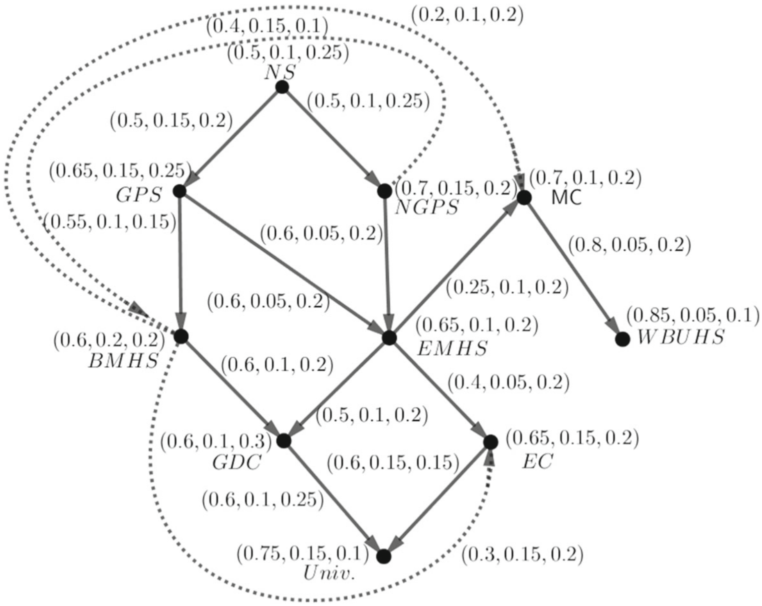


Fig. 11 PFD of the institutions on the basis of infrastructure

5 percent and non existence is 30 percent, i.e., the DTMS, DAMS and DFMS of the species fox is (0.4, 0.05, 0.3). Similarly we can consider for the other species. The DTMS of each directed edge between species and feed represents the likeliness to eat, DAMS represents indeterminacy of likeliness to eat and DFMS represents unlikeliness to eat of feed for predators (see Fig. 13a). It is seen that if goat and bird are removed from this food cycle, then fox must be extinct. As a result, the count of grass, grain and grasshopper will be increased. Thus, we evaluate the food cycle with the help of 2-SPFCG.

7.2.2 Decision making

We have, $\aleph^+(fox) \cap \aleph^+(owl) = \{(grain, (0.35, 0.05, 0.25)), (grasshopper, (0.35, 0.05, 0.2))\}$, $\aleph^+(fox) \cap \aleph^+(bird) = \{(grass, (0.4, 0.05, 0.2)), (grain, (0.4, 0.05, 0.2))\}$, $\aleph^+(owl) \cap \aleph^+(bird) = \{(grain, (0.35, 0.05, 0.2))\}$ (see Table 5). Therefore, there is an edge between fox and owl; fox and bird; owl and bird in the 2-SPFCG. This indicates there is a 2-step competition in the PFCG. The DTMS, DAMS and DFMS of this edges are respectively (0.14, 0.0025, 0.06), (0.16, 0.0025, 0.06) and (0.35, 0.005, 0.05) in Fig. 13b. Hence there is a 2-step competition between Fox and owl, fox and bird, owl and bird on the basis of feeds in ecosystem.

Table 4 2-step picture fuzzy out neighborhood of Fig. 11

$v \in V$	$\aleph_2^+(v)$
NS	$\{(\text{BMHS},(0.5,0.1,0.2)), (\text{EMHS},(0.5,0.05,0.2)), (\text{BMHS},(0.4,0.1,0.25)), (\text{EMHS},(0.5,0.05,0.25))\}$
GPS	$\{(\text{GDC},(0.55,0.1,0.2)), (\text{EC},(0.3,0.1,0.2)), (\text{MC},(0.2,0.1,0.2)),$ $(\text{GDC},(0.5,0.05,0.2)), (\text{EC},(0.4,0.05,0.2)), (\text{MC},(0.25,0.05,0.2))\}$
NGPS	$\{(\text{GDC},(0.5,0.05,0.2)), (\text{EC},(0.4,0.05,0.2)), (\text{MC},(0.25,0.05,0.2)),$ $(\text{GDC},(0.4,0.1,0.2)), (\text{EC},(0.3,0.15,0.2)), (\text{MC},(0.2,0.1,0.2))\}$
BMHS	$\{(\text{Univ},(0.6,0.1,0.25)), (\text{Univ},(0.3,0.15,0.2)), (\text{WBUHS},(0.2,0.05,0.2))\}$
EMHS	$\{(\text{Univ},(0.5,0.1,0.25)), (\text{Univ},(0.4,0.05,0.2)), (\text{WBUHS},(0.25,0.05,0.2))\}$
GDC	$\emptyset$
EC	$\emptyset$
MC	$\emptyset$
Univ	$\emptyset$
WBUHS	$\emptyset$

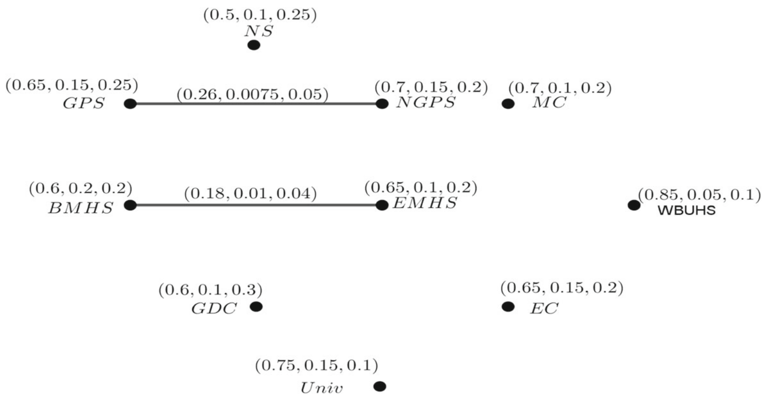


Fig. 12 Corresponding 2-SPFCG

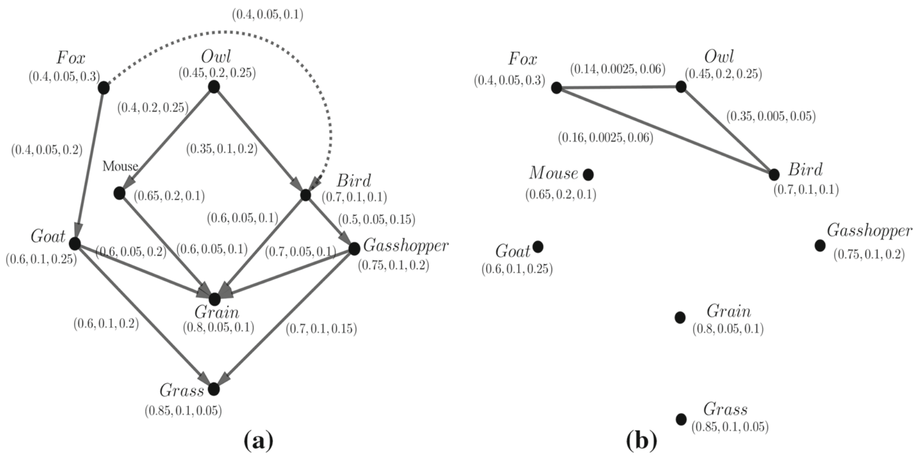


Fig. 13 **a** PFD of ecosystem and **b** Corresponding PFCG

7.3 An application of PFCH in business market

7.3.1 Construction of model

The concept of PFCHs can be used successfully in different domains of applications. One of its application is in business market. Consider a PFD as shown in Fig. 14a representing the competition among seven brand automobile companies—Tata Motors ($C1$), Hero Moto Corp. Ltd ($C2$), Bajaj Auto Ltd ($C3$), Honda Moto Co. Ltd ($C4$), Tayota Motor Corp. ($C5$), Maruti Suzuki ($C6$), Mahindra Ltd ($C7$) in the global industry. Due to globalization, companies strive to manufacture their products with some facilities such as unique designs using modern technology, sophisticated electronic functions, safety features, comfort, low fuel consumption and lower prices. In the business market there always arise a competitive situation as several companies manufacture identical products. So all companies want to attract consumer's attention with their product facilities. Let us consider all companies as

Table 5 2-step picture fuzzy out neighborhood of Fig. 13

$v \in V$	$\aleph_2^+(v)$
Fox	$\{(\text{grass},(0.4,0.05,0.2)),(\text{grain},(0.4,0.05,0.2)),(\text{grain},(0.4,0.05,0.1)),(\text{grasshopper},(0.4,0.05,0.15))\}$
Owl	$\{(\text{grain},(0.4,0.05,0.25)),(\text{grain},(0.35,0.05,0.2)),(\text{grasshopper},(0.35,0.05,0.2))\}$
Bird	$\{(\text{grass},(0.5,0.05,0.15)),(\text{grain},(0.5,0.05,0.15))\}$
Goat	$\emptyset$
Mouse	$\emptyset$
Grasshopper	$\emptyset$
Grain	$\emptyset$
Grass	$\emptyset$

vertices of the digraph. Suppose the degree of product of C1 maintaining above facilities is 80 percent, indeterminacy of facilities is 10 percent and less facilities is 10 percent, i.e., the DTMS, DAMS and DFMS of C1 is (0.8, 0.1, 0.1). Similarly we can consider for the other vertices. The directed edges $\overrightarrow{(C1, C2)}$ indicates that the products of C1 have extra facilities than C2. Suppose the degree of products of C1 having more facilities is 70%, indeterminacy of facilities is 10 percent and less facilities is 10 percent than the products of C2, i.e., the DTMS, DAMS and DFMS of this edge is (0.7, 0.1, 0.1) and similarly for the other edges as shown in Fig. 14a. Here, we evaluate the competition among companies with the help of PFCH.

7.3.2 Decision making

We have, $\aleph^+$(C1)∩ $\aleph^+$(C2)∩ $\aleph^+$(C4)=$\{(C3,(0.3,0.1,0.25))\}$, $\aleph^+$(C1)∩ $\aleph^+$(C7)=$\{(C2,(0.6,0.1,0.1))\}$ and $\aleph^+$(C5)∩ $\aleph^+$(C6)=$\{(C1,(0.5,0.1,0.1))\}$ (see Table 6). Therefore, hyperedges of the PFCH are $E_1 = \{C5, C6\}$, $E_2 = \{C1, C7\}$ and $E_3 = \{C1, C2, C4\}$. The DTMS, DAMS and DFMS of E_1, E_2 and E_3 are respectively (0.25, 0.01, 0.06), (0.36, 0.01, 0.02) and (0.21, 0.01, 0.05) showing in Fig. 14b. Hence there are competitions between pair of companies ($C5$, $C6$), ($C1$, $C7$) and also competition among group of companies ($C1$, $C2$, $C4$) on the basis of products with facilities in business market.

7.4 An application of PFCH in job competition

7.4.1 Construction of model

Let us consider a PFD (see Fig. 15a) representing the competition among applicants for a railway-job. Let the set of five applicants $\{A_1, A_2, A_3, A_4, A_5\}$ applying against the set of four job-vacancies $\{StationManager(J_1), Driver(J_2), Technician(J_3), TTE(J_4)\}$. All of these are aching as vertices of the digraph. Suppose the DTMS, DAMS and DFMS of each applicant represents his/her degree of good qualification, indeterminacy of qualification and poor qualification, respectively. Let the degree of good qualification of A_1 is 60 percent, indeterminacy of qualification is 20 percent and poor qualification is 10 percent,

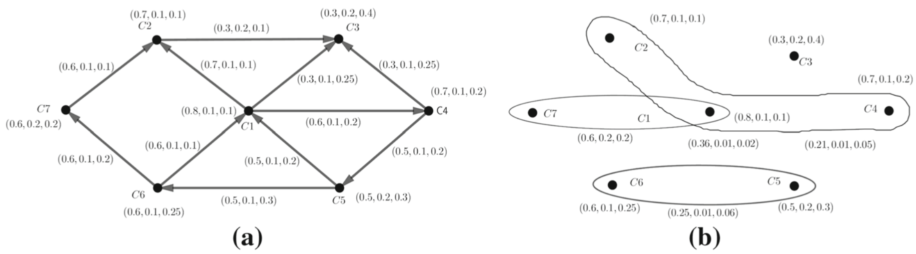


Fig. 14 **a** PFD in business market and **b** corresponding PFCH

Table 6 Picture fuzzy out neighborhood of Fig. 14

$v \in V$	$\aleph^{+}(v)$
$C1$	$\{(C2,(0.7,0.1,0.1)),(C3,(0.3,0.1,0.25)),(C4,(0.6,0.1,0.2))\}$
$C2$	$\{(C3,(0.3,0.1,0.2))\}$
$C3$	$\emptyset$
$C4$	$\{(C3,(0.3,0.1,0.25)),(C5,(0.5,0.1,0.2))\}$
$C5$	$\{(C1,(0.5,0.1,0.2)),(C6,(0.5,0.1,0.3))\}$
$C6$	$\{(C1,(0.6,0.1,0.1)),(C7,(0.6,0.1,0.3))\}$
$C7$	$\{(C2,(0.6,0.1,0.1))\}$

i.e., the DTMS, DAMS and DFMS of A_1 is (0.6, 0.2, 0.1) and similarly for the other applicants. Again, suppose the DTMS, DAMS and DFMS of each vacancy represents its degree of strong criteria, indeterminacy of criteria and weak criteria, respectively. Let the degree of strong criteria for J_1 is 70 percent, indeterminacy of the criteria is 10 percent and weak criteria is 20 percent, i.e., the DTMS, DAMS and DFMS of J_1 is (0.7, 0.1, 0.2) and similarly for the other vacancies. The DTMS, DAMS and DFMS of each directed edge between an applicant and vacancy represents the candidate's eligibility, indeterminacy of eligibility and non-eligibility for a particular vacancy as shown in Fig. 15a.

7.4.2 Decision making

Here $\aleph^{+}(A_1)\cap\aleph^{+}(A_2)=\{(J_4,(0.5,0.1,0.3))\}$, $\aleph^{+}(A_1)\cap\aleph^{+}(A_3)\cap\aleph^{+}(A_5)=\{(J_2,(0.4,0.1,0.25))\}$, $\aleph^{+}(A_3)\cap\aleph^{+}(A_4)\cap\aleph^{+}(A_5)=\{(J_1,(0.5,0.1,0.2))\}$ (see Table 7). Therefore, hyperedges of the PFCH are $E_1=\{A_1,A_2\}$, $E_2=\{A_1,A_3,A_5\}$ and $E_3=\{A_3,A_4,A_5\}$. The DTMS, DAMS and DFMS of E_1, E_2 and E_3 are respectively (0.25, 0.02, 0.09), (0.24, 0.01, 0.05) and (0.3, 0.01, 0.04) showing in Fig. 15b. Hence there are competitions among the applicants (A_1,A_3,A_5), (A_3,A_4,A_5) and (A_1,A_2) on the basis of their eligibility for the vacancies.

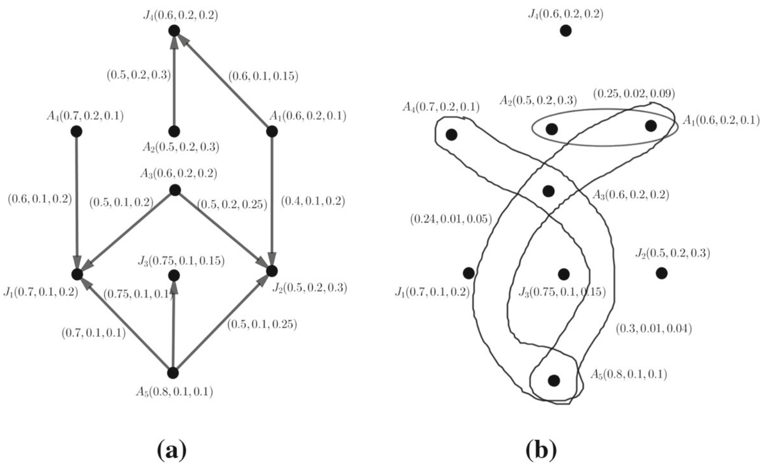


Fig. 15 **a** PFD of job competition and **b** corresponding PFCH

8 Comparative study with existing papers

In existing papers on fuzzy CGs, all information are collected in fuzzy sense. But, when information are in picture fuzzy sense, the existing models are not suitable to handle such information. In these scenario, our currently developed models play an important role.

Samanta and Pal (2013), Samanta et al. (2014, 2015), and Samanta and Sarkar (2018) studied many variations of CGs with fuzzy information for the first time. Sahoo and Pal (2015) proposed a model by considering each vertex and edge with IF information and determined competition among the species in food cycle. But, all problems of feed-predator can not be modeled using these CGs as the measurement of competitions was taken as IF sense in that paper. In IFSs, the membership and non-membership values are considered only. So these models are not applicable when the model is considered in other environment like in picture fuzzy environment. In our present work, we consider another parameter called neutral membership value and it will be effectively useful to model many more real-world problems. So, our study is the extension of the study of the above works.

9 Limitations of *m*-SPFCG model

Some limitations of this study are listed below:

- *m*-SPFCG models are only applicable to those problems with picture fuzzy information.
- *m*-SPFCG models do not demonstrate all the competitions of real-world problems.
- In a PFD, if the number of vertices is less than the value of *m*, then we can not find any competition between objects through *m*-SPFCG models.
- *m*-SPFCG models are unable to find any group-wise competition among three or more objects.

Table 7 Picture fuzzy out neighborhood of Fig. 15

$v \in V$	$\aleph^{+}(v)$
A_1	$\{(J_2, (0.4, 0.1, 0.2)), (J_4, (0.6, 0.1, 0.15))\}$
A_2	$\{(J_4, (0.5, 0.2, 0.3))\}$
A_3	$\{(J_1, (0.5, 0.1, 0.2)), (J_2, (0.5, 0.2, 0.25))\}$
A_4	$\{(J_1, (0.6, 0.1, 0.2))\}$
A_5	$\{(J_1, (0.7, 0.1, 0.1)), (J_2, (0.5, 0.1, 0.25)), (J_3, (0.75, 0.1, 0.1))\}$

10 Conclusion

In this study, the powerful tool of fuzziness is applied to generalize the notion of CGs under the picture fuzzy environment. Our proposed picture fuzzy models provided more legibility, flexibility and suitability to the system as compared with the models in other fields. The methods of construction of several types of PFCGs, and PFHs using PFON and PFCN are studied here. A formula is suggested for the strength of feeds and established few results over strong feeds. Also strong relations between PFkCHs and PFkNHs are established. This study will help to measure the strength of competitions in real-world problems. Our proposed models have been applied in real field competitions for representation of fuzziness in different domains including identification of species-feed relations in ecosystem, competitions between institutions in education system, competition in business market and job competition between applicants, which motivates the idea introduced in this study. In future, we will extend this work to (1) Picture fuzzy tolerance competition graphs (2) m-Step Picture fuzzy tolerance competition graphs and (3) Picture fuzzy tolerance competition hypergraphs, etc.

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