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m-polar neutrosophic soft mapping with application to multiple personality disorder and its associated mental disorders

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Abstract

Multiple personality disorder (MPD) or dissociative identity disorder is the mental disease in which one can observe the existence of two or more than two personalities in a single person. We define the controversies nearby the diagnosis of MPD with its associated mental disorders. We discuss the various symptoms of MPD, dissociative amnesia, depersonalization or derealization disorder, and major depression disorder. After this exploration, we perceive that these disorders enclose parallel symptoms and it is difficult to identify the accurate type of disorder with its severeness. Since in experimental diagnosis the indeterminacy and falsity parts are often neglected. Due to this problem, we cannot see the accuracy in the patient's improvement record and cannot predict the duration of treatment. To eradicate these boundaries, we present the m-polar neutrosophic soft set (MPNSS) and m-polar neutrosophic soft mapping (MPNS-mapping) with its inverse mapping. These notions are proficient and valuable to diagnose the disorder appropriately by connecting it with the mathematical modeling. The connection of m-polar neutrosophic set (MPNS) with the soft set characterizes a relation among patients, symptoms, and treatments which decreases the complexity of the case study. We build a chart based on a fuzzy interval [0, 1] to range the types of disorders. We establish an algorithm based on MPNS-mapping to identify the disease appropriately and to select the finest treatment for the corresponding disease of every patient. At last, we introduce the generalized MPNS-mapping which will helps a doctor to save the patient's improvement record and to predict the period of treatment until the disease is cured.

Keywords m-polar neutrosophic soft set · m-polar neutrosophic soft mapping and its properties · Multiple personality disorder (MPD/DID) and its associated mental disorders · Decision-making

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1 Introduction

Psychological disorder is a social psychological problem that causes substantial pain or damage to human working. Presently, psychological sickness is one of the five main syndromes instigating incapacity, accounting for more than 30% of entire infirmities in a generation (Noor et al. 2012; Sayarifard and Ghadirian 2013). According to the report of the "World Health Organization" (WHO) (Noor et al. 2012) in 2002, 500 million individuals were suffering from some kind of psychological disorder. "Mental Health Literacy" (MHL) is a subcategory of "health literacy" and it was first familiarized by Australian researchers (Lakdawala and Vankar 2016; Sayarifard and Ghadirian 2013), which represents the information and principles about psychological illnesses. Preliminary definition indicates that MHL has seven mechanisms:

1. The capability to diagnose precise disorder.
2. Awareness that how to seek psychological strength information.
3. Awareness of risk dynamics of psychological sickness.
4. Awareness of reasons of psychological sickness.
5. Awareness of personal-treatment.
6. Awareness of expert's aid available.
7. Behavior that stimulate acknowledgment and appropriate help-seeking.

These dynamics can be categorized into further types to elaborate on the study of psychological disorders. There are several categories of psychological disorders that influence the diverse parts of the human brain. Many researchers introduced and studied its types with diverse cases having psychological disorders. There are several segments of the human brain and these segments are connected to different actions of the human body. The segments of the human brain that are connected with the short term memory are from the limbic system especially the amygdala and the dorsolateral prefrontal. The segment connected with the long term memory is the hippocampus. All these regions are affected by MPD.

In recent years, many researchers studied and established novel techniques to diagnose psychological disorders with its effects, causes, and treatments. The correlations of MHL among Iranian female students with psychological features were presented by Bahrami et al. (2019). Alonso (2011) studied some common psychological and physical conditions of patients suffering from mental disorders. The effects, causes, and treatments of patients suffering from MPD/DID has been studied and explored by various researchers (Ashraf et al. 2016; Allen and Movius 2000; Morton 2018; Nissen et al. 1988; Rutkofsky et al. 2017). They established the actual effects of the disease and its relation to other mental disorders. Mathematicians started to solve problems of medical sciences by using fuzzy logic and its associated hybrid structures. They started to relate the case study with the mathematical logic by using linguistic terms and linguistic variables then solve these problems by using mathematical modeling. Innocent and Jhon (2004) presented computer-aided fuzzy medical diagnosis. Kovalerchukab et al. (1997) used fuzzy logic in breast cancer diagnosis.

The novel conception of neutropsychic personality was described by Smarandache (2018) by connecting neutrosophic reasoning and establishing psychological philosophy, human nature, memories, and human disposition. He set out various outcomes and illustrations in this book to correspond human personality and nature to psychological theory of Neutropsyche. Neutropsyche is the philosophical theory that incorporates the neutrosophy and neutrosohic hypotheses to research the consciousness or soul. The Neutrosophic Theory of Psychology. This is focused on triadic psychological neutrosophic principles , processes, thoughts, and type hypotheses "($\langle A\rangle$, $\langle neutA\rangle$, $\langle antiA\rangle$)", such as "(positive , neutral, negative)", and so on. We can observe numerous implementations of this principle, such as awesome behavior and attitude, ignorance, negative behavior; deciding to act, delayed, deciding not to respond; sensitive, modest, insensitive, etc. The refinement of neutrosophic values can be observed as Fig. 1.

Christianto and Smarandache (2019) provided a comprehensive analysis of seven aspects of neutrosophical philosophy such as cultural psychology, theorizing finance, dispute management, scientific philosophy, etc. Farahani et al. (2015) presented an ADHD case study and compared combined overlap block fuzzy cognitive maps (COBFCM) and combined overlap block neutrosophic cognitive map (COBNCM) to find the hidden patterns and indeterminacies in psychological causal models. Using multi-polarity and parameterizations, we relate these theoretical theories to the neutrosophic group. We may use it in MPD as "positive, neutral and negative" consequences according to different levels in those three classes. MPD or DID is one of the most sensitive and serious types of psychological disorders. Initially, people think that the patients suffering from MPD have affected by some negative spiritual power or negative energy. After that many researchers worked on this disease and explored it with its effects, causes, and treatments. Due to the experience and awareness gained by this exploration people starts to discover and understand the actual causes of MPD. Then many artists worked on some movies and serials related to the patients suffering from MPD e.g, split, glass, the three faces of eve, identity, borderline and ishaq zahe naseeb etc. It is challenging to diagnose the genuine type of disorder because various symptoms look parallel to each other. On the other hand, the selection of best treatment and calculation of time duration for the treatment is difficult to evaluate. This drawback is due to the lack of information in the input data of the patient. In the field of medication, the input data does not give any information about the indeterminacy and falsity or dissatisfaction grades with the parameterizations. So we propose the novel idea of MPNSS and MPNS-mapping to handle these types of medical diagnosis and decision-making problems.

1.1 Background and decision-making based hypothetical data interpretation

In real life complications, we encounter several situations, which contain vagueness and obscurities due to unsatisfactory knowledge and incompatible data. To handle these difficulties Zadeh (1965) originated the idea of fuzzy sets and logics in 1965. A fuzzy set is an independent abstraction of crisp set theory to handle ambiguities and hesitations. The concept of linguistic variable was introduced by Zadeh (1975). He said that the linguistic variable is a variable whose values are sentences or words in an artificial or ordinary language. If these words are expressed by fuzzy sets defined over a reference set, then the variable is called a fuzzy linguistic variable (Zadeh 1975). Atanassov (1984); Atanassov and Stoeva (1983); Atanassov (1986) proposed the concept of intuitionistic fuzzy set (IFS) as an extension of fuzzy set by introducing the concepts of membership grades (denoted by $\mu(\mathcal{U})$)and non-membership grades (denoted by $\nu(\mathcal{U})$) along with the constraint that sum of these two grades must not exceed unity. In certain real life applications, we deal with the difficulties having indeterminacy in their environment.

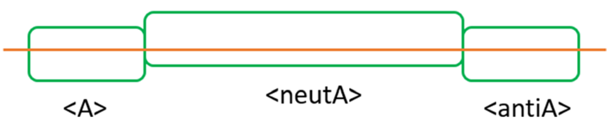

Fig. 1 Neutrosophic refinement

In that case, we cannot relate the problem with the fuzzy and IFSs by using mathematical modeling. If we use these models then results are ambiguous and inaccurate due to the lack of information. Due to this drawback, Smarandache (1998) established the idea of neutrosophic set having membership, indeterminacy and non-membership degrees. We take the values from the subsets of the interval $]^{-}0, 1^{+}[$ for neutrosophic sets. It is very difficult to use these values in the daily life problems. Accordingly, we use the interval [0, 1] for evaluation of degrees and decision analysis in the context of neutrosophic set. Wang et al. (2010) established some novel ideas on neutrosophic sets. The beauty of this structure is that all the grades are independent to each other. To deal with the bipolar nature of alternatives, Zhang (1994, 1998), Zhang and Zhang (2004) initiated the idea of bipolar fuzzy set. It is suitable for the input data which have bipolarity having positive and negative properties. After that Chen et al. (2014) introduced the notion of m-polar fuzzy set (MPFS), which is generalized model of bipolar fuzzy set. MPFS deals with the knowledge having multiple properties in its nature. We can assemble the heavy data containing multi-polarity by using MPFS. Soft set was originated by Molodtsov (1999) in 1999 for categorizing uncertainties by using parameterizations. Some novel operations of soft set theory were established by Ali et al. (2009). Ali and Shabir (2014) studied the logic connectives of soft sets and fuzzy soft sets such as implications, t-norms and t-conorms. Maji et al. (2003) introduced some results on soft set theory. Çağman et al. (2011) developed some new results with its applications of fuzzy soft set theory. Deli et al. (2015) established a hybrid structure named as bipolar neutrosophic set and presented its applications in daily life decision-making problems. Jose and Kuriaskose (2014) introduced score functions, accuracy functions and aggregation operators of intuitionistic fuzzy sets with applications. Hashmi and Riaz (2020) established the idea of Pythagorean m-polar fuzzy sets and presented Pythagorean m-polar fuzzy dombi's aggregation operator to the censuses process. Hashmi et al. (2020) introduced the hybrid structure of m-polar neutrosophic set (MPNS) as an abstraction of bipolar neutrosophic set by combining MPFSs and neutrosophic sets. They developed new algorithms to deal with the problems in medical sciences and for clustering of information data.

Riaz and Hashmi (2018, 2019a, b, c) introduced various results on the fixed points of neutrosophic soft mapping with applications. They established cubic m-polar fuzzy aggregation operators and presented multi-attribute group decision-making (MAGDM) to solve agribusiness problems. They established the new concept of linear Diophantine fuzzy sets (LDFSs) as an extension of q-rung orthopair fuzzy sets (Ali 2018; Yager 2017), Pythagorean fuzzy sets (Yager and Abbasov 2013; Yager 2013, 2014) and intuitionistic fuzzy sets. They introduced the novel structures of Pythagorean m-polar fuzzy soft rough sets (PMPFSRSs) and soft rough Pythagorean m-polar fuzzy sets (SRPMPFSs). They established new algorithms based on LDFSs, PMPFSRSs and SRPMPFSs to solve decision-making problems. Riaz and Naeem (2016), Riaz and Tehrim (2019) and Riaz and Tehrim (2020) introduced the idea of measurable soft mappings and they established bipolar soft mappings with its applications to diagnose bipolar mental disorder with its treatments. They introduced cubic bipolar fuzzy aggregation operators to solve MAGDM problems. Riaz and Tehrim (2020) introduced a robust extension of VIKOR method for bipolar fuzzy sets using connection numbers of SPA theory based metric spaces. Chen and Tan (1994) used vague set theory to developed fuzzy decision-making technique. Tversky and Kahneman (1992) presented some modifications in the prospect theory for the presentation of ambiguities in cumulative manner. Feng et al. (2019, 2010) established some algorithms in the environment of fuzzy soft sets to handle decision problems. They presented a novel view on generalized intuitionistic fuzzy soft sets (GIFSSs) with the help of numerical examples. Demirci (1999) presented fuzzy functions and its fundamental properties. Majumdar and Samanta (2010) invented some results on soft mappings. Kharal and Ahmad (2009, 2011) established mappings on soft classes and fuzzy soft classes. Bashir and Salleh (2013) studied intuitionistic fuzzy soft classes and established its mappings with illustrations. Shen et al. (2012) presented some modified results on intuitionistic fuzzy mappings. Jiang et al. (2020) developed an algorithm for medical diagnosis utilizing a MADM technique to covering fuzzy rough sets dependent on variable precision. Mu et al. (2020) presented new groups of multi-granulated, fuzzy rough sets dependent coverings and related implementations of various decision-making system attribute models. Zhang and Zhan (2019) discovered fuzzy soft β-covering fuzzy rough sets and similar implementations for decision taking. Zhang et al. (2020) built an application focused on CVPIFRS models to solve biomedical problems using intuitionistic fuzzy TOPSIS process. Zhan and Alcantud (2019a, 2019b) introduced a novel form of soft rough cover and a review of soft sets reduction parameters and relevant algorithms. They have developed their expertise in issues related to decision-making. Zhan and Wang (2019) developed some forms of rough sets dependent on soft coverings with its implementations. Zhan et al. (2020) implemented a CPFRS-based PF-TOPSIS system and an application for unusual emergency occurrences.

1.2 Motivation, highlights and focus of the study

These mappings have numerous applications in decision-making techniques. But due to the lack of information in the input data we cannot handle indeterminacy parts with the parameterizations. So, we establish the hybrid structure of MPNSS with its mapping and inverse mapping to diagnose the MPD and its associated psychological disorders in the patients. This model is proficient and superior to others because it collects the knowledge about the membership, indeterminacy and non-membership parts of patient's disease with its parameterizations. We can talk about all the factors of disease with multiple criteria. Multi-polarity helps us to deal with the multiple personalities in the patients. We set a chart of mental disorders in the fuzzy interval [0, 1] with its severeness. The presented algorithm deals with the diagnosis of disorder and then give an optimal decision about the best treatment. It can develops an improvement chart history of patient and give a prediction about the time duration in which the disease is cured. This is one of the generalized model and use to handle various decision-making problems.

Because of the close association with human existence and neutrosophy, we expand this research in neutrosophic and its various properties using multi-criteria. All the factors for personality development are categorizations of neutrosophical values and their sub-categories. Various of them were discussed by Smarandache (2018) and some of them are listed in the Table 1.

He applies all these variables to the neutrsophic set in his work, but in this manuscript we apply these ideas for the creation of human identity to the neutrosophy and multi-polarity under defined parameterizations. For instance, if we consider a person's memory then it can be classified into such three terms as: "unconscious, aconscious and conscious". He also cites several scholars' study to split it down into different groups. Such catagories can be linked to the multi-polarity under a parameter of an "individuals memory" (see Table 2). They may transform other mentioned variables into MPNSNs in the same model. It explicitly connects the knowledge details of a patient suffering from MDP with MPNSSs and their mapping. The further detail can be observed in Smarandache (2018).

The arrangement of this manuscript is schematized as follows: Sect. 2 provides some rudimentary concepts of neutrosophic sets, MPFSs, MPNSs and its operations, score functions and accuracy functions. In Sect. 3, we explore about the MPD and its associated psychological

Table 1 Lifespan personality development based on neutrosophic values

$\langle AntiA \rangle$	$\langle NeutA \rangle$	$\langle A \rangle$
Unconscious	Aconscious	Conscious
Underego	Ego	Superego
Inferiority	Normal standard	Superiority complex
Surrealistic	Semi-realistic	Realistic
Pleasureful and painless	Semi-pleasurefull and semi-painful	Painful
Immoral	Semi-moral	Moral
Desires	Takes a decision	Restrictions
Biological	Bio-social	Social
Imperfection	Semi-imperfection and semi-perfection	Perfection
Unorganized	Semi-organized	Organized
Provides energy	Provides directions	Provides norms
Radical	In between radical and conservative	Conservative
In the short term	In the middle term	In the long term
Force	Force and influence	Influence

Table 2 Neutrosophic memory based on multi-polarity

Neutrosophy	Type of memory	Multiplicity
$\langle AntiA \rangle$	Unconscious	Personal, collective and group preconscious, subconscious, semiconscious
$\langle NeutA \rangle$	Aconscious	Semiunconscious, subunconscious, preunconscious, personal, collective and group
$\langle A \rangle$	Conscious	Personal, collective and group

disorders. We discuss about the causes, symptoms and effects of these mental disorders with all of its properties. We present the motivation of this proposed method by connecting our proposed model with the real life applications and decision-making techniques. We present semantic assessment of proposed model with the existing structures. We present various properties of MPNSS and MPNS-mapping with the help of illustrations. In Sect. 4, We establish the methodology of created algorithm and set the ranges of MPD with its associated psychological disorders with in the interval [0, 1]. The proposed algorithm represents the diagnosis of disease, selection of suitable treatment, and improvement chart history of every patient. We present a numerical example with the case study in the field of medical science and establish the results by using proposed algorithm. We present a brief comparison to highlight the consistency, superiority, validity and flexibility of proposed technique. Finally, we conclude our results and research in Sect. 5.

2 Background

In this segment, we discuss about some rudimentary ideas including fuzzy, soft, bipolar fuzzy, neutrosophic, MPFSs and MPNSs. We construct the new structure of m-polar neutrosophic soft set (MPNSS) by using some primary components. In the entire manuscript, we use $\mathcal{Q}$ as a universal or reference set. We use $\dot{\mathcal{T}}, \dot{\mathcal{I}}$ and $\dot{\mathcal{F}}$ as a membership grade, indeterminacy grade and non-membership grade for the alternatives respectively and Δ as an indexing set.

Definition 2.1 (Zadeh 1965) For the reference set $\mathcal{Q}$, a fuzzy set (FS) $\mathfrak{F}$ can be represented by a mapping $\sigma : \mathcal{Q} \rightarrow [0, 1]$, where $\sigma(\mathcal{U})$ for every $\mathcal{U} \in \mathcal{Q}$, represents the membership degree of that object to which that element related to $\mathfrak{F}$. It can be scripted as;

$$\mathfrak{F} = \{(\mathcal{U}, \sigma(\mathcal{U})) : \mathcal{U} \in \mathcal{Q}\}$$

Definition 2.2 (Molodtsov 1999) For the reference set $\mathcal{Q}$ with the set of attributes $\mathcal{A}$, the soft set is scripted by a mapping $\mathcal{J} : \mathcal{G} \rightarrow \dot{P}(\mathcal{Q})$ with $\mathcal{G} \subseteq \mathcal{A}$ and it can be represented as

$$(\mathcal{J}, \mathcal{G}) = \mathcal{J}_{\mathcal{G}} = \{(\wp, \mathcal{J}(\wp)) : \wp \in \mathcal{G}; \mathcal{J}(\wp) \subseteq \dot{P}(\mathcal{Q})\}$$

where $\dot{P}(\mathcal{Q})$ represents the power set of $\mathcal{Q}$.

Definition 2.3 (Smarandache 1998) A neutrosophic set $\mathfrak{P}$ in $\mathcal{Q}$ is represented by using the degrees of membership $\dot{\mathcal{T}}$, indeterminacy $\dot{\mathcal{I}}$ and non-membership $\dot{\mathcal{F}}$. $\dot{\mathcal{T}}(\mathcal{U})$, $\dot{\mathcal{I}}(\mathcal{U})$ and $\dot{\mathcal{F}}(\mathcal{U})$ are elements of $]0^-, 1^+[$ for the alternative $\mathcal{U}$. It can be scripted as

$$\mathfrak{P} = \{(\mathcal{U}, \langle \dot{\mathcal{T}}(\mathcal{U}), \dot{\mathcal{I}}(\mathcal{U}), \dot{\mathcal{F}}(\mathcal{U}) \rangle) : \mathcal{U} \in \mathcal{Q}; \dot{\mathcal{T}}(\mathcal{U}), \dot{\mathcal{I}}(\mathcal{U}), \dot{\mathcal{F}}(\mathcal{U}) \in]0^-, 1^+[\}$$

satisfying the constraint $0^- \leq \dot{\mathcal{T}}(\mathcal{U}) + \dot{\mathcal{I}}(\mathcal{U}) + \dot{\mathcal{F}}(\mathcal{U}) \leq 3+$.

Definition 2.4 (Zhang 1994, 1998; Zhang and Zhang 2004) A bipolar fuzzy set (BFS) $\mathcal{K}$ in $\mathcal{Q}$ can be scripted as

$$\mathcal{K} = \{(\mathcal{U}, \langle \sigma^+(\mathcal{U}), \sigma^-(\mathcal{U}) \rangle) : \mathcal{U} \in \mathcal{Q}\}$$

where $\sigma^+(\mathcal{U}) \in [0, 1]$ signifies the truth or positive grade and $\sigma^-(\mathcal{U}) \in [-1, 0]$ signifies the opposite or negative grade for the alternatives of $\mathcal{Q}$.

Definition 2.5 (Chen et al. 2014) An m-polar fuzzy set (MPFS) is generalized model of BFS. The mapping $\mathfrak{C} : \mathcal{Q} \rightarrow [0, 1]^m$ signifies the MPFS $\mathfrak{C}$ in $\mathcal{Q}$ and denoted by

$$\mathfrak{C} = \{\langle \mathcal{U}, P_\alpha o \Lambda(\mathcal{U}) \rangle : \mathcal{U} \in \mathcal{Q}; \alpha = 1, 2, 3, \ldots, m\}$$

where and $P_\alpha : [0, 1]^m \rightarrow [0, 1]$ is the αth projection ($\alpha \in m$).

Definition 2.6 (Hashmi et al. 2020) An object $\mathcal{M}_{\mathfrak{N}}$ in a reference set $\mathcal{Q}$ is called MPNS, if it can be scripted as

$$\mathcal{M}_{\mathfrak{N}} = \{\left(\mathcal{U}, \langle \dot{\mathcal{T}}_\alpha(\mathcal{U}), \dot{\mathcal{I}}_\alpha(\mathcal{U}), \dot{\mathcal{F}}_\alpha(\mathcal{U}) \rangle\right) : \mathcal{U} \in \mathcal{Q}, \alpha = 1, 2, 3, \ldots, m\}$$

or

$$\mathcal{M}_{\mathfrak{N}} = \{\mathcal{U}, \left(\langle \dot{\mathcal{T}}_1(\mathcal{U}), \dot{\mathcal{I}}_1(\mathcal{U}), \dot{\mathcal{F}}_1(\mathcal{U})\rangle, \langle \dot{\mathcal{T}}_2(\mathcal{U}), \dot{\mathcal{I}}_2(\mathcal{U}), \dot{\mathcal{F}}_2(\mathcal{U})\rangle, \dots, \langle \dot{\mathcal{T}}_m(\mathcal{U}), \dot{\mathcal{I}}_m(\mathcal{U}), \dot{\mathcal{F}}_m(\mathcal{U})\rangle\right) : \mathcal{U} \in \mathcal{Q}\}$$

where $\dot{\mathcal{T}}_\alpha, \dot{\mathcal{I}}_\alpha, \dot{\mathcal{F}}_\alpha : \mathcal{Q} \to [0,1]$ and $0 \le \dot{\mathcal{T}}_\alpha(\mathcal{U}) + \dot{\mathcal{I}}_\alpha(\mathcal{U}) + \dot{\mathcal{F}}_\alpha(\mathcal{U}) \le 3$; $\alpha = 1,2,3,\dots,m$. This constraint represents that all the three grades $\dot{\mathcal{T}}_\alpha(\mathcal{U}), \dot{\mathcal{I}}_\alpha(\mathcal{U})$ and $\dot{\mathcal{F}}_\alpha(\mathcal{U})$ are independent and signifies the positiveness, indeterminacy and negativeness of the alternative respectively under multi-polarity of the information. The assembling of all MPNSs in $\mathcal{Q}$ can be scripted as $MPN(\mathcal{Q})$.

The notion $\dot{\mathcal{N}} = (\langle t_\alpha, i_\alpha, f_\alpha\rangle; \alpha = 1,2,3,\dots,m)$ is said to be an m-polar neutrosophic number (MPNN) satisfying the constraint $0 \le t_\alpha, i_\alpha, f_\alpha \le 3$.

Example 2.7 Let $\mathcal{Q} = \{\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3\}$ represents an assembling of some mobile phones. The 5-polar neutrosophic set (4PNS) in $\mathcal{Q}$ can be scripted as

$$\begin{aligned}\mathcal{M}_{\mathfrak{N}} = &\{(\mathcal{U}_1, \langle 0.352, 0.273, 0.462\rangle, \langle 0.355, 0.362, 0.247\rangle, \langle 0.577, 0.546, 0.245\rangle, \langle 0.553, 0.534, 0.414\rangle,\\ &\langle 0.345, 0.656, 0.431\rangle), (\mathcal{U}_2, \langle 0.456, 0.454, 0.865\rangle, \langle 0.455, 0.856, 0.324\rangle, \langle 0.234, 0.457, 0.456\rangle,\\ &\langle 0.678, 0.344, 0.445\rangle, \langle 0.346, 0.676, 0.123\rangle), (\mathcal{U}_3, \langle 0.213, 0.346, 0.567\rangle, \langle 0.346, 0.678, 0.523\rangle,\\ &\langle 0.123, 0.436, 0.456\rangle, \langle 0.657, 0.679, 0.322\rangle, \langle 0.634, 0.235, 0.534\rangle)\}\end{aligned}$$

In 5PNS the multi-polarity (m = 5) of each alternative $\mathcal{U}$ signifies its some specific property or quality according to the information data such as

$$\begin{aligned}1 &= \text{Affordable},\\ 2 &= \text{Long lasting battery},\\ 3 &= \text{Extra storage},\\ 4 &= \text{Good camera quality},\\ 5 &= \text{Metallic body}.\end{aligned}$$

We have neutrosophic grades to signifies the positiveness, indeterminacy and negativeness of alternatives corresponding to each criteria (m = 5). The data can be evaluated by the suggestion of an expert using linguistic terms. In $\mathcal{M}_{\mathfrak{N}}$ the triplet $\langle 0.352, 0.273, 0.462\rangle$ for $\mathcal{U}_1$ represents that the mobile phone $\mathcal{U}_1$ has 35.2% positiveness, 27.3% indeterminacy and 46.2% negativeness for the attribute ”affordable”. On the same pattern, we can observe remaining grades for other alternatives and attributes.

Definition 2.8 (Hashmi et al. 2020) Now we study some operations for MPNSs, which we will use later for further modifications.

Let $\mathcal{M}_{\mathfrak{N}}, \mathcal{M}_{\mathfrak{N}_\wp} \in MPN(\mathcal{Q})$, where $\mathcal{M}_{\mathfrak{N}} = \{\left(\mathcal{U}, \langle \dot{\mathcal{T}}_\alpha(\mathcal{U}), \dot{\mathcal{I}}_\alpha(\mathcal{U}), \dot{\mathcal{F}}_\alpha(\mathcal{U})\rangle\right) : \mathcal{U} \in \mathcal{Q}, \alpha = 1,2,3,\dots,m\}$ and $\mathcal{M}_{\mathfrak{N}_\wp} = \{\left(\mathcal{U}, \langle {}^\wp\dot{\mathcal{T}}_\alpha(\mathcal{U}), {}^\wp\dot{\mathcal{I}}_\alpha(\mathcal{U}), {}^\wp\dot{\mathcal{F}}_\alpha(\mathcal{U})\rangle\right) : \mathcal{U} \in \mathcal{Q}, \wp \in \Delta, \alpha = 1,2,3,\dots,m\}$, then:

(i) $\mathcal{M}^c_{\mathfrak{N}} = \{\left(\mathcal{U}, \langle \dot{\mathcal{F}}_\alpha(\mathcal{U}), 1 - \dot{\mathcal{I}}_\alpha(\mathcal{U}), \dot{\mathcal{T}}_\alpha(\mathcal{U})\rangle\right) : \mathcal{U} \in \mathcal{Q}, \alpha = 1,2,3,\dots,m\}$

(ii) $\mathcal{M}_{\mathfrak{N}_1} = \mathcal{M}_{\mathfrak{N}_2} \Leftrightarrow \langle {}^1\dot{\mathcal{T}}_\alpha(\mathcal{U}), {}^1\dot{\mathcal{I}}_\alpha(\mathcal{U}), {}^1\dot{\mathcal{F}}_\alpha(\mathcal{U})\rangle = \langle {}^2\dot{\mathcal{T}}_\alpha(\mathcal{U}), {}^2\dot{\mathcal{I}}_\alpha(\mathcal{U}), {}^2\dot{\mathcal{F}}_\alpha(\mathcal{U})\rangle; \mathcal{U} \in \mathcal{Q},\ \alpha = 1,2,3,\dots,m$

(iii) $\mathcal{M}_{\mathfrak{N}_1} \subseteq \mathcal{M}_{\mathfrak{N}_2} \Leftrightarrow {}^1\dot{\mathcal{T}}_\alpha(\mathcal{U}) \le {}^2\dot{\mathcal{T}}_\alpha(\mathcal{U}), {}^1\dot{\mathcal{I}}_\alpha(\mathcal{U}) \ge {}^2\dot{\mathcal{I}}_\alpha(\mathcal{U}), {}^1\dot{\mathcal{F}}_\alpha(\mathcal{U}) \ge {}^2\dot{\mathcal{F}}_\alpha(\mathcal{U}); \mathcal{U} \in \mathcal{Q},\ \alpha = 1,2,3,\dots,m$

(iv) $\bigcup_\wp \mathcal{M}_{\mathfrak{N}_\wp} = \{(\mathcal{U}, \left\langle \sup_\wp {}^\wp\dot{\mathcal{T}}_\alpha(\mathcal{U}), \inf_\wp {}^\wp\dot{\mathcal{I}}_\alpha(\mathcal{U}), \inf_\wp {}^\wp\dot{\mathcal{F}}_\alpha(\mathcal{U})\right\rangle); \mathcal{U} \in \mathcal{Q}, \wp \in \Delta, \alpha = 1,2,3,\dots,m\}$

(v) $\bigcap_\wp \mathcal{M}_{\mathfrak{N}_\wp} = \{(\mathcal{U}, \left\langle \inf_\wp {}^\wp\dot{\mathcal{T}}_\alpha(\mathcal{U}), \sup_\wp {}^\wp\dot{\mathcal{I}}_\alpha(\mathcal{U}), \sup_\wp {}^\wp\dot{\mathcal{F}}_\alpha(\mathcal{U})\right\rangle); \mathcal{U} \in \mathcal{Q}, \wp \in \Delta, \alpha = 1,2,3,\dots,m\}$

Example 2.9 We consider two 4PNSs $\mathcal{M}_{\mathfrak{N}_1}$ and $\mathcal{M}_{\mathfrak{N}_2}$, which can be represented as Table 3.

Table 3 4PNSs

$\mathcal{Q}$	$4PNSs$
$\mathcal{M}_{\mathfrak{R}_1}$	$\{(\mathcal{U}_1, \langle 0.611, 0.111, 0.251\rangle, \langle 0.821, 0.631, 0.111\rangle, \langle 0.721, 0.381, 0.591\rangle, \langle 0.211, 0.321, 0.411\rangle), (\mathcal{U}_2, \langle 0.443, 0.244, 0.211\rangle, \langle 0.434, 0.122, 0.322\rangle, \langle 0.865, 0.333, 0.111\rangle, \langle 0.765, 0.232, 0.652\rangle)\}$
$\mathcal{M}_{\mathfrak{R}_2}$	$\{(\mathcal{U}_1, \langle 0.321, 0.621, 0.511\rangle, \langle 0.831, 0.111, 0.921\rangle, \langle 0.521, 0.431, 0.391\rangle, \langle 0.181, 0.931, 0.821\rangle), (\mathcal{U}_2, \langle 0.112, 0.221, 0.111\rangle, \langle 0.653, 0.221, 0.234\rangle, \langle 0.766, 0.232, 0.233\rangle, \langle 0.876, 0.233, 0.122\rangle)\}$

Table 4 4PNSs

$\mathcal{Q}$	$4PNSs$
$\mathcal{M}^c_{\mathfrak{R}_1}$	$\{(\mathcal{U}_1, \langle 0.251, 0.889, 0.611\rangle, \langle 0.111, 0.369, 0.821\rangle, \langle 0.591, 0.619, 0.721\rangle, \langle 0.411, 0.679, 0.211\rangle) (\mathcal{U}_2, \langle 0.211, 0.756, 0.443\rangle, \langle 0.322, 0.878, 0.434\rangle, \langle 0.111, 0.667, 0.865\rangle, \langle 0.652, 0.768, 0.765\rangle)\}$
$\mathcal{M}_{\mathfrak{R}_1} \cup \mathcal{M}_{\mathfrak{R}_2}$	$\{(\mathcal{U}_1, \langle 0.611, 0.111, 0.251\rangle, \langle 0.831, 0.111, 0.111\rangle, \langle 0.721, 0.381, 0.391\rangle, \langle 0.211, 0.321, 0.411\rangle) (\mathcal{U}_2, \langle 0.443, 0.221, 0.111\rangle, \langle 0.653, 0.122, 0.234\rangle, \langle 0.865, 0.232, 0.111\rangle, \langle 0.876, 0.232, 0.122\rangle)\}$
$\mathcal{M}_{\mathfrak{R}_1} \cap \mathcal{M}_{\mathfrak{R}_2}$	$\{(\mathcal{U}_1, \langle 0.321, 0.621, 0.511\rangle, \langle 0.821, 0.631, 0.921\rangle, \langle 0.521, 0.431, 0.591\rangle, \langle 0.181, 0.931, 0.821\rangle) (\mathcal{U}_2, \langle 0.112, 0.244, 0.211\rangle, \langle 0.434, 0.221, 0.322\rangle, \langle 0.766, 0.333, 0.233\rangle, \langle 0.765, 0.233, 0.652\rangle)\}$

By using Definition 2.8, we evaluate some operations for both 4PNSs and the results can be represented as Table 4.

Definition 2.10 (Hashmi et al. 2020) For the MPNN $\dot{\mathcal{N}} = \left(\langle t_\alpha, i_\alpha, f_\alpha\rangle; \alpha = 1, 2, 3, \ldots, m\right)$ the score functions are given as:

$$\dot{\pounds}_1(\dot{\mathcal{N}}) = \frac{1}{2m}\left(m + \sum_{\alpha=1}^{m}(t_\alpha - 2i_\alpha - f_\alpha)\right); \quad \dot{\pounds}_1(\dot{\mathcal{N}}) \in [0, 1]$$

$$\dot{\pounds}_2(\dot{\mathcal{N}}) = \frac{1}{m}\sum_{\alpha=1}^{m}(t_\alpha - 2i_\alpha - f_\alpha); \quad \dot{\pounds}_2(\dot{\mathcal{N}}) \in [-1, 1]$$

If the two MPNNs produce the same score values, then for further ranking we will use improved score or accuracy functions defined as

$$\dot{\pounds}_3(\dot{\mathcal{N}}) = \frac{1}{2m}\left(m + \sum_{\alpha=1}^{m}\left((t_\alpha - 2i_\alpha - f_\alpha)(2 - t_\alpha - f_\alpha)\right)\right); \quad \dot{\pounds}_3(\dot{\mathcal{N}}) \in [-1, 1]$$

In the case, when $t_\alpha + f_\alpha = 1; \forall \alpha = 1, 2, \ldots, m$, then $\dot{\pounds}_3(\dot{\mathcal{N}})$ reduces to $\dot{\pounds}_1(\dot{\mathcal{N}})$.

Definition 2.11 If $\dot{\mathcal{N}}_1$ and $\dot{\mathcal{N}}_2$ are two MPNNs, then the following results hold for score values:

(a) If $\dot{\pounds}_1(\dot{\mathcal{N}}_1) \succ \dot{\pounds}_1(\dot{\mathcal{N}}_2)$ then $\dot{\mathcal{N}}_1 \succ \dot{\mathcal{N}}_2$.
(b) If $\dot{\pounds}_1(\dot{\mathcal{N}}_1) = \dot{\pounds}_1(\dot{\mathcal{N}}_2)$ then
(1) If $\dot{\pounds}_2(\dot{\mathcal{N}}_1) \succ \dot{\pounds}_2(\dot{\mathcal{N}}_2)$ then $\dot{\mathcal{N}}_1 \succ \dot{\mathcal{N}}_2$.
(2) If $\dot{\pounds}_2(\dot{\mathcal{N}}_1) = \dot{\pounds}_2(\dot{\mathcal{N}}_2)$ then

(i) If $\dot{\pounds}_3(\dot{\mathcal{N}}_1) \succ \dot{\pounds}_3(\dot{\mathcal{N}}_2)$ then $\dot{\mathcal{N}}_1 \succ \dot{\mathcal{N}}_2$.
(ii) If $\dot{\pounds}_3(\dot{\mathcal{N}}_1) \prec \dot{\pounds}_3(\dot{\mathcal{N}}_2)$ then $\dot{\mathcal{N}}_1 \prec \dot{\mathcal{N}}_2$.
(iii) If $\dot{\pounds}_3(\dot{\mathcal{N}}_1) = \dot{\pounds}_3(\dot{\mathcal{N}}_2)$ then $\dot{\mathcal{N}}_1 \sim \dot{\mathcal{N}}_2$.

3 m-polar neutrosophic soft set (MPNSS)

In this section, we present the notion of m-polar neutrosophic soft set (MPNSS). This hybrid structure is the amalgamation of MPFS, soft set and neutrosophic set. We create this model to deal with the ambiguities having multi-polarity, positiveness, indeterminacy and negativeness in the input information for the alternatives under the effect of parameterizations. In neutrosophic set, we can just talk about the relation of a single attribute with the alternatives having neutrosophy in its nature. On the other hand, if we want to deal with the problems having multiple properties of the object then neutrosophic set does not go in that respect. For the same role, if we handle the difficulties in decision-making problems only with the MPFS, then we can freely assign the multiple grades to the objects under multiplicity. We can only target the positiveness of objects and do not get any evidence about the falsity and indeterminacy parts. In the hybrid structure of MPNS, we can deal with the multiple criteria, positiveness, indeterminacy and negativeness of the alternatives, but we do not have any information about the parameterizations. Parameterizations are important to deal with the uncertainties and we get superior results under the effect of parameterizations of the input information. So, to fill out the research cavity and to remove these difficulties, we construct this hybrid model of MPNSS. It handle the uncertainness and ambiguities in the presence of multi-polarity, positiveness, indeterminacy and negativeness of alternatives under the suitable parameterizations. So, this is authentic and general concept and one can see its impact in the whole manuscript, especially in Sect. 4. In this part, we discuss some of its operations, mapping and inverse mapping with the help of illustrations.

Definition 3.1 For the reference set $\mathcal{Q}$ and set of attributes $\mathcal{A}$. If $\mathcal{G} \subseteq \mathcal{A}$, then we define a mapping $\mathcal{Y} : \mathcal{G} \to MPN(\mathcal{Q})$, where $MPN(\mathcal{Q})$ is an assembling of all MPN-subsets of $\mathcal{Q}$. Then $\mathcal{Y}_{\mathcal{G}}$ or $(\mathcal{Y}, \mathcal{G})$ is said to be an MPNSS in $\mathcal{Q}$ and can be scripted as

$$\mathcal{Y}_{\mathcal{G}} = \left\{ \mathcal{Y}_{\wp} = \{\mathcal{U}, \langle {}^{\alpha}\dot{\mathcal{T}}_{\wp}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{I}}_{\wp}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{F}}_{\wp}(\mathcal{U})\rangle\} : \mathcal{U} \in \mathcal{Q}, \wp \in \mathcal{G}; \alpha = 1, 2, 3, \dots, m \right\}$$

where ${}^{\alpha}\dot{\mathcal{T}}_{\wp}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{I}}_{\wp}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{F}}_{\wp}(\mathcal{U}) \in [0, 1]$ satisfying the constraint $0 \le {}^{\alpha}\dot{\mathcal{T}}_{\wp}(\mathcal{U}) + {}^{\alpha}\dot{\mathcal{I}}_{\wp}(\mathcal{U}) + {}^{\alpha}\dot{\mathcal{F}}_{\wp}(\mathcal{U}) \le 3, \forall \alpha = 1, 2, 3, \dots, m$.

The assembling of all MPNSSs over over the reference set $\mathcal{Q}$ and set of attributes $\mathcal{A}$ is represented as $MPNS(\mathcal{Q}_{\mathcal{A}})$ and is said to be the class of all MPNSSs.

Example 3.2 Let $\mathcal{Q} = \{\mathcal{U}_1, \mathcal{U}_2\}$ be the assembling of patients suffering from diarrhea, typhoid and skin infections. $\mathcal{A} = \{\wp_1, \wp_2, \wp_3\}$ be the collection of different antibiotics used to cure diarrhea, typhoid and skin infection given as

$$\wp_1 = \text{Antibiotic X}$$
$$\wp_2 = \text{Antibiotic Y}$$
$$\wp_3 = \text{Antibiotic Z}$$

As we know that sometimes a single antibiotic is use for multi-purposes or to cure multiple diseases e.g antibiotic X can be used for bacterial infections, middle ear infections, pneumonia, bone and joint infections, skin infections and endocarditis etc.

So for the given input data doctor can suggest $\wp_1, \wp_2$ and $\wp_3$ to the listed patients for their health problems. In this case "m" (multi-polarity) represents multiple diseases appearing in the patients. Then we construct (for m=3) 3PNSS in $\mathcal{Q}$ and $\mathcal{G} = \{\wp_1, \wp_2\} \subseteq \mathcal{A}$ scripted as

$$\begin{aligned}\mathcal{Y}_{\mathcal{G}} = \{\mathcal{Y}_{\wp_1} &= \{(\mathcal{U}_1, \langle 0.68, 0.13, 0.14\rangle, \langle 0.76, 0.21, 0.23\rangle, \langle 0.83, 0.11, 0.13\rangle),\\ &(\mathcal{U}_2, \langle 0.73, 0.18, 0.21\rangle, \langle 0.71, 0.24, 0.23\rangle, \langle 0.81, 0.12, 0.18\rangle)\},\\ \mathcal{Y}_{\wp_2} &= \{(\mathcal{U}_1, \langle 0.73, 0.13, 0.25\rangle, \langle 0.67, 0.21, 0.25\rangle, \langle 0.83, 0.17, 0.21\rangle),\\ &\mathcal{U}_2, \langle 0.81, 0.13, 0.23\rangle, \langle 0.61, 0.31, 0.35\rangle, \langle 0.63, 0.31, 0.28\rangle)\}\}\end{aligned}$$

In this 3PNSS we take "m=3" (for three diseases diarrhea, typhoid and skin infection). For the patient $\mathcal{U}_1$ the first triplet $\langle 0.68, 0.13, 0.14\rangle$, shows that $\wp_1$ (antibiotic X) has 68% good effects, 13% neutral and 14% bad effects for the disease diarrhea. The next two triplets for $\mathcal{U}_1$ represents the same effect for the diseases typhoid and skin infection respectively. On the same pattern, we can observe the next numerical terms given in 3PNSS.

Definition 3.3 The MPNSS scripted as $\mathcal{Y}_{\phi} = \{\mathcal{Y}_{\wp} = \{(\mathcal{U}, \underbrace{\langle 0,1,1\rangle, \langle 0,1,1\rangle, \ldots, \langle 0,1,1\rangle}_{\text{m times}})\} : \mathcal{U} \in \mathcal{Q}, \wp \in \mathcal{A}\}$ is called null MPNSS in the reference set $\mathcal{Q}$ and the set of attributes $\mathcal{A}$, i.e ${}^{\alpha}\dot{\mathcal{T}}_{\wp}(\mathcal{U}) = 0, {}^{\alpha}\dot{\mathcal{I}}_{\wp}(\mathcal{U}) = 1, {}^{\alpha}\dot{\mathcal{F}}_{\wp}(\mathcal{U}) = 1$; $\forall \mathcal{U} \in \mathcal{Q}, \wp \in \mathcal{A}; \alpha = 1, 2, 3, \ldots, m$.

Definition 3.4 The MPNSS scripted as $\mathcal{Y}_{\mathcal{Q}} = \{\mathcal{Y}_{\wp} = \{(\mathcal{U}, \underbrace{\langle 1,0,0\rangle, \langle 1,0,0\rangle, \ldots, \langle 1,0,0\rangle}_{\text{m times}})\} : \mathcal{U} \in \mathcal{Q}, \wp \in \mathcal{A}\}$ is called absolute MPNSS in the reference set $\mathcal{Q}$ and the set of attributes $\mathcal{A}$, i.e ${}^{\alpha}\dot{\mathcal{T}}_{\wp}(\mathcal{U}) = 1, {}^{\alpha}\dot{\mathcal{I}}_{\wp}(\mathcal{U}) = 0, {}^{\alpha}\dot{\mathcal{F}}_{\wp}(\mathcal{U}) = 0$; $\forall \mathcal{U} \in \mathcal{Q}, \wp \in \mathcal{A}; \alpha = 1, 2, 3, \ldots, m$.

Definition 3.5 Let $\mathcal{Y}_{\mathcal{G}_1}$ and $\mathcal{Y}_{\mathcal{G}_2}$ are MPNSSs in $\mathcal{Q}$ and $\mathcal{G}_1, \mathcal{G}_2 \subseteq \mathcal{A}$, then $\mathcal{Y}_{\mathcal{G}_1}$ is subset of $\mathcal{Y}_{\mathcal{G}_2}$ if

(i) $\mathcal{G}_1 \subseteq \mathcal{G}_2$;
(ii) ${}^{\alpha}\dot{\mathcal{T}}_{\mathcal{G}_1}(\mathcal{U}) \leq {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{G}_2}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{G}_1}(\mathcal{U}) \geq {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{G}_2}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{G}_1}(\mathcal{U}) \geq {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{G}_2}(\mathcal{U}); \mathcal{U} \in \mathcal{Q}, \alpha = 1, 2, 3, \ldots, m$.

Definition 3.6 Let $\mathcal{Y}_{\mathcal{G}_1}, \mathcal{Y}_{\mathcal{G}_2} \in MPNS(\mathcal{Q}_{\mathcal{A}})$. Then intersection of $\mathcal{Y}_{\mathcal{G}_1}$ and $\mathcal{Y}_{\mathcal{G}_2}$ can be scripted as $\mathcal{Y}_{\mathcal{G}}$, where $\mathcal{G} = \mathcal{G}_1 \cap \mathcal{G}_2 \neq \phi$ and $\mathcal{Y}^{\mathcal{G}_1}_{\wp} \cap \mathcal{Y}^{\mathcal{G}_2}_{\wp} = \mathcal{Y}^{\mathcal{G}}_{\wp} : \forall \wp \in \mathcal{G}$. $\mathcal{Y}^{\mathcal{G}_1}_{\wp}$ and $\mathcal{Y}^{\mathcal{G}_2}_{\wp}$ are MPNSs in $\mathcal{Q}$.

Definition 3.7 Let $\mathcal{Y}_{\mathcal{G}_1}, \mathcal{Y}_{\mathcal{G}_2} \in MPNS(\mathcal{Q}_{\mathcal{A}})$. Then union of $\mathcal{Y}_{\mathcal{G}_1}$ and $\mathcal{Y}_{\mathcal{G}_2}$ can be scripted as $\mathcal{Y}_{\mathcal{G}}$, where $\mathcal{G} = \mathcal{G}_1 \cup \mathcal{G}_2$ and $\mathcal{Y}^{\mathcal{G}_1}_{\wp} \cup \mathcal{Y}^{\mathcal{G}_2}_{\wp} = \mathcal{Y}^{\mathcal{G}}_{\wp} : \forall \wp \in \mathcal{G}$. $\mathcal{Y}^{\mathcal{G}_1}_{\wp}$ and $\mathcal{Y}^{\mathcal{G}_2}_{\wp}$ are MPNSs in $\mathcal{Q}$.

Definition 3.8 Let $MPNS(\mathcal{Q}_{\mathcal{A}})$ and $MPNS(\mathcal{R}_{\mathcal{B}})$ are two classes over the reference set $\mathcal{Q}$ and $\mathcal{R}$ corresponding to the assembling of attributes $\mathcal{A}$ and $\mathcal{B}$ respectively. We consider two mathematical functions $\eta : \mathcal{Q} \to \mathcal{R}$ and $\xi : \mathcal{A} \to \mathcal{B}$, defining a mapping $\dot{\delta} = (\eta, \xi) : MPNS(\mathcal{Q}_{\mathcal{A}}) \to MPNS(\mathcal{R}_{\mathcal{B}})$ for the m-polar neutrosophic soft set (MPNSS) $\mathcal{Y}_{\mathcal{G}} \in MPNS(\mathcal{Q}_{\mathcal{A}})$ with $\mathcal{G} \subseteq \mathcal{A}$. The image of $\mathcal{Y}_{\mathcal{G}}$ under $\dot{\delta} = (\eta, \xi)$ is $\dot{\delta}(\mathcal{Y}_{\mathcal{G}})$ which is an MPNSS in $MPNS(\mathcal{R}_{\mathcal{B}})$. Mathematically, it can be scripted as

$$\begin{aligned}&\dot{\delta}(\mathcal{Y}_{\mathcal{G}})\\&\quad = \left\{\mathcal{Y}_{\wp'}\right\} = \{(\mathcal{U}', \langle {}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}})}(\wp')(\mathcal{U}'), {}^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}})}(\wp')(\mathcal{U}'), {}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}})}(\wp')(\mathcal{U}')\rangle) : \mathcal{U}' \in \mathcal{R}, \wp' \in \xi(\mathcal{A}) \subseteq \mathcal{B}\}\end{aligned}$$

where $\alpha = 1, 2, 3, \ldots, m$ and

$${}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}})}(\wp')(\mathcal{U}') = \begin{cases} \bigvee\limits_{\mathcal{U} \in \eta^{-1}(\mathcal{U}')} \left(\bigvee\limits_{\wp \in \xi^{-1}(\wp') \cap \mathcal{G}} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}_{\wp}} \right)(\mathcal{U}); & \text{if}, \eta^{-1}(\mathcal{U}') \neq \phi \text{ and } \xi^{-1}(\wp') \cap \mathcal{G} \neq \phi \\ 0; & \text{otherwise} \end{cases} \tag{1}$$

$${}^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}})}(\wp')(\mathcal{U}') = \begin{cases} \bigwedge\limits_{\mathcal{U} \in \eta^{-1}(\mathcal{U}')} \left(\bigwedge\limits_{\wp \in \xi^{-1}(\wp') \cap \mathcal{G}} {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}_{\wp}} \right)(\mathcal{U}); & \text{if}, \eta^{-1}(\mathcal{U}') \neq \phi \text{ and } \xi^{-1}(\wp') \cap \mathcal{G} \neq \phi \\ 1; & \text{otherwise} \end{cases} \tag{2}$$

$${}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}})}(\wp')(\mathcal{U}') = \begin{cases} \bigwedge\limits_{\mathcal{U} \in \eta^{-1}(\mathcal{U}')} \left(\bigwedge\limits_{\wp \in \xi^{-1}(\wp') \cap \mathcal{G}} {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}_{\wp}} \right)(\mathcal{U}); & \text{if}, \eta^{-1}(\mathcal{U}') \neq \phi \text{ and } \xi^{-1}(\wp') \cap \mathcal{G} \neq \phi \\ 1; & \text{otherwise} \end{cases} \tag{3}$$

Combining Eqs. (1), (2) and (3) we get

$$\dot{\delta}(\mathcal{Y}_{\mathcal{G}})(\wp')(\mathcal{U}') = \begin{cases} \bigcup\limits_{\mathcal{U} \in \eta^{-1}(\mathcal{U}')} \left(\bigcup\limits_{\wp \in \xi^{-1}(\wp') \cap \mathcal{G}} \mathcal{Y}_{\wp} \right)(\mathcal{U}); & \text{if}, \eta^{-1}(\mathcal{U}') \neq \phi \text{ and } \xi^{-1}(\wp') \cap \mathcal{G} \neq \phi \\ \mathcal{Y}_{\phi}; & \text{otherwise} \end{cases} \tag{4}$$

Then $\dot{\delta}(\mathcal{Y}_{\mathcal{G}})$ is called image of $\mathcal{Y}_{\mathcal{G}}$ under the mapping $\dot{\delta}$. We can calculate this image by using (4).

Definition 3.9 Let $MPNS(\mathcal{Q}_{\mathcal{A}})$ and $MPNS(\mathcal{R}_{\mathcal{B}})$ are two classes over the reference set $\mathcal{Q}$ and $\mathcal{R}$ corresponding to the assembling of attributes $\mathcal{A}$ and $\mathcal{B}$ respectively. We consider two functions $\eta : \mathcal{Q} \to \mathcal{R}$ and $\xi : \mathcal{A} \to \mathcal{B}$, defining a mapping $\dot{\delta} = (\eta, \xi) : MPNS(\mathcal{Q}_{\mathcal{A}}) \to MPNS(\mathcal{R}_{\mathcal{B}})$ as follows: if $\mathcal{Y}_{\mathcal{G}'}$ is an MPNSS in $MPNS(\mathcal{R}_{\mathcal{B}})$ for $\mathcal{G}' \subseteq \mathcal{B}$, then we have an MPNSS $\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'})$ in $MPNS(\mathcal{Q}_{\mathcal{A}})$, which can be obtained as

$$\begin{aligned}&\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'}) = \{\mathcal{Y}_{\wp}\\&\quad = \{(\mathcal{U}, \langle {}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}})}(\wp)(\mathcal{U}), {}^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}})}(\wp)(\mathcal{U}), {}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}})}(\wp)(\mathcal{U})\rangle) : \mathcal{U} \in \mathcal{Q}, \wp \in \xi^{-1}(\mathcal{B}) \subseteq \mathcal{A}\Big\}\end{aligned}$$

where $\alpha = 1, 2, 3, \ldots, m$ and

$${}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}})}(\wp)(\mathcal{U}) = \begin{cases} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if}, \xi(\wp) \in \mathcal{G}' \\ 0; & \text{otherwise} \end{cases} \tag{5}$$

$$^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}})}(\wp)(\mathcal{U}) = \begin{cases} ^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if}, \xi(\wp) \in \mathcal{G}' \\ 1; & \text{otherwise} \end{cases} \tag{6}$$

$$^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}})}(\wp)(\mathcal{U}) = \begin{cases} ^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if}, \xi(\wp) \in \mathcal{G}' \\ 1; & \text{otherwise} \end{cases} \tag{7}$$

From (5), (6) and (7) we can write that

$$\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}})(\wp)(\mathcal{U}) = \begin{cases} \mathcal{Y}_{\xi(\wp)}(\eta(\mathcal{U})); & \text{if}, \xi(\wp) \in \mathcal{G}' \\ \mathcal{Y}_{\phi}; & \text{otherwise} \end{cases} \tag{8}$$

Example 3.10 Consider the reference sets $\mathcal{Q} = \{\mathcal{U}_1, \mathcal{U}_2\}$ and $\mathcal{R} = \{\mathcal{U}'_1, \mathcal{U}'_2\}$. Let $\mathcal{A} = \mathcal{G} = \{\wp_1, \wp_2, \wp_3\}$ and $\mathcal{B} = \mathcal{G} = \{\wp'_1, \wp'_2, \wp'_3\}$ be the corresponding collection of decision variables respectively. Suppose that $MPNS(\mathcal{Q}_{\mathcal{A}})$ and $MPNS(\mathcal{R}_{\mathcal{B}})$ are two classes of MPNSSs. Then we define mappings $\eta : \mathcal{Q} \to \mathcal{R}$ and $\xi : \mathcal{A} \to \mathcal{B}$ given as

$$\begin{aligned} &\xi(\wp_1) = \wp'_1, \quad \xi(\wp_2) = \wp'_2, \quad \xi(\wp_3) = \wp'_2 \\ &\eta(\mathcal{U}_1) = \mathcal{U}'_2, \quad \eta(\mathcal{U}_2) = \mathcal{U}'_1 \end{aligned}$$

Let $\mathcal{Y}_{\mathcal{G}}$ and $\mathcal{Y}'_{\mathcal{G}}$ be two 2-polar neutrosophic soft sets in $MPNS(\mathcal{Q}_{\mathcal{A}})$ and $MPNS(\mathcal{R}_{\mathcal{B}})$ respectively, given as

$$\mathcal{Y}_{\mathcal{G}} = \left\{ \begin{array}{l} \mathcal{Y}_{\wp_1} = \{(\mathcal{U}_1, \langle 0.58, 0.51, 0.43\rangle, \langle 0.71, 0.25, 0.11\rangle), (\mathcal{U}_2, \langle 0.56, 0.31, 0.21\rangle, \langle 0.68, 0.31, 0.28\rangle)\}, \\ \mathcal{Y}_{\wp_2} = \{(\mathcal{U}_1, \langle 0.43, 0.21, 0.23\rangle, \langle 0.58, 0.61, 0.38\rangle), (\mathcal{U}_2, \langle 0.56, 0.61, 0.21\rangle, \langle 0.67, 0.21, 0.38\rangle)\}, \\ \mathcal{Y}_{\wp_3} = \{(\mathcal{U}_1, \langle 0.71, 0.21, 0.34\rangle, \langle 0.47, 0.38, 0.21\rangle), (\mathcal{U}_2, \langle 0.83, 0.12, 0.18\rangle, \langle 0.41, 0.38, 0.11\rangle)\} \end{array} \right\}$$

$$\mathcal{Y}'_{\mathcal{G}} = \left\{ \begin{array}{l} \mathcal{Y}_{\wp'_1} = \{(\mathcal{U}'_1, \langle 0.48, 0.21, 0.31\rangle, \langle 0.51, 0.38, 0.41\rangle), (\mathcal{U}'_2, \langle 0.51, 0.38, 0.21\rangle, \langle 0.48, 0.38, 0.17\rangle)\}, \\ \mathcal{Y}_{\wp'_2} = \{(\mathcal{U}'_1, \langle 0.38, 0.11, 0.23\rangle, \langle 0.78, 0.43, 0.21\rangle), (\mathcal{U}'_2, \langle 0.68, 0.23, 0.18\rangle, \langle 0.73, 0.48, 0.35\rangle)\}, \\ \mathcal{Y}_{\wp'_3} = \{(\mathcal{U}'_1, \langle 0.73, 0.21, 0.23\rangle, \langle 0.86, 0.13, 0.21\rangle), (\mathcal{U}'_2, \langle 0.68, 0.41, 0.43\rangle, \langle 0.73, 0.51, 0.61\rangle)\} \end{array} \right\}$$

Under the mapping $\dot{\delta} : MPNS(\mathcal{Q}_{\mathcal{A}}) \to MPNS(\mathcal{R}_{\mathcal{B}})$ we find the image of MPNSS $\mathcal{Y}_{\mathcal{G}}$ as follows:

$$\begin{aligned}
&\dot{\delta}(\mathcal{Y}_{\mathcal{G}})(\wp'_1)(\mathcal{U}'_1)\\
&\quad=\bigcup_{\mathcal{U}\in\eta^{-1}(\mathcal{U}'_1)}\left(\bigcup_{\wp\in\xi^{-1}(\wp'_1)\cap\mathcal{G}}\mathcal{Y}_{\wp}\right)(\mathcal{U})\\
&\quad=\bigcup_{\mathcal{U}\in\eta^{-1}(\mathcal{U}'_1)}\left(\bigcup_{\wp\in\{\wp_1\}}\mathcal{Y}_{\wp}\right)(\mathcal{U})\\
&\quad=\bigcup_{\mathcal{U}\in\{\mathcal{U}_2\}}\left(\{(\mathcal{U}_1,\langle 0.58,0.51,0.43\rangle,\langle 0.71,0.25,0.11\rangle),(\mathcal{U}_2,\langle 0.56,0.31,0.21\rangle,\langle 0.68,0.31,0.28\rangle)\}\right)\\
&\quad=(\langle 0.56,0.31,0.21\rangle,\langle 0.68,0.31,0.28\rangle)\\
&\dot{\delta}(\mathcal{Y}_{\mathcal{G}})(\wp'_2)(\mathcal{U}'_1)\\
&\quad=\bigcup_{\mathcal{U}\in\eta^{-1}(\mathcal{U}'_1)}\left(\bigcup_{\wp\in\xi^{-1}(\wp'_2)\cap\mathcal{G}}\mathcal{Y}_{\wp}\right)(\mathcal{U})\\
&\quad=\bigcup_{\mathcal{U}\in\eta^{-1}(\mathcal{U}'_1)}\left(\bigcup_{\wp\in\{\wp_2,\wp_3\}}\mathcal{Y}_{\wp}\right)(\mathcal{U})\\
&\quad=\bigcup_{\mathcal{U}\in\{\mathcal{U}_2\}}\left(\{(\mathcal{U}_1,\langle 0.71,0.21,0.23\rangle,\langle 0.58,0.38,0.21\rangle),(\mathcal{U}_2,\langle 0.83,0.12,0.18\rangle,\langle 0.67,0.21,0.11\rangle)\}\right)\\
&\quad=(\langle 0.83,0.12,0.18\rangle,\langle 0.67,0.21,0.11\rangle)
\end{aligned}$$

Similarly, we can find other values

$$\begin{aligned}
\dot{\delta}(\mathcal{Y}_{\mathcal{G}})(\wp'_1)(\mathcal{U}'_2)&=(\langle 0.58,0.51,0.43\rangle,\langle 0.71,0.25,0.11\rangle)\\
\dot{\delta}(\mathcal{Y}_{\mathcal{G}})(\wp'_2)(\mathcal{U}'_2)&=(\langle 0.71,0.21,0.23\rangle,\langle 0.58,0.31,0.21\rangle)\\
\dot{\delta}(\mathcal{Y}_{\mathcal{G}})(\wp'_3)(\mathcal{U}'_1)&=(\langle 0.00,1.00,1.00\rangle,\langle 0.00,1.00,1.00\rangle)\\
\dot{\delta}(\mathcal{Y}_{\mathcal{G}})(\wp'_3)(\mathcal{U}'_2)&=(\langle 0.00,1.00,1.00\rangle,\langle 0.00,1.00,1.00\rangle)
\end{aligned}$$

Hence we obtain $\dot{\delta}(\mathcal{Y}_{\mathcal{G}})$, which is an image of $\mathcal{Y}_{\mathcal{G}}$ under MPNS-mapping as follows:

$$\dot{\delta}(\mathcal{Y}_{\mathcal{G}})=\left\{\begin{array}{l}
\mathcal{Y}_{\wp'_1}=\{(\mathcal{U}'_1,\langle 0.56,0.31,0.21\rangle,\langle 0.68,0.31,0.28\rangle),(\mathcal{U}'_2,\langle 0.85,0.51,0.43\rangle,\langle 0.71,0.25,0.11\rangle)\},\\
\mathcal{Y}_{\wp'_2}=\{(\mathcal{U}'_1,\langle 0.83,0.12,0.18\rangle,\langle 0.67,0.21,0.11\rangle),(\mathcal{U}'_2,\langle 0.71,0.21,0.23\rangle,\langle 0.58,0.38,0.21\rangle)\},\\
\mathcal{Y}_{\wp'_3}=\{(\mathcal{U}'_1,\langle 0.00,1.00,1.00\rangle,\langle 0.00,1.00,1.00\rangle),(\mathcal{U}'_2,\langle 0.00,1.00,1.00\rangle,\langle 0.00,1.00,1.00\rangle)\}
\end{array}\right\}$$

Now, pre-image of $\mathcal{Y}'_{\mathcal{G}}$ is calculated as follows:

$$\begin{aligned}
\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}})(\mathcal{U}_1)(\wp_1)&=\mathcal{Y}_{\xi(\wp_1)}(\eta(\mathcal{U}_1))\\
&=\mathcal{Y}_{\wp'_1}(\mathcal{U}'_2)\\
&=(\langle 0.51,0.38,0.21\rangle,\langle 0.48,0.38,0.17\rangle)\\
\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}})(\mathcal{U}_2)(\wp_1)&=\mathcal{Y}_{\xi(\wp_1)}(\eta(\mathcal{U}_2))\\
&=\mathcal{Y}_{\wp'_1}(\mathcal{U}'_1)\\
&=(\langle 0.48,0.21,0.38\rangle,\langle 0.51,0.38,0.41\rangle)\\
\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}})(\mathcal{U}_1)(\wp_2)=\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}})(\mathcal{U}_1)(\wp_3)&=(\langle 0.68,0.23,0.18\rangle,\langle 0.73,0.48,0.35\rangle)\\
\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}})(\mathcal{U}_2)(\wp_2)=\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}})(\mathcal{U}_2)(\wp_3)&=(\langle 0.38,0.11,0.23\rangle,\langle 0.78,0.43,0.21\rangle)
\end{aligned}$$

Thus the inverse image of $\mathcal{Y}'_{\mathcal{G}}$ can be written as

$$\dot{\delta}^{-1}(\mathcal{Y}'_{\mathcal{G}}) = \left\{ \begin{array}{l} \mathcal{Y}_{\wp_1} = \{(\mathcal{U}_1, \langle 0.51, 0.38, 0.21\rangle, \langle 0.48, 0.31, 0.17\rangle), (\mathcal{U}_2, \langle 0.48, 0.21, 0.38\rangle, \langle 0.51, 0.38, 0.41\rangle)\}, \\ \mathcal{Y}_{\wp_2} = \{(\mathcal{U}_1, \langle 0.68, 0.23, 0.18\rangle, \langle 0.73, 0.48, 0.35\rangle), (\mathcal{U}_2, \langle 0.38, 0.11, 0.23\rangle, \langle 0.78, 0.43, 0.21\rangle)\}, \\ \mathcal{Y}_{\wp_3} = \{(\mathcal{U}_1, \langle 0.68, 0.23, 0.18\rangle, \langle 0.73, 0.48, 0.35\rangle), (\mathcal{U}_2, \langle 0.38, 0.11, 0.23\rangle, \langle 0.78, 0.43, 0.21\rangle)\} \end{array} \right\}$$

Remark

(i) If η and ξ are injective MPNS-mappings then $\dot{\delta} = (\eta, \xi)$ is also injective.
(ii) An MPNS-mapping $\dot{\delta} = (\eta, \xi)$ is surjective if η and ξ are surjective MPNS-mappings.
(iii) An MPNS-mapping $\dot{\delta} = (\eta, \xi)$ is bijective if η and ξ are bijective MPNS-mappings.

Example 3.11 Let $\mathcal{Q} = \{\mathcal{U}_1, \mathcal{U}_2\}$ and $\mathcal{R} = \{\mathcal{U}'_1, \mathcal{U}'_2\}$ be the reference sets and $\mathcal{A} = \mathcal{G} = \{\wp_1, \wp_2\}$ with $\mathcal{B} = \mathcal{G}' = \{\wp'_1, \wp'_2\}$ be the corresponding collection of set of attributes respectively. Suppose that $MPNS(\mathcal{Q}_{\mathcal{A}})$ and $MPNS(\mathcal{R}_{\mathcal{B}})$ are two classes of MPNSSs. Then the mappings $\eta : \mathcal{Q} \rightarrow \mathcal{R}$ and $\xi : \mathcal{A} \rightarrow \mathcal{B}$ can be defined as

$$\xi(\wp_1) = \wp'_2, \quad \xi(\wp_2) = \wp'_1 \quad \eta(\mathcal{U}_1) = \mathcal{U}'_2 \quad \eta(\mathcal{U}_2) = \mathcal{U}'_1$$

Let $\mathcal{Y}_{\mathcal{G}}$ and $\mathcal{Y}'_{\mathcal{G}}$ be two 3PNSSs in $MPNS(\mathcal{Q}_{\mathcal{A}})$ and $MPNS(\mathcal{R}_{\mathcal{B}})$ respectively, given as

$$\dot{\delta}(\mathcal{Y}_{\mathcal{G}}) = \left\{ \begin{array}{l} \mathcal{Y}_{\wp_1} = \{(\mathcal{U}_1, \langle 0.58, 0.21, 0.41\rangle, \langle 0.38, 0.59, 0.35\rangle, \langle 0.78, 0.18, 0.31\rangle), \\ \qquad (\mathcal{U}_2, \langle 0.73, 0.18, 0.31\rangle, \langle 0.81, 0.21, 0.41\rangle, \langle 0.51, 0.23, 0.18\rangle)\}, \\ \mathcal{Y}_{\wp_2} = \{(\mathcal{U}_1, \langle 0.73, 0.41, 0.38\rangle, \langle 0.81, 0.23, 0.17\rangle, \langle 0.73, 0.17, 0.24\rangle), \\ \qquad (\mathcal{U}_2, \langle 0.38, 0.11, 0.18\rangle, \langle 0.31, 0.12, 0.11\rangle, \langle 0.54, 0.21, 0.31\rangle)\}, \end{array} \right\}$$

$$\dot{\delta}(\mathcal{Y}'_{\mathcal{G}}) = \left\{ \begin{array}{l} \mathcal{Y}_{\wp'_1} = \{(\mathcal{U}'_1, \langle 0.38, 0.21, 0.11\rangle, \langle 0.51, 0.43, 0.38\rangle, \langle 0.67, 0.25, 0.18\rangle), \\ \qquad (\mathcal{U}'_2, \langle 0.41, 0.21, 0.11\rangle, \langle 0.87, 0.21, 0.17\rangle, \langle 0.38, 0.21, 0.11\rangle)\}, \\ \mathcal{Y}_{\wp'_2} = \{(\mathcal{U}'_1, \langle 0.41, 0.21, 0.11\rangle, \langle 0.58, 0.23, 0.17\rangle, \langle 0.38, 0.17, 0.23\rangle), \\ \qquad (\mathcal{U}'_2, \langle 0.73, 0.41, 0.21\rangle, \langle 0.81, 0.37, 0.18\rangle, \langle 0.58, 0.21, 0.18\rangle)\}, \end{array} \right\}$$

It is obvious that $\dot{\delta} = (\eta, \xi) : MPNS(\mathcal{Q}_{\mathcal{A}}) \rightarrow MPNS(\mathcal{R}_{\mathcal{B}})$ is bijective MPNS-mapping.

Definition 3.12 Consider that $\dot{\delta} = (\eta, \xi) : MPNS(\mathcal{Q}_{\mathcal{A}}) \rightarrow MPNS(\mathcal{R}_{\mathcal{B}})$ is an MPNS-mapping, where $\mathcal{Y}_{\mathcal{G}_1}$ and $\mathcal{Y}_{\mathcal{G}_2}$ are MPNSSs over $MPNS(\mathcal{Q}_{\mathcal{A}})$ for $\wp' \in \mathcal{G}'$ and $\mathcal{U}' \in \mathcal{R}$ then we can write the following

$$\left(\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1}) \cup \dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})\right)(\wp')(\mathcal{U}') = \dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})(\wp')(\mathcal{U}') \cup \dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})(\wp')(\mathcal{U}')$$
$$\left(\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1}) \cap \dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})\right)(\wp')(\mathcal{U}') = \dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})(\wp')(\mathcal{U}') \cap \dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})(\wp')(\mathcal{U}')$$

Definition 3.13 Consider that $\dot{\delta} = (\eta, \xi) : MPNS(\mathcal{Q}_{\mathcal{A}}) \rightarrow MPNS(\mathcal{R}_{\mathcal{B}})$ is an MPNS-mapping, where $\mathcal{Y}_{\mathcal{G}'_1}$ and $\mathcal{Y}_{\mathcal{G}'_2}$ are MPNSSs over $MPNS(\mathcal{R}_{\mathcal{B}})$ for $\wp \in \mathcal{G}$ and $\mathcal{U} \in \mathcal{Q}$ then we can write the following

$$\left(\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1}) \cup \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_2})\right)(\wp)(\mathcal{U}) = \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1})(\wp)(\mathcal{U}) \cup \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_2})(\wp)(\mathcal{U})$$
$$\left(\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1}) \cap \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_2})\right)(\wp)(\mathcal{U}) = \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1})(\wp)(\mathcal{U}) \cap \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_2})(\wp)(\mathcal{U})$$

Theorem 3.14 *Consider that $\mathcal{Y}_{\mathcal{G}_1}, \mathcal{Y}_{\mathcal{G}_2}$ and $\mathcal{Y}_{\mathcal{G}_3} \in MPNS(\mathcal{Q}_{\mathcal{A}})$ are MPNSSs. Then for the mapping $\dot{\delta} = (\eta, \xi) : MPNS(\mathcal{Q}_{\mathcal{A}}) \rightarrow MPNS(\mathcal{R}_{\mathcal{B}})$, we can define the following:*

(i) $\dot{\delta}(\mathcal{Y}_{\phi}) = \mathcal{Y}_{\phi}$,
(ii) $\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1} \cup \mathcal{Y}_{\mathcal{G}_2}) = \dot{\delta}(\mathcal{Y}_{\mathcal{G}_1}) \cup \dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})$,
(iii) $\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1} \cap \mathcal{Y}_{\mathcal{G}_2}) \subseteq \dot{\delta}(\mathcal{Y}_{\mathcal{G}_1}) \cap \dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})$,
(iv) $\mathcal{Y}_{\mathcal{G}_1} \subseteq \dot{\delta}^{-1}(\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1}))$. *The equality does not hold if* $\eta : \mathcal{Q} \to \mathcal{R}$ *is an injective mapping.*
(v) If $\mathcal{Y}_{\mathcal{G}_1} \subseteq \mathcal{Y}_{\mathcal{G}_2}$ *then* $\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1}) \subseteq \dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})$.

Proof

(i) This is obvious.
(ii) Suppose that for $\wp' \in \xi(\mathcal{A}) \subseteq \mathcal{B}$ and $\mathcal{U}' \in \mathcal{R}$, we want to show that $\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1} \cup \mathcal{Y}_{\mathcal{G}_2})(\wp')(\mathcal{U}') = \dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})(\wp')(\mathcal{U}') \cup \dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})(\wp')(\mathcal{U}')$. We can write that $\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1} \cup \mathcal{Y}_{\mathcal{G}_2})(\wp')(\mathcal{U}') = \dot{\delta}(\mathcal{K}_{\mathcal{G}_1 \cup \mathcal{G}_2})(\wp')(\mathcal{U}')$.

$$
{}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}(\mathcal{K}_{\mathcal{G}_1 \cup \mathcal{G}_2})}(\wp')(\mathcal{U}') = \begin{cases} \max\limits_{\mathcal{U} \in \eta^{-1}(\mathcal{U}')} \left(\max\limits_{\wp \in \xi^{-1}(\wp') \cap (\mathcal{G}_1 \cup \mathcal{G}_2)} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{K}_{\wp}} \right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}') \neq \phi, \xi^{-1}(\wp') \cap (\mathcal{G}_1 \cup \mathcal{G}_2) \neq \phi \\ 0; & \text{otherwise} \end{cases}
$$

$$
{}^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}(\mathcal{K}_{\mathcal{G}_1 \cup \mathcal{G}_2})}(\wp')(\mathcal{U}') = \begin{cases} \min\limits_{\mathcal{U} \in \eta^{-1}(\mathcal{U}')} \left(\min\limits_{\wp \in \xi^{-1}(\wp') \cap (\mathcal{G}_1 \cup \mathcal{G}_2)} {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{K}_{\wp}} \right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}') \neq \phi, \xi^{-1}(\wp') \cap (\mathcal{G}_1 \cup \mathcal{G}_2) \neq \phi \\ 1; & \text{otherwise} \end{cases}
$$

$$
{}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}(\mathcal{K}_{\mathcal{G}_1 \cup \mathcal{G}_2})}(\wp')(\mathcal{U}') = \begin{cases} \min\limits_{\mathcal{U} \in \eta^{-1}(\mathcal{U}')} \left(\min\limits_{\wp \in \xi^{-1}(\wp') \cap (\mathcal{G}_1 \cup \mathcal{G}_2)} {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{K}_{\wp}} \right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}') \neq \phi, \xi^{-1}(\wp') \cap (\mathcal{G}_1 \cup \mathcal{G}_2) \neq \phi \\ 1; & \text{otherwise} \end{cases}
$$

$$
\dot{\delta}(\mathcal{K}_{\mathcal{G}_1 \cup \mathcal{G}_2})(\wp')(\mathcal{U}') = \begin{cases} \bigcup\limits_{\mathcal{U} \in \eta^{-1}(\mathcal{U}')} \left(\bigcup\limits_{\wp \in \xi^{-1}(\wp') \cap (\mathcal{G}_1 \cup \mathcal{G}_2)} \mathcal{K}_{\wp} \right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}') \neq \phi, \xi^{-1}(\wp') \cap (\mathcal{G}_1 \cup \mathcal{G}_2) \neq \phi \\ \mathcal{Y}_{\phi}; & \text{otherwise} \end{cases}
$$

where

$$
\mathcal{K}_{\wp} = \begin{cases} \mathcal{Y}_{\wp}^{\mathcal{G}_1}; & \text{if } \wp \in \mathcal{G}_1 - \mathcal{G}_2 \\ \mathcal{Y}_{\wp}^{\mathcal{G}_2}; & \text{if } \wp \in \mathcal{G}_2 - \mathcal{G}_1 \\ \mathcal{Y}_{\wp}^{\mathcal{G}_1} \cup \mathcal{Y}_{\wp}^{\mathcal{G}_2}; & \text{if } \wp \in \mathcal{G}_1 \cap \mathcal{G}_2 \end{cases}
$$

for some $\wp \in \xi^{-1}(\wp') \cap (\mathcal{G}_1 \cup \mathcal{G}_2)$. For non-trivial case we can write that

$$
\dot{\delta}(\mathcal{K}_{\mathcal{G}_1 \cup \mathcal{G}_2})(\wp')(\mathcal{U}') = \bigcup_{\mathcal{U} \in \eta^{-1}(\mathcal{U}')} \left(\bigcup \begin{cases} \mathcal{Y}_{\wp}^{\mathcal{G}_1}(\mathcal{U}); & \text{if } \wp \in (\mathcal{G}_1 - \mathcal{G}_2) \cap \xi^{-1}(\wp') \\ \mathcal{Y}_{\wp}^{\mathcal{G}_2}(\mathcal{U}); & \text{if } \wp \in (\mathcal{G}_2 - \mathcal{G}_1) \cap \xi^{-1}(\wp') \\ (\mathcal{Y}_{\wp}^{\mathcal{G}_1} \cup \mathcal{Y}_{\wp}^{\mathcal{G}_2})(\mathcal{U}); & \text{if } \wp \in \mathcal{G}_1 \cap \mathcal{G}_2 \cap \xi^{-1}(\wp') \end{cases} \right) \quad (9)
$$

Now for right-hand side we can write that $\dot{\delta}(\mathcal{K}_{\mathcal{G}_1\cup\mathcal{G}_2})(\wp')(\mathcal{U}') = \dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})(\wp')(\mathcal{U}') \cup \dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})(\wp')(\mathcal{U}')$

$$\begin{aligned}
&{}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})}(\wp')(\mathcal{U}') \cup {}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})}(\wp')(\mathcal{U}') = \left(\max_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\ \max_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_1}}(\mathcal{U})\right)\\
&\bigvee\left(\max_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\ \max_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_2}}(\mathcal{U})\right)\\
&= \max_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\ \max_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)}\left(\max\left({}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_1}}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_2}}(\mathcal{U})\right)\right) \qquad (10)\\
&= \max_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\max\begin{cases} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_1}}(\mathcal{U}); & \text{if } \wp\in(\mathcal{G}_1-\mathcal{G}_2)\cap\xi^{-1}(\wp')\\ {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_2}}(\mathcal{U}); & \text{if } \wp\in(\mathcal{G}_2-\mathcal{G}_1)\cap\xi^{-1}(\wp')\\ \max({}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_1}}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_2}}(\mathcal{U})); & \text{if } \wp\in\mathcal{G}_1\cap\mathcal{G}_2\cap\xi^{-1}(\wp')\end{cases}\right)
\end{aligned}$$

$$\begin{aligned}
&{}^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})}(\wp')(\mathcal{U}') \cup {}^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})}(\wp')(\mathcal{U}') = \left(\min_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\ \min_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_1}}(\mathcal{U})\right)\\
&\bigwedge\left(\min_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\ \min_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_2}}(\mathcal{U})\right)\\
&= \min_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\ \min_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)}\left(\min\left({}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_1}}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_2}}(\mathcal{U})\right)\right) \qquad (11)\\
&= \min_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\min\begin{cases} {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_1}}(\mathcal{U}); & \text{if } \wp\in(\mathcal{G}_1-\mathcal{G}_2)\cap\xi^{-1}(\wp')\\ {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_2}}(\mathcal{U}); & \text{if } \wp\in(\mathcal{G}_2-\mathcal{G}_1)\cap\xi^{-1}(\wp')\\ \min({}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_1}}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_2}}(\mathcal{U})); & \text{if } \wp\in\mathcal{G}_1\cap\mathcal{G}_2\cap\xi^{-1}(\wp')\end{cases}\right)
\end{aligned}$$

$$\begin{aligned}
&{}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})}(\wp')(\mathcal{U}') \cup {}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})}(\wp')(\mathcal{U}') = \left(\min_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\ \min_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_1}}(\mathcal{U})\right)\\
&\bigwedge\left(\min_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\ \min_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_2}}(\mathcal{U})\right)\\
&= \min_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\ \min_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)}\left(\min\left({}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_1}}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_2}}(\mathcal{U})\right)\right) \qquad (12)\\
&= \min_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\min\begin{cases} {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_1}}(\mathcal{U}); & \text{if } \wp\in(\mathcal{G}_1-\mathcal{G}_2)\cap\xi^{-1}(\wp')\\ {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_2}}(\mathcal{U}); & \text{if } \wp\in(\mathcal{G}_2-\mathcal{G}_1)\cap\xi^{-1}(\wp')\\ \min({}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_1}}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}_{\wp}^{\mathcal{G}_2}}(\mathcal{U})); & \text{if } \wp\in\mathcal{G}_1\cap\mathcal{G}_2\cap\xi^{-1}(\wp')\end{cases}\right)
\end{aligned}$$

from Eqs. (10), (11) and (12), we get that

$$\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})(\wp')(\mathcal{U}') \cup \dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})(\wp')(\mathcal{U}') = \bigcup_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\bigcup\begin{cases} \mathcal{Y}_{\wp}^{\mathcal{G}_1}(\mathcal{U}); & \text{if } \wp\in(\mathcal{G}_1-\mathcal{G}_2)\cap\xi^{-1}(\wp')\\ \mathcal{Y}_{\wp}^{\mathcal{G}_2}(\mathcal{U}); & \text{if } \wp\in(\mathcal{G}_2-\mathcal{G}_1)\cap\xi^{-1}(\wp')\\ (\mathcal{Y}_{\wp}^{\mathcal{G}_1}\cup\mathcal{Y}_{\wp}^{\mathcal{G}_2})(\mathcal{U}); & \text{if } \wp\in\mathcal{G}_1\cap\mathcal{G}_2\cap\xi^{-1}(\wp')\end{cases}\right) \qquad (13)$$

On comparing equation (9) and (13) we can write that $\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1} \cup \mathcal{Y}_{\mathcal{G}_2}) = \dot{\delta}(\mathcal{Y}_{\mathcal{G}_1}) \cup \dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})$.

(iii) Suppose that for $\wp' \in \xi(\mathcal{A}) \subseteq \mathcal{B}$ and $\mathcal{U}' \in \mathcal{R}$, we can write that $\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1}) \cap \mathcal{Y}_{\mathcal{G}_2})(\wp')(\mathcal{U}') = \dot{\delta}(\mathcal{K}_{\mathcal{G}_1\cap\mathcal{G}_2})(\wp')(\mathcal{U}')$. Now by using definition we can write that

$$
\begin{aligned}
&{}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}(\mathcal{K}_{\mathcal{G}_1\cap\mathcal{G}_2})}(\wp')(\mathcal{U}')\\
&= \begin{cases} \max\limits_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\max\limits_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{K}_{\wp}}\right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}')\neq\phi, \xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)\neq\phi\\ 0; & \text{otherwise}\end{cases}\\
&{}^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}(\mathcal{K}_{\mathcal{G}_1\cap\mathcal{G}_2})}(\wp')(\mathcal{U}')\\
&= \begin{cases} \min\limits_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\min\limits_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{K}_{\wp}}\right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}')\neq\phi, \xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)\neq\phi\\ 1; & \text{otherwise}\end{cases}\\
&{}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}(\mathcal{K}_{\mathcal{G}_1\cap\mathcal{G}_2})}(\wp')(\mathcal{U}')\\
&= \begin{cases} \min\limits_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\min\limits_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{K}_{\wp}}\right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}')\neq\phi, \xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)\neq\phi\\ 1; & \text{otherwise}\end{cases}\\
&\dot{\delta}(\mathcal{K}_{\mathcal{G}_1\cap\mathcal{G}_2})(\wp')(\mathcal{U}')\\
&= \begin{cases} \bigcup\limits_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\bigcup\limits_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} \mathcal{K}_{\wp}\right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}')\neq\phi, \xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)\neq\phi\\ \mathcal{Y}_{\phi}; & \text{otherwise}\end{cases}
\end{aligned}
$$

where $\mathcal{K}_{\wp} = \mathcal{Y}_{\mathcal{G}_1}\cap\mathcal{Y}_{\mathcal{G}_2}$

$$
\begin{aligned}
&\dot{\delta}(\mathcal{K}_{\mathcal{G}_1\cap\mathcal{G}_2})(\wp')(\mathcal{U}')\\
&= \bigcup_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\bigcup_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} (\mathcal{Y}_{\mathcal{G}_1}\cap\mathcal{Y}_{\mathcal{G}_2})\right)(\mathcal{U})\\
&= \bigcup_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\bigcup_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} (\mathcal{Y}_{\mathcal{G}_1}(\mathcal{U})\cap\mathcal{Y}_{\mathcal{G}_2})(\mathcal{U})\right)\\
&\subseteq \left(\bigcup_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\bigcup_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} (\mathcal{Y}_{\mathcal{G}_1}(\mathcal{U})\right)\right)\cap\left(\bigcup_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\bigcup_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1\cap\mathcal{G}_2)} (\mathcal{Y}_{\mathcal{G}_2}(\mathcal{U})\right)\right)\\
&= \dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})(\wp')(\mathcal{U}')\cap\dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})(\wp')(\mathcal{U}')\\
&= (\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})\cap\dot{\delta}(\mathcal{Y}_{\mathcal{G}_2}))(\wp')(\mathcal{U}')
\end{aligned}
$$

(iv) It is obvious.

(v) We suppose that $\wp'\in\xi(\mathcal{A})\subseteq\mathcal{B}$ and $\mathcal{U}'\in\mathcal{R}$ by definition of mapping we can write that

$$
{}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})}(\wp')(\mathcal{U}') = \begin{cases} \max\limits_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\max\limits_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1)} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}_{\wp}}\right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}')\neq\phi, \xi^{-1}(\wp')\cap(\mathcal{G}_1)\neq\phi\\ 0; & \text{otherwise}\end{cases} \tag{14}
$$

$$ {}^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})}(\wp')(\mathcal{U}') = \begin{cases} \min\limits_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\min\limits_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1)} {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}_{\wp}}\right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}')\neq\phi, \xi^{-1}(\wp')\cap(\mathcal{G}_1)\neq\phi \\ 1; & \text{otherwise} \end{cases} \tag{15} $$

$$ {}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})}(\wp')(\mathcal{U}') = \begin{cases} \min\limits_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\min\limits_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1)} {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}_{\wp}}\right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}')\neq\phi, \xi^{-1}(\wp')\cap(\mathcal{G}_1)\neq\phi \\ 1; & \text{otherwise} \end{cases} \tag{16} $$

Given is that $\mathcal{Y}_{\mathcal{G}_1} \subseteq \mathcal{Y}_{\mathcal{G}_2}$ which implies that ${}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})} \leq {}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})}, {}^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})} \geq {}^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})}, {}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})} \geq {}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})}$. We combining (14), (15) and (16), we get

$$ \begin{aligned} \dot{\delta}(\mathcal{Y}_{\mathcal{G}_1})(\wp')(\mathcal{U}') &= \bigcup_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\left(\bigcup_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1)}(\mathcal{Y}_{\mathcal{G}_1})\right)(\mathcal{U}) \\ &= \bigcup_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\bigcup_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_1)}\mathcal{Y}_{\mathcal{G}_1}(\mathcal{U}) \\ &\subseteq \bigcup_{\mathcal{U}\in\eta^{-1}(\mathcal{U}')}\bigcup_{\wp\in\xi^{-1}(\wp')\cap(\mathcal{G}_2)}\mathcal{Y}_{\mathcal{G}_2}(\mathcal{U}) \\ &= \dot{\delta}(\mathcal{Y}_{\mathcal{G}_2})(\wp')(\mathcal{U}') \end{aligned} $$

□

Theorem 3.15 *Consider that $\mathcal{Y}_{\mathcal{G}'_1}, \mathcal{Y}_{\mathcal{G}'_2}$ and $\mathcal{Y}_{\mathcal{G}'_3} \in MPNS(\mathcal{R}_{\mathcal{B}})$ are MPNSSs. Then for the mapping $\dot{\delta} = (\eta, \xi) : MPNS(\mathcal{Q}_{\mathcal{A}}) \to MPNS(\mathcal{R}_{\mathcal{B}})$, we can define the following:*

(i) $\dot{\delta}^{-1}(\mathcal{Y}_{\phi}) = \mathcal{Y}_{\phi}$,
i(i) $\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1} \cup \mathcal{Y}_{\mathcal{G}'_2}) = \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1}) \cup \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_2})$,
(iii) $\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1} \cap \mathcal{Y}_{\mathcal{G}'_2}) \subseteq \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1}) \cap \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_2})$,
(iv) $\dot{\delta}(\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1})) \subseteq \mathcal{Y}_{\mathcal{G}'_1}$. *The equality does not hold if $\eta : \mathcal{Q} \to \mathcal{R}$ and $\xi : \mathcal{A} \to \mathcal{B}$ is an surjective functions.*
(v) If $\mathcal{Y}_{\mathcal{G}'_1} \subseteq \mathcal{Y}_{\mathcal{G}'_2}$ *then* $\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1}) \subseteq \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_2})$.

Proof

(i) This is Obvious.
(ii) We suppose that $\wp \in \mathcal{A}$ and $\mathcal{U} \in \mathcal{Q}$, $\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1} \cup \mathcal{Y}_{\mathcal{G}'_2})(\wp)(\mathcal{U}) = \dot{\delta}^{-1}(\mathcal{K}_{\mathcal{G}'_1\cup\mathcal{G}'_2})(\wp)(\mathcal{U}) = \mathcal{K}(\xi(\wp))(\eta(\mathcal{U}))$, where $\xi(\wp) \in (\mathcal{G}'_1 \cup \mathcal{G}'_2), \eta(\mathcal{U}) \in \mathcal{R}$. By definition of inverse MPNS-mapping we can write that

$$ {}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}^{-1}(\mathcal{K}_{\mathcal{G}'_1\cup\mathcal{G}'_2})}(\wp)(\mathcal{U}) = \begin{cases} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{K}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if}, \xi(\wp) \in (\mathcal{G}'_1 \cup \mathcal{G}'_2) \\ 0; & \text{otherwise} \end{cases} \tag{17} $$

$$^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}^{-1}(\mathcal{K}_{\mathcal{G}_1' \cup \mathcal{G}_2'})}(\wp)(\mathcal{U}) = \begin{cases} ^{\alpha}\dot{\mathcal{I}}_{\mathcal{K}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if}, \xi(\wp) \in (\mathcal{G}_1' \cup \mathcal{G}_2') \\ 1; & \text{otherwise} \end{cases} \tag{18}$$

$$^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}^{-1}(\mathcal{K}_{\mathcal{G}_1' \cup \mathcal{G}_2'})}(\wp)(\mathcal{U}) = \begin{cases} ^{\alpha}\dot{\mathcal{F}}_{\mathcal{K}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if}, \xi(\wp) \in (\mathcal{G}_1' \cup \mathcal{G}_2') \\ 1; & \text{otherwise} \end{cases} \tag{19}$$

$$\dot{\delta}^{-1}(\mathcal{K}_{\mathcal{G}_1' \cup \mathcal{G}_2'})(\wp)(\mathcal{U}) = \begin{cases} \mathcal{K}_{\xi(\wp)}(\eta(\mathcal{U})); & \text{if}, \xi(\wp) \in (\mathcal{G}_1' \cup \mathcal{G}_2') \\ \mathcal{K}_{\phi}; & \text{otherwise} \end{cases} \tag{20}$$

where

$$\mathcal{K}_{\xi(\wp)} = \begin{cases} \mathcal{Y}^{\mathcal{G}_1'}_{\xi(\wp)}; & \text{if } \xi(\wp) \in \mathcal{G}_1' - \mathcal{G}_2' \\ \mathcal{Y}^{\mathcal{G}_2'}_{\xi(\wp)}; & \text{if } \xi(\wp) \in \mathcal{G}_2' - \mathcal{G}_1' \\ \mathcal{Y}^{\mathcal{G}_1'}_{\xi(\wp)} \cup \mathcal{Y}^{\mathcal{G}_2'}_{\xi(\wp)}; & \text{if } \xi(\wp) \in \mathcal{G}_1' \cap \mathcal{G}_2' \end{cases}$$

For non-trivial case we can write that

$$\dot{\delta}^{-1}(\mathcal{K}_{\mathcal{G}_1' \cup \mathcal{G}_2'})(\wp)(\mathcal{U}) = \begin{cases} \mathcal{Y}^{\mathcal{G}_1'}_{\xi(\wp)}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_1' - \mathcal{G}_2') \\ \mathcal{Y}^{\mathcal{G}_2'}_{\xi(\wp)}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_2' - \mathcal{G}_1') \\ \mathcal{Y}^{\mathcal{G}_1'}_{\xi(\wp)} \cup \mathcal{Y}^{\mathcal{G}_2'}_{\xi(\wp)}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_1' \cap \mathcal{G}_2') \end{cases} \tag{21}$$

Now for union of inverse MPNS-mapping we get $(\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}_1'}) \cup \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}_2'}))(\wp)(\mathcal{U}) = \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}_1'})(\wp)(\mathcal{U}) \cup \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}_2'})(\wp)(\mathcal{U})$

$$\begin{aligned} &^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}_1'})}(\wp)(\mathcal{U}) \cup {}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}_2'})}(\wp)(\mathcal{U}) = \max\left({}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}^{\mathcal{G}_1'}_{\xi(\wp)}}(\eta(\mathcal{U})), {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}^{\mathcal{G}_2'}_{\xi(\wp)}}(\eta(\mathcal{U}))\right) \\ &= \begin{cases} ^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}^{\mathcal{G}_1'}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_1 - \mathcal{G}_2) \\ ^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}^{\mathcal{G}_2'}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_2 - \mathcal{G}_1) \\ \max({}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}^{\mathcal{G}_1'}_{\xi(\wp)}}, {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}^{\mathcal{G}_2'}_{\xi(\wp)}})(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_1 \cap \mathcal{G}_2) \end{cases} \end{aligned} \tag{22}$$

$$\begin{aligned} &^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}_1'})}(\wp)(\mathcal{U}) \cup {}^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}_2'})}(\wp)(\mathcal{U}) = \min\left({}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}^{\mathcal{G}_1'}_{\xi(\wp)}}(\eta(\mathcal{U})), {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}^{\mathcal{G}_2'}_{\xi(\wp)}}(\eta(\mathcal{U}))\right) \\ &= \begin{cases} ^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}^{\mathcal{G}_1'}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_1 - \mathcal{G}_2) \\ ^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}^{\mathcal{G}_2'}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_2 - \mathcal{G}_1) \\ \min({}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}^{\mathcal{G}_1'}_{\xi(\wp)}}, {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}^{\mathcal{G}_2'}_{\xi(\wp)}})(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_1 \cap \mathcal{G}_2) \end{cases} \end{aligned} \tag{23}$$

$$
{}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1})}(\wp)(\mathcal{U}) \cup {}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_2})}(\wp)(\mathcal{U}) = \min\left({}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}^{\mathcal{G}'_1}_{\xi(\wp)}}(\eta(\mathcal{U})), {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}^{\mathcal{G}'_2}_{\xi(\wp)}}(\eta(\mathcal{U}))\right)
$$

$$
= \begin{cases} {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}^{\mathcal{G}'_1}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_1 - \mathcal{G}_2) \\ {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}^{\mathcal{G}'_2}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_2 - \mathcal{G}_1) \\ \min({}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}^{\mathcal{G}'_1}_{\xi(\wp)}}, {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}^{\mathcal{G}'_2}_{\xi(\wp)}})(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_1 \cap \mathcal{G}_2) \end{cases} \tag{24}
$$

from Eqs. (22), (23) and (24), we get that

$$
\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1})(\wp)(\mathcal{U}) \cup \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_2})(\wp)(\mathcal{U}) = \begin{cases} \mathcal{Y}^{\mathcal{G}'_1}_{\xi(\wp)}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_1 - \mathcal{G}_2) \\ \mathcal{Y}^{\mathcal{G}'_2}_{\xi(\wp)}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_2 - \mathcal{G}_1) \\ (\mathcal{Y}^{\mathcal{G}'_1}_{\xi(\wp)} \cup \mathcal{Y}^{\mathcal{G}'_2}_{\xi(\wp)})(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}_1 \cap \mathcal{G}_2) \end{cases} \tag{25}
$$

On comparing equation (21) and (25) we can write that $\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1} \cup \mathcal{Y}_{\mathcal{G}'_2}) = \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1}) \cup \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_2})$.

(iii) Suppose that for $\wp \in \mathcal{A}$ and $\mathcal{U} \in \mathcal{Q}$, we can write that $\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1}) \cap \mathcal{Y}_{\mathcal{G}'_2})(\wp)(\mathcal{U}) = \dot{\delta}^{-1}(\mathcal{K}_{\mathcal{G}'_1 \cap \mathcal{G}'_2})(\wp)(\mathcal{U})$. Now by using definition we can write that

$$
{}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}^{-1}(\mathcal{K}_{\mathcal{G}'_1 \cap \mathcal{G}'_2})}(\wp)(\mathcal{U}) = \begin{cases} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{K}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}'_1 \cap \mathcal{G}'_2) \\ 0; & \text{otherwise} \end{cases}
$$

$$
{}^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}^{-1}(\mathcal{K}_{\mathcal{G}'_1 \cap \mathcal{G}'_2})}(\wp)(\mathcal{U}) = \begin{cases} {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{K}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}'_1 \cap \mathcal{G}'_2) \\ 1; & \text{otherwise} \end{cases}
$$

$$
{}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}^{-1}(\mathcal{K}_{\mathcal{G}'_1 \cap \mathcal{G}'_2})}(\wp)(\mathcal{U}) = \begin{cases} {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{K}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in (\mathcal{G}'_1 \cap \mathcal{G}'_2) \\ 1; & \text{otherwise} \end{cases}
$$

$$
\dot{\delta}^{-1}(\mathcal{K}_{\mathcal{G}'_1 \cap \mathcal{G}'_2})(\wp)(\mathcal{U}) = \begin{cases} \mathcal{K}_{\xi(\wp)}(\wp)(\mathcal{U}); & \text{if } \xi(\wp) \in (\mathcal{G}'_1 \cap \mathcal{G}'_2) \\ \mathcal{K}_{\phi}; & \text{otherwise} \end{cases}
$$

where $\mathcal{K}_{\xi(\wp)} = \mathcal{Y}^{\mathcal{G}'_1}_{\xi(\wp)} \cap \mathcal{Y}^{\mathcal{G}'_2}_{\xi(\wp)}$

$$
\begin{aligned}
\dot{\delta}^{-1}(\mathcal{K}_{\mathcal{G}'_1 \cap \mathcal{G}'_2})(\wp)(\mathcal{U}) &= \mathcal{K}_{\xi(\wp)}(\eta(\mathcal{U})) \\
&= (\mathcal{Y}^{\mathcal{G}'_1}_{\xi(\wp)} \cap \mathcal{Y}^{\mathcal{G}'_2}_{\xi(\wp)})(\eta(\mathcal{U})) \\
&= \mathcal{Y}^{\mathcal{G}'_1}_{\xi(\wp)}(\eta(\mathcal{U})) \cap \mathcal{Y}^{\mathcal{G}'_2}_{\xi(\wp)}(\eta(\mathcal{U})) \\
&= \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1})(\wp)(\mathcal{U}) \cap \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_2})(\wp)(\mathcal{U}) \\
&= (\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1}) \cap \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_2}))(\wp)(\mathcal{U})
\end{aligned}
$$

(iv) It is obvious.

(v) We suppose that $\wp \in \mathcal{A}$ and $\mathcal{U} \in \mathcal{Q}$ by definition of mapping we can write that

$$
{}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1})}(\wp)(\mathcal{U}) = \begin{cases} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}^{\mathcal{G}'_1}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in \mathcal{G}'_1 \\ 0; & \text{otherwise} \end{cases} \tag{26}
$$

$$^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1})}(\wp)(\mathcal{U}) = \begin{cases} ^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}^{\mathcal{G}'_1}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in \mathcal{G}'_1 \\ 1; & \text{otherwise} \end{cases} \tag{27}$$

$$^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1})}(\wp)(\mathcal{U}) = \begin{cases} ^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}^{\mathcal{G}'_1}_{\xi(\wp)}}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in \mathcal{G}'_1 \\ 1; & \text{otherwise} \end{cases} \tag{28}$$

We combining (26), (27) and (28) and we get

$$\dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1})(\wp)(\mathcal{U}) = \begin{cases} \mathcal{Y}^{\mathcal{G}'_1}_{\xi(\wp)}(\eta(\mathcal{U})); & \text{if } \xi(\wp) \in \mathcal{G}'_1 \\ \mathcal{Y}_{\phi}; & \text{otherwise} \end{cases} \tag{29}$$

$$\begin{aligned} \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_1})(\wp)(\mathcal{U}) &= \mathcal{Y}^{\mathcal{G}'_1}_{\xi(\wp)}(\eta(\mathcal{U})) \\ &\subseteq \mathcal{Y}^{\mathcal{G}'_2}_{\xi(\wp)}(\eta(\mathcal{U})) \\ &= \dot{\delta}^{-1}(\mathcal{Y}_{\mathcal{G}'_2})(\wp)(\mathcal{U}) \end{aligned}$$

□

Definition 3.16 An MPNS-relation over $\mathcal{Q} \times \mathcal{R}$ can be scripted as

$$\mathcal{H} = \{(\mathcal{U},\mathcal{U}'), \langle {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{H}}(\mathcal{U},\mathcal{U}'), {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{H}}(\mathcal{U},\mathcal{U}'), {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{H}}(\mathcal{U},\mathcal{U}')\rangle : (\mathcal{U},\mathcal{U}') \in \mathcal{Q} \times \mathcal{R}, \alpha = 1, 2, 3, \dots, m\}$$

where ${}^{\alpha}\dot{\mathcal{T}}_{\mathcal{H}}(\mathcal{U},\mathcal{U}'), {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{H}}(\mathcal{U},\mathcal{U}'), {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{H}}(\mathcal{U},\mathcal{U}') \in [0, 1]$ are called positiveness, indeterminacy and negativeness respectively. The constraint $0 \leq {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{H}}(\mathcal{U},\mathcal{U}') + {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{H}}(\mathcal{U},\mathcal{U}') + {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{H}}(\mathcal{U},\mathcal{U}') \leq 3$ holds for each α. The assembling of all MPNS-relations can be represented as $MPN(\mathcal{Q} \times \mathcal{R})$.

Definition 3.17 Let $\mathcal{M}_1 \in MPN(\mathcal{Q} \times \mathcal{R})$ and $\mathcal{M}_2 \in MPN(\mathcal{R} \times \mathcal{S})$, then max-min composition of $\mathcal{M}_1$ and $\mathcal{M}_2$ can be denoted as $\mathcal{M}_1 \circ \mathcal{M}_2$ and defined as

$$\begin{aligned} &\mathcal{M}_1 \circ \mathcal{M}_2 \\ &= \{(\mathcal{U},\mathcal{U}'), \langle {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{M}_1 \circ \mathcal{M}_2}(\mathcal{U},\mathcal{U}'), {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{M}_1 \circ \mathcal{M}_2}(\mathcal{U},\mathcal{U}'), {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{M}_1 \circ \mathcal{M}_2}(\mathcal{U},\mathcal{U}')\rangle : \mathcal{U} \in \mathcal{Q}, \mathcal{U}'' \in \mathcal{S}, \alpha = 1, 2, 3, \dots, m\} \end{aligned}$$

where

$$\begin{aligned} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{M}_1 \circ \mathcal{M}_2}(\mathcal{U},\mathcal{U}') &= \max_{\mathcal{U}' \in \mathcal{R}} \left\{\min({}^{\alpha}\dot{\mathcal{T}}_{\mathcal{M}_1}(\mathcal{U},\mathcal{U}'), {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{M}_2}(\mathcal{U}',\mathcal{U}''))\right\} \\ {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{M}_1 \circ \mathcal{M}_2}(\mathcal{U},\mathcal{U}') &= \min_{\mathcal{U}' \in \mathcal{R}} \left\{\max({}^{\alpha}\dot{\mathcal{I}}_{\mathcal{M}_1}(\mathcal{U},\mathcal{U}'), {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{M}_2}(\mathcal{U}',\mathcal{U}''))\right\} \\ {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{M}_1 \circ \mathcal{M}_2}(\mathcal{U},\mathcal{U}') &= \min_{\mathcal{U}' \in \mathcal{R}} \left\{\max({}^{\alpha}\dot{\mathcal{F}}_{\mathcal{M}_1}(\mathcal{U},\mathcal{U}'), {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{M}_2}(\mathcal{U}',\mathcal{U}''))\right\} \end{aligned}$$

4 Application of MPNS-mappings to MPD/DID and its associated mental disorders

In this part, we discuss about the physiognomies of MPD (or DID) and its associated psychological disorders. We examine the reasons, symptoms, diagnosis and treatment of corresponding mental disorders. We introduce the novel idea of MPNSS and its associated mapping, inverse mapping with corresponding properties. We present that how our proposed structure is appropriate to set an agenda for MPD and its associated psychological disorders.

4.1 Exploration of multiple personalities disorder (MPD and its associated psychological disorder)

The analytical study of psychological disorders and mathematical modeling have a countless significance in the field of psychology and biomedical engineering. Psychological disorder is the behaviorial and mental issue that origins substantial distress or damage of personal working. To find out a suitable mathematical framework, we study the features of different kinds of psychological diseases. In medical sciences, there are different kinds of disorders but here we discuss about the three main psychological disorders defined in the "Diagnostic and Statistical Manual of Mental Disorders (DSM-5, 5th Edition)", printed by the "American Psychiatric Association":

- "Dissociative identity disorder (DID)/multiple personality disorder (MPD)".
- "Dissociative amnesia (DA)".
- "Depersonalization/derealization disorder (DD)".

4.1.1 Multiple personality disorder

In this type, the patient starts to "switching" between diverse personalities. Patient feels that he may controlled by different characters and two or more persons are chatting or surviving inside his head. He talks about the different identities and history of each character individually with unique name, personal features, different voice, gender and motions. Sometimes, the Patients suffering from MPD also have dissociative amnesia.

4.1.2 Dissociative amnesia

The major indication of this disorder is memory loss. This type of loss is diverse and severe as compared to the normal vagueness. Patient forgets the personal information, persons and events of his life, particularly from a stressful time. The attack of DA frequently happens unexpectedly and last for hours, days or months.

4.1.3 Depersonalization/derealization disorder

This disorder includes a constant sense of impartiality, detecting your moods, views and activities from a distance like someone watching a movie (depersonalization). Patient feels that the objects, persons and world is vague, illusory and imaginary (derealization). The Symptoms are intensely disturbing, may last only a few minutes or come and go over many years.

There are some mutual causes and symptoms of these disorders appearing in the patient. We listed here some symptoms connected to these disorders.

- “Depression”
- “Mood swings”
- “Suicidal tendencies”
- “Sleep disorders (insomnia, night terrors, and sleep walking)”
- “Anxiety, panic attacks, and phobias (flashbacks, reactions to stimuli or triggers)”
- “Alcohol and drug abuse”
- “Compulsions and rituals”
- “Psychotic-like symptoms (including auditory and visual hallucinations)”
- “Eating disorders”
- “Memory loss (amnesia) of certain time periods, events, people and personal information”
- “A sense of being detached from yourself and your emotions”
- “A perception of the people and things around you as distorted and unreal”
- “A blurred sense of identity”
- “Significant stress or problems in your relationships, work or other important areas of your life”
- “Inability to cope well with emotional or professional stress”
- “Mental health problems, such as depression, anxiety, and suicidal thoughts and behaviors”

There are several causes behind these disorders such as, sexual, emotional or physical abuse, trauma in childhood, war, accidents, natural disasters, loss of some loved one, etc. There are diverse techniques of treatment and diagnosis of these disorders. By conducting physical and psychiatric test or by using diagnostic conditions in the DSM-5 a doctor can diagnose the type of disorder. Dissociative disorders may include various types of treatments, but generally include psychiatric therapy, adjunctive therapy, hypnotherapy and medication. The MRI figures of brain having psychological disorders are given in Figs. 3 and 4.

Our proposed structure of MPNSS is most general and appropriate for these types of diseases. We can handle and diagnose the disorders by using exiting theories but they have their own boundaries (see Table 5). Due to these drawbacks, we cannot assemble the complete input information of a patient and it disturbs our concluding results. But our proposed model can completely covert the patient history into mathematical language without any loss of information and we get superior results for diagnosis and treatment of the patient. In Table 5 we present the semantic analysis of our proposed model with the existing theories. It clearly shows that our hybrid structure is generic, valid and strong as compared to existing methodologies and can handle these types of problems in a good way. The comparison of brain of the normal subject with disordered brain is given in Fig. 2.

In the next subsection, we talk about about the methodology, which we will use for our mathematical modeling. We construct the novel algorithm based on MPNS-mapping to diagnose the disease, to find out the finest treatment and progress of treatment episodes.

Table 5 Semantic comparison of suggested technique with some existing structures

Set theories	Advantages	Semantic disadvantages
Fuzzy sets (Zadeh 1965)	Contribute knowledge about specific property	Dose not give information about the indeterminacy and falsity of multiple criteria
Interval valued fuzzy sets (Zadeh 1975)	Truth based fuzzy interval	Does not contain multi-polar constancy with falsity and indeterminacy portions
Intuitionistic fuzzy sets (Atanassov 1984; Atanassov and Stoeva 1983; Atanassov 1986)	Detect vagueness with single criteria	Does not update about multiple criteria with indeterminacy
Neutrosophic sets (Smarandache 1998)	Contain indeterminacy about a specific property	Does not characterize multiple properties
Bipolar fuzzy sets (Zhang 1994, 1998; Zhang and Zhang 2004)	Represents bipolarity	Does not suitable in human thinking containing multiplicity and indeterminacy
m-polar fuzzy sets (Chen et al. 2014)	Give multi-polar information about the alternatives	Does not create connections for falsity and indeterminacy parts
Bipolar neutrosophic sets (Deli et al. 2015)	Represents bipolarity and indeterminacy with uncertainty	Does not suitable in human thinking containing multiplicity
m-polar neutrosophic sets (Hashmi et al. 2020)	Suitable for human thinking contains multiplicity, indeterminacy and uncertainity	Lengthy and heavy calculations in decision-making

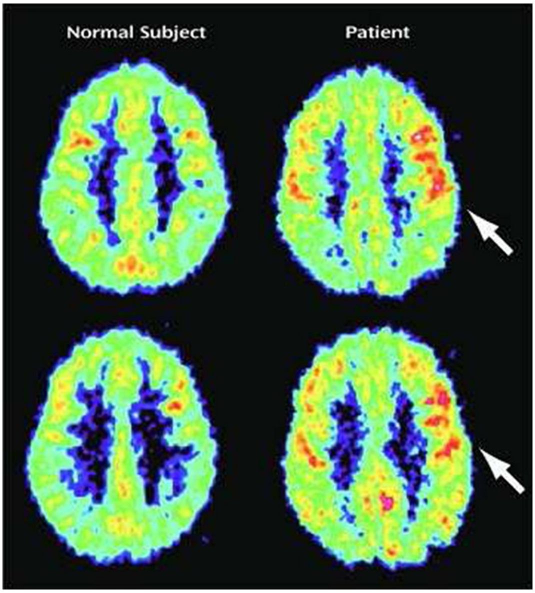

Fig. 2 Comparison of brain of normal subject with disordered brain. *Source*: "https://sites.google.com/site/brainandabnormalbehavior/_/rsrc/1283374182827/week1/pet.jpg"

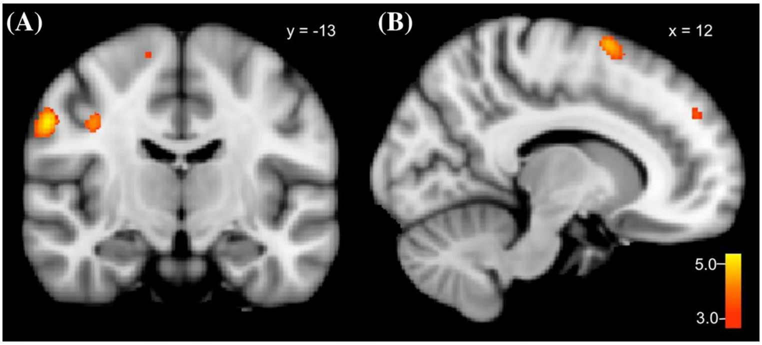

Fig. 3 Resting state activity in MPD/DID depending upon dissociative part. *Source*: "https://doi.org/10.1371/journal.pone.0098795.g001"

4.2 Methodology

4.2.1 Pre-step

A psychiatrist face several complications, when he wants to diagnose a psychological disorder of a patient, due to the parallel symptoms of psychological disorders. It is very hard to catch the difference among these categories. It means that these type of difficulties contain uncertainties and vagueness, so MPNSS is suitable to handle such kind of input information. Firstly, we set the fuzzy interval [0, 1] for different kinds of mental disorders to connect verbal information into mathematical language. For different types of disorder, we set a chart for the assessment of authentic type of psychological disorder. This chart is given in Table 6.

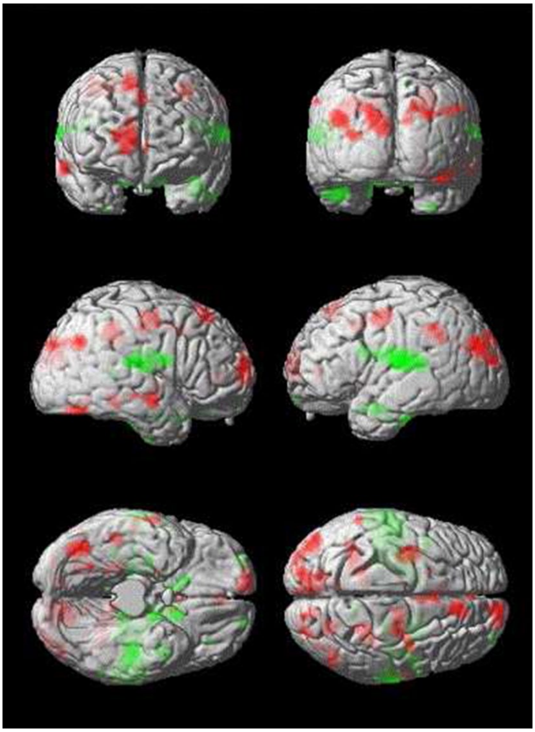

Fig. 4 Psychobiological characteristics of MPD/DID. *Source*: "https://ars.els-cdn.com/content/image/1-s2.0-S000632230600388X-gr3.jpg"

Table 6 Diagnosis chart of psychological disorders

Types of disorder	Different ranges of [0, 1]
Dissociative identity disorder (DID) or Multiple personality disorder (MPD)	[0.5, 1]
Dissociative amnesia (DA)	[0.3, 0.5)
Depersonalization (D) or derealization disorder (DD)	[0.2, 0.3)
Major depression disorder (MDD)	[0.1, 0.2)
No mental disorder	[0, 0.1)

Since, every disorder has its own concentration with the passage of time. To assemble the healthier input history of a patient, every psychiatrist want at least his 2–3 weeks data corresponding to the appearing symptoms for well diagnosis. We set additional chart of circumstances and their weekly concentration to diagnose the psychological disorder. This chart is given in Table 7. We have three phases for each type of disorder one is severe disorder, second is moderate disorder and third is mild or low disorder. In the case of moderate MPD (M-MPD) the fuzzy value are in between the interval [0.65, 0.70) for first week. For 2-3 weeks it value goes to [0.70, 0.75) and for more than 3 weeks fuzzy values goes to [0.75, 0.80). On the same pattern we can observe all the

Table 7 Associated circumstances and their weekly concentration to diagnose psychological disorder

Conditions	≤ 1 week	2–3 weeks	> 3 weeks
Severe MPD (S-MPD)	$00.80 \leq \mathcal{U} < 00.90$	$00.90 \leq \mathcal{U} < 1$	$=1$
Moderate MPD (M-MPD)	$00.65 \leq \mathcal{U} < 00.70$	$00.70 \leq \mathcal{U} < 00.75$	$00.75 \leq \mathcal{U} < 00.80$
Mild or low MPD (L-MPD)	$00.50 \leq \mathcal{U} < 00.55$	$00.55 \leq \mathcal{U} < 00.60$	$00.60 \leq \mathcal{U} < 00.65$
Severe DA (S-DA)	$00.43 \leq \mathcal{U} < 00.45$	$00.45 \leq \mathcal{U} < 00.47$	$00.47 \leq \mathcal{U} < 00.50$
Moderate DA (M-DA)	$00.36 \leq \mathcal{U} < 00.38$	$00.38 \leq \mathcal{U} < 00.40$	$00.40 \leq \mathcal{U} < 00.43$
Mild or low DA (L-DA)	$00.30 \leq \mathcal{U} < 00.32$	$00.32 \leq \mathcal{U} < 00.34$	$00.34 \leq \mathcal{U} < 00.36$
Severe DD (S-DD)	$00.27 \leq \mathcal{U} < 00.28$	$00.28 \leq \mathcal{U} < 00.29$	$00.29 \leq \mathcal{U} < 00.30$
Moderate DD (M-DD)	$00.24 \leq \mathcal{U} < 00.25$	$00.25 \leq \mathcal{U} < 00.26$	$00.26 \leq \mathcal{U} < 00.27$
Mild or low DD (L-DD)	$00.20 \leq \mathcal{U} < 00.22$	$00.22 \leq \mathcal{U} < 00.23$	$00.23 \leq \mathcal{U} < 00.24$
Severe MDD (S-MDD)	$00.17 \leq \mathcal{U} < 00.18$	$00.18 \leq \mathcal{U} < 00.19$	$00.19 \leq \mathcal{U} < 00.20$
Moderate MDD (M-MDD)	$00.14 \leq \mathcal{U} < 00.15$	$00.15 \leq \mathcal{U} < 00.16$	$00.16 \leq \mathcal{U} < 00.17$
Mild or low MDD (L-MDD)	$00.10 \leq \mathcal{U} < 00.12$	$00.12 \leq \mathcal{U} < 00.13$	$00.13 \leq \mathcal{U} < 00.14$
No mental disorder (NMD)	$00.00 \leq \mathcal{U} < 00.10$	$00.00 \leq \mathcal{U} < 00.10$	$00.00 \leq \mathcal{U} < 00.10$

phases of psychological disorders in the form of fuzzy sub-interval. The flow chart of different ranges allotted to these conditions is given in Fig. 5.

4.2.2 Algorithm

Step 1 We identify the MPD. Let $\mathcal{Q} = \{\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \ldots, \mathcal{U}_n\}$ be an assembling of patients suffering from psychological disorders and $\mathcal{G} = \mathcal{A} = \{\wp_1, \wp_2, \wp_3, \ldots, \wp_v\}$ be the assembling of symptoms of psychological disorders.A psychiatrist build "t" number of weeks diagnosis chart (which can be scripted as MPNSSs) by the help of a mathematician using linguistic terms. This chart will help us to find the appropriate disease of the patient. The MPNSs chart provided by the doctor after primary assessment at εth times can be scripted as

$$\mathcal{Y}^{\varepsilon}_{\mathcal{G}} = \left\{ \mathcal{Y}^{\varepsilon}_{\wp} = \{\mathcal{U}, \langle {}^{\alpha}\dot{\mathcal{T}}^{\varepsilon}_{\wp}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{I}}^{\varepsilon}_{\wp}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{F}}^{\varepsilon}_{\wp}(\mathcal{U}) \rangle\} \,:\, \mathcal{U} \in \mathcal{Q}, \wp \in \mathcal{G}, \alpha = 1, 2, 3, \ldots, m \right\}$$

where ${}^{\alpha}\dot{\mathcal{T}}^{\varepsilon}_{\wp}(\mathcal{U}), {}^{\alpha}\dot{\mathcal{I}}^{\varepsilon}_{\wp}(\mathcal{U})$ and ${}^{\alpha}\dot{\mathcal{F}}^{\varepsilon}_{\wp}(\mathcal{U})$ are satisfaction, indeterminacy and dissatisfaction grades of MPD, DA, DD and MDD for kth symptoms and lth patients respectively. $(\alpha = 1, 2, 3, \ldots, m; l = 1, 2, 3, \ldots, n; k = 1, 2, 3, \ldots, v; \varepsilon = 1, 2, 3, \ldots, t)$. We take MPNS-union of all input charts to aggregate the initial input data of all patients.

Step 2 We assume that $\mathcal{G}' = \mathcal{B} = \{\wp'_1, \wp'_2, \wp'_3, \ldots, \wp'_{v'}\}$ be the assembling of associated symptoms ("initial symptoms covering related basic symptoms"). We consider an MPNSS ("doctor allocating weights by keeping in mind that patients ε number week estimation of basic symptoms") based on major or initial indications of the patients.

Step 3 We construct a mapping defining as $\eta : \mathcal{Q} \to \mathcal{Q}$ and $\xi : \mathcal{G} \to \mathcal{G}'$ defined as follows $\eta(\mathcal{U}_l) = \mathcal{U}_l$, $\xi(\wp_k) = \wp'_{k'}$ ("depends upon the relationship between basic and primary symptoms").

Consider MPNS-mapping $\dot{\delta} = (\eta, \xi) : MPNS(\mathcal{Q}_{\mathcal{A}}) \to MPNS(\mathcal{R}_{\mathcal{B}})$ defined as

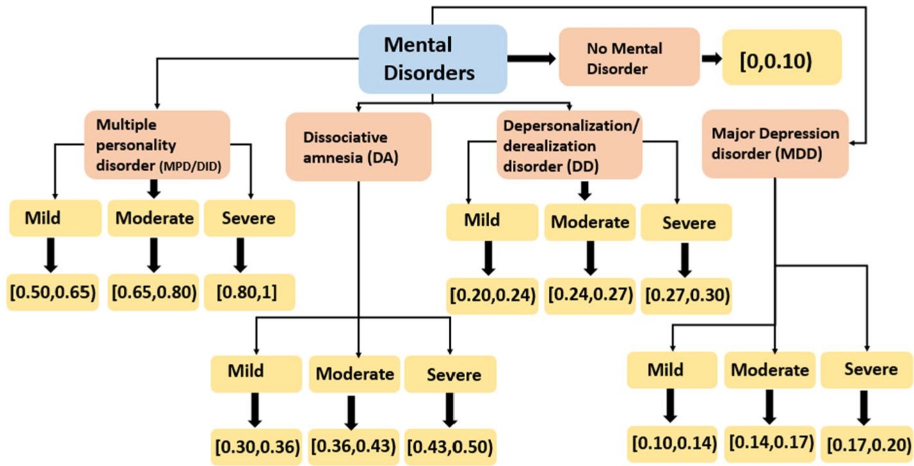

Fig. 5 Flow chart of different ranges corresponding to the listed conditions of mental disorders

$${}^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}})}(\wp')(\mathcal{U}) = |{}^{\alpha}\dot{\mathcal{T}}_{\wp'_{k'}}| \begin{cases} \max\limits_{\mathcal{U}\in\eta^{-1}(\mathcal{U})} \left(\max\limits_{\wp\in\xi^{-1}(\wp')\cap\mathcal{G}} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}_{\wp}} \right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}) \neq \phi, \xi^{-1}(\wp')\cap\mathcal{G}\neq\phi \\ 0; & \text{otherwise}\end{cases}$$

$${}^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}})}(\wp')(\mathcal{U}) = |{}^{\alpha}\dot{\mathcal{I}}_{\wp'_{k'}}| \begin{cases} \min\limits_{\mathcal{U}\in\eta^{-1}(\mathcal{U})} \left(\min\limits_{\wp\in\xi^{-1}(\wp')\cap\mathcal{G}} {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}_{\wp}} \right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}) \neq \phi, \xi^{-1}(\wp')\cap\mathcal{G}\neq\phi \\ 1; & \text{otherwise}\end{cases}$$

$${}^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}})}(\wp')(\mathcal{U}) = |{}^{\alpha}\dot{\mathcal{F}}_{\wp'_{k'}}| \begin{cases} \min\limits_{\mathcal{U}\in\eta^{-1}(\mathcal{U})} \left(\min\limits_{\wp\in\xi^{-1}(\wp')\cap\mathcal{G}} {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}_{\wp}} \right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}) \neq \phi, \xi^{-1}(\wp')\cap\mathcal{G}\neq\phi \\ 1; & \text{otherwise}\end{cases}$$

where ${}^{\alpha}\dot{\mathcal{T}}_{\wp'_{k'}}$, ${}^{\alpha}\dot{\mathcal{I}}_{\wp'_{k'}}$ and ${}^{\alpha}\dot{\mathcal{F}}_{\wp'_{k'}}$ are associated weights from $\mathcal{Y}_{\mathcal{G}'}$. Obtain the image of $\mathcal{Y}^{\xi}_{\mathcal{G}}$ under the defined mapping $\dot{\delta}$, which can be written as $\mathcal{Y}'_{\mathcal{G}'}$.

Step 4 Then relate the results of obtaining set with the values given in Table 7 and build the pre-diagnosis table from which we can notice the accuracy of concluding results.

Step 5 We estimate the score values of the obtaining MPNSSs and take average of every score value corresponding to associated symptoms. Then we conclude our final result from Table 6. The score values can be calculated by using Definition 2.10.

Step 6 We suppose that $\mathcal{G}' = \{\wp'_1, \wp'_2, \wp'_3, \ldots, \wp'_{\nu'}\}$ be the collection of associated symptoms and $\mathcal{G}'' = \{\wp''_1, \wp''_2, \wp''_3, \ldots, \wp''_{\nu''}\}$ be an assembling of possible treatments then we can construct $\mathcal{Y}_{\mathcal{G}''}$.

Step 7 We use max–min composition over $\mathcal{Y}'_{\mathcal{G}'}$ and $\mathcal{Y}_{\mathcal{G}''}$ and obtain $\mathcal{Y}_{\mathcal{G}_{\simeq\simeq\simeq}}$ by using Definition 3.17.

Step 8 We select the treatment having extra benefits and fewer bad effects.

We do the following steps for tracking the improvement record of the patient.

Step 9 We define a novel generalized mapping $\dot{\delta}' = (\eta', \xi') : \mathcal{Q}^{p-1}_{\mathcal{G}''} \to \mathcal{Q}^{1}_{\mathcal{G}''}$ where $\eta' : \mathcal{Q}^{p-1} \to \mathcal{Q}^{1}$ and $\xi' : (\mathcal{G}'')^{p-1} \to \mathcal{G}''$ given as

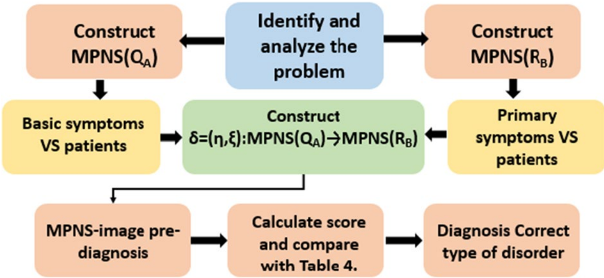

Fig. 6 Flow chart of diagnosis of MPD and its associated mental disorders

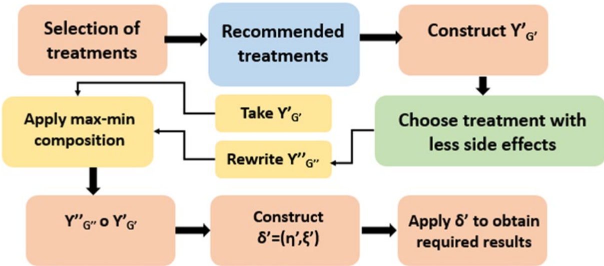

Fig. 7 Flow chart of treatments and improvement record for the patients

$$\eta'(\mathcal{U}_1) = \mathcal{U}_1, \eta'(\mathcal{U}_2) = \mathcal{U}_2, \ldots, \eta'(\mathcal{U}_n) = \mathcal{U}_n$$
$$\xi'(\wp''_1) = \wp''_1, \xi'(\wp''_2) = \wp''_2, \ldots, \xi'(\wp''_{v''}) = \wp''_{v''}$$

and can be computed as

$$\mathcal{Q}^p_{\mathcal{G}''} = \dot{\delta}'(\mathcal{Q}^{p-1}_{\mathcal{G}''})(\wp'')(\mathcal{U})$$
$$= \frac{1}{p}\begin{cases} \bigcup_{\pi\in\eta'^{-1}(\mathcal{U})}\left(\bigcup_{\beta\in\xi'^{-1}(\wp'')\cap\mathcal{G}''} \mathcal{Q}^{p-1}_{\mathcal{G}''}\right)(\pi); & \text{if } \eta'^{-1}(\mathcal{U}) \neq \phi, \xi'^{-1}(\wp'') \cap \mathcal{G}'' \neq \phi \\ \mathcal{Y}_\phi; & \text{otherwise} \end{cases}$$

where $p = 2, 3, 4, \ldots$ is the "number of treatments episodes" and $\wp'' \in \xi(\mathcal{G}') \subseteq \mathcal{G}'', \mathcal{U} \in \mathcal{Q}^1, \pi \in \mathcal{Q}^{p-1}, \beta \in (\mathcal{G}'')^{p-1}$.

Step 10 Repeat the step 9 again and again until we archived our required results. The flow chart diagrams of proposed algorithm for diagnosis, treatment and improvement record are given in Figs. 6 and 7.

4.3 Case study and numerical example

In this portion we are applying our suggested algorithm to a psychological scenario. In this case the input samples are gathered and translated with the aid of a psychologist into mathematical language. Next we pick the group of patients and the doctor prescribed the associated MPD symptoms. Under the supervision of a psychiatrist, we build the descriptive map of psychiatric conditions (Table 6) and relevant situations and their weekly focus for diagnosing psychological illness (Table 7). According to these designed linguistic terms, we convert the appearing symptoms according to the severity of the mental disorder. One may enter the patient 's initial data into this model and accurately determine the particular form of the disorder. We also described some of the relevant therapies the doctor has recommended for the related disorders. The suggested algorithm provides us an optimal choice based on the class of disorder. The best part is a generalized mapping of the patient's recovery. This mapping provides us a suitable criteria and efficient recovery graphs for each patient individually according to the type of disorder. For this process, we use information from different psychologically disordered patients to gather the evidence and use our models to apply it to descriptive concepts and statistical formulation. A psychiatrist acknowledges all the outcomes derived from evaluation, treatment, and rehabilitation.

Now we consider that four patients visits to a psychiatrist and he wants to diagnose the kind of psychological disorder in every patient according to their situations. Frequently, the symptoms of several types of psychological disorders are parallel to each other, so it is challenging to diagnose the accurate type with its phases. After some number of episodes and a comprehensive physical test doctor rules out the some dynamics. He operated all the conceivable factors such as comprehensive history of patients, genetics, neurological ("episode of high stress, such as the death of loved one or other traumatic event, structure and functioning of brain and anxiety disorder"), physical illness etc.

Step 1 Let $\mathcal{Q} = \{\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \mathcal{U}_4\}$ be an assembling of four patients and $\mathcal{G} = \mathcal{A} = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7\}$ be the group of symptoms of the patients, which can be estimated by the psychiatrist after a complete psychological checkup. These calculated symptoms are composed after some episodes of patient with the doctor and conversation of psychiatrist to the friends and family members of the patients. The attributes of set $\mathcal{G}$ can be characterized as

$$\wp_1 = \text{"feeling of euphoric, exultant and somtimes sorrowful,"}$$
$$\wp_2 = \text{"full energized meanwhile decreased levels of activity,"}$$
$$\wp_3 = \text{"disturbed sleep due to racing thoughts, unusual active as well as eat too little or too much,"}$$
$$\wp_4 = \text{"full activated mood on other hand disturbed mood,"}$$
$$\wp_5 = \text{"most takative and touchy as well as cannot enjoy anything,"}$$
$$\wp_6 = \text{"think about risky things like death or suicide,"}$$
$$\wp_7 = \text{"existance of multiple personalities in different time slots according to different situations,"}$$

According to the initial input data, one can observe the symptoms of three ($m = 3$) personalities among all considered patients. We can construct a chart of two ($\varepsilon = 2$) weeks with the input data collected by psychiatrist given as $\mathcal{Y}^{\varepsilon}_{\mathcal{G}} \in MPNS(\mathcal{Q}_{\mathcal{A}})$. We assign satisfaction, abstinence and dissatisfaction grades to every patient corresponding to the symptoms appearing for three personalities in every individual.

First and second week chart is given as (30) and (31) respectively, which are 3PNSSs. Now we take 3PNS-union over the $\mathcal{Y}^1_{\mathcal{G}}$ and $\mathcal{Y}^2_{\mathcal{G}}$. The resultant 3PNSS $\cup\mathcal{Y}^{\varepsilon}_{\mathcal{G}}$ is given as set (32).

Step 2 Consider that $\mathcal{B} = \mathcal{G}' = \{\wp'_1, \wp'_2, \wp'_3, \wp'_4\}$ be an assembling of connected symptoms of MPD/DID, where

$$\wp'_1 = \text{"Mood symptoms,"}$$
$$\wp'_2 = \text{"Behavioral disorder,"}$$
$$\wp'_3 = \text{"Though disorder,"}$$
$$\wp'_4 = \text{"Multiple personalities symptoms,"}$$

Then $MPNS(\mathcal{Q}_{\mathcal{B}})$ be the collection of all MPNSSs in $\mathcal{Q}$ and $\mathcal{B} = \mathcal{G}'$. Doctor assign the weight to the connected symptoms corresponding to the collected data of patients and we convert verbal information into mathematical language into the form of 3PNSS given as chart (33).

Step 3 We define two mappings $\eta : \mathcal{Q} \to \mathcal{Q}$ and $\xi : \mathcal{G} \to \mathcal{G}'$ given as

$$\begin{aligned} &\eta(\mathcal{U}_1) = \mathcal{U}_1, \quad \eta(\mathcal{U}_2) = \mathcal{U}_2, \eta(\mathcal{U}_3) = \mathcal{U}_3, \quad \eta(\mathcal{U}_4) = \mathcal{U}_4 \\ &\xi(\wp_1) = \wp'_1, \quad \xi(\wp_4) = \wp'_1, \quad \xi(\wp_2) = \wp'_2, \quad \xi(\wp_5) = \wp'_2 \\ &\xi(\wp_3) = \wp'_3, \quad \xi(\wp_6) = \wp'_3, \quad \xi(\wp_7) = \wp'_4 \end{aligned}$$

Then we can define an MPNS-mapping $\dot{\delta} = (\eta, \xi) : MPNS(\mathcal{Q}_{\mathcal{A}}) \to MPNS(\mathcal{R}_{\mathcal{B}})$ for $\alpha = 1, 2, 3$ given as

$$^{\alpha}\dot{\mathcal{T}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}})}(\wp')(\mathcal{U}) = |^{\alpha}\dot{\mathcal{T}}_{\wp'_{k'}}| \begin{cases} \max\limits_{\mathcal{U}\in\eta^{-1}(\mathcal{U})} \left(\max\limits_{\wp\in\xi^{-1}(\wp')\cap\mathcal{G}} {}^{\alpha}\dot{\mathcal{T}}_{\mathcal{Y}_{\wp}} \right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}) \neq \phi, \xi^{-1}(\wp') \cap \mathcal{G} \neq \phi \\ 0; & \text{otherwise} \end{cases}$$

$$^{\alpha}\dot{\mathcal{I}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}})}(\wp')(\mathcal{U}) = |^{\alpha}\dot{\mathcal{I}}_{\wp'_{k'}}| \begin{cases} \min\limits_{\mathcal{U}\in\eta^{-1}(\mathcal{U})} \left(\min\limits_{\wp\in\xi^{-1}(\wp')\cap\mathcal{G}} {}^{\alpha}\dot{\mathcal{I}}_{\mathcal{Y}_{\wp}} \right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}) \neq \phi, \xi^{-1}(\wp') \cap \mathcal{G} \neq \phi \\ 1; & \text{otherwise} \end{cases}$$

$$^{\alpha}\dot{\mathcal{F}}_{\dot{\delta}(\mathcal{Y}_{\mathcal{G}})}(\wp')(\mathcal{U}) = |^{\alpha}\dot{\mathcal{F}}_{\wp'_{k'}}| \begin{cases} \min\limits_{\mathcal{U}\in\eta^{-1}(\mathcal{U})} \left(\min\limits_{\wp\in\xi^{-1}(\wp')\cap\mathcal{G}} {}^{\alpha}\dot{\mathcal{F}}_{\mathcal{Y}_{\wp}} \right)(\mathcal{U}); & \text{if } \eta^{-1}(\mathcal{U}) \neq \phi, \xi^{-1}(\wp') \cap \mathcal{G} \neq \phi \\ 1; & \text{otherwise} \end{cases}$$

Now we find the image of $\cup\mathcal{Y}^{\epsilon}_{\mathcal{G}}$ given as $\mathcal{Y}'_{\mathcal{G}'}$ in chart (34) by using the above mapping $\dot{\delta}$.

Step 4 We compare the chart (34) with the Table 7 and we get the chart of initial diagnosis (Table 10). We will use this chart later to check accuracy of our results. The indeterminacy parts and dissatisfaction grades have lowest fuzzy values, which represents that patients are highly effected.

Step 5 Now we calculate the score values of 3-polar neutrosophic numbers (3PNNs) from chart (34) or from Table 10 for every patient corresponding to their connected symptoms. We use Definition 2.10 to calculate these score values and then take average of all values for every individual patient. For example, the average score of patient $\mathcal{U}_1$ can be calculated as

$$\text{average score} = \frac{0.7416 + 0.7016 + 0.7616 + 0.6750}{4} = 0.7199$$

Similarly, we can observe it for the others and it can be scripted as Table 8.

Now we compare our results obtained in Table 8 with the diagnosis chart of mental disorders given in Table 6. Comparison shows that patients $\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3$ are diagnosed with MPD/DID and patient $\mathcal{U}_4$ is diagnosed with DA.

Table 8 Score values of patients data corresponding to the connected symptoms

Patients	$\mathcal{Y}'_{\wp'_1}$	$\mathcal{Y}'_{\wp'_2}$	$\mathcal{Y}'_{\wp'_3}$	$\mathcal{Y}'_{\wp'_4}$	Total average score
$\mathcal{U}_1$	0.7416	0.7016	0.7616	0.6750	0.7199
$\mathcal{U}_2$	0.6733	0.7416	0.6816	0.7000	0.6991
$\mathcal{U}_3$	0.6966	0.6983	0.7316	0.6616	0.6970
$\mathcal{U}_4$	0.6166	0.3800	0.4733	0.3566	0.4566

Step 6 After diagnosis the actual type of disease of every patient, the doctor suggested some medication and psychiatric therapies to the patients. Now we construct the 3PNSS according to the doctor's recommendation with the appropriate treatment corresponding to the kind of disorder. Let $\mathcal{G}' = \{\wp'_1, \wp'_2, \wp'_3, \wp'_4\}$ be an assembling of associated symptoms of psychological disorders and $\mathcal{G}'' = \{\wp''_1, \wp''_2, \wp''_3\}$ be an assembling of treatments suggested by psychiatrist, where

$$\wp''_1 = \text{“Electroconvulsive therapy with high potency medication”}$$
$$\wp''_2 = \text{“Congnitive behavior therapy with mild medication”}$$
$$\wp''_3 = \text{“Some psychotherapies with moderate potency medication”}$$

Now we construct $\mathcal{Y}_{\mathcal{G}''} \in MPNS(\mathcal{G}'_{\mathcal{G}''})$ given as chart (35). In chart (35) the grades are given according to the history of every patient. The satisfaction grades represents the positive impacts of treatment for each type, the indeterminacy grades shows the neutral effects of each type and falsity grades represents the side effects of treatments for each type of mental disorder with its symptoms.

Step 7 We perform MPNS max-min composition between $\mathcal{Y}_{\mathcal{G}''}$ and $\mathcal{Y}'_{\mathcal{G}'}$ and obtain the relation between recommended treatments and patients in the form of 3PNSS $\mathcal{Y}_{\mathcal{G}''} \circ \mathcal{Y}'_{\mathcal{G}'} = \mathcal{Y}_{\mathcal{G}_{\simeq\simeq\simeq}}$. It can be represented as Table 11.

Step 8 The treatment with extra benefits and fewer bad effects is appropriate for the patients. So, we calculate the score values by using Definition 2.10 of each 3PNN corresponding to the treatments for every patient. The score values corresponding to the treatments for every patient are given in Table 9.

From Table 9 it is clear that treatment $\wp''_3$ is best for the treatment of every patient, because it has the maximum score values as compared to other treatments. The selection of treatment may be different for different patients in other cases. The final decision is depending upon the condition of patient according to his previous medical history and type of disease.

Step 9 The episodes of every patient is depends upon the type of disease and history of that patient. One can repeat the episodes until disease is cured completely. We can see the progress of every patient by using the MPNS-mapping. Let $\dot{\delta}' = (\eta', \xi') : \mathcal{Q}^{p-1}_{\mathcal{G}''} \to \mathcal{Q}^1_{\mathcal{G}''}$, where $\eta' : \mathcal{Q}^{p-1} \to \mathcal{Q}^1$ and $\xi' : (\mathcal{G}'')^{p-1} \to \mathcal{G}''$ defined as

$$\eta'(\mathcal{U}_1) = \mathcal{U}_1, \quad \eta'(\mathcal{U}_2) = \mathcal{U}_2, \quad \eta'(\mathcal{U}_3) = \mathcal{U}_3, \quad \eta'(\mathcal{U}_4) = \mathcal{U}_4$$
$$\xi'(\wp''_1) = \wp''_1, \quad \xi'(\wp''_2) = \wp''_2, \quad \xi'(\wp''_3) = \wp''_3$$

then set $\mathcal{Q}^1_{\mathcal{G}''}$ is given as chart (36). The MPNS-mapping is given as

Table 9 Score values of the patients corresponding to every treatment

Patients	$\wp''_1$	$\wp''_2$	$\wp''_3$	Maximum score	Selected treatment
$\mathcal{U}_1$	00.46833	00.50660	00.56500	00.56500	$\wp''_3$
$\mathcal{U}_2$	00.46833	00.50660	00.56330	00.56330	$\wp''_3$
$\mathcal{U}_3$	00.46333	00.51500	00.56160	00.56160	$\wp''_3$
$\mathcal{U}_4$	00.27333	00.31330	00.40000	00.40000	$\wp''_3$

$$\mathcal{Q}^{p}_{\mathcal{G}''} = \dot{\delta}'(\mathcal{Q}^{p-1}_{\mathcal{G}''})(\wp'')(\mathcal{U}) = \frac{1}{p}\begin{cases} \bigcup_{\pi\in\eta'^{-1}(\mathcal{U})}\left(\bigcup_{\beta\in\xi'^{-1}(\wp'')\cap\mathcal{G}''} \mathcal{Q}^{p-1}_{\mathcal{G}''}\right)(\pi); & \text{if } \eta'^{-1}(\mathcal{U}) \neq \phi, \xi'^{-1}(\wp'')\cap\mathcal{G}'' \neq \phi \\ \mathcal{Y}_{\phi}; & \text{otherwise} \end{cases}$$

where $p = 2, 3, 4, \ldots$ is the "number of treatments episodes" and $\wp'' \in \xi(\mathcal{G}') \subseteq \mathcal{G}'', \mathcal{U} \in \mathcal{Q}^1, \pi \in \mathcal{Q}^{p-1}, \beta \in (\mathcal{G}'')^{p-1}$. For the first episode we have $p = 1$ and the chart of $\mathcal{Q}^1_{\mathcal{G}''}$ is given as (36). For second episode of treatment, we have $p = 2$ and the chart of $\mathcal{Q}^2_{\mathcal{G}''}$ is given as (37). Similarly, for $p = 3$ (third episode), we apply the MPNS-mapping $\dot{\delta}'$ on the resultant MPNSS $\mathcal{Q}^2_{\mathcal{G}''}$ and get $\mathcal{Q}^3_{\mathcal{G}''}$ given in chart (38).

Now from this mapping, we observe that membership grades are approaching to zero that means effects and symptoms of disease are reducing of the corresponding patient after treatment. The indeterminacy grades are also approaching to zero that represent the neutral and unaffected dynamics of the treatment are decreasing day by day with every episode. The non-membership grades are approaching to zero that represents the side effects of every treatment corresponding to each patient are decreasing after every episode of treatment. On the similar pattern if we continue the process of treatments and episodes then after some time patient will enter to the regular domain. The period of a single episode is according to the treatment suggested by the doctor.

Step 10 We repeat the application in **step 9** again and again until we attained our required results for the patients. The bar charts of progress of every patient is given in Figs. 8, 9, 10 and 11.

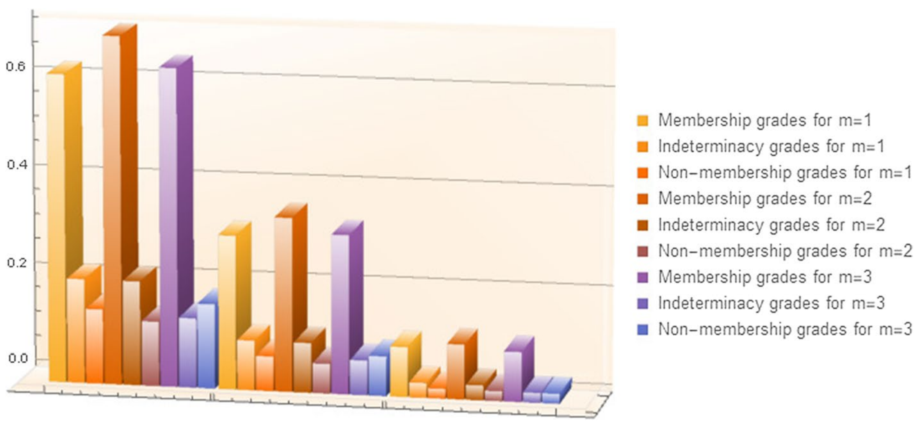

Fig. 8 Progress chart of patient $\mathcal{U}_1$

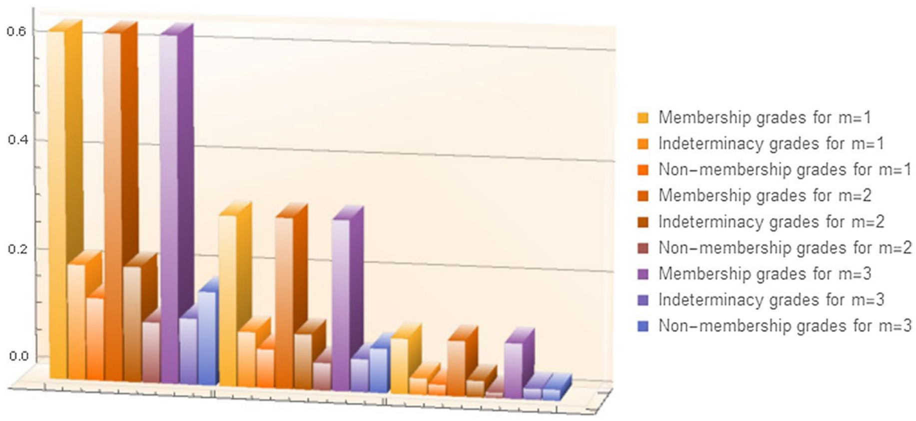

Fig. 9 Progress chart of patient $\mathcal{U}_2$

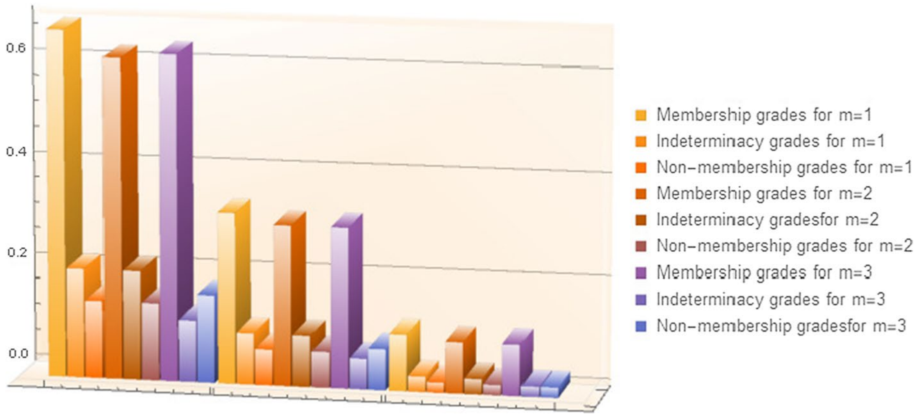

Fig. 10 Progress chart of patient $\mathcal{U}_3$

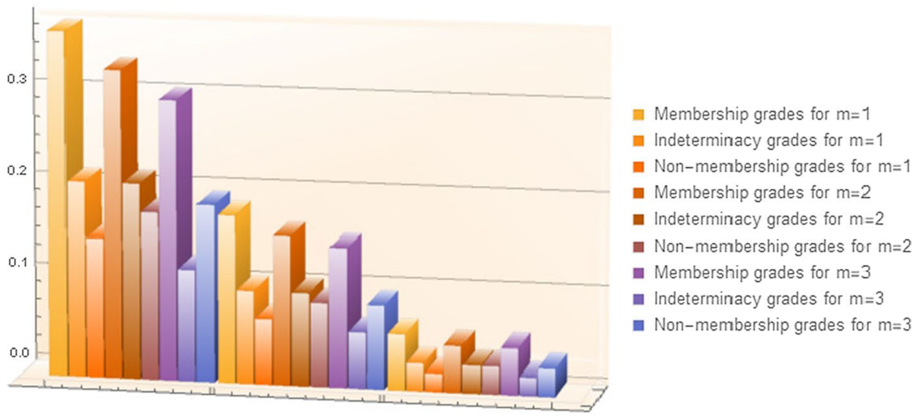

Fig. 11 Progress chart of patient $\mathcal{U}_4$

$$\mathcal{Y}^1_{\mathcal{G}} = \left\{ \begin{array}{l} \mathcal{Y}_{\wp_1} = \{(\mathcal{U}_1, \langle 00.87, 00.21, 00.31\rangle, \langle 00.73, 00.21, 00.41\rangle, \langle 00.68, 00.31, 00.21\rangle), \\ \quad (\mathcal{U}_2, \langle 00.83, 00.31, 00.41\rangle, \langle 00.68, 00.21, 00.41\rangle, \langle 00.85, 00.41, 00.51\rangle), \\ \quad (\mathcal{U}_3, \langle 00.86, 00.41, 00.51\rangle, \langle 00.56, 00.11, 00.61\rangle, \langle 00.78, 00.21, 00.41\rangle), \\ \quad (\mathcal{U}_4, \langle 00.51, 00.21, 00.53\rangle, \langle 00.41, 00.11, 00.61\rangle, \langle 00.31, 00.21, 00.81\rangle)\}, \\ \mathcal{Y}_{\wp_2} = \{(\mathcal{U}_1, \langle 00.76, 00.21, 00.31\rangle, \langle 00.81, 00.41, 00.31\rangle, \langle 00.61, 00.31, 00.21\rangle), \\ \quad (\mathcal{U}_2, \langle 00.81, 00.21, 00.17\rangle, \langle 00.91, 00.43, 00.31\rangle, \langle 00.73, 00.41, 00.31\rangle), \\ \quad (\mathcal{U}_3, \langle 00.73, 00.41, 00.21\rangle, \langle 00.73, 00.21, 00.31\rangle, \langle 00.68, 00.31, 00.11\rangle), \\ \quad (\mathcal{U}_4, \langle 00.41, 00.21, 00.61\rangle, \langle 00.31, 00.11, 00.71\rangle, \langle 00.38, 00.17, 00.81\rangle)\}, \\ \mathcal{Y}_{\wp_3} = \{(\mathcal{U}_1, \langle 00.76, 00.21, 00.31\rangle, \langle 00.81, 00.41, 00.31\rangle, \langle 00.61, 00.31, 00.21\rangle), \\ \quad (\mathcal{U}_2, \langle 00.81, 00.21, 00.17\rangle, \langle 00.91, 00.43, 00.31\rangle, \langle 00.73, 00.41, 00.31\rangle), \\ \quad (\mathcal{U}_3, \langle 00.73, 00.41, 00.21\rangle, \langle 00.73, 00.21, 00.31\rangle, \langle 00.68, 00.31, 00.11\rangle), \\ \quad (\mathcal{U}_4, \langle 00.41, 00.21, 00.61\rangle, \langle 00.31, 00.11, 00.71\rangle, \langle 00.38, 00.17, 00.81\rangle)\}, \\ \mathcal{Y}_{\wp_4} = \{(\mathcal{U}_1, \langle 00.81, 00.11, 00.21\rangle, \langle 00.73, 00.18, 00.31\rangle, \langle 00.68, 00.17, 00.31\rangle), \\ \quad (\mathcal{U}_2, \langle 00.78, 00.21, 00.31\rangle, \langle 00.68, 00.31, 00.41\rangle, \langle 00.73, 00.41, 00.21\rangle), \\ \quad (\mathcal{U}_3, \langle 00.68, 00.31, 00.21\rangle, \langle 00.73, 00.41, 00.31\rangle, \langle 00.83, 00.41, 00.31\rangle), \\ \quad (\mathcal{U}_4, \langle 00.31, 00.11, 00.41\rangle, \langle 00.21, 00.19, 00.71\rangle, \langle 00.31, 00.14, 00.81\rangle)\}, \\ \mathcal{Y}_{\wp_5} = \{(\mathcal{U}_1, \langle 00.76, 00.21, 00.43\rangle, \langle 00.86, 00.31, 00.46\rangle, \langle 00.69, 00.41, 00.59\rangle), \\ \quad (\mathcal{U}_2, \langle 00.68, 00.21, 00.31\rangle, \langle 00.78, 00.31, 00.21\rangle, \langle 00.83, 00.41, 00.31\rangle), \\ \quad (\mathcal{U}_3, \langle 00.78, 00.18, 00.21\rangle, \langle 00.86, 00.31, 00.21\rangle, \langle 00.91, 00.21, 00.41\rangle), \\ \quad (\mathcal{U}_4, \langle 00.83, 00.21, 00.31\rangle, \langle 00.41, 00.31, 00.51\rangle, \langle 00.31, 00.23, 00.68\rangle)\}, \\ \mathcal{Y}_{\wp_6} = \{(\mathcal{U}_1, \langle 00.68, 00.31, 00.41\rangle, \langle 00.76, 00.21, 00.11\rangle, \langle 00.81, 00.21, 00.43\rangle), \\ \quad (\mathcal{U}_2, \langle 00.76, 00.43, 00.51\rangle, \langle 00.68, 00.23, 00.45\rangle, \langle 00.76, 00.51, 00.42\rangle), \\ \quad (\mathcal{U}_3, \langle 00.87, 00.42, 00.18\rangle, \langle 00.68, 00.25, 00.38\rangle, \langle 00.73, 00.43, 00.38\rangle), \\ \quad (\mathcal{U}_4, \langle 00.77, 00.38, 00.27\rangle, \langle 00.83, 00.25, 00.18\rangle, \langle 00.74, 00.51, 00.43\rangle)\}, \\ \mathcal{Y}_{\wp_7} = \{(\mathcal{U}_1, \langle 00.91, 00.21, 00.38\rangle, \langle 00.83, 00.21, 00.31\rangle, \langle 00.74, 00.41, 00.31\rangle), \\ \quad (\mathcal{U}_2, \langle 00.86, 00.41, 00.38\rangle, \langle 00.73, 00.43, 00.31\rangle, \langle 00.68, 00.41, 00.51\rangle), \\ \quad (\mathcal{U}_3, \langle 00.73, 00.41, 00.38\rangle, \langle 00.86, 00.37, 00.44\rangle, \langle 00.77, 00.22, 00.44\rangle), \\ \quad (\mathcal{U}_4, \langle 00.33, 00.11, 00.48\rangle, \langle 00.48, 00.11, 00.68\rangle, \langle 00.38, 00.18, 00.73\rangle)\} \end{array} \right\} \tag{30}$$

Table 10 Pre-diagnosis chart for MPD and its associated mental disorders

Pre-diagnosis	Patients	3-polar neutrosophic numbers
$\mathcal{Y}_{\wp'_1}$	$\mathcal{U}_1$	$(\langle M-MPD, NMD, NMD\rangle, \langle L-MPD, NMD, L-MDD\rangle, \langle L-MPD, NMD, NMD\rangle)$
	$\mathcal{U}_2$	$(\langle L-MPD, NMD, NMD\rangle, \langle M-MPD, NMD, M-MDD\rangle, \langle M-DA, NMD, NMD\rangle)$
	$\mathcal{U}_3$	$(\langle L-MPD, NMD, NMD\rangle, \langle L-MPD, NMD, L-MDD\rangle, \langle L-MPD, NMD, NMD\rangle)$
	$\mathcal{U}_4$	$(\langle M-DA, NMD, NMD\rangle, \langle L-MPD, NMD, NMD\rangle, \langle L-DA, NMD, NMD\rangle)$
$\mathcal{Y}_{\wp'_2}$	$\mathcal{U}_1$	$(\langle L-MPD, NMD, NMD\rangle, \langle L-MPD, NMD, NMD\rangle, \langle L-MPD, NMD, NMD\rangle)$
	$\mathcal{U}_2$	$(\langle M-MPD, NMD, NMD\rangle, \langle M-MPD, NMD, NMD\rangle, \langle L-MPD, NMD, NMD\rangle)$
	$\mathcal{U}_3$	$(\langle M-MPD, NMD, NMD\rangle, \langle L-MPD, NMD, L-MDD\rangle, \langle M-MPD, L-MDD, NMD\rangle)$
	$\mathcal{U}_4$	$(\langle L-DA, NMD, S-MDD\rangle, \langle M-MDD, NMD, M-DA\rangle, \langle L-MDD, NMD, S-DA\rangle)$
$\mathcal{Y}_{\wp'_3}$	$\mathcal{U}_1$	$(\langle L-MPD, NMD, NMD\rangle, \langle M-MPD, NMD, NMD\rangle, \langle L-MPD, NMD, NMD\rangle)$
	$\mathcal{U}_2$	$(\langle L-MPD, NMD, L-MDD\rangle, \langle L-MPD, NMD, NMD\rangle, \langle L-MPD, NMD, M-MDD\rangle)$
	$\mathcal{U}_3$	$(\langle M-MPD, NMD, NMD\rangle, \langle L-MPD, NMD, NMD\rangle, \langle L-MPD, NMD, M-MDD\rangle)$
	$\mathcal{U}_4$	$(\langle L-DA, NMD, M-MDD\rangle, \langle M-DD, NMD, NMD\rangle, \langle M-MDD, L-MPD, S-MDD\rangle)$
$\mathcal{Y}_{\wp'_4}$	$\mathcal{U}_1$	$(\langle L-MPD, NMD, NMD\rangle, \langle L-MPD, NMD, L-MDD\rangle, \langle L-MPD, M-MDD, L-MDD\rangle)$
	$\mathcal{U}_2$	$(\langle L-MPD, L-MDD, M-MDD\rangle, \langle L-MPD, NMD, NMD\rangle, \langle L-MPD, NMD, NMD\rangle)$
	$\mathcal{U}_3$	$(\langle M-MPD, NMD, M-MDD\rangle, \langle L-MPD, NMD, M-MDD\rangle, \langle L-MPD, NMD, M-MDD\rangle)$
	$\mathcal{U}_4$	$(\langle L-DD, NMD, L-DD\rangle, \langle L-DD, NMD, S-DA\rangle, \langle L-MPD, L-MDD, S-DD\rangle)$

$$\mathcal{Y}_{\mathcal{G}}^{2}=\left\{\begin{array}{l} \mathcal{Y}_{\wp_1}=\{(\mathcal{U}_1,\langle 00.86,00.13,00.43\rangle,\langle 00.73,00.18,00.43\rangle,\langle 00.86,00.17,00.38\rangle),\\ \qquad(\mathcal{U}_2,\langle 00.76,00.13,00.45\rangle,\langle 00.86,00.32,00.41\rangle,\langle 00.68,00.18,00.37\rangle),\\ \qquad(\mathcal{U}_3,\langle 00.84,00.13,00.37\rangle,\langle 00.74,00.11,00.38\rangle,\langle 00.69,00.31,00.41\rangle),\\ \qquad(\mathcal{U}_4,\langle 00.41,00.31,00.81\rangle,\langle 00.38,00.21,00.73\rangle,\langle 00.31,00.17,00.68\rangle)\},\\ \mathcal{Y}_{\wp_2}=\{(\mathcal{U}_1,\langle 00.86,00.31,00.41\rangle,\langle 00.78,00.11,00.31\rangle,\langle 00.83,00.21,00.31\rangle),\\ \qquad(\mathcal{U}_2,\langle 00.78,00.41,00.21\rangle,\langle 00.67,00.23,00.43\rangle,\langle 00.73,00.33,00.41\rangle),\\ \qquad(\mathcal{U}_3,\langle 00.78,00.31,00.41\rangle,\langle 00.86,00.23,00.46\rangle,\langle 00.77,00.41,00.33\rangle),\\ \qquad(\mathcal{U}_4,\langle 00.21,00.18,00.68\rangle,\langle 00.31,00.18,00.68\rangle,\langle 00.41,00.13,00.73\rangle)\},\\ \mathcal{Y}_{\wp_3}=\{(\mathcal{U}_1,\langle 00.78,00.31,00.23\rangle,\langle 00.87,00.13,00.33\rangle,\langle 00.77,00.13,00.41\rangle),\\ \qquad(\mathcal{U}_2,\langle 00.73,00.41,00.33\rangle,\langle 00.88,00.21,00.41\rangle,\langle 00.78,00.13,00.33\rangle),\\ \qquad(\mathcal{U}_3,\langle 00.88,00.33,00.21\rangle,\langle 00.77,00.41,00.31\rangle,\langle 00.88,00.23,00.44\rangle),\\ \qquad(\mathcal{U}_4,\langle 00.21,00.13,00.68\rangle,\langle 00.31,00.13,00.77\rangle,\langle 00.34,00.17,00.86\rangle)\},\\ \mathcal{Y}_{\wp_4}=\{(\mathcal{U}_1,\langle 00.83,00.13,00.23\rangle,\langle 00.73,00.17,00.43\rangle,\langle 00.81,00.23,00.68\rangle),\\ \qquad(\mathcal{U}_2,\langle 00.73,00.23,00.31\rangle,\langle 00.81,00.31,00.41\rangle,\langle 00.73,00.23,00.51\rangle),\\ \qquad(\mathcal{U}_3,\langle 00.81,00.19,00.31\rangle,\langle 00.71,00.23,00.47\rangle,\langle 00.83,00.41,00.43\rangle),\\ \qquad(\mathcal{U}_4,\langle 00.31,00.23,00.67\rangle,\langle 00.31,00.29,00.78\rangle,\langle 00.41,00.12,00.83\rangle)\},\\ \mathcal{Y}_{\wp_5}=\{(\mathcal{U}_1,\langle 00.73,00.31,00.23\rangle,\langle 00.86,00.32,00.41\rangle,\langle 00.67,00.31,00.28\rangle),\\ \qquad(\mathcal{U}_2,\langle 00.83,00.31,00.41\rangle,\langle 00.78,00.11,00.31\rangle,\langle 00.67,00.21,00.41\rangle),\\ \qquad(\mathcal{U}_3,\langle 00.89,00.32,00.51\rangle,\langle 00.82,00.41,00.37\rangle,\langle 00.81,00.31,00.41\rangle),\\ \qquad(\mathcal{U}_4,\langle 00.12,00.31,00.68\rangle,\langle 00.21,00.41,00.73\rangle,\langle 00.31,00.51,00.89\rangle)\},\\ \mathcal{Y}_{\wp_6}=\{(\mathcal{U}_1,\langle 00.81,00.21,00.41\rangle,\langle 00.76,00.31,00.43\rangle,\langle 00.69,00.31,00.42\rangle),\\ \qquad(\mathcal{U}_2,\langle 00.78,00.12,00.51\rangle,\langle 00.68,00.31,00.68\rangle,\langle 00.73,00.15,00.42\rangle),\\ \qquad(\mathcal{U}_3,\langle 00.79,00.15,00.61\rangle,\langle 00.85,00.21,00.42\rangle,\langle 00.88,00.35,00.41\rangle),\\ \qquad(\mathcal{U}_4,\langle 00.81,00.33,00.42\rangle,\langle 00.83,00.38,00.43\rangle,\langle 00.79,00.28,00.39\rangle)\},\\ \mathcal{Y}_{\wp_7}=\{(\mathcal{U}_1,\langle 00.86,00.31,00.43\rangle,\langle 00.76,00.37,00.51\rangle,\langle 00.78,00.39,00.61\rangle),\\ \qquad(\mathcal{U}_2,\langle 00.81,00.32,00.42\rangle,\langle 00.76,00.11,00.32\rangle,\langle 00.68,00.29,00.43\rangle),\\ \qquad(\mathcal{U}_3,\langle 00.83,00.38,00.43\rangle,\langle 00.76,00.21,00.38\rangle,\langle 00.71,00.23,00.41\rangle),\\ \qquad(\mathcal{U}_4,\langle 00.35,00.12,00.71\rangle,\langle 00.41,00.11,00.68\rangle,\langle 00.43,00.21,00.69\rangle)\} \end{array}\right\} \tag{31}$$

Table 11 Chart between primary symptoms and recommended treatments

$\mathcal{Y}_{\mathcal{G}''} \circ \mathcal{Y}_{\mathcal{G}'}$	Patients	3-polar neutrosophic numbers	Patients	3-polar neutrosophic numbers
$\mathcal{Y}_{\wp''_1}$	$\mathcal{U}_1$	$(\langle 00.70, 00.21, 00.31\rangle, \langle 00.60, 00.31, 00.21\rangle, \langle 00.64, 00.21, 00.15\rangle)$	$\mathcal{U}_2$	$(\langle 00.62, 00.21, 00.31\rangle, \langle 00.69, 00.31, 00.21\rangle, \langle 00.63, 00.21, 00.15\rangle)$
	$\mathcal{U}_3$	$(\langle 00.67, 00.21, 00.31\rangle, \langle 00.62, 00.31, 00.21\rangle, \langle 00.62, 00.21, 00.15\rangle)$	$\mathcal{U}_4$	$(\langle 00.37, 00.21, 00.31\rangle, \langle 00.33, 00.31, 00.41\rangle, \langle 00.30, 00.21, 00.18\rangle)$
$\mathcal{Y}_{\wp''_2}$	$\mathcal{U}_1$	$(\langle 00.70, 00.21, 00.15\rangle, \langle 00.62, 00.25, 00.25\rangle, \langle 00.64, 00.21, 00.18\rangle)$	$\mathcal{U}_2$	$(\langle 00.67, 00.21, 00.15\rangle, \langle 00.66, 00.25, 00.25\rangle, \langle 00.63, 00.21, 00.18\rangle)$
	$\mathcal{U}_3$	$(\langle 00.70, 00.21, 00.15\rangle, \langle 00.62, 00.25, 00.25\rangle, \langle 00.69, 00.21, 00.18\rangle)$	$\mathcal{U}_4$	$(\langle 00.37, 00.21, 00.19\rangle, \langle 00.33, 00.25, 00.31\rangle, \langle 00.30, 00.21, 00.28\rangle)$
$\mathcal{Y}_{\wp''_3}$	$\mathcal{U}_1$	$(\langle 00.62, 00.21, 00.15\rangle, \langle 00.70, 00.21, 00.13\rangle, \langle 00.64, 00.14, 00.17\rangle)$	$\mathcal{U}_2$	$(\langle 00.63, 00.21, 00.15\rangle, \langle 00.63, 00.21, 00.11\rangle, \langle 00.63, 00.12, 00.17\rangle)$
	$\mathcal{U}_3$	$(\langle 00.67, 00.21, 00.15\rangle, \langle 00.62, 00.21, 00.15\rangle, \langle 00.63, 00.12, 00.17\rangle)$	$\mathcal{U}_4$	$(\langle 00.37, 00.21, 00.15\rangle, \langle 00.33, 00.21, 00.18\rangle, \langle 00.30, 00.12, 00.19\rangle)$

$$\cup\mathcal{Y}_{\mathcal{G}}^{\varepsilon}=\left\{\begin{array}{l}\mathcal{Y}_{\wp_1}=\{(\mathcal{U}_1,\langle 00.87,00.13,00.31\rangle,\langle 00.73,00.18,00.41\rangle,\langle 00.86,00.17,00.21\rangle),\\ \quad(\mathcal{U}_2,\langle 00.83,00.13,00.41\rangle,\langle 00.86,00.21,00.41\rangle,\langle 00.85,00.18,00.37\rangle),\\ \quad(\mathcal{U}_3,\langle 00.86,00.13,00.37\rangle,\langle 00.74,00.11,00.38\rangle,\langle 00.78,00.21,00.41\rangle),\\ \quad(\mathcal{U}_4,\langle 00.51,00.21,00.53\rangle,\langle 00.41,00.11,00.61\rangle,\langle 00.31,00.17,00.61\rangle)\},\\ \mathcal{Y}_{\wp_2}=\{(\mathcal{U}_1,\langle 00.86,00.21,00.31\rangle,\langle 00.81,00.11,00.38\rangle,\langle 00.83,00.21,00.21\rangle),\\ \quad(\mathcal{U}_2,\langle 00.81,00.21,00.17\rangle,\langle 00.91,00.23,00.31\rangle,\langle 00.73,00.33,00.31\rangle),\\ \quad(\mathcal{U}_3,\langle 00.78,00.31,00.21\rangle,\langle 00.86,00.21,00.31\rangle,\langle 00.77,00.31,00.11\rangle),\\ \quad(\mathcal{U}_4,\langle 00.41,00.18,00.61\rangle,\langle 00.31,00.11,00.68\rangle,\langle 00.41,00.13,00.73\rangle)\},\\ \mathcal{Y}_{\wp_3}=\{(\mathcal{U}_1,\langle 00.81,00.31,00.21\rangle,\langle 00.87,00.12,00.31\rangle,\langle 00.84,00.13,00.24\rangle),\\ \quad(\mathcal{U}_2,\langle 00.73,00.41,00.31\rangle,\langle 00.88,00.21,00.21\rangle,\langle 00.78,00.13,00.41\rangle),\\ \quad(\mathcal{U}_3,\langle 00.89,00.21,00.21\rangle,\langle 00.77,00.16,00.31\rangle,\langle 00.88,00.11,00.44\rangle),\\ \quad(\mathcal{U}_4,\langle 00.31,00.11,00.68\rangle,\langle 00.31,00.13,00.77\rangle,\langle 00.34,00.17,00.73\rangle)\},\\ \mathcal{Y}_{\wp_4}=\{(\mathcal{U}_1,\langle 00.83,00.11,00.21\rangle,\langle 00.73,00.17,00.31\rangle,\langle 00.81,00.17,00.31\rangle),\\ \quad(\mathcal{U}_2,\langle 00.73,00.21,00.31\rangle,\langle 00.81,00.31,00.41\rangle,\langle 00.73,00.23,00.21\rangle),\\ \quad(\mathcal{U}_3,\langle 00.81,00.19,00.21\rangle,\langle 00.73,00.23,00.31\rangle,\langle 00.83,00.41,00.31\rangle),\\ \quad(\mathcal{U}_4,\langle 00.31,00.11,00.41\rangle,\langle 00.31,00.19,00.17\rangle,\langle 00.41,00.12,00.81\rangle)\},\\ \mathcal{Y}_{\wp_5}=\{(\mathcal{U}_1,\langle 00.76,00.21,00.23\rangle,\langle 00.86,00.31,00.41\rangle,\langle 00.69,00.31,00.28\rangle),\\ \quad(\mathcal{U}_2,\langle 00.83,00.21,00.31\rangle,\langle 00.78,00.11,00.21\rangle,\langle 00.83,00.21,00.31\rangle),\\ \quad(\mathcal{U}_3,\langle 00.89,00.18,00.21\rangle,\langle 00.86,00.21,00.31\rangle,\langle 00.91,00.31,00.41\rangle),\\ \quad(\mathcal{U}_4,\langle 00.83,00.21,00.31\rangle,\langle 00.41,00.31,00.51\rangle,\langle 00.31,00.23,00.68\rangle)\},\\ \mathcal{Y}_{\wp_6}=\{(\mathcal{U}_1,\langle 00.86,00.21,00.41\rangle,\langle 00.76,00.21,00.11\rangle,\langle 00.81,00.21,00.42\rangle),\\ \quad(\mathcal{U}_2,\langle 00.78,00.12,00.51\rangle,\langle 00.68,00.23,00.45\rangle,\langle 00.76,00.15,00.42\rangle),\\ \quad(\mathcal{U}_3,\langle 00.87,00.15,00.18\rangle,\langle 00.85,00.21,00.38\rangle,\langle 00.88,00.35,00.38\rangle),\\ \quad(\mathcal{U}_4,\langle 00.81,00.33,00.27\rangle,\langle 00.83,00.25,00.18\rangle,\langle 00.79,00.28,00.39\rangle)\},\\ \mathcal{Y}_{\wp_7}=\{(\mathcal{U}_1,\langle 00.91,00.21,00.38\rangle,\langle 00.83,00.21,00.31\rangle,\langle 00.78,00.39,00.31\rangle),\\ \quad(\mathcal{U}_2,\langle 00.86,00.32,00.38\rangle,\langle 00.76,00.11,00.31\rangle,\langle 00.68,00.29,00.43\rangle),\\ \quad(\mathcal{U}_3,\langle 00.83,00.38,00.38\rangle,\langle 00.86,00.21,00.38\rangle,\langle 00.77,00.22,00.41\rangle),\\ \quad(\mathcal{U}_4,\langle 00.35,00.11,00.48\rangle,\langle 00.48,00.11,00.68\rangle,\langle 00.43,00.18,00.69\rangle)\}\end{array}\right\} \tag{32}$$

$$\mathcal{Y}_{\mathcal{G}} = \left\{\begin{array}{l} \mathcal{Y}_{\wp'_1} = \{(\mathcal{U}_1, \langle 00.81, 00.21, 00.11\rangle, \langle 00.76, 00.31, 00.41\rangle, \langle 00.63, 00.11, 00.21\rangle), \\ \quad (\mathcal{U}_2, \langle 00.73, 00.21, 00.11\rangle, \langle 00.81, 00.31, 00.41\rangle, \langle 00.43, 00.51, 00.41\rangle), \\ \quad (\mathcal{U}_3, \langle 00.68, 00.41, 00.33\rangle, \langle 00.71, 00.32, 00.41\rangle, \langle 00.75, 00.31, 00.28\rangle), \\ \quad (\mathcal{U}_4, \langle 00.73, 00.21, 00.18\rangle, \langle 00.81, 00.31, 00.21\rangle, \langle 00.74, 00.21, 00.11\rangle)\}, \\ \mathcal{Y}_{\wp'_2} = \{(\mathcal{U}_1, \langle 00.67, 00.32, 00.41\rangle, \langle 00.73, 00.51, 00.21\rangle, \langle 00.63, 00.28, 00.17\rangle), \\ \quad (\mathcal{U}_2, \langle 00.81, 00.32, 00.21\rangle, \langle 00.73, 00.21, 00.41\rangle, \langle 00.68, 00.21, 00.31\rangle), \\ \quad (\mathcal{U}_3, \langle 00.79, 00.43, 00.37\rangle, \langle 00.66, 00.43, 00.38\rangle, \langle 00.76, 00.36, 00.45\rangle), \\ \quad (\mathcal{U}_4, \langle 00.41, 00.51, 00.63\rangle, \langle 00.38, 00.21, 00.81\rangle, \langle 00.32, 00.18, 00.71\rangle)\}, \\ \mathcal{Y}_{\wp'_3} = \{(\mathcal{U}_1, \langle 00.73, 00.21, 00.18\rangle, \langle 00.81, 00.31, 00.41\rangle, \langle 00.73, 00.41, 00.21\rangle), \\ \quad (\mathcal{U}_2, \langle 00.81, 00.31, 00.43\rangle, \langle 00.68, 00.41, 00.38\rangle, \langle 00.74, 00.51, 00.41\rangle), \\ \quad (\mathcal{U}_3, \langle 00.76, 00.38, 00.21\rangle, \langle 00.73, 00.31, 00.21\rangle, \langle 00.68, 00.41, 00.38\rangle), \\ \quad (\mathcal{U}_4, \langle 00.41, 00.51, 00.61\rangle, \langle 00.31, 00.61, 00.41\rangle, \langle 00.21, 00.71, 00.51\rangle)\}, \\ \mathcal{Y}_{\wp'_4} = \{(\mathcal{U}_1, \langle 00.68, 00.21, 00.18\rangle, \langle 00.73, 00.31, 00.43\rangle, \langle 00.83, 00.41, 00.38\rangle), \\ \quad (\mathcal{U}_2, \langle 00.73, 00.41, 00.38\rangle, \langle 00.83, 00.21, 00.18\rangle, \langle 00.93, 00.18, 00.21\rangle), \\ \quad (\mathcal{U}_3, \langle 00.81, 00.21, 00.37\rangle, \langle 00.73, 00.43, 00.41\rangle, \langle 00.69, 00.21, 00.41\rangle), \\ \quad (\mathcal{U}_4, \langle 00.61, 00.51, 00.47\rangle, \langle 00.43, 00.51, 00.68\rangle, \langle 00.31, 00.68, 00.41\rangle)\} \end{array}\right\} \tag{33}$$

$$\mathcal{Y}'_{\mathcal{G}'} = \left\{\begin{array}{l} \mathcal{Y}'_{\wp'_1} = \{(\mathcal{U}_1, \langle 00.70, 00.02, 00.02\rangle, \langle 00.55, 00.05, 00.12\rangle, \langle 00.54, 00.01, 00.04\rangle), \\ \quad (\mathcal{U}_2, \langle 00.60, 00.02, 00.03\rangle, \langle 00.69, 00.06, 00.16\rangle, \langle 00.36, 00.09, 00.08\rangle), \\ \quad (\mathcal{U}_3, \langle 00.58, 00.05, 00.06\rangle, \langle 00.52, 00.03, 00.12\rangle, \langle 00.62, 00.06, 00.08\rangle), \\ \quad (\mathcal{U}_4, \langle 00.37, 00.02, 00.07\rangle, \langle 00.33, 00.03, 00.03\rangle, \langle 00.30, 00.02, 00.06\rangle)\}, \\ \mathcal{Y}'_{\wp'_2} = \{(\mathcal{U}_1, \langle 00.57, 00.06, 00.09\rangle, \langle 00.62, 00.05, 00.06\rangle, \langle 00.52, 00.05, 00.03\rangle), \\ \quad (\mathcal{U}_2, \langle 00.67, 00.06, 00.03\rangle, \langle 00.66, 00.02, 00.08\rangle, \langle 00.56, 00.04, 00.09\rangle), \\ \quad (\mathcal{U}_3, \langle 00.70, 00.07, 00.07\rangle, \langle 00.56, 00.09, 00.11\rangle, \langle 00.69, 00.11, 00.04\rangle), \\ \quad (\mathcal{U}_4, \langle 00.34, 00.09, 00.19\rangle, \langle 00.15, 00.02, 00.41\rangle, \langle 00.13, 00.02, 00.48\rangle)\}, \\ \mathcal{Y}'_{\wp'_3} = \{(\mathcal{U}_1, \langle 00.62, 00.04, 00.03\rangle, \langle 00.70, 00.03, 00.04\rangle, \langle 00.61, 00.05, 00.05\rangle), \\ \quad (\mathcal{U}_2, \langle 00.63, 00.03, 00.13\rangle, \langle 00.59, 00.08, 00.07\rangle, \langle 00.57, 00.06, 00.16\rangle), \\ \quad (\mathcal{U}_3, \langle 00.67, 00.05, 00.03\rangle, \langle 00.62, 00.04, 00.06\rangle, \langle 00.59, 00.04, 00.14\rangle), \\ \quad (\mathcal{U}_4, \langle 00.33, 00.05, 00.16\rangle, \langle 00.25, 00.07, 00.07\rangle, \langle 00.16, 00.12, 00.19\rangle)\}, \\ \mathcal{Y}'_{\wp'_4} = \{(\mathcal{U}_1, \langle 00.61, 00.04, 00.06\rangle, \langle 00.60, 00.06, 00.13\rangle, \langle 00.64, 00.15, 00.11\rangle), \\ \quad (\mathcal{U}_2, \langle 00.62, 00.13, 00.14\rangle, \langle 00.63, 00.02, 00.05\rangle, \langle 00.63, 00.05, 00.09\rangle), \\ \quad (\mathcal{U}_3, \langle 00.67, 00.07, 00.14\rangle, \langle 00.62, 00.09, 00.15\rangle, \langle 00.53, 00.04, 00.16\rangle), \\ \quad (\mathcal{U}_4, \langle 00.21, 00.05, 00.22\rangle, \langle 00.20, 00.05, 00.46\rangle, \langle 00.13, 00.12, 00.28\rangle)\} \end{array}\right\} \tag{34}$$

$$\mathcal{Y}'_{\mathcal{G}} = \left\{ \begin{array}{l} \mathcal{Y}_{\wp_1''} = \{(\wp_1', \langle 00.90, 00.21, 00.31\rangle, \langle 00.83, 00.31, 00.41\rangle, \langle 00.85, 00.21, 00.18\rangle), \\ (\wp_2', \langle 00.50, 00.40, 00.43\rangle, \langle 00.45, 00.38, 00.53\rangle, \langle 00.53, 00.41, 00.38\rangle), \\ (\wp_3', \langle 00.40, 00.60, 00.55\rangle, \langle 00.38, 00.61, 00.54\rangle, \langle 00.45, 00.51, 00.38\rangle), \\ (\wp_4', \langle 00.80, 00.21, 00.31\rangle, \langle 00.73, 00.41, 00.21\rangle, \langle 00.95, 00.21, 00.15\rangle)\}, \\ \mathcal{Y}_{\wp_2''} = \{(\wp_1', \langle 00.70, 00.50, 00.45\rangle, \langle 00.65, 00.25, 00.31\rangle, \langle 00.67, 00.30, 00.45\rangle), \\ (\wp_2', \langle 00.90, 00.21, 00.15\rangle, \langle 00.85, 00.31, 00.25\rangle, \langle 00.80, 00.25, 00.20\rangle), \\ (\wp_3', \langle 00.60, 00.30, 00.25\rangle, \langle 00.55, 00.40, 00.38\rangle, \langle 00.57, 00.32, 00.51\rangle), \\ (\wp_4', \langle 00.86, 00.21, 00.31\rangle, \langle 00.89, 00.31, 00.41\rangle, \langle 00.93, 00.21, 00.18\rangle)\}, \\ \mathcal{Y}_{\wp_3''} = \{(\wp_1', \langle 00.50, 00.45, 00.38\rangle, \langle 00.45, 00.38, 00.54\rangle, \langle 00.47, 00.50, 00.61\rangle), \\ (\wp_2', \langle 00.60, 00.40, 00.38\rangle, \langle 00.58, 00.41, 00.38\rangle, \langle 00.63, 00.51, 00.61\rangle), \\ (\wp_3', \langle 00.90, 00.21, 00.15\rangle, \langle 00.91, 00.21, 00.18\rangle, \langle 00.94, 00.14, 00.18\rangle), \\ (\wp_4', \langle 00.81, 00.31, 00.21\rangle, \langle 00.94, 00.21, 00.11\rangle, \langle 00.95, 00.12, 00.17\rangle)\} \end{array} \right\} \tag{35}$$

$$\mathcal{Q}^1_{\mathcal{G}''} = \mathcal{Y}_{\mathcal{G}''} \circ \mathcal{Y}'_{\mathcal{G}'} = \left\{ \begin{array}{l} \mathcal{Y}_{\wp_1''} = \{(\mathcal{U}_1, \langle 00.70, 00.21, 00.31\rangle, \langle 00.60, 00.31, 00.21\rangle, \langle 00.64, 00.21, 00.15\rangle), \\ (\mathcal{U}_2, \langle 00.62, 00.21, 00.31\rangle, \langle 00.69, 00.31, 00.21\rangle, \langle 00.63, 00.21, 00.15\rangle), \\ (\mathcal{U}_3, \langle 00.67, 00.21, 00.31\rangle, \langle 00.62, 00.31, 00.21\rangle, \langle 00.62, 00.21, 00.15\rangle), \\ (\mathcal{U}_4, \langle 00.37, 00.21, 00.31\rangle, \langle 00.33, 00.31, 00.41\rangle, \langle 00.30, 00.21, 00.18\rangle)\}, \\ \mathcal{Y}_{\wp_2''} = \{(\mathcal{U}_1, \langle 00.70, 00.21, 00.15\rangle, \langle 00.62, 00.25, 00.25\rangle, \langle 00.64, 00.21, 00.18\rangle), \\ (\mathcal{U}_2, \langle 00.67, 00.21, 00.15\rangle, \langle 00.66, 00.25, 00.25\rangle, \langle 00.63, 00.21, 00.18\rangle), \\ (\mathcal{U}_3, \langle 00.70, 00.21, 00.15\rangle, \langle 00.62, 00.25, 00.25\rangle, \langle 00.69, 00.21, 00.18\rangle), \\ (\mathcal{U}_4, \langle 00.37, 00.21, 00.19\rangle, \langle 00.33, 00.25, 00.31\rangle, \langle 00.30, 00.21, 00.28\rangle)\}, \\ \mathcal{Y}_{\wp_3''} = \{(\mathcal{U}_1, \langle 00.62, 00.21, 00.15\rangle, \langle 00.70, 00.21, 00.13\rangle, \langle 00.64, 00.14, 00.17\rangle), \\ (\mathcal{U}_2, \langle 00.63, 00.21, 00.15\rangle, \langle 00.63, 00.21, 00.11\rangle, \langle 00.63, 00.12, 00.17\rangle), \\ (\mathcal{U}_3, \langle 00.67, 00.21, 00.15\rangle, \langle 00.62, 00.21, 00.15\rangle, \langle 00.63, 00.12, 00.17\rangle), \\ (\mathcal{U}_4, \langle 00.37, 00.21, 00.15\rangle, \langle 00.33, 00.21, 00.18\rangle, \langle 00.30, 00.12, 00.19\rangle)\} \end{array} \right\} \tag{36}$$

$$\mathcal{Q}^2_{\mathcal{G}''} = \left\{ \begin{array}{l} \mathcal{Y}_{\wp_1''} = \{(\mathcal{U}_1, \langle 00.35, 00.10, 00.15\rangle, \langle 00.30, 00.15, 00.10\rangle, \langle 00.32, 00.10, 00.07\rangle), \\ (\mathcal{U}_2, \langle 00.31, 00.10, 00.15\rangle, \langle 00.35, 00.15, 00.10\rangle, \langle 00.31, 00.10, 00.07\rangle), \\ (\mathcal{U}_3, \langle 00.33, 00.10, 00.15\rangle, \langle 00.31, 00.15, 00.10\rangle, \langle 00.31, 00.10, 00.07\rangle), \\ (\mathcal{U}_4, \langle 00.18, 00.10, 00.15\rangle, \langle 00.16, 00.15, 00.20\rangle, \langle 00.15, 00.10, 00.09\rangle)\}, \\ \mathcal{Y}_{\wp_2''} = \{(\mathcal{U}_1, \langle 00.35, 00.10, 00.07\rangle, \langle 00.31, 00.12, 00.12\rangle, \langle 00.32, 00.10, 00.09\rangle), \\ (\mathcal{U}_2, \langle 00.33, 00.10, 00.07\rangle, \langle 00.33, 00.12, 00.12\rangle, \langle 00.31, 00.10, 00.09\rangle), \\ (\mathcal{U}_3, \langle 00.35, 00.10, 00.07\rangle, \langle 00.31, 00.12, 00.12\rangle, \langle 00.34, 00.10, 00.09\rangle), \\ (\mathcal{U}_4, \langle 00.18, 00.10, 00.09\rangle, \langle 00.16, 00.12, 00.15\rangle, \langle 00.15, 00.10, 00.14\rangle)\}, \\ \mathcal{Y}_{\wp_3''} = \{(\mathcal{U}_1, \langle 00.31, 00.10, 00.07\rangle, \langle 00.35, 00.10, 00.06\rangle, \langle 00.32, 00.07, 00.08\rangle), \\ (\mathcal{U}_2, \langle 00.31, 00.10, 00.07\rangle, \langle 00.31, 00.10, 00.05\rangle, \langle 00.31, 00.06, 00.08\rangle), \\ (\mathcal{U}_3, \langle 00.33, 00.10, 00.07\rangle, \langle 00.31, 00.10, 00.07\rangle, \langle 00.31, 00.06, 00.08\rangle), \\ (\mathcal{U}_4, \langle 00.18, 00.10, 00.07\rangle, \langle 00.16, 00.10, 00.09\rangle, \langle 00.15, 00.06, 00.09\rangle)\} \end{array} \right\} \tag{37}$$

$$\mathcal{Q}^3_{\mathcal{G}''} = \left\{ \begin{array}{l} \mathcal{Y}_{\wp''_1} = \{(\mathcal{U}_1, \langle 00.11, 00.03, 00.05\rangle, \langle 00.10, 00.05, 00.03\rangle, \langle 00.10, 00.03, 00.02\rangle), \\ \quad (\mathcal{U}_2, \langle 00.10, 00.03, 00.05\rangle, \langle 00.11, 00.05, 00.03\rangle, \langle 00.10, 00.03, 00.02\rangle), \\ \quad (\mathcal{U}_3, \langle 00.11, 00.03, 00.05\rangle, \langle 00.10, 00.05, 00.03\rangle, \langle 00.10, 00.03, 00.02\rangle), \\ \quad (\mathcal{U}_4, \langle 00.06, 00.03, 00.05\rangle, \langle 00.05, 00.05, 00.06\rangle, \langle 00.05, 00.03, 00.03\rangle)\}, \\ \mathcal{Y}_{\wp''_2} = \{(\mathcal{U}_1, \langle 00.11, 00.03, 00.02\rangle, \langle 00.10, 00.04, 00.04\rangle, \langle 00.10, 00.03, 00.03\rangle), \\ \quad (\mathcal{U}_2, \langle 00.11, 00.03, 00.02\rangle, \langle 00.11, 00.04, 00.04\rangle, \langle 00.10, 00.03, 00.03\rangle), \\ \quad (\mathcal{U}_3, \langle 00.11, 00.03, 00.02\rangle, \langle 00.10, 00.04, 00.04\rangle, \langle 00.11, 00.03, 00.03\rangle), \\ \quad (\mathcal{U}_4, \langle 00.06, 00.03, 00.03\rangle, \langle 00.05, 00.04, 00.05\rangle, \langle 00.05, 00.03, 00.04\rangle)\}, \\ \mathcal{Y}_{\wp''_3} = \{(\mathcal{U}_1, \langle 00.10, 00.03, 00.02\rangle, \langle 00.11, 00.03, 00.02\rangle, \langle 00.10, 00.02, 00.02\rangle), \\ \quad (\mathcal{U}_2, \langle 00.10, 00.03, 00.02\rangle, \langle 00.10, 00.03, 00.01\rangle, \langle 00.10, 00.02, 00.02\rangle), \\ \quad (\mathcal{U}_3, \langle 00.11, 00.03, 00.02\rangle, \langle 00.10, 00.03, 00.02\rangle, \langle 00.10, 00.02, 00.02\rangle), \\ \quad (\mathcal{U}_4, \langle 00.06, 00.03, 00.02\rangle, \langle 00.05, 00.03, 00.03\rangle, \langle 00.05, 00.02, 00.03\rangle)\} \end{array} \right\} \tag{38}$$

4.4 Discussion and comparison analysis

In this subsection, we discuss and compare our suggested approach with its accuracy.

1. In this algorithm, we add multiple weeks because the psychological patient cannot be diagnose perfectly after a single episode. The MPNSS and its union offers maximum information data about the patient and we can estimate the associated symptoms with its severeness.
2. We observe that the association between the associated and basic indications with its weights are significant in every episode of patient. If we select only basic symptoms then results will be imprecise.
3. The pre-diagnosis chart gives us the accuracy of our technique. With the passage of time, we can see the vibrant symptoms of disorders in the patients with number of episodes. These episodes are important to diagnose the authentic kind of disorder and give us a suitable optimal decision in mathematical modeling.
4. For the secondary stage, we can see choose the treatment for the patients according to their type of disorder and its severeness. By using the score function, we can give the ranking of the selected treatments.
5. In the third phase, we use a generalized MPNS-mapping to see the patients progress record. With every episode, all the grades are decreasing to zero which means that the symptoms of disease, neutral effects of medication with therapies and side effects are decreasing. This criteria shows the improvement of patient with the passage of time.
6. If any patient cannot get its convergence to the improvement in one episode then, we can use inverse MPNS-mapping to get back him on the preceding episode and then start the treatment from here again.
7. This technique is useful for a large number of patients with multiple diseases and multiple criteria under the effect of parameterizations. This work is consistent and proficient to handle such type of medical and decision-making problems.

Now we equate our suggested methods with those methodologies already in use. We also reviews on numerous publications and books on psychiatric problems and human behavior and find that our theoretical paradigm is much better than these current models and can cope easily and efficiently with the complexities. Smarandache (2018) developed different psychological consequences and linked neutrosophic theory to human actions, memory, and temperaments. Christianto and Smarandache (2019) provided a comprehensive analysis of seven aspects of neutrosophical philosophy such as cultural psychology, theorizing finance, dispute management, scientific philosophy, etc. Farahani et al. (2015) presented an ADHD case study and compared combined overlap block fuzzy cognitive maps (COB-FCM) and combined overlap block neutrosophic cognitive map (COBNCM) to find the hidden patterns and indeterminacies in psychological causal models. Using multi-polarity and parameterizations, we relate these theoretical theories to the neutrosophic group. We may use it in MPD as "positive, neutral and negative" consequences according to different levels in those three classes. We can note the various definitions owing to the multiplicity of alternatives and can expand the disorder more precisely and reliably. The soft parameterizations provide us with a multi-valued mapping for coping with uncertainty and classifying the disorder according to its signs and indications.

We discovered more about the neutrosophic temperaments in Smarandache (2018). The "temperament" is part of the Characteristic Personality on two measurements: sustainable/inconsistent and extroverted/introverted. According to the study there are four kinds of temperaments such as

1. Sanguine (optimistic),
2. Choleric (angry),
3. Melancholic (sad),
4. Phlegmatic (lethargic)

But the dimensions of psychology differ according to time, sex, circumstance, background, climate etc. They 're always special for a single person. So we will categorize the details more according to the personalities and the diseases. We use the MPNSS for this purpose, and further classify all kinds into subcategories. The description of these categories can be seen as an MPNSS in Fig. 12.

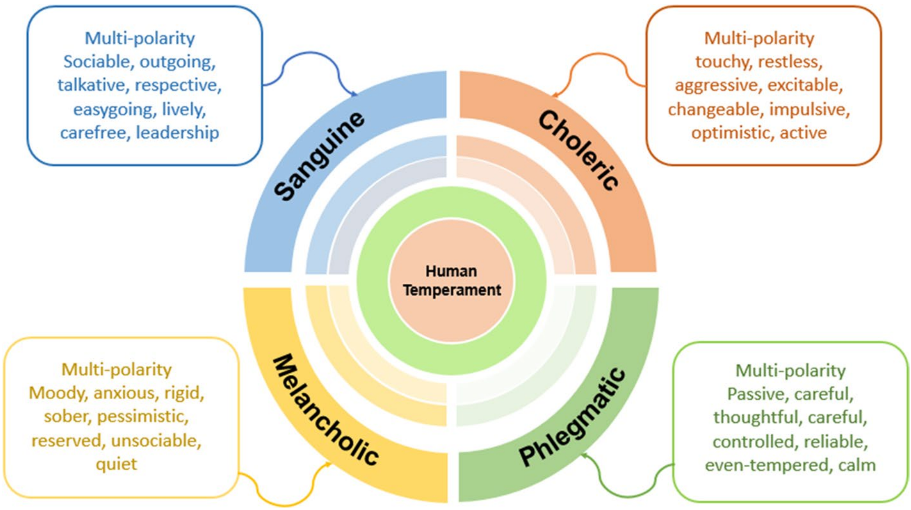

Fig. 12 Classical diagram of human temperaments

In Fig. 12 we note that we have four parameters for the alternative “temperament”. The more sub-categories reflect the multi-polarity of the proposed problem, which can be addressed using MPNSS under the influence of soft parameterizations and multi-polarity.

In 2019 Riaz and Tehrim (2019) designed a novel technique with reference to bipolar disorder on bipolar fuzzy soft mappings. They utilized bipolar fuzzy soft set (BFSS) in this article to explain the psychology and disorders of humans. The MPNSS is the annex of BFSS, with certain necessary conditions. BFSS only manages the bipolar disorders input data detail. But if we choose the bipolar fuzzy soft mappings for our problem, then tests would be unreliable due to the lack of knowledge in the input assessments. Yet as we translate data through the new method we will cope with MPD’s various assets and related behavioral illnesses owing to its multi-polarity. Under the comparison to some current techniques, our proposed model and its mapping is real, robust and effective.

5 Conclusion

In our manuscript, we have studied the MPD and its associated mental disorders. We have proposed a novel technique to diagnose the disorder of patient by analyzing its basic and associated symptoms. For this purpose, we have introduced MPNS-mapping with its inverse mapping and some useful operations with its properties. We have constructed an algorithm having three phases: Firstly, we use our structure to diagnose the actual type of disorder in the patients. Secondly, we have estimated the ranking of suitable treatments for the patients according to the severeness of disorder by using MPNS-mapping. At third stage, we have constructed generalized MPNS-mapping to see the patient’s improvement record and to predict the time duration of patient’s treatment until he entered in his normal domain. In the area of neurological syndromes, this technique is helpful and effective to diagnose the diseases. It helps to amass the data at a large scale having multi polarity. Comparison shows that proposed algorithm is superior, easy to handle, valid, strong and flexible to solve the decision-making problems. In future, we will extend our research in the environment of new hybrid structures of fuzzy, soft and rough sets.

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