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ERRATUM

Erratum to: Stability of Diffusion Coefficients in an Inverse Problem for the Lotka-Volterra Competition System

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The construction of the weight function $\psi(x)$ in (2.1) and Assumption 3.1 are inconsistent. Therefore Assumption 3.1 on the weight function $\psi(x)$ should be corrected as follows:

Assumption 3.1 *Suppose* $A_i^\theta \in C^3(\Omega)$, $i = 1, 2$ *be a real valued function such that* A_i^θ, $i = 1, 2$ *and all its derivatives up to order three are bounded and also it satisfies* $|\nabla\psi \cdot \nabla A_i^\theta| \geq \sigma > 0$, $i = 1, 2$ *on* $\overline{\Omega\backslash\omega_0}$, *where* $\omega_0 \Subset \omega \Subset \Omega$.

This assumption does not change the results in Sect. 2. In Sect. 3, the statement after (3.1), "Let us remark that … arguments in [19]." should be removed and Lemma 3.1 should be replaced by the following

Lemma 3.1 *Consider the first order partial differential operator* $P_0 h = \nabla A^\theta \cdot \nabla h$, *where* A^θ *satisfies Assumption 3.1. Then there exists a constant* $C > 0$, *such that for sufficiently large* λ *and* s, *the following inequality holds*:

$$s^2\lambda^2 \int_\Omega \phi^\theta e^{-2s\alpha^\theta} |h|^2 dx \leq C\left(\int_\Omega \frac{1}{\phi^\theta} e^{-2s\alpha^\theta} |P_0 h|^2 dx + s^2\lambda^2 \int_{\omega_0} \phi^\theta e^{-2s\alpha^\theta} |h|^2 dx\right),$$

with $\theta \in (0, T)$ *and for* $h \in H_0^1(\Omega)$.

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Proof Let us consider $v = e^{-s\alpha^\theta} h$ and so $Q_0 v = e^{-s\alpha^\theta} P_0(e^{s\alpha^\theta} v) = sv P_0 \alpha^\theta + P_0 v$, where we note that $h \in H_0^1(\Omega)$. Then by integration by parts with respective to space variable, we obtain

$$\begin{aligned}\int_\Omega \frac{1}{\phi^\theta}(Q_0 v)^2 dx &= s^2 \int_\Omega \frac{1}{\phi^\theta}(P_0\alpha^\theta)^2 v^2 dx + \int_\Omega \frac{1}{\phi^\theta}(P_0 v)^2 dx + 2s\int_\Omega \frac{1}{\phi^\theta} v (P_0\alpha^\theta)(P_0 v) dx \\ &= s^2\lambda^2 \int_\Omega \phi^\theta (\nabla A^\theta \cdot \nabla\psi)^2 v^2 dx + \int_\Omega \frac{1}{\phi^\theta}(P_0 v)^2 dx \\ &\quad -2s\lambda \int_\Omega (\nabla A^\theta \cdot \nabla \psi)(\nabla A^\theta \cdot \nabla v) v dx \\ &\geq s^2\lambda^2 \int_\Omega \phi^\theta |\nabla\psi \cdot \nabla A^\theta|^2 |v|^2 dx + s\lambda \int_\Omega \nabla(P_0 \psi \nabla A^\theta)|v|^2 dx.\end{aligned}$$

Using Assumption 3.1, one can get

$$s^2\lambda^2 \int_\Omega \phi^\theta |\nabla\psi \cdot \nabla A^\theta|^2 |v|^2 dx \geq \sigma^2 s^2 \lambda^2 \left(\int_\Omega \phi^\theta |v|^2 dx - \int_{\omega_0} \phi^\theta |v|^2 dx \right)$$

and hence

$$s^2\lambda^2\sigma^2 \left(\int_\Omega \phi^\theta |v|^2 dx - \int_{\omega_0} \phi^\theta |v|^2 dx \right) \leq \int_\Omega \frac{1}{\phi^\theta} |Q_0 v|^2 dx + s\lambda \int_\Omega |\nabla(P_0 \psi \nabla A^\theta)||v|^2 dx.$$

Again from Assumption 3.2, we have

$$\begin{aligned} s^2\lambda^2\sigma^2 \int_\Omega e^{-2s\alpha^\theta} \phi^\theta |h|^2 dx &\leq c_1 T^2 s\lambda \int_\Omega e^{-2s\alpha^\theta} \phi^\theta |h|^2 dx + \int_\Omega e^{-2s\alpha^\theta} \frac{1}{\phi^\theta} |P_0 h|^2 dx \\ &\quad + s^2\lambda^2\sigma^2 \int_{\omega_0} e^{-2s\alpha^\theta} \phi^\theta |h|^2 dx. \end{aligned}$$

Finally, taking $\lambda \geq 1$, $s \geq 2c_1 T^2/\sigma^2$ one can conclude the lemma. □

The Proposition 3.1 also needs some minor revision.

Proposition 3.1 *Let* (B_1, B_2, B_3) *be the solutions of* (1.3) *and* u, v, w *be the solutions of* (3.1). *Suppose all the conditions of Theorem* 2.1 *and Assumption* 3.1 *are satisfied. Then there exists a constant* $C(\gamma_1, \gamma_2, \delta, \sigma) > 0$, *such that for sufficiently large* s *and* λ, *the following estimate holds*:

$$\begin{aligned} & s^2\lambda^2 \int_\Omega \phi^\theta e^{-2s\alpha^\theta} \left(|f|^2 + |g|^2 + |\nabla f|^2 + |\nabla g|^2\right) dx \\ & \leq C(\gamma_1, \gamma_2, \delta, \sigma) \sum_{i=1}^{7} \mathcal{E}_i(\theta) + C s^2 \lambda^2 \int_{\omega_0} \phi^\theta e^{-2s\alpha^\theta} \left(|f|^2 + |g|^2 + |\nabla f|^2 + |\nabla g|^2\right) dx, \end{aligned}$$

for any $f, g \in H_0^2(\Omega)$, *where the functions* $\mathcal{E}_i$, $i = 1, \ldots, 7$ *are given in* (3.2).

Proof From the value of the solutions satisfying (3.1a) at $t = \theta$ and the definition of F, we arrive at

$$P_0 f = \nabla \widetilde{A}_1^\theta \cdot \nabla f = u^\theta - \nabla(d_1 \nabla B_1^\theta) + a_1^\theta B_1^\theta + a_2^\theta B_2^\theta + a_3^\theta B_3^\theta - f \Delta \widetilde{A}_1^\theta,$$

and therefore, using Lemma 3.1, we obtain

$$\begin{aligned}s^2\lambda^2\int_\Omega \phi^\theta e^{-2s\alpha^\theta}|f|^2dx &\le C\Big(\int_\Omega \frac{1}{\phi^\theta}e^{-2s\alpha^\theta}|P_0 f|^2dx + s^2\lambda^2\int_{\omega_0}\phi^\theta e^{-2s\alpha^\theta}|f|^2dx\Big)\\ &\le C\int_\Omega \frac{1}{\phi^\theta}e^{-2s\alpha^\theta}\Big(|u^\theta|^2+|\nabla d_1|^2|\nabla B_1^\theta|^2+|d_1|^2|\Delta B_1^\theta|^2\\ &\quad +|a_1^\theta|^2|B_1^\theta|^2+|a_2^\theta|^2|B_2^\theta|^2+|a_3^\theta|^2|B_3^\theta|^2+|f\Delta\widetilde{A}_1^\theta|^2\Big)dx\\ &\quad + Cs^2\lambda^2\int_{\omega_0}\phi^\theta e^{-2s\alpha^\theta}|f|^2dx\\ &\le C(\gamma_1,\delta)(\mathcal{E}_1(\theta)+\mathcal{E}_5(\theta))+Cs^2\lambda^2\int_{\omega_0}\phi^\theta e^{-2s\alpha^\theta}|f|^2dx.\end{aligned}$$

From the solutions satisfying (3.1b) at $t=\theta$ and the definition of G, we have

$$P_0 g=\nabla\widetilde{A}_2^\theta\cdot\nabla g = v^\theta-\nabla(d_2\nabla B_2^\theta)+a_4^\theta B_2^\theta+a_5^\theta B_3^\theta+a_6^\theta B_1^\theta-g\Delta\widetilde{A}_2^\theta$$

whence it follows that

$$\begin{aligned}s^2\lambda^2\int_\Omega \phi^\theta e^{-2s\alpha^\theta}|g|^2dx &\le C\Big(\int_\Omega \frac{1}{\phi^\theta}e^{-2s\alpha^\theta}|P_0 g|^2dx + s^2\lambda^2\int_{\omega_0}\phi^\theta e^{-2s\alpha^\theta}|g|^2dx\Big)\\ &\le C\int_\Omega \frac{1}{\phi^\theta}e^{-2s\alpha^\theta}\Big(|v^\theta|^2+|\nabla d_2|^2|\nabla B_2^\theta|^2+|d_2|^2|\Delta B_2^\theta|^2\\ &\quad +|a_4^\theta|^2|B_2^\theta|^2+|a_5^\theta|^2|B_3^\theta|^2+|a_6^\theta|^2|B_1^\theta|^2+|g\Delta\widetilde{A}_2^\theta|^2\Big)dx\\ &\quad + Cs^2\lambda^2\int_{\omega_0}\phi^\theta e^{-2s\alpha^\theta}|g|^2dx\\ &\le C(\gamma_2,\delta)(\mathcal{E}_2(\theta)+\mathcal{E}_6(\theta))+Cs^2\lambda^2\int_{\omega_0}\phi^\theta e^{-2s\alpha^\theta}|g|^2dx.\end{aligned}$$

On the other hand, it is easy to see from the expression $P_0 f$ that

$$P_0\nabla f=\nabla u^\theta-\Delta(d_1\nabla B_1^\theta)+\nabla(a_1^\theta B_1^\theta)+\nabla(a_2^\theta B_2^\theta)+\nabla(a_3^\theta B_3^\theta)-\nabla(f\Delta\widetilde{A}_1^\theta)-\nabla f\Delta\widetilde{A}_1^\theta$$

and similarly, we also have

$$P_0\nabla g=\nabla v^\theta-\Delta(d_2\nabla B_2^\theta)+\nabla(a_6^\theta B_1^\theta)+\nabla(a_4^\theta B_2^\theta)+\nabla(a_5^\theta B_3^\theta)-\nabla(g\Delta\widetilde{A}_2^\theta)-\nabla g\Delta\widetilde{A}_2^\theta.$$

Thus it follows again, from Lemma 3.1 and the computations similar to preceding estimates, that

$$\begin{aligned}s^2\lambda^2\int_\Omega \phi^\theta e^{-2s\alpha^\theta}(|\nabla f|^2+|\nabla g|^2)dx &\le C(\gamma_1,\gamma_2,\delta)(\mathcal{E}_3(\theta)+\mathcal{E}_4(\theta)+\mathcal{E}_7(\theta))\\ &\quad + C\Big(s^2\lambda^2\int_{\omega_0}\phi^\theta e^{-2s\alpha^\theta}(|\nabla f|^2+|\nabla g|^2)dx\Big).\end{aligned}$$

The proof is thus concluded by combining the above three estimates. □

One can easily see that the proofs of Lemma 3.2 and Lemma 3.3 are not affected by the new Assumption 3.1. Finally the main result stated in Theorem 3.1 should read as follows

Theorem 3.1 *Let* (B_1, B_2, B_3) *be the solutions of* (1.3). *Suppose all the assumptions of Theorem* 2.1 *hold true and* $f, g \in H_0^2(\Omega)$. *In addition, suppose that Assumptions* 3.1 *and* 3.2 *are also satisfied. Then there exists a constant* C *with* $C(\Omega, \omega, T, a_0, \delta, \mu_1, \mu_2, \mu_3, \gamma_1, \gamma_2, \sigma) > 0$, *such that for sufficiently large* $\lambda \geq \lambda_1$ *and* $s \geq s_4$, *the following estimate holds*:

$$\int_\Omega (|f|^2 + |g|^2 + |\nabla f|^2 + |\nabla g|^2)dx$$

$$\leq C \int_{\omega_0} (|f|^2 + |g|^2 + |\nabla f|^2 + |\nabla g|^2)dx + C\left(\int_{Q_\omega} (|\partial_t B_2|^2 + |\partial_t B_3|^2)dtdx\right.$$

$$\left. + \int_\Omega \left(\sum_{i=1}^{3} (|B_i^\theta|^2 + |\nabla B_i^\theta|^2) + \sum_{i=1}^{2} (|\Delta B_i^\theta|^2 + |\nabla(\Delta B_i^\theta)|^2)\right)dx\right).$$

It is easy to see that the proof of Theorem 3.1 remains as such except the last integral term in Proposition 3.1, that is, $s^2\lambda^2 \int_{\omega_0} \phi^\theta e^{-2s\alpha^\theta} (|f|^2 + |g|^2 + |\nabla f|^2 + |\nabla g|^2)dx$ should be added on the right hand side of four main estimates in the proof.

Moreover, the correction in the main estimate of Theorem 3.1 should be carried out for the statement of the stability result given in (1.4).