



A behavioral approach to inconsistencies in intertemporal choices with the Analytic Hierarchy Process methodology

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Abstract

The framework of this paper is behavioral finance and, more specifically, the analysis of the main anomalies (delay, magnitude and sign effects) present in the processes of intertemporal choice. To the extent of our knowledge, only the delay effect (also known as decreasing impatience) has been discriminated between moderately and strongly decreasing impatience. However, taking into account that anomalies must be explained from a psychological point of view, the main objective of this paper is to relate the aforementioned paradoxes with the four categories of temperaments (artisan, guardian, idealist and rational) by using the sixteen personality types derived from the Myers–Briggs Type Indicator and the Behavioral Investor Types. To do this, we will use the Analytic Hierarchy Process methodology in order to detect the different levels of impatience through the so-called hyperbolic factor. Indeed, the main contribution of this paper refers to an empirical application which complements the theoretical analysis.

Keywords AHP · Delay effect · Discount Function · Hyperbolic Factor · Impatience · Magnitude effect · Preference · Sign effect

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1 Introduction

One of the main topics in the field of intertemporal choice is inconsistency. The seminal Discounted Utility Model (Samuelson 1952) assumes that the intertemporal utility function $U^t(c_t, \dots, c_T)$, defined on consumption (c_t, \dots, c_T) profiles at time t , under assumptions of completeness, continuity and transitivity, can be described by the following formula:

$$U^t(c_t, \dots, c_T) = \sum_{k=0}^{T-t} u(c_{t+k})F(k), \quad (1)$$

where:

- $F(0) = 1$;
- $F(k)$ represents the discount function of the individual, interpreted as the relative weight associated with the period from t to $t + k$;
- t is the time at which the evaluation takes place;
- c_{t+k} refers to the resources consumed during the $(t + k)$ -th period; and
- $u(c_{t+k})$ is the instantaneous cardinal utility function due to consumption in the $(t + k)$ -th period.

According to this model, individuals base their decision-making on associating a certain level of “utility” to each choice-related consequence: the value assigned turns out to be the lower the longer the time interval from the immediate moment of the decision to the time at which it is used. This decrease is one of the main elements of the Samuelson’s model and it is enclosed in the parameter defined as discount rate whose trend determines the preferences of individuals. Initially, the discount function was assumed to be an exponential function, defined, for every amount x and every time period t , as:

$$F(x, t) := x\delta^t, \quad 0 < \delta < 1. \quad (2)$$

This expression contains two types of additivity: with respect to time (Cruz Rambaud and González Fernández 2019) and with respect to the amount (Cruz Rambaud et al. 2018). On the one hand, the additivity with respect to amount (also called the property of linearity):

$$F(x + y, t) = F(x, t) + F(y, t) \quad (3)$$

makes all amounts to have the same discount rate:

$$r(x, t) := \frac{x - F(x, t)}{xt} = \frac{1 - \delta^t}{t}. \quad (4)$$

On the other hand, if additivity with respect to time also holds:

$$F(x, t + s) = F(F(x, t), s) \quad (5)$$

then the instantaneous discount rate is constant:

$$\delta(x, t) := \lim_{t \rightarrow 0} r(x, t) = -\ln \delta, \quad (6)$$

by indicating constant preferences over time. From a behavioral perspective, the term time consistency indicates the hypothesis that a person's preferences are always the same regardless of when the choice is made. However, the preferences shown by individuals do not respect this paradigm and, usually, $r(x, t)$ and $\delta(x, t)$ depend on x and t . In this context, the most usual anomalies are the delay and the magnitude effects, and also a particular case of the magnitude effect: the sign effect.

The magnitude effect means that the discount rate is decreasing with respect to the amounts involved in the study. The sign effect is a particular case when comparing the discount rates of positive and negative amounts. Finally, the delay effect or decreasing impatience means that the discount rate is decreasing with respect to the time period. To the extent of our knowledge, only two types of decreasing impatience have been analyzed, viz the so-called moderately and the strongly decreasing impatience (Rohde 2019). In this way, the manifestation of these behavioral anomalies has led to the need to formulate new expressions for the discount rate. Thus, the hyperbolic model arose with the aim to best describe not only the dynamic evolution of preferences, a change labeled as "preference reversal", but also the way in which people report the results according to the time and nature of the result itself.

Summarizing, as it is difficult to maintain a certain consistency, the results of experimental research do not always conform to the predictions of the exponential model. The main objective of this paper is to classify the different levels of impatience with respect to time and amount by using the AHP methodologies.

This paper is organized as follows. After this Introduction, which includes the framework, the objectives and the methodology of the paper, Sect. 2 presents the concepts of discount function, impatience and the main types of decreasing impatience, viz moderately and strongly decreasing impatience. The introduction of the hyperbolic factor will be of vital importance for the subsequent development of this paper. Section 3 justifies embedding this paper in the context of behavioral finance by introducing the Temperament Theory and the Myers–Briggs Type Indicator (MBTI), relating its sixteen personality types with the four categories of temperaments and, later, with the Behavioral Investor Types (BITs). In order to associate the main anomalies in intertemporal choice (delay, magnitude and sign effects) with the four categories of individual temperaments, Sect. 4 will be devoted to the use of the Analytic Hierarchy Process (AHP) in detecting the different levels of impatience according to the so-called hyperbolic factor. As an empirical application, this methodology is applied to a sample of 52 people in Sect. 5. Finally, Sect. 6 summarizes and concludes.

2 Preliminaries

2.1 Impatience in intertemporal choices

The term “impatience” was introduced in 1930 by Fisher (1930) when referring to the preference to anticipate future satisfactions. Recently, the concept of decreasing impatience has been associated with situations where discount rates decrease.

Before continuing, we are going to introduce the following definitions (Cruz Rambaud and González Fernández 2019).

Definition 1 A discount function in one variable is defined as a map

$$\begin{aligned} f : \mathbb{R}^+ &\longrightarrow \mathbb{R} \\ t &\mapsto f(t) \end{aligned}$$

such that:

1. $f(0) = 1$;
2. $f(t) > 0$, for every t ; and
3. $f(t)$ is strictly decreasing.

Definition 2 Let f be a discount function in one variable. The patience associated with the function in a range $[t_1, t_2]$ ($t_1 < t_2$) is defined as the ratio $\frac{f(t_2)}{f(t_1)}$.

We note that, from Definition 1, it follows that $0 < \frac{f(t_2)}{f(t_1)} < 1$.

Definition 3 Let $f(t)$ be a discount function in one variable. The impatience associated with the function in a range $[t_1, t_2]$ ($t_1 < t_2$) is defined as $1 - \frac{f(t_2)}{f(t_1)}$.

Theorem 1 Let $f_1(t)$ and $f_2(t)$ be two discount functions. The following conditions are equivalent:

- (i) The ratio $\frac{f_2(t)}{f_1(t)}$ is increasing;
- (ii) The impatience corresponding to $f_1(t)$ is greater than the impatience corresponding to $f_2(t)$; and
- (iii) If $f_1(t)$ and $f_2(t)$ are differentiable, $\delta_1(t) > \delta_2(t)$, for every t , where $\delta(t) := -\frac{d \ln f(t)}{dt}$.

The proof of this theorem is left to the reader.

2.2 Decrease in impatience

The introduction to decreasing impatience will be made by using preference relations. The bridge between discount functions and preference relations can be found in Fishburn and Rubinstein (1982): If order, monotonicity, continuity, impatience, and separability hold, and the set of rewards X is an interval, then there are continuous real-valued functions u on X and f on the time interval T such that

$$(y, r) \succeq (z, s) \text{ if, and only if, } u(y)f(r) \geq u(z)f(s). \quad (7)$$

Additionally, $u(0) = 0$ and u is increasing, whilst f is decreasing and positive.

Definition 4 The preference \preceq shows decreasing (resp. strictly decreasing) impatience (DI) if $(x, s) \sim (y, t)$ ($0 < x < y$) implies $(x, s + \sigma) \preceq (y, t + \sigma)$ (resp. $(x, s + \sigma) < (y, t + \sigma)$), for every $\sigma > 0$.

Definition 5 The preference \preceq shows a decreasing impatience greater than \preceq^* if $(x, s) \sim (y, t)$ ($0 < x < y$), $(x, s + \sigma) \sim (y, t + \sigma + \rho)$ ($\sigma > 0$ and $\rho > 0$) and $(\alpha, s) \sim^* (\beta, t)$ ($0 < \alpha < \beta$) implies $(\alpha, s + \sigma) \preceq^* (\beta, t + \sigma + \rho)$.

Prelec (2004) has experienced an equivalence between the selection of dominated results, i.e., not optimal from any temporal point of view and the decrease in the degree of impatience. In this sense, if, for example, an individual prefers an apple today to two tomorrow and two apples between a year and a day rather than one in a year (Thaler and Sunstein 2014), the decrease in impatience can be seen as a reflection of the irrationality underlying the reversal of preferences. Due to the importance of the relationship between inconsistency and impatience, there have been numerous studies aimed at finding a measure of the degree of DI shown by a preference.

Rohde (2010) proposed a tool to measure DI which does not need knowledge of the discount function and does not assume any information about the utility. Consider the following indifference relationships:

$$(x, s) \sim (y, t)$$

($0 < x < y$) and

$$(x, s + \sigma) \sim (y, t + \sigma + \tau)$$

($\sigma > 0$ and $\tau > 0$).

Definition 6 Given the indifference pair $(x, s) \sim (y, t)$ and $(x, s + \sigma) \sim (y, t + \sigma + \tau)$, the hyperbolic factor is defined by

$$H(s, t, \sigma, \tau) := \frac{\tau - \sigma}{t\sigma - s\tau}.$$

Theorem 2 An indifference pair can be constructed as follows:

- Step (I). Fix $y > 0$ and fix s, t ($s < t$) and $\tau > 0$;
- Step (II). Find x such that $(x, s) \sim (y, t)$; and
- Step (III). Find $\sigma > 0$ such that $(x, s + \sigma) \sim (y, t + \tau)$.

In these conditions, one has:

- $\tau - \sigma = 0$ if, and only if, the impatience is constant;
- $\tau - \sigma > 0$ if, and only if, the impatience is decreasing; and
- $\tau - \sigma < 0$ if, and only if, the impatience is increasing.

3 Decision-making processes and personality theories

3.1 Why personalized behavioral finance?

Studies on behavioral finance have shown that most people, compared to their financial schedules, show a lack of self-control and a hyperbolic discount. This means that each individual has a limited capacity in the management of money and in the design of his future. Individual knowledge in the financial field can improve the autonomous management of investment and savings processes but it must also be considered that, in addition to the level of competence, there are other factors that condition the behavior of an individual such as emotions and the propensity to behavioral distortions (Cruz Rambaud and Ventre 2017; Cruz Rambaud et al. 2018).

In this regard, financial education plays an important role but, since not everyone is interested in learning or deepening the financial subjects, it is right that advisors should use their notions taking into account that the choice cannot ignore the emotional and social factors of the person to whom they are offering service. Moreover, even those who have a good command of the necessary topics may still not be able to get a good enough overview to not miss the key aspects (Linciano and Soccorso, 2017).

What emerges from the observations just made is that to improve financial advice, with the aim of improving the quality of individual choices, there is a need for personalized advice and, therefore, “personalized behavioral finance”. This approach aims to understand the most common mistakes of investors through personality theory. In fact, if the individual dynamics of the decision-making process are not clear, any action aimed at facilitating the understanding of the problem and the selection of the best alternative, whether by a consultant or through financial education, could be ineffective. The purpose is to create a structure which allows to build personalized strategies providing for each individual, based on his personality, which distortion and preference is most inclined. In this way, it would be possible to exploit its weaknesses to present financial information in a “fair” way.

3.2 Temperament theory and MBTI

Temperament Theory was realized by Keirsey and Bates (1984) and states that the behavior of each individual is determined by his/her temperament in four categories of individuals:

- Artisan: it is defined as the temperament which acts. They stand out in the fields of art, business and politics; moreover, they are spontaneous and lovers of freedom.
- Guardian: they are cautious, reliable and disciplined, concrete and organized. They have a strong sense of responsibility and duty.
- Idealist: they are interested in personal growth and development, distinguishing themselves by their ability to inspire and clarify.
- Rational: recognized as the theoretical temperament, they are skeptical and attracted by problem solving.

On the other hand, the Myers–Briggs Type Indicator (hereinafter, MBTI), developed by Katharine Briggs and her daughter Isabel Briggs (Myers 1980), is intended to

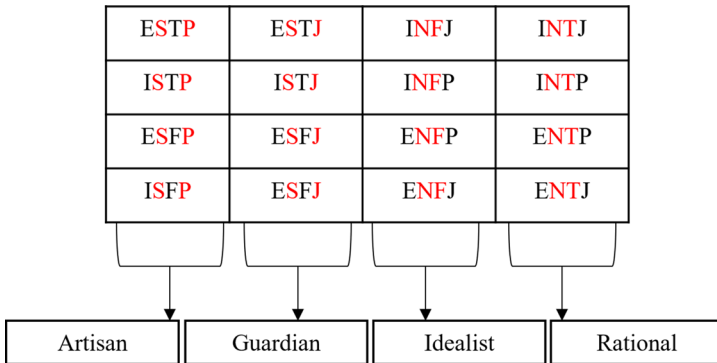


Fig. 1 Relationship between the four temperaments and the MBTI

schematize the way an individual relates to the world. It is a questionnaire which identifies psychological characteristics based on Carl Gustav Jung's theory of psychological types (Jung 1923).

MBTI distinguishes one among sixteen personalities, defined by four dichotomous pairs:

- Extroversion/Introversion (E/I), which describes how to interact with others. For example, whilst extroverts like to feel part of a group trying to stand out for their ideas, introverts tend not to share their information.
- Feeling/Intuition (S/N), which indicates how people collect information. Intuitive people, in fact, tend to collect information through sensations and inspiration as opposed to individuals who give greater weight to the senses.
- Thought/Feeling (T/F), which expresses the way they make decisions. The preference for thought indicates the tendency to a scientific approach whilst those who prefers feeling consider the choices also in relation to others favoring harmony to objectivity.
- Judgment/Perception (J/P). Those who prefer judgment are decisive and well-organized, contrarily to those who are adaptable and spontaneous, characteristics of those who prefer perception. This couple indicates the orientation that characterizes the lifestyle.

According to Jung's theory, people are predisposed to use and enhance one of the two preferences for each pair, thus obtaining sixteen personality types. These theories are interrelated: Keirsey, by dividing the four temperaments into two categories, each with two variants, obtained sixteen resulting temperaments related to the sixteen types of MBTI (see Fig. 1).

3.3 Relationship between personality and decision-making style

To effectively work with people, it is necessary to first understand how people deal with decisions as everyone, in fact, wants and must be treated according to their personality. In addition, it must be considered that the way of approaching a goal is also influenced by personality traits. Thus, the multiple differences between temperaments suggest to

us the impossibility of building a strategy that, while working in financial terms, gives the same degree of emotional and personal satisfaction to all individuals.

Since financial personality has implications for financial behavior (McKenna et al. 2003), it is important to note that, in some cases, the first may differ from the general personality. Pompian (2012) crossed Keirse's theory with behavioral finance studies by developing four Behavioral Investor Types (hereinafter, BITs): conservative, follower, independent and accumulator.

The objective is to link temperaments to the most frequent cognitive biases which will be covered in the next section. Although there is no precise correspondence, the following intersections are emphasized:

- Guardians are mostly conservatives because they aim to protect their heritage.
- Idealists are often followers as, not being interested in financial matters, they tend to follow the advice of others.
- Rationals generally correspond to independents.
- Artisans are accumulators because they are interested in accumulating wealth.

3.4 Behavioral biases and BITs

Behavioral finance studies show that, when we have to make decisions, we behave much more irrationally than we thought. Kahneman (2011) explains that at the basis of this phenomenon there are cognitive and emotional biases, called behavioral biases, which influence our attitude leading us to unintentionally choices which are far from optimal.

Biases are divided into emotional and cognitive: the first biases are related to distortions of decision-making due to factors concerning the emotional sphere (e.g., impulse or intuition); the second biases are defined as “mental shortcuts” which man creates to help himself understand reality and, therefore, cognitive biases develop during the interpretation and processing of information. The knowledge of cognitive biases can help replace one's mental patterns with less deceptive patterns. Likewise, recognizing emotional biases is a first step in improving awareness and self-control.

The effects of biases on investors are varied and interrelated. For example, loss aversion (emotional bias) in behavioral literature is the main cause of the strong attachment to the current status-quo. This leads investors to be unwilling to change their investments even though they are no longer fit for context.

The most important thing about BIT classification is that everyone is prone to behavioral fouls than the others. In this regard, it is particularly interesting to note that conservatives, like guardians, are not only risk averse but also the most risk averse. In fact, Kahneman and Tversky (1979) have proved that, on average, a loss weighs about twice as much as a gain of the same size but, for conservatives, gravity is perceived much more.

3.5 Personalized communication strategies

Since individual BITs are characterized by typical biases, educators and consultants need to consider the various differences with the aim of developing effective com-

munication techniques for each individual. McKenna et al. (2003) have developed strategies for Keirsey's different temperaments. For guardians, for example, as they prefer the use of senses and give weight to judgment, authority must be the basis of communication for counselors and educators to gain trust and respect. On the other hand, artisans are completely the opposite of what has just been described: they do not like plans and intend to use money rather than manage it. For idealists, bearing in mind that they are uninterested in accumulating money, it is important that their investments are sensible, that is, they need to understand who the financial plans are addressed to with all the information related to the impact on the environment and society. Finally, rationalists are good at planning and, as they are more interested in thinking about different plans rather than setting goals, they are particularly attracted to consultants challenging their ideas. They love complexity and, therefore, effective communication must be based on graphs, tables and equations.

4 Mathematical decision support models

4.1 The analytic hierarchy process

Decision theory is a branch of applied mathematics in which complex decision-making processes are analyzed and resolved through decision support models. This allows to select the best choices to achieve a goal if external constraints are imposed, such as states of nature. Multicriteria decision analysis (MCDA) is part of the broad landscape of decision theory and deals with the evaluation of multiple criteria which contribute to the structuring of complex problems MCDA thus allows decisions to be defined based on the -individual, collective, social, economic, and ecological- often competing components that, from different perspectives, drive decision making (Maturro and Ventre 2009a, b).

Saaty's Analytic Hierarchy Process (AHP) (1980, 2008) is a multi-criteria decision support technique, and a method of helping planning and designing, applied in social, political, and economic areas. The process we are going to describe is precisely applied mathematics, based on the alternation between rationality and common sense.

The AHP is based on a representation of the problem in terms of oriented graphs (Knuth 1973) because it is first necessary to build a hierarchical structure to represent the elements involved in the decision-making problem.

Definition 7 An oriented graph is a pair $G = (V, A)$, where:

- V is a non-empty set of elements, called vertices; and
- A is a set of ordered pairs of vertices, called arcs.

Vertices are indicated in Latin letters and, for each arc (u, v) , u and v represent the initial and final vertices, respectively.

Definition 8 An ordered tuple of vertices (v_1, v_2, \dots, v_n) , with $n > 1$, is a path of length $n - 1$, if each pair (v_i, v_{i+1}) , $i = 1, 2, \dots, n - 1$, is an arc of G .

Table 1 Scale of evaluations

Value	Interpretation
1	<i>i</i> and <i>j</i> are equally important
3	<i>i</i> is a little more important than <i>j</i>
5	<i>i</i> is a quite more important than <i>j</i>
7	<i>i</i> is definitely more important than <i>j</i>
9	<i>i</i> is absolutely more important than <i>j</i>
1/3	<i>i</i> is a little less important than <i>j</i>
1/5	<i>i</i> is a quite less important than <i>j</i>
1/7	<i>i</i> is definitely less important than <i>j</i>
1/9	<i>i</i> is absolutely less important than <i>j</i>

After creating this structure, we are going to determine a preference ratio of the elements of each level over all those of the previous level (see Table 1). The matrices we get from comparisons are called pairs comparison matrices and meet the following:

- If the vertex *i* assumes the value *x* in comparison with the vertex *j* with respect to an element of the top level, then the vertex *j* will assume the value $1/x$ in comparison with the vertex *i* with respect to the same element of the top level.
- If the comparison between two equally important alternatives is achieved with the value 1, it follows that the diagonal of each matrix will always be composed entirely of units.

Once constructed the matrices, we must proceed with the calculation of the weights related to the elements of each level. The importance of this step lies in the fact that it gives us information about the relevance of matrices. Weights must satisfy the following condition of normality:

$$\omega_1 + \omega_2 + \dots + \omega_n = 1.$$

Our hypothesis is the following: If the decision-makers knew all the actual weights of the comparisons, then:

$$A = \begin{pmatrix} \omega_1/\omega_1 & \omega_1/\omega_2 & \dots & \omega_1/\omega_n \\ \omega_2/\omega_1 & \omega_2/\omega_2 & \dots & \omega_2/\omega_n \\ \vdots & \vdots & \ddots & \vdots \\ \omega_n/\omega_1 & \omega_n/\omega_2 & \dots & \omega_n/\omega_n \end{pmatrix}.$$

Observe that each arrow of this matrix is a multiple of $(1/\omega_1, 1/\omega_2, \dots, 1/\omega_n)$, whereby the matrix has rank 1 and by writing

$$\omega = (1/\omega_1, 1/\omega_2, \dots, 1/\omega_n)^T,$$

one has the following chain of equivalent equalities:

$$- a_{ij} = \frac{\omega_i}{\omega_j}, i, j = 1, 2, \dots, n.$$

- $a_{ij} \frac{\omega_j}{\omega_i} = 1, i, j = 1, 2, \dots, n.$
- $\sum_{j=1}^n [a_{ij} \omega_j] / \omega_i = n.$

Last equation means that n is an eigenvalue of A and that w is the associated eigenvector. Since the diagonal elements are all 1, the trace of A is $tr(A) = \alpha_1 + \alpha_2 + \dots + \alpha_n = n$, with $\alpha_1, \alpha_2, \dots, \alpha_n$ the eigenvalues of A . As n is an eigenvalue of A , all the others must necessarily be null.

Definition 9 The constraint $a_{ij}a_{jk} = a_{ik}$, for every i, j and k , is called the consistency condition for the pair comparison matrix.

Definition 10 A pair comparison matrix is said to be consistent if it satisfies the consistency condition.

In practice, the decision-makers do not know the vector w in the sense that the actual values a_{ij} could move away from ω_i/ω_j , making the matrices inconsistent. The closer the value of a_{ij} is to ω_i/ω_j , the closer the value n will be to the maximum eigenvalue, so that all the others are null. The weight vector associated with the maximum eigenvalue which satisfies the condition of normality indicates an estimate of the weight vector which will be more precise the smaller the difference $\alpha_{max} - n$ which, in general, is positive.

Definition 11 The degree of consistency (CI) of matrix A is the ratio $\frac{\alpha_{max} - n}{n - 1}$ and indicates the “proximity to consistency”.

Saaty suggests the matrices for which $CI < 1/10$. The next step will be to set local priorities and then to obtain, through the sum of the relative weights of each level, a hierarchy of elements of the last level. Local evaluations represent an estimate of the relative importance of hierarchy elements relative to any element at the top level. This step must consider the following conditions:

- The scores are non-negative real numbers such that the sum of the weights relative to the arcs outgoing from the same vertex must be 1.
- The score assigned to the arc (u, v) indicates the extent to which the vertex v satisfies u .
- The score of a path is the product of the weights related to the arcs that form the path itself.
- For each vertex v , different from 1, the score $p(v)$ is the sum of the scores of all paths which start at the level 1 vertex and arrive to v .

4.2 The AHP method for intertemporal choices

The purpose of this subsection is to apply Saaty’s method to build a hierarchy of intertemporal perspectives which considers the relationship between personality traits and investor errors. Section 3 covered topics in favor of the idea that personality knowledge is useful for predicting behavioral biases.

The first level of the AHP will be the objective in which the customer is interested: the more detailed the definition of the purpose, the more parameters can be considered

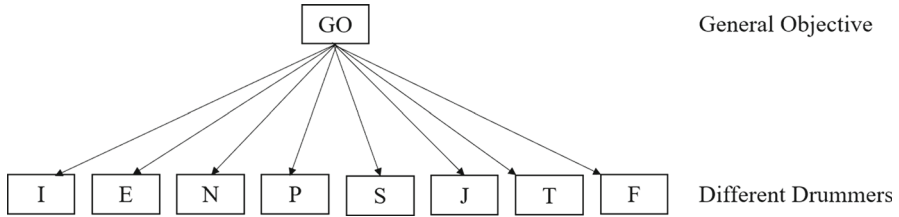


Fig. 2 Structure of the experiment: levels 1 and 2

Table 2 Comparison of strokes by preference ratio over objective

GO	I	E	N	P	S	J	T	F
I	1	I/E	I/N	I/P	I/S	I/J	I/T	I/F
E	E/I	1	E/N	E/P	E/S	E/J	E/T	E/F
N	N/I	N/E	1	N/P	N/S	N/J	N/T	N/F
P	P/I	P/E	P/N	1	P/S	P/J	P/T	P/F
S	S/I	S/E	S/N	S/P	1	S/J	S/T	S/F
J	J/I	J/E	J/N	J/P	J/S	1	J/T	J/F
T	T/I	T/E	T/N	T/P	T/S	T/J	1	T/F
F	F/I	F/E	F/N	F/P	F/S	F/J	F/T	1

to enrich the structure, such as initial wealth, lifestyle, and social context. However, since our purpose is to study how much personality “weighs” in decision-making, we will neglect the factors related to the goal itself and this will be indicated simply by “GO” (general objective). What interests us from this point of view is that the same goal can be differently achieved by the subjects, according to their personality and, since it is not possible to determine what is the right way among the many, as there is no personality fairer than the others, the second level will consist of the eight elements of the four alternative pairs of Myers’ theory. The level in question will be referred to as “Different drummers” to recall the preface to the book “Please Understand Me II. Temperament Character Intelligence” (Keirsey 1998) in which the author writes: “If you do not want what I want, please try not to tell me that my want is wrong. Or if my beliefs are different from yours, at least pause before you set out to correct them. Or if my emotion seems less or more intense than yours, given the same circumstances, try not to ask me to feel other than I do. [. . .] I may be your spouse, your parent, your offspring, your friend, your colleague. But whatever our relation, this I know: You and I are fundamentally different and both of us have to march to our own drummer.”

The structure designed so far is as shown in Fig. 2.

To construct the matrix of comparisons between all pairs in level 2 with respect to the overall objective, we will use “The Keirsey temperament Sorter II” (Keirsey 1998), a quiz of 70 questions with two options for each question. At the end of the test, we will get Table 2 which will indicate a score for each element of the dichotomic pair (E/I; N/S; T/F; J/P). Each pair has a total score of 20, except E/I which scores a maximum of 10 points. After converting the E/I pair proportionally into twentieths, the matrix will be obtained by relating two to two all eight strokes as follows:

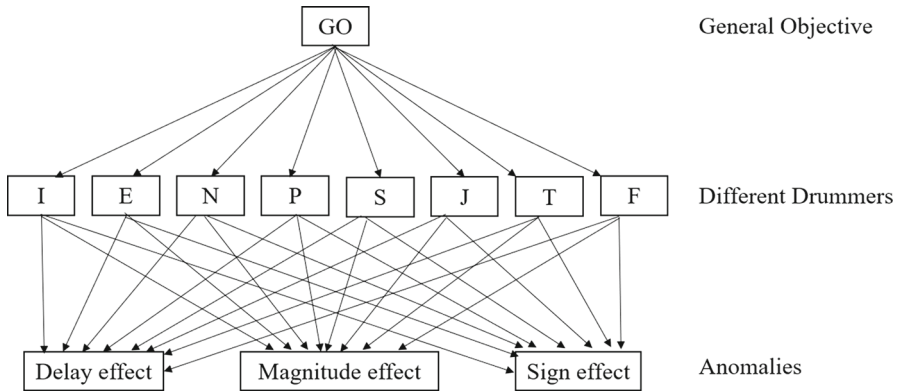


Fig. 3 Structure of the experiment: levels 1, 2 and 3

Table 3 Matrix of comparisons to pairs of anomalies

Anomalies	Delay effect	Magnitude effect	Sign effect
Delay effect	1	DI(d)/DI(m)	DI(d)/DI(s)
Magnitude effect	DI(m)/DI(d)	1	DI(m)/DI(s)
Sign effect	DI(s)/DI(d)	DI(s)/DI(m)	1

At this point, remembering that personality traits affect financial personality and that different investors are subject to different behavioral biases, the third level will include some of the anomalies found in intertemporal choices. In Sect. 1, we have proved that the decrease in impatience encompasses the relationship which exists between the psychological motivations behind the anomalies and the hyperbolic discounting applied by the agents. Inserting the anomalies just mentioned in the graph of Fig. 3 we get the third level of the hierarchy indicated as “anomalies”.

In order to determine the weights of level three with respect to the elements of level two, we will proceed as follows. For each individual undergoing the personality test, the degree of decrease in impatience with the anomalies will be calculated by using the considerations of Sect. 1 adapted to ad hoc quizzes. For each element of level 2, will be calculated the type of decrease in impatience of the subjects who have that trait as the dominant feature, proceeding to the comparison in pairs as indicated by Table 3.

On the other hand, the last level consists of several investment strategies by assuming that these can be seen as a combination of anomalies. When considering two strategies, weights can be indirectly derived by making a statistics evaluation of the subjective weights of each interviewed person. Subjective weights will be calculated by solving the system for the structure shown in Fig. 4.

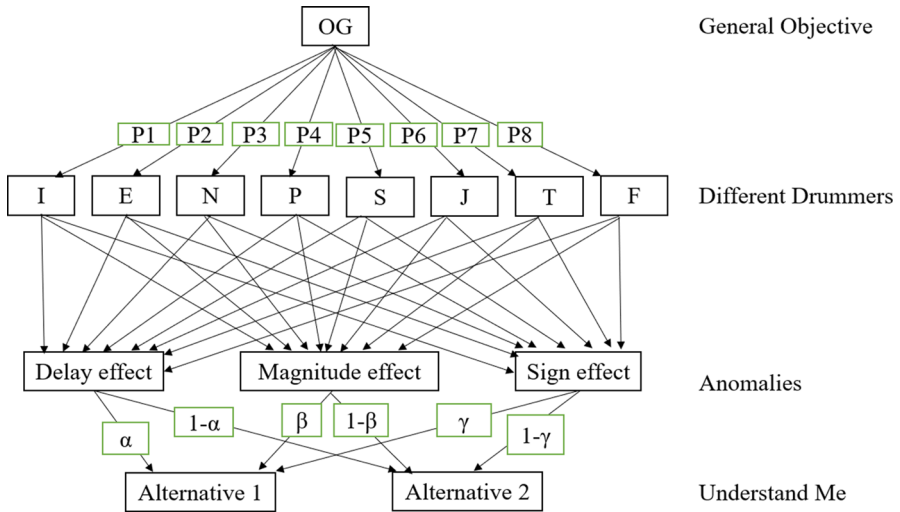


Fig. 4 Structure of the experiment: levels 1, 2, 3 and 4

The system associated to the structure in Figure 4 is the following:

$$\begin{cases}
 \alpha \sum_{i=1}^8 p_{i,1} + \beta \sum_{i=1}^8 p_{i,2} + \gamma \sum_{i=1}^8 p_{i,3} = \text{Preference A1} \\
 (1 - \alpha) \sum_{i=1}^8 p_{i,1} + (1 - \beta) \sum_{i=1}^8 p_{i,2} + (1 - \gamma) \sum_{i=1}^8 p_{i,3} = \text{Preference A2} \\
 \text{Preference A1} + \text{Preference A2} = 1
 \end{cases}$$

where:

- p_{ij} , with $j = 1, 2, 3$, indicates the weight of the j -th anomaly with respect to the i -th tract;
- α, β and γ are individual unknown weights; and
- preferences A1 and A2 are expressed in terms of comparison with a real value between 0 and 10, converted to cents.

The reason whereby the last level was referred to as “Understand me” is that, once level 3 and level 4 weights are set through a first phase of interviews, they do not vary from person to person. Therefore, by subjecting new individuals to the temperament quiz, their weights will allow us to anticipate for each the global weight of alternatives A1 and A2.

In this way, those involved in financial advice can know *a priori* the client’s propensity both in terms of behavioral errors and in terms of preference, diversifying the assistance as he sees fit.

5 Experimental phase

5.1 Structure of the experiment and objectives

In this subsection, we are going to propose two different objectives linked to each other. Firstly, we would like to know whether the observations made in Sect. 1 have an experimental value. In this regard, we remind you that the anomalies we want to investigate are:

- delay effect;
- magnitude effect; and
- sign effect.

In particular, we expect the hyperbolic factor to take higher values for less important figures, for shorter and shorter intervals and to be conditioned by the sign of the considered amounts.

The study will be based on the analysis of the degree of impatience through the hyperbolic factor, measure proposed by Rohde (2010) and presented in Theorem 2.

At the same time we will study the differences and similarities between Keirsey's four temperaments to understand how people deal with decisions. These considerations are aimed at determining the weights of each anomaly for individual behavioral categories.

The questionnaire submitted to the candidates was divided into two sections. The first part consisted of the questionnaire "The Keirsey FourTypes Sorter", taken from the book "Please Understand Me II. Temperament Character Intelligence" (Keirsey 1998) (see Fig. 5). The quiz consisted of 16 questions with 4 alternatives to order according to the following indication on the top of the text: "For each item, rank-order the four choices. Mark the response most like you as #1; less like you, #2; still less like you, #3; & least like you, #4. Put your numbers next to the corresponding letters." (Keirsey 1998).

This test, unlike the questionnaire "The Keirsey temperament Sorter II" (Keirsey 1998) indicated in subsection 4.3, was considered more suitable for administration for the following considerations:

- Shorter times: the request to order the alternatives of the 16 questions was met on average within 15 minutes per candidate. Instead, the estimated time for the questionnaire of 70 questions is about 40 minutes.
- Less effort: although both tests were laborious, the length and structure of the 70 applications require greater effort, for example in terms of concentration.
- Final result: whereas the test of 70 questions indicates at the end the dominant trait on 16 total strokes, the quiz with 16 questions identifies the main temperament compared to the 4 possible. In this way, the study of the behavioral differences of each temperament was possible even with a small number of individuals.

At the end of the test for each candidate, a table similar to that shown in Fig. 6 can be obtained.

The main temperament was finally calculated according to the following "Scoring Directions: First, in the numbered columns above, record your rankings (1 to 4) for

- 1. **I'd rather study**
 _ (a) arts & crafts
 _ (b) literature & humanities
 _ (c) business & finance
 _ (d) science & engineering
- 2. **I feel best about myself when**
 _ (a) I'm graceful in action
 _ (b) I'm en rapport with someone
 _ (c) I'm rock-solid dependable
 _ (d) I exercise my ingenuity
- 3. **In mood I'm more often**
 _ (a) excited & stimulated
 _ (b) enthusiastic & inspired
 _ (c) cautious & prudent
 _ (d) calm & detached
- 4. **I keep coming back to**
 _ (a) perfecting my craft
 _ (b) helping others affirm themselves
 _ (c) helping others do right
 _ (d) figuring out how things work
- 5. **Coming right down to it I tend to be**
 _ (a) practical & opportunistic
 _ (b) compassionate & altruistic
 _ (c) dutiful & diligent
 _ (d) efficient & pragmatic
- 6. **I respect myself more for**
 _ (a) being bold & adventurous
 _ (b) being kind-hearted & of good will
 _ (c) doing good deeds
 _ (d) being autonomous & independent
- 7. **I'm more inclined to trust**
 _ (a) impulses & whims
 _ (b) intuitions & intimations
 _ (c) customs & traditions
 _ (d) pure reason & formal logic
- 8. **I'm sometimes eager to**
 _ (a) make an impression & have impact
 _ (b) lose myself in romantic dreams
 _ (c) be a valued & legitimate member
 _ (d) make a scientific breakthrough
- 9. **I'm in a life-long search for more**
 _ (a) thrills & adventures
 _ (b) self-understanding
 _ (c) safety & security
 _ (d) efficient methods of operation
- 10. **In facing the future**
 _ (a) I bet something lucky will turn up
 _ (b) I believe in people's innate goodness
 _ (c) you just can't be too careful
 _ (d) it's best to keep a wary eye
- 11. **If it were possible I'd like to become**
 _ (a) an artistic virtuoso
 _ (b) a wise prophet
 _ (c) a chief executive
 _ (d) a technological genius
- 12. **I'd do best in a job working with**
 _ (a) tools & equipment
 _ (b) human resources development
 _ (c) materiel & services
 _ (d) systems & structures
- 13. **As a guide to action I look primarily at**
 _ (a) immediate advantages
 _ (b) future possibilities
 _ (c) past experience
 _ (d) necessary & sufficient conditions
- 14. **I'm most self-confident when I'm**
 _ (a) adaptable & flexible
 _ (b) genuine & authentic
 _ (c) honorable & respectable
 _ (d) strong-willed & resolute
- 15. **I appreciate it when others**
 _ (a) surprise me with generosity
 _ (b) recognize my true self
 _ (c) express their gratitude
 _ (d) ask me for my rationale
- 16. **When thinking about misfortune**
 _ (a) I usually laugh it off
 _ (b) I often wonder why
 _ (c) I try to make the best of it
 _ (d) I view it from a wide perspective

Fig. 5 Test “The Keirsey FourTypes Sorter”

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16		
a																		A
b																		I
c																		G
d																		R

Fig. 6 Table of scores

each of the 16 items. Second, add the numbers across each of the four rows (a, b, c, d) and place the sums in the boxes at the far right. Third, circle the letter (A, I, G, or R) beside the lowest sum. Fourth, A stands for Artisan (SP), I for Idealist (NF), G for Guardian (SJ), R for Rational (NT).” (Keirsey 1998).

Regarding the formulation of the questionnaire for the calculation of the hyperbolic factor, the idea is to construct indifference pairs as indicated by Theorem 2 using the hyperbolic factor as a measure of the alteration of the degree of impatience and, since we are mainly interested in understanding the extent of the variation, it will sometimes be considered only the module of the result obtained.

The first question, built with the purpose of studying the delay effect, results in the pair of indifferences $(x, s) \sim (y, t)$ and $(x, s + \sigma) \sim (y, t + \tau)$, remembering that the exponential discount predicts the condition $\sigma = \tau$. As specified in step I of

Theorem 2, we have therefore fixed $s = 0$, $t = 6$, $\tau = 12$, and $y = \text{€}500$. To derive the value of the x , respondents were asked:

You must receive a sum of €500 in 6 months, not before the set date but, alternatively, there is the possibility to immediately collect a certain result by reducing the total to be collected. To accept the offer, how much at least do you want to receive today?

Once obtained the value of x and then the first indifference $(x, s) \sim (y, t)$, step III of Theorem 2 was simulated as follows:

You must receive a sum of €500 in 18 months, not before the set date but you are given the opportunity to anticipate the application and collect a result of x instead of €500. To accept the offer, how long do you want to receive the x -digit?

Once obtained the value of σ , all the parameters necessary for the calculation of the hyperbolic factor have been identified:

$$H(0, 6, 500, 12) = \frac{12 - \sigma}{\sigma}.$$

Leaving unchanged s , t and τ , we changed the initial figure by lowering it to a total of €50 in order to study which temperament manifests a steeper discount function for lower amounts. In fact, although the hyperbolic factor turns out to be independent of y (Rohde 2010), we believe that σ can be influenced by this variable:

$$H(0, 6, 50, 12) = \frac{12 - \sigma}{\sigma}.$$

Leaving the sum of €50 and $s = 0$, the value of t and τ have been changed in the third question in order to study how the hyperbolic factor varies in case of involving periods closer to the present. By reusing the same setting for demand, we can analyze the propensity to hyperbolic discounting:

$$H(0, 1, 50, 1) = \frac{1 - \sigma}{\sigma}.$$

Finally, in order to study how impatience varies based on the sign associated with the utility of the amounts, respondents answered the following questions:

You must pay a sum of €500 in 6 months, not before the set date but you are given the option to pay a lower result as long as the payment is paid on the same day. What is a figure that you think you could offer to accept your offer?

and

You must pay a sum of €500 in 18 months, not before the set date but you are given the option to pay a lower result of x instead of €500. How long would you commit to paying the payment?

To further investigate this latest phenomenon, participants were asked to propose a figure that would be right to pay off the debt on the day, again obtaining the first indifference $(x, 0) \sim (-500, 6)$.

Table 4 Distribution of the sample

Temperament	Distribution (%)
Rational	38.5
Idealist	26.9
Guardian	19.2
Artisan	15.4

Table 5 Values for hyperbolic factor $H(0, 6, 500, 12)$

Temperament	Minimum value (distribution)	Maximum value (distribution)	Median
Artisan	0.50 (12.5%)	66.50 (25.00%)	7.83
Guardian	0.00 (10%)	66.50 (20%)	2.83
Idealist	0.00 (7%)	28.40 (7.14%)	1.83
Rational	-0.02 (5%)	66.50 (25.00%)	1.83

The quiz was given to a sample of 52 people between the ages of 18 and 65, of whom 48.08% were women. The percentage of distribution of the sample in relation to the four temperaments is shown in Table 4.

The interviews were administered mainly in telematic mode. During the first phase of the test, i.e., the part aimed at identifying the dominant personality trait, the candidates were free to answer calmly to ensure a good understanding of the proposed questions, also favored through the sharing of the text. In addition, for younger respondents, the presence of a “trusted” person was requested in order to correct and protect the veracity of answers. In this respect, in case of disagreement, the responses suggested by the partner were considered. The aim of this was to reduce self-assessment errors which could occur because of indecision. Instead, during the phase involved in collecting the data necessary for the calculation of the hyperbolic factor, the interviewed individuals were alone and constantly urged to communicate their preferences as soon as possible. In fact, due to the low complexity of the proposed intertemporal perspectives and the impossibility of simulating the effect of the passage of time in a single interview, the tension generated by the haste in the answers was necessary to obtain a considerable manifestation of behavioral distortions, indispensable for analysis.

5.2 Analysis of results

Delay effect. The reference value for the analysis of the relationships between the four temperaments and the construction of the AHP is the median. Remember that this parameter is a position index which divides the distribution into two parts and is therefore recommended when the data have a high variability: the minimum and maximum values shown in Table 5 justify its choice for each temperament.

The major median corresponds to the temperament of artisans in line with their behavioral characteristics: the strong impulsivity which characterizes them is trans-

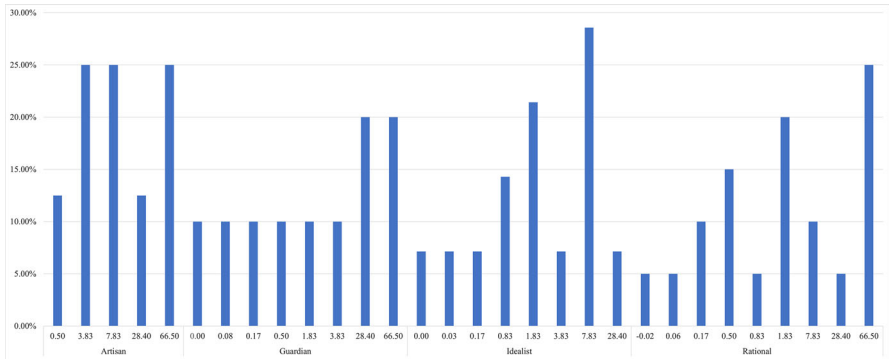


Fig. 7 Distribution of values for $H(0, 6, 500, 12)$

Table 6 Comparison of hyperbolic factors $H(0, 6, 50, 12)$ and $H(0, 6, 500, 12)$

Values	$H(0, 6, 500, 12)$	$H(0, 6, 50, 12)$
Minimum value (distribution)	0.00 (3.85%) M (0%), F(100%)	0.00 (3.85%) M (50%), F(50%)
Maximum value (distribution)	66.50 (17.31%) M (55.56%), F(44.44%)	66.50 (40.38%) M (61.90%), F(38.10%)
Median	2.83	18.12

lated into a high degree of decrease in impatience. On the other hand, idealists and rationals have the lower value of median. However, whilst only 7% of idealists reach the maximum hyperbolic factor with the measure 28.40, rationals show the same maximum value and distribution as artisans. Figure 7 summarizes the distributions for each value in the four categories.

The largest distribution, at 28.57%, is in the category of idealists for the value $H = 7.83$, a result which agrees with their general lack of interest in financial matters.

Finally, observe that the highest percentage of a measure of the hyperbolic factor is manifested by guardians: their love of self-control results in an exponential discount function.

Magnitude effect. Our aim in this paragraph is to see if the increase in figures increases, the patience of the agent increases because he is willing to wait longer. In fact, a psychological explanation for the phenomenon is to associate the smaller figures with an impending consumption by expecting a greater hyperbolic factor. Therefore, the objective is to analyze how the hyperbolic factor varies as the figures considered vary, by analyzing whether, for digits with the same ratio, the difference between them plays a significant role or not. Let us start with a general analysis and then focus on the four temperaments.

The distribution of the minimum value remains unchanged when less important amounts are considered and, in the same case, the distribution of the maximum value doubles. The median of hyperbolic factors for the amount of €50 is about 6 times greater than that related to the amount of €500, in line with forecasts (see Table 6).

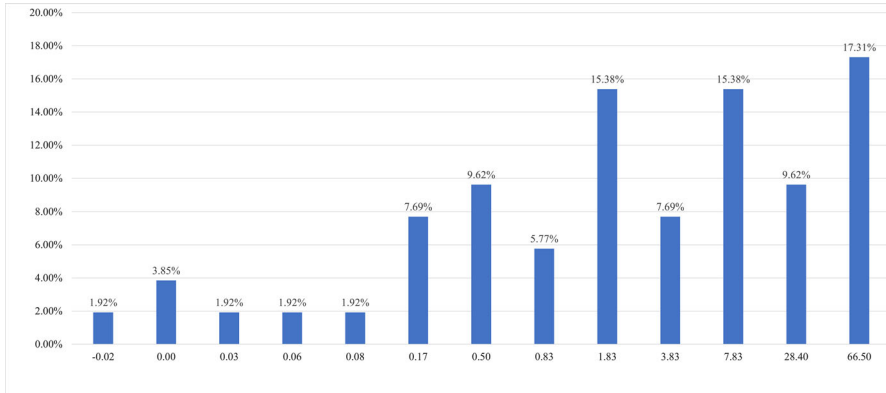


Fig. 8 Distribution of hyperbolic factor $H(0, 6, 500, 12)$

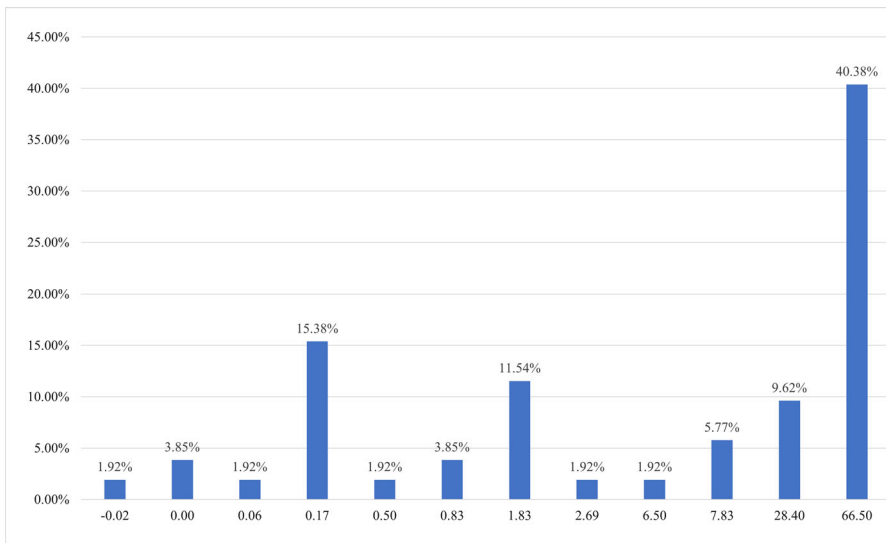


Fig. 9 Distribution of hyperbolic factor $H(0, 6, 50, 12)$

The following graphs point out how the distribution varies according to the figures considered: observe that, whilst for $H(0, 6, 500, 12)$ (see Fig. 8) there are multiple peaks, for $H(0, 6, 50, 12)$ (see Fig. 9) the graph is more homogeneous with a single superscript at the maximum value.

In order to investigate the influence of the difference between the amounts considered, the ratio of the hyperbolic factor calculated for an initial sum of €50 to €500 was analyzed in the specific case where $\frac{500}{x_{500}} = \frac{50}{x_{50}}$. The sample showing this condition was 28.85% over the total of which:

- 46.67% showed no change in the value of the hyperbolic factor (57.7% were men);
- 13.34% showed a steeper discount for larger amounts (100% women); and

Table 7 Values for the hyperbolic factor $H(0, 6, 50, 12)$, first stroke

Temperament	Minimum value (distribution)	Maximum value (distribution)	Median
Artisan	2.69 (12.50%)	66.50 (62.50%)	28.4
Guardian	0.00 (10%)	66.50 (30%)	18.12
Idealist	0.06 (7.14%)	66.50 (35.71%)	4.83
Rational	-0,02 (5%)	66.50 (40.00%)	4.17

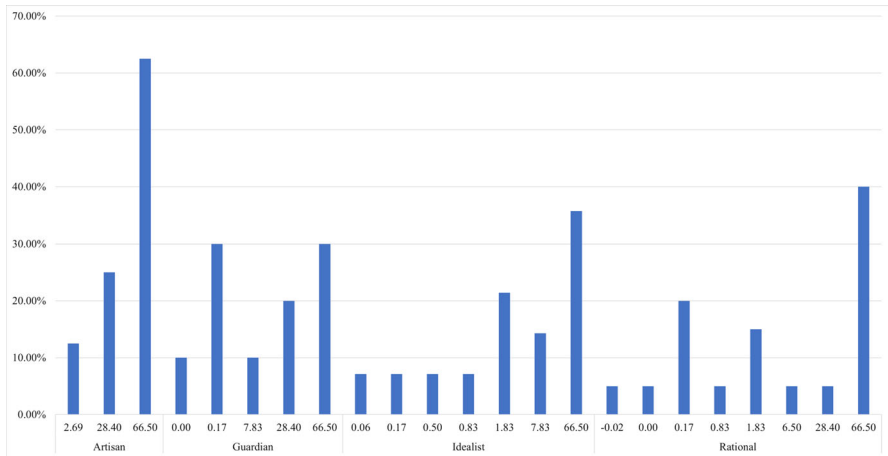


Fig. 10 Distribution of hyperbolic factor $H(0, 6, 50, 12)$ by dominant trait

– 40% showed a steeper discount for smaller amounts (of which 66.67% were women).

It would therefore be interesting to understand, from a behavioral point of view, the trait which characterizes 46.67% of the sample that is not inclined to this attitude. In addition, women are more sensitive to this phenomenon.

As far as temperaments are concerned, let us begin by presenting Table 7.

We can immediately observe that not only the median values increased for all temperaments but also the distributions related to the maximum value have increased. Particularly interesting is the case of idealists who, from a maximum of 28.40, have reached the value 66.50 (see Fig. 10). Artisans, on the other hand, are temperament with greater distribution of maximum value: their preference for immediate gratification is evident. For further observations, we present the graph of distributions for each temperament.

Guardians represent a temperament with the least distribution of maximum value. However, their corresponding median is 6.4 times higher than the one calculated for €500, the highest increase among the four temperaments. This is not surprising at all. In fact, guardians represent conservative investors, prone to saving and moderated in expenses: in front of smaller figures, it is not worth taking the risk of the greatest expectation, as if small amounts were categorized into expenses and not savings. Mental accounting is in fact one of the cognitive biases to which this category is

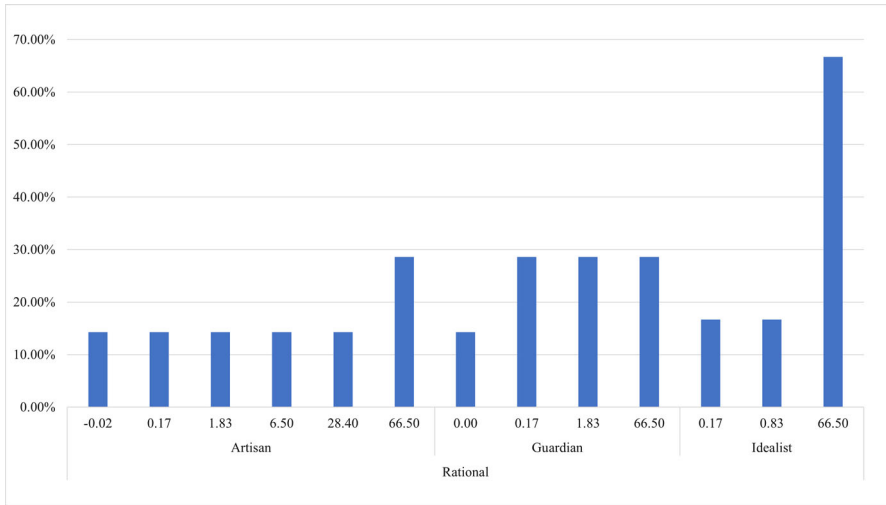


Fig. 11 Distribution of the hyperbolic factor $H(0, 6, 50, 12)$ of category NT according to the second dominant section

Table 8 Values related to the ratio and differences of outcomes

Temperament	Median $\frac{500}{x}$	Median $\frac{50}{x}$	Median $500 - x$	Median $50 - x$
Artisan	1.67	2	200	25
Guardian	1.67	1.67	150	20
Idealist	1.63	1.67	150	20
Rational	1.25	1.25	100	10

most inclined. The rational median is the one that has been least influenced: unlike idealists, a rational investor has shown himself to be less impulsive. To investigate the relationship between rationals and idealists, we studied the distribution of the first ones according to the second trait (see Fig. 11).

As a result, those who present the idealist as a second trait have a 66.67% chance of being more impulsive and prone to immediate gratification: this result reflects the combination of the bias of excess trust, precisely of rationals, and the general disinterest, precisely of idealists.

Let us now present the data necessary to analyze the phenomenon in terms of the ratio and difference of amounts. Although the data do not show high variability, in order to be consistent with the study carried out so far, we will still take into account the median value (see Table 8).

We can immediately observe that:

- the ratio remains mainly constant, the increase in the decrease in impatience is therefore necessarily linked to a decrease in the difference in amounts; and
- the relationship between magnitude effect and temperament exists and cannot be overlooked (see Table 9). Idealists and guardians, for example, respond to

Table 9 Ratio $H(0, 6, 50, 12)$ to $H(0, 6, 500, 12)$

Temperament	Median €50	Median €500	Report
Artisan	28.4	7.83	3.62
Guardian	18.12	2.83	6,4
Idealist	4.83	1.83	2.63
Rational	4.17	1.83	2.28

Table 10 Distribution of the ratio of $H(50)$ to $H(500)$ in the case $\frac{500}{x_{500}} = \frac{50}{x_{50}}$

$H(50)/H(500)$	Distribution (%)
0.03	6.67
0.33	6.67
1.00	46.67
2.34	6.67
8.49	20.00
36.27	13.33

differences (150, 20) differently: the former with an increase in the median for the hyperbolic factor at about €500 by about 3 times greater whilst the latter have an increase of 6.4 times.

To further investigate the influence of the difference between the amounts considered for each temperament, the relationships between the hyperbolic factor calculated for an initial sum of €50 and €500 in the specific case in which $\frac{500}{x_{500}} = \frac{50}{x_{50}}$. The Table 10 shows the distribution of the $H(0, 6, 50, 12)/H(0, 6, 500, 12)$ ratio for the part of the sample with this condition (about 29%).

Intrigued by 46.67% of value 1, the results were analyzed in relation to the dominant temperament, concluding that of these 71.43% are rational, as expected (see Fig. 12).

In line with the observations previously made on the relationship between rationals and idealists, only 20% of rationals under consideration present the category of idealists as their second temperament (see Fig. 13).

This confirms that, in addition to the dominant trait, it is important to consider the extent to which the other traits are present in the subject. For example, a rational who presents the idealist as the second dominant trait has a different inclination in this situation than a rational who presents the guardian as the second dominant trait.

Hyperbolic discount and interval effect. In this paragraph, we are interested to understand how the steepness of the discount function varies quantitatively for prospectuses which are timelessly more immediate. Let us therefore draw attention to the comparison between $H(0, 6, 50, 12)$ and $H(0, 1, 50, 1)$ (see Table 11).

Observe that the hyperbolic factor for the shortest prospectus has a maximum value of almost a half of the maximum value for larger elevations, whilst distributions remain mostly unchanged. It follows that those who serve hyperbolically apply most of the decrease in impatience in the first period from “today” to “2 months”. This confirms that the rate at which the discount function decreases, decreases over time. If this were

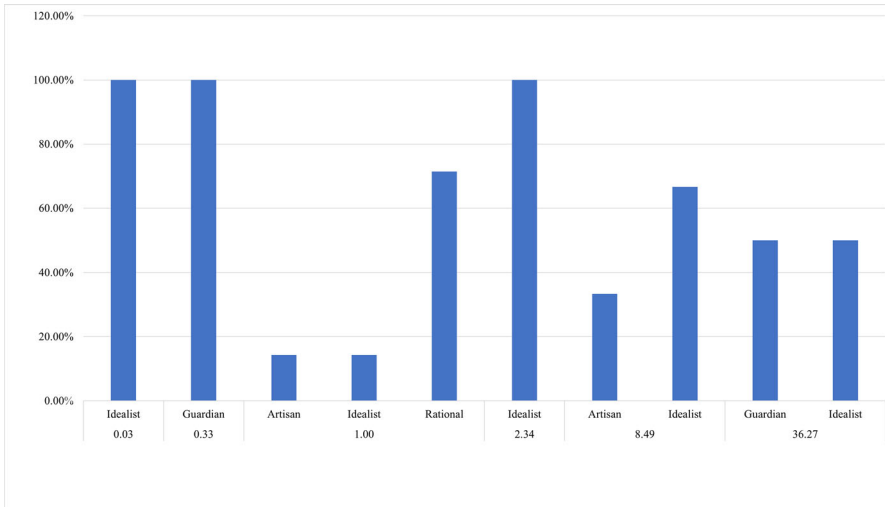


Fig. 12 Distribution of the ratio of $H(50)$ to $H(500)$ in the case $\frac{500}{x500} = \frac{50}{x50}$, for each temperament

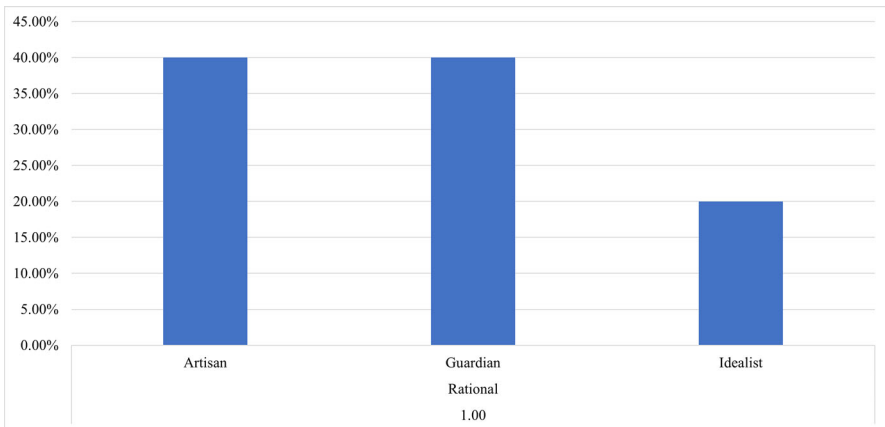


Fig. 13 Distribution of $H(50)/H(500) = 1$ for rationals with respect to the second dominant section

Table 11 Comparison between the hyperbolic factors $H(0, 1, 50, 1)$ and $H(0, 6, 50, 12)$

Values	$H(0, 1, 50, 1)$	$H(0, 6, 50, 12)$
Minimum value (distribution)	0.00 (7.69%) M (25.00%), F(75.00%)	0.00 (3.85%) M (50%), F(50%)
Maximum value (distribution)	32.33 (48.08%) M (64.00%), F(36.00%)	66.50 (40.38%) M (61.90%), F(38.10%)
Median	13.29	18.12

Table 12 Medians of $H(0, 6, 50, 12)$ and $H(0, 1, 50, 1)$ for each temperament

Temperament	Median $H(0, 1, 50, 1)$	Median $H(0, 6, 50, 12)$
Artisan	32.33	28.4
Guardian	3.00	18.12
Idealist	17.67	4.83
Rational	8.32	4.17

Table 13 Hyperbolic factor numerator $H(0, 1, 50, 1)$

Sign	Distribution (%)
Positive	90.39
Negative	1.92
Null	7.69

Table 14 Hyperbolic factor numerator $H(0, 6, 50, 12)$

Sign	Distribution (%)
Positive	94.23
Negative	1.92
Null	3.85

not the case, we would have had to obtain for the prospectus from “today” to “12 months” a hyperbolic factor equal to 6 times the hyperbolic factor $H(0, 6, 50, 12)$.

This prediction is about manifested only by the guardian category, as shown by Table 12.

As the length of the interval increases, the discount therefore decreases (interval effect). In particular, for both elevations, most of the sample has a positive term to the numerator, indicating a decreasing impatience over time:

- Case in which $t = 1$ and $\tau = 1$ (see Table 13).
- Case in which $t = 6$ and $\tau = 12$ (see Table 14).

Sign effect. Changing the sign of the amounts fixed, most respondents presented a variation in the hyperbolic factor as shown in Fig. 14.

Thus, only by changing the sign, 78.85% presented a different value of the hyperbolic factor of which only 41.48% showed a higher figure for negative amounts. While expecting the opposite, it must be borne in the way that, in a payment situation, the impatience of the debtor and the individual who is to receive the debt is added up by lowering the degree of decrease in total impatience. Nevertheless, by explicitly comparing the values $H(0, 6, 500, 12)$ and $H(0, 6, -500, 12)$, we note that the discount function for negative amounts is much steeper: the percentage of distribution of the maximum value has almost doubled compared to the case of positive utilities and the median has a higher value (see Table 15).

Figure 15 shows that the distribution in the case of negative amounts is more homogeneous. The unique peak relative to the maximum value emphasizes that all temperaments are affected by the sign.

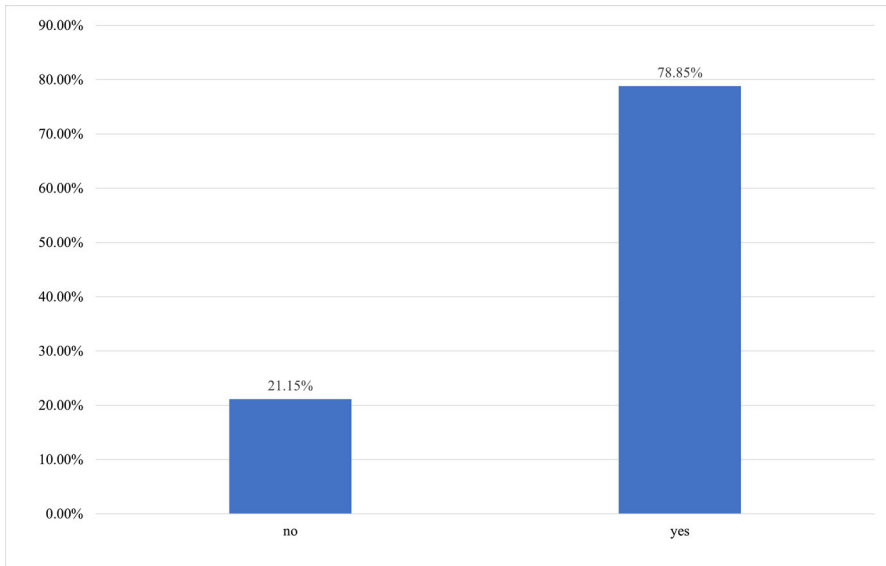


Fig. 14 Distribution of the sample which presented different values for $H(500)$ and $H(-500)$

Table 15 Comparison between the hyperbolic factors $H(0, 6, 500, 12)$ and $H(0, 6, -500, 12)$

Values	$H(0, 6, 500, 12)$	$H(0, 6, -500, 12)$
Minimum value (distribution)	0.00 (3.85%) M (0.00%), F(100.00%)	0.00 (7.69%) M (75%), F(25%)
Maximum value (distribution)	66.50 (17.31%) M (55.56%), F(44.44%)	66.50 (32.69%) M (58.82%), F(41.18%)
Median	2.83	3.17

Table 16 Values of $H(0, 6, -500, 12)$ for each temperament

Temperament	Minimum value (distribution)	Minimum value (distribution)	Median $H(-500)$	Median $H(500)$
Artisan	0.03 (25.00%)	66.50 (62.50%)	66.50	7.83
Guardian	0.00 (20%)	7.83 (20%)	2.83	2.83
Idealist	0.00 (7.14%)	66.50 (14.29%)	1.83	1.83
Rational	0.00 (5%)	66.50 (50.00%)	31.17	1.83

To analyze how the four temperaments relate to goods with negative utility, we report the values shown in Table 16.

We immediately observe that artisans, being among the four temperaments the most flexible in accepting losses, present the greatest distribution for the maximum value: facing the thought of payment in a serene way for them prevails the interest of paying as soon as possible to “take your mind off”. What is interesting is that guardians show

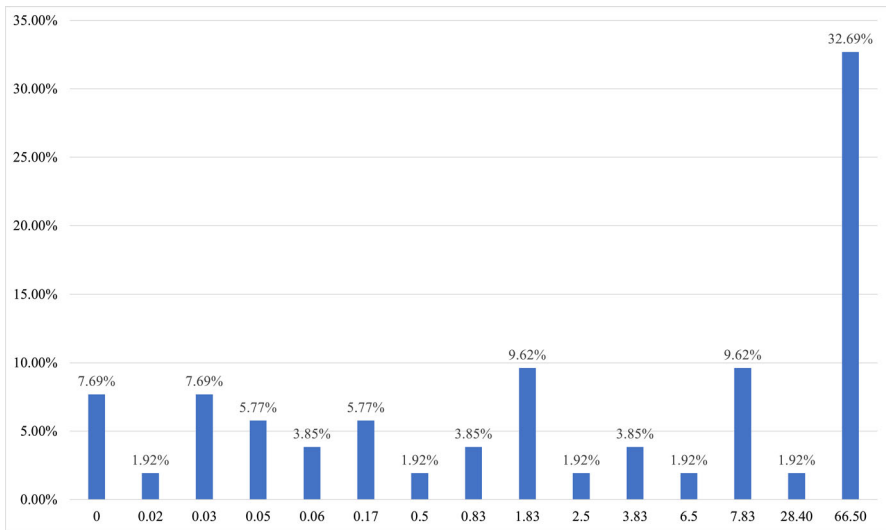


Fig. 15 Distribution of $H(0, 6, -500, 12)$

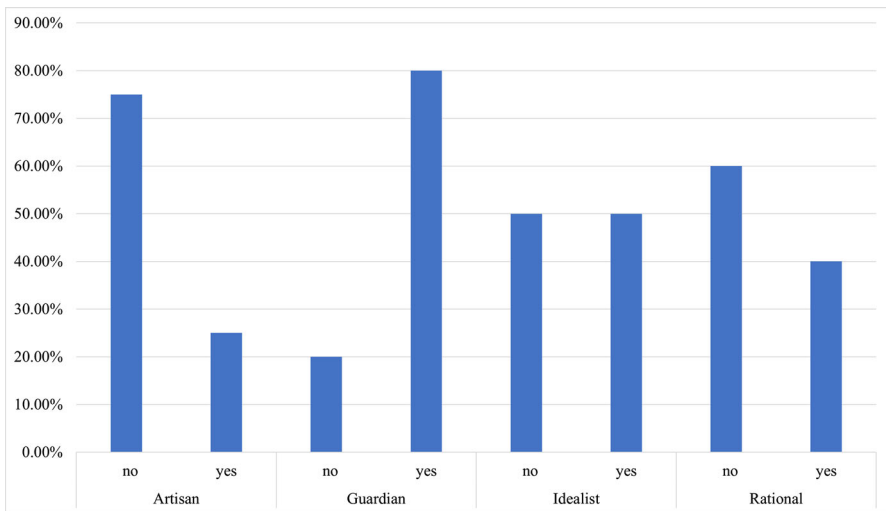


Fig. 16 Distribution of those postponing payment

the lowest maximum value: this category, remembering that their aims are to protect the assets, suffers more from payment as it sees it as a reduction in one’s assets, rather than the thought of someone waiting for the payment itself. In fact, to investigate this phenomenon, since the median does not change, we have analyzed which of the four temperaments tends to wait longer for payment. We expect the guardians, because of the bias of the status quo, to postpone the alteration of their initial wealth. Figure 16 confirms our assumptions.

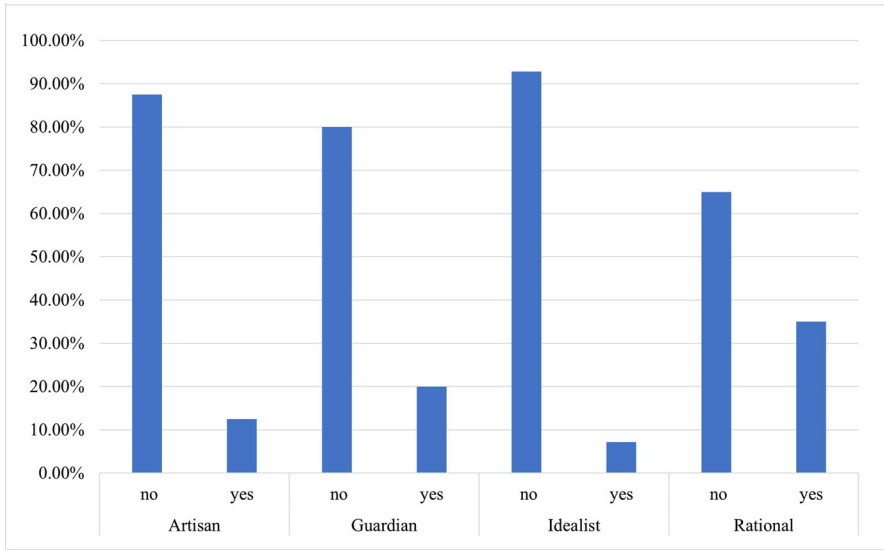


Fig. 17 Distribution of those who would unconsciously increase the amount of payment

Table 17 Matrix of comparisons to pairs of anomalies

Anomalies	Delay effect	Magnitude effect	Sign effect
Delay effect	1	$\frac{Me(H(0,6,500,12))}{Me(H(0,6,50,12))}$	$\frac{Me(H(0,6,500,12))}{Me(H(0,6,-500,12))}$
Magnitude effect	$\frac{Me(H(0,6,50,12))}{Me(H(0,6,500,12))}$	1	$\frac{Me(H(0,6,50,12))}{Me(H(0,6,-500,12))}$
Sign effect	$\frac{Me(H(0,6,-500,12))}{Me(H(0,6,500,12))}$	$\frac{Me(H(0,6,-500,12))}{Me(H(0,6,50,12))}$	1

To conclude, we note that, among the four temperaments, the highest percentage of individuals who have proposed a larger sum for payment corresponds to the category of rationals: the bias of over-confidence makes in this circumstance the rationals more irrational than other temperaments (see Fig. 17).

5.3 Construction of the weights of anomalies

For the construction of the weights of the hierarchy, the pairs comparison matrices were calculated by using the medians of the hyperbolic factors obtained during the analysis of the results as shown in Table 17.

We report the Excel sheet with the construction of the arrays: on the left, are the actual values of the reports whilst, on the right, the approximations for the comparison (see Fig. 18).

Using the Business Performance Management website—AHP-OS (<https://bpmsg.com/ahp/>), the weights and consistency indices for each temperament were calculated. The results are obtained as shown in Figs. 19, 20, 21 and 22.

The structure of the AHP then becomes as shown in Fig. 23.

Artisan	Delay	Mag	Sign		Artisan	Delay	Mag	Sign		Artisan	Delay	Mag	Sign
Delay	1.00	0.28	0.12		Delay	1	0.25	0.11		Delay	1	0.1	0.12
Mag	3.63	1.00	0.43		Mag	4	1	0.50		Mag	11.4065	1	1.40
Sign	8.49	2.34	1.00		Sign	9	2	1.00		Sign	8.14751	0.714286	1.00
Guardian	Delay	Mag	Sign		Guardian	Delay	Mag	Sign		Guardian	Delay	Mag	Sign
Delay	1.00	0.16	1.00		Delay	1.00	0.17	1.00		Delay	1.00	0.17	1.00
Mag	6.40	1.00	6.40		Mag	6.00	1.00	6.00		Mag	6.00	1.00	6.00
Sign	1.00	0.16	1.00		Sign	1.00	0.17	1.00		Sign	1.00	0.17	1.00
Idealist	Delay	Mag	Sign		Idealist	Delay	Mag	Sign		Idealist	Delay	Mag	Sign
Delay	1.00	0.38	1.00		Delay	1.00	0.33	1.00		Delay	1.00	0.33	1.00
Mag	2.64	1.00	2.64		Mag	3.00	1.00	3.00		Mag	3.00	1.00	3.00
Sign	1.00	0.38	1.00		Sign	1.00	0.33	1.00		Sign	1.00	0.33	1.00
Rational	Delay	Mag	Sign		Rational	Delay	Mag	Sign		Rational	Delay	Mag	Sign
Delay	1.00	0.44	0.06		Delay	1.00	0.50	0.11		Delay	1.00	0.50	0.11
Mag	2.28	1.00	0.13		Mag	2.00	1.00	0.13		Mag	2.00	1.00	0.13
Sign	17.03	7.47	1.00		Sign	9.00	8.00	1.00		Sign	9.00	8.00	1.00

Fig. 18 Matrices of the pair comparison of anomalies with respect to 4 temperaments

Artisan

Priorities

These are the resulting weights for the criteria based on your pairwise comparisons:

Cat		Priority	Rank	(+)	(-)
1	Delay	7.2%	3	0.3%	0.3%
2	Mag	30.1%	2	1.2%	1.2%
3	Sign	62.2%	1	2.5%	2.5%

Number of comparisons = 3
 Consistency Ratio CR= 0.2%

Decision Matrix

	1	2	3
1	1	0.25	0.11
2	4.00	1	0.52
3	9.00	2.00	1

Principal eigen value=3.002
 Eigenvector solution: 2 iterations, delta =1.8E-8

Fig. 19 Weight construction for the artisans

Guardian

Priorities

These are the resulting weights for the criteria based on your pairwise comparisons:

Cat		Priority	Rank	(+)	(-)
1	Delay	12.5%	2	0.0%	0.0%
2	Mag	75.0%	1	0.0%	0.0%
3	Sign	12.5%	2	0.0%	0.0%

Number of comparisons = 3
 Consistency Ratio CR= 0,0%

Decision Matrix

	1	2	3
1	1	0.17	1.00
2	6.00	1	6.00
3	1.00	0.17	1

Principal eigen value=3.000
 Eigenvector solution: 2 iterations, delta =00E+0

Fig. 20 Weight construction for the guardians

Idealist

Priorities

These are the resulting weights for the criteria based on your pairwise comparisons:

Cat		Priority	Rank	(+)	(-)
1	Delay	20.0%	2	0.0%	0.0%
2	Mag	60.0%	1	0.0%	0.0%
3	Sign	20.0%	2	0.0%	0.0%

Number of comparisons = 3
 Consistency Ratio CR= 0,0%

Decision Matrix

	1	2	3
1	1	0.33	1.00
2	3.00	1	3.00
3	1.00	0.33	1

Principal eigen value=3.000
 Eigenvector solution: 1 iterations, delta =6.2E-33

Fig. 21 Weight construction for the idealists

Rational

Priorities

These are the resulting weights for the criteria based on your pairwise comparisons:

Cat	Priority	Rank	(+)	(-)
1 Delay	7.4%	3	1.4%	1.4%
2 Mag	12.2%	2	2.3%	2.3%
3 Sign	80.4%	1	15.4%	15.4%

Number of comparisons = 3
 Consistency Ratio CR= 3,9%

Decision Matrix

	1	2	3
1	1	0.5	0.11
2	2.00	1	0.12
3	9.00	8.00	1

Principal eigen value=3.037
 Eigenvector solution: 3 iterations, delta =6.9E-8

Fig. 22 Weight construction for the rationals

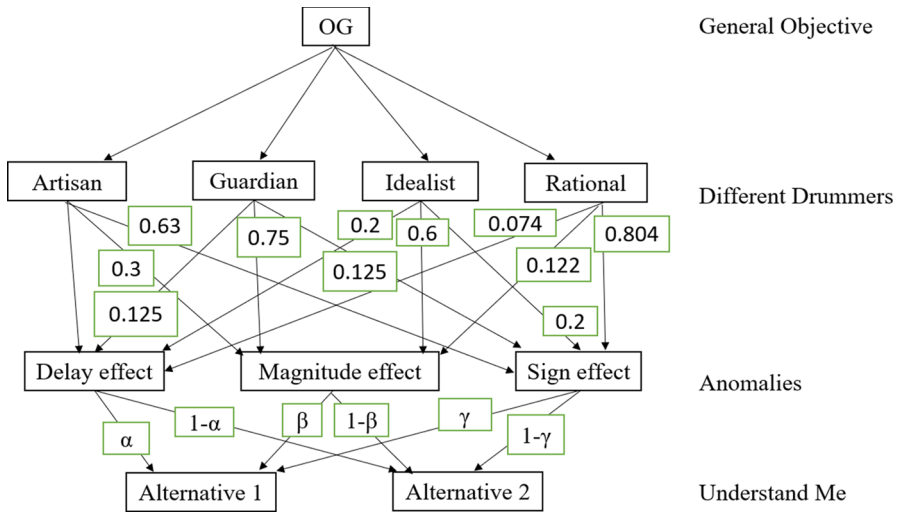


Fig. 23 Final structure of the AHP

Table 18 Matrix of comparisons to pairs of Different Drummers

GO	Artisan	Guardian	Idealist	Rational
Artisan	1	G/A	I/A	R/A
Guardian	A/G	1	I/G	R/G
Idealist	A/I	G/I	1	R/I
Rational	A/R	G/R	I/R	1

Before concluding we observe that, unlike the structure that had 8 strokes, the quiz used in the experimental phase predicts the lowest score for the main temperament. Therefore, for the determination of the weights of the Different Drummers level the matrix is shown in Table 18:

6 Conclusions

The limit of rationality is specific to human nature: we do things that we really do not want to do and the principles which guide our choices change continuously, in relation to external and internal factors. In addition, we are all different drummers, and each temperament is more or less prone to particular behavioral distortions. If, instead of focusing on what is right, we shift our attention to what is best for us, we will be able to accept the consequences of our decisions more calmly. In fact, we have seen how, due to factors such as behavioral biases and lack of self-control, we are less rational than we expected.

In Thaler and Sunstein (2014), it is presented the concept of “libertarian paternalism”: from the moment people fail to identify and make correct and useful decisions, it is right to intervene with small “gentle pushes”. In particular, the authors write: “We consider ourselves paternalistic as we think it is permissible for the architects of choices to try to influence the behaviors of individuals in order to make their lives longer, healthier and better.”

The idea is therefore to make the most of the opportunity to have a positive, that is, paternal, impact on the lives of people living in a society which manipulates individuals. In this regard, decision theory allows a personalized architecture of choice: by expanding the structure with the insertion of elements such as age, sex, education and wealth, it is possible to reach the weights of every nuance which constitutes our person.

Such an approach can be applied not only to financial advice but also to any type of advice. In effect, several mechanisms such as dependence on smoking, alcohol, food, and procrastination are just some of the social phenomena to which the theory of decisions and intertemporal choices can give strong support. Mathematics and psychology combine, thus, to achieve the same goal: the well-being of people.

Summarizing, the main contribution of this paper is the use of the AHP methodology in the treatment of inconsistency in intertemporal choices.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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