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A New Wall Stress Equation for Aneurysm-Rupture Prediction

Z. LI and C. KLEINSTREUER

Department of Mechanical and Aerospace Engineering, North Carolina State University, Raleigh, NC

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Abstract—Aneurysms, especially in the abdominal aorta (AAA), are prone to rupture, and hence a reliable and easy-to-use predictor is most desirable. Based on clinical observations and numerical analyses, a semi-empirical equation for the peak AAA-wall stress has been developed. It can be readily used for AAA-rupture predictions or can be integrated into more elaborate AAA-assessment models.

Keywords—Laplace structure equation, Abdominal aortic aneurysm, Semi-empirical wall stress equation, AAA-rupture prediction.

INTRODUCTION

An aneurysm is a focal dilation of a blood vessel to greater than twice its normal diameter. Aneurysms are most commonly found in large arteries (aortic, iliac, and femoral); however, they have been reported in smaller arteries such as the radial or coronary arteries as well. The etiology of aneurysm is currently believed to be multi-factorial with atherosclerosis contributing the greatest part to the disease process. Other causes may include infectious etiologies, traumatic injury, chronic lung diseases, genetic disorders, smoking, and biomechanical factors such as hypertension, disturbed blood flow, and wall tissue degradation.[6] The prevalence of aneurysms is greatest in the infra-renal abdominal aorta and it has been estimated that there are 3 million people with undiagnosed abdominal aortic aneurysms (AAAs). If left untreated, nearly all aneurysms continue to enlarge and eventually rupture.[18] Treatment options involve replacement of the diseased artery segment with a synthetic tube and, until this past decade, were solely performed with standard open surgical technique, which is considered to be a major procedure with significant risk to the patient.

Thus, reliable risk estimation of AAA rupture is an important goal which has been pursued by some researchers (e.g., Raghavan *et al.*, 2000[14]; Vorp *et al.*, 1998[20]; Di Martino *et al.*, 2001[3]; Thurbrikar *et al.*, 2001[17]; Wang *et al.*, 2002[21]; Wilson *et al.*, 2003[22]; Sonesson *et al.*, 1999[15]; Cappeller *et al.*, 1997[1]; Hatakeyama *et al.*, 2001[8]; Limet *et al.*, 1991[11]; Ouriel *et al.*, 1992[13]; Stenbaek *et al.*, 2000[16]; Vardulaki *et al.*, 1998[19]; among others). Generally, the cut-off maximum AAA diameter (i.e., the "$d_{\text{AAA,max}} \geq 5$ cm" criterion) and the AAA expansion rate (i.e., $\dot{d}_{\text{AAA,max}} 0.5$ cm/year) are employed as the key risk factors. However, up to 23% of AAAs rupture when $d_{\text{AAA,max}} \geq 5$ cm, which implies that other biomechanical factors are equally or even more important. Indeed, knowledge of a patient's maximum wall stress is probably most important because when the actual wall stress reaches the patient's yield stress, the aneurysm wall ruptures.[3,6,14,20] The problem is that a patient's yield stress is usually unknown and the maximum wall stress *in vivo* is always impossible to measure. In fact, coupled fluid–structure interaction models may presently be the only way to generate reliable wall stress data.[3,10,23]

In this paper, we propose a semi-empirical extension of the Laplace equation to compute the aneurysm wall stress based on routine pressure and geometric measurements. This new wall stress equation can be directly used for possible rupture prediction or may be a factor in a more elaborate rupture-risk assessment program.

THEORY

The original Laplace equation,

$$\sigma = \frac{pr}{ct} \tag{1}$$

(where σ is the average wall stress, p the pressure load, r the radius and t the wall thickness, while $c = 1$ is for cylinders and $c = 2$ for spheres), greatly overestimates or underestimates actual aneurysm wall stresses because of the many underlying assumptions.[3–6] In order to provide a useful, i.e., accurate and easy-to-calculate, predictor of the maximum wall stress in common AAAs (of Fig. 1), Eq. (1) was extended based on observed evidence and computational analyses. The functional form of $\sigma_{\max} = \sigma_{\max}$ (systolic pressure, AAA-geometry parameters) was obtained via trial and error. Specifically, the existence of an intra-luminal thrombus (ILT), the influence of asymmetry, and the nonlinear

Address correspondence to C. Kleinstreuer, Department of Mechanical and Aerospace Engineering, North Carolina State University, Raleigh, NC 27695-7910. Electronic mail: ck@eos.ncsu.edu; Website: www.mae.ncsu.edu/research/ck_CFPDlab/index.html

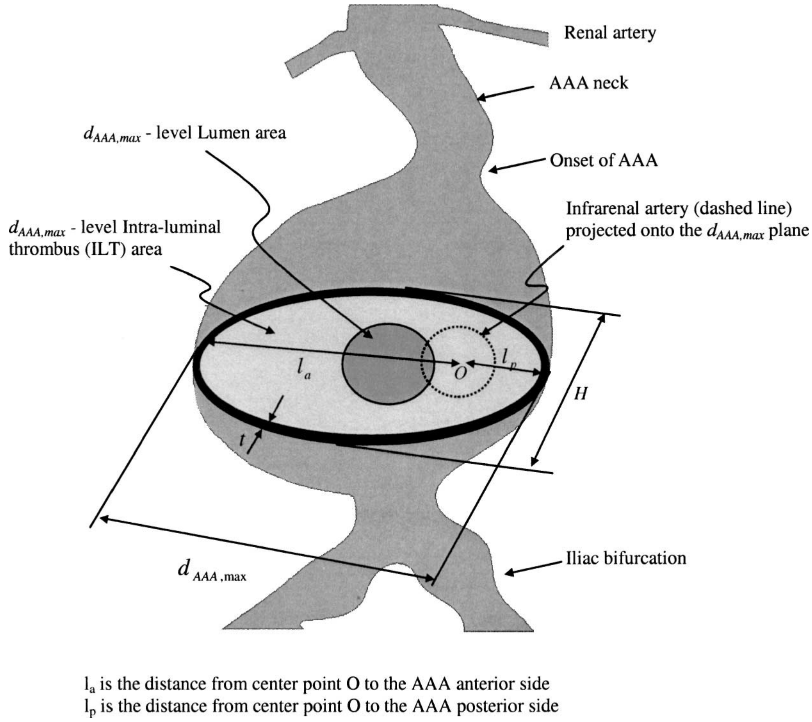

FIGURE 1. AAA schematic.

relationship between wall stress, blood pressure, and AAA diameter were taken into account. The tissue properties are, in the average, implicit in the pressure–stress relation. To derive the new equation, all coefficients and exponents in Eq. (2) were determined by statistical analyses and curve fitting of clinical/experimental data and computer modeling results considering different AAA parameters. A second set of 10 data sources (see Table 1) was employed to test the accuracy of the $\sigma_{\max}$-correlation.

$$\sigma_{\max} = 0.006 \frac{(1 - 0.68\alpha) e^{0.0123(0.85 p_{\text{sys}} + 19.5 d_{\text{AAA,max}})}}{t^{0.63} \beta^{0.125}} \quad \text{in (MPa)} \tag{2}$$

where $\sigma_{\max}$ is the maximum wall stress which occurs most frequently at a location two-thirds of the maximum AAA diameter, the area ratio $\alpha = \frac{A_{\text{ILT,max}}}{A_{\text{AAA,max}}}$; the asymmetry index $\beta = \frac{l_{\text{p}}}{l_{\text{a}}}$, where l_{p} and l_{a} are the distances from the center point O to the posterior and the anterior (Fig. 1); p_{sys} is the systolic blood pressure (mmHg), $d_{\text{AAA,max}}$ the maximum AAA diameter (cm), and t the wall thickness at the location of $d_{\text{AAA,max}}$ (cm). Specifically, $A_{\text{AAA,max}}$ and $A_{\text{ILT,max}}$ are the transverse areas of the AAA and intra-luminal thrombus (ILT) at the $d_{\text{AAA,max}}$-location, respectively. For imaging techniques not providing area measurements, the transverse area can be approximately calculated as

$$A_{\text{AAA,max}} = \frac{\pi d_{\text{AAA,max}} H}{4} \tag{3}$$

where H is the in-plane axis normal to the $d_{\text{AAA,max}}$-measurement plane (see Fig. 1). The lumen area $A_{\text{lumen,max}}$ may be calculated similarly; then, the ILT area is given as

$$A_{\text{ILT,max}} = A_{\text{AAA,max}} - A_{\text{lumen max}} \tag{4}$$

If the wall thickness t is difficult to obtain from CT-scans, it may be approximated with a curve-fitted correlation:[17]

$$t = 3.9 \left(\frac{d_{\text{AAA,max}}}{2} \right)^{-0.2892} \quad \text{in (mm)}, \tag{5}$$

where the unit of $d_{\text{AAA,max}}$ is (mm).

One can see that Eq. (2) not only represents the nonlinear correlation between wall stress and blood pressure, diameter, and wall thickness, but also takes into account the effects of ILT and AAA asymmetry.

NOTES

1. To some extent, Eq. (2) integrates the stress concentration effect caused by asymmetry. However, for seriously distorted geometries, the evaluation of maximum wall stress magnitude (and location) is too complicated to predict with a simple equation.
2. The ILT material property is assumed to be uniform. Even though the ILT close to the luminal surface is most newly formed and tends to be "older" away from the luminal surface toward the AAA wall. Di Martino and Vorp[2] found that the use of mean ILT properties as opposed to the exact patient-specific parameters

TABLE 1. Comparisons with the new wall stress equation.

AAA model	p (mmHg)	$d_{AAA,max}$ (cm)	Thickness t (cm)	α	β	Maximum stress σ (MPa)	New wall stress equation [Eq. (2)] Stress (MPa)	Error (%)	Laplace equation [Eq. (1)] (cylinder) Stress (MPa)	Error (%)
Fillinger *et al.*[5,6]	120	6.7	0.19	0	0.4	0.32	0.335	4.7	0.281	12.2
	130	5.5	0.19	0	0.4	0.3	0.278	7.3	0.25	16.7
Wang *et al.*[21]	128	6.1	0.184	0.54	0.3	0.19	0.208	9.5	0.282	48.4
	155	6.4	0.175	0.3	0.9	0.35	0.343	2.0	0.277	20.9
Vorp *et al.*[20]	120	6	0.15	0	0.3	0.33	0.34	3.0	0.319	3.3
Raghavan *et al.*[14]	115	5.2	0.19	0	0.7	0.23	0.21	8.7	0.209	9.1
	188	5.5	0.19	0	0.9	0.43	0.46	6.9	0.362	5.8
Thurbrikar *et al.*[17]	120	5.86	0.104	0	0.5	0.37	0.389	5.1	0.449	21.4
	120	5.86	0.158	0	0.5	0.28	0.299	6.7	0.296	5.7
Li and Kleinstreuer[10]	120	5.0	0.05	0.15	1	0.43	0.412	4.2	0.798	85.6

would result in a maximum error of only 5% in wall stress prediction.

3. Residual stress is neglected because its magnitude is much smaller than the mechanical stress caused by blood pressure in the artery wall. Matsumoto and Sato[12] found the maximum residual stress to be only 3% of the mechanical stress in the arterial wall. Holzapfel *et al.*[9] indicated that the residual stress may decrease the inner wall stress, while it may increase stress at the outer wall. Thus, the total change of mean stress across the wall in the presence of residual stress is not significant.

RESULTS

Comparisons with Computer Simulations

Comparisons between numerical analyses as well as the semi-empirical correlation and the original Laplace equation are shown in Figs. 2–6. It can be seen that the new wall stress equation is in good agreement with numerical simulations obtained by Li and Kleinstreuer,[10] who evaluated $d_{AAA,max}$, t, p, A_{ILT}/A_{AAA}, and l_p/l_a, assuming quasi-steady flow. Especially in the presence of ILTs, asymmetric AAA geometries and small wall thickness, the new wall stress equation produces much better results than the simple Laplace equation [Eq. (1)].

Comparisons with Clinical Observations

In order to test the broader validity of the new wall stress equation, we calculated the wall stresses, using data from 10 different clinical and numerical AAA models, and compared the results with Eq. (2) to these published data sets. As shown in Table 1, the maximum error using the new wall stress equation is 9.5%, whereas the Laplace equation generates a maximum error of 85.6%.

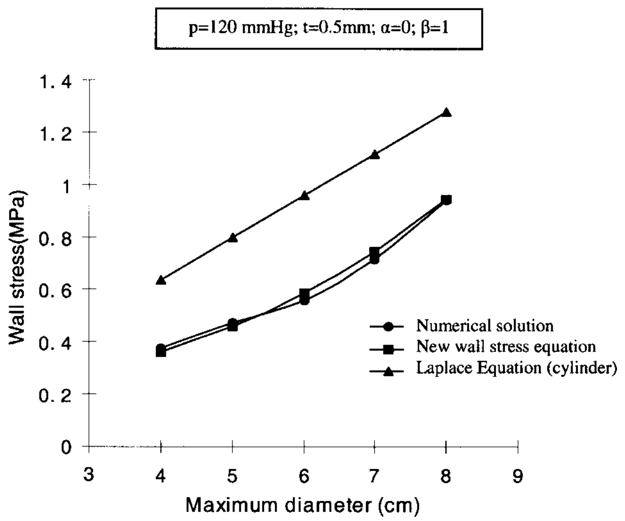

FIGURE 2. Relationship between wall stress and maximum diameter.

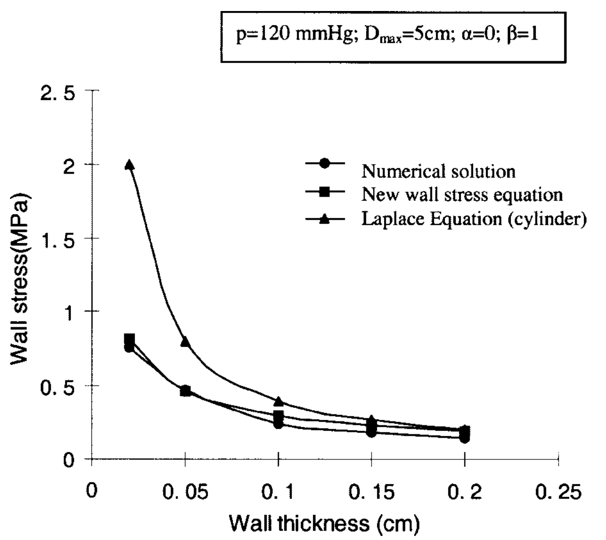

FIGURE 3. Relationship between wall stress and wall thickness.

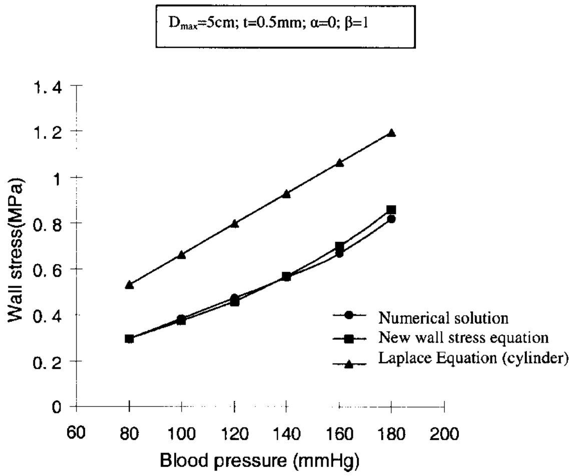

FIGURE 4. Relationship between wall stress and blood pressure.

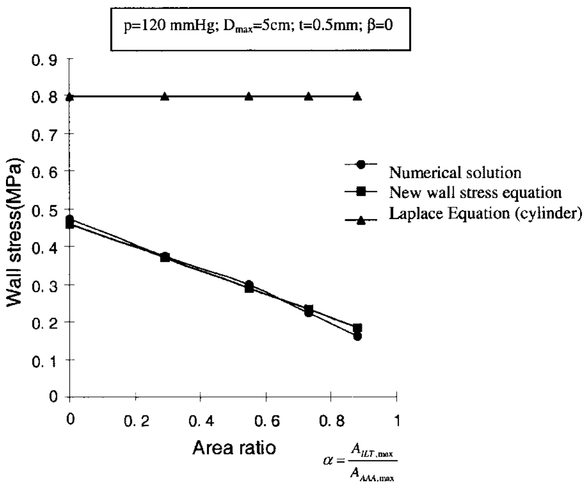

FIGURE 5. Relationship between wall stress and area ratio $\left(\alpha = \frac{A_{ILT,max}}{A_{AAA,max}}\right)$.

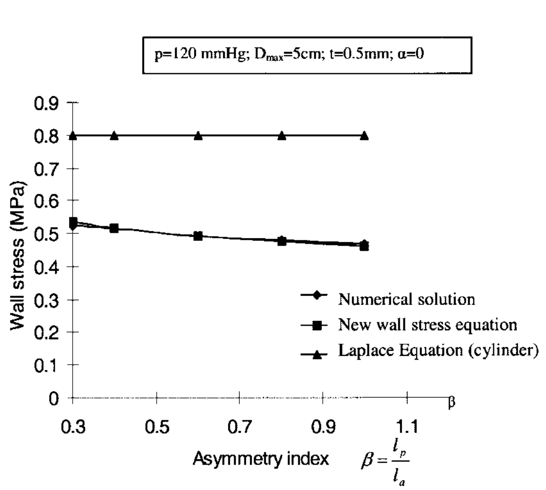

FIGURE 6. Relationship between wall stress and asymmetry index $\left(\beta = \frac{l_p}{l_a}\right)$.

DISCUSSION AND CONCLUSIONS

There still exist limitations inherent in the semi-empirical wall stress equation [Eq. (2)]. First of all, it works better for AAAs which do not have seriously distorted shapes which may cause large stress concentrations. Secondly, while the maximum wall stress can be estimated, its actual location is still unknown. Thirdly, the stress in the AAA wall is taken as equal in radial direction (i.e., wall-thickness direction), and the ILT material property distribution in the AAA sac is assumed to be uniform. The main advantages are that Eq. (2) is easy-to-use, quite accurate, representative, and that its parameters can be readily calculated based on data from ultrasound, MRI and/or CT scans.

REFERENCES

[1]Cappeller, W. A., H. Engelmann, S. Blechschmidt, M. Wild, and L. Lauterjung. Possible objectification of a critical maximum diameter for elective surgery in abdominal aortic aneurysms based on one- and three-dimensional ratios. *J. Cardiovasc. Surg.* 38:623–628, 1997.

[2]Di Martino, E. S., and D. A. Vorp. Effect of variation in intraluminal thrombus constitutive properties on abdominal aortic aneurysm wall stress. *Ann. Biomed. Eng.* 31(7):804–809, 2003.

[3]Di Martino, E. S., G. Guadagni, A. Fumero, and G. Ballerini. Fluid–structure interaction within realistic three-dimensional models of the aneurysmatic aorta as a guidance to assess the risk of rupture of the aneurysm. *Med. Eng. Phys.* 23:647–655, 2001.

[4]Elger, D. F., D. M. Blackketter, R. S. Budwig, and K. H. Johansen. The influence of shape on the stresses in model abdominal aortic aneurysms. *J. Biomech. Eng. (ASME)* 118:326–332, 1996.

[5]Fillinger, M. F., M. L. Raghavan, P. Marra, L. Cronenwett, and E. Kennedy. In vivo analysis of mechanical wall stress and abdominal aortic aneurysm rupture risk. *J. Vasc. Surg.* 36:589–596, 2002.

[6]Fillinger, M. F., P. S. Marra, M. L. Raghavan, and E. F. Kennedy. Prediction of rupture in abdominal aortic aneurysm during observation: Wall stress versus diameter. *J. Vasc. Surg.* 37:724–732, 2003.

[7]Hall, A. J., E. F. G. Busse, D. J. McCarville, and J. J. Burgess. Aortic wall tension as a predictive factor for abdominal aortic aneurysm rupture: Improving the selection of patients for abdominal aortic aneurysm repair. *Ann. Vasc. Surg.* 14:152–157, 2000.

[8]Hatakeyama, T., H. Shigematsu, and T. Muto. Risk factors for rupture of abdominal aortic aneurysm based on three dimensional study. *J. Vasc. Surg.* 33:453–461, 2001.

[9]Holzapfel, G. A., T. C. Gasser, and R. W. Ogden. A new constitutive framework for arterial wall mechanics and a comparative study of material models. *J. Elasticity* 61(1–3):1–48, 2000.

[10]Li, Z., and C. Kleinstreuer. Computational analysis of fluid–structure interactions in a stented aneurysm model. Computer Methods in Biomechanics and Biomedical Engineering, Vol. 8 (2005).

[11]Limet, R., N. Sakalihassan, and A. Albert. Determination of the expansion rate and incidence of rupture of abdominal aortic aneurysms. *J. Vasc. Surg.* 14:540–548, 1991.

[12]Matsumoto, T., and M. Sato. Analysis of stress and strain distribution in the artery wall consisted of layers with different

elastic modulus and opening angle. *JSME Int. J. C-Mech. SY* 45(4):906–912, 2002.

[13]Ouriel, K., R. M. Green, C. Donayre, C. K. Shortell, J. Elliott, and J. A. Deweese. An evaluation of new methods of expressing aortic aneurysm size: Relationship to rupture. *J. Vasc. Surg.* 15:12–20, 1992.

[14]Raghavan, M., D. Vorp, M. Federle, M. Makaroun, and M. Webster. Wall stress distribution on three-dimensionally reconstructed models of human abdominal aortic aneurysm. *J. Vasc. Surg.* 31:760–769, 2000.

[15]Sonesson, B., T. Sandgren, and T. Lanne. Abdominal aortic aneurysm wall mechanics and their relation to risk of rupture. *Eur. Vasc. Endovasc. Surg.* 18:487–493, 1999.

[16]Stenbaek, J., B. Kalin, and J. Swedenborg. Growth rate of thrombus may be a better predictor of rupture than diameter in patients with abdominal aortic aneurysm. *Eur. Vasc. Endovasc. Surg.* 20:466–469, 2000.

[17]Thurbrikar, M., J. Al-Soudi, and F. Robicsek. Wall stress studies of abdominal aortic aneurysm in a clinical model. *Ann. Vasc. Surg.* 3:355–366, 2000.

[18]Uflacker, R., and J. Robison. Endovascular treatment of abdominal aortic aneurysms: A review. *Eur. Radiol.* 11:739–753, 2001.

[19]Vardulaki, K. A., T. C. Prevost, N. M. Walker, N. E. Day, A. B. M. Wilmink, C. R. G. Quick, H. A. Ashton, and A. P. Scott. Growth rates and risk of rupture of abdominal aortic aneurysms. *Br. J. Surg.* 85:1674–1680, 1998.

[20]Vorp, D. A., M. L. Raghavan, and M. W. Webster. Mechanical wall stress in abdominal aortic aneurysm: Influence of diameter and asymmetry. *J. Vasc. Surg.* 27:632–639, 1998.

[21]Wang, D., M. Makaroun, M. Webster, and D. A. Vorp. Effect of intraluminal thrombus on wall stress in patient specific model of abdominal aortic aneurysm. *J. Vasc. Surg.* 3:598–604, 2002.

[22]Wilson, K. A., A. J. Lee, P. R. Hoskins, F. G. Fowkers, C. V. Ruckley, and A. W. Bradbury. The relationship between aortic wall distensibility and rupture of infrarenal abdominal aortic aneurysm. *J. Vasc. Surg.* 37:112–117, 2003.

[23]Zhao, S. Z., X. Y. Xu, and M. W. Collins. The numerical analysis of fluid–solid interactions for blood flow in arterial structures. Part 2. Development of coupled fluid–solid algorithms. *Proc. Inst. Mech. Eng. Part H, J. Eng. Med.* 212:241–252, 1998.