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Risk assessment in discrete production processes considering uncertainty and reliability: Z-number multi-stage fuzzy cognitive map with fuzzy learning algorithm

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Abstract
The Failure Mode and Effects Analysis (FMEA) technique due to its proactive nature can identify failures and their causes as well as potential effects, and provide preventive/controlling measures before they occur. Nevertheless, some of the shortcomings of the FMEA technique like lack of a mental framework for considering the relationships between risks, lack of systematic perspective in confronting with risks, and weakness of Risk Priority Number (RPN) score in mathematical basis and disregarding the uncertainty of problem reduce the reliability of the outputs. In this study, an approach based on the Multi-Stage Fuzzy Cognitive Map and the Z-number theory (Z-MSFCM) is proposed to simultaneously consider the concept of uncertainty and reliability in quantities of risk factors and the weights of causal relationships in the MSFCM. Besides, a novel learning approach for Z-MSFCM has been applied based on the combination of the Particle Swarm Optimization (PSO) and S-shaped transfer function (PSO-STF) to preserve the uncertain environment of the problem. The proposed approach has been applied in a manufacturing automotive parts company and results indicate that: first, Z-MSFCM by considering the causal relationships between risks and their uncertainty and reliability in comparison with traditional RPN can provide better process-oriented insight into the impact of risks on the system; and second, the PSO-STF has high potential in generating solutions with high separability compared to Nonlinear Hebbian Learning and PSO algorithms. To put it differently, the mentioned advantages of the proposed approach can help decision-makers to analyze the problem with high reliability.

Keywords Failure mode and effects analysis · Multi-stage fuzzy cognitive map · Z-number theory · Fuzzy learning algorithm · Risk assessment

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1 Introduction

Uncertainty and increased competitiveness of organizations have created many challenges for various industries in recent years. To effectively manage these challenges, various approaches have been developed. Identifying and managing risk is one of the main approaches to improve the effectiveness of the organization (Ayyub 2014). In other words, the risk management process is carried out to ensure decision-makers that all risks are formally identified, ranked, monitored, prevented, or mitigated. Consequently, the application of different types of qualitative and quantitative risk evaluation approaches in various organizations has been expanded (Vose 2008). These techniques are used to anticipate and prevent risks (failures), imperfections, and deficiencies that can arise in designing a product or a product manufacturing process.

One of the prevailing ways to achieve this goal is the Failure Mode and Effects Analysis (FMEA) technique. It is a systematic tool based on team members' cooperation and the pre-occurrence prevention principle. It is used to identify failures, causes, potential effects, and preventive and control measures in a system, design, process, and/or service before they occur (Hu et al. 2019). One of the advantages of this technique is its functionality compared to other reactive modes. In fact, in the FMEA, the goal is to prevent the damage that may occur in the future to save cost and time. Traditionally, by calculating risk priority number (RPN) which is derived by multiplying three risk factors, failure modes in FMEA are ranked. These three priority factors are severity (S), occurrence (O), and detection (D). Here, O indicates the likelihood of the failure, S indicates the seriousness of the effect of the failure, and D is the possibility of not detecting the failure. Each of the three risk factors is scored using a 10-point scale which is available in FMEA literature (Hu et al. 2019; Liu et al. 2013). Despite the positive features of this technique, some of its shortcomings can reduce the reliability of the results. Among the disadvantages of the traditional FMEA due to its conventional RPN score, it can be referred to disregarding the uncertainties in the values of the risk factors (Baghery et al. 2018), the mathematical equation for computing RPN does not have a completely scientific basis (Hu et al. 2019), failing in creating enough distinction between failure priorities and not considering the causal relationships between failures (Rezaee et al. 2018), and finally misleading the concentration of the FMEA team's efforts into struggling with failures with low severity instead of critical ones (Liu et al. 2013). On the other hand, numerous changes in the business environment have caused a systematic and holistic attitude instead of a non-holistic attitude to risk management (Rezaee et al. 2017).

The objective of this research is to present a novel approach by extended Multi-Stage Fuzzy Cognitive Map (MSFCM), and the parameter estimation of FMEA methods based on the Z-number theory to increase the reliability of the FMEA outcomes. In this study, by using the Z-number theory which considers the uncertainties of the experts and allocates their reliability to estimate the fuzzy parameters (Zadeh 2011), uncertainty and reliability in the process of quantifying risk assessment factors in the traditional FMEA are considered simultaneously. Besides, with a systematic perspective and the step-by-step approach, using the extended MSFCM based on the Z-number theory (Z-MSFCM), unlike the FMEA technique, it has been attempted to consider the causal relationships in the prioritization of the failures. The reason is that some failures can exacerbate critical situations by affecting other parts, and identifying these root failures can help the management team to deal with this problem. Also, when there is a step-by-step approach, especially in manufacturing processes, the occurrence of one failure leads to the occurrence of other failures in the next stages and a holistic perspective leads to unreliable results (Rezaee et al. 2018). In this study, by introducing a new fuzzy learning algorithm based on MSFCM, it has been endeavored to overcome one of the most fundamental shortcomings of the conventional RPN score, the lack of distinction between the assigned ratings to the failures. For the first time, Z-MSFCM is trained with triangular fuzzy numbers instead of crisp numbers to maintain the uncertain environment of the problem and to avoid eliminating the ambiguity of failures. Z-MSFCM outputs by fuzzy numbers help decision-makers to ensure the accuracy of the generated results. Another taken step to increase the quality of decision making is proposing a modified learning algorithm. It is based on the combination of the Particle Swarm Optimization (PSO) and S-shaped transfer function (PSO-STF) which generates solutions with high separability. The Mean Squared Error (MSE) function is implemented as the fitness function of the proposed approach which can improve the quality of the generated solutions by minimizing their errors. The proposed algorithm can reduce dependence on experts' opinions and increase the separability of the generated solutions. In this approach failures with high scores will have priority in the ranking. To validate the feasibility of the proposed approach, prioritizing the failures in the production line of a manufacturing automotive parts company is implemented and its results are compared with some traditional methods.

The rest of this study is organized as follows: In Sect. 2, some research is reviewed in two subsections as FMEA technique, Fuzzy Cognitive Map (FCM) method, and its learning algorithms. In Sect. 3, theoretical foundations of the FMEA technique, conventional FCM versus MSFCM methods, and Z-numbers theory are expressed. In Sect. 4, the proposed approach of this study is provided. In Sect. 5, the analysis of results from the implementation of the proposed approach in a case study is presented. Finally, in Sect. 6, the conclusions and development suggestions of this study are provided.

2 Literature review

In this section, the literature background in developing the FMEA technique is presented. This section exclusively reviews novel researches in Multi-Criteria Decision-Making (MCDM) and FCM realms. First, the earliest research is introduced, and then the most recent research in the development of the FMEA is reviewed, and compared with the proposed approach. Moreover, a brief introduction to the FCM learning algorithms has been prepared.

FMEA in the process is a systematic approach to recognize and impede the failure in the product and its process. Additionally, it is an analytical approach that depends on the prevention rule. Before the occurrence of failure, it is applied to recognize the potential causes of failure (Rezaee et al. 2017). Firstly, this technique developed as a formal design methodology in the 1960s by the aerospace industry (Bowles and Peláez 1995). It has been proven that FMEA is a practical and potent tool in evaluating potential failures and preventing them from outbreaking (Ravi Sankar and Prabhu 2001). Traditionally, risk assessment in FMEA is accomplished by developing an RPN score. Nonetheless, the RPN score exhibits some critical shortcomings when traditional FMEA is applied in real-world cases. Therefore, it has been suggested a lot of alternative methods in the FMEA literature resolve some of the drawbacks of the conventional RPN score and to implement it more effectively (Liu et al. 2013). Determining the priority of failure modes in FMEA needs MCDM analysis as a multifaceted challenge. Due to the involvement of multiple risk factors in the evaluation and prioritization of failure modes, FMEA can be counted as an MCDM problem (Liu et al. 2019a; Karunathilake et al. 2020). Kutlu and Ekmekçioglu (2012) with the combination of two practical methods, Fuzzy Analytic Hierarchy Process (AHP) and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), used linguistic variables for determining S, O, and D factors. Liu et al. (2012) used the extended VIKOR method for selecting the most severe failure modes for assessing the risk of general anesthesia process under fuzzy environment. Furthermore, Liu et al. (2014) proposed a new risk priority model for the risk assessment in rotor blades for an aircraft turbine based on D-numbers, and Grey Relational Projection (GRP). Wang et al. (2016a) proposed an improved Failure Mode Effects and Criticality Analysis (FMECA) in a feed system case by the Decision Making Trial and Evaluation Laboratory (DEMATEL) method. Mohsen and Fereshteh (2017) in a geothermal power plant case used the concept of the Z-number, Shannon entropy concept, and the fuzzy VIKOR technique to rank and prioritize the failure modes based on the minimum individual regret and the maxi group utility. In another study, after implementing the Process Failure Mode And Effects Analysis (PFMEA) technique, Baghery et al. (2018) applied interval Data Envelopment Analysis (DEA) and Grey Relational Analysis (GRA) to prioritize and analyze all failures. Lo and Liou (2018) proposed a new model that uses the BWM method in combination with the grey theory for FMEA in an international electronics company.

One of the main shortcomings of the FMEA is that it does not consider the relationships between the failures. In some cases, a failure may have causal relationships with other failures, and by relieving a failure, other failures may be eliminated. Hence, for the first time, Peláez and Bowles (1996) implemented FCM to overcome this shortcoming of FMEA. Baykasoğlu and Gölcük (2020) endeavored to develop fuzzy FMEA by proposing a hybrid multi-attribute decision-making model by combining Fuzzy Preference Programming (FPP), FCM, and fuzzy Graph-Theoretical Matrix Approach (GTMA) at a software company. Erbay and Özkan (2018) used FCM to consider causal relationships in fuzzy FMEA in a real-world IT case study. One of the most important steps in constructing FCM is learning algorithm. The main motivation for developing learning algorithms for FCM is due to two problems: First, the FCM cannot be made without the presence of an expert to build the knowledge map. Second, human experts' knowledge can be subjective, and frequently it could consist of biases and errors (Salmeron et al. 2019). There are three main learning algorithms for FCM: (1) Hebbian-based algorithms; (2) Population-based algorithms; (3) Hybrid algorithms (Papageorgiou 2011). Hebbian-based algorithms are based on Hebbian law and some of the most prominent algorithms in this group include Differential Hebbian Learning (DHL) (Dickerson and Kosko 1994), Active Hebbian Learning (AHL) (Papageorgiou et al. 2004), and Nonlinear Hebbian Learning (NHL) (Papageorgiou et al. 2003). Population-based algorithms are based on evolutionary algorithms like PSO (Parsopoulos et al. 2003), Genetic Algorithm (GA) (Khan et al. 2004), and chaotic Simulated Annealing (SA) (Alizadeh and Ghazanfari 2009). The last group of learning algorithms for FCM are hybrid algorithms which are a combination of Hebbian-based and population-based algorithms like NHL-Differential Evolution (DE) (Papageorgiou and Groumpos 2005), and NHL-Real Coded Genetic Algorithm (RCGA) (Peng et al. 2015).

So far, many studies have been proposed to prioritize the failures in different industries. These studies have attempted to consider most of the characteristics and factors that influence risk ranking to make the decision-making process more accurate and rational. Table 1 summarizes some of these studies and identifies the characteristics that are intended to prioritize the failures. Finally, the characteristics considered in this study are compared to other studies.

Table 1 A review of recent research on hybrid approaches based on the FMEA

Author(s)	Methods	Case study	Specification-characteristic			
			Uncertainty	Reliability	Causality	DM's opinion
Chemweno et al. (2015)	ANP	European process industries and European automated guided vehicle			✓	High
Wang et al. (2016b)	COPRAS, ANP	Healthcare industry	✓		✓	High
Rezaee et al. (2017)	MSFCM	Automobile parts industry			✓	Medium
Tian et al. (2018)	Fuzzy BWM, VIKOR	Grinding wheel system	✓			High
Bian et al. (2018)	TOPSIS, D-number	Aviation industry	✓			High
Fattahi and Khalilzadeh (2018)	Fuzzy MULTIMOORA, AHP	Steel industry	✓			High
Rezaee et al. (2018)	MSFCM	Food industry			✓	High
Nie et al. (2018)	BWM, COPRAS	Water gasification system	✓			High
Hu et al. (2019)	GRA, TOPSIS	Healthcare industry	✓			High
Li et al. (2019)	TOPSIS, rough set theory	A steam valve system	✓			Medium
Mangeli et al. (2019)	Revised fuzzy TOPSIS, LFPP, SVM	Copper leaching factory	✓			Medium
Liu et al. (2019b)	DEMATEL, AHP	Rotary switch	✓			High
Huang et al. (2019)	Z-number projection model, TOPSIS	Aircraft landing system	✓	✓		High
Fattahi et al. (2020)	Fuzzy AHP, Fuzzy weighted MULTIMOORA	Steel industry	✓			High
Boral et al. (2020)	Fuzzy AHP, FMAIRCA	Benchmark example	✓			High
Yazdi et al. (2020)	Fuzzy BWM	Supercritical water gasification	✓			High
Sagnak et al. (2020)	Fuzzy AHP, Fuzzy TODIM	Manufacturing industry	✓			High
Das et al. (2020)	Z-VIKOR, Shannon entropy	Crane operation	✓	✓		High
Chen et al. (2020)	Fuzzy MULTIMOORA, OWGA	Manufacturing industry	✓			High
Proposed Approach	MSFCM, Z-number theory	Automobile parts industry	✓	✓	✓	Medium

3 Preliminaries

The purpose of this study is to present a novel approach of PFMEA-MSFCM based on the Z-numbers theory to evaluate and prioritize failures. To this end, the theoretical foundations of the used methods in this proposed approach are discussed in this section.

3.1 Conventional FCM versus its multi-stage form

Conventional FCMs were presented by Kosko (1986) to illustrate the causal relationship between concepts and analyze inference patterns. FCM combines artificial neural networks and fuzzy logic. It is practical for modeling some sort of dynamic system called complex adaptive systems (Salmeron et al. 2017).

The concepts and weights of the edges are usually considered in the interval [0, 1] and [−1, 1], respectively. The connection strength between two nodes c_j and c_i is equal to w_{ji}, where w_{ji} can takes any value in the range of −1 to 1. There can be three types of causal relationships between concepts in the FCMs (Felix et al. 2019):

1. $w_{ji} > 0$, positive causality between the concepts of c_i and c_j,
2. $w_{ji} < 0$, negative causality between the concepts c_j and c_i,
3. $w_{ji}=0$, there is no relationship between the concepts c_j and c_i.

Together, these concepts represent the state vector $A = [A_1, A_2, \ldots, A_n]$. The state vector A must be repeatedly transmitted through the weight matrix to evolve the system. Therefore, the following rule is proposed to compute the state vector A for each concept of c_i at iteration t using Eq. (1).

$$A_i^t = f\left(A_i^{t-1} + \sum_{j \neq i, j=1}^{N} A_j^{t-1} \cdot W_{ji} \right) \tag{1}$$

where A^t is the value of concept c_i at iteration t, A_i^{t-1} is the value of concept c_i at the iteration $t-1$ and the W_{ji} is the weight of the connection from concept c_j to concept c_i. The function $f_{(x)}$ is a threshold function to convert the result in the interval [0, 1] or [−1, 1]. Bueno and Salmeron (2009) exhibited that the unipolar sigmoid function has the best performs among other threshold functions (see Eq. 2).

$$f_{(x)} = \frac{1}{1 + e^{-\lambda . x}} \tag{2}$$

where x is defined as the value of $A_i(k)$ at the equilibrium point and λ is a real positive number that models the slope of the function. Using Eqs. (1) and (2) the successive states of the vector can be computed. After each simulation, the new state vector is obtained with new values, and this process continues until equilibrium is reached (Salmeron et al. 2019). A simple FCM is presented in Fig. 1.

Based on the given description, a learning algorithm needs to implement the mentioned points about the FCM. Learning algorithms are implemented to increase decision-making reliability. Consequently, by applying such algorithms, the problem of non-convergence is solved and the efficiency of FCM is increased (Rezaee et al. 2018).

Conventional FCMs and presented learning algorithms in the past had deficient in some applications. Rezaee et al. (2017) for the first time introduced the concept of MSFCM.

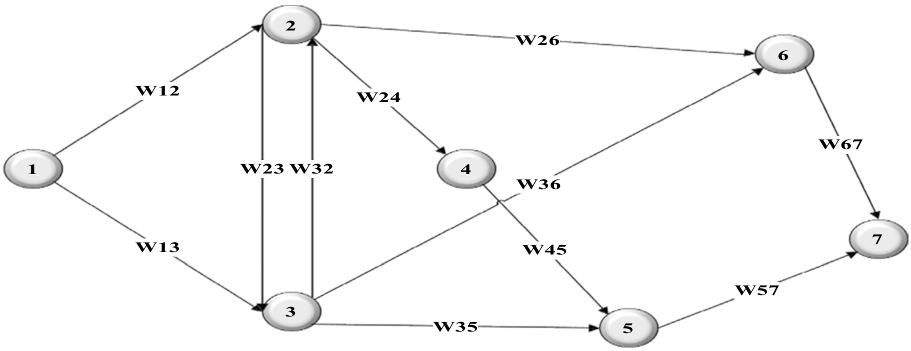

Fig. 1 An FCM with 7 nodes and 11 arcs

This method has the potential to evaluate complex systems with large amounts of data processed in the real world. FCMs are usually not considered as a process and merely assess the overall behavior of a system. However, MSFCM has the capability of considering all stages of a process and considering the relationship between the concepts of each stage and other stages. Accordingly, it can validate the decision-making process. MSFCM consists of several conventional FCMs, each is considered as a stage in the larger process. Each stage has several concepts that have internal causal relationships with each other. Also, they have external causal relationships with other concepts of various stages. In this study, each stage is considered as a step of the manufacturing process. Meanwhile, each main concept is considered as the corresponding stage's risks (see Fig. 2).

Figure 2 shows an MSFCM which has four stages. As it is clear, each stage has own concepts and causal relationships. Moreover, it has causal relationships with other stages. The concept of *C*1-1 is concept number 1 in stage 1, which affects concept *C*1-2, i.e. an internal causal relationship. This concept also has an arc to the concept *C*2-1, indicating its effect as an external causal relationship. In this way, the MSFCM can show the relationships of the other stages by describing the relationships within each stage.

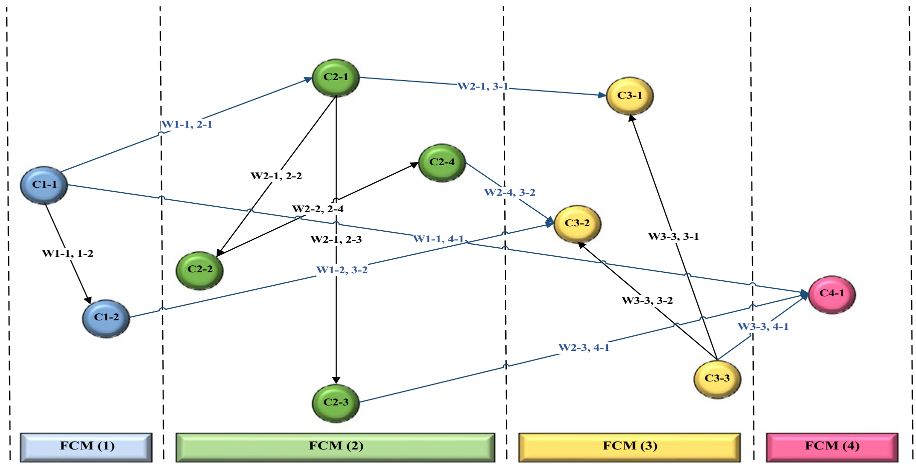

Fig. 2 A four-stage fuzzy cognitive map

3.2 Z-number theory

Zadeh (2011) for the first time proposed a new concept, the Z-number, which is a fuzzy number pair (A, R) (Kang et al. 2018a). The concept of Z-number is intended to serve as a basis for calculating numbers that are not reliable at all (Aboutorab et al. 2018). Z-numbers are expressed as a pair in which A and R are constraints to describing Z-numbers' behavior. A is usually a fuzzy set while R describes the degree of certainty. The degree of confidence may be expressed as a probability density function or fuzzy set. In this case, $Z = \{x|x \in A$ with a certain degree equal to $R\}$. To clarify the concept of Z-number, an industrial forecasting model can be used to announce production rate:

("production rate in a manufacturing unit is 105 parts for next two months", very likely)

This expression can be described as "X is $Z = (A, R)$", where X is the variable of "production rate", A is a fuzzy set that describes the production rate "105 parts for next 2 months", and R is the fuzzy probability of A to describe the degree of certainty in an event which is "very likely" (see Eq. 3) (Aliev et al. 2015).

$$R(X) = X \quad \text{is } A, \tag{3}$$

It is known as probability constraint and A has the role of probability distribution X. Specifically, it can be stated that:

$$R(X) : X \quad \text{is } A \rightarrow Poss(X = u) = \mu_A(u) \tag{4}$$

where μ_A is the membership function of A, and u is the general value of X. μ_A can be described as a constraint that is associated with $R(X)$, meaning that $\mu_A(u)$ is the degree that u satisfies this constraint (see Eq. 5).

$$R(X) : X \quad \text{is } p \tag{5}$$

In which p is the probability density function of X (Zadeh 2011):

$$R(X) : X \quad \text{is } p \rightarrow \Pr ob(u \leq X \leq u + du) = p(u)du \tag{6}$$

Zadeh (2011) underlines that declaring the problem described by Z-numbers is comparatively easy, but solving problems by Z-numbers is difficult in terms of computation. Hence, many researchers have endeavored to solve this problem (Qiao et al. 2019a). Two ways have been investigated to solve the mentioned problem of Z-numbers. The first one is to dispose of Z-numbers straightforward with the operations of them. The other is to transform Z-numbers into other forms of fuzzy numbers or crisp numbers (Song et al. 2020). For the first group there are studies like arithmetic computations on Z-numbers (Aliev et al. 2017), and measuring uncertainty for Z-numbers (Kang et al. 2018b). These methods follow the classical fuzzy theory and probability theory, which often still maintain the high complexity of the operation process (Qiao et al. 2019b). In the second group, researchers try to develop a transformation based approach to address Z-numbers, in which Z-numbers are translated into some corresponding fuzzy numbers (Yaakob and Gegov 2016; Tian et al. 2020). It should be mention that although converting Z-number to the fuzzy number or crisp number can beneficially simplify the complex computational operations of Z-number, it may cause significant information loss during information conversion and consequently reduce the benefits of using Z-information. (Shen et al. 2019).

4 The proposed approach for PFMEA

This section presents the proposed Z-MSFCM approach and describes how to implement and train it. By combining the logic of Z-numbers, the PFMEA technique, and MSFCM, it seeks to enhance the accuracy of MSFCM in an uncertain environment. Furthermore, in this approach, by introducing a new fuzzy learning algorithm it is endeavored to reduce the dependence on experts' opinions more effectively, and enhance the separability of the solutions.

4.1 Z-MSFCM

One of the most important reasons that lead to introduce MSFCM is that failures are interconnected, and failures at one stage may cause failure at the next stages. Hence, by monitoring preceding stages, failures can be eliminated at the next stages. To this end, each of the failures is considered as an FCM concept. The MSFCM considers the relationships between failures at different stages and can exhibit very promising and powerful performance. However, the nature of the failures is ambiguous and unpredictable, and many factors affect it. Thus, the exact time for the occurrence of the failure cannot be determined. On the other hand, even with high experience and precise analyzes, the severity of a failure cannot be accurately determined because the failure severity can be varied depending on different factors. The defective product may reach the customer despite various corrective actions and inspections. However, there is a need for a more accurate scoring method in the uncertainty environment. The Z-number is a very appropriate approach for scoring risk factors in the PFMEA technique. It can provide reliability for each score, along with fuzzy scoring. For this purpose, Z-numbers are used in this study to score SOD factors and the causal relationships between the failures.

At first, after organizing the FMEA team, the team explores the failures involved in a process. After identifying the process failures, the SOD factor scores are assigned to each failure using Z-numbers, for which the scores in Tables 2 and 3 are used.

For example, when the FMEA team detects the value of "slight severity" (C) and "medium reliability" (M) for a failure, the Z-number value is as follows:

$$Z = [(0.2, 0.3, 0.4), (0.3, 0.5, 0.7)]$$

In this study, to convert Z-numbers into triangular fuzzy numbers, the study of Kang et al. (2012) has been used. For converting Z-numbers to fuzzy numbers, suppose that:

Table 2 Normal FMEA factor levels

Linguistic variables	Membership function
A	(0.1, 0.1, 0.2)
B	(0.1, 0.2, 0.3)
C	(0.2, 0.3, 0.4)
D	(0.3, 0.4, 0.5)
E	(0.4, 0.5, 0.6)
F	(0.5, 0.6, 0.7)
G	(0.6, 0.7, 0.8)
H	(0.7, 0.8, 0.9)
I	(0.8, 0.9, 1.0)
J	(0.9, 1.0, 1.0)

Table 3 Conversion rules for reliability linguistic variables (Aboutorab et al. 2018)

Linguistic variables	Membership function
Very low (VL)	(0.0, 0.0, 0.3)
Low (L)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
High (H)	(0.5, 0.7, 0.9)
Very high (VH)	(0.7, 1.0, 1.0)

$$\{\tilde{A} = (x, u_{\tilde{A}})|x \in [0, 1]\} \tag{7}$$

$$\{\tilde{R} = (x, u_{\tilde{R}})|x \in [0, 1]\} \tag{8}$$

The reliability of Z-numbers is converted into the crisp number as follows:

$$\alpha = \frac{\int x\mu_\beta dx}{\int \mu_\beta dx} \tag{9}$$

Then, crisp values of reliability are applied to the constraint:

$$\tilde{Z}^\alpha = \{(x, \mu_{\tilde{A}^\alpha})|\mu_{\tilde{A}^\alpha}(x) = \alpha\mu_{\tilde{A}}(x), x \in [0, 1])\} \tag{10}$$

Considering the previous example, which had a "slight severity" with "medium reliability", converting the Z-number into fuzzy number as follows:

First, reliability is converted into a crisp number:

$$\alpha = \frac{\int x\mu_\beta dx}{\int \mu_\beta dx} = 0.5$$

Then, the obtained crisp number is applied to the constraint:

$$\tilde{Z}^\alpha = (0.2, 0.3, 0.4;0.5)$$

Finally, Z-number converts into a fuzzy number:

$$\tilde{Z}' = (0.2 \times \sqrt{0.5}, 0.3 \times \sqrt{0.5}, 0.4 \times \sqrt{0.5}) = (0.14142, 0.21213, 0.28284)$$

Similarly, other values are converted into fuzzy numbers (See Table 4).

After determining the SOD factor scores for each failure, the failures are considered as concepts of FCM, and SOD factors are considered as the input concepts to the target concept. Under this assumption, MSFCM experts depict the map and identify causal relationships between concepts (both internal and external). Experts in this section use Z-numbers to score the causal relationships of each process failure, SOD factors, and target concepts. Figure 3 shows an overview of one stage of Z-MSFCM-PFMEA. In this figure, for example, $C_Z3 - 1$ represents the first failure in the third stage, which has been assigned a score according to Z-numbers. Its SOD factors have been rated by Z-numbers, and the strength of causal relationships between the factors and failure have been allocated by Z-numbers. $W_Z3 - 3, 3 - 1$ represents the causal strength of the third concept in stage three, to the first concept in stage three in the form of Z-numbers.

Table 4 Converted Z-numbers into fuzzy numbers

Linguistic variables	Transfer function	Linguistic variables	Transfer function
A, VL	(0.0315, 0.0315, 0.0630)	F, VL	(0.1575, 0.1890, 0.2205)
A, L	(0.0548, 0.0548, 0.1095)	F, L	(0.2739, 0.3286, 0.3834)
A, M	(0.0707, 0.0707, 0.1414)	F, M	(0.3536, 0.4243, 0.4950)
A, H	(0.0837, 0.0837, 0.1673)	F, H	(0.4184, 0.5020, 0.5857)
A, VH	(0.0951, 0.0951, 0.1903)	F, VH	(0.4757, 0.5708, 0.6660)
B, VL	(0.0315, 0.0630, 0.0945)	G, VL	(0.1890, 0.2205, 0.2520)
B, L	(0.0548, 0.1095, 0.1643)	G, L	(0.3286, 0.3834, 0.4382)
B, M	(0.0707, 0.1414, 0.2121)	G, M	(0.4243, 0.4950, 0.5657)
B, H	(0.0837, 0.1673, 0.2510)	G, H	(0.5020, 0.5857, 0.6694)
B, VH	(0.0951, 0.1903, 0.2854)	G, VH	(0.5708, 0.6660, 0.7611)
C, VL	(0.0630, 0.0945, 0.1260)	H, VL	(0.2205, 0.2520, 0.2835)
C, L	(0.1095, 0.1643, 0.2191)	H, L	(0.3834, 0.4382, 0.4929)
C, M	(0.1414, 0.2121, 0.2828)	H, M	(0.4950, 0.5657, 0.6364)
C, H	(0.1673, 0.2510, 0.3347)	H, H	(0.5857, 0.6694, 0.7530)
C, VH	(0.1903, 0.2854, 0.3806)	H, VH	(0.6660, 0.7611, 0.8563)
D, VL	(0.0945, 0.1260, 0.1575)	I, VL	(0.2520, 0.2835, 0.3150)
D, L	(0.1643, 0.2191, 0.2739)	I, L	(0.4382, 0.4929, 0.5477)
D, M	(0.2121, 0.2828, 0.3536)	I, M	(0.5657, 0.6364, 0.7071)
D, H	(0.2510, 0.3347, 0.4184)	I, H	(0.6694, 0.7530, 0.8367)
D, VH	(0.2854, 0.3806, 0.4757)	I, VH	(0.7611, 0.8563, 0.9514)
E, VL	(0.1260, 0.1575, 0.1890)	J, VL	(0.2835, 0.3150, 0.3150)
E, L	(0.2191, 0.2739, 0.3286)	J, L	(0.4929, 0.5477, 0.5477)
E, M	(0.2828, 0.3536, 0.4243)	J, M	(0.6364, 0.7071, 0.7071)
E, H	(0.3347, 0.4184, 0.5020)	J, H	(0.7530, 0.8367, 0.8367)
E, VH	(0.3806, 0.4757, 0.5708)	J, VH	(0.8563, 0.9514, 0.9514)

4.2 The proposed fuzzy learning algorithm

As noted before, the nature of the failure is uncertain and ambiguous. So far, two important steps have been taken to maintain this uncertainty. First, Z-numbers are used to estimate the parameters of the PFMEA technique. This considers both failures' uncertainty and the reliability of the fuzzy number. Second, the MSFCM method, which is capable of considering causal relationships between each failure. The last step, in this case, is applying a fuzzy learning algorithm to accurately rank the failures. Traditional FCMs are trained with crisp numbers but in this study, MSFCM is trained for the first time by fuzzy numbers to maintain the uncertainty of the problem. By training MSFCM with fuzzy numbers, the output of the map instead of the crisp number becomes triangular fuzzy numbers. The output of the map in fuzzy numbers ensures that the failure rating process is performed with more confidence. The input of MSFCM consists of causal relationships between the failures and the importance of each failure in the form of fuzzy numbers. The whole learning process of the algorithm is based on fuzzy numbers. Now, to prioritize the failures involved in the production process, the proposed approach is applied to the problem.

In this regard, PSO-STF modified learning algorithm is proposed which is based on the combination of the PSO algorithm and the S-shaped transfer function. PSO is based

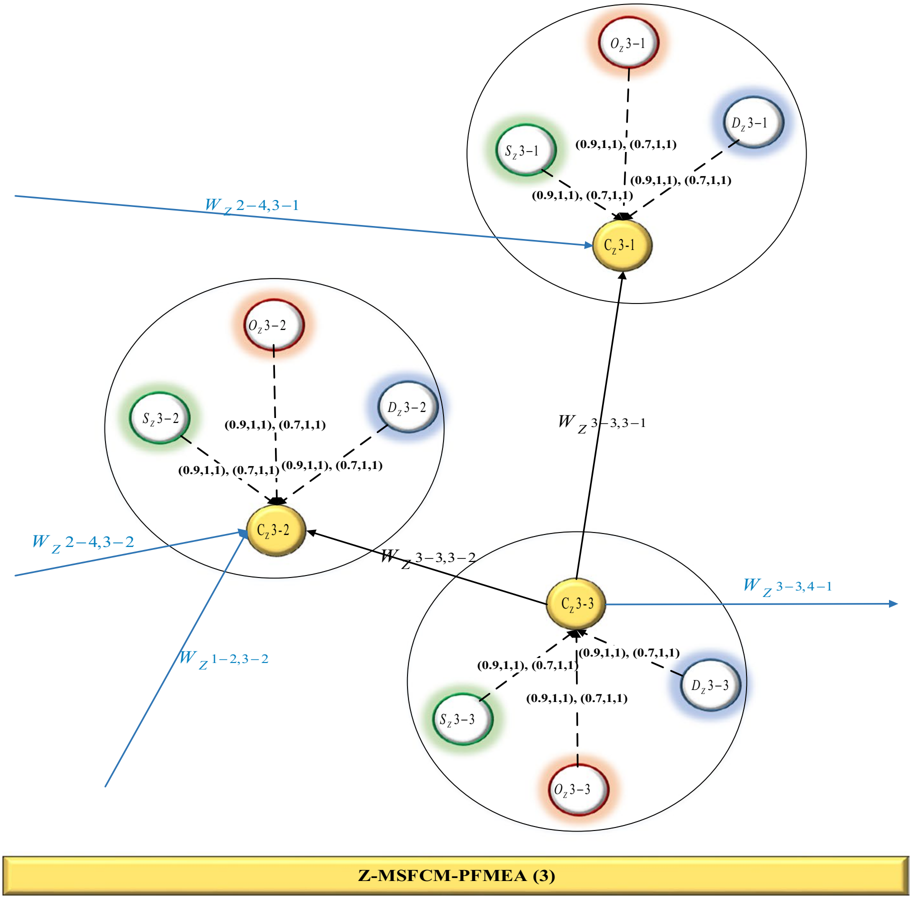

Fig. 3 An overview of the third stages of the Z-MSFCM-PFMEA approach

on artificial intelligence and selected due to its efficiency and effectiveness on an excess of applications in science and engineering, and its straightforward applicability (Juneja and Nagar 2016; Parsopoulos et al. 2003). The experimental results demonstrate that the PSO can converge quickly (Shi and Eberhart 1999) because the PSO algorithm does not involve selection operation or mutation calculation so the search can be performed by repeatedly varying particle's speed (Juneja and Nagar 2016). The performance of PSO is not susceptible to the population size, and PSO scales well (Shi and Eberhart 1999). In the PSO, by learning from the group's experiences, particles fly only to proper areas. Moreover, the PSO is based on simple calculations, and with the development of newer evaluation techniques, they are being carried out easily (Juneja and Nagar 2016). The PSO algorithm achieves the optimal solution by generating random populations and evaluating them with a fitness function. PSO algorithm avoids producing unjustified solutions by limiting the generated solutions in the feasible region. This feature of the PSO algorithm causes generating solutions that do not have acceptable separability which is spread in a short interval. On the other hand, decision-makers need a method to help them make reliable decisions. To overcome this shortcoming, the PSO-STF is proposed. The PSO algorithm generates new solutions based on two main equations:

$$v_i(t+1) = w * v_i(t) + c_1 * rand() * (pbest_i(t) - x_i(t)) + c_2 * rand() * (gbest(t) - x_i(t)) \tag{11}$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{12}$$

c_1 and c_2 in Eq. (11) are acceleration constants that refer to the weighting of the stochastic acceleration terms that pull each particle toward *pbest* (personal best) and *gbest* (global best) positions, *rand()* is a random variable that is generated by uniform distribution between 0 and 1, w is inertia weight, x refers to the position vector, and v velocity vector (Kennedy and Eberhart 1995).

The next step to improve the accuracy of the algorithm is using an appropriate and accurate transfer function to improve the accuracy of the generated solutions with high separability. Transfer functions are a very important factor in FCMs to generate appropriate solutions. The S-shaped transfer function is one of the functions that is used in image processing (Bansal et al. 2008), transportation planning decisions (Peidro and Vasant 2011), and MCDM (Vasant and Bhattacharya 2007). In this research, this function is used as the transfer function of the problem's learning algorithm. The S-shaped curve (see Fig. 4) is dependent on two parameters a and b that determine the two upper and lower boundaries of the slope of the curve (see Eq. 13).

$$f(x;a,b) = \begin{cases} 0, & x \le a \\ 2\left(\frac{x-a}{b-a}\right)^2, & a \le x \le \frac{a+b}{2} \\ 1 - 2\left(\frac{x-b}{b-a}\right)^2, & \frac{a+b}{2} \le x \le b \\ 1, & x \ge b \end{cases} ; \quad a < b \tag{13}$$

On the other hand, the MSE function is used as the fitness function of the PSO-STF. This function attempts at each iteration of the algorithm to reduce the error of the generated solutions. To evaluate the robustness of the proposed algorithm, it will be compared with both the NHL and PSO algorithms. To train Z-MFCM-PFMEA, the PSO-STF obtains the failure scores as fuzzy numbers and optimizes the results by updating random populations for the

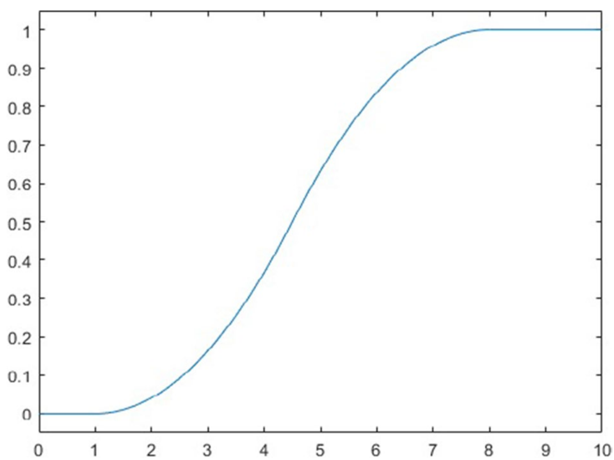

Fig. 4 S-shaped transfer function

weights of the causal relationships and combining them with the values of the SOD factors. The pseudo-code of the proposed learning algorithm based on the combination of S-shaped transfer function and PSO algorithm has been presented as follows:

The proposed fuzzy learning algorithm pseudo-code
Initialization phase: Determining the population (swarm) size (Weight matrix), the maximum number of algorithm iteration, initial position (particle) x and velocities v, and c_1, c_2, and w. **Repeat:** **Transfer function:** Leading the generated random solutions and concept values to the S-shaped transfer function. **Evaluating:** Evaluating each particle's value according to the MSE objective function. **Termination of the desired value check:** If the desired value is obtained, exit the loop **Discovering the personal best:** Find the best-generated solution for each particle **if** $fitness[(x) < (P_{best})]$: Update velocity and position according to equations (4), (5) **else** **Discovering the global best** Find the best-generated solution for all of the generated particles **if** $fitness[(P_{best}) < (G_{best})]$: It is the best-generated solution **else** **Update the velocities** Update the velocity of each particle according to the equation (4) **Update positions** Update the position of each particle according to the equation (5) **Until:** stopping criterion is met.

Once the Z-MSFCM-PFMEA steady state is reached, the initial values of each failure are prepared for the implementation of the Z-MSFCM. In this section, according to the MSFCM approach, the algorithm presented in this study is implemented for Z-MSFCM. As explained above, the first stage of the Z-MSFCM is trained, and after reaching the steady state, the values of each concept are considered as the final score of each failure. If any of the concepts is a pre-requisite for other concepts at the next stage, they will be considered as one of the Z-MSFCM concepts at that stage. Likewise, all stages of the Z-MSFCM are executed to obtain the final failure scores. Once all the failures have been achieved, the failures are ranked. Because the final values of points are obtained as triangular fuzzy numbers, the mean and variance of triangular fuzzy numbers according to Eqs. (14) and (15) are used to rank them (Baghery et al. 2018). This approach is based on the (Zimmermann 2011) and risks with higher mean value will achieve lower ranks which exhibit their cruciality and higher priority. For risks with the same mean value, risk with a higher variance will have a higher priority. If the obtained fuzzy score for any risk is considered as $\tilde{S}_i = \left(S_i^l, S_i^m, S_i^u\right)$, its mean and variance will be calculated as follows ($i = 1, \ldots, n$):

$$\overline{X}(\tilde{S}) = \frac{1}{3}(S_i^l + S_i^m + S_i^u) \tag{14}$$

$$\sigma(\tilde{S}_i) = \frac{1}{18}\left[(S_i^l)^2 + (S_i^m)^2 + (S_i^u)^2 - (S_i^l S_i^m) - (S_i^l S_i^u) - (S_i^m S_i^u)\right] \tag{15}$$

In this respect, S_i^l represents the lower bound of the fuzzy number, S_i^m represents the middle bound of the fuzzy number, and S_i^u represents the upper bound of the fuzzy number. The process of implementing the proposed approach step by step is illustrated in Fig. 5.

5 Analysis of results

In this section, the proposed approach is applied in a real-world case study. In the first subsection, the implementation process is described, and in the second subsection, the proposed approach is compared with some traditional algorithms and the results are analyzed.

5.1 Implementation

To validate the proposed approach, the involved failures in the process of manufacturing automotive parts in an active company are evaluated. In this regard, the existing 11-stage process is intended to produce the "Left room arm of Peugeot 405" (see Fig. 6) which a lot of failures have been reported by customers in this product. Besides, the FMEA team in this research to provide data in the proposed approach has been organized a multidisciplinary group. This group includes senior management representative, production manager, and quality control manager. Based on the implementation of the PFMEA technique, 34 failures have been identified during the process.

After identifying failures by the team, the SOD scores are assigned to them according to Tables 2 and 3. The stages of the process failure names and their SOD scores are shown in Table 5. Conversion of Z-numbers to fuzzy numbers is also achieved from Table 4, and the results of which are shown in Table 6. Now, according to the proposed approach, SOD factors are first considered as FCM concepts and each failure is considered as a target concept for SOD.

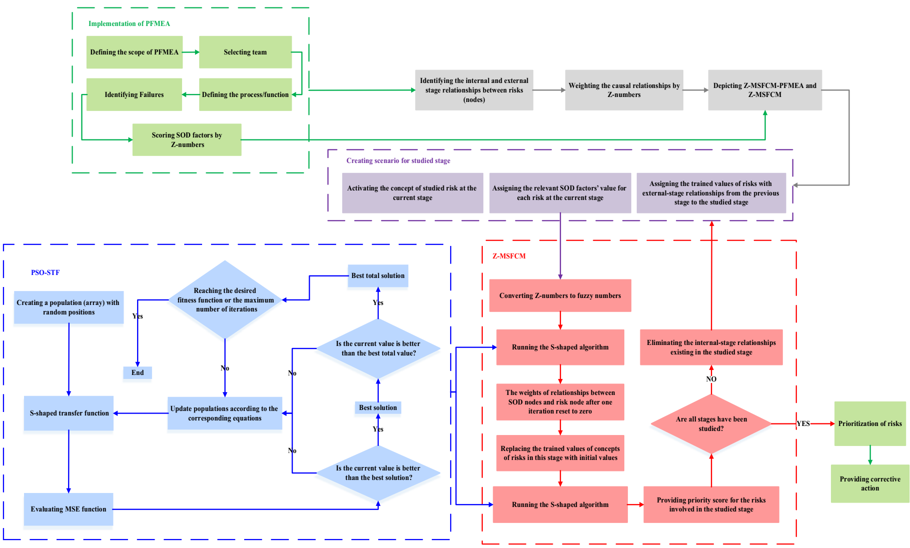

Fig. 5 The MSFCM-PFMEA approach for prioritizing failure

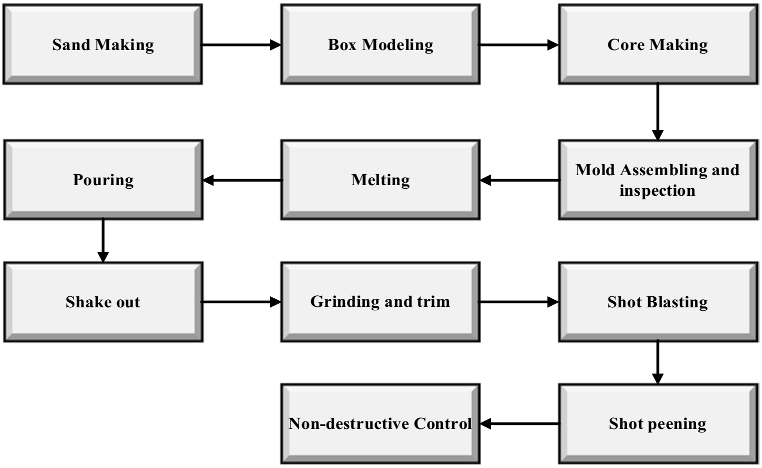

Fig. 6 Manufacturing stages in auto parts case study

To implement the proposed approach, an MSFCM should be depicted. In this regard, the FMEA team determines causal relationships between failures (FCM concepts) and determines the strength of each relationship between concepts by Z-numbers. The Z-numbers are then converted into fuzzy numbers to implement the algorithm according to Table 4. Figure 7 shows an overview of the MSFCM, and "Appendix 1" shows the causal relationships between concepts and their strength after converting from Z-numbers into fuzzy numbers.

To obtain initial values of the concepts, SOD factors are considered as the concepts, and each failure as to the target concept, and Z-MSFCM-PFMEA is executed for each failure. After reaching the steady state, the basic concepts of each failure are obtained and the Z-MSFCM approach can be implemented. In the MSFCM approach, in addition to the concepts within each stage, they have causal relationships with other concepts in the next stages. This FCM is trained step-by-step. At first, each phase of the MSFCM is trained, and after reaching the steady state, the other stages are trained. The crucial point in implementing MSFCM is that the concepts at the previous stages can be one of the concepts in the current stage. This important point is due to the nature of the problem, and because of affecting a risk on the other risk(s) in other stages, its effect on the other risk(s) should be considered. To evaluate the performance of the proposed algorithm, this method has been compared with the NHL and PSO algorithms. The parameters of the PSO and PSO-STF are the same for a fair comparison and MSE function has been implemented as the fitness function for both algorithms. The maximum number of iterations and population size is set to 400 and 50, respectively. Clerc and Kennedy (2002) generalized the model of the PSO algorithm, containing a set of coefficients to control the system's convergence tendencies. Their approach is implemented in this study, and the rest of the PSO parameters are set based on Eq. (16). The constriction coefficients are $\phi_1 = \phi_2 = 2.05$ and $\Phi = \phi_1 + \phi_2$. The value χ is attained based on Eq. (16) and Φ. The inertia weight, ω, is set to χ. The acceleration coefficients, c_1 and c_2, are obtained as $c_1 = \phi_1 \times \chi$ and $c_2 = \phi_2 \times \chi$.

Table 5 List of failures in the Left room arm of Peugeot 405 manufacturing process

Num. stage	Process name	Failure name	Symbol	S		O		D	
				Constraint	Reliability	Constraint	Reliability	Constraint	Reliability
1	Sand making	Low active clay percentage/ low Bentonite	F1, 1	(0.7, 0.8, 0.9)	(0.5, 0.7, 0.9)	(0.3, 0.4, 0.5)	(0.5, 0.7, 0.9)	(0.2, 0.3, 0.4)	(0.5, 0.7, 0.9)
		Low humidity	F1, 2	(0.6, 0.7, 0.8)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)	(0.3, 0.4, 0.5)	(0.7, 1.0, 1.0)
		Mix low time	F1, 3	(0.7, 0.8, 0.9)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)	(0.3, 0.4, 0.5)	(0.7, 1.0, 1.0)
		Small amount of new sand	F1, 4	(0.6, 0.7, 0.8)	(0.3, 0.5, 0.7)	(0.1, 0.2, 0.3)	(0.3, 0.5, 0.7)	(0.3, 0.4, 0.5)	(0.3, 0.5, 0.7)
		High amounts of the soil	F1, 5	(0.7, 0.8, 0.9)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)
		The high temperature of the old sand	F1, 6	(0.6, 0.7, 0.8)	(0.5, 0.7, 0.9)	(0.1, 0.2, 0.3)	(0.5, 0.7, 0.9)	(0.1, 0.2, 0.3)	(0.5, 0.7, 0.9)
		The little charcoal powder	F1, 7	(0.5, 0.6, 0.7)	(0.3, 0.5, 0.7)	(0.2, 0.3, 0.4)	(0.3, 0.5, 0.7)	(0.3, 0.4, 0.5)	(0.3, 0.5, 0.7)
2	Box Molding	High percentage of clay	F2, 1	(0.7, 0.8, 0.9)	(0.3, 0.5, 0.7)	(0.2, 0.3, 0.4)	(0.3, 0.5, 0.7)	(0.1, 0.2, 0.3)	(0.3, 0.5, 0.7)
		Low vibration and press	F2, 2	(0.6, 0.7, 0.8)	(0.7, 1.0, 1.0)	(0.3, 0.4, 0.5)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)
		Low air pressure	F2, 3	(0.7, 0.8, 0.9)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)	(0.3, 0.4, 0.5)	(0.7, 1.0, 1.0)
		Inappropriate ventilation	F2, 4	(0.7, 0.8, 0.9)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)
		Inappropriate filtering	F2, 5	(0.7, 0.8, 0.9)	(0.7, 1.0, 1.0)	(0.3, 0.4, 0.5)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)
		Inappropriate core assembling	F2, 6	(0.6, 0.7, 0.8)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)
		Clash of the mold wall	F2, 7	(0.7, 0.8, 0.9)	(0.5, 0.7, 0.9)	(0.2, 0.3, 0.4)	(0.5, 0.7, 0.9)	(0.1, 0.2, 0.3)	(0.5, 0.7, 0.9)
3	Core Making	High adhesive	F3, 1	(0.6, 0.7, 0.8)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)
		High gas production	F3, 2	(0.7, 0.8, 0.9)	(0.7, 1.0, 1.0)	(0.3, 0.4, 0.5)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)
4	Mold assembling and inspection	Non centered pin and bush	F4, 1	(0.7, 0.8, 0.9)	(0.5, 0.7, 0.9)	(0.1, 0.2, 0.3)	(0.5, 0.7, 0.9)	(0.2, 0.3, 0.4)	(0.5, 0.7, 0.9)
5	Melting	Non-melting refine of the ladle furnace	F5, 1	(0.7, 0.8, 0.9)	(0.5, 0.7, 0.9)	(0.1, 0.2, 0.3)	(0.5, 0.7, 0.9)	(0.1, 0.2, 0.3)	(0.5, 0.7, 0.9)
		Improper charging	F5, 2	(0.8, 0.9, 1.0)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)
		A failure during the melting temperature control	F5, 3	(0.8, 0.9, 1.0)	(0.5, 0.7, 0.9)	(0.1, 0.2, 0.3)	(0.5, 0.7, 0.9)	(0.2, 0.3, 0.4)	(0.5, 0.7, 0.9)

Table 5 (continued)

Num. stage	Process name	Failure name	Symbol	S		O		D	
				Constraint	Reliability	Constraint	Reliability	Constraint	Reliability
		Incorrect calculation of melting	F5, 4	(0.8, 0.9, 1.0)	(0.5, 0.7, 0.9)	(0.3, 0.4, 0.5)	(0.5, 0.7, 0.9)	(0.2, 0.3, 0.4)	(0.5, 0.7, 0.9)
		Failure in sampling	F5, 5	(0.9, 1.0, 1.0)	(0.5, 0.7, 0.9)	(0.2, 0.3, 0.4)	(0.5, 0.7, 0.9)	(0.1, 0.2, 0.3)	(0.5, 0.7, 0.9)
		Not following the correct procedure of germination	F5, 6	(0.9, 1.0, 1.0)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)
		High inoculation temperature of germinal	F5, 7	(0.9, 1.0, 1.0)	(0.5, 0.7, 0.9)	(0.2, 0.3, 0.4)	(0.5, 0.7, 0.9)	(0.2, 0.3, 0.4)	(0.5, 0.7, 0.9)
		High and low rate of germinal	F5, 8	(0.8, 0.9, 1.0)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)	(0.3, 0.4, 0.5)	(0.7, 1.0, 1.0)
6	Pouring	Low speed of pouring	F6, 1	(0.9, 1.0, 1.0)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)
		The low temperature of melting	F6, 2	(0.9, 1.0, 1.0)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)
		The lack of uniformity in pouring	F6, 3	(0.6, 0.7, 0.8)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)
7	Shake out	Failure to comply with damping time	F7, 1	(0.5, 0.6, 0.7)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)
		Carelessness of the operators	F7, 2	(0.6, 0.7, 0.8)	(0.5, 0.7, 0.9)	(0.2, 0.3, 0.4)	(0.5, 0.7, 0.9)	(0.1, 0.2, 0.3)	(0.5, 0.7, 0.9)
8	Shot Blasting	Improper adjustment shot blasting time	F8, 1	(0.6, 0.7, 0.8)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)	(0.1, 0.2, 0.3)	(0.7, 1.0, 1.0)
9	Grinding and trim	Improper grinding and trim	F9, 1	(0.7, 0.8, 0.9)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)
10	Shot peening	Low shot peening	F10, 1	(0.8, 0.9, 1.0)	(0.3, 0.5, 0.7)	(0.2, 0.3, 0.4)	(0.3, 0.5, 0.7)	(0.1, 0.2, 0.3)	(0.3, 0.5, 0.7)
11	Non-destructive Control	No sign for tested parts	F11, 1	(0.8, 0.9, 1.0)	(0.7, 1.0, 1.0)	(0.2, 0.3, 0.4)	(0.7, 1.0, 1.0)	(0.3, 0.4, 0.5)	(0.7, 1.0, 1.0)

Table 6 Final values of failures' SOD factors after converting into triangular fuzzy numbers

	Risk factors value		
	S	O	D
F1, 1	(0.590, 0.670, 0.760)	(0.260, 0.340, 0.420)	(0.170, 0.260, 0.340)
F1, 2	(0.580, 0.670, 0.780)	(0.200, 0.290, 0.390)	(0.290, 0.390, 0.480)
F1, 3	(0.670, 0.770, 0.860)	(0.096, 0.200, 0.290)	(0.290, 0.390, 0.480)
F1, 4	(0.510, 0.590, 0.670)	(0.084, 0.170, 0.260)	(0.260, 0.340, 0.420)
F1, 5	(0.670, 0.770, 0.860)	(0.096, 0.200, 0.290)	(0.096, 0.200, 0.290)
F1, 6	(0.510, 0.590, 0.670)	(0.084, 0.170, 0.260)	(0.084, 0.170, 0.260)
F1, 7	(0.360, 0.430, 0.500)	(0.170, 0.260, 0.340)	(0.260, 0.340, 0.420)
F2, 1	(0.590, 0.670, 0.760)	(0.170, 0.260, 0.340)	(0.084, 0.170, 0.260)
F2, 2	(0.580, 0.670, 0.780)	(0.290, 0.390, 0.480)	(0.200, 0.290, 0.390)
F2, 3	(0.670, 0.770, 0.860)	(0.200, 0.290, 0.390)	(0.290, 0.390, 0.480)
F2, 4	(0.670, 0.770, 0.860)	(0.100, 0.200, 0.290)	(0.096, 0.200, 0.290)
F2, 5	(0.670, 0.770, 0.860)	(0.290, 0.390, 0.480)	(0.200, 0.290, 0.390)
F2, 6	(0.580, 0.670, 0.780)	(0.100, 0.200, 0.290)	(0.096, 0.200, 0.290)
F2, 7	(0.590, 0.670, 0.760)	(0.170, 0.260, 0.340)	(0.084, 0.170, 0.260)
F3, 1	(0.580, 0.670, 0.780)	(0.200, 0.290, 0.390)	(0.200, 0.290, 0.390)
F3, 2	(0.670, 0.770, 0.860)	(0.290, 0.390, 0.480)	(0.200, 0.290, 0.390)
F4, 1	(0.590, 0.670, 0.760)	(0.084, 0.170, 0.260)	(0.170, 0.260, 0.340)
F5, 1	(0.590, 0.670, 0.760)	(0.084, 0.170, 0.260)	(0.084, 0.170, 0.260)
F5, 2	(0.770, 0.860, 0.960)	(0.200, 0.290, 0.390)	(0.096, 0.200, 0.290)
F5, 3	(0.770, 0.860, 0.960)	(0.084, 0.170, 0.260)	(0.170, 0.260, 0.340)
F5, 4	(0.770, 0.860, 0.960)	(0.260, 0.340, 0.420)	(0.170, 0.260, 0.340)
F5, 5	(0.640, 0.710, 0.710)	(0.170, 0.260, 0.340)	(0.084, 0.170, 0.260)
F5, 6	(0.860, 0.960, 0.960)	(0.200, 0.290, 0.390)	(0.096, 0.200, 0.290)
F5, 7	(0.640, 0.710, 0.710)	(0.170, 0.260, 0.340)	(0.170, 0.260, 0.340)
F5, 8	(0.770, 0.860, 0.960)	(0.200, 0.290, 0.390)	(0.290, 0.390, 0.480)
F6, 1	(0.860, 0.960, 0.960)	(0.100, 0.200, 0.290)	(0.096, 0.200, 0.290)
F6, 2	(0.860, 0.960, 0.960)	(0.100, 0.200, 0.290)	(0.200, 0.290, 0.390)
F6, 3	(0.580, 0.670, 0.780)	(0.200, 0.290, 0.390)	(0.096, 0.200, 0.290)
F7, 1	(0.480, 0.580, 0.680)	(0.200, 0.290, 0.390)	(0.200, 0.290, 0.390)
F7, 2	(0.510, 0.590, 0.670)	(0.170, 0.260, 0.340)	(0.084, 0.170, 0.260)
F8, 1	(0.580, 0.670, 0.780)	(0.200, 0.290, 0.390)	(0.096, 0.200, 0.290)
F9, 1	(0.670, 0.770, 0.860)	(0.200, 0.290, 0.390)	(0.200, 0.290, 0.390)
F10, 1	(0.570, 0.640, 0.710)	(0.170, 0.260, 0.340)	(0.084, 0.170, 0.260)
F11, 1	(0.770, 0.860, 0.960)	(0.200, 0.290, 0.390)	(0.290, 0.390, 0.480)

$$\chi = \frac{2}{\Phi - 2 + \sqrt{\Phi^2 - 4\Phi}} \tag{16}$$

All of the learning algorithms are trained with triangular fuzzy numbers and the PSO and PSO-STF have been executed 50 times independently, and the solutions with the lowest objective function have been collected. The optimal results are shown in Table 7. Training MSFCM by fuzzy numbers is because of the uncertain environment in the problem,

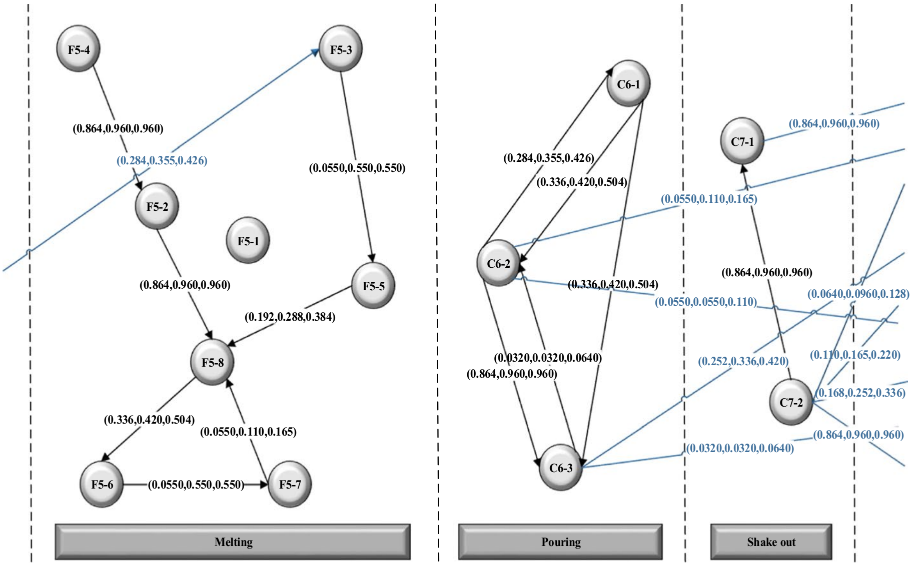

Fig. 7 The overview of the MSFCM based on the Z-number theory

and to avoid disregarding this important factor in the problem, and to guarantee the validity of the results. As stated above, the mean and variance of the fuzzy numbers are used to prioritize the failures. If the mean of the concepts is equal, the variance of those concepts will be calculated.

5.2 Comparisons and discussions

Table 7 shows the obtained final scores for the failures by the trained FCM algorithms. Obtained results for the NHL algorithm show that the average value of three failures (F11,1 in the Non-destructive Control stage, F1,3 in the Sand making stage and F9,1 in Grinding and trim stage) have the highest values, indicating that these failures are critical. The range of obtained scores by the NHL algorithm based on Z-MSFCM training is [0.2262, 0.9995], which is a broad and acceptable interval for the obtained solutions. The evaluation of the generated solutions by this algorithm has a significant separability which is acceptable for the solutions. The obtained mean values from the NHL algorithm have been plotted as a scatter plot (see Fig. 8) and their position compare to the regression line is shown. This graph evaluates the resolution of the obtained results by the dispersion of values. The NHL scatter plot shows good behavior and can be considered as acceptable performance. The examination of the largest and smallest produced solutions shows that the range of produced solutions is in a considerable interval. However, one of the problems of Hebbian-based algorithms in neural networks learning is the lack of convergence of the algorithm when there is a correlation between the input vectors or when they are independent. Further, they are not orthogonal which this issue does not lead to convergence based on the minimum squares of errors (Rezaee et al. 2017). Besides, the NHL algorithm with high probability tends to trap in the local minimum which is another shortcoming of this algorithm. Hence, their solutions are not reliable for decision-makers. Therefore, or reducing

Table 7 Obtained optimal results from FCM learning algorithms

	NHL (based on Hebbian rule)						PSO (based on evolutionary algorithms)						PSO-STF (PSO with S-shaped transfer function)					
	Score			Mean	Variance	Rank	Score			Mean	Variance	Rank	Score			Mean	Variance	Rank
F1,1	0.7064	0.7527	0.7958	0.7517	0.00034	19	0.6828	0.7183	0.7552	0.7188	0.00022	19	0.3597	0.4169	0.4623	0.4130	0.00045	20
F1,2	0.8971	0.9289	0.9387	0.9216	0.00008	6	0.7417	0.7446	0.7715	0.7526	0.00005	9	0.6772	0.7025	0.7456	0.7085	0.00020	8
F1,3	0.9631	0.9934	0.9948	0.9838	0.00006	2	0.7598	0.7836	0.8127	0.7854	0.00012	7	0.7610	0.8989	0.9227	0.8609	0.00128	5
F1,4	0.5668	0.6395	0.7255	0.6440	0.00106	29	0.7427	0.7440	0.7533	0.7467	0.00001	11	0.3578	0.5904	0.7300	0.5594	0.00590	14
F1,5	0.8306	0.8935	0.9345	0.8862	0.00046	7	0.7945	0.8078	0.8197	0.8074	0.00003	4	0.8998	0.9063	0.9124	0.9062	0.00001	4
F1,6	0.7483	0.8040	0.8486	0.8003	0.00043	15	0.7554	0.7699	0.7753	0.7669	0.00002	8	0.6242	0.6451	0.7799	0.6831	0.00120	10
F1,7	0.6171	0.6449	0.6757	0.6459	0.00015	28	0.6818	0.6839	0.6907	0.6855	0.00001	28	0.2147	0.3905	0.4038	0.3364	0.00186	23
F2,1	0.1380	0.2254	0.3151	0.2262	0.00131	31	0.5976	0.6237	0.6327	0.6180	0.00006	31	0.0457	0.0470	0.0948	0.0625	0.00014	31
F2,2	0.7893	0.8032	0.8284	0.8070	0.00007	14	0.7115	0.7431	0.7520	0.7356	0.00008	15	0.4547	0.5286	0.6760	0.5531	0.00212	15
F2,3	0.8473	0.8516	0.8587	0.8526	0.00001	10	0.7119	0.7457	0.7491	0.7356	0.00008	16	0.3979	0.5363	0.5703	0.5015	0.00139	18
F2,4	0.7138	0.7243	0.7407	0.7263	0.00004	23	0.6887	0.7027	0.7050	0.6988	0.00002	25	0.2967	0.3444	0.4269	0.3560	0.00073	22
F2,5	0.6908	0.7297	0.7826	0.7344	0.00036	22	0.7857	0.8041	0.8149	0.8016	0.00004	5	0.7212	0.7861	0.8297	0.7790	0.00050	6
F2,6	0.6907	0.7436	0.7940	0.7428	0.00045	21	0.7837	0.7997	0.8107	0.7981	0.00004	6	0.7498	0.7546	0.8212	0.7752	0.00027	7
F2,7	0.6974	0.7237	0.7513	0.7242	0.00013	24	0.6657	0.6703	0.6765	0.6709	0.00001	29	0.2021	0.2946	0.3164	0.2711	0.00062	29
F3,1	0.8135	0.8135	0.8135	0.8135	0.00001	11	0.7077	0.7100	0.7122	0.7100	0.00001	20	0.2616	0.2896	0.3115	0.2876	0.00011	27
F3,2	0.7346	0.7742	0.8145	0.7745	0.00027	17	0.6763	0.7217	0.7219	0.7067	0.00012	21	0.3522	0.4129	0.6232	0.4628	0.00338	19
F4,1	0.9488	0.9489	0.9491	0.9490	0.00001	5	0.7122	0.7329	0.7480	0.7311	0.00006	18	0.6324	0.6415	0.6530	0.6423	0.00002	12
F5,1	0.6591	0.6591	0.6591	0.6591	0.00001	27	0.6591	0.6591	0.6591	0.6591	0.00001	30	0.2499	0.2499	0.2499	0.2499	0.00001	30
F5,2	0.8119	0.8122	0.8125	0.8122	0.00001	13	0.6866	0.6951	0.7230	0.7016	0.00007	23	0.2673	0.3172	0.4126	0.3324	0.00091	25
F5,3	0.6591	0.6591	0.6591	0.6591	0.00001	27	0.6591	0.6591	0.6591	0.6591	0.00001	30	0.2499	0.2499	0.2499	0.2499	0.00001	30
F5,4	0.6591	0.6591	0.6591	0.6591	0.00001	27	0.6591	0.6591	0.6591	0.6591	0.00001	30	0.2499	0.2499	0.2499	0.2499	0.00001	30
F5,5	0.6613	0.7525	0.7523	0.7221	0.00047	25	0.6942	0.6961	0.7100	0.7001	0.00002	24	0.2783	0.2984	0.3725	0.3164	0.00042	26
F5,6	0.7402	0.7518	0.7710	0.7544	0.00005	18	0.6772	0.7055	0.7300	0.7043	0.00012	22	0.2756	0.3549	0.4472	0.3593	0.00123	21
F5,7	0.6689	0.6862	0.7058	0.6870	0.00006	26	0.6723	0.6899	0.7053	0.6892	0.00005	27	0.2793	0.2878	0.4404	0.3359	0.00137	24
F5,8	0.8640	0.8789	0.8925	0.8785	0.00004	8	0.7316	0.7342	0.7420	0.7360	0.00001	14	0.6524	0.6529	0.7723	0.6926	0.00080	9

Table 7 (continued)

	NHL (based on Hebbian rule)						PSO (based on evolutionary algorithms)						PSO-STF (PSO with S-shaped transfer function)					
	Score			Mean	Variance	Rank	Score			Mean	Variance	Rank	Score			Mean	Variance	Rank
F6,1	0.7665	0.7800	0.8035	0.7834	0.00006	16	0.7086	0.7411	0.7554	0.7351	0.00010	17	0.4354	0.5407	0.6275	0.5346	0.00155	17
F6,2	0.7314	0.7423	0.7678	0.7472	0.00006	20	0.7378	0.7506	0.7620	0.7502	0.00003	10	0.4032	0.6001	0.6243	0.5426	0.00246	16
F6,3	0.8690	0.8759	0.8884	0.8778	0.00002	9	0.7131	0.7459	0.7549	0.7380	0.00009	13	0.5792	0.6164	0.7397	0.6451	0.00118	11
F7,1	0.8135	0.8135	0.8135	0.8135	0.00001	12	0.6724	0.6747	0.7208	0.6893	0.00013	26	0.2545	0.2617	0.3146	0.2770	0.00018	28
F7,2	0.6591	0.6591	0.6591	0.6591	0.00001	27	0.6591	0.6591	0.6591	0.6591	0.00001	30	0.2499	0.2499	0.2499	0.2499	0.00001	30
F8,1	0.9555	0.9625	0.9713	0.9631	0.00002	4	0.8451	0.8601	0.8724	0.8592	0.00004	3	0.9349	0.9391	0.9618	0.9453	0.00004	3
F9,1	0.9403	0.9689	0.9808	0.9634	0.00008	3	0.9012	0.9042	0.9113	0.9056	0.00001	2	0.9739	0.9745	0.9805	0.9763	0.00001	2
F10,1	0.5305	0.5812	0.6567	0.5895	0.00068	30	0.7300	0.7439	0.7530	0.7423	0.00003	12	0.5931	0.6324	0.6558	0.6271	0.00017	13
F11,1	0.9994	0.9994	0.9995	0.9995	0.00001	1	0.9006	0.9288	0.9294	0.9196	0.00005	1	0.9979	0.9987	1.0000	0.9989	0.00001	1

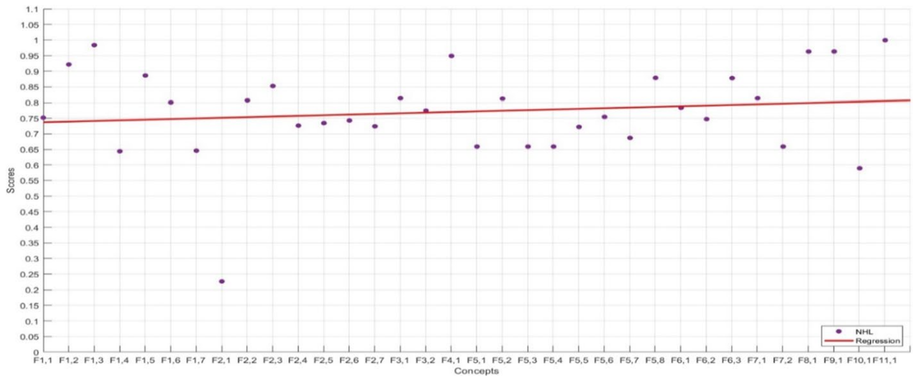

Fig. 8 Scatter plot of obtained values from the NHL algorithm

the probability of reaching the local minimum, population-based learning algorithms have been implemented.

Furthermore, the obtained results for the PSO algorithm shows that the average value of three failures (F11,1 in the Non-destructive Control stage, F9,1 in Grinding and trim stage and F8,1 in Shot Blasting stage). The obtained solutions by the PSO algorithm are in the range of [0.618, 0.9196], indicating that the interval is short. The other generated solutions in this interval are not sufficiently resolute and have a lack of validation to use them. The proximity of the generated solutions by this algorithm impedes its accurate rating and validity, and experts are unable to easily distinguish between different failures. According to Fig. 9, it can be seen that the PSO algorithm does not show good dispersion and the accumulation of most of the generated solutions near the regression line indicates the poor separability of this algorithm. The biggest and smallest generated solutions are in a short-range, which undermines the validity of this method. The main reason for the poor separability of this method is the PSO algorithm's attempt to justify the solutions, which leads to the convergence of the solutions.

On the other hand, according to the PSO-STF, the average of the three failures (F11,1 in the Non-destructive Control stage, F9,1 in Grinding and trim stage, and F8,1 in Shot Blasting stage) have the highest values. The separability of this method shows considerable

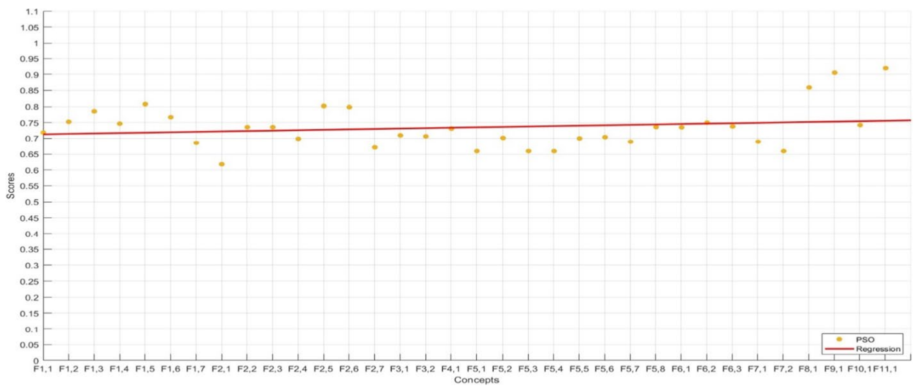

Fig. 9 Scatter plot of obtained values by PSO algorithm

performance. Generated solutions in the interval [0.0625, 0.9989] show better performance rather than the NHL algorithm. Its variability in producing solutions are more appropriate than the NHL algorithm. The strength of this algorithm in producing distinctive solutions facilitates the decision-making process. Moreover, differences between the failures are clear and understandable. The S-shaped scatter plot (Fig. 10) shows the best performance among the investigated algorithms. The generated solutions are generally far from the regression line which indicates that this method has a considerable separability. The domain of solutions, which can be inferred from the biggest and smallest generated solutions, approves the strongness of this method. Based on the S-shaped transfer function equation, it diffuses the generated solution into a more precise domain which does not change the nature of the generated solutions. Because the PSO algorithm generates solutions in the feasible area it cannot generate solutions with high separability in some cases and their real impact is disregarded. High separability is a very important factor in decision making and especially in the risk assessment. However, the PSO-STF can bold the real impact of the generated solutions. In fact, by implementing the S-shaped transfer function the solution will be valid in the optimization perspective, and also they will have acceptable separability to be used in the decision-making process.

In Fig. 11, the generated solutions by all three algorithms have been compared. This graph clearly shows that PSO-STF significantly performs better than the other two methods. The generated solutions by the PSO algorithm cannot compete with the NHL and PSO-STF. The PSO algorithm exhibits almost a linear behavior which indicates the closeness of the generated solutions by this algorithm. The proximity of the solutions prevents the experts and decision-makers from distinguishing between different failures and relying on the results. A comparison between NHL and PSO-STF shows that the PSO-STF performs better than the NHL algorithm. The NHL and PSO-STF produce a broader range of solutions. This being proven by comparing the biggest and smallest generated solutions by the two algorithms. By comparing the domains of two algorithms the PSO-STF has better performance. Also, the fluctuations of the solutions indicate that the PSO-STF has more tendency to generate varied solutions. The fluctuation of the PSO-STF solutions is more than the fluctuation of the NHL algorithm and its tendency to follow a nonlinear behavior is far greater than the NHL algorithm. Besides, the

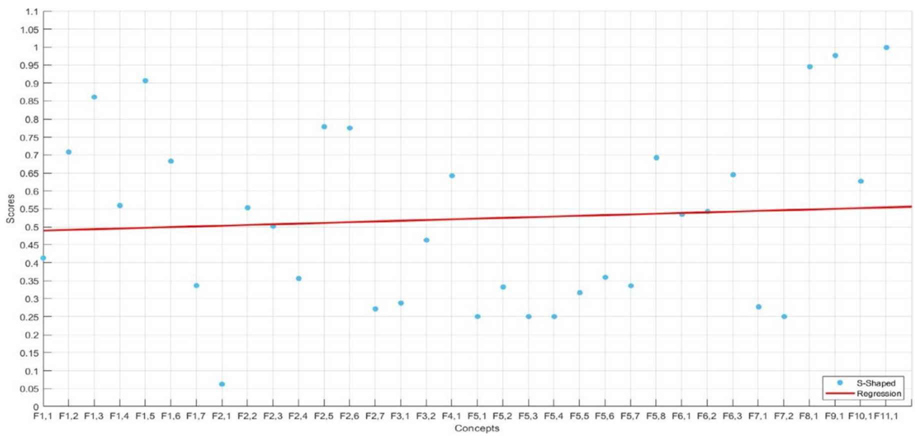

Fig. 10 Scatter plot of the obtained values from the PSO-STF

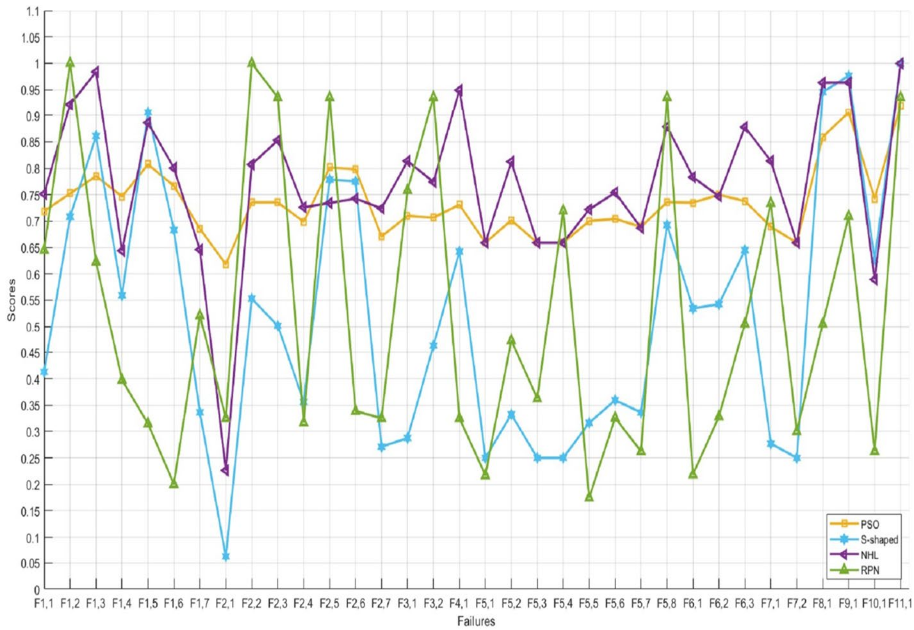

Fig. 11 Comparison of the generated solutions by FCM learning algorithms

distance of the generated solutions by the PSO-STF is greater than the NHL algorithm. The fluctuations in the solutions give assurance to experts and decision-makers that they can rely on their accuracy. The PSO-STF helps experts to distinguish between criticality and degree of failure by providing appropriate distinctions. They can understand the differences between the failures and thus, the decision-making process becomes easier and tangible.

According to the same result, the remarkable feature of the PSO-STF is the closeness of the generated solutions by that algorithm to the experts' opinion and their experiences. According to the experts' opinion, this algorithm is the closest one to the real experiences. The three failures, F11,1, F9,1, and F8,1, respectively, were selected as critical failures by the PSO-STF. These failures had been identified as the most critical failures by experts as well. Therefore, the PSO-STF can satisfy the most important need of the experts which is the high separability. It also makes an understandable distinction between solutions. Finally, the results of the traditional PFMEA technique are compared with the results of other algorithms. First, the mean of the obtained fuzzy values for the sod factors have been extracted based on Table 6. Then, it is compared with the results of the learning algorithms. As can be seen from the results of Fig. 11, the results of the PFMEA method are different from other Z-MSFCM algorithms. In other words, the prioritization of failures by this score does not correspond to other learning algorithms. Failure F1,2 and F2,2 are the most significant failures in the traditional PFMEA, whereas in other learning algorithms these failures are not recognized as a critical failure. The main reason for this incident is ignoring the causal relationships between the failures at different stages by the traditional PFMA. Using the MSFCM approach can help prioritize failures by considering the causal relationships between failures.

6 Conclusions and further studies

The main aim of this research is to present a novel approach based on MSFCM, PFMEA, and Z-number theory for prioritizing failures in the prone environment of the automotive spare parts industry to failures with ambiguous and uncertain nature. Another objective of this study is to provide a fuzzy learning algorithm for MSFCM to make the distinction between the priority of failures more tangible by generating solutions with high separability. In the first step of the proposed approach, the failure of the manufacturing automotive parts process is identified by the FMEA team and the SOD factors of each failure are calculated with Z-numbers. Then, the MSFCM is constructed with Z-numbers and the training phase of the map is done with a new modified algorithm based on fuzzy numbers to protect the uncertain environment of the problem. The proposed algorithm is based on the combination of the PSO algorithm and S-shaped transfer function which can generate solutions with high separability. The obtained results from this algorithm in comparison with the NHL and PSO algorithms show high performance in generating valid solutions with high separability. The results show that the domain of the PSO-STF's solutions is broader than the other two algorithms. The variety of solutions that this algorithm generates inside the interval has a significant advantage over the other two algorithms. Besides, comparing the results of the proposed approach with conventional methods such as the traditional FMEA demonstrates the high performance of the Z-MSFCM algorithms. The proposed algorithm has advantages in creating sufficient differentiation in prioritization of failures, considering causal relationships, and applying uncertainty in the risk assessment process in comparison with the FMEA technique. For future research, it is suggested that using other evolutionary algorithms for FCM training. The S-shape transfer function showed a good performance in the separability of the solutions. It is suggested that new transfer functions be used for FCM training and the results can be compared with the S-shaped solutions. Also, the use of other fitness functions can significantly improve the accuracy of the generated solutions. For more appropriately preserving the information of the Z-numbers some approaches have been introduced in the study which can be applied in future studies. Meanwhile, various industries have the potential to cause unpredictable failures, including electronics manufacturing industries that can be investigated.

Appendix 1

See Table 8.

Table 8 Final values of the weights of the relationships between the concepts based on fuzzy numbers

Failure	F1,1	F1,2	F1,3	F1,4	F1,5	F1,6	F1,7	F2,1
F1,1	(0,0,0)	(−0.960, −0.960, −0.864)	(0.864,0.960,0.960)	(−0.960, −0.960, −0.864)	(0.110,0.165,0.220)	(0.284,0.355,0.426)	(0.284,0.355,0.426)	(−0.960, −0.960, −0.864)
F1,2	(0,0,0)	(0,0,0)	(0.864,0.960,0.960)	(−0.960, −0.960, −0.864)	(0.284,0.355,0.426)	(0.284,0.355,0.426)	(0,0,0)	(−0.504, −0.420, −0.336)
F1,3	(0,0,0)	(0.864,0.960,0.960)	(0,0,0)	(0.864,0.960,0.960)	(0.284,0.355,0.426)	(0.284,0.355,0.426)	(0,0,0)	(−0.426, −0.355, −0.284)
F1,4	(−0.420, −0.336, −0.252)	(−0.504, −0.420, −0.336)	(0.960,0.960,0.960)	(0,0,0)	(0.384,0.480,0.576)	(−0.576, −0.480, −0.384)	(−0.576, −0.480, −0.384)	(−0.504, −0.420, −0.336)
F1,5	(0.284,0.355,0.426)	(0.864,0.960,0.960)	(0.864,0.960,0.960)	(0.284,0.355,0.426)	(0,0,0)	(0,0,0)	(0,0,0)	(0.284,0.355,0.426)
F1,6	(0.284,0.355,0.426)	(0.864,0.960,0.960)	(0.864,0.960,0.960)	(0.336,0.420,0.504)	(0.384,0.480,0.576)	(0,0,0)	(0,0,0)	(−0.504, −0.420, −0.336)
F1,7	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(−0.504, −0.420, −0.336)	(0,0,0)	(0,0,0)	(−0.504, −0.420, −0.336)
F2,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,3	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,4	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,5	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,6	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,7	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F3,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F3,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F4,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,3	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,4	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,5	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,6	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,7	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,8	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F6,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F6,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F6,3	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

Table 8 (continued)

Failure	F1,1	F1,2	F1,3	F1,4	F1,5	F1,6	F1,7	F2,1
F7,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F7,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F8,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F9,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F10,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F11,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

Failure	F2,2	F2,3	F2,4	F2,5	F2,6	F2,7	F3,1	F3,2	F4,1
F1,1	(0.284,0.355,0.426)	(0.864,0.960,0.960)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,2	(0.284,0.355,0.426)	(0.284,0.355,0.426)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,3	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(−0.420, −0.336, −0.252)	(0,0,0)	(0,0,0)	(0,0,0)
F1,4	(0.284,0.355,0.426)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,5	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,6	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.588,0.672,0.756)	(0,0,0)	(0,0,0)	(0,0,0)
F1,7	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.864,0.960,0.960)	(−0.504, −0.420, −0.336)	(0,0,0)
F2,1	(−0.504, −0.420, −0.336)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,2	(0,0,0)	(0,0,0)	(0,0,0)	(0.055,0.110,0.165)	(0.055,0.110,0.165)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,3	(0,0,0)	(0,0,0)	(0,0,0)	(0.055,0.055,0.110)	(0.055,0.110,0.165)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,4	(0,0,0)	(0,0,0)	(0,0,0)	(0.142,0.213,0.284)	(0.142,0.213,0.284)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,5	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,6	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.084,0.168,0.252)	(0,0,0)
F2,7	(0,0,0)	(0,0,0)	(0.284,0.355,0.426)	(0.055,0.110,0.165)	(0.055,0.110,0.165)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F3,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.672,0.768,0.864)	(0,0,0)
F3,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

Table 8 (continued)

Failure	F2,2	F2,3	F2,4	F2,5	F2,6	F2,7	F3,1	F3,2	F4,1
F4,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,3	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,4	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,5	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,6	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,7	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,8	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F6,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F6,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F6,3	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F7,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F7,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F8,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F9,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F10,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F11,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

Failure	F5,1	F5,2	F5,3	F5,4	F5,5	F5,6	F5,7	F5,8	F6,1	F6,2
F1,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,3	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,4	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,5	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,6	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,7	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

Table 8 (continued)

Failure	F5,1	F5,2	F5,3	F5,4	F5,5	F5,6	F5,7	F5,8	F6,1	F6,2
F2,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,3	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,4	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,5	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.284,0.355,0.426)	(0,0,0)
F2,6	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,7	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F3,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F3,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F4,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.864,0.960,0.960)	(0,0,0)	(0,0,0)
F5,3	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.055,0.550,0.550)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,4	(0,0,0)	(0.864,0.960,0.960)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F5,5	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.192,0.288,0.384)	(0,0,0)	(0,0,0)
F5,6	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.084,0.168,0.252)	(0,0,0)	(0,0,0)	(0,0,0)
F5,7	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.055,0.110,0.165)	(0,0,0)	(0,0,0)
F5,8	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.336,0.420,0.504)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F6,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.336,0.420,0.504)
F6,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.284,0.355,0.426)	(0,0,0)
F6,3	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.032,0.032,0.064)
F7,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F7,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F8,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

Table 8 (continued)

Failure	F5,1	F5,2	F5,3	F5,4	F5,5	F5,6	F5,7	F5,8	F6,1	F6,2
F9,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F10,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F11,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

Failure	F6,3	F7,1	F7,2	F8,1	F9,1	F10,1	F11,1
F1,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,3	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,4	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,5	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(− 0.960, − 0.960, − 0.864)	(0,0,0)
F1,6	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F1,7	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F2,1	(0,0,0)	(0,0,0)	(0,0,0)	(0.864,0.960,0.960)	(0.864,0.960,0.960)	(0,0,0)	(0,0,0)
F2,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(− 0.960, − 0.960, − 0.864)	(0,0,0)	(0,0,0)
F2,3	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(− 0.504, − 0.420, − 0.336)	(0,0,0)	(0,0,0)
F2,4	(0,0,0)	(0,0,0)	(0,0,0)	(0.110,0.165,0.220)	(0.864,0.960,0.960)	(0.284,0.355,0.426)	(0.864,0.960,0.960)
F2,5	(0,0,0)	(0,0,0)	(0,0,0)	(0.284,0.355,0.426)	(0.110,0.165,0.220)	(0,0,0)	(0,0,0)
F2,6	(0,0,0)	(0,0,0)	(0,0,0)	(0.336,0.420,0.504)	(0.336,0.420,0.504)	(0,0,0)	(0.096,0.192,0.288)
F2,7	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.284,0.355,0.426)	(0,0,0)	(0.355,0.426,0.497)
F3,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F3,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F4,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.756,0.840,0.840)	(0,0,0)	(0,0,0)
F5,1	(0,0,0)	(0,0,0)	(0,0,0)	(0.639,0.710,0.710)	(0.864,0.960,0.960)	(0,0,0)	(0,0,0)
F5,2	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.864,0.960,0.960)
F5,3	(0,0,0)	(0,0,0)	(0,0,0)	(0.220,0.275,0.330)	(0.336,0.420,0.504)	(0,0,0)	(0,0,0)
F5,4	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.864,0.960,0.960)

Table 8 (continued)

Failure	F6,3	F7,1	F7,2	F8,1	F9,1	F10,1	F11,1
F5,5	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.864,0.960,0.960)
F5,6	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.864,0.960,0.960)
F5,7	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.864,0.960,0.960)
F5,8	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.864,0.960,0.960)
F6,1	(0.336,0.420,0.504)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F6,2	(0.864,0.960,0.960)	(0,0,0)	(0,0,0)	(0,0,0)	(0.055,0.110,0.165)	(0,0,0)	(0.055,0.055,0.110)
F6,3	(0,0,0)	(0,0,0)	(0,0,0)	(0.252,0.336,0.420)	(0.032,0.032,0.064)	(0,0,0)	(0,0,0)
F7,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.864,0.960,0.960)
F7,2	(0,0,0)	(0.864,0.960,0.960)	(0,0,0)	(0.0640,0.0960,0.128)	(0.110,0.165,0.220)	(0.168,0.252,0.336)	(0.864,0.960,0.960)
F8,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(−0.504, −0.420, −0.336)	(0,0,0)	(0,0,0)
F9,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0.055,0.110,0.165)	(0,0,0)
F10,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
F11,1	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)

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