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RESEARCH ARTICLE

Dynamic optimal hedge ratio design when price and production are stochastic with jump

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Abstract

In this paper, we focus on the farmer's risk income when using commodity futures, when price and output processes are randomly correlated and represented by jump-diffusion models. We evaluate the expected utility of the farmer's wealth and determine the optimal consumption rate and hedging position at each point in time given the harvest timing and state variables. We find a closed form for the optimal consumption and positioning rate in the case of an investor with CARA utility. This result (see Table 3.3) is a generalization of the result of Ho (J Financ 39:351–376, 1984), which considers the special case in which price and output are diffusion models.

Keywords Jump-diffusion process · Futures · stochastic dynamic programming · Lévy measure · Risk management

JEL Classifications Q14 · Q12 · D81 · G13 · G52

1 Introduction

Among the many individuals or organizations that must address risk in agriculture are: Farmers, agricultural traders, commercial enterprises that sell to or buy from farmers, agricultural research personnel, and policy makers and planners.

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Agricultural traders who buy from farmers may well take into account farmers' willingness to reduce market risks. For example, some of them will be willing to accept a lower price for their production if the buyer is willing to offer a futures contract at a guaranteed price.

Buying or selling derivatives can be useful to reduce price risk for both future and future products. The most important examples are hedging commodities in the futures market or buying or selling call or put options (see Sakong et al. 1993), depending on the farmer's hedging needs (see Anderson and Danthine 1980 and Anderson and Danthine 1983). For more details on futures hedging under price or production risks, see Moschini and Hennessy (2001), Harvey Lapan and Moschini (1991), Lapan and Moschini (1994) and Lioui and Poncet (1996, 2003). It is also important to consider the risk aversion of farmers (see Morgenstern and Von Neumann 1953 and Lien and Hardaker 2001 for more details). The absolute risk aversion function can be categorized according to its evolution with respect to the increase in wealth, such as increasing absolute risk aversion (IARA), absolute risk aversion (CARA), or decreasing absolute risk aversion (DARA).

Constant risk aversion (CARA) means that preferences remain unchanged when a constant amount is added to or subtracted from all payments.

Anderson and Danthine (1983), Marcus and Modest (1984), Ho (1984), and Hey (1987) first develop dynamic hedging models in which producers are assumed to be able to revise their hedge position during the growth period. Ho (1984) allows hedging positions to be continuously adjusted over time. Karp (1988) extended Anderson and Danthine's models to include stochastic production. Karp (1987) developed a continuous model similar to that of Ho (1984). Unfortunately, the Ho (1984) paper assumes that the evolution of commodity prices follows a continuous Brownian-type diffusion process. In fact, Brownian processes do not really take into account the occurrence of jumps or strong turbulence in the evolution of the price of commodities.

In this paper, we study a normative model of the farmer's optimal hedging strategy in relation to his consumption behavior in a continuous time jump-diffusion framework. Some works, such as Rolfo (1980), have analyzed the farmer's hedging strategies in a single-period context. However, as noted in Ho (1984), a one-period model imposes some fairly restrictive conditions on the individual's behavior.

Our contribution is to revisit Thomas Ho's work in the same context, taking into account jumps in price trends. Under our new assumptions regarding the stochastic evolution of price and quantity, added to those of Ho (1984), we obtain a coverage ratio (see the last row of Table 3.3) that is quite different from that found in Ho's work.

This paper is divided into four sections. The Sect. 2 examines the optimal portfolio and determines the design of its optimal decisions, the Sect. 3 provides the optimal hedge ratio, and the Sect. 4 contains the final conclusions and remarks, as well as some possible orientations for the management of agricultural risks.

2 Model

2.1 Assumptions

A-1 • The farmer's optimal behavior is determined in the context of a continuous-time finite horizon. The harvest time T is assumed to be known.
• At the beginning of the period (the planting season), the farmer makes his production decision on the quantity of crops, Q bushels, to be produced. At the end of the period (the harvest season), he sells the entire crop at the current spot price, P (per bushel), so that his farm income is PQ.

A-2 • During the production period (time between the planting and harvest seasons) the farm income is subject to two sources of uncertainty:

1. the price at which the crop will be sold (Price risk), P.
2. The quantity of the crop which will be sold (output risk) Q.

• At each time t, the farmer forms expectations of the spot price P_t and output Q_t at harvest, and then at the next instant, with more information, the farmer revises the expectations:
• P_t and Q_t are Itô-Lévy processes.

$$dQ_t = \sigma_Q Q_t dZ_t. \tag{1}$$

$$\frac{dP_t}{P_{t_-}} = \sigma_P d\omega_t + (e^J - 1)dN_t - \lambda E(e^J - 1)dt \tag{2}$$

$$dZ_t d\omega_t = \rho dt, \tag{3}$$

where σ_Q and σ_P are constant instantaneous standard deviations of output or spot price, ρ is the instantaneous correlation coefficient, dZ_t and $d\omega_t$ are standardized Wiener processes, and dN_t is a Poisson process with arbitrarily distributed jump amplitude J. The symbol E in front of $(e^J - 1)$ stands for the expected value under physical measure and the constant variable λ is the jump intensity of the Poisson process. The three stochastic processes N_t, ω_t and J are independent of each other. For more details see Zhang et al. (2012) and Ho (1984).

A-3 There are frictions in the real sector.

i) During the production period, the farmer cannot influence the production level by further investment (buying more land acreage) or disinvestment (abandoning land or selling parts of the farm).
ii) Substantial agency costs prevent the farmer from shifting the uncertainty of farm income to the market by issuing shares.

A-4 • The futures market is assumed to be perfect:

* Participants can trade cheaply and continuously.
* The futures contracts are perfectly divisible.
* The contracts have the settlement date at the time of harvest.

* Mark-to-market settlements occur continuously over time so that the net value of the contract always remains zero.

• Each contract calls for a delivery of one bushel. Those holding long positions promise to take delivery of the underlying crop and make payment at the futures price of the contract; those holding short positions promise to deliver the underlying commodity and receive payment at the futures price of the contract at maturity.

• Let F_t denote the equilibrium settlement price on the contract at time t. F_t is stochastic, and it is characterized by $F_t = P_t$ for more details see Mahul and Vermersch (2000). Thus, $dF_t = dP_t$.

A-5 The farmer has a cash account: W_t that may be positive (negative) on which a constant rate of interest r is received (paid). Any net cash flow resulting from a position in futures contracts or from consumption is deposited into or withdrawn from this cash account. Therefore, the borrowing rate is equal to the lending interest rate.

A-6 During the period, at each instant t, the farmer has to determine optimal consumption rate: c_t^* and the futures position: x_t^* such that the expected additive utility of consumption is maximized.
$\max_{c,x} E[\int_0^T U(c,t)dt + B(Y_T, T)]$ with $Y_T = P_T Q_T + W_T$, where Y_t is the total wealth, $P_t Q_t$ is the crop, W_t is the cash account value at time t, $B(.,T)$ is the terminal utility of wealth and is assumed to be a concave function, and $U(c,t)$ is an instantaneous utility function for consumption such that : $\begin{cases} \frac{\partial U}{\partial c} > 0 \\ \frac{\partial^2 U}{\partial c^2} < 0. \end{cases}$

2.2 Objective function and the optimal decisions

- Let x_t denote the number of contracts held by the farmer at time t in a short position.
- dF_t denotes the variation due to an increase in the settlement price of the futures.
- $x_t dF_t$: denotes the amount, in cash, to pay at the clearing corporation by the farmer.
- The change in the farmers cash account is the sum of three cash flows:
  - the interest earned from the cash account: $rW_t dt$,
  - the consumption: $-c_t dt$,
  - the mark-to-market settlement of his futures position.
- This is summarized by the following budget constraint equation

$$dW_t = (rW_t - c_t)dt - x_t dF_t. \tag{4}$$

- To derive optimal decisions, x_t^* and c_t^*, we use the Bellman stochastic dynamic programming technique. The objective function

$$J^T(W, F, Q, t) = \max_{c_t, x_t, t<T} E_t\left[\int_t^T U(c,s)ds + B(Y_T, T)\right] \tag{5}$$

where E_t is an expectation operator, conditional on $W(t) = W$, $F(t) = F$, $Q(t) = Q$ and $c_s > 0$.

- J^T is the solution of Hamilton-Jacobi-Bellmann equation

$$\max_{c,x}[dJ^T + U(c,t)dt] = 0 \tag{6}$$

satisfying the boundary condition

$$J^T(W, F, Q, T) = B(F_T Q_T + W_T, T). \tag{7}$$

- The boundary condition suggests that the optimized derived utility equals to, (at time T), the terminal utility of wealth, since the consumption equal to zero when the wealth equal to zero to. This is the initial value to our dynamic programming. Thus, this problem is a time backward one (see Beckmann and Czudaj 2013 for further details).

3 Solution

3.1 Evaluation of dynamics dW_t, dY_t and dJ^T

Let us evaluate dW_t.
$\begin{cases} dF_t = dP_t \\ F_t = P_t, \end{cases}$ implies

$$\frac{dF_t}{F_{t\_}} = \sigma_F d\omega_t + (e^J - 1)dN_t - \lambda E(e^J - 1)dt, \tag{8}$$

and

$$dZd\omega = \rho dt. \tag{9}$$

Substituting dF_t into (4), we obtain

$$dW_t = [rW_t - c_t + x_t\lambda F_{t\_}E(e^J - 1)]dt - x_t F_{t\_}\sigma_F d\omega_t - x_t F_{t\_}(e^J - 1)dN_t \tag{10}$$

Let us evaluate the dynamic dY_t of the wealth farmer Y_t.

$$Y_t = P_t Q_t + W_t = F_t Q_t + W_t \tag{11}$$

implies

$$\begin{aligned} dY_t =& dF_t Q_t + dW_t \\ =& F_t dQ_t + Q_t dF_t + d[F_t, Q_t] + dW_t \\ =& F_t dQ_t + Q_t dF_t + d < F_t, Q_t >^c + \Delta F_s \Delta Q_s + dW_t, \end{aligned} \tag{12}$$

where $d < F_t, Q_t >^c = \rho\sigma_F\sigma_Q F_{t-}Q_t dt$ and $\Delta Q_s = 0$, then (12) takes the form:

$$\begin{aligned} dY_t &= [(rW_t - c_t) - \lambda(Q_t - x_t)E(e^J - 1)F_{t-} + \rho\sigma_F\sigma_Q F_{t-}Q_t]dt \\ &\quad + \sigma_F(Q_t - x_t)F_{t-}d\omega_t + \sigma_Q F_{t-}Q_t dZ_t \\ &\quad + (Q_t - x_t)F_{t-}(e^J - 1)dN_t \end{aligned} \tag{13}$$

Let (ω^1, ω^2) be two independent standard Wiener Processes satisfying: $\omega_t = \omega_t^1$ and $Z_t = \rho\omega_t^1 + \sqrt{1-\rho^2}\omega_t^2$. Thus (13) takes the form:

$$\begin{aligned} dY_t &= \underbrace{[(rW_t - c_t) - \lambda(Q_t - x_t)E(e^J - 1)F_t + (\rho\sigma_F\sigma_Q F_t Q_t)]}_{\alpha_1} dt \\ &\quad + \underbrace{[(Q_t - x_t)F_t\sigma_F + \rho\sigma_Q F_t Q_t]}_{\alpha_2} d\omega_t^1 + \underbrace{\sigma_Q F_t Q_t\sqrt{1-\rho^2}}_{\alpha_3} d\omega_t^2 \\ &\quad + (Q_t - x_t)F_t(e^J - 1)dN_t \end{aligned} \tag{14}$$

Deriving dJ^T, we obtain:

$$\begin{aligned} dJ^T &= J_t^T dt + J_Y^T \alpha_1 dt + \frac{1}{2}J_{Y^2}^T(\alpha_2^2 dt + \alpha_3^2 dt) \\ &\quad + \lambda dt \int [J^T(Y_t + (Q_t - x_t)F_t(e^J - 1)z, t) - J^T(Y_t, t)]\nu(dz), \end{aligned} \tag{15}$$

as $E_t(d\omega_t^i) = 0$ for $i = 1, 2$.
Now consider the dynamic programming equation.
Substituting (15) into $\max_{c,x}[dJ^T + U(c,t)dt] = 0$, we obtain this following result:

$$\begin{aligned} \max_{c_t, x_t} \Bigg(& J_t^T + J_Y^T[(rW_t - c_t) - \lambda(Q_t - x_t)E(e^J - 1)F_t + (\rho\sigma_F\sigma QF_tQ_t)] \\ & + \frac{1}{2}J_{Y_t^2}^T F_t^2[(Q_t - x_t)^2\sigma_F^2 + 2\rho(Q_t - x_t)\sigma_F\sigma_Q Q_t + \sigma_Q^2 Q_t^2] \\ & + \lambda \int [J^T(Y_t + (Q_t - x_t)F_t(e^J - 1)z, t) - J^T(Y_t, t)]\nu(dz) + U(c_t)\Bigg) \\ & = 0. \end{aligned} \tag{16}$$

As Eq. (16) exists, the optimal decisions (c^*, x^*) satisfy the first order conditions.

3.2 Evaluation of c_t^* and x_t^*

Let us evaluate c_t^*.
The first order condition with respect to c is:

$$-J_Y^T + U_c(c, t) = 0 \tag{17}$$

thus, the optimal consumption rate is determined such that the marginal utility of consumption equates the marginal derived utility in wealth.
We obtain:

$$U_c(c^*, t) = J_Y^T(Y, t). \tag{18}$$

This result was obtained by Ho (1984) in case of no jump into dynamic processes.

Hence c^* is determined independently of the hedging decisions:

$$c^* = \left[\frac{\partial U}{\partial c}\right]^{-1} \left[\frac{\partial J^T(Y, t)}{\partial Y}\right]. \tag{19}$$

Let us evaluate x_t^*.
Let us write the second condition of first order and derive the optimal position x_t^*. To do this, we use utility functions defined by: $U(c, t) = e^{-\beta t} V(c)$ and $G(Y, t) = e^{-\beta t} L(Y)$ where V is the utility function of consumption, L is the utility function of wealth. The choice of U and G (as product of two functions with separable variables) is motivated by the fact that the consumption and wealth processes are Markov.
More, we suppose that these two utility functions are exponential type: $V(c) = -\frac{1}{q}\exp(-qc)$ and $L(y) = -\frac{K}{q}\exp(-rqy)$, where $q > 0$ and K are positive constants.
Let us evaluate c_t^* in this case.
since $\frac{\partial L(y)}{\partial y} = rK\exp(-rqy) = -rqL(y)$ and $\frac{\partial^2 L(y)}{\partial y^2} = r^2q^2L(y)$, thus (19) takes the form:

$$c^* = rY - \frac{1}{q}\log(rK). \tag{20}$$

Substituting these two functions into (16), we obtain:

$$\begin{aligned} 0 = \max_{c_t, x_t} \Bigg(& V(c_t) - \beta L(Y_t) - rqL(Y_t)\left[(rW_t - c_t)\right. \\ & \left. + \rho\sigma_F\sigma_Q F_t Q_t - \lambda(Q_t - x_t)E(e^J - 1)F_t\right] \\ & + \frac{1}{2}r^2q^2L(Y_t)\left[(Q_t - x_t)^2\sigma_F^2F_t^2 + 2\rho(Q_t - x_t)\sigma_F\sigma_Q F_t^2 Q_t + \sigma_Q^2F_t^2Q_t^2\right] \\ & + \lambda\int\left[e^{-rq[(Q_t - x_t)F_t(e^J - 1)z]}L(Y_t) - L(Y_t)\right]\nu(dz)\Bigg). \end{aligned} \tag{21}$$

We divide this term by $rqL(Y_t)$. Since $qL(Y_t) < 0$, thus the max is replaced by the min operator and (21) takes the form:

$$\begin{aligned} 0 = \min_{c_t, x_t} \Bigg(& \frac{V(c_t)}{rqL(Y_t)} - \frac{\beta}{rq} - \left[(rW_t - c_t) + \rho\sigma_F\sigma_Q F_t Q_t - \lambda(Q_t - x_t)E(e^J - 1)F_t\right] \\ & + \frac{1}{2}rq\left[(Q_t - x_t)^2\sigma_F^2F_t^2 + 2\rho(Q_t - x_t)\sigma_F\sigma_Q F_t^2 Q_t + \sigma_Q^2F_t^2Q_t^2\right] \\ & + \frac{\lambda}{rq}\int\left[e^{-rq[(Q_t - x_t)F_t(e^J - 1)z]} - 1\right]\nu(dz)\Bigg). \end{aligned} \tag{22}$$

Let us evaluate K :
We deduce its value by replacing (c_t, x_t) by (c_t^*, x_t^*) into (22). So we obtain:

$$\begin{aligned}K &= \frac{1}{r}\exp\Bigg(1-\frac{\beta}{r}-q\left[(rW_t-rqY)+\rho\sigma_F\sigma_Q F_t Q_t-\lambda(Q_t-x_t)E(e^J-1)F_t\right]\\ &\quad+\frac{1}{2}rq^2\left[(Q_t-x_t)^2\sigma_F^2F_t^2+2\rho(Q_t-x_t)\sigma_F\sigma_Q F_t^2 Q_t+\sigma_Q^2F_t^2Q_t^2\right]\\ &\quad+\frac{\lambda}{r}\int\left[e^{-rq[(Q_t-x_t)F_t(e^J-1)z]}-1\right]\nu(dz)\Bigg).\end{aligned} \tag{23}$$

To evaluate x_t^*, we distinguish two case: $\lambda = 0$ and $\lambda \neq 0$.

First case: $\lambda = 0$, i.e, the dynamic of price is pure diffusive process.
The condition of first order, i.e, the derivative of second term of Eq. (22) with respect to x is zero, applying it to (22) we obtain:

$$rq[(Q-x)\sigma_F^2F^2+\rho\sigma_Q\sigma_F F^2 Q]=0 \tag{24}$$

so

$$x_{t,P}^* = \left(1+\rho\frac{\sigma_Q}{\sigma_F}\right)Q. \tag{25}$$

Remark 3.1 The optimal hedge ratio $\frac{x_{t,P}^*}{Q} = (1+\rho\frac{\sigma_Q}{\sigma_F})$ depends on the correlation ρ between the two uncertainties, the price P and the quantity Q. If the correlation is negative, the farmer's revenue is less uncertain, thus the farmer would not hedge his entire position in futures market.

Second case: $\lambda \neq 0$.
As Aït-Sahalia et al. (2009), we choose a Levy measure to obtain a closed form.
Consider the Levy measure defined by: $\nu(dz) = \beta e^{-\eta z}\mathbf{I}_{\{z\geq 0\}}dz$ where β and η are strictly positive constant. This measure satisfies $\int_{\mathbb{R}}\min(1,|z|)\nu(dz) < \infty$
The calculation of integral term into (22) gives:

$$\begin{aligned}\frac{\lambda}{rq}\int[e^{-rq(Q_t-x_t)F_t(e^J-1)z}-1]\nu(dz) &= \frac{\lambda}{rq}\int_0^{+\infty}[e^{-rq(Q_t-x_t)F_t(e^J-1)z}-1]ce^{-\eta z}dz\\ &= \frac{\lambda\beta}{rq}\int_0^{+\infty}[e^{-[rq(Q_t-x_t)F_t(e^J-1)+\eta]z}-e^{-\eta z}]dz\\ &= \frac{\lambda\beta}{rq}\Big[\frac{1}{rq(Q_t-x_t)F_t(e^J-1)+\eta}-\frac{1}{\eta}\Big].\end{aligned} \tag{26}$$

The second condition of first order applying to (22) give us:

$$\begin{aligned}&-\lambda E(e^J-1)F_t-rq(Q_t-x_t)\sigma_F^2F_t^2-rq\rho\sigma_F\sigma_Q F_t^2Q_t\\ &+\lambda\beta\Big[\frac{F_t(e^J-1)}{(rq(Q_t-x_t)F_t(e^J-1)+\eta)^2}\Big]=0.\end{aligned} \tag{27}$$

(27) is an implicit form of cubic equation in $(Q_t - x_t)$ contains x_t. Otherwise x_t is solution of (27).
Setting $S = (Q_t - x_t)$, (27) takes the form $aS^3 + bS^2 + cS + d = 0$ and setting $X = S + \frac{b/a}{3}$, the equation takes the form $X^3 + pX + q = 0$. The discriminant is $\Delta = \frac{p^3}{27} + \frac{q^2}{4}$. We solve in case where $\Delta < 0$ (this implies that $p < 0$). The equation has three real solutions. Since we minimize in the objective function , we consider the smallest solution defined by:

$$(Q_t - x_t^*) = -\frac{b/a}{3} + \sqrt{\frac{-4p}{3}}cos(\frac{1}{3}arccos(-q\sqrt{\frac{-27}{4p^3}} - \frac{2\pi}{3})) \tag{28}$$

Replacing p and q by its values we obtain:

$$x_t^* = (1 + \frac{1}{3}\rho\frac{\sigma_Q}{\sigma_F})Q + \Theta \tag{29}$$

where $\Theta = \frac{2\eta}{3rqF} + \frac{\lambda E(e^J-1)}{rq\sigma_F^2 F}(e^J - 1) + \Big(+ \frac{4}{9}\frac{\rho^2\sigma_Q^2 Q^2}{\sigma_F^2} + \frac{8}{3}\big(\frac{2}{3}(e^J - 1) - 1\big)\frac{\eta\lambda E(e^J-1)}{r^2q^2\sigma_F^2 F^2} - \frac{8\eta\rho\sigma_Q Q}{3rq\sigma_F F} + \big(\frac{16}{3} - \frac{4}{(e^J-1)}\big)\frac{\eta^2}{3r^2q^2F^2} + \frac{16\rho\sigma_Q Q\eta}{9\sigma_F rqF} + \frac{8\lambda\rho\sigma_Q QE(e^J-1)}{9rq\sigma_F^3 F}(e^J-1) + \frac{4\lambda^2E^2(e^J-1)}{9r^2q^2\sigma_F^4F^2}(e^J - 1)^3\Big)^{\frac{1}{2}} cos(\frac{1}{3}arccos(-q\sqrt{\frac{-27}{4p}} - \frac{2\pi}{3}))$, which depends on the choice of jump component.

3.3 Comparison with Ho et al.

	Thomas HO et al.	Nyassoke et al.
Empirical facts	Pure diffusion model	Jump-diffusion model
Great and sudden variations in prices	No price jumps.	Generic property of the model
Markets are incomplete; some risks can not be hedged	Markets are complete	Markets are incomplete
Some strategies are better	All strategies lead to zero residual risk depending of choice of measure	Hedging is obtained by solving a optimization portfolio problem
Utility function of wealth	CARA type	CARA type
Optimal consumption	$c^* = [\frac{\partial U}{\partial c}]^{-1}\left[\frac{\partial J^T(Y,t)}{\partial Y}\right]$	$c^* = \left[\frac{\partial U}{\partial c}\right]^{-1}\left[\frac{\partial J^T(Y,t)}{\partial Y}\right]$
Optimal position	$x_P^* = \left(1 + \rho\frac{\sigma_Q}{\sigma_F}\right)Q$	$x_t^* = \left(1 + \frac{1}{3}\rho\frac{\sigma_Q}{\sigma_F}\right)Q + \Theta$

4 Conclusion

Our continuous-time investment-consumption model determines the optimal instantaneous consumption and optimal hedging position using futures for a farmer who wishes to manage income risk (price risk and production uncertainty). We assume that the price process is a jump-diffusion process, which generalizes Ho (1984) who assumed a diffusion process for prices. We show that optimal instantaneous consumption is unchanged in both models. Assuming that the farmer's preferences for consumption and wealth are represented by exponential utility functions, we determine a closed form solution for the optimal hedge ratio. Our principal result is the last line of the Table 3.3

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