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RESEARCH

Locally conformally Kähler spaces and proper open morphisms

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Abstract
In this paper, we prove a stability result for the non-Kähler geometry of locally conformally Kähler (lcK) spaces with singularities. Specifically, we find sufficient conditions under which the image of an lcK space by a holomorphic mapping also admits lcK metrics, thus extending a result by Varouchas about Kähler spaces.

Keywords Kähler space · Locally conformally Kähler space · Stability

Mathematics Subject Classification 32S45 · 32J27 · 32C15 · 53C55

1 Introduction

While the strongest geometric results on complex manifolds may be obtained in the pure Kähler setting, the requirement of the existence of such a metrics in the compact case imposes great topological and geometric restrictions, and thus Kähler manifolds are relatively rare. That is why in the last decades there have been many efforts to find suitable replacements by relaxing, in various ways, the Kähler condition, and looking at non-Kähler Hermitian metrics whose existence is more common but can also lead to nice properties for the manifold and, ideally, classification results. One of the most intensely studied metrics are *locally conformally Kähler* (*lcK* for short).

A Hermitian metric ω on the complex manifold X is called *locally conformally Kähler (lcK)* if for every point $x \in X$, there exists an open neighborhood $U \ni x$ and a smooth function $f : U \to \mathbb{R}$ such that $d(e^{-f}\omega) = 0$ *i.e.* $e^{-f}\omega$ is Kähler on U. This is equivalent to

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$d\omega = \theta \wedge \omega$, where θ is a closed 1-form, called the Lee form of the lcK metric ω. If the Lee form θ is exact, which is equivalent to saying that the function f can be defined globally on X, then ω is called *globally conformally Kähler (gcK)*. If ω is lcK, but not gcK, we call it *pure lcK*. These metrics were first defined and studied by Vaisman [8], where he also proved a characterization theorem for lcK manifolds: a complex manifold X admits an lcK metric if and only if its universal cover $\widetilde{X}$ admits a Kähler metric $\widetilde{\omega}$ on which the deck group acts by homotheties *i.e.* such that for every $\gamma \in \mathrm{Deck}_X(\widetilde{X})$, we have $\gamma^*\widetilde{\omega} = e^{c_\gamma}\widetilde{\omega}$. Some years later, in [9] he proved that Kähler and pure lcK metrics cannot coexist on compact complex manifolds, with respect to the same complex structure. This is one motivation for studying lcK structures preferentially on compact manifolds, as here we have a clear separation between Kähler and lcK geometry. Since Vaisman published these two papers, there has been great progress in understanding compact lcK manifolds and many useful theorems were obtained. For a recent coprehensive overview of the development of lcK geometry, one may check [4].

In contrast to the abundance of results about Kähler manifolds, the lack of an all-around good definition of (p, q) differential forms on singular spaces (see [3] for a comparison study of different generalizations) made the study of Kähler spaces more difficult. Grauert [2] was the first to give a definition for Kähler metrics on complex analytic spaces. A more restrictive definition was used by Varouchas in [10–12] to obtain some results about modifications of Kähler spaces. Among them, he proves that if some conditions are satisfied, then the image of Kähler space by a holomorphic, proper mapping also admits Kähler metrics.

In [5], we adapted Grauert's idea of using families of plurisubharmonic functions and compatibility conditions to define lcK metrics on complex analytic spaces, and to prove that the characterization theorem involving the universal cover is still true for lcK spaces, exactly in the same form. Also, in [6], we proved that Vaisman's theorem on the dichotomy *Kähler – lcK* remains true for compact lcK spaces, with the additional assumption that the space is locally irreducible, and also gave an example which shows that the local irreducibility condition cannot be dropped. Although all the results of [5] and [6] are proved using Grauert's definition of Kähler metrics (and its adaptation to lcK metrics), they all remain true for Varouchas' definition, with the same proof, the additional condition of pluriharmonic differences being easily verified.

In this paper, we give an lcK version of Varouchas' stability results from [12]. More precisely, we prove:

Theorem 3.1 *Let (X, ω, θ) be an lcK space of pure dimension and X' be a normal space, such that there exists $p : X \to X'$ holomorphic, proper, open and surjective. Assume that* $\ker p_* \subset \ker \theta$. *Then, X' also admits lcK metrics.*

The strategy for our proof is to lift p to a morphism $\widetilde{p} : \widetilde{X} \to \widetilde{X}'$ from a covering space $\widetilde{X}$ of X onto the universal cover $\widetilde{X}'$ of X', and then make use of Varouchas' methods [12] for $\widetilde{p}$ to obtain a Kähler metric on $\widetilde{X}'$. As we need to integrate differential forms on the fibers of $\widetilde{p}$, these must be compact and Kähler for the method to work. Thus, for $\widetilde{p}$ to still be a proper mapping and its fibers to be Kähler, we need to impose the additional condition $\ker p_* \subset \ker \theta$. This is done so that $\mathrm{Deck}_{X'}(\widetilde{X}')$ acts by positive homotheties on the newly constructed Kähler metric on $\widetilde{X}'$. Finally, the characterization theorem for lcK spaces mentioned above yields that X' has lcK metrics.

In Sect. 2 we give all the definitions and the results we use. Section 3 is devoted to proving our new result.

2 Preliminaries

2.1 Kähler and lcK metrics

Firstly, we recall the definitions for Kähler and lcK metrics on complex analytic spaces.

Definition 2.1 Let X be a complex analytic space.

(K) A *Kähler metric* on X is the equivalence class $\widehat{(U_i, \varphi_i)_{i\in I}}$ of a family such that $(U_i)_{i\in I}$ is an open cover of X, $\varphi_i : U_i \to \mathbb{R}$ is $\mathcal{C}^\infty$ and strictly psh, and $\varphi_i - \varphi_j = \mathrm{Re}(h_{ij})$ on $U_{ij} = U_i \cap U_j$, for every $i, j \in I$, where h_{ij} is holomorphic. Two such families are equivalent if their union verifies the compatibility condition on the intersections, described above.

(lcK) An *lcK metric* on X is the equivalence class $\widehat{(U_i, \varphi_i, f_i)_{i\in I}}$ of a family such that $(U_i)_{i\in I}$ is an open cover of X, $\varphi_i : U_i \to \mathbb{R}$ is $\mathcal{C}^\infty$ and strictly psh, $f_i : U_i \to \mathbb{R}$ is smooth, and $e^{f_i - f_j}\varphi_i - \varphi_j = \mathrm{Re}(h_{ij})$ on $U_{ij} = U_i \cap U_j$, for every $i, j \in I$. As before, two such families are equivalent if their union verifies the compatibility condition on the intersections.

Remark The definition of Kähler metrics on complex spaces was first introduced by Grauert in [2, p.346]. In his definition, it is required only that $\varphi_i - \varphi_j = \mathrm{Re}(h_{ij})$, where h_{ij} is holomorphic on $U_{ij} \cap X_{\mathrm{reg}}$.

Definition 2.1 is Varouchas' definition [12, p.23]. It requires that $\varphi_i - \varphi_j = \mathrm{Re}(h_{ij})$, where h_{ij} is holomorphic on U_{ij} (including the singular points). Hence, it is more restrictive that the one given by Grauert, but they coincide if X is normal. Since we use extensively Varouchas' results from [12], we also follow his definition of Kähler metric throughout this article.

For lcK forms on singular spaces we also want to define the analogue of its associated Lee form. For this, we have the following:

Definition 2.2

- Let X be a topological space and consider $(U_i, f_i)_{i\in I}$, consisting of an open cover $(U_i)_{i\in I}$ of X and a family of continuous functions $f_i : U_i \to \mathbb{R}$ such that $f_i - f_j$ is locally constant on $U_i \cap U_j$, for all $i, j \in I$. The class

$$\theta = \widehat{(U_i, f_i)_{i\in I}} \in \check{\mathrm{H}}^0\left(X, \mathscr{C}/\underline{\mathbb{R}}\right)$$

is called a *topologically closed* 1*-form* (*TC* 1*-form*).
- We say that a TC 1-form θ is *exact* if $\theta = \widehat{(X, f)}$ for a continuous function $f : X \to \mathbb{R}$. In this case, we make the notation $\theta = df$.
- Let $\omega = \widehat{(U_i, \varphi_i, f_i)_{i\in I}}$ be an lcK metric on a complex space X. Then, the TC 1-form $\theta = \widehat{(U_i, f_i)_{i\in I}}$ is called the *Lee form* of ω. If θ is exact, then ω is called *globally conformally Kähler (gcK)*.

Pushforward and stability

We assemble below a few results we will need later.

The first is a theorem ([7, p.330 (III)]) which gives necessary and sufficient conditions under which a holomorphic mapping of complex spaces has pure and equal dimensional fibers.

Theorem 2.3 *Let $p : X \to Y$ be a holomorphic and surjective mapping of complex spaces, with Y locally irreducible. Then, p is an open mapping if and only if $\dim_x p^{-1}(p(x)) = \dim_x X - \dim_{p(x)} Y$ for every $x \in X$.*

The next result ([12, Chap. II, Lemma 3.1.2] combined with [12, Chap. I, Section 3.3]) shows that analytical properties of functions are preserved by pushforward through an open finite map.

Lemma 2.4 *Consider $p : X \to X'$ a finite, open and surjective morphism of complex spaces. If φ is psh, strictly psh, holomorphic or pluriharmonic on X, then*

$$p_*\varphi(x') = \sum_{x \in p^{-1}(x')} \varphi(x)$$

has the corresponding properties on X'.

As to the pushforward through a map which is not finite, the summation above is naturally replaced by integration on the fibers. Firstly, we need the following sufficient condition for geometric flatness [12, Section 3.3, Prop. 3.3.1].

Proposition 2.5 *Suppose that $p : X \to X'$ is a morphism of complex spaces such that, for some fixed $m \geq 0$, the following conditions are verified:*

- *π is proper, open and surjective;*
- *all fibers of π are of pure dimension m;*
- *X' is reduced;*
- *X' is normal.*

Then π is geometrically flat.

Geometric flatness is a notion we do not use directly, but we need it for connecting the previous proposition with the next one, which is the part that we need from [12, Chap. I, Proposition 3.4.1], combined with [1, Théorème principal]. It says that positivity and holomorphicity are again preserved by pushforward via a holomorphic map with good properties. Also, in what follows, we use the definitions of differential forms on complex spaces as given in [12, Chap. I, Section 1].

Proposition 2.6 *Consider $p : X \to X'$ holomorphic, proper, geometrically flat, with m-dimensional fibers and $\varphi \in A^{m,m}(X)$. Define*

$$p_*\varphi(x') = \int_{p^{-1}(x')} \varphi.$$

Then,

(i) *if $\varphi = \overline{\varphi}$ and $\mathrm{i}\partial\overline{\partial}\varphi \geq 0$, then $p_*\varphi$ is psh.*
(ii) *if $\varphi = \overline{\varphi}$ and $\mathrm{i}\partial\overline{\partial}\varphi \gg 0$, then $p_*\varphi$ is s.psh.*
(iii) *if $\overline{\partial}\varphi = 0$, then $p_*\varphi$ is holomorphic.*

The key result in proving the stability theorems on projections of Kähler spaces is the following ([12, Theorem 3]):

Theorem 2.7 *Let (X, ω) be a Kähler space and $m \geq 0$ an integer. Then there exist open sets $U_\alpha \subset X$ ($\alpha \in A$) and $U^j_{\alpha\beta} \subset U_\alpha \cap U_\beta$ ($j \in J_{\alpha\beta}$), which depend only on X and m alone such that:*

(i) *Any compact m-dimensional analytic subset of X is contained in some* U_α.
(ii) *Any compact m-dimensional analytic subset of* $U_\alpha \cap U_\beta$ *is contained in some* $U^j_{\alpha\beta}$.
(iii) *There exist elements* $\chi_\alpha \in A^{m,m}(U_\alpha, \mathbb{R})$ *such that*

$$\omega^{m+1} = \mathrm{i}\partial\overline{\partial}\chi_\alpha.$$

(iv) *There exist elements* $\tau^j_{\alpha\beta} \in A^{m,m}(U^j_{\alpha\beta})$ *such that*

$$\overline{\partial}\tau^j_{\alpha\beta} = 0 \text{ and } (\chi_\alpha - \chi_\beta)_{\restriction U_{\alpha\beta}} = \tau^j_{\alpha\beta} + \overline{\tau}^j_{\alpha\beta}.$$

(v) *The* $\tau^j_{\alpha\beta}$ *are* $\overline{\partial}$*-closed representatives of elements* $\xi^j_{\alpha\beta} \in H^m(U^j_{\alpha\beta}, \Omega^m)$.

3 The main result

In this section, we prove our main result on the existence of lcK metrics on images of lcK spaces. In the particular case of finite mappings, our previous result [5, Thm.4.1] says that if $p : X \to X'$ is holomorphic and finite, and X' admits lcK metrics, then X is also admits lcK metrics. Our theorem, in the case of 0-dimensional fibers, is a kind of reciprocal of this result, in the sense of giving information about the image of an lcK space, instead of the preimage. However, we need some additional conditions to be verified for our proof to work, for which we introduce the following notations: for a mapping $p : X \to X'$, denote $p_* : \pi_1(X) \to \pi_1(X')$ the induced morphism, and for a TC 1-form θ on a complex space X, we denote

$$\ker\theta = \left\{\gamma \in \pi_1(X) \mid \int_\gamma \theta = 0\right\},$$

where the integral $\int_\gamma \theta$ is defined as in [5].

Theorem 3.1 *Let* (X, ω, θ) *be an lcK space of pure dimension and* X' *be a normal space, such that there exists* $p : X \to X'$ *holomorphic, proper, open and surjective. Assume that* $\ker p_* \subset \ker\theta$. *Then,* X' *also admits lcK metrics.*

Proof Denote $\pi' : \widetilde{X}' \to X'$ the universal cover of X' and consider $\widetilde{X} = X \times_{X'} \widetilde{X}'$ to be the pull-back of the universal cover $\pi' : \widetilde{X}' \to X'$ along p. Then, we have the following commutative diagram:

$$\begin{array}{ccc} \widetilde{X} & \xrightarrow{\widetilde{p}} & \widetilde{X}' \\ \pi \downarrow & & \downarrow \pi' \\ (X, \omega, \theta) & \xrightarrow[p]{} & X' \end{array}$$

where $\widetilde{p}$ is also holomorphic, proper, open and surjective, and π is a cover of X. Moreover, since X' is normal, $\widetilde{X}'$ is also normal. Since X' is normal, it is locally irreducible, hence, by Remmert's open mapping 2.3, all the fibers of p have pure dimension m. Also, by taking the pull-back metric, $(\widetilde{X}, \pi^*\omega, \pi^*\theta)$ is an lcK space.

Next, we should note that our assumption $\ker p_* \subset \ker\theta$ is equivalent to $\pi^*\theta$ being exact by elementary covering space theory. Indeed, for any $\widetilde{\gamma} \in \pi_1(\widetilde{X})$, we have firstly that $\widetilde{p}_*\widetilde{\gamma} = 0$ as $\widetilde{X}'$ is simply connected, so $\pi'_*\widetilde{p}_*\widetilde{\gamma} = 0$. Equivalently, as the diagram is commutative, $\pi_*\widetilde{\gamma} \in \ker p_*$. On the other hand $\int_{\widetilde{\gamma}} \pi^*\theta = \int_{\pi_*\widetilde{\gamma}} \theta$, so $\pi^*\theta$ is exact *i.e.* $\int_{\widetilde{\gamma}} \pi^*\theta = 0$ for any $\widetilde{\gamma} \in \pi_1(\widetilde{X})$ if and only if $\ker p_* \subset \ker \theta$.

Thus $\pi^*\theta = d\widetilde{f}$ for a smooth function $\widetilde{f}$ on $\widetilde{X}$. This $\widetilde{f}$ also verifies $\widetilde{f} \circ \xi = \widetilde{f} - c_\xi$ for each $\xi \in H := \mathrm{Deck}_X(\widetilde{X})$, where $c_\xi \in \mathbb{R}$. Subsequently, $e^{-\widetilde{f}}\pi^*\omega$ is a Kähler metric on $\widetilde{X}$. If $\omega = \widehat{(U_i, \varphi_i, f_i)_{i\in I}}$, then

$$\widetilde{\omega} = e^{-\widetilde{f}}\pi^*\omega = \widehat{(U_i^\eta, \varphi_i^\eta)_{i\in I, \eta\in H}},$$

where $\pi^{-1}(U_j) = \cup_{\eta\in H} U_i^\eta$ is a disjoint union, and $\varphi_i^\eta = e^{f_i\circ\pi - \widetilde{f}}\varphi_i \circ \pi_{\restriction U_i^\eta}$. A simple calculation shows that $\xi^*\widetilde{\omega} = e^{c_\xi}\widetilde{\omega}$ for each $\xi \in H$.

We now split the proof into two cases for clarity's sake, although addressing them separately is not strictly necessary (see the comment at the end).

Case 1: $m = 0$. This means that π is a finite mapping. For this step of the proof, we use the methods of Varouchas [12] to construct a Kähler metric on $\widetilde{X}'$. Note that since $\widetilde{p}$ is holomorphic, finite, open and surjective, it is a ramified covering, so there exists an analytic subset with empty interior $\widetilde{R} \subset \widetilde{X}$ such that $\widetilde{p}_{\restriction \widetilde{X}\setminus\widetilde{R}} : \widetilde{X}\setminus\widetilde{R} \to \widetilde{X}'\setminus\widetilde{R}'$ is an unramified covering of finite degree k. For every $(i, \eta) \in I \times H$, consider $\widetilde{p}_*\varphi_i^\eta : V_i^\eta = \widetilde{p}(U_i^\eta) \to \mathbb{R}$ to be the unique continuous function for which

$$\widetilde{p}_*\varphi_i^\eta(x') = \sum_{x\in\widetilde{p}^{-1}(x')} \varphi_i^\eta(x)$$

on $\widetilde{X} \setminus \widetilde{R}$. By [12, Lemma 3.1.2], the functions $\{\widetilde{p}_*\varphi_i^\eta\}_{i\in I,\eta\in H}$ are strictly psh. They are also continuous, of class $\mathcal{C}^\infty$ outside R', and the differences are pluriharmonic outside $R' \cup \widetilde{p}(X_{\mathrm{sing}})$. Moreover, we have $\xi^*\widetilde{p}_*\varphi_i^\eta = e^{c_\xi}\widetilde{p}_*\varphi_i^{\xi^{-1}\eta}$ for every $\xi \in \mathrm{Deck}_{X'}(\widetilde{X}') \simeq \mathrm{Deck}_X(\widetilde{X}) = H$. Next, we apply [12, Thm.1] to obtain a Kähler metric

$$\widetilde{\tau}' = \widehat{(V_i^\eta, \psi_i^\eta)_{i\in I,\eta\in H}}$$

on $\widetilde{X}'$, with $\mathcal{C}^\infty$ strictly psh functions ψ_i^η. Since for a fixed $i \in I$, the family of open sets $\{\widetilde{p}(U_i^\eta)\}_{\eta\in H}$ are mutually disjoint, this can be done such that the property $\psi_i^\eta \circ \xi = e^{c_\xi}\psi_i^{\xi^{-1}\eta}$ for every $\eta \in H$ is verified by these new psh functions. Finally, this shows that for every $\xi \in H = \mathrm{Deck}_{X'}(\widetilde{X}')$, we have $\xi^*\widetilde{\tau}' = e^{c_\xi}\widetilde{\tau}'$, and by [5, Thm.3.10], X' admits lcK metrics.

Case 2: $m \geq 1$. There exists an open cover $(V_j)_{j\in J}$ of X' such that:

- $(\pi')^{-1}(V_j) = \bigcup_{\eta\in H} V_j^\eta$ for every $j \in J$ and for every $j \in J$, $(V_j^\eta)_{\eta\in H}$ are mutually disjoint, and $\xi^{-1}(V_j^\eta) = V^{\xi^{-1}\eta}$ for any $\xi, \eta \in H$
- $U_j^\eta := \widetilde{p}^{-1}(V_j^\eta)$, for every $j \in J, \eta \in H$, and $\xi^{-1}(U_j^\eta) = U^{\xi^{-1}\eta}$ for any $\xi, \eta \in H$.

If the sets $(V_j)_{j\in J}$ were chosen sufficiently small, then, by 2.7, for every $j \in J, \eta \in H$, there exists an (m, m)-form χ_j^η on U_j^η such that

$$\widetilde{\omega}^{m+1}_{\restriction U_j^\eta} = \mathrm{i}\partial\overline{\partial}\chi_j^\eta.$$

Let $\xi \in H$. Since, $\xi^*\widetilde{\omega} = e^{c_\xi}\widetilde{\omega}$, we have

$$\begin{aligned}
\mathrm{i}\partial\overline{\partial}(\xi^*\chi_j^\eta) &= \xi^*(\mathrm{i}\partial\overline{\partial}\chi_j^\eta) = \xi^*(\widetilde{\omega}^{m+1}_{\restriction U_j^\eta}) = (\xi^*\widetilde{\omega})^{m+1}_{\restriction U_j^{\xi^{-1}\eta}} \\
&= e^{(m+1)c_\xi}\widetilde{\omega}^{m+1}_{\restriction U_j^{\xi^{-1}\eta}} = e^{(m+1)c_\xi}\mathrm{i}\partial\overline{\partial}(\chi_j^{\xi^{-1}\eta}) = \mathrm{i}\partial\overline{\partial}(e^{(m+1)c_\xi}\chi_j^{\xi^{-1}\eta}).
\end{aligned}$$

Hence, we can chose χ_j^η such that they verifiy $\xi^*\chi_j^\eta = e^{(m+1)c_\xi}\chi_j^{\xi^{-1}\eta}$ for any $\xi, \eta \in H$. Then, we define $\widetilde{p}_*\chi_j^\eta : V_j^\eta \to \mathbb{R}$,

$$\widetilde{p}_*\chi_j^\eta(\widetilde{x}') = \int_{\widetilde{p}^{-1}(\widetilde{x}')} \chi_j^\eta.$$

By the above property, we obtain $\xi^*\widetilde{p}_*\chi_j^\eta = e^{(m+1)c_\xi}\widetilde{p}_*\chi_j^{\xi^{-1}\eta}$ for any $\xi, \eta \in H$. Moreover, Proposition 2.5 together with Proposition 2.6 ensure that the functions $\widetilde{p}_*\chi_j^\eta$, $j \in J, \eta \in H$, are s.psh (but not necessarily $\mathcal{C}^\infty$) and the difference of any two such functions is pluriharmonic. Finally, applying [12, Thm.1], we obtain a Kähler metric

$$\widetilde{\tau}' = \widehat{(V_j^\eta, \psi_j^\eta)_{j\in J, \eta\in H}}$$

on $\widetilde{X}'$. As in **Case 1**, the choices in the proof of [12, Thm.1] can be made such that the property $\xi^*\widetilde{\tau}' = e^{c_\xi}\widetilde{\tau}'$, for every $\xi \in H$ is satisfied, which means that X' admits lcK metrics. □

Remark Please note that treating the finite and positive-dimensional fibers as separate cases is in fact artificial and only done for clarity. Indeed, note that for the mapping $\widetilde{p} : \widetilde{X} \longrightarrow \widetilde{X}'$, summing along the fiber in the first case or integrating on it in the second are just instances of the same trace operator

$$\mathrm{Trace}_{\widetilde{p}} : \widetilde{p}_*\mathcal{O}_{\widetilde{X}} \longrightarrow \mathcal{O}_{\widetilde{X}'}.$$

□

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Declarations

Conflict of interest The authors declare no competing interests.

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