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A review and analysis of "graph theoretical-matrix permanent" approach to decision making with example applications

Adil Baykasoglu

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Abstract In this paper, a multiple attribute decision making technique is explained in detail and its existing applications are reviewed/analyzed first time in the literature. The technique was originated from combinatorial mathematics and it is based on the graph theory and matrix algebra and has some desirable properties like "ability to model criteria interactions", "ability to generate hierarchical models etc." for modeling and solving complex decision making problems. In order to enable a better understanding of the technique, two illustrative examples (new graduates' industry sector preferences and supermarket location selection) with crisp and fuzzy values are also modeled and solved in the present study.

Keywords Multiple attribute decision making · Graph theory · Matrix permanents · Combinatorics · Fuzzy variables

1 Introduction

Many different state of the art methods were proposed in the literature to handle and solve multiple attribute decision making (MADM) problems like AHP, ANP, TOPSIS, PROMETHEE, VIKOR, ELECTRE, GRA, LINMAP, Conjoint Analysis, Multi-Attribute Utility Theory etc. These techniques were extensively applied to numerous diverse decision making problems. For example; Kahraman et al. (2004) employed Analytical Hierarchy Process (AHP) with fuzzy data in order to compare of catering service companies in Turkey; Kayakutlu and Buyukozkan (2011) proposed an Analytical Network Process (ANP) based approach in order to explore, illustrate and assess the performance factors for 3PL companies through a managerial view; Buyukozkan et al. (2008) conducted the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) with fuzzy data to selection of the strategic alliance partner selection problem in logistics value chains; Dereli et al. (2010) developed a PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations)

A. Baykasoglu (✉)
Department of Industrial Engineering, Faculty of Engineering, Dokuz Eylul University, Izmir, Turkey
e-mail: baykasoglu@gmail.com

based approach for personnel selection; Chen and Wang (2009) employed VIKOR method for IS/IT outsourcing project partner selection problem; Hokkanen et al. (1995) employed ELECTRE (Elimination et Choice Translating Reality) method for selecting a solid waste management system; Xu et al. (2007) used GRA (Grey Relational Analyses) approach for conflict reassignment; Bereketli et al. (2011) developed a fuzzy LINMAP (LINear programming technique for Multidimensional Analysis of Preference) approach to evaluate treatment strategies for waste electrical and electronic equipments; Yoo and Ohta (1995) discussed the optimal pricing and product planning for new multi-attribute products based on the utility values of attribute levels measured by the conjoint analysis; Hatush and Skitmore (1998) applied Multi-Attribute Utility Theory (MAUT) to contractor selection problem in construction industry.

The advantages and disadvantages of these MADM techniques were also discussed in depth by many authors in the literature (Baykasoglu et al. 2009; Parkan and Wu 2000). In this paper a relatively new and not very well known approach for modeling and solving MADM problems is explained in detail and its existing applications are reviewed. This new approach is based on the graph theory and matrix algebra and has some desirable properties (like ability to model interactions, ability to structure problems hierarchically etc.) for modeling and solving complex decision making problems. The technique does not have a standard name in the literature for this reason it is named as GT-MP-DM (Graph Theory Matrix Permanent approach to Decision Making) in this paper. In order to enable better understanding, we present the application of the method on two example problems as well. We believe that GT-MP-DM will gain high acceptance and further improved by the research community for modeling and solving decision making problems.

The remainder of the paper is organized as follows: In Sect. 2 an overview of matrix permanents which is the core of the GT-MP-DM is given; in Sect. 3 the methodological steps of GT-MP-DM are presented; in Sect. 4 bibliometric analysis and literature review on GT-MP-DM is provided; Sect. 5 presents two example applications for GT-MP-DM with crisp and fuzzy data; finally the paper concludes in Sect. 6.

2 An overview of matrix permanents

Permanents were introduced in 1812 simultaneously by Binet and Cauchy (Minc 1978). Permanent is similar to determinant with a basic difference, in permanents the signatures of the permutations are not taken into account (*all signatures are taken as positive*). Due to this property no information is lost in permanents. Permanents are mainly used in combinatorial mathematics. Permanent describes the number of perfect matching in a bipartite graph. Let $A = (a_{ij})$ be an $m \times n$ matrix, $m \leq n$. The permanent of A, written as $Per(A)$ is defined as (Minc 1978):

$$Per(A) = \sum_{\sigma} a_{1\sigma(1)} a_{2\sigma(2)}, \ldots, a_{m\sigma(m)} \tag{1}$$

where, the summation extends over all one-to-one functions from $\{1, \ldots, m\}$ to $\{1, \ldots, n\}$. The sequence $a_{1\sigma(1)}, \ldots, a_{m\sigma(m)}$ is called a diagonal of A and the product $a_{1\sigma(1)} \ldots a_{m\sigma(m)}$ is a diagonal product of A. Thus the permanent of A is the sum of all diagonal products of A.

The permanent is much more difficult to compute than the determinant. The determinant can be computed in polynomial time by using Gaussian elimination. The permanent cannot be computed by using Gaussian elimination. Moreover, computing the permanent of a 0–1 matrix (matrix whose entries are 0 or 1) is #P-complete (Nourani and Andersen 1999). When

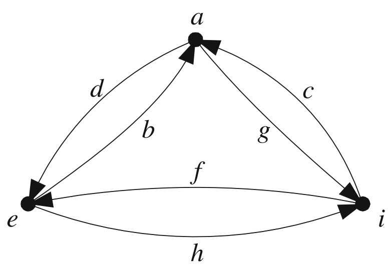


Fig. 1 Digraph representation of matrix A

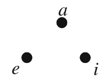
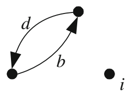
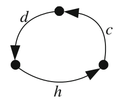
Group	Number of terms	Terms	Sub-graphs
1	1	*aei*	
2	0	-	No self loops
3	3	*bdi* *ceg* *afh*	 … etc.
4	2	*cdh* *bfg*	 … etc.

Fig. 2 Graphical interpretation of terms (sub-graphs) in $Per(A)$

the entries of the matrix are nonnegative, however, the permanent can be computed approximately in probabilistic polynomial time, up to an error of εM, where M is the value of the permanent and $\varepsilon > 0$ is arbitrary. In computing permanent of matrices Ryser formula which is given in Eq. 2 is generally used (Minc 1978).

$$Per(A) = (-1)^n \sum_{S \subseteq \{1,\ldots,n\}} (-1)^{|S|} \prod_{i=1}^{n} \sum_{j \in S} a_{ij} \tag{2}$$

where, the sum is over all subsets of $\{1, \ldots, n\}$ and $|S|$ is the number of elements in S. As an example of computing permanent consider the following 3×3 matrix (Glynn 2010):

$$A = \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$$

Matrix A can also be presented as a digraph as shown in Fig. 1 for visual interpretation. The classical Eq. (1) which makes use of all permutations in S_3 is:

$$Per(A) = aei + bfg + cdh + afh + bdi + ceg.$$

$Per(A)$ contains 6(3!) terms. These terms can be grouped into 4(3+1) as shown in Fig. 2.

Whereas Ryser's Eq. (2) gives:

$$Per(A) = (a+b+c)(d+e+f)(g+h+i) - (a+b)(d+e)(g+h) \\ -(a+c)(d+f)(g+i) - (b+c)(e+f)(h+i) + adg + beh + cfi$$

Vardi (1991) proposed a *Mathematica* function which programs Ryser's formula in *Mathematica* software. By using this function permanents of matrices can be computed in *Mathematica* software. The function is defined as follows (Weisstein 2010):

```
Permanent[m_List]:=With[{v=Array[x,Length[m]]},
        Coefficient[Times@@(m.v),Times@@v]]
```

Unlike the determinant, the permanent has no easy geometrical interpretation, therefore their usage are not very common in practical applications. Benjamin and Cameron (2005) provided a very nice example, which might be very useful geometrical interpretation of permanents. Their example is repeated here in order to provide a better understanding to readers.

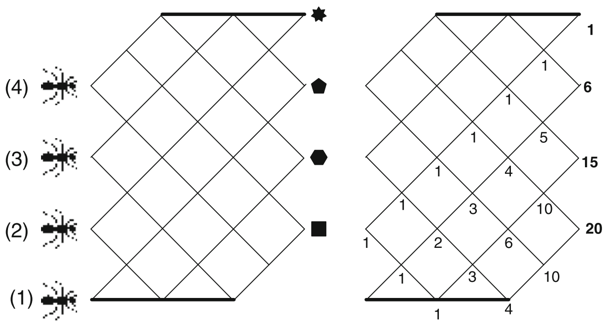
2.1 Benjamin and Cameron's example

Assume that there are four determined ants simultaneously walking along the edges of a picnic table graph as shown in Fig. 3a. The ants can only move right in order to reach four different morsels. How many ways can these ants simultaneously reach to different morsels?

In order to compute 4-paths, first the number of ways that each ant can reach to each morsel is found (see Fig. 3b for ant-2). Then this information is stored in a square matrix A whose entries are the number of ways that ant i can reach to morsel j (see Fig. 3c).

Then the permanent of this matrix produces the answer, which is 171361. If you are interested with the number of ways that ants can simultaneously reach to different morsels where no two paths intersects, you need to simply compute the determinant of matrix A (Benjamin and Cameron 2005). The answer is 889.

Several computer programs were proposed in the literature in order to compute the permanent of matrices (Shriver et al. 1969; Nijenhuis and Wilf 1975).



$$A = \begin{pmatrix} 14 & 6 & 1 & 0 \\ \mathbf{20} & \mathbf{15} & \mathbf{6} & \mathbf{1} \\ 15 & 20 & 15 & 6 \\ 6 & 15 & 20 & 14 \end{pmatrix}$$

(a) **(b)** **(c)**

Fig. 3 Benjamin and Cameron's example in explaining permanent "adopted from Benjamin and Cameron (2005)"

3 Application of GT-MP-DM to MCDM problems: the methodology

In this section detailed explanations about the GT-MP-DM approach in modeling and solving crisp and fuzzy MCDM problems are given. The methodology is explained in a step by step manner in order to facilitate a better understanding.

Step-1: Identify criteria, sub-criteria and alternatives which are essential to MCDM problem at hand. Structure the MCDM problem as shown in Fig. 4 in order to enable a better communication and presentation. As it can be seen from Fig. 4 it is possible to model a MCDM problem which has a hierarchical structure of criteria and interactions between criteria by using the GT-MP-DM approach.

Step-2: Determine the "relative importance" (or interaction) between criteria and "the scores of alternatives" for each criterion. The scores of alternatives can be obtained from standard or specified tests if quantitative objective values are available. Otherwise a ranked value judgment on a scale from 0 to 10 can be adopted similar to Rao and Gandhi (2002).

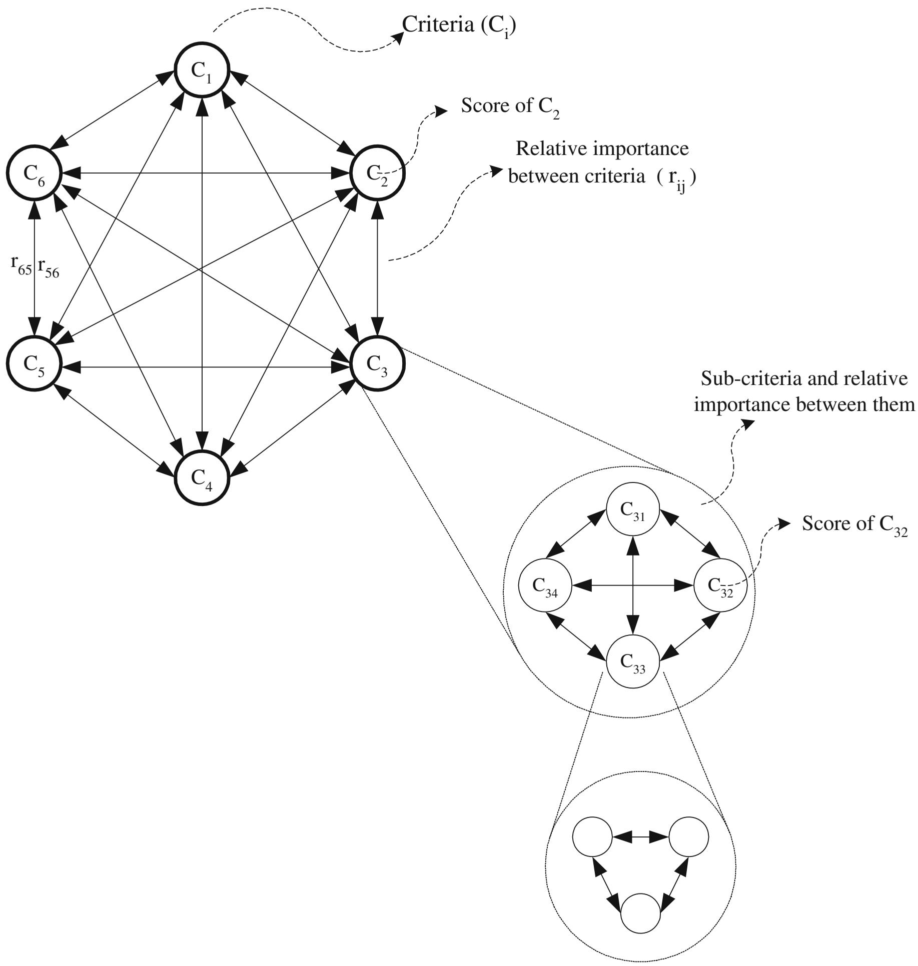


Fig. 4 Structure of MCDM problem for GT-MP-DM approach

Table 1 Qualitative scores for alternatives

Qualitative measure	Crisp score
Exceptionally low	0
Extremely low	1
Very low	2
Low	3
Below average	4
Average	5
Above average	6
High	7
Very high	8
Extremely high	9
Exceptionally high	10

For this purpose Table 1 can be used. However, it is possible that for an alternative some of the criteria values can be quantitative. Moreover, these quantitative values can have different units. In such cases it is necessary to convert or normalize the quantitative values on the same scale as qualitative values (i.e., 0–1). If the value of an alternative C_i has a range $\{C_{il}, C_{iu}\}$, the value 0 is assigned to lowest range (C_{il}) and 1 is assigned to highest range (C_{iu}). The other intermediate values (C_{ii}) can be determined by using Eq. 3 (Rao and Gandhi 2002).

$$C_i = (C_{ii} - C_{il})/(C_{iu} - C_{il}) \tag{3}$$

Equation 3 is suitable for beneficial criteria. Therefore for non-beneficial case the value of 0 is assigned to highest value (C_{iu}) and 1 is assigned to lowest value (C_{il}). The other intermediate values can be determined by using Eq. 4.

$$C_i = (C_{iu} - C_{ii})/(C_{iu} - C_{il}) \tag{4}$$

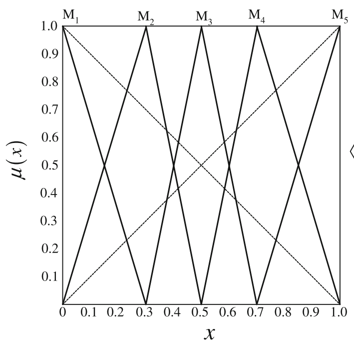
After computing all criteria values ($i = 1, \ldots, N$) for an alternative we can define a criteria rating matrix (Ψ) for that alternative. This is an $N * N$ square diagonal matrix as given by Eq. 5.

$$[\Psi] = \begin{bmatrix} C_{11} & 0 & \ldots & 0 \\ 0 & C_{22} & \ldots & 0 \\ . & \ldots & . & . \\ 0 & \ldots & 0 & C_{nn} \end{bmatrix} \tag{5}$$

Relative importance (or interaction) between criteria r_{ij} can also be assigned a value between 0 and 1 based on six scales as shown in Table 2. The values of r_{ij} can be obtained from the decision maker. In Table 2 $r_{ji} = 1 - r_{ij}$, but, it is not compulsory that $r_{ij}$and r_{ji} are interrelated. It may follow $r_{ji} = 1 - r_{ij} or r_{ji} = 1/r_{ij}$ or can be evaluated independently. It is also not necessary that the inter influence of the criteria to be equal. Thus the relative importance (interaction) matrix β as given by Eq. 6 can be symmetrical or non-symmetrical (Garg et al. 2006).

Table 2 Relative importance of criteria

Class definition	r_{ij}	$r_{ji} = 1 - r_{ij}$
Two criteria equally important	0.5	0.5
One criteria slightly more important than other	0.6	0.4
One criteria more important than other	0.7	0.3
One criteria very important than other	0.8	0.2
One criteria exceptionally important than other	0.9	0.1
One criteria most important, other not important	1.0	0.0



$$\mu_{M_1}(x) = \begin{cases} 1, & x = 0 \\ (0.3 - x)/0.3 & 0 \le x \le 0.3 \end{cases}$$

$$\mu_{M_2}(x) = \begin{cases} (x - 0)/0.25 & 0 \le x \le 0.25 \\ (0.5 - x)/0.25 & 0.25 \le x \le 0.5 \end{cases}$$

$$\Leftrightarrow \quad \mu_{M_3}(x) = \begin{cases} (x - 0.3)/0.2 & 0.3 \le x \le 0.5 \\ (0.7 - x)/0.2 & 0.5 \le x \le 0.7 \end{cases} \quad \Leftrightarrow$$

$$\mu_{M_4}(x) = \begin{cases} (x - 0.5)/0.25 & 0.5 \le x \le 0.75 \\ (1 - x)/0.25 & 0.75 \le x \le 1 \end{cases}$$

$$\mu_{M_5}(x) = \begin{cases} (x - 0.7)/0.3 & 0.7 \le x \le 1 \\ 1, & x = 1 \end{cases}$$

Linguistic term (M_i)	Crisp score
Much Less important (M_1)	0.115
Less important (M_2)	0.295
Equally important (M_3)	0.495
More important (M_4)	0.695
Very important (M_5)	0.895

Fig. 5 Linguistic terms to fuzzy number conversation (5-point scale) for comparing criteria

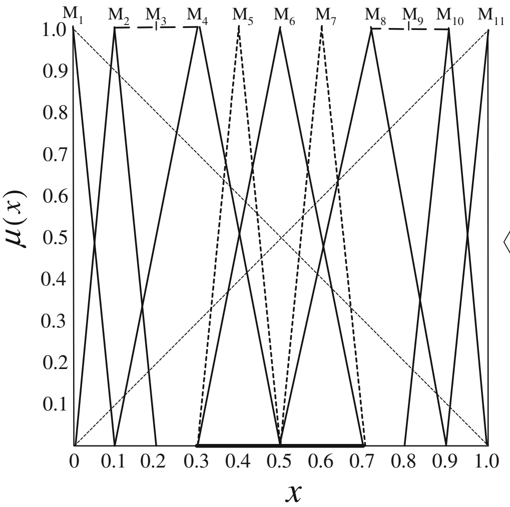
$$[\beta] = \begin{bmatrix} 0 & r_{12} & \ldots & r_{1n} \\ r_{21} & 0 & \ldots & r_{2n} \\ . & \ldots & . & . \\ r_{n1} & \ldots & \ldots & 0 \end{bmatrix} \tag{6}$$

Fuzzy set theory can also be used to determine relative importance (or interaction) r_{ij} between criteria and criteria scores of alternatives C_i linguistically. In this paper a five-point fuzzy scale is used for evaluating the relative importance and an eleven-point scale is used for evaluation criteria scores for alternatives as shown in Figs. 5 and 6. The procedure that was proposed by Chen and Hwang (1992) can then be used to convert fuzzy numbers (linguistic evaluations) into crisp scores. The crisp scores for a fuzzy number M can be obtained as follows: Given maximizing and minimizing sets such as:

$$\mu_{\max}(x) = \begin{cases} x & 0 \le x \le 1 \\ 0 & otherwise \end{cases}, \quad \mu_{\min}(x) = \begin{cases} 1 - x & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

Their fuzzy max and min are defined in such a manner that absolute locations of fuzzy numbers can be automatically incorporated in the comparison cases. The left utility score of each fuzzy number M_i is defined as $\mu_L(M_i) = Sup_x\left[\mu_{\min}(x) \wedge \mu_{M_i}(x)\right]$. The $\mu_L(M_i)$ score is a unique real number in [0,1]. It is the maximum membership value of intersection of M_i and fuzzy min. The right utility score can be obtained similarly: $\mu_R(M_i) = Sup_x\left[\mu_{\max}(x) \wedge \mu_{M_i}(x)\right]$.$\mu_R(M_i)$ is a unique real number in [0,1]. Given the left and right scores, total score of M_i is defined as: $\mu_T(M_i) = [\mu_R(M_i) + 1 - \mu_L(M_i)]/2$.

Based on this procedure crisp score for linguistic terms (M_i) can be obtained as shown in Figs. 5 and 6. These scores can also be used in constructing Ψ and β matrices if decision problem is modeled under fuzziness.



Linguistic term (M_i)	Crisp score
Exceptionally low (M_1)	0.045
Extremely low (M_2)	0.135
Very low (M_3)	0.255
Low (M_4)	0.335
Below average (M_5)	0.420
Average (M_6)	0.500
Above average (M_7)	0.590
High (M_8)	0.665
Very high (M_9)	0.745
Extremely high (M_{10})	0.865
Exceptionally high (M_{11})	0.955

Fig. 6 Linguistic terms to fuzzy number conversation (11-point scale) for evaluating criteria scores for alternatives

Step 3: After obtaining criteria rating matrix (Ψ) for an alternative and relative importance (interaction) matrix β between criteria the next step is to form alternative evaluation matrix (ξ) which is given by Eq. 7.

$$\xi = \psi + \beta = \begin{bmatrix} C_1 & r_{12} & r_{13} & .. & .. & r_{1n} \\ r_{21} & C_2 & r_{23} & .. & .. & r_{2n} \\ r_{31} & r_{32} & C_3 & .. & .. & r_{3n} \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ r_{n1} & r_{n2} & r_{n3} & .. & .. & C_n \end{bmatrix} \tag{7}$$

The permanent of matrix ξ, $per(\xi)$ gives the rating for the alternative. For each alternative $per(\xi)$ must be calculated and arranged in an descending order. The alternative with the highest $per(\xi)$ value is the best alternative. $per(\xi)$ can be calculated as explained in the previous section by using Eq. 2. An expanded version of this equation is given in Eq. 8.

$$per(\xi) = \prod_{i=1}^{N} C_i + \sum_{i,j,\ldots,N} (r_{ij}r_{ji})C_kC_l\ldots C_N + \sum_{i,j,\ldots,N} (r_{ij}r_{jk}r_{ki} + r_{ik}r_{kj}r_{ji})C_lC_n\ldots C_N$$

$$+ \left\{ \sum_{i,j,\ldots,N} (r_{ij}r_{ji})(r_{kl}r_{lk})C_nC_m\ldots C_N \right.$$

$$\left. + \sum_{i,j,\ldots,N} (r_{ij}r_{jk}r_{kl}r_{li} + r_{il}r_{lk}r_{kj}r_{ji})C_nC_m\ldots C_N \right\}$$

$$+ \left[\sum_{i,j,\ldots,N} (r_{ij}r_{ji})(r_{kl}r_{\ln}r_{nk} + r_{kn}r_{nl}r_{lk})C_mC_o\ldots C_N \right.$$

$$+\sum_{i,j,\ldots,N}(r_{ij}r_{jk}r_{kl}r_{\ln}r_{ni}+r_{in}r_{nl}r_{lk}r_{kj}r_{ji})C_m C_o\ldots C_N\Bigg]+\ldots \tag{8}$$

4 Bibliometric analysis and literature review

Bibliometric analysis is one of the most widely used research approach for detecting the state of the art in a particular field. The method is able to utilize quantitative analysis and statistics in order to describe the patterns of publications within a given period or body of literature (Dereli et al. 2011). Researchers usually employ bibliometric analysis to evaluate a field of study or to ascertain influences and the relationships of several distinct fields. One of the basic ways of conducting bibliometric analysis is to search for publications listed in the Social Science Citation Index, the Science Citation Index, the Arts and Humanities Citation Index and other available databases. With this purpose in mind, we have collected the papers on GT-MP-DM approach through a very detailed search from the Thomson Reuters' Web of Science/Knowledge (WoS/K) with Conference Proceedings, consisting Science Citation Index-Expanded (SCI-E), Social Science Citation Index (SSCI), Conference Proceedings Citation Index- Science (CPCI-S) and Conference Proceedings Citation Index- Social Science & Humanities (CPCI-SSH) databases. Google scholar and many other search engines are also utilized with many different keywords and keyword combinations in order find out all of the relevant material on GT-MP-DM approach.

The annual number of paper published in the scientific literature on GT-MP-DM approach is depicted in Fig. 7. As it can be seen from this Figure the first application is appeared in 1991. From 1991 to 2005 very few papers were appeared in the literature. After 2005 there is a considerable increase in the number of publications. However, the number of publications is still very low in comparison to other MCDM techniques, like AHP, ANP, TOPSIS etc.

The brief summaries of the 76 papers published on GT-MP-DM approach are as follows: The paper presented by Gandhi et al. (1991) is most probably the first paper which presents and makes use of GT-MP-DM approach in modeling and solving a decision making problem

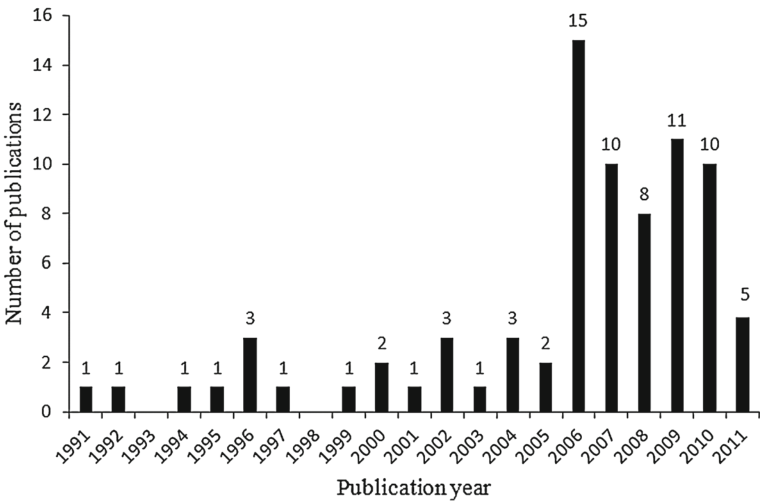


Fig. 7 The annual number of GT-MP-DM publications (80 publications in total)

with multiple and interrelated attributes. In their paper they presented how the reliability of a mechanical and hydraulic system can be modeled by making use of GT-MP-DM approach. The proposed a matrix permanent index which can be used optimum selection of a system at the initial stage of design.

Gandhi and Agrawal (1992) proposed a GT-MP-DM based approach for failure mode and effect analyses (FMEA) of mechanical and hydraulic systems. Their approach is based on development of an index which is actually a permanent function of interrelated elements of the studied mechanical system. This index can also be used for comparing alternative system designs based on FMEA.

Gandhi and Agrawal (1992) developed an analysis procedure which is based on GT-MP-DM approach for analyzing wear characteristics of mechanical components. Their approach analyses wear characteristics of mechanical systems and proposes a permanent function based index for their quantification. Their approach can also be used for optimum selection of subsystems based on wear characteristics.

Venkatasamy and Agrawal (1995) proposed a GT-MP-DM based approach for structural analyses of automobile vehicles in terms of their main components, sub-components and elements. Their main purpose was to perform an evaluation which can lead to selection/identification of an optimum structural design of an automobile system.

Singh and Sekhon (1996) proposed a GT-MP-DM approach for developing expressions for strip-layout selectivity index which can help to evaluate and rank any given set of metal strip-layout alternatives. They presented their approach by several case studies and provided a computer program which can be used to solve similar problems in industry.

Venkatasamy and Agrawal (1996) employed GT-MP-DM approach for modeling and solving classical automobile selection problem. They considered fuel used, consumption, maximum speed, cost and #passengers as criteria in their model. They provided an example which details their approach.

Gandhi and Agrawal (1996) proposed a GT-MP-DM based approach for failure cause analyses for manufacturing processes. They presented an example for welding processes as well. They developed a failure causality digraph which models failure contributing events and their interactions in terms of cause-effect relation. Their procedure can be used for identification, comparison and evaluation of failure causes in manufacturing processes, specifically for welding processes.

Venkatasamy and Agrawal (1997) developed a quality evaluation system for automobile vehicles. They determined 12 quality characteristics like performance, reliability, durability, conformance, serviceability etc. for evaluation and comparing alternative vehicles. Their model is based on GT-MP-DM approach where interrelationships between quality characteristics were presented by a digraph and they are quantified by employing a permanent index.

Wani and Gandhi (1999) developed a procedure which is based on GT-MP-DM approach for evaluation of maintainability index of mechanical systems. They determined the features that characterize or ease in maintenance of a mechanical system are named as maintainability attributes. Interrelations between these attributes like accessibility, disassembly, standardization, simplicity, identification etc. were modeled by using digraphs and quantified by using permanent functions. They also provided some examples from bushing.

Sehgal et al. (2000) developed an index for optimal selection of rolling element bearings for mechanical design applications. They employed GT-MP-DM approach in developing their index. Many characteristics related to material, shape, lubricant, size, motion etc. which are important for reliability evaluation were considered and their interrelationships were modeled by using digraphs. Afterwards, a permanent formulation is used to derive an index for selecting and ranking roller element bearings.

Rao (2000) employed GT-MP-DM approach for developing a performance evaluation system for technical education institutions. His model can also be used for ranking of the technical institutions.

Rao and Gandhi (2001) proposed a methodology to select a metal cutting fluid for a given machining operation by using GT-MP-DM approach. They identified metal cutting fluid attributes and their relative importance for a given machining operation. These attributes and their interrelationships were modeled and evaluated by using digraphs and permanent function.

Rao and Gandhi (2002) proposed a GT-MP-DM approach for machinability evaluation of work materials for a given machining operation. Their approach considers many criteria simultaneously like tool wear, specific energy consumed and surface roughness etc. for complete evaluation of machinability. The proposed approach can also be used to select/rank the best work-tool combination for a given machining operation.

Rao and Gandhi (2002) developed a GT-MP-DM based methodology for failure cause analyses of machine tools. Their approach takes in to account failure contributing events and their interactions in terms of cause-effects relationship by employing a digraph and develops a permanent function for ranking failure causes of a machine tool.

Wani and Gandhi (2002) suggested a GT-MP-DM based procedure for maintainability desgin and evaluation of mechanical systems based on tribology. They identified tribo-features of mechanical systems which can be used to characterize maintainability and modeled the relationships between them by using digraphs. Afterwards permanent function based indexes were derived in order to be able to compare alternative systems design in terms of maintainability.

Mohan et al. (2003) developed a systematic approach for system and performance modeling of a coal based steam power plant. They utilized GT-MP-DM approach in order to model the interaction between main and sub-components of power plant. Their methodology can used for optimum selection, benchmarking, maintenance strategy analyses, selection and performance comparison of alternative power plant designs as well.

Mohan et al. (2004) proposed a GT-MP-DM based approach for devising a maintenance strategy for coal-based steam power plant equipment. They defined several criteria and sub-criteria along with their interactions like maintenance effort, loss of production, safety, reliability etc. by employing digraphs and matrix permanents for devising, analyzing and comparing alternative maintenance strategies.

Rao (2004) proposed a matrix permanent index for evaluating alternative environmentally conscious manufacturing programs. He employed several criteria like cost, quality, recyclability, process waste reduction, packaging waste reduction and compliance for evaluating and comparing alternative manufacturing programs.

Grover et al. (2004) developed a TQM index by utilizing GT-MP-DM approach. In their approach five characteristics namely human factors, behavioral factors, use of tools/techniques, non behavioral factors and functional areas and their interactions are modeled by using digraphs. Afterwards a permanent index is proposed which can be used to quantify degree of TQM concepts implementation in an industry.

Kulkarni (2005) proposed a TQM index by utilizing GT-MP-DM approach. In their approach five characteristics namely infrastructure, top management support, strategic planning, employee empowerment, customer satisfaction and their interactions are modeled by using digraphs. Afterwards a permanent index is proposed which can be used to quantify success of implementation of TQM in an industry.

Grover et al. (2005, 2006) presented two papers on the development of TQM indexes by utilizing GT-MP-DM approach. In these studies the focus was on the evaluation of human factors on TQM implementation success. They identified various factors and sub-factors related to human issues from different perspectives and their interaction in deriving permanent indexes to quantify TQM success.

Rao (2006a,b,c,d) presented a fuzzy GT-MP-DM approach to develop an index to select the most suitable location for production facilities. He utilized classical location criteria like cost of land, cost of energy, cost of raw material, cost of transportation, cost of labor, nearness to market and their interaction in deriving a permanent index for facility location.

Garg et al. (2006) employed GT-MP-DM approach in developing a permanent index which can be used to compare alternative power plants for electricity generation. They compared thermal, wind and hydraulic power plants in terms of capital cost, electricity generation cost and plant load factor.

Rao (2006a,b,c,d) proposed a permanent index by utilizing fuzzy GT-MP-DM approach for machine group selection in flexible manufacturing systems applications. He evaluated alternative machine groups in terms of floor space requirement, total number of machines, productivity, required product quality and number of AGVs.

Rao (2006a,b,c,d) proposed a permanent index by utilizing fuzzy GT-MP-DM approach for evaluating and selection of flexible manufacturing systems. He used total cost, floor space required, number of employees, throughput time, product-mix flexibility, and routing flexibility as selection attributes in developing the GT-MP-DM model.

Rao (2006a,b,c,d) employed fuzzy GT-MP-DM approach for developing a procedure for material selection. He actually used an example form the literature and presented how GT-MP-DM approach can be used to solve the same example. Seven factors namely, toughness index, yield strength, Young's modulus, density, thermal expansion, thermal conductivity, and specific heat were used to rank/evaluate seven material alternatives for manufacturing of cryogenic storage thank for transportation of liquid nitrogen.

Prabhakaran et al. (2006a,b,c,d) employed GT-MP-DM approach for developing a conceptual composite product design methodology to industry by considering major design aspects like environment, reliability, cost, manufacturability, maintainability etc. and their interactions. They also presented the application of their approach to resin transfer molding process design.

Prabhakaran et al. (2006a,b,c,d) utilized GT-MP-DM approach for developing a quality index for quality modeling and analyses of polymer composite products. They considered five main criteria namely; resin system, reinforcement system, tooling system, product design, processing equipment and their interaction in developing the quality index which can be used to evaluate, select composite product systems.

Mohan et al. (2006) developed a real-time efficiency index by utilizing GT-MP-DM approach for steam power plants. They also presented how their approach can be implemented systematically on a real system.

Rao and Padmanabhan (2006) employed GT-MP-DM approach for developing a procedure for robot selection for manufacturing applications. They actually used an example form the literature and presented how GT-MP-DM approach can be used to solve the same example. Six factors namely, purchase cost, load capacity, velocity, repeatability, number of degrees of freedom, and man-machine interface were used to rank/evaluate robot alternatives for a specific manufacturing application.

Prabhakaran et al. (2006a,b,c,d) utilized GT-MP-DM approach for developing a permanent index for conceptual modeling and analyses of polymer composite products. They considered many design factors/sub-factors and their interaction in developing the index which can be used to evaluate, select polymer composite product systems.

Zhong et al. (2006) proposed a systematic procedure by using GT-MP-DM approach for machinability evaluation of ceramic materials. In their approach three criteria namely hardness, fracture toughness and elastic modules and their interactions were used to develop a material selection index. They also provided an example application.

Prabhakaran et al. (2006a,b,c,d) utilized GT-MP-DM approach for developing a permanent index for structural modeling and analyses of composite products. They considered many design factors/sub-factors and their interaction in developing the index which can be used to evaluate, select composite product systems.

Kaur et al. (2006) proposed a supply chain coordination index by using GT-MP-DM which can be used to evaluate several coordination mechanisms like coordination by contracts, coordination by information sharing, coordination by using information technology and coordination by collaboration and their interaction.

Ionica and Edelhauser (2006) proposed a GT-MP-DM based approach for evaluating customer supplier relationships within a total quality management context. They considered, “improving external communication”, “increasing the accessible market segment”, “consolidating the relations with the main clients” and “improving the relations with the main suppliers” as the main criteria. Criteria interactions are modeled via digraph and permanent index is used to evaluate alternative policies.

Garg et al. (2007a,b) employed fuzzy GT-MP-DM approach for developing a quality index which can be used to quantify quality characteristics of a thermal plant. They used 12 criteria like performance, automation, reliability, durability, conformance, serviceability, technical expertise, availability, maintainability, safety, environmental effect and life-cycle cost for deriving the quality index.

Grover and Singh (2007) proposed a matrix permanent index for evaluating total quality management level in organizations in relation to use of quality management tools and techniques. In their approach they considered “problem identification tools”, “data analysis tools”, “graphical tools”, “creativity tools”, “companywide techniques”, “productivity improvement tools”, “decision making tools” as the main groups and developed an index by also considering interrelationships between these groups.

Rao and Padmanabhan (2007) presented a methodology for rapid prototyping process selection for a given product by using GT-MP-DM approach. They actually modified one of the previous approaches from the literature and presented how this problem can be handled by GT-MP-DM. In their study, six main criteria namely; accuracy, surface roughness, tensile strength, elongation, cost of part and build time and their interaction were considered in developing a selection index.

Faisal et al. (2007) developed an information risk index by using GT-MP-DM approach. This index can be used to quantify information risk factors in supply chains. In developing this index they used for main criteria and their interactions namely, information security/breakdown risk, forecast risk, intellectual property rights risk, IS/IT outsourcing risks.

Faisal et al. (2007) proposed an agility improvement index for supply chains by analyzing the factors and enablers which are critical for enhanced agility. They utilized GT-MP-DM approach for developing such an index.

Faisal et al. (2007) proposed an agility index for supply chains by considering the factors which are critical for supply chain agility. They utilized GT-MP-DM approach for developing the index. Virtual integration, process alignment, network integration, market sensitivity and their interactions are considered as the main factors in developing the index.

Faisal et al. (2007) developed a risk mitigation index for supply chains by using GT-MP-DM approach. They considered 12 factors namely, “information sharing”, “supply chain agility”, “aligning incentives”, “strategic risk planning”, “risk sharing”, “thrust among partners", "collaborative relationships", "information security", "corporate social responsibility", "knowledge about risk in supply chain", "continual risk assessment and analyses", and "loss assessment metrics for the supply chain" in developing their index.

Garg et al. (2007a,b) proposed a GT-MP-DM based approach for reliability modeling and evaluation of tribo-mechanical systems. They used mechanical system parameters for developing their index which can be used to make selection between alternative system designs in terms of their reliability characteristics.

Mohan et al. (2007) proposed a real time commercial availability index for steam power plants by using GT-MP-DM approach. Their approach can also be used for techno-economic evaluation of new power plant investment proposals or in evaluating proposals for renovation/modernization of old power plants.

Upadhyay and Agrawal (2007) proposed a systematic approach in order to model, evaluate and analyze intelligent mobile learning environments by making use of GT-MP-DM approach. They identified five main systems like; "intelligent tutoring system", "multi-agent intelligent system", "mobile dimension system", "environment and human aspect system", "mobile agent system" and their sub-system along with interaction between these sub-systems in evaluating alternative mobile learning environments.

Rao (2008) proposed an environmental impact assessment index by making use of fuzzy GT-MP-DM approach. This index can be used to quantify and rank alternative manufacturing processes for producing a given product in terms of their environmental impacts. He considered solid waste, energy consumption and waste water as the parameters in developing the index similar to other studies in the literature.

Upadhyay (2008) proposed a systematic procedure which is based on GT-MP-DM approach for analyses of object oriented software systems. The proposed approach can be useful if applied at the early stages of software development life cycle in order to avoid pitfalls in the quality of final product.

Thakkar et al. (2008) proposed an extensive methodology for evaluating buyer-supplier relationships in supply chains by making use of GT-MP-DM approach. The proposed approach can also be used to compare supply chain relationships of SMEs. In developing their model many factors (like; business growth-long term perspective, mutual understanding and closeness, meeting customer/market requirement, role in decision making, risk/profit sharing) and sub-factors along with their interrelationships were considered and modeled with digraphs.

Singh and Agrawal (2008) proposed an extensive methodology for evaluating manufacturing systems in terms of their various structural characteristics like input sub-system, management sub-system, manufacturing process sub-system, support sub-system, output sub-system and their related sub-systems by making use of GT-MP-DM approach. Their approach can be very useful in analyses, comparison and selection of alternative manufacturing systems.

Mohan et al. (2008) proposes a real-time reliability index for steam power plants by making use of GT-MP-DM approach. The proposed index integrates various systems, sub-systems characteristics of steam power plants.

Kumar and Agrawal (2008) proposed a GT-MP-DM based approach for modeling and analyses of electroplating systems. Their method indentifies sub-systems and their interactions and can generate alternative design solutions. The developed indexes can be used for optimal selection from alternative system designs.

Bhosle and Basu (2008) developed a structural approach by using GT-MP-DM approach for effective assessment of product life cycle cost. Many cost factors like operating, maintenance, acquisition etc. were considered in developing their methodology.

Babu et al. (2008) developed a GT-MP-DM approach for evaluating quality of resin transfer molded products. They identified 15 factors like resin viscosity, resin gel time etc. and their interaction in developing a quality index.

Chakladar et al. (2009) employed GT-MP-DM approach for non-tradition process selection for a given application. They also provided a computer program which automated computations. They considered various factors like power requirement, surface finish, work material type etc. in proposing a permanent index for making a selection.

Rao and Parnichkun (2009) proposed a FMS selection index by making use of fuzzy GT-MP-DM approach. I developing such an index they considered several criteria like total cost, floor space requirement, quality results, ease of use, competitiveness, and expandability and interactions between these criteria.

Qureshi et al. (2009) proposed a permanent index which can be used to select third party logistic service providers. They also provided a real case study problem to present working of their approach.

Baykasoglu (2009a,b) proposed flexibility indexes for manufacturing systems by making use of matrix permanents with crisp and fuzzy evaluations. His approach can be used to quantify machine and manufacturing systems flexibility from several perspectives by considering versatility, performance and probability.

Qureshi et al. (2009) proposed a permanent index which can be used to select logistic service providers. They considered capability, compatibility, flexibility in operation and delivery, financial stability, geographic spread and range of services as criteria which were evaluated by using fuzzy scores. They also provided an example application.

Darvish et al. (2009) developed a contractor selection index by making use of GT-MP-DM approach. Their approach can be used to select most suitable contractor for a given construction project. They considered several criteria like work experience, technology and equipments, experience, financial stability, quality, reputation etc. in developing their index.

Jaya and Thanushkodi (2009) proposed a systematic approach based on GT-MP-DM for analyses of computer aided diagnosis systems. They considered all major attributes responsible for development and implementation along with interactions between constituents by using the GT-MP-DM approach.

Paramasivam and Senthil (2009) developed a matrix permanent index for analyses and evaluation of product designs through design aspects. Design for manufacturing, design for assembly, design for environment, design for safety, design for reliability, design for maintenance, design for aesthetic features, design for economy and design for ergonomics were considered as product design aspects.

Prince and Agrawal (2009) proposed a GT-MP-DM based approach for structural modeling and integrative analyses of micromechanical systems product. They provided examples for practical applications and presented possible ways of using their approach for comparing alternative systems.

Upadhyay et al. (2009) proposed a systematic procedure by employing GT-MP-DM approach for modeling and analyses of component based software systems. Their approach can be useful in analyzing component based software at the architecture level.

Anand and Wani (2010) proposed a GT-MP-DM based approach for product life-cycle modeling and evaluation at the conceptual design stage. They actually identified life-cycle design attributes and used them to evaluate a life-cycle design index. Their index is useful in assessing the relative life-cycle design value of product design alternatives.

Yadav et al. (2010) developed an operational-economic index by using matrix permanents for evaluation and selection of power plants in terms of operational-economic factors.

They developed this index by considering capital cost, heat rate, net efficiency, operation-maintenance cost, availability, reliability and project time parameters and interaction between these parameters.

Wagner and Neshat (2010) developed a vulnerability index for supply chains by making use of GT-MP-DM approach. They classified supply chain vulnerability factors under three category such as "demand side", "supply side" and "supply chain structure characteristic" factors in developing their index and compared supply chain vulnerability characteristics of different industries.

Upadhyay et al. (2010) proposed a maintainability index for software components by utilizing GT-MP-DM approach. In their approach characteristics like customizability, testability and changeability and related sub factors were used to develop the maintainability index. They also validated their index with several tests and applications.

Rao and Padmanabhan (2010) developed an end-of-life selection index for the selection of best product end of life scenario. In their approach several factors like logistics costs, emissions, energy consumption etc. which are important for end-of-life strategy formulation are considered in developing the selection index. They also presented an example for the application of the proposed index.

Raj et al. (2010) proposed a GT-MP-DM based approach for analyzing intensity of barriers for FMS implementation in industry. In their approach the factors namely; behavioral barriers, technical barriers, operational barriers, financial barriers, strategic barriers and supply chain barriers along with their sub-factors and interactions were considered in developing an index for evaluation.

Raj and Attri (2010) developed an index by making use of GT-MP-DM approach for quantification of barriers to implementing total quality management system in organizations. In their approach seven barriers namely; lack of top management commitment and involvement, instability of senior managers, union resistance, poor interdepartmental relations, nonparticipation and apathy of employees, poor quality infrastructure facilities and inadequate tools and equipment were considered and interaction between them were taken into account for index development.

Paramasivam et al. (2010) employed GT-MP-DM approach for developing an index to be used in the optimal selection of jigs/fixtures for a given application. They identified eleven factors namely; location, clamping, loading/unloading, stability and rigidity, fool proof, provision for indexing, provision for tool guidance, weight, safety, coolant supply, and cost for developing the index for selection. They also considered factor interactions and provided practical examples for implementation.

Paramasivam et al. (2010) employed GT-MP-DM approach for developing an index to be used in the optimal selection of milling machine tool. They identified six factors namely; price, weight, power, spindle speed, spindle diameter and stroke length for developing the index for selection. They also considered factor interactions and provided practical examples for implementation.

Kumar et al. (2010) proposed a systematic approach by making use of GT-MP-DM approach for structural modeling and analyses of an effluent treatment process for electroplating. They also presented examples to validate their approach.

Jangra et al. (2011a) developed a performance evaluation method for carbide compacting die manufactured by wire EDM my making use of GT-MP-DM approach. They considered several factors which are important for die performance such as die material, machine tool, tool electrode, geometry of die and machining operation in developing performance evaluation index.

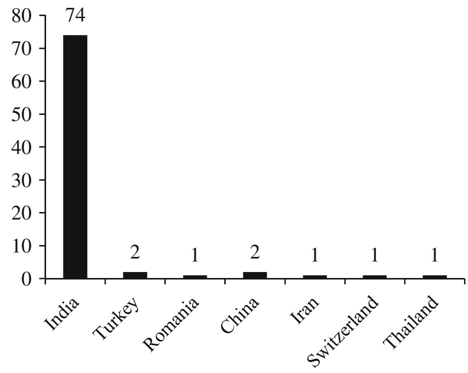


Fig. 8 Country origin’s of GT-MP-DM publications

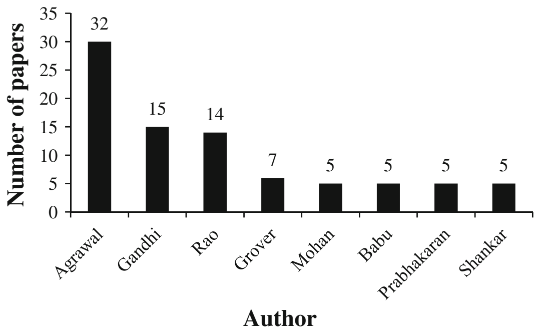


Fig. 9 The most publishing authors on GT-MP-DM

Jangra et al. (2011b) proposed a digraph and matrix based method in order to evaluate the machinability of tungsten carbide composite with wire EDM. In their study factors affecting the machinability and their interactions are analyzed by developing a mathematical model which makes use of digraph and matrix method.

Kumar et al. (2011) proposed a design index by making use of GT-MP-DM approach for concurrent design of electroplating system for X-abilities.

Kiran et al. (2011) developed a design index by making use of GT-MP-DM approach for design of a mechatronic system.

Gadakh and Shinde (2011) developed a selection procedure by making use of GT-MP-DM approach for cutting parameters determination in milling operations.

Some analyses on these 80 papers are also carried out. In Fig. 8 the origin of the 76 papers are presented. As it can be seen from this figure almost all of the papers were originated from India, there are just a few papers from other countries. As it can also be seen from Fig. 9, Prof. Agrawal, Prof. Gandhi and Prof. Rao are the leading authors who utilized the GT-MP-DM approach in modeling and solving decision making problems.

Finally a classification of application areas of GT-MP-DM approach is also provided in Fig. 10. Problems in industrial and mechanical engineering domains are most frequently considered. Problems specific to manufacturing, manufacturing systems, quality, supply chain management are the ones which are usually modeled and solved by GT-MP-DM approach. In “Appendix” a summary of the applied problems is also given.

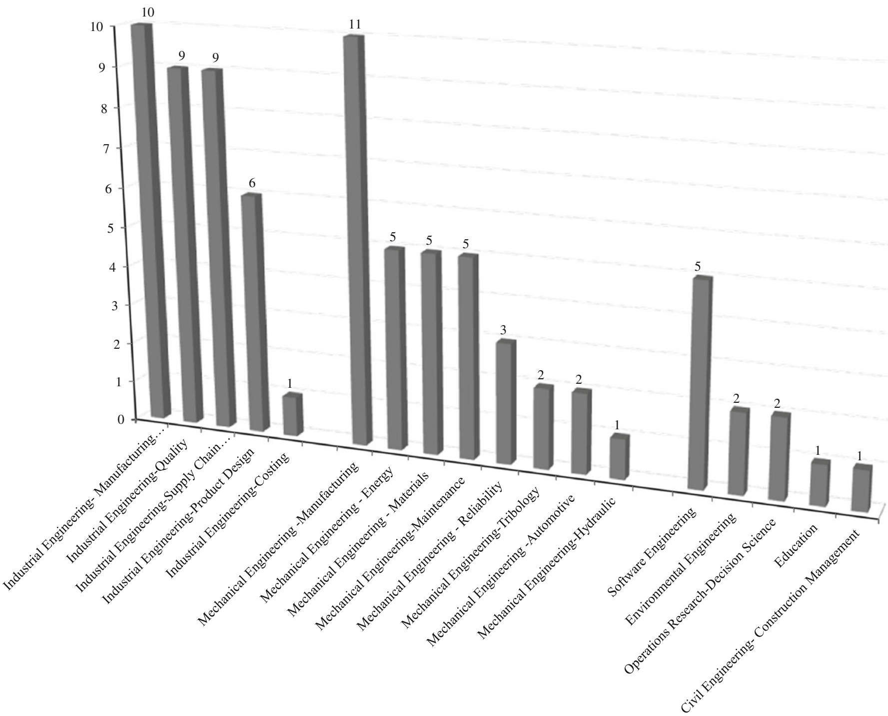


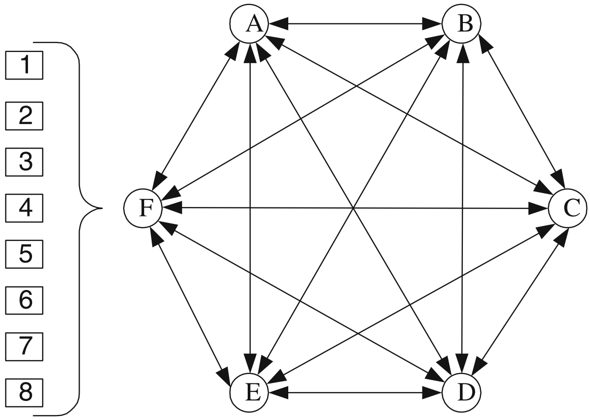
Fig. 10 Classification of GT-MP-DM publications based on main application areas

5 Illustrative examples

In this section, two case study problems which were previously studied and presented in conferences by the author and his research team are presented. The first problem is about industrial sector preferences of industrial engineering students. The problem was previously modeled with analytical hierarchy process (AHP). The second problem is about supermarket location selection. This problem was also modeled with AHP previously.

5.1 Example-1: industrial sector preferences of industrial engineering students

It was aimed to investigate the industrial sector preferences of final year industrial engineering students studying at Erciyes University. The study was repeated for the final year industrial engineering students studying at University of Gaziantep later. The reason for initiating such a study was to provide some data for improving curriculum at the industrial engineering department (Tapkan et al. 2007). For that purpose surveys were carried out in order to determine criteria and students' preferences for these criteria. The original study analysis was performed by using AHP technique. The data obtained from this study is rearranged to be used for GT-MP-DM approach. Based on the GT-MP-DM methodology as described in the previous section, the problem's graphical structure is presented in Fig. 11 where criteria and alternatives are depicted. The relative importance between criteria and alternatives scores were collected as crisps scores. The averaged and normalized alternative's scores and relative importance between criteria (i.e., Ψ and β matrices) are presented in Fig. 12.



Criteria
A: Income level
B: Prestige
C: Relevance to IE practice
D: Working conditions
E: Carrier opportunities
F: Location

Alternatives
1: Health
2: Finance
3: Consultancy
4: Logistics
5: Academic
6: Research & Development
7: Manufacturing
8: Information Technology

Fig. 11 Structural model of sector preferences of industrial engineering students

After determining Ψ and β matrices as shown in Fig. 12 the permanent of evaluation matrices can be computed by using Eq. 8 as follows:

$$
\begin{aligned}
[\xi]_1 &= [\Psi]_1 + [\beta]; \ \text{per}\,([\xi]_1)\ 2.743 \\
[\xi]_2 &= [\Psi]_2 + [\beta]; \ \text{per}\,([\xi]_2) = 7.0653 \\
[\xi]_3 &= [\Psi]_3 + [\beta]; \ \text{per}\,([\xi]_3) = 5.9111 \\
[\xi]_4 &= [\Psi]_4 + [\beta]; \ \text{per}\,([\xi]_4) = 1.2441 \\
[\xi]_5 &= [\Psi]_5 + [\beta]; \ \text{per}\,([\xi]_5) = 3.843 \\
[\xi]_6 &= [\Psi]_6 + [\beta]; \ \text{per}\,([\xi]_6) = 6.5528 \\
[\xi]_7 &= [\Psi]_7 + [\beta]; \ \text{per}\,([\xi]_7) = 2.3401 \\
[\xi]_8 &= [\Psi]_8 + [\beta]; \ \text{per}\,([\xi]_8) = 8.0778
\end{aligned}
$$

Based on this results Information technology sector is the most favorable sector for the students, finance sector ranks second, research & development is third etc.

5.2 Example-2: supermarket location selection

It was aimed to select the best possible location for a supermarket-chain which was decided to open a new branch in Gaziantep, Turkey. The study was performed by the author and his research group and presented in a conference paper (Baykasoglu et al. 2003) from which more details about the problem can be obtained. In this paper we decided to present how this

$$[\beta]=\begin{array}{cc} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A\\ B\\ C\\ D\\ E\\ F \end{array} & \begin{bmatrix} 0 & 0.5 & 0.6 & 0.7 & 0.7 & 0.9 \\ 0.5 & 0 & 0.6 & 0.7 & 0.6 & 1 \\ 0.4 & 0.4 & 0 & 0.8 & 0.6 & 0.9 \\ 0.3 & 0.3 & 0.2 & 0 & 0.5 & 1 \\ 0.3 & 0.4 & 0.4 & 0.5 & 0 & 0.9 \\ 0.1 & 0 & 0.1 & 0 & 0.1 & 0 \end{bmatrix} \end{array}$$

$$[\Psi]_1=\begin{array}{cc} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A\\ B\\ C\\ D\\ E\\ F \end{array} & \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 \end{bmatrix} \end{array} \quad [\Psi]_2=\begin{array}{cc} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A\\ B\\ C\\ D\\ E\\ F \end{array} & \begin{bmatrix} 0.67 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.83 \end{bmatrix} \end{array}$$

$$[\Psi]_3=\begin{array}{cc} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A\\ B\\ C\\ D\\ E\\ F \end{array} & \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.75 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.67 \end{bmatrix} \end{array} \quad [\Psi]_4=\begin{array}{cc} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A\\ B\\ C\\ D\\ E\\ F \end{array} & \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0 \end{bmatrix} \end{array}$$

$$[\Psi]_5=\begin{array}{cc} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A\\ B\\ C\\ D\\ E\\ F \end{array} & \begin{bmatrix} 0.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \end{bmatrix} \end{array} \quad [\Psi]_6=\begin{array}{cc} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A\\ B\\ C\\ D\\ E\\ F \end{array} & \begin{bmatrix} 0.67 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.83 \end{bmatrix} \end{array}$$

$$[\Psi]_7=\begin{array}{cc} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A\\ B\\ C\\ D\\ E\\ F \end{array} & \begin{bmatrix} 0.33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.75 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.33 \end{bmatrix} \end{array} \quad [\Psi]_8=\begin{array}{cc} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A\\ B\\ C\\ D\\ E\\ F \end{array} & \begin{bmatrix} 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 \end{bmatrix} \end{array}$$

Fig. 12 Problem data example-1

problem can be modeled by using a fuzzy GT-MP-DM methodology. The problem has five main criteria and several sub-criteria as shown in Table 3. Three candidate locations were determined and evaluated by the team which was composed of experts about the subject. The evaluations were made by using linguistic (fuzzy) variables as defined in the previous section. The overall graphical model is presented in Fig. 13.

Table 3 Criteria and sub-criteria for example-2

A: Regional population and its structure
A_1: Population density
A_2: Income level of people
B: Site condition
B_1: Site shape and size
B_2: Proximity to other supermarkets
B_3: Warehouse availability
B_4: Suitability of land for expansion
B_5: Infrastructure
C: Parking
C_1: Availability of parking area
C_2: Capacity of parking area
D: Transportation
D_1: Availability and frequency of public transportation
D_2: Distance to highways
D_3: Cost of transportation
D_4: Distance to city centre
E: Policies of regional government
E_1: Tax policy
E_2: Conditions for renting and ownership
E_3: Security service
E_4: Quality and availability of fire, health and garbage collection services

The permanent of evaluation matrices for alternative location 1 ($L1$) based on main criteria and sub-criteria $(per([\xi]_{L1}^{A}), \ldots, per([\xi]_{L1}))$ can be computed by using Eq. 8 as follows:

$$[\xi]_{L1}^{A} = \begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} A_1 & A_2 \end{array} \\ \begin{bmatrix} \mathbf{0.665} & 0.495 \\ 0.495 & \mathbf{0.665} \end{bmatrix} \end{array}, \quad [\xi]_{L1}^{B} = \begin{array}{c} \\ B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{array} \begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{bmatrix} \mathbf{0.420} & 0.695 & 0.495 & 0.695 & 0.495 \\ 0.295 & \mathbf{0.420} & 0.115 & 0.295 & 0.115 \\ 0.495 & 0.895 & \mathbf{0.335} & 0.695 & 0.495 \\ 0.295 & 0.695 & 0.295 & \mathbf{0.335} & 0.295 \\ 0.495 & 0.895 & 0.495 & 0.695 & \mathbf{0.665} \end{bmatrix} \end{array},$$

$$[\xi]_{L1}^{C} = \begin{array}{c} \\ C_1 \\ C_2 \end{array} \begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} \mathbf{0.335} & 0.495 \\ 0.495 & \mathbf{0.255} \end{bmatrix} \end{array}, \quad [\xi]_{L1}^{D} = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \end{array} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{bmatrix} \mathbf{0.665} & 0.895 & 0.695 & 0.895 \\ 0.115 & \mathbf{0.420} & 0.115 & 0.495 \\ 0.295 & 0.895 & \mathbf{0.500} & 0.695 \\ 0.115 & 0.495 & 0.295 & \mathbf{0.500} \end{bmatrix} \end{array},$$

$$[\xi]_{L1}^{E} = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \end{array} \begin{array}{c} \begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \end{array} \\ \begin{bmatrix} \mathbf{0.665} & 0.295 & 0.495 & 0.295 \\ 0.695 & \mathbf{0.865} & 0.695 & 0.695 \\ 0.495 & 0.295 & \mathbf{0.665} & 0.295 \\ 0.695 & 0.295 & 0.695 & \mathbf{0.745} \end{bmatrix} \end{array}$$

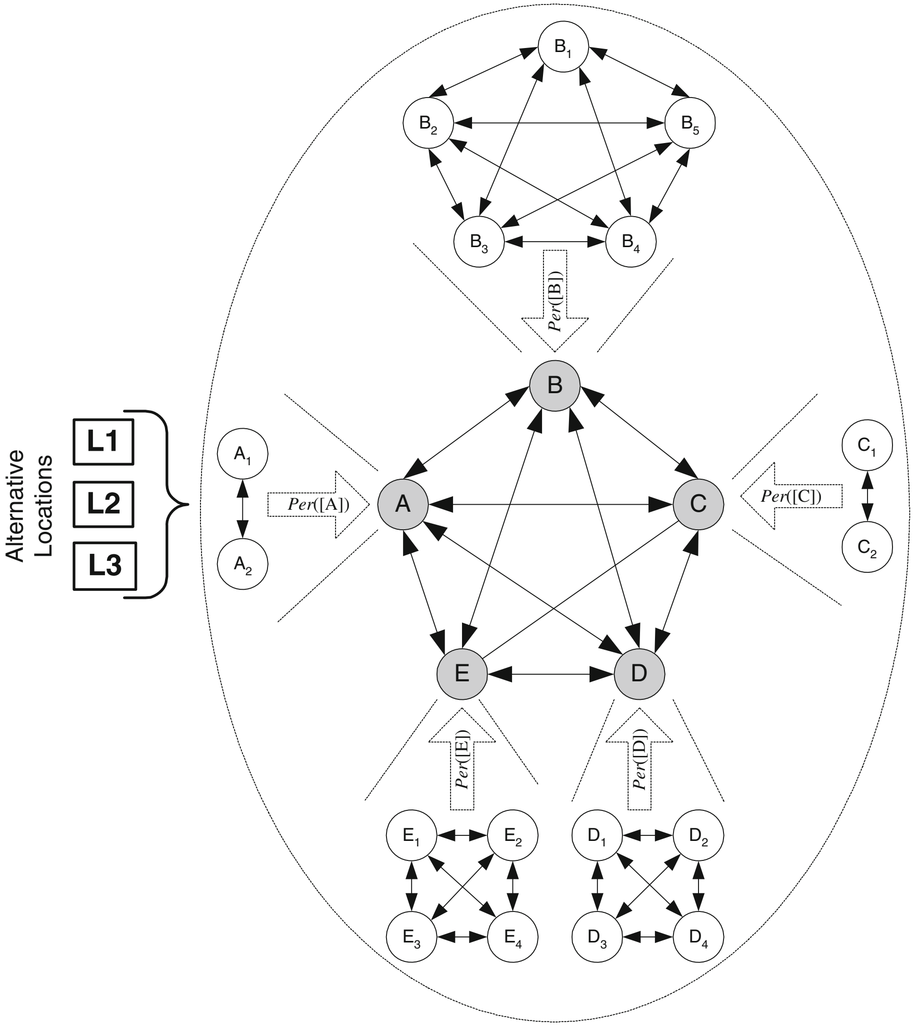


Fig. 13 Structural model of supermarket location selection problem

$$[\xi]_{L1} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} per([\xi]^{A}_{L1}) & 0.695 & 0.695 & 0.695 & 0.695 \\ 0.295 & per([\xi]^{B}_{L1}) & 0.695 & 0.495 & 0.695 \\ 0.295 & 0.295 & per([\xi]^{C}_{L1}) & 0.295 & 0.495 \\ 0.295 & 0.495 & 0.695 & per([\xi]^{D}_{L1}) & 0.695 \\ 0.295 & 0.295 & 0.495 & 0.295 & per([\xi]^{E}_{L1}) \end{bmatrix} \end{array}$$

$per([\xi]^{A}_{L1}) = 0.6873$, $per([\xi]^{B}_{L1}) = 2.0043$, $per([\xi]^{C}_{L1}) = 0.3305$, $per([\xi]^{D}_{L1}) = 0.8382$, $per([\xi]^{E}_{L1}) = 1.9886$ and $per([\xi]_{L1}) = 9.2187$

Similar computations can be carried out for location alternative 2 ($L2$) as follows:

$$[\xi]^A_{L2} = \begin{array}{c} \\ A_1 \\ A_2 \end{array}\!\!\begin{array}{c} \begin{array}{cc} A_1 & A_2 \end{array} \\ \begin{bmatrix} \mathbf{0.665} & 0.495 \\ 0.495 & \mathbf{0.420} \end{bmatrix} \end{array}, \quad [\xi]^B_{L2} = \begin{array}{c} \\ B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{bmatrix} \mathbf{0.590} & 0.695 & 0.495 & 0.695 & 0.495 \\ 0.295 & \mathbf{0.500} & 0.115 & 0.295 & 0.115 \\ 0.495 & 0.895 & \mathbf{0.590} & 0.695 & 0.495 \\ 0.295 & 0.695 & 0.295 & \mathbf{0.745} & 0.295 \\ 0.495 & 0.895 & 0.495 & 0.695 & \mathbf{0.665} \end{bmatrix} \end{array},$$

$$[\xi]^C_{L2} = \begin{array}{c} \\ C_1 \\ C_2 \end{array}\!\!\begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} \mathbf{0.865} & 0.495 \\ 0.495 & \mathbf{0.865} \end{bmatrix} \end{array}, \quad [\xi]^D_{L2} = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{bmatrix} \mathbf{0.665} & 0.895 & 0.695 & 0.895 \\ 0.115 & \mathbf{0.665} & 0.115 & 0.495 \\ 0.295 & 0.895 & \mathbf{0.500} & 0.695 \\ 0.115 & 0.495 & 0.295 & \mathbf{0.335} \end{bmatrix} \end{array},$$

$$[\xi]^E_{L2} = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \end{array} \\ \begin{bmatrix} \mathbf{0.500} & 0.295 & 0.495 & 0.295 \\ 0.695 & \mathbf{0.335} & 0.695 & 0.695 \\ 0.495 & 0.295 & \mathbf{0.665} & 0.295 \\ 0.695 & 0.295 & 0.695 & \mathbf{0.665} \end{bmatrix} \end{array}$$

$$[\xi]_{L2} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} per([\xi]^A_{L2}) & 0.695 & 0.695 & 0.695 & 0.695 \\ 0.295 & per([\xi]^B_{L2}) & 0.695 & 0.495 & 0.695 \\ 0.295 & 0.295 & per([\xi]^C_{L2}) & 0.295 & 0.495 \\ 0.295 & 0.495 & 0.695 & per([\xi]^D_{L2}) & 0.695 \\ 0.295 & 0.295 & 0.495 & 0.295 & per([\xi]^E_{L2}) \end{bmatrix} \end{array}$$

$per([\xi]^A_{L2}) = 0.5243$, $per([\xi]^B_{L2}) = 3.0069$, $per([\xi]^C_{L2}) = 0.9932$, $per([\xi]^D_{L2}) = 0.8871$, $per([\xi]^E_{L2}) = 1.2944$ and $per([\xi]_{L2}) = 12.5913$

Computations for location alternative 3 ($L3$) are as follows:

$$[\xi]^A_{L3} = \begin{array}{c} \\ A_1 \\ A_2 \end{array}\!\!\begin{array}{c} \begin{array}{cc} A_1 & A_2 \end{array} \\ \begin{bmatrix} \mathbf{0.745} & 0.495 \\ 0.495 & \mathbf{0.500} \end{bmatrix} \end{array}, \quad [\xi]^B_{L3} = \begin{array}{c} \\ B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{bmatrix} \mathbf{0.420} & 0.695 & 0.495 & 0.695 & 0.495 \\ 0.295 & \mathbf{0.500} & 0.115 & 0.295 & 0.115 \\ 0.495 & 0.895 & \mathbf{0.420} & 0.695 & 0.495 \\ 0.295 & 0.695 & 0.295 & \mathbf{0.335} & 0.295 \\ 0.495 & 0.895 & 0.495 & 0.695 & \mathbf{0.745} \end{bmatrix} \end{array},$$

$$[\xi]^C_{L3} = \begin{array}{c} \\ C_1 \\ C_2 \end{array}\!\!\begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} \mathbf{0.420} & 0.495 \\ 0.495 & \mathbf{0.335} \end{bmatrix} \end{array}, \quad [\xi]^D_{L3} = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{bmatrix} \mathbf{0.865} & 0.895 & 0.695 & 0.895 \\ 0.115 & \mathbf{0.590} & 0.115 & 0.495 \\ 0.295 & 0.895 & \mathbf{0.500} & 0.695 \\ 0.115 & 0.495 & 0.295 & \mathbf{0.865} \end{bmatrix} \end{array},$$

$$[\xi]^E_{L3} = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \end{array} \\ \begin{bmatrix} \mathbf{0.500} & 0.295 & 0.495 & 0.295 \\ 0.695 & \mathbf{0.335} & 0.695 & 0.695 \\ 0.495 & 0.295 & \mathbf{0.665} & 0.295 \\ 0.695 & 0.295 & 0.695 & \mathbf{0.665} \end{bmatrix} \end{array}$$

$$[\xi]_{L3} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} per([\xi]_{L3}^{A}) & 0.695 & 0.695 & 0.695 & 0.695 \\ 0.295 & per([\xi]_{L3}^{B}) & 0.695 & 0.495 & 0.695 \\ 0.295 & 0.295 & per([\xi]_{L3}^{C}) & 0.295 & 0.495 \\ 0.295 & 0.495 & 0.695 & per([\xi]_{L3}^{D}) & 0.695 \\ 0.295 & 0.295 & 0.495 & 0.295 & per([\xi]_{L3}^{E}) \end{bmatrix} \end{array}$$

$per([\xi]_{L3}^{A})=0.6175$, $per([\xi]_{L3}^{B}) = 2.2497$, $per([\xi]_{L3}^{C})=0.3857$, $per([\xi]_{L3}^{D})=1.2868$,

$per([\xi]_{L3}^{E}) = 1.2944$ and $per([\xi]_{L3}) = 9.4832$

Based on these computations we can conclude that $L2$ is the best location alternative and $L1$ is the worst alternative.

6 Conclusions

In this paper a comprehensive review of a multiple attribute decision making technique which is named as GT-MP-DM is provided. A detailed explanation of the technique along with some of its theoretical foundations and existing applications are also given. GT-MP-DM was originated from combinatorial mathematics (*mainly from graph theory and matrix algebra*) and has some very desirable properties for modeling and solving complex decision making problems. Some of the main advantages of GT-MP-DM approach were stated as follows by Rao (2006a,b,c,d, 2007) and Darvish et al. (2009):

- The computational procedure used in GT-MP-DM approach is relatively simple compared to the most other multiple attribute decision making methods.
- Unlike most of the other MADM methods, GT-MP-DM approach incorporates the interdependencies like ANP.
- GT-MP-DM approach enables a more critical analysis than most of the other MADM techniques since any number of quantitative and qualitative attributes can be considered.
- In the matrix permanent procedure, a small variation in the attribute values leads to a considerable difference in the result. Therefore it is easier to rank the alternatives in the descending order with clear cut difference.
- Additionally, GT-MP-DM procedure not only provides the analysis of the alternatives, but also enables the visualization of various criteria present and their interrelations, using the graphical representation.
- The usage of the matrix permanent concept helps in better appreciation of the criteria and it characterizes the considered decision making problem as it contains all possible structural components of the criteria and their relative importance.

In the present paper, two illustrative examples with crisp and fuzzy data are also provided in the paper in order to enable better understating of the technique and it's potential.

In comparison to other MCDM techniques, the applications of GT-MP-DM are rare and it is used only by a few researchers in the literature. We believe that this technique will gain high acceptance in the research community and it will be developed further and applied to many other problems.

Several future studies can be considered for GT-MP-DM approach, like investigating its possible integration with several existing MCDM methods and other analyses techniques (like fuzzy cognitive maps etc.) can be very interesting. Comparing GT-MP-DM approach with several other MCDM methods can be a very interesting study. Some more work is also required for modeling MCDM under uncertainty with GT-MP-DM approach.

Appendix

See Table 4.

Table 4 A summary of the problems modeled and solved by GT-MP-DM approach

	Reference	General application domain	Studied problem
1	Gandhi et al. (1991)	Mechanical Engineering—Hydraulic	Hydraulic system selection based on reliability analysis
2	Gandhi and Agrawal (1992)	Mechanical Engineering—Maintenance	FMEA analysis of alternative hydraulic systems
3	Gandhi and Agrawal (1994)	Mechanical Engineering—Tribology	Wear analyses of mechanical systems
4	Venkatasamy and Agrawal (1995)	Mechanical Engineering—Automotive	Comparison and analyses of automobile vehicles based on structural analyses
5	Singh and Sekhon (1996)	Mechanical Engineering—Manufacturing	Evaluation/selection of alternative metal stamping layouts
6	Venkatasamy and Agrawal (1996)	Operations Research—Decision Science	Automobile selection
7	Gandhi and Agrawal (1996)	Mechanical Engineering—Manufacturing	Analysis, identification, comparison and evaluation of failure causes in manufacturing processes
8	Venkatasamy and Agrawal (1997)	Mechanical Engineering—Automotive	Comparison, analysis and evaluation of quality of automobile vehicles
9	Wani and Gandhi (1999)	Mechanical Engineering—Maintenance	Evaluation/ranking of maintainability of mechanical components-systems
10	Sehgal et al. (2000)	Mechanical Engineering—Reliability	Reliability evaluation and selection/ranking of rolling element bearing for an application
11	Rao (2000)	Education	Performance evaluation/ranking of technical institutions

Table 4 continued

	Reference	General application domain	Studied problem
12	Rao and Gandhi (2001)	Mechanical Engineering—Tribology	Selection, identification and comparison of metal cutting fluids
13	Rao and Gandhi (2002)	Mechanical Engineering—Manufacturing	Machinability evaluation of work materials for a given operation
14	Rao and Gandhi (2002)	Mechanical Engineering—Maintenance	Failure cause analyses of machine tools
15	Wani and Gandhi (2002)	Mechanical Engineering—Maintenance	Developing a procedure for maintainability evaluation of mechanical systems
16	Mohan et al. (2003)	Mechanical Engineering—Energy	Performance analyses of coal-based steam power plant
17	Mohan et al. (2004)	Mechanical Engineering—Maintenance	Maintenance strategy analyses for coal-based steam power plant equipment
18	Rao (2004)	Environmental Engineering	Evaluating alternative environmentally conscious manufacturing programs
19	Grover et al. (2004)	Industrial Engineering—Quality	Evaluation of TQM (Total Quality Management) in industry
20	Kulkarni (2005)	Industrial Engineering—Quality	TQM success/implementation evaluation in industry
21	Grover et al. (2005)	Industrial Engineering—Quality	Identification and measurement of human resources contribution to TQM
22	Grover et al. (2006)	Industrial Engineering—Quality	Analyzing the effects of human factors in TQM
23	Rao (2006a,b,c,d)	Operations Research—Decision Science	Plant location selection
24	Garg et al. (2006)	Mechanical Engineering—Energy	Selecting the most suitable electricity generation power plant alternative
25	Rao (2006a,b,c,d)	Industrial Engineering—Manufacturing Systems	Machine group selection for flexible manufacturing systems (FMS) applications
26	Rao (2006a,b,c,d)	Industrial Engineering—Manufacturing Systems	Evaluating and selecting of FMS
27	Rao (2006a,b,c,d)	Mechanical Engineering—Materials	Material selection for a specific application
28	Prabhakaran et al. (2006a,b,c,d)	Industrial Engineering—Product Design	Conceptual composite product design
29	Prabhakaran et al. (2006a,b,c,d)	Mechanical Engineering—Materials	Quality modeling and analyses of polymer products

Table 4 continued

	Reference	General application domain	Studied problem
30	Mohan et al. (2006)	Mechanical Engineering—Energy	Efficiency analyses of steam power plant
31	Rao and Padmanabhan (2006)	Industrial Engineering—Manufacturing Systems	Robot selection for a manufacturing application
32	Prabhakaran et al. (2006a,b,c,d)	Mechanical Engineering—Materials	Conceptual modeling and analyses of polymer products
33	Zhong et al. (2006)	Mechanical Engineering—Materials	Machinability evaluation of ceramic materials
34	Prabhakaran et al. (2006a,b,c,d)	Mechanical Engineering—Materials	Structural modeling/ evaluation of composite materials
35	Kaur et al. (2006)	Industrial Engineering—Supply Chain Management	Evaluating different mechanisms for supply chain coordination
36	Ionica and Edelhauser (2006)	Industrial Engineering—Quality	Evaluating customer-supplier relationship with a TQM context
37	Garg et al. (2007a,b)	Industrial Engineering—Quality	Quality evaluation of a thermal power plant
38	Grover and Singh (2007)	Industrial Engineering—Quality	Evaluating the use of quality improvement tools by organizations
39	Rao and Padmanabhan (2007)	Mechanical Engineering—Manufacturing	Rapid prototyping process selection
40	Faisal et al. (2007a,b,c,d)	Industrial Engineering—Supply Chain Management	Quantification of information management risks in supply chains
41	Faisal et al. (2007a,b,c,d)	Industrial Engineering—Supply Chain Management	Analyzing/evaluating supply chain agility enablers
42	Faisal et al. (2007a,b,c,d)	Industrial Engineering—Supply Chain Management	Developing a supply chain agility index
43	Faisal et al. (2007a,b,c,d)	Industrial Engineering—Supply Chain Management	Quantification of risk mitigation in supply chains
44	Garg et al. (2007a,b)	Mechanical Engineering—Reliability	Reliability evaluation of tribo-mechanical systems
45	Mohan et al. (2007)	Mechanical Engineering—Energy	Quantification of real-time commercial availability of steam power plant
46	Upadhyay and Agrawal (2007)	Software Engineering	Modeling and analyses of mobile learning environment
47	Rao (2008)	Environmental Engineering	Environmental impact assessment of manufacturing processes
48	Upadhyay (2008)	Software Engineering	Modeling and analyses of object oriented software systems

Table 4 continued

	Reference	General application domain	Studied problem
49	Thakkar et al. (2008)	Industrial Engineering—Supply Chain Management	Evaluation of buyer-supplier relationship in supply chains
50	Singh and Agrawal (2008)	Industrial Engineering—Manufacturing Systems	Structural and integrative evaluation, analyses and comparison of manufacturing systems
51	Mohan et al. (2008)	Mechanical Engineering—Reliability	Development of a real-time reliability index for a steam power plant
52	Kumar and Agrawal (2008)	Mechanical Engineering—Manufacturing	Structural modeling and analyses of electroplating systems for optimal system selection
53	Bhosle and Basu (2008)	Industrial Engineering—Costing	Assessing product life cycle cost
54	Babu et al. (2008)	Industrial Engineering—Quality	Quality analyses and evaluation of resin transfer molded products
55	Chakladar et al. (2009)	Industrial Engineering—Manufacturing Systems	Non-tradition process selection for an application
56	Rao and Parnichkun (2009)	Industrial Engineering—Manufacturing Systems	Flexible manufacturing system selection
57	Qureshi et al. (2009)	Industrial Engineering—Supply Chain Management	Selection of third party logistics (3PL) providers
58	Baykasoglu (2009a)	Industrial Engineering—Manufacturing Systems	Quantifying flexibility in manufacturing systems
59	Baykasoglu (2009b)	Industrial Engineering—Manufacturing Systems	Quantifying flexibility in manufacturing systems under fuzziness
60	Qureshi et al. (2009)	Industrial Engineering—Supply Chain Management	Selection of logistics service providers
61	Darvish et al. (2009)	Civil Engineering—Construction Management	Contractor selection
62	Jaya and Thanushkodi (2009)	Software Engineering	Structural modeling and analyses of computer aided diagnosis system
63	Paramasivam and Senthil (2009)	Industrial Engineering—Product Design	Analyses and evaluation of product design through design aspects
64	Prince and Agrawal (2009)	Mechanical Engineering—Manufacturing	Structural modeling and integrative analyses of micromechanical systems product
65	Upadhyay et al. (2009)	Software Engineering	Modeling and analyses of component based software
66	Anand and Wani (2010)	Industrial Engineering—Product Design	Product life-cycle design modeling and evaluation

Table 4 continued

	Reference	General application domain	Studied problem
67	Yadav et al. (2010)	Mechanical Engineering—Energy	Evaluation and selection of power plants in terms of operational economics
68	Wagner and Neshat (2010)	Industrial Engineering—Supply Chain Management	Assessing vulnerability of supply chains
69	Upadhyay et al. (2010)	Software Engineering	Maintainability index development for software components
70	Rao and Padmanabhan (2010)	Industrial Engineering—Product Design	Selection of best product end of life scenario
71	Raj et al. (2010)	Industrial Engineering—Manufacturing Systems	Evaluating the intensity of barriers for FMS implementation
72	Raj and Attri (2010)	Industrial Engineering—Quality	Quantifying barriers for total quality management implementation
73	Paramasivam et al. (2010)	Mechanical Engineering—Manufacturing	Design selection of jigs and fixtures for a given application
74	Paramasivam et al. (2010)	Industrial Engineering—Manufacturing Systems	Machine tool selection problem
75	Kumar et al. (2010)	Mechanical Engineering—Manufacturing	Structural modeling and analyses of effluent treatment process for electroplating
76	Jangra et al. (2011a)	Mechanical Engineering—Manufacturing	Performance evaluation of carbide compacting die
77	Jangra et al. (2011b)	Mechanical Engineering—Manufacturing	Evaluating the machinability of tungsten carbide composite with wire EDM
78	Kumar et al. (2011)	Industrial Engineering—Product Design	Concurrent design of electroplating system
79	Kiran et al. (2011)	Industrial Engineering—Product Design	Concurrent design of a mechatronic system
80	Gadakh and Shinde (2011)	Mechanical Engineering—Manufacturing	Selection of cutting parameters in side milling

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