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RESEARCH ARTICLE

Uncertainty in firm valuation and a cross-sectional misvaluation measure

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Abstract
The degree of uncertainty associated with the value of a company plays a relevant role in valuation analysis. We propose an original and robust methodology for company market valuation, which replaces the traditional point estimate of the conventional Discounted Cash Flow model with a probability distribution of fair values that convey information about both the expected value of the company and its intrinsic uncertainty. Our methodology depends on two main ingredients: an econometric model for company revenues and a set of firm-specific balance sheet relations that are estimated using historical data. We explore the effectiveness and scope of our methodology through a series of statistical exercises on publicly traded U.S. companies. At the firm level, we show that the fair value distribution derived with our methodology constitutes a reliable predictor of the company's future abnormal returns. At the market level, we show that a long-short valuation (LSV) factor, built using buy-sell recommendations based on the fair value distribution, contains information not accessible through the traditional market factors. The LSV factor significantly increases the explanatory and the predictive power of factor models estimated on portfolios and individual stock returns.

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1 Introduction

Among the several models proposed and explored by the large literature on firm valuation, the discounted cash flow model (DCF) is probably the most fundamental direct valuation method, widely used by sell-side financial analysts and practitioners (Brown et al. 2015, see e.g.) In DCF valuation, one starts by determining the stream of future cash flows of a company and then computes their present value through an appropriately defined discount rate. The discount rate is meant to capture two different effects: the time value of money and the uncertainty of future cash flows. In fact, primarily due to the intrinsic difficulty of estimating the future cash flows of a company, the value provided by DCF is likely to be affected by a considerable amount of uncertainty. For instance, in Viebig et al. (2008), the authors acknowledge that (emphasis is our) "Being intellectually honest, financial analysts can at best determine ranges or *distribution* of possible fundamental financial values but not exact price targets for stocks, as future revenue growth rates, future operating margins, and other inputs which go into DCF models cannot be predicted with certainty." Starting from similar considerations, existing work has highlighted the need to develop probabilistic and statistical tools to extend the conventional DCF method to include some measure of uncertainty associated with the estimated value (Casey 2001). To the best of our knowledge, despite its practical relevance, this problem has been the subject of surprisingly few academic studies. The general suggestion has been to perform Monte Carlo simulations of the underlying (accounting) variables starting from historically estimated correlation matrices (French and Gabrielli 2005; Damodaran 2007). This approach is similar to Monte Carlo procedures commonly used by analysts in investment banking studies (see Koller et al. 2010). For instance, in Ali et al. (2010), Gimpelevich (2011), and Samis and Davis (2014) both scenario-based and simulation-based analyses are used, together with the DCF, for investment decisions in real-estate projects or for the evaluation of a specific market sector.

In this paper, we propose a new, general, and theoretically grounded valuation method, the Stochastic Discounted Cash Flow (SDCF), that replaces the traditional point estimate of the conventional DCF method with a proper random variable. The basic idea of the SDCF is to consider a suitably defined probability space that can describe a company's future cash flow dynamics. Should the true cash flow process be known, the value computed by the standard DCF would be precisely the expectation of the SDCF random variable. The reliability of the method depends on the goodness of the data generating process that describes the flow of cash flows. We rely on two empirical observations to obtain a satisfactory prediction of future cash flows. The first observation is that the dynamics of revenues, which are the basic source of the company's cash flow, is characterised by the presence of a substantially volatile idiosyncratic component. The second observation is that even if, from an accounting point of view, the cash flow is a reconstructed variable that depends on a set of other, more fundamental variables (e.g. amortisation, cost of debts, and taxes), all interacting and affecting the final realised cash flow in different degrees, the structural relationship among these variables results stable in time. The main methodological novelty of our approach is merging these two observations into a three-step procedure to derive a prediction model for future cash flows. First, a set of econometric models are estimated at the firm level, their efficiency is individually and independently compared for each firm, and the best model of each firm is used in a Monte Carlo procedure to obtain the distribution of future revenues. Second, all other accounting variables that enter into the final definition of the company's cash flow are estimated as "margins" on the revenues by using historical data. Finally, the obtained data generating process is used in a controlled Monte Carlo simulation to derive a probability distribution for the company's fair value. The details of the model and its estimation are discussed in Sect. 2.

The fair value distribution can be used to obtain both an estimate of the expected fair value of the company and its degree of uncertainty. To explore the information content of the fair value distribution, we build a volatility-adjusted mispricing indicator, defined as the difference between the market price of the company and its expected fair value divided by the standard deviation of the fair value distribution. Under the assumption that the company's future market prices will eventually adjust to re-absorb the company's present misvaluation, in Sect. 3 we run a series of empirical exercises to investigate the relation between our mispricing indicator and market returns. We start with a firm-level investigation. We find that the mispricing indicator has significant predictive power for one-quarter ahead excess returns when used to augment the linear factor model commonly used in financial applications (Fama and French 1993, 2015, see, e.g., the Fama-French three-factor model,) and other control variables. To further assess the reliability of our mispricing indicator, we sort stocks into (appropriately defined) quantiles based on the empirical distribution function of the individual firm indicator, and we construct Buy, Hold and Sell portfolios according to this quantile splitting. By comparing the equally weighted daily returns of these portfolios, we observe that the Buy portfolio earns a gross return that is consistently and significantly higher than that of the Sell portfolio.

Motivated by the evidence at the firm level, in Sect. 4 we explore whether and to what extent our mispricing indicator has some predictive power when augmenting traditional market factor models. We form a long-short valuation factor (*LSV*) by measuring the returns of a factor-mimicking portfolio that goes long on the most recommended (undervalued) stocks and short in the less recommended (overvalued) stocks. Our exercise is similar to that performed by Hirshleifer and Jiang (2010) for the *UMO* factor and by Chang et al. (2013) for the *MSV* factor. The *LSV* factor, when added to the Fama French five factor model (Fama and French 2015) augmented by the momentum factor introduced in Carhart (1997), as well as by the *UMO* factor of Hirshleifer and Jiang (2010)[1], is not redundant in describing average returns, both in the cross-section of portfolio and individual stock returns. This fact confirms the ability of our indicator to capture a previously unexplained contribution to the company's

[1] At the moment of writing we were not able to construct or find the *MSV* factor.

mispricing. Sect. 5 collects some final remarks and suggestions for possible model extensions.

2 The valuation model and its estimation

Following the Unlevered Free Cash Flow (UFCF) approach[2] and considering all random quantities defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathcal{P})$, we define the enterprise value of a company as the following random variable:

$$V_0(\omega) = \sum_{t=1}^{T} \frac{CF_t(\omega)}{(1+k)^t} + \frac{CF_T(\omega)(1+g)}{(1+k_{TV})^T(k_{TV}-g)}, \quad \forall \omega \in \Omega, \tag{1}$$

where k is the constant short-term discount factor, k_{TV} is the long-term (terminal value) discount factor, CF_t is the stochastic cash flow at date t in the future, and we have assumed that there exist a $T > 0$ and a constant rate g, with $0 < g < k_{TV}$, such that $CF_{t+1} = (1+g)CF_t$, $\forall\, t \geq T$. Taking the expected value with respect to the proper measure P, one recovers the traditional point estimate of the company's present value $v_0 = \mathbb{E}_P[V_0]$. The fair value of the equity $V_0^{Eq}(\omega)$ is obtained by subtracting the current value of the debt from $V_0(\omega)$.

The cash flow CF_t in (1) is the sum of the operating cash flow $\overline{CF}_t$, which includes depreciation and amortisation, and the variation of working capital, WC_t. We assume that both quantities can be expressed as margins with respect to contemporaneous revenues, $\overline{CF}_t = \alpha\, REV_t$, and $WC_t = \beta\, REV_t$, so that

$$CF_t = (\alpha - \beta)REV_t + \beta REV_{t-1}. \tag{2}$$

The distribution of future cash flows necessary to compute (1) can now be obtained from a revenue forecasting model.

2.1 The estimation of margins and discount factors

Our analysis covers the period from January 2009 to December 2017. We estimate the margins in (2) reconstructing the operating cash flow and working capital from the Eikon Datastream database made available by Thomson Reuters. The margin α of operating cash flows over revenues is computed for each company in each quarter, estimating

$$\begin{cases} \overline{CF}_{i,t} & = \alpha\, REV_{i,t} + u_t \\ u_t & = \varepsilon_t - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}, \quad \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2), \end{cases} \tag{3}$$

[2] There are two alternative definitions of cash flow that are typically used in DCF models: the Unlevered Free Cash Flow (UFCF) (or Free Cash Flow to Firm) and the Levered Free Cash Flow (LCFC) (or Free Cash Flow to Equity). They differ mainly in their treatment of corporate expenses.

over the previous four quarters (the initial period is FQ4 1992-FQ1 2009), setting

$$\overline{CF}_t = (EBITDA_t - D\&A_t)(1 - \tau_0) + D\&A_t - CAPEX_t, \tag{4}$$

where $EBITDA$ stands for earnings before interest, taxes, depreciation, and amortisation, $D\&A$ for depreciation and amortisation, τ_0 for the marginal tax rate, and $CAPEX$ for capital expenditures. The number of lags q is decided using the Akaike Information Criterion. The model is estimated assuming independent and normally distributed residuals. The Kolmogorov–Smirnov test fails to reject the assumption of normality for estimated residuals in about the 56% of the firms in our universe at the 0.01 level, and there is no evidence of serial correlations for about 72% of the firms using the Ljung-Box statistic. Given the simplicity of the model, we consider these performances acceptable. Instead, the quantity β is estimated each year for each company, averaging the historical ratio of working capital over revenues in the tree previous year (the initial period 2006–2009).

Our initial sample comprises all non-financial companies included in the S& P 500 for the entire period. We reject firms with insufficient observations or missing data (mainly $EBITDA$ and $CAPEX$) and financial companies, as they are subject to industry-specific regulations that make the reconstruction of past free cash flow from revenues extremely complicated, if not meaningless. We remain with a sample of 182 firms. We discard another 32 firms for which the coefficient of determination (R^2) of (3) is less than 10%, remaining with 150 firms for which the R^2 of the above regression is, on average, higher than 0.9.[3]

Our cash flows streams are random variables and we discount them with a conventional DCF model rate (see e.g. Ali et al. 2010; Razgaitis 2009; French and Gabrielli 2005; Dayananda et al. 2002). [4] Specifically, for the short-term discount rate, we follow the Weighted Average Cost of Capital (WACC) approach and set

$$k = k_e w_e + k_d w_d + k_p w_p, \quad w_e + w_d + w_p = 1, \tag{5}$$

where k_e is the cost of equity, k_d the after-tax cost of debt, k_p the cost of preferred stocks, and w_e, w_d and w_p are the related weights. These values are provided directly by Eikon, and Datastream every quarter. The long-term discount rate k_{TV} is computed by considering the fixed corporate tax rate instead of the individual tax rate, although the difference is minimal for all companies and all years considered. The perpetual growth rate g is equal to the 5-year T bond rate obtained from the FRED database (Federal Reserve Bank in St. Louis).

2.2 The revenues model

The revenue dynamic of each company is estimated by comparing three alternative econometric models. Let $y_t = \log(REV_t)$. Model 1 is the stationary model defined

[3] The survival bias potentially induced by our selection criteria is ineffectual as we are only interested in the relative performance of firms in the selected sample.

[4] If we were interested in a point estimate of the present value, as in Casey (2001), the riskless rate would have been more appropriate.

by

$$\left(1-\sum_{k=1}^{p} a_k L^k\right)(1-L)y_t = \varepsilon_t, \quad \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2),$$

where L is the usual lag operator, that is, $Ly_t = y_{t-1}$. Model 2 is the local-level model defined by

$$\begin{aligned} &\text{(Observation equation)} \quad y_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2), \\ &\text{(Local level)} \quad \mu_{t+1} = \mu_t + \eta_t, \quad \eta_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\eta^2). \end{aligned}$$

Model 3 is a local linear trend model defined by

$$\begin{aligned} &\text{(Observation equation)} \quad y_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2), \\ &\text{(Local trend)} \quad \mu_{t+1} = \mu_t + \nu_t + \eta_t, \quad \eta_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\eta^2), \\ &\text{(Time-varying slope)} \quad \nu_{t+1} = \nu_t + \zeta_t, \quad \zeta_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\zeta^2). \end{aligned}$$

Model 1 assumes that an AR(p) model describes the first difference in logarithmic revenues. The lag p is decided according to the AIC. Model 2 and Model 3 are estimated in their state-space form, using the Kalman filter, to obtain both an estimate of the parameters and of the time series of the latent state variables (see, e.g., Harvey 1990 and Durbin and Koopman 2012 for further details). Models are estimated using trailing twelve-month data so that we can safely neglect seasonal characteristics in our estimates. The model selection procedure is made up of two steps. First, we check if the log-revenue time series is stationary. If it is the case, we select Model 1. If it is not the case, we estimate both Model 2 and Model 3. Then, since Model 2 is nested in Model 3, we use the likelihood ratio test to select the best between the two. "Appendix A" reports an analysis of the goodness of fit of the three models together with a performance comparison against a simple AR(1) model.

The econometric models described in this section can be replaced by direct bootstrapping of historical revenues or revenue growth rates. A selection of the analysis in Sects. 3 and 4 is replicated using these models in "Appendix B". The resulting fair pricing distributions generally have greater support. As a consequence, the results are less clear-cut, albeit qualitatively similar.

2.3 The fair value distribution

Once a revenue model has been estimated and selected, future revenues are generated by Monte Carlo sampling from the model. Using estimated margins and discount factors, future revenues generate a distribution for the value of the company V_0 in (1). Finally, the distribution of the equity values V_0^{Eq} is obtained from the distribution of

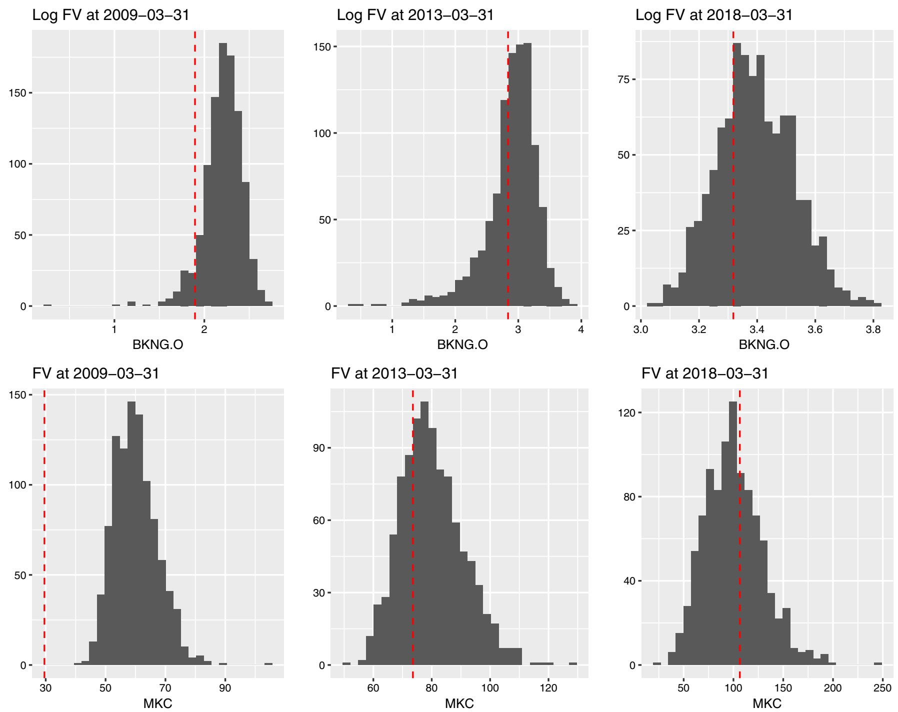

Fig. 1 The distribution of the logarithm of the fair value for Booking Holdings Inc. (ticker BKNG) and of the fair value for McCormick & Company (ticker MKC) computed at different dates. Dotted lines indicate the market (log) price at the evaluation date

the company values by setting

$$V_0^{Eq}(\omega) = V_0(\omega) - (TD - CsI + MI + PS)\,,$$

where TD stands for total debt, used as a proxy for the market value of debt (consistent with the assumption of the data provider and Damodaran 2007), CsI for cash and short-term investments, MI for minority interest and PS for preferred stocks. The fair values of the equity are divided by the number of outstanding shares of the company to obtain the fair value distribution, which can now be compared with the corresponding stock price. Figure 1 shows two examples of the logarithm of the fair value distribution for Booking Holdings Inc. (ticker BKNG) and McCormick & Company (ticker MKC) computed on different dates. Dotted lines indicate the market price at the evaluation date.

Finally, we drop from the sample ten firms for which we observe a negative estimated fair value distribution in some quarters immediately after the financial crisis of 2008-2009. Thus, the database that we use in the following analysis is made up of $N = 140$ firms. According to the Industry Classification Benchmark (ICB) taxonomy, we have 17 firms in both the Oil & Gas (ICB 1) and the Basic Material (ICB 1000) sector, 44 Industrial firms (ICB 2000), 22 Consumer Good firms (ICB 3000),

Table 1 Percentage of the stocks of our universe relative to the S&P500 for each ICB sector

ICB sector	(%)	ICB sector	(%)
1 and 1000	36.17	5000	16.90
2000	49.44	6000	100.00
3000	37.93	7000	25.00
4000	36.54	9000	30.19

Table 2 Proportion of stocks in each ICB sector relative to both our universe (first and second column) and the S&P500 (third and fourth column)

Our Universe				S&P 500			
ICB sector	(%)	ICB sector	(%)	ICB sector	(%)	ICB sector	(%)
1 and 1000	12.14	5000	8.57	1 and 1000	11.72	5000	17.71
2000	31.43	6000	2.14	2000	22.19	6000	0.75
3000	15.72	7000	5.00	3000	14.46	7000	6.98
4000	13.57	9000	11.43	4000	12.97	9000	13.22

19 Healthcare firms (ICB 4000), 12 firms in the Consumer Service sector (ICB 5000), three firms in the Telecommunication sector (ICB 6000), 7 Utilities firms (ICB 7000) and 16 Technology Firms (ICB 9000). Table 1 reports, for each ICB sector, the percentage of stocks in our universe relative to the number of firms in the same sector of the S&P 500 index. To check for possible sample distortion introduced by our selection criteria, Table 2 displays the percentage of stocks in each ICB sector relative to both our universe (first two columns) and the S&P 500 (second two columns). Together, Tables 1 and 2 show that the final sample exhibits substantial heterogeneity in terms of industrial sectors and reflects the composition of the index.

3 Mispricing indicator

Let p_t^i be the closing log price of stock i on the day t and μ_t^i and σ_t^i the empirical mean and standard deviation of the log-fair value distribution of the same company at the same date, obtained from the bootstrapping procedure based on our SDCF method. As a mispricing indicator of the company i at time t, we take

$$z_t^i = \frac{p_t^i - \mu_t^i}{\sigma_t^i}, \tag{6}$$

that is, the log difference between the company's expected fair value and its price, divided by the standard deviation of the log-fair value distribution. In our indicator, the absolute level of mispricing, $|p_t^i - \mu_t^i|$, is amplified when the valuation procedure is less uncertain.

We expect an appropriate mispricing indicator to be related with future market adjustments, as prices of the undervalued companies grow more than those of the overvalued ones. With this hypothesis, we test the predictive power of our indicator with respect to future expected price returns.

3.1 Cross-section analysis

First, we assess whether the individual mispricing indicator z_t^i possesses significant predictive power for the excess return one quarter ahead when used to augment factor models.

To this end, we regress stocks excess returns on the z-scores and a set of control variables in a panel fixed-effect model. For each month t, let $R_{i,t}^{\mathrm{EX}}$ be the monthly excess return of the firm i over the risk-free rate $R_{F,t}$. We consider the following model:

$$\begin{aligned} R_{i,t}^{\mathrm{EX}} = \alpha_i &+ \beta_i^M \,(R_{M,t} - R_{F,t}) + \beta_i^{\mathrm{SMB}}\, SMB_t + \beta_i^{\mathrm{HML}}\, HML_t \\ &+ \gamma_1\, z_{t-3}^i + \gamma_2\, R_{t-1}^i + \gamma_3\, R_{t-12,t-2}^i + \gamma_4\, \log(ME_t^i) \\ &+ \gamma_5\, \log(BM_t^i) + \gamma_6\, ACC_t^i + \gamma_7\, AG_t^i + \gamma_8\, DE_t^i + e_{i,t}\,, \quad i \in \{1, \ldots, N\}\,, \end{aligned} \tag{7}$$

where $R_{M,t} - R_{F,t}$, SMB_t and HML_t are respectively the market factor, the size factor, and the book-to-market factor of the Fama-French three-factor model; z_{t-3}^i is the z-score of the firm i computed averaging the daily z-scores in the previous quarter; R_{t-1}, $R_{t-12,t-2}$ are the last month return and the return from month $t-12$ to $t-2$; ME is the market equity; BM is the book-to-market ratio; ACC are the operating accruals; AG is the asset growth; DE is the leverage ratio and $e_{i,t}$ an idiosyncratic error term. The results are reported in Table 3 for seven different models with an increasing number of controls. The estimated γ's, i.e., the common effect of the mispricing score, are statistically significant and with a negative sign, regardless of the number and type of control variables considered. In other terms, undervalued (resp. overvalued) stocks are, on average, consistently characterised by higher (resp. lower) future excess returns. This observation confirms the idea that our z-score represents a measure of mispricing, which is reabsorbed by the market over time, while the price gradually converges to the company's fundamental value. Notice that each explanatory variable in (7) is cross-sectionally normalised to have mean 0 and standard deviation 1. With some precautions due to possible cross-correlation effects which might be neglected, this allows for a direct comparison of the regression coefficients. The picture that emerges from Table 3 is that, among all the considered regressors, the three effects that seem to be more persistent are those of the z score, the last one-month return and the book-to-market ratio.

3.2 Portfolio analysis

To further validate the ability of the z-score indicator to anticipate future market performance, we sort stocks into quantiles based on the z-score empirical distribution function at the beginning of each semester and then construct Buy, Hold and Sell port-

Table 3 Results for monthly fixed-effect time series regressions in (7)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
z-score	− 0.280***	− 0.331***	− 0.500***	− 0.485***	− 0.503***	− 0.475***	− 0.473***
	[− 3.318]	[− 3.892]	[− 6.134]	[− 5.979]	[− 6.159]	[− 5.836]	[− 5.797]
R_{t-1}		− 0.460***	− 0.500***	− 0.524***	− 0.500***	− 0.519***	− 0.518***
		[− 3.539]	[− 3.849]	[− 4.006]	[− 3.849]	[− 3.915]	[− 3.910]
$R_{t-12,t-2}$		− 0.184	− 0.200	− 0.199	− 0.200	− 0.212	− 0.207
		[− 1.368]	[− 1.486]	[− 1.470]	[− 1.488]	[− 1.554]	[− 1.518]
$\log(ME)$			− 0.693*	− 0.730*	− 0.670*	− 0.764**	− 0.784**
			[− 2.523]	[− 2.544]	[− 2.438]	[− 2.854]	[− 2.777]
$\log(BM)$			1.174***	1.157***	1.163***	1.198***	1.207***
			[8.853]	[8.802]	[8.743]	[8.232]	[8.269]
ACC				0.176			0.203
				[1.248]			[1.444]
AG					− 0.032		− 0.027
					[− 0.592]		[− 0.506]
DE						− 0.126**	− 0.129**
						[− 3.028]	[− 3.112]
Adj. $R^2(\%)$	34.157	34.586	34.897	35.177	34.894	35.464	35.350
No. obs.	15540	15540	15429	15207	15429	14652	14541

Coefficients significant at 5%, 1% and 0.1% level are marked with ‘$*$’, ‘$**$’ and ‘$***$’ respectively. T-ratio based on HAC robust standard errors is reported in parentheses. The Eikon database provides all control variables. The analysis is from April 2009 to June 2018. The regressors are rescaled to have mean 0 and standard deviation 1 in cross-section

Table 4 Descriptive statistics of portfolios based on z-score ranking

	Sell	Hold	Buy	Our Universe
Avg. Number of Firms	56	28	56	140
Avg. Market Cap. (%)	34	16	50	100
Avg. Annual Return(%)	17.63	18.07	20.83	19.04
Sharpe Ratio	1.05	1.15	1.43	1.24
Sortino Ratio	1.36	1.48	1.87	1.59

For each portfolio, the average number of firms, the average percentage of market capitalization with respect to our universe, the average annual log return, and the annualized Sharpe and Sortino ratios are reported

folios according to this quantile-based splitting. Specifically, let $\rho(\alpha)$ be the quantile function at level α of the empirical distribution of the z-scores z^i, $i \in \{1, \dots, N\}$. If $z^i < \rho(0.4)$ firm i is assigned to the Buy portfolio, if $z^i \geq \rho(0.6)$ firm i is assigned to the Sell portfolio and if $\rho(0.4) < z^i < \rho(0.6)$ it is assigned to the Hold portfolio. The Buy and Sell portfolios contain the same number of firms, while the Hold portfolio contains half that number. For each portfolio, we compute the equally weighted daily return and compare its performance with the Our universe portfolio, defined as the equally weighted portfolio of all stocks in our universe. The results are reported in Table 4. The Sharpe (1994) and Sortino and Price (1994) ratios associated with the portfolio Buy are 1.43 and 1.87 respectively, which are higher than those of the Sell, Hold and Our universe portfolios. The same conclusions hold for the average annual return. Using the test discussed in Ledoit and Wolf (2008) and Ardia and Boudt (2018), we found a significant difference between the Sharpe ratios of the Buy and the Our universe portfolios, with a t-$Statistic$ of 2.98 and a p-$value$ of $3 \cdot 10^{-3}$. This cross-sectional investigation confirms the explanatory power of our mispricing measure, as portfolios built using undervalued firms perform better than portfolios made of overvalued firms or the portfolio containing all firms of our reference universe.

In summary, the statistical analysis performed in this section reveals that our mispricing indicator might be able to explain a significant portion of the company's future excess returns. A relevant question that remains to be addressed is how much of this predictive power is retained when our indicator is confronted with other possible sources of excess returns, as identified in the literature. We need to understand if, and to what extent, the information revealed by our indicator represents a genuinely new contribution to the analysis of market dynamics that is not already contained in other variables the literature proposes as possible explanatory factors of stocks performances. This investigation is the focus of the next section.

4 The valuation factor

To assess the predictive power of our misvaluation indicator with respect to future stock performance, we revert to factor model analysis. We consider a misvaluation factor LSV (Long Short Valuation) whose value at each day t is given by the difference between the equally-weighted return of a portfolio that goes long on the undervalued stocks and short on the overvalued ones. Therefore, the LSV factor is computed as the difference between the Buy and Sell portfolios discussed in Sect. 3.2. In the period from April 1, 2009 to September 28, 2018, this factor earns a slightly significant positive average return of 2.7% (t-$Statistic = 1.49$ and p-$value = 0.14$) and has an annual Sharpe ratio of 0.48.

In the next section, we compare LSV with other commonly considered factors affecting stock return, namely the market factor, defined as the difference between the market return R_M and the risk-free interest rate R_F, the size factor (SMB), the book-to-market factor (HML), the momentum factor (MOM), the profitability factor (robust minus weak) (RMW) and the investment factor (CMA). See Fama and French (2015) and Carhart (1997) for a discussion of how these factors are built.[5] In addition, we will investigate the relationship of LSV with the UMO factor, recently proposed in Hirshleifer and Jiang (2010) as a possible way to capture the presence of persistent long-term company misvaluation.

In Sect. 4.2 we will use LSV to augment standard factor models and explore its relative merits using the Fama-MacBeth regression framework (Fama and French 1992; Fama and MacBeth 1973).

4.1 Comparing LSV with other market factors

Table 5 reports the Pearson's correlation coefficient between LSV and the other factors considered during the sample period computed using daily returns. Our factor seems to share some information content with the $R_M - R_F$, SMB, MOM, and RMW factors. Conversely, its correlation with the UMO factor does not appear significant throughout the entire period of analysis. A visual inspection of the scatter plots, reported in Fig. 2, confirms the apparent lack of correlation between LSV and UMO and a strong correlation of LSV with both $R_M - R_F$ and SMB. At this stage, the correlation with MOM seems instead due to few extreme observations (like the one with RMW, which is not reported for brevity).

The orthogonality of the UMO and LSV factors, emerging from Table 5 and confirmed in Fig. 2, seems peculiar due to their shared claim of capturing the presence of market misvaluation. To understand this finding, it is helpful to look at the time profile of the two factors. In Fig. 3, we plot the absolute value of the daily logarithmic return of the UMO and LSV factor rescaled by their mean and standard deviation. This absolute variation can be interpreted as a measure of the contribution og the factor to explaining the market dynamics (Chang et al. 2013). As can be seen, the UMO factor identifies a high value of market misvaluation in the period between 2015 and 2016 while, according to the LSV factor, the period characterised by the most misvalued market prices is between 2010 and 2011 . The inspection of the time profile reveals that the two misvaluation factors are, in some sense, complementary and they seem to capture different phenomena. In fact, their different behaviour in the period covered by our analysis can be traced back to their definitions. The UMO factor is market-oriented and is based on stocks classification that looks at market operations (equity and

[5] Data are taken from the Kenneth R. French Data Library, https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table 5 Pearson correlation of market factors

	LSV	$R_M - R_F$	SMB	HML	MOM	RMW	CMA	UMO
LSV	–	− 0.405	− 0.338	− 0.035	− 0.226	0.229	0.042	0.094
$R_M - R_F$		–	0.386	0.230	− 0.120	− 0.461	− 0.038	− 0.263
SMB			–	0.103	− 0.099	− 0.358	0.025	− 0.320
HML				–	− 0.421	− 0.298	0.477	0.149
MOM					–	0.092	− 0.069	0.039
RMW						–	− 0.050	0.461
CMA							–	0.288
UMO								–

$R_M - R_F$ is the market factor, SMB the size factor, HML the book-to-market factor, MOM the momentum factor, RMW the profitability factor, CMA the investment factor and UMO the financing-based misvaluation factor of Hirshleifer and Jiang (2010). The sample size is 2392, reduced to 1953 for the UMO factor

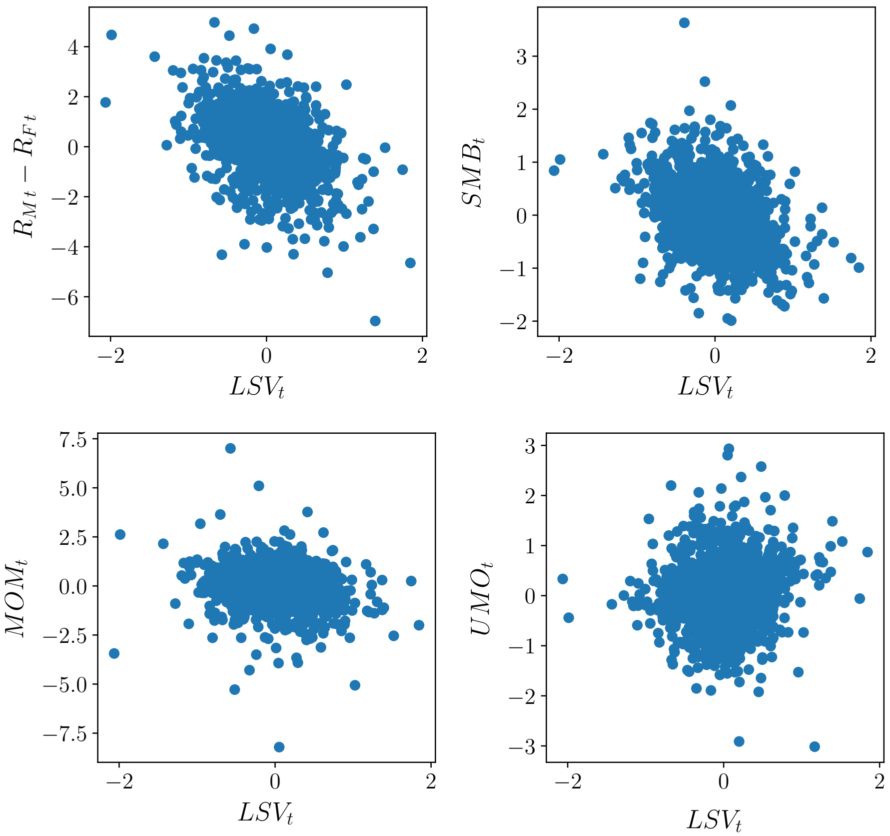

Fig. 2 Scatter plot between LSV_t and a selection of other factors

debt offerings and buy-backs) in the previous two years. This explains its correlation with the HML factor, whose value is derived by looking at the book-to-market ratio.[6] Conversely, the LSV factor, based on the misvaluation indicators built from individual balance sheet data and revenues forecast, is more orientated toward the company's operating performance. On the eve of the 2008 financial crisis, the scope of market operations and consequently the variability of the UMO factor, was dramatically reduced. The subsequent liquidity crisis induced a significant misvaluation in several sectors, which led to the increase in turbulence of the LSV factor observed in the years 2009–2011.

Interestingly, even though revenues play an important role in the definition of both the LSV and HML factors, their correlation is weak. This suggests that expenses and investments play an important role in the construction of our mispricing indicator. However, the time profile of the HML factor, reported in the bottom panel of Fig. 3, is similar to that of the LSV factor.

[6] In Table 2 of Hirshleifer and Jiang (2010), the authors report a positive correlation of 0.65 between these two factors.

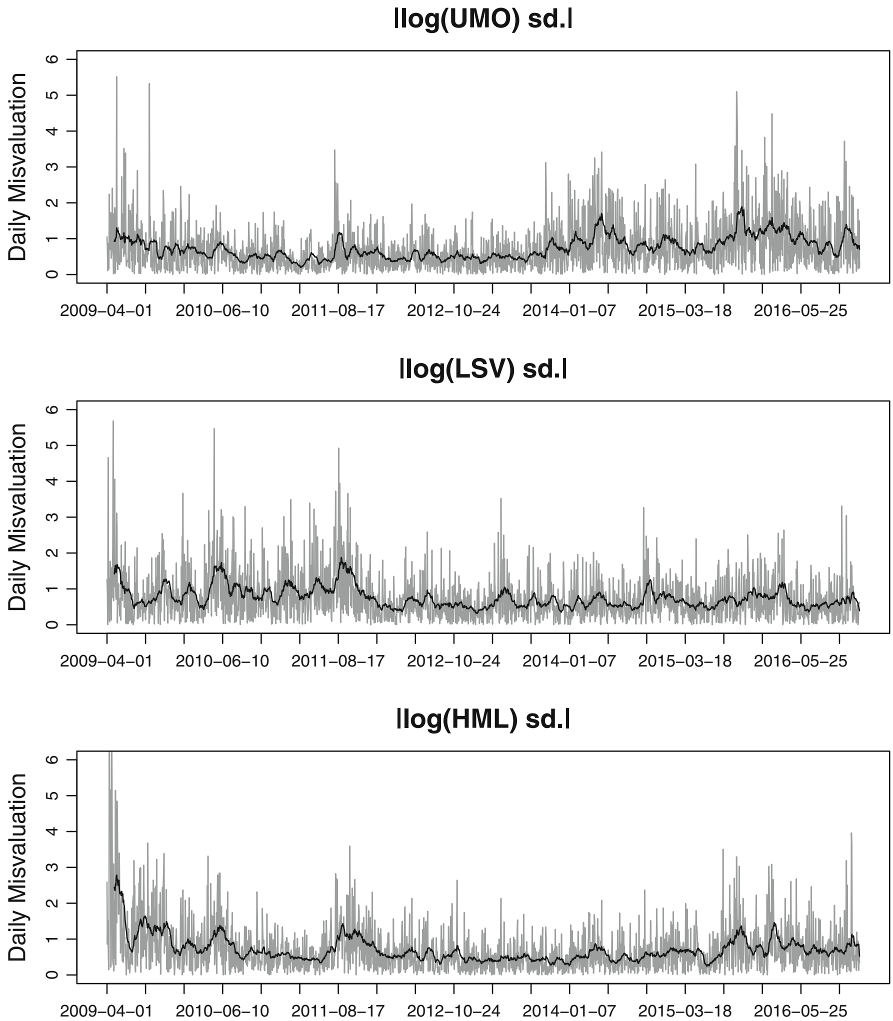

Fig. 3 Absolute value of the daily logarithmic return of the UMO, LSV, and HML factors rescaled by their mean and standard deviation. The dark solid lines are the moving averages over the past 20 days

Next, we move to a multivariate analysis, regressing LSV over the other factors. Consider the general model

$$\begin{aligned} LSV_t = \beta_0 + \beta^{\mathrm{M}}(R_{M,t} - R_{F,t}) + \beta^{\mathrm{SMB}} SMB_t + \beta^{\mathrm{HML}} HML_t + \beta^{\mathrm{MOM}} MOM_t \\ + \beta^{\mathrm{RMW}} RMW_t + \beta^{\mathrm{CMA}} CMA_t + \beta^{UMO} UMO_t + e_t, \end{aligned} \tag{8}$$

where e_t is a zero-mean residual. The regression results, in various model configurations, are shown in Table 6.

Both the intercept value, significantly different from zero at any conventional level in any setting, and the relatively small adjusted R^2, which is only between 0.20 and 0.30

Table 6 Summary of the daily regressions of LSV on $R_M - R_F$, SMB, HML, MOM, RMW, CMA and UMO factors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	0.021**	0.020**	0.021**	0.021**	0.020**	0.025***	0.022**
	[3.160]	[3.269]	[3.115]	[3.168]	[3.242]	[3.303]	[3.240]
$R_M - R_F$	− 0.125***	− 0.122***	− 0.122***	− 0.126***	− 0.117***	− 0.132***	− 0.120***
	[− 8.286]	[− 10.043]	[− 7.843]	[− 8.087]	[− 9.316]	[− 9.040]	[− 9.082]
SMB	− 0.141***	− 0.155***	− 0.137***	− 0.141***	− 0.156***	− 0.178***	− 0.187***
	[− 6.716]	[− 8.927]	[− 6.572]	[− 6.795]	[− 8.941]	[− 7.912]	[− 9.604]
HML	0.059*	− 0.025	0.062*	0.063°	− 0.042	0.075**	− 0.020
	[2.376]	[− 1.213]	[2.368]	[1.955]	[− 1.636]	[2.704]	[− 0.602]
MOM		− 0.135***			− 0.138***		− 0.134***
		[− 7.935]			[− 8.045]		[− 7.380]
RMW			0.026		0.005		0.070°
			[0.933]		[0.214]		[1.934]
CMA				− 0.015	0.056		0.058
				[− 0.303]	[1.462]		[1.274]
UMO						− 0.069***	− 0.072***
						[− 3.656]	[− 3.679]
Adj. R^2(%)	20.912	28.561	20.928	20.890	28.655	23.661	31.192
No. obs.	2392	2392	2392	2392	2392	1953	1953

T-ratio based on HAC robust standard errors is reported in parentheses. Coefficients significant at 10%, 5%, 1% and 0.1% level are marked with '∘', '∗', '∗∗' and '∗ ∗ ∗', respectively. The analysis is from April 1, 2009, to September 28, 2018, when the UMO factor is not included. Otherwise, it is from April 1, 2009, to December 30, 2016 in all models, suggest that the LSV factor captures yet another anomaly that is hardly explained by well-accepted risk factors. Multivariate analysis confirms the correlation of LSV with the market, SMB, and MOM factors. On the contrary, RMW, HML and CMA are orthogonal to LSV. When the interaction of different factors is taken into account, the correlation between LSV and UMO is negative and statistically significant at any conventional level. This is further evidence of the different nature of the two factors.

4.2 The LSV beta and the cross-section of portfolio (Abnormal) returns

We now turn to our primary task in this section, which is testing, through factor model analysis, how well LSV explains average abnormal returns in the cross-section of portfolios. We select the 25 Fama-French portfolios formed on size and book-to-market, and we examine the effect of the LSV factor and other market factors by computing the average premium using the Fama-MacBeth regression framework (Fama and MacBeth 1973; Fama and French 1992). As observed by Hirshleifer and Jiang (2010) for the UMO factors, we expect to obtain more stable loadings on portfolios that are formed based on possible mispricing measures. A positive relation between abnormal returns and LSV factor loadings would suggest the existence of a systematic stock misvaluation positively captured by our indicator. In other words, a positive (negative) loading on LSV signals a systematic under- (over-)valuation (Hirshleifer and Jiang 2010; Chang et al. 2013).

To analyse both the explanatory and predictive power of the loadings of LSV, we investigate this relation in an in-sample and out-of-sample setting.[7] Table 7 exhibits in-sample Fama-MacBeth results based on monthly abnormal returns, computed using the Fama and French three-factor model, of the 25 size-BM portfolios. In addition to LSV, we consider the five traditional Fama and French, the momentum, and the UMO factors as potential confounding explanatory variables. As expected, given the nature of the portfolios considered, the SMB and HML factors are never significant. The monthly average premium of the LSV factor is always positive and significantly different from zero when all factors are considered; see Columns (3) – (6). Remarkably, this remains true also for the model in Column (7), where we consider an orthogonalised misvaluation factor, $LSV^{\perp}$, defined as the sum of the intercept and residuals extracted from the regression in (8), that is, $LSV^{\perp} = \beta_0 + e_t$. By construction, the orthogonalised misvaluation factor has zero correlation with the Fama-French, MOM and UMO factors. Note that the loadings of the UMO are concordant with the loadings of the LSV and $LSV^{\perp}$ factors. The "misvaluation" these factors are built to capture, although different in nature, is still consistent in predicting higher (lower) returns for undervalued (overvalued) stocks.

Then we move to an out-of-sample analysis using a 60 day rolling window updated every 30 days: for each portfolio at each date, the loadings on the considered factors are estimated from a time-series regression using daily excess returns over the previous

[7] We replicated the same analysis using portfolios excess returns instead of abnormal returns. The in-sample results are similar. In the out-of-sample case, we did not find sufficient statistical evidence for all factors premia. Results are available upon request.

Table 7 Fama-MacBeth monthly regressions on the cross-section of portfolio abnormal returns

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	0.330	1.132*	1.765***	1.838***	1.863***	2.228***	2.241***
	[1.510]	[2.683]	[5.035]	[5.385]	[4.862]	[6.262]	[6.270]
LSV	0.313		0.492*	0.747**		0.762**	
	[1.608]		[2.273]	[3.346]		[3.729]	
$LSV^{\perp}$							0.543*
							[2.812]
$R_M - R_F$		− 1.015*	− 1.812***	− 1.750***	− 1.897***	− 2.180***	− 2.193***
		[− 2.618]	[− 5.435]	[− 5.362]	[− 5.081]	[− 6.466]	[− 6.479]
SMB		− 0.153	− 0.062	− 0.001	− 0.033	− 0.037	− 0.036
		[− 1.331]	[− 0.704]	[− 0.014]	[− 0.333]	[− 0.441]	[− 0.424]
HML		0.136	− 0.001	− 0.026	0.056	− 0.002	− 0.001
		[1.221]	[− 0.009]	[− 0.274]	[0.543]	[− 0.018]	[− 0.006]
MOM			− 0.536		− 0.618	− 0.449	− 0.446
			[− 1.135]		[− 1.246]	[− 1.046]	[− 1.037]
RMW			0.808***		0.568**	0.611***	0.606***
			[5.427]		[3.281]	[4.091]	[4.069]
CMA			− 0.640**		− 0.345	− 0.355*	− 0.353°
			[− 3.096]		[− 1.593]	[− 1.911]	[− 1.904]
UMO				1.423***	1.104***	1.315***	1.319***
				[6.397]	[4.537]	[5.895]	[5.903]
Adj. R^2(%)	6.203	25.957	65.719	65.219	63.382	73.090	73.116

The dependent variables are the monthly abnormal returns of 25 portfolios based on size and book-to-market, which are computed using Fama and French three-factor model on excess returns of equally-weighted portfolios. The independent variables are the loadings on the factors LSV, $LSV^{\perp}$ $R_M - R_F$, SMB, HML, MOM, RMW, CMA and UMO. T-ratio based on robust standard errors to HAC are reported in parentheses. Coefficients resulting to be significant at 10%, 5%, 1% and 0.1% level are marked with '$\circ$', '$*$', '$**$' and '$***$' respectively. The analysis is from April 2009 to September 2018. When the UMO factor is considered the analysis is from April 2009 to December 2016

60 days. Then, the future abnormal returns of each portfolio are computed by regressing the equally-weighted excess returns on the Fama and French three-factor model over the following 30 days. The estimated abnormal returns and factor loadings are then used as dependent and independent variables, respectively, in the cross-sectional regressions. Table 8 reports the average premia of the out-of-sample analysis and the related statistics. The market factor of Fama and French is the only factor that possesses a strongly significant premium for all model specifications. Note that the LSV premium is always positive, and it results significant when all other factors are considered, column (6).

4.3 The LSV beta and the cross-section of individual stock returns

Generally, factor loadings on individual stocks tend to be unstable, and their comparison is challenging. To study the novel information content of our factor, and following the approach in Hirshleifer and Jiang (2010), we examine the loadings of the $LSV^{\perp}$ factor, obtained by removing from LSV all the information collinear to other market factors. We estimate the $LSV^{\perp}$ betas from daily excess returns using the following model over 100 days for each firm i of the S&P500 index:

$$\begin{aligned} R_{i,t} - R_{F,t} = \alpha_i + \beta_i^{M}\,(R_{M,t} - R_{F,t}) + \beta_i^{SMB}\,SMB_t + \beta_i^{HML}\,HML_t + \\ + \beta_i^{UMO}\,UMO_t + \beta_i^{LSV}\,LSV_t^{\perp}. \end{aligned} \quad (9)$$

We then sort firms in ten deciles of increasing β^{LSV}. For each decile, Table 9 reports the average value of β^{LSV} in the decile, the annual average return, on the succeeding 30 days, of the equally-weighted portfolio built using the firms of the decile, and the related average abnormal returns computed using $CAPM$ ($\alpha_{CAPM+UMO}$), Fama and French three-factor model ($\alpha_{FF3+UMO}$), and Carhart four-factor model($alpha_{FF3+MOM+UMO}$). Each model is augmented with the UMO factor. As a benchmark, the last row reports the corresponding performance of the equally-weighted portfolio that includes all stocks.

Even if a one-way ANOVA cannot reject the hypothesis of equality for the returns reported in the third column (F-stat=0.02), their statistical significance tends to increase with the ranking based on β^{LSV}. This can be interpreted as a signal of more persistent performances by the firms in the higher classes. This effect becomes more evident when we consider the corresponding abnormal returns. In fact, only the three higher classes earn a statistically positive abnormal return greater than the benchmark abnormal return of the equally-weighted portfolio of all stocks. For example, the annual percentage of abnormal return calculated using the Carhart four-factor model augmented with UMO is 4.493% and 1.758% for H and All classes, respectively. Moreover, we observe that abnormal returns remain stable and statistically significant among the $CAPM$, Fama and French three-factor model and Carhart four-factor model (augmented with UMO). The second column of Table 9 reports, for each decile, the average post ranking loading β^{LSV}_{post}, computed using model (9) over the firms in the

Table 8 Out of sample Fama-MacBeth daily regressions on the cross-section of portfolio abnormal returns

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	0.027**	0.083***	0.077***	0.094***	0.084***	0.079***	0.079***
	[2.968]	[4.446]	[4.440]	[4.603]	[4.217]	[3.916]	[3.916]
LSV	0.011°		0.017°	0.014		0.028*	
	[1.857]		[1.711]	[1.481]		[2.537]	
$LSV^{\perp}$							0.016
							[1.394]
$R_M - R_F$		− 0.080***	− 0.078***	− 0.091***	− 0.085***	− 0.080***	− 0.080***
		[− 4.350]	[− 4.487]	[− 4.436]	[− 4.159]	[− 3.903]	[− 3.903]
SMB		− 0.001	0.004	− 0.001	0.004	0.004	0.004
		[− 0.317]	[1.317]	[− 0.204]	[0.832]	[0.982]	[0.982]
HML		0.005	0.001	0.003	0.006	0.003	0.003
		[1.256]	[0.364]	[0.581]	[1.262]	[0.744]	[0.744]
MOM			− 0.014		− 0.020	− 0.019	− 0.019
			[− 0.779]		[− 1.059]	[− 0.920]	[− 0.920]
RMW			0.014°		0.010	0.012	0.012
			[1.938]		[1.108]	[1.190]	[1.190]
CMA			0.011		0.021*	0.024**	0.024**
			[1.437]		[2.279]	[2.667]	[2.667]
UMO				0.023*	0.024°	0.026°	0.026°
				[2.118]	[1.820]	[1.885]	[1.885]
Avg. R^2(%)	4.807	16.568	29.480	28.953	32.993	34.273	34.273

The dependent variables are the abnormal returns of 25 portfolios based on size and book-to-market, which are computed using Fama and French three-factor model on excess returns of equally-weighted portfolios. The independent variables are the loadings on the factors LSV, $LSV^{\perp}$ $R_M - R_F$, SMB, HML, MOM, RMW, CMA and UMO. The time-series averages of the cross-sectional regression coefficients are reported. The Avg. R^2 is the time series average of the adjusted R^2 across the full sample period. T-ratio based on robust standard errors to HAC is reported in parentheses. Coefficients significant at 10%, 5%, 1% and 0.1% level are marked with '∘', '∗', '∗∗' and '∗ ∗ ∗', respectively. The analysis is from April 1, 2009, to September 28, 2018. When the UMO factor is considered the analysis is from April 1, 2009 to December 30, 2016

Table 9 Average performance of equally-weighted portfolios of firms in the different deciles of a ranking based on $LSV^{\perp}$ loadings

	β^{LSV}	β^{LSV}_{post}	Ret	$\alpha_{CAPM+UMO}$	$\alpha_{FF3+UMO}$	$\alpha_{FF3+MOM+UMO}$
L	− 1.436	− 0.666	14.615°	− 0.698	− 0.643	− 0.562
			[1.798]	[− 0.253]	[− 0.247]	[− 0.216]
2	− 0.759	− 0.406	17.296*	2.166	2.284	2.310
			[2.377]	[1.158]	[1.290]	[1.299]
3	− 0.475	− 0.252	15.578*	0.743	0.845	0.869
			[2.289]	[0.507]	[0.570]	[0.585]
4	− 0.268	− 0.086	14.332*	0.296	0.223	0.211
			[2.251]	[0.207]	[0.157]	[0.148]
5	− 0.100	− 0.030	14.769*	0.938	1.022	1.027
			[2.425]	[0.692]	[0.763]	[0.769]
6	0.053	0.021	15.443**	2.316	2.331	2.315
			[2.643]	[1.625]	[1.624]	[1.618]
7	0.209	0.066	14.352*	1.456	1.436	1.401
			[2.567]	[1.078]	[1.059]	[1.039]
8	0.380	0.122	15.378**	2.876°	2.884°	2.840°
			[2.852]	[1.862]	[1.857]	[1.848]
9	0.601	0.206	15.679**	3.850*	3.861*	3.791*
			[2.940]	[2.044]	[2.043]	[2.017]
H	1.155	0.412	15.796**	4.246°	4.436°	4.493°
			[2.583]	[1.656]	[1.682]	[1.693]
H-L	2.591	1.082	1.181	6.605°	6.738°	6.715°
			[0.256]	[1.653]	[1.648]	[1.697]
All	− 0.062	− 0.061	15.327*	1.708*	1.757*	1.758*
			[2.511]	[2.104]	[2.242]	[2.215]

The loadings are computed over 100 days and firms are sorted in deciles for the next 30 days. The post-ranking β^{LSV}_{post} loadings are estimated using all firms in each decile. Abnormal returns are computed using $CAPM$ model, Fama and French three-factor model, and Carhart four-factor model, augmented with the UMO factor. The H-L row corresponds to a portfolio that is long on the higher decile (H) and short on the lower (L). The last row (All) is the performance when all deciles are merged. T-ratio based on robust standard errors to HAC is reported in parentheses. Coefficients significant at 10%, 5%, 1% and 0.1% level are marked with '$\circ$', '$*$', '$**$' and '$***$' respectively. The data sample is constituted by all S&P500 firms from April 1, 2009 to December 30, 2016

decile. The post and pre ranking loadings are strongly correlated, suggesting a high degree of persistence among the $LSV^{\perp}$ loadings over a 30 days window.[8]

In conclusion, the analyses of this Section reveal the presence of relevant information captured by the LSV factor which is complementary with respect to the information made available by other market factors. This is more evident at firm

[8] We replicate the analysis of Table 9 excluding from the S&P500 sample the 140 firms employed in the construction of the LSV factor. The results are essentially unchanged.

level than in the portfolio aggregate, even if we find a significant positive explanatory relation between LSV loadings and portfolio abnormal returns.

5 Final remarks and possible extensions

This paper proposes a novel valuation framework, the Stochastic Discount Cash Flow method (SDCF), rooted in fundamental analysis and based on an econometric forecasting model of future firm cash flow. The framework can be seen as a generalisation of the DCF model of firm valuation, in which the traditional point estimate is replaced with an estimated probability distribution of fair values. In this way, one can derive both an estimate of the fair value of a company and a measure of the degree of uncertainty associated with it. In fact, we show that a simple volatility-adjusted misvaluation indicator, derived from the estimated fair value distribution, has predictive power with respect to future returns of stocks. Furthermore, by longing undervalued stocks and shortening overvalued stocks, we are able to build a misvaluation factor, the long-short valuation LSV factor, which captures novel information not accounted for by previously explored market factors. Our new factor possesses a significant explanatory power of realised abnormal returns of both portfolios and individual stocks. The factor based on the mispricing indicator that we propose differs from other factors recently explored in the literature. Hirshleifer and Jiang (2010) introduce a misvaluation factor using special market operations (e.g., repurchase, new issue of equity and debt) the company underwent in the previous two years. In Chang et al. (2013) the misvaluation of the company is captured by the residual of a sector-wise regression of the company's past returns on a set of market factors and a few key firm-specific financial indicators. In both cases, the resulting misvaluation indicator is strictly related to firm market dynamics and emerges from the comparison of relative performances, in the long or short term, of different stocks. Conversely, our indicator is based on the comparison of firm's fundamental value, estimated starting from balance sheet data, and prevailing market prices. The most challenging step in the construction of our indicator is the identification of a reliable statistical model capable of forecasting the future cash flows of individual companies. We built it by introducing a structural model that links the dynamics of several accounting variables with that of revenues. This step proved to be essential. Forecasting cash flow by bootstrapping from historical data, an approach suggested by several authors in the literature, tends to produce fair value distributions with extremely ample supports that spoil the subsequent analysis. We model the revenues dynamics of individual firms using a robust econometric procedure characterised by model selection based on relative likelihood. This step can be simplified by performing a bootstrap on historical revenues or revenue growth rates. We tested these alternatives and they proved to be more reliable than directly bootstrapping cash flows. However, they do not achieve the same clear-cut results obtained with the econometric models.

The present study can be extended in several directions. An obvious step is to exploit the information provided by the misvaluation indicator to derive portfolio recommendations for individual stocks and build specific, profit-seeking investment strategies. Another relatively straightforward application of the SDCF methodology is comparing the fair value distribution obtained with the fair value implied by the price distribution of call/put options on company stock. This comparison could shed light on the process by which the temporary misvaluation captured by our indicator is progressively eliminated by market price adjustments. In the forecasting of future cash flows, the univariate model for revenues that we have adopted could be replaced by multivariate time-series models, possibly exploiting the residual cross-sectional information available when explicitly considering the temporal dynamics of different balance sheet variables.

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Appendix A: Revenues model checks

As a goodness-of-fit measure for Model 1 we consider the usual coefficient of determination R^2. For Models 2 and 3 we consider an adjusted version R^2_{adj} that takes into account the estimates of the state variable. Let $\bar{y}$ be the sample average of the logarithmic revenues computed over a time period of length T, $\hat{\mu}_t$ the Kalman filter prediction of the state variable μ at time t, and K the number of explanatory variables. Define $SS_{res} = \sum_{t=1}^{T}(y_t - \hat{\mu}_t^2)$ and $SS_{tot} = \sum_{t=1}^{T}(y_t - \bar{y})^2$. Then $R^2_{adj} = 1 - (1 - R^2)(T - 1)/(T - K - 1)$.

Regardless of the selected model, we find that R^2 and R^2_{adj} are, on average, greater than 0.90 throughout the period considered and throughout the sample. For 71% of the companies, Model 1 turns out to be the selected model. Of the remaining, only 1.5% of the firms have a logarithmic revenue process that is well described by Model 2. To see why adopting different models is necessary, look at the revenue trajectories reported in Fig. 4. The firm in the left panel displays a roughly stationary log growth rate in the period, which is well captured by Model 1. The firm in the right panel, instead, displays a varying log growth rate, which is well described by the state variable approach in Model 2 or 3.

We also run a normality and independence test on the one-step-ahead forecast errors of Models 2 and 3. In 74% of the firms, the Kolmogorov–Smirnov test cannot reject

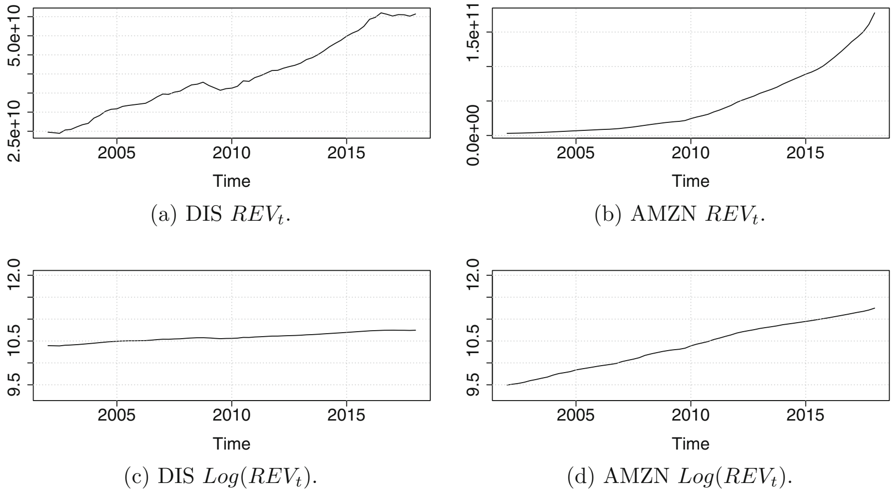

Fig. 4 Two examples of observed revenue dynamics. The dynamics on the left panels (DIS, ticker DIS) is best described by Model 1. The dynamics on the right panels (Amazon, ticker AMZN) is best described by Model 3

the assumption of normality for the distribution of errors. Ljung-Box tests with lag 1 and lag 10 do not reject the hypothesis of a lack of autocorrelation for 76% and 78% of companies, respectively, throughout the period and throughout the sample.

To gauge their out-of-sample predictive power, we compare the performance of the three alternative models against an autoregressive model of order one, AR(1). We consider the mean square error over a rolling window of 20 quarters of the predictions obtained with our models MSE_j, $j = 1, 2, 3$, and with an AR(1) model, MSE'. The cross-sectional average of $\log MSE_j/MSE'$ is always negative and is significantly different from zero at the 1% level for all quarters, with the exception of the last quarter of 2009 and the first of 2010, for any j. Throughout the period, our models perform better than a simple AR(1) model in the 65% of the companies considered.

In conclusion, the proposed framework has a good performance both in-sample and out-of-sample. It represents an acceptable trade-off between high goodness-of-fit and practical feasibility.

Appendix B: Alternative revenues models

In this section, we replicate the analysis of Sects. 3 and 4 replacing the revenue model described in Sect. 2.2 with bootstrap methods, keeping all other model specifications equal. We consider two alternative methods. In the first method, we perform a Crystal Ball prediction on the revenue growth rate (CB-GR, henceforth) and forecast, at time t, the logarithmic revenues k steps ahead with $y_{t+k} = y_t + kr_t$, where r_t is bootstrapped from a uniform random variable having support over the empirical distribution of historical revenues growth rates. This method assumes that revenues grow at a constant rate, tuned to historical observations. In the second method (B-HR, henceforth) we independently bootstrap future revenues from the empirical distribution of historical revenues. This method assumes a complete lack of autocorrelation in the revenue process. Both CB-GR and B-HR can be seen as extreme cases of our econometric model. We estimate them on the same rolling-window sample used for the estimation of the latter. In each case, we discard firms that present a negative fair-value distribution. Thus, the samples are not identical. We end up with 138 stocks in the CB-GR case and 124 stocks in the B-HR case.

Table 10 reports the results for the panel fixed-effects regression (7) where the excess returns of the cross section of the stocks are regressed on the z-scores computed with the CB-GR (*Top panel*), and with the B-HR (*Bottom panel*) methods, in addition to a set of control variables. The results can be compared with those of the econometric model in Table 3. Only the estimated common effects of the mispricing indicator, that is, the coefficients γ_1' s, are reported in Table 10. They all appear statistically significant and with a negative sign, regardless of the number and type of control variables considered. The good news is that our z score mispricing indicator appears rather robust, as it seems to represent some measure of misvaluation independently from the choice of the method adopted to forecast revenues.

The situation is different for portfolio analysis (see Sect. 3.2). The average Sharpe ratio of the Buy portfolios built based on CB-GR or B-HR, 1.3 and 1.38, respectively, does not differ significantly from that of the corresponding universe, 1.23 and 1.25, respectively. The adoption of less structured models for revenue forecasting seems to degrade the overall quality of the derived portfolios. However, the results remain qualitatively consistent: both the Sharpe Ratio and the Avg. Annual Return(%) of the Buy portfolios are greater than those of the Sell portfolios.

We then replicated the analyses in Sect. 4, computing the LSV factor from the z-score obtained with the CB-GR and B-HR methods, denoted as $LSV^{CB\text{-}GR}$ and $LSV^{B\text{-}HR}$ respectively. Table 11 reports the Pearson correlation between the new factors and the factors that are commonly considered to affect the returns of stocks. $LSV^{B\text{-}HR}$ has a correlation similar to that of LSV, whereas $LSV^{CB\text{-}GR}$ presents an opposite correlation sign with HML, MOM, CMA, and UMO. Adopting different methods for forecasting future revenues has an impact on the general behaviour of the factor. In particular, the increased support of the fair price distribution induced by assuming perfectly autocorrelated revenues induces significant differences in the relative degree of misvaluation across firms. This is confirmed by performing the multivariate analysis obtained by regressing the factors $LSV^{CB\text{-}GR}$ and $LSV^{B\text{-}HR}$ over the other factors considered. The results are reported in Tables 12 and 13 respectively. Although the intercepts remain positive in both cases, they are less significant than those obtained with our econometric model, reported in Table 6. The same conclusions are obtained by replicating the results of the Fama-MacBeth monthly regressions on the cross-section of portfolio abnormal returns, reported in Table 14, and by computing the performance of deciles based on the different LSV loadings.

In conclusion, the mispricing indicator and the LSV factor built using it, seem to retain some of their qualities even when simplified (and extreme) revenue forecasting models are assumed. However, the econometric model for the revenues proposed in Sect. 2.2 significantly improves the degree of misvaluation the factor can capture and its overall significance.

Table 10 Results of the monthly fixed-effect time series regressions in Equation 7, using the z-score computed from the SDCF approach where CB-GR and B-HR are employed, respectively

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
z-score$^{CB\text{-}GR}$	− 0.244**	− 0.324***	− 0.573***	− 0.556***	− 0.573***	− 0.549***	− 0.545***
	[− 2.898]	[− 3.869]	[− 7.516]	[− 7.314]	[− 7.517]	[− 7.181]	[− 7.142]
Adj. R^2(%)	34.242	34.704	35.046	35.339	35.042	35.642	35.527
z-score$^{B\text{-}HR}$	− 0.336***	− 0.392***	− 0.587***	− 0.581***	− 0.589***	− 0.572***	− 0.566***
	[− 3.672]	[− 4.281]	[− 7.044]	[− 6.989]	[− 7.070]	[− 6.880]	[− 6.839]
Adj. R^2(%)	37.637	37.947	38.255	38.139	38.252	38.654	38.535

Coefficients significant at 1% and 0.1% level are marked with ‘**’ and ‘***’ respectively. T-ratio based on HAC robust standard errors is reported in parentheses

Table 11 Comparison between Pearson correlation of the misvaluation factor *LSV* built using CB-GR and B-HR methods and other market factors

	LSV	$R_M - R_F$	SMB	HML	MOM	RMW	CMA	UMO
$LSV^{CB\text{-}GR}$	–	-0.159	-0.061	-0.112	0.048	0.005	-0.075	-0.119
$LSV^{B\text{-}HR}$	–	-0.378	-0.263	0.127	-0.226	0.215	0.259	0.207

Table 12 Summary of the daily regressions of LSV, when CB-GR is employed, on the factors $R_M - R_F$, SMB, HML, MOM, RMW, CMA and UMO

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	0.010°	0.010°	0.011*	0.010*	0.011*	0.011°	0.011*
	[1.913]	[1.916]	[2.132]	[2.016]	[2.222]	[1.906]	[1.978]
$R_M - R_F$	− 0.035***	− 0.035***	− 0.044***	− 0.038***	− 0.047***	− 0.045***	− 0.046***
	[− 5.443]	[− 5.445]	[− 6.590]	[− 5.802]	[− 7.039]	[− 6.325]	[− 6.450]
SMB	0.001	0.001	− 0.011	0.002	− 0.009	− 0.017	− 0.015
	[0.001]	[0.005]	[− 0.958]	[0.145]	[− 0.743]	[− 1.215]	[− 1.181]
HML	− 0.032**	− 0.031*	− 0.041***	− 0.016	− 0.025°	0.007	0.016
	[− 2.742]	[− 2.549]	[− 3.480]	[− 1.347]	[− 1.827]	[0.474]	[0.985]
MOM		0.001			0.002		0.010
		[0.067]			[0.230]		[0.942]
RMW			− 0.079***		− 0.077***		− 0.003
			[− 4.587]		[− 4.296]		[− 0.141]
CMA				− 0.054*	− 0.051*		− 0.014
				[− 2.482]	[− 2.320]		[− 0.501]
UMO						− 0.081***	− 0.081***
						[− 6.505]	[− 5.287]
Adj. R^2(%)	2.873	2.833	3.714	3.126	3.891	4.914	4.852
No. obs.	2392	2392	2392	2392	2392	1953	1953

T-ratio based on HAC robust standard errors is reported in parentheses. Coefficients significant at 10%, 5%, 1% and 0.1% level are marked with '∘', '∗', '∗∗' and '∗ ∗ ∗', respectively. Compare with Table 6

Table 13 Summary of the daily regressions of LSV, when B-HR is employed, on the factors $R_M - R_F$, SMB, HML, MOM, RMW, CMA and UMO

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	0.016**	0.015**	0.015*	0.015*	0.013**	0.020	0.017**
	[2.622]	[2.642]	[2.483]	[2.435]	[2.319]	[2.987]	[2.629]
$R_M - R_F$	− 0.142***	− 0.140***	− 0.133***	− 0.130***	− 0.117***	− 0.147***	− 0.125***
	− [11.324]	− [13.275]	− [10.648]	− [9.089]	− [10.491]	− [11.913]	− [12.021]
SMB	− 0.087***	− 0.097***	− 0.076***	− 0.093***	− 0.098***	− 0.099***	− 0.111***
	− [5.007]	− [6.299]	− [4.379]	− [5.354]	− [6.345]	− [5.215]	− [6.904]
HML	0.166***	0.108***	0.175***	0.110***	0.035	0.164***	0.050°
	[10.048]	[6.953]	[10.202]	[3.831]	[1.591]	[8.438]	[1.907]
MOM		− 0.093***			− 0.107***		− 0.104***
		− [7.452]			− [9.266]		− [8.708]
RMW			0.082**		0.058*		0.087**
			[2.898]		[2.337]		[2.613]
CMA				0.197***	0.249***		0.234***
				[5.027]	[8.219]		[6.649]
UMO						0.005	− 0.025
						[0.327]	− [1.538]
Adj. R^2(%)	22.084	25.880	22.533	24.064	29.275	24.846	30.972
No. obs.	2392	2392	2392	2392	2392	1953	1953

T-ratio based on HAC robust standard errors is reported in parentheses. Coefficients significant at 10%, 5%, 1% and 0.1% level are marked with '$\circ$', '$*$', '$**$' and '$***$', respectively. Compare with Table 6

Table 14 Fama-MacBeth monthly regressions on the cross-section of portfolio abnormal returns when CB-GR and B-HR are employed

	LSV	$LSV+FF3+MOM$	$LSV+FF3+UMO$	$LSV+FF5+MOM+UMO$	$LSV^{\perp}+FF5+MOM+UMO$
$LSV_{CB\text{-}GR}$	0.026	0.033	0.354°	0.228	
	[0.142]	[0.156]	[1.962]	[1.125]	
$LSV^{\perp}_{CB\text{-}GR}$					0.210
					[1.012]
$LSV_{B\text{-}HR}$	0.403*	0.289	0.608*	0.688*	
	[2.509]	[0.990]	[2.298]	[2.580]	
$LSV^{\perp}_{B\text{-}HR}$					0.464°
					[1.871]
LSV	0.313	0.492*	0.747**	0.762**	
	[1.608]	[2.273]	[3.346]	[3.729]	
$LSV^{\perp}$					0.543*
					[2.812]

The dependent variables are the monthly abnormal returns of 25 portfolios based on size and book-to-market, which are computed using Fama and French three-factor model on excess returns of equally-weighted portfolios. The independent variables are the loadings on the factors LSV, $LSV^{\perp}$ $R_M - R_F$, SMB, HML, MOM, RMW, CMA and UMO. T-ratio based on robust standard errors to HAC is reported in parentheses. Coefficients significant at 10%, 5%, 1% and 0.1% level are marked with '∘', '∗', '∗∗' and '∗ ∗ ∗', respectively. The last row reports the coefficients of the reference models in Table 7

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