




# Heterogeneous beliefs, monetary policy, and stock price volatility

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## Abstract

In this paper, I build a two-agent New Keynesian model in which households with subjective and objective beliefs about capital gains from stock prices exist. The former type of households constructs their beliefs about expected capital gains by Bayesian learning from observed growth rates of stock prices. In a homogenous agent model with only subjective beliefs, the effect of the interest rate on stock prices tends to be unrealistically strong. I show how the presence of heterogeneity improves second moments of stock prices with realistic moments of business cycle properties. This quantitative improvement in stock price behaviors allows me to conduct a realistic analysis of how the stance of monetary policy affects stock price volatilities. Strong inertia of monetary policy provides the stability of stock prices. This is because the near-term real interest rate has dominant effects on stock prices under the presence of subjective beliefs since the presence limits the forward-looking nature in pricing stocks. However, because output depends on the expected path of the real interest rate in the forward-looking manner, strong monetary policy inertia does not necessarily provide stabilities of stock prices and output at the same time.

**Keywords** Stock price · Asset pricing · Heterogeneity · Subjective belief · Monetary policy · New Keynesian

**JEL Classification** D83 · D84 · E44 · E52 · G12 · G14

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## 1 Introduction

In this study, I build a two-agent New Keynesian model with subjective and objective beliefs about capital gains from stock prices. I show how the presence of the two agents improves second moments of stock prices with realistic moments of business cycle properties, such as output, inflation, and interest rates, which is considered to be the benefits over a homogenous agent model either with subjective or objective beliefs. To reach policy implications based on the model, I then analyze how the stance of monetary policy affects stock price volatilities under the presence of heterogeneity.

The motivation to develop a heterogeneous model is as follows. The actual stock price responses to monetary policy shocks are larger than what plain rational expectation models usually produce. Besides, studies that consider stock prices in New Keynesian models under rational expectations often assume high relative risk aversion rates than what are normally calibrated in dynamic stochastic general equilibrium models to generate large swings of stock prices as in De Paoli et al. (2010). On the other hand, having realistic moments of stock prices and business cycles at the same time is not easily attainable in a homogenous model with subjective beliefs because the impacts of interest rate shocks on stock prices are unrealistically large as in Adam and Merkel (2018).<sup>1</sup> These difficulties that rational models or homogenous models with subjective beliefs have in explaining stock price volatility warrant the importance of developing a heterogeneous model.

In addition to the issue regarding stock price volatility in response to monetary policy shocks, empirical studies show that the effects of unexpected monetary policy shocks on stock prices change over time. Laopodis (2013) finds that the nature of a dynamic relationship between monetary policy and the stock market was different in each of operating regimes under three chairmen of the Federal Reserve Board (pre-Volcker, Volcker, Greenspan). Paul (2019) empirically shows that stock market reactions to monetary policy are time-varying. However, existing studies have not reached a clear consensus on why the reactions of stock prices to monetary policy shocks change over time. These empirical studies imply the importance to study the impacts of the stance of monetary policy on stock prices in a model that can generate realistic moments of stock prices and business cycle properties at the same time. Understanding the relationship between the stance of monetary policy and stock price behaviors contributes to the discussion on whether monetary policy inertia (“gradualism”) helps reduce financial market volatilities in terms of stock market.<sup>2</sup>

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<sup>1</sup> See Challe and Giannitsarou (2014) for a summary of empirical studies about the effect of monetary policy shock on stock prices including Bernanke and Kuttner (2005).

<sup>2</sup> Srouf (2001) lists an argument that the large surprises in short-term interest rates can cause volatility in financial markets as one reason for smoothing interest rates. Rudebusch (2006) examines a discussion about a rationale for policy gradualism, which is a desire to reduce the volatility in asset prices. González-Páramo (2006) argues that a gradual monetary policy could reduce the likelihood of financial market disruptions. In actual policymaking, FOMC Secretariat (1994) records that there were discussions led by chairman Alan Greenspan on whether a 25 basis point policy tightening was preferable to a 50 basis point tightening because some members considered the larger move to have a higher probability of cracking financial markets. To this question, Bernanke (2004) gives no decisive conclusion on whether gradualism of monetary policy provides stability of financial market or asset prices.

To generate realistic moments of stock prices and business cycle properties, and to investigate how the stance of monetary policy affects stock price volatilities, I develop a New Keynesian model with the two types of households with subjective and objective beliefs about capital gains from stock prices, respectively. Households consume goods, supply labor, and save their wealth in stocks and bonds. Households with subjective beliefs construct their expectations about capital gains by Bayesian learning from observed stock price growth rates. The subjective probability belief does not equal the objective probability density as they emerge in equilibrium. The population shares of the subjective and objective households are exogenously given. I use a general equilibrium model because under nominal rigidity monetary policy affects the real interest rates and pricing kernels, while partial equilibrium models usually assume that the consumption path and consequently pricing kernel are exogenously given. Studies using Bayesian learning about stock price growth, such as a partial equilibrium model of Adam et al. (2017) or general equilibrium models of Adam and Merkel (2018), Oshima (2019), and Winkler (2019) do not consider heterogeneity. I design the model so that the effects of the existence of subjective households are minimal, i.e., it affects only stock prices to focus on the analysis of stock price behaviors while business cycle properties are kept very standard.

The main findings are as follows. First, in the general equilibrium model with heterogeneous beliefs, the overall theoretical second moments match the data well without assuming a high risk aversion rate or high habit formation. The model is successful in generating stock price volatilities close to the data with realistic moments of business cycle properties such as output and inflation. Considering both subjective and objective beliefs is beneficial because it enhances moment matching compared to a model with only subjective beliefs or objective beliefs.

Second, the model can generate empirically plausible stock price drops in response to an interest rate shock. Challe and Giannitsarou (2014) argue that a 100 basis point increase in the nominal interest rate is associated with a 2.2–9% decrease in real stock prices empirically. The model shows a 9.3% decrease in stock prices at the timing of nominal interest rate shock of 100 basis points (annual term) in impulse response analysis. The average response over four quarters at and after the shock is a 4.5% decrease. This cannot be attained in a homogenous subjective belief model because the impacts of interest rate shocks on stock prices are unrealistically large in a homogenous model. For example, Adam and Merkel (2018), Winkler (2019), and Oshima (2019) allow only small interest rate volatilities or shocks to have realistic stock price volatilities.

Third, monetary policy inertia helps stabilize stock prices because it reduces the volatility of the near-term real interest rate, which is a key variable to explain stock prices under the presence of subjective beliefs. Under a positive productivity shock, for example, the decrease in today's nominal interest rate in response to the shock becomes gradual and small when monetary policy is persistent. This is because the Taylor rule with high interest rate smoothing implies that the policy rate reacts weakly to real-time changes in inflation, which results in low volatility of today's real interest rates and hence stock prices. This supports the discussion about "gradualism" of monetary policy. Strong monetary policy inertia does not necessarily provide stabilities of output and stock prices at the same time because the presence of subjective beliefs limits the

forward-looking nature in pricing stocks, while output depends on the expected path of the real interest rate. This discrepancy implies the necessity for monetary policy to consider the path of the interest rate to stabilize both the financial and non-financial variables of the economy under the presence of heterogeneous beliefs.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents my model. Section 4 discusses the quantitative results based on the model. Section 5 investigates the relation between deep parameters of monetary policy and stock price volatilities. Section 6 concludes this study and discusses future extensions.

## 2 Related literature

Theoretical studies which explain the volatility puzzle based on rational expectation are, for example, long-run risks by Bansal and Yaron (2004) and habit formation by Campbell and Cochrane (1999). Studies which depart from rational expectation are, for example, Timmermann (1993) and Collin-Dufresne et al. (2016). Both consider investors who form expectations about fundamentals by learning. Choi and Mertens (2013), Barberis et al. (2015), and Hirshleifer et al. (2015) assume that investors form their expectations about fundamentals by extrapolation. As another strand of belief-based approach, there is research focusing on subjective beliefs about asset prices or returns such as De Long et al. (1990) and Lansing (2010).

The work of Adam et al. (2017), which my model mainly refers to, can be categorized as this type of research. They consider the homogenous investors with subjective beliefs about stock price growth (capital gains) under exogenous consumption and dividend processes. They assume that investors who know the fundamentals have subjective beliefs about stock price growth and do not have knowledge about a pricing function of stock price mapping from the fundamentals. The investors' expectations of capital gains are influenced by the capital gains observed in the past. In Adam et al. (2017), consumption and dividends are given by simple stochastic processes, whereas in this model they are generated through a New Keynesian model. Therefore, this model is suitable for investigating the relations between deep parameters regarding monetary policy and stock price volatilities.

Some studies consider stock price in New Keynesian models assuming rational expectations such as De Paoli et al. (2010). To generate large swings of stock prices, rational expectation models tend to assume higher relative risk aversion rates than what are normally calibrated in dynamic stochastic general equilibrium models.<sup>3</sup> A New Keynesian model by Wei (2009) does not have a high relative risk aversion rate. However, Wei (2009) argues that a highly persistent exogenous monetary policy shock, which is not usual in the literature, is necessary to have enough variations of marginal utility of consumption. My model can generate realistic stock price volatility without a high relative risk aversion rate or a highly persistent exogenous monetary policy shock.

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<sup>3</sup> De Paoli et al. (2010) and Challe and Giannitsarou (2014) assume 5 for relative risk aversion rate. The prominent work by Jermann (1998), which is a real model, also assumes 5.

Departing from rational expectation, another strand of research studies subjective beliefs about stock price growth in a general equilibrium model context. Adam and Merkel (2018) propose a real business cycle model to explain business cycles and stock price volatility. Investors/households homogeneously hold subjective beliefs about capital gains following Adam et al. (2017). By regarding the capital price as stock price, this model generates feedback effects of the stock price on output fluctuation. In reality, the capital price itself does not show as large volatility as the stock price. The risk-free rate volatility in their model is quite low compared to the actual data while other theoretical moments match data well.

Winkler (2019) analyzes the stock price movements and business cycles in his New Keynesian model with homogenous agents who have subjective beliefs about capital gains. The belief structures are similar to Adam et al. (2016) and Adam et al. (2017).<sup>4</sup> Stock holders are risk-neutral in his model while households are risk-averse with access to the risk-free bond market. This allows to reduce the effects of interest rates on stock prices and making the near-term dividend effects important. In stock pricing, the current dividend plays a major role in determining the stock price and the interest rate affects the stock price mainly through the dividend paid to stock holders as firms' costs in the balance sheets. Even under this setting, the stock price reaction to interest rate movements is stronger than what the usual empirical research implies. Oshima (2019) studies the stock price volatility and finds that monetary policy parameters are the keys to stock price volatility. However, he assumes homogenous subjective households and cannot generate stock price reactions to interest rate shocks with realistic sizes.

Some studies in the literature consider heterogeneous beliefs in asset pricing. Harrison and Kreps (1978) show that different beliefs about future states and short-selling restrictions lead to higher stock prices because of option values. Scheinkman and Xiong (2003) assume heterogeneity in signals under short-selling restriction. Their model also derives option values in stock price. Shiller et al. (1984) provide a model with rational and noise traders. Brock and Hommes (1998) investigate an asset pricing model with heterogeneous beliefs. Agents select different beliefs or predictors of the future price of a risky asset based on their past performances. However, because these models assume endowment economies, it is difficult to find the relations between stock price and monetary policy.

There is a growing strand of the literature to study macroeconomic questions including monetary policy transmission in New Keynesian models with heterogeneous agents; such models are often referred to as Heterogeneous Agent New Keynesian (HANK) models. These models often assume idiosyncratic shocks to individual income, incomplete market, and financial constraints. Kaplan et al. (2018) study the transmission mechanism of monetary policy focusing on empirically realistic effects of unexpected cuts of interest rates on consumption. Another strand of the literature studies Two Agent New Keynesian (TANK) model, in which two types of agents exist with different accessibilities to the financial market; these studies include Campbell and Mankiw (1990), Galí et al. (2007), Bilbiie (2008), and Debortoli and Galí (2017).

<sup>4</sup> Caines and Winkler (2018) study housing prices in a New Keynesian model with learning about housing price capital gains. Because housing stock quantity is directly included in households' utility, housing price has wealth effects on business cycles.

My two-agent model is heterogeneous in beliefs about stock price growth. This is different from the settings of usual TANK models in which heterogeneity comes from access to financial markets. However, similarly to initial motivations of HANK and TANK, I incorporate two types of agents to provide empirically realistic responses of stock prices to interest rate shocks.

### 3 Model

The model is built on a standard New Keynesian model. I assume Rotemberg type price adjustment costs. Firms pay dividends to the households, which are output minus investment cash flows and price adjustment costs. Households consume goods, supply labor, and save their wealth in stocks and bonds. Two types of households exist, those with subjective and objective beliefs about stock price growth. The population share of the subjective and objective households is exogenously given. The belief structures of the subjective households follow Adam et al. (2017), a model with exogenous consumption and dividend streams, and Adam and Merkel (2018), a real business cycle model.<sup>5</sup>

#### 3.1 Households

Households with subjective beliefs form their beliefs about stock price growth based on Kalman filtering with expected capital gain as a state variable and realized capital gain as an observed variable. They do not know other households' beliefs and do not know the pricing function mapping fundamentals to stock prices while they know fundamentals (given stock prices).

The subjective household's maximization problem basically follows the "internal rationality" discussed by Adam and Marcet (2011) and Adam et al. (2017). Internal rationality requires that agents make fully optimal decisions given a well-defined system of subjective probability beliefs about payoff-relevant external variables that are beyond their control including stock prices. That is, internal rationality means standard utility maximization given subjective beliefs about variables that are beyond their control.<sup>6</sup> In this study, following Adam et al. (2017), households with subjective beliefs do not know the stock price function derived from fundamental variables, and choose the optimal plans of stock holdings and consumption given the subjective belief about capital gains under the probability measure " $P$ ." They form this by learning from observed past growth rates of stock prices. Whatever the agents' expectations about stock price growth are, the stock price level and consumption plans satisfy the Euler equation with subjective expectations of stock price growths.

<sup>5</sup> The belief structure in Winkler (2019) is also similar. However, Winkler (2019) assumes that agents have "conditionally model-consistent expectations". Conditionally model-consistent expectations are consistent with all equilibrium conditions of the model, except those that would convey knowledge of the price that clears the asset market, when agents solve for the perceived law of motion. On the other hand, this study follows Adam and Merkel (2018) in which market clearing conditions are known by agents.

<sup>6</sup> On the other hand, external rationality postulates that agents' subjective probability belief equals the objective probability density of external variables as they emerge in equilibrium.

Key assumptions about the belief structure are as follows. (i) Homogeneity of the households with subjective beliefs is not common knowledge among the subjective households. In addition, they do not know the preferences of households with objective beliefs. This knowledge structure prevents the households from mapping the fundamentals to market-based stock prices based only on their own preferences under any population share of the two types of households and hence enables the subjective expectations of capital gains to deviate from objective expectations.

(ii) I have additional assumptions because of heterogeneity of beliefs in this model, whereas in Adam et al. (2017) and Adam and Merkel (2018) agents are homogenous. Households with objective beliefs know preferences of the subjective households and their homogeneity, other objective households' preferences and their homogeneity, and the population share of each household in the economy. Because of this belief structure, objective households can apply the law of iterated expectations in stock pricing, given the presence of subjective households.

### 3.1.1 Households with subjective beliefs

The infinitely lived representative household with subjective beliefs makes decisions on consumption, savings in stock and risk-free bonds, and labor supply. I assume that the household's expectations about wages and dividends are rational. The household's expectation of the growth rate of stock prices is subjective following Adam et al. (2017). The household's utility in each period is presented by the following function with consumption habit formation,

$$U(C_{s,t}, L_{s,t}) = \frac{1}{1-\gamma} (C_{s,t} - \phi C_{s,t-1})^{1-\gamma} - \chi \frac{L_{s,t}^{1+\varphi}}{1+\varphi}, \tag{1}$$

where  $C_{s,t}$  is consumption at time  $t$ ,  $L_{s,t}$  is labor at time  $t$ ,  $\gamma$  is the rate of relative risk aversion,  $\phi$  is the parameter of habit formation,  $\chi$  is the weight assigned to labor, and  $\varphi$  is the inverse of Frisch elasticity.  $s$  is an index for households with subjective beliefs.

The budget constraint of the household is given by the following equation. The households have access to the financial market via stocks and bonds.

$$\begin{aligned} S_{s,t} p_t^s + C_{s,t} + p_t^s \frac{\zeta^S}{2} (S_{s,t} - S_{ss})^2 + B_{s,t} + \frac{\zeta^B}{2} (B_{s,t} - B_{ss})^2 \\ = S_{s,t-1} (p_t^s + d_t) + w_t L_{s,t} + R_t^f B_{s,t-1}. \end{aligned} \tag{2}$$

$S_{s,t}$  is stock holdings at time  $t$ ,  $p_t^s$  is real stock price at time  $t$ ,  $B_{s,t}$  is real bond holding at time  $t$ ,  $R_t^f$  is real interest rate at time  $t$ ,  $d_t$  is real dividend at time  $t$ , and  $w_t$  is real wage at time  $t$ .  $S_{ss}$  is steady state stock holdings.  $B_{ss}$  is steady state bond holdings.  $p_t^s \frac{\zeta^S}{2} (S_{s,t} - S_{ss})^2$  and  $\frac{\zeta^B}{2} (B_{s,t} - B_{ss})^2$  are the stock and bond adjustment costs, respectively.<sup>7</sup>  $\zeta^S$  is the parameter of stock adjustment cost and  $\zeta^B$  is the parameter of

<sup>7</sup> Both adjustment costs have positive values when the stock and bond holdings deviate from steady state levels. I assume these costs to incorporate demand for stock and bond from the two types of households.

bond adjustment cost.<sup>8</sup> Subjective households are under the constraint of minimum and maximum stock holding positions. This allows one to have maximums in their optimization problem under subjective beliefs. In equilibrium, this constraint is not binding over the entire time path.

$$\underline{S}_s \leq S_{s,t} \leq \overline{S}_s. \quad (3)$$

The household maximization problem is given by

$$\begin{aligned} \max E_0^P \sum_{t=0}^{\infty} \delta^t \exp(Z_t) U(C_{s,t}, L_{s,t}) \\ \text{subject to (2) and (3),} \end{aligned} \quad (4)$$

where  $E_0^P$  denotes the subjective expectation operator at time 0. This setting basically follows Adam et al. (2017).  $\delta \in (0, 1)$  is the time preference rate.  $\exp(Z_t)$  denotes the preference shock. The preference shock process is formulated with persistency parameter  $\rho_Z$  as

$$Z_t = \rho_Z Z_{t-1} + \epsilon_t^Z, \quad (5)$$

where  $\epsilon_t^Z$  represents an i.i.d. stochastic shock regarding the preference shock process.

The first-order conditions with respect to  $C_{s,t}$  and  $S_{s,t}$  are given by

$$\lambda_{s,t} = \exp(Z_t)(C_{s,t} - \phi C_{s,t-1})^{-\gamma} - \delta \phi E_t^P \left[ \exp(Z_{t+1})(C_{s,t+1} - \phi C_{s,t})^{-\gamma} \right] \quad (6)$$

and

$$1 + \zeta^S (S_{s,t} - S_{ss}) = \delta E_t^P \left[ \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \left( \frac{p_{t+1}^s}{p_t^s} + \frac{d_{t+1}}{p_t^s} \right) \right], \quad (7)$$

where  $\lambda_{s,t}$  represents the Lagrange multiplier for (2). The expectation operator of this Euler equation is governed by subjective beliefs, as explained later. The bond Euler equation is

$$1 + \zeta^B (B_{s,t} - B_{ss}) = \delta E_t^P \left[ \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \right] R_t^f. \quad (8)$$

The first-order condition with respect to labor supply is

$$\chi L_{s,t}^\varphi = \lambda_{s,t} w_t. \quad (9)$$

<sup>8</sup> These representations of stock and bond adjustment costs are parsimonious ways to aggregate two types of agents by having a mathematical representation similar to that obtained by the mean-variance optimization under constant absolute risk aversion utility. For example, see Brock and Hommes (1998) and Hanson and Stein (2015). Intuitively speaking,  $\zeta^S$  and  $\zeta^B$  represent the variance of expected returns of stocks and bonds, respectively.



### 3.1.2 Subjective expectation of stock price growth

The households' expectations of stock price growth are subjective and use the Kalman filter to form the belief. I assume that households perceive stock prices to evolve according to

$$\frac{p_{t+1}^s}{p_t^s} = \beta_{t+1} + \epsilon_{t+1}. \tag{10}$$

where  $\epsilon_{t+1}$  is the transitory shock to price growth,  $\epsilon_{t+1} \sim N(0, \sigma_\epsilon^2)$ .  $\beta$  is the unobserved persistent price growth component following Adam et al. (2017). The persistent component of stock prices drifts according to

$$\beta_{t+1} = \beta_t + \nu_{t+1}, \tag{11}$$

where  $\nu_{t+1}$  is the innovation to price growth,  $\nu_{t+1} \sim N(0, \sigma_\nu^2)$ . The Kalman filter implies that the beliefs are given by

$$\beta_{t+1} \sim N(m_t, \sigma_\beta^2), \tag{12}$$

where  $m_t$  is the conditional expectation of  $\beta_{t+1}$  and  $\sigma_\beta^2$  is the steady state Kalman filter uncertainty.

I assume that  $\epsilon$  and  $\nu$  are independent and the variances of each shock satisfy

$$\sigma_\nu^2 \ll \sigma_\epsilon^2, \tag{13}$$

so that the Kalman gain for the persistent component of stock price growth becomes small.<sup>9</sup> With the optimal constant gain  $g$ , the Kalman gain process becomes

$$m_t = m_{t-1} + g \left( \frac{p_t^s}{p_{t-1}^s} - m_{t-1} \right). \tag{14}$$

The optimal constant Kalman gain  $g$  is given by

$$g = \frac{\sigma_\beta^2}{\sigma_\beta^2 + \sigma_\epsilon^2}, \tag{15}$$

and the steady state uncertainty  $\sigma_\beta$  is calculated as

$$\sigma_\beta^2 = \frac{\sigma_\nu^2 + \sqrt{(\sigma_\nu^2)^2 + 4\sigma_\nu^2\sigma_\epsilon^2}}{2}. \tag{16}$$

<sup>9</sup> The reason why a small value is necessary for the optimal Kalman gain  $g$  is that when  $g$  is not small enough, the Blanchard-Kahn condition is not satisfied in general equilibrium, as mentioned later. From (15) and (16) shown shortly,  $g$  becomes large if I do not assume (13). The calibrated value for  $g$  in this model is  $\frac{1}{150}$ .

Internal rationality assumes that subjective agents make fully optimal decisions given a well-defined system of subjective probability beliefs about stock prices. I assume that subjective households construct and update their expectations of capital gains at time  $t + 1$ ,  $\frac{p_{t+1}^s}{p_t^s}$ , by using  $m_t$  in (14) under the subjective probability measure  $P$ . Under this assumption with (7), the Euler equation becomes

$$1 + \zeta^S(S_{s,t} - S_{ss}) = \delta E_t^P \left[ \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \left( m_t + \frac{d_{t+1}}{p_t^s} \right) \right]. \quad (17)$$

$m_t$  evolves according to (14). Because (14) includes only the present and past variables,  $m_t$  is not a stochastic variable at time  $t$ . Internal rationality implies fully rational utility maximization given the subjective beliefs about stock price growth,  $m_t$ . Therefore, I have

$$1 + \zeta^S(S_{s,t} - S_{ss}) = \delta E_t \left[ \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \right] m_t + \delta E_t \left[ \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \frac{d_{t+1}}{p_t^s} \right], \quad (18)$$

where  $E_t$  denotes the rational expectation operator given the subjective beliefs of stock price growth.

The equation obtained by substituting (14) for  $m_t$  in (18) implies simultaneous determination of the price beliefs and prices, and could generate multiple solutions of stock price because this equation is a quadratic function of the stock price  $p_t^s$ . In this study, I use the log-linearization around the steady state to solve this model. Therefore, I can avoid the simultaneity problem by setting a steady state at a point that economically makes sense, even though the learning process is set as (14). However, as often used in the learning literature such as in Eusepi and Preston (2011) and Evans and Honkapohja (2001), I can also consider another version of the Kalman gain process by modifying the observation timing as

$$m_t = m_{t-1} + g \left( \frac{p_{t-1}^s}{p_{t-2}^s} - m_{t-1} \right). \quad (19)$$

Besides avoiding the simultaneity problem, due to superiority in data fitting, I use the belief system addressed in (19) instead of (14) in this study.

### 3.1.3 Households with objective belief

The optimization problem of the infinitely lived representative household with objective beliefs is standard. The households' expectation about the growth rate of the stock price is objective. The household's utility in each period is presented by the following function with consumption habit formation:

$$U(C_{o,t}, L_{o,t}) = \frac{1}{1 - \gamma} (C_{o,t} - \phi C_{o,t-1})^{1-\gamma} - \chi \frac{L_{o,t}^{1+\varphi}}{1 + \varphi}, \quad (20)$$

where  $C_{o,t}$  is consumption at time  $t$  and  $L_{o,t}$  is labor at time  $t$ .  $o$  is the index for households with objective beliefs.

The budget constraint of the household is given by the following equation,

$$\begin{aligned}
 S_{o,t} p_t^s + C_{o,t} + p_t^s \frac{\zeta^S}{2} (S_{o,t} - S_{ss})^2 + B_{o,t} + \frac{\zeta^B}{2} (B_{o,t} - B_{ss})^2 \\
 = S_{o,t-1} (p_t^s + d_t) + w_t L_{o,t} + R_t^f B_{o,t-1},
 \end{aligned}
 \tag{21}$$

where  $S_{o,t}$  is the stock holdings by the households at time  $t$  and  $B_{o,t}$  is the bond holdings by the households at time  $t$ . The household’s maximization problem is given by

$$\begin{aligned}
 \max E_0 \sum_{t=0}^{\infty} \delta^t \exp(Z_t) U(C_{o,t}, L_{o,t}) \\
 \text{subject to (21)},
 \end{aligned}
 \tag{22}$$

where  $E_0$  denotes the objective expectation operator at time 0. The first-order conditions with respect to  $C_{o,t}$  and  $S_{o,t}$  are given by

$$\lambda_{o,t} = \exp(Z_t) (C_{o,t} - \phi C_{o,t-1})^{-\gamma} - \delta \phi E_t [\exp(Z_{t+1}) (C_{o,t+1} - \phi C_{o,t})^{-\gamma}]
 \tag{23}$$

and

$$1 + \zeta^S (S_{o,t} - S_{ss}) = \delta E_t \left[ \frac{\lambda_{o,t+1}}{\lambda_{o,t}} \left( \frac{p_{t+1}^s}{p_t^s} + \frac{d_{t+1}}{p_t^s} \right) \right],
 \tag{24}$$

where  $\lambda_{o,t}$  represents the Lagrange multiplier for (21). The bond Euler equation is

$$1 + \zeta^B (B_{o,t} - B_{ss}) = \delta E_t \left[ \frac{\lambda_{o,t+1}}{\lambda_{o,t}} \right] R_t^f.
 \tag{25}$$

The first-order condition with respect to labor supply is

$$\chi L_{o,t}^\varphi = \lambda_{o,t} w_t.
 \tag{26}$$

### 3.1.4 Market clearing and stock demand

The stock market clearing with  $\alpha$  as the population share of subjective households is

$$\alpha S_{s,t} + (1 - \alpha) S_{o,t} = S_t.
 \tag{27}$$

In addition, I assume that stock supply is unity,

$$S_t = 1 \forall t.
 \tag{28}$$

From (18), (24), (27), and (28), I obtain the stock market clearing condition as

$$1 = \alpha \delta \left\{ E_t \left[ \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \right] m_t + E_t \left[ \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \frac{d_{t+1}}{p_t^s} \right] \right\} + (1 - \alpha) \delta E_t \left[ \frac{\lambda_{o,t+1}}{\lambda_{o,t}} \left( \frac{p_{t+1}^s}{p_t^s} + \frac{d_{t+1}}{p_t^s} \right) \right]. \tag{29}$$

The bond market clearing condition becomes

$$\alpha B_{s,t} + (1 - \alpha) B_{o,t} = 0. \tag{30}$$

From (8), (25), and (30), the bond market clearing condition becomes

$$1 = \delta R_t^f E_t \left[ \alpha \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \right] + \delta R_t^f E_t \left[ (1 - \alpha) \frac{\lambda_{o,t+1}}{\lambda_{o,t}} \right]. \tag{31}$$

The nominal risk-free rate satisfies

$$1 = \delta R_t^n E_t \left[ \alpha \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \frac{P_t}{P_{t+1}} \right] + \delta R_t^n E_t \left[ (1 - \alpha) \frac{\lambda_{o,t+1}}{\lambda_{o,t}} \frac{P_t}{P_{t+1}} \right]. \tag{32}$$

### 3.1.5 Log-linearized stock price equation

Before moving to the firm sector’s settings, I intuitively discuss how stock prices are formulated in the model. To summarize, stock prices react strongly to near-term information under subjective beliefs, while in a rational expectation case, they are determined by the infinite future information about dividends and stochastic discount factors. To observe this, I show a log-linearized version of the stock price equation. I substitute (19) for  $m_t$  in (29) and log-linearize this. Eliminating  $\lambda_s$  and  $\lambda_o$  in this equation using the log-linearized equation of (31) yields the stock price equation as

$$\hat{p}_t^s = -\frac{\delta^{-1}}{\delta^{-1} - \alpha} \hat{R}_t^f + \frac{\delta^{-1} - 1}{\delta^{-1} - \alpha} E_t \hat{d}_{t+1} + \frac{\alpha(1 - g)}{\delta^{-1} - \alpha} \hat{m}_{t-1} + \frac{\alpha g}{\delta^{-1} - \alpha} (\hat{p}_{t-1}^s - \hat{p}_{t-2}^s) + \frac{1 - \alpha}{\delta^{-1} - \alpha} E_t \hat{p}_{t+1}^s. \tag{33}$$

$\hat{x}$  denotes the log deviation from the steady state value of  $x$ . Again,  $\alpha$  is the population share of households with subjective beliefs. This equation implies that the real interest rate at time  $t$  affects the stock price significantly when  $\alpha$  is close to 1 because  $\frac{\delta^{-1}}{\delta^{-1} - \alpha}$  is large. The impact of the interest rate is much larger than that of dividends. In contrast, as  $\alpha$  becomes close to 0, this impact becomes weaker because  $\frac{\delta^{-1}}{\delta^{-1} - \alpha}$  becomes smaller. In addition, future interest rates and dividends become more important as  $\alpha$  becomes closer to 0 because the last term of (33) includes  $\hat{p}^s$  at time  $t + 1$ . The level of  $g$  determines how sensitively the stock price moves in response to near-term past capital gain.

### 3.2 Firms

Monopolistically competitive intermediate goods firms maximize their profits. The intermediate goods firms pay price adjustment costs which are paid by final goods following Rotemberg (1982) when they change their prices. They hire labor from the household, own capital, and conduct capital formations. Capital formations require investment adjustment costs.  $j \in [0, 1]$  is an intermediate goods firm index. Competitive final goods producers produce final goods by aggregating intermediate goods. The final goods are transformed into consumption goods and investment goods costlessly.

The final goods sector is perfectly competitive and transforms intermediate goods into final goods for consumption and investment by the CES production function,

$$Y_t = \left( \int_0^1 Y_t(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}, \tag{34}$$

where  $Y_t$  is aggregate output of final goods,  $Y_t(j)$  is output of intermediate goods, and  $\eta (> 1)$  is the demand elasticity parameter. The profit maximization of the final goods firm becomes

$$\max_{Y_t(j)} P_t \left( \int_0^1 Y_t(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} - \int_0^1 P_t(j) Y_t(j) dj, \tag{35}$$

where  $P_t(j)$  is an intermediate good price. This generates a downward-sloping demand for intermediate goods. The intermediate goods demand is set as

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\eta} Y_t. \tag{36}$$

The maximization problem of the firm producing intermediate goods  $j$  is given by

$$\max E_t \sum_{i=0}^{\infty} M_{t+i} \{d_{t+i}(j)\}, \tag{37}$$

where  $M_{t+i}$  is the pricing kernel under the presence of the two types of households who own the firm sector.  $d_{t+i}(j)$  denotes the real dividend of firm  $j$ . The real dividend cash flow is governed by

$$d_t(j) = \frac{P_t(j)}{P_t} Y_t(j) - w_t L_t(j) - I_t(j) - \frac{\zeta^P}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t, \tag{38}$$

where  $I_t(j)$  is investment of firm  $j$ , and  $\zeta^P$  is the price adjustment cost parameter.

Capital formation is defined as

$$K_t(j) = (1 - \psi) K_{t-1}(j) + \exp(v_t) I_t(j) \left( 1 - \frac{\zeta^I}{2} \left( \frac{I_t(j)}{I_{t-1}(j)} - 1 \right)^2 \right), \tag{39}$$

where  $K_t(j)$  is capital of firm  $j$ .  $\psi \in (0, 1]$  is the depreciation rate of capital,  $\frac{\zeta^I}{2} \left(\frac{I_t(j)}{I_{t-1}(j)} - 1\right)^2$  is the investment adjustment cost, and  $\zeta^I$  is a parameter of investment adjustment costs.  $v_t$  represents the investment-specific technology shock process at time  $t$ . The shock process is formulated with the persistency parameter of  $\rho_V$  as

$$v_t = \rho_V v_{t-1} + \epsilon_t^v, \tag{40}$$

where  $\epsilon_t^v$  represents an i.i.d. stochastic shock regarding the investment specific technology shock process. The production technology of intermediate goods is given by

$$Y_t(j) = \exp(A_t) K_{t-1}(j)^\xi L_t(j)^{1-\xi}, \tag{41}$$

where  $A_t$  represents the technology level at time  $t$ .  $\xi \in (0, 1)$  is the capital share. The productivity shock process is formulated with the persistency parameter  $\rho_A$  and an i.i.d. shock to productivity  $\epsilon_t^A$  as

$$A_t = \rho_A A_{t-1} + \epsilon_t^A. \tag{42}$$

The Lagrangian of this problem can be set as

$$\begin{aligned} \mathcal{L}_t = & E_t \sum_{i=0}^{\infty} M_{t+i} \left\{ \left( \frac{P_{t+i}(j)}{P_{t+i}} \right)^{1-\eta} Y_{t+i} - w_{t+i} L_{t+i}(j) - I_{t+i}(j) \right. \\ & - \frac{\zeta^P}{2} \left( \frac{P_{t+i}(j)}{P_{t+i-1}(j)} - 1 \right)^2 Y_{t+i} \\ & + \Omega_{t+i}(j) (\exp(A_{t+i}) K_{t+i-1}(j)^\xi L_{t+i}(j)^{1-\xi} - \left\{ \frac{P_{t+i}(j)}{P_{t+i}} \right\}^{-\eta} Y_{t+i}) \\ & + q_{t+i}(j) ((1 - \psi) K_{t+i-1}(j) + \exp(v_{t+i}) I_{t+i}(j) \\ & \left. \left( 1 - \frac{\zeta^I}{2} \left( \frac{I_{t+i}(j)}{I_{t+i-1}(j)} - 1 \right)^2 \right) - K_{t+i}(j)) \right\}, \tag{43} \end{aligned}$$

where  $\Omega_{t+i}(j)$  and  $q_{t+i}(j)$  are the Lagrange multipliers. I assume that the stochastic discount factor to discount firms' cash flow is the weighted average of those of the two types of households. Define  $\Lambda$  as

$$\begin{aligned} \Lambda_t &= \frac{M_{t+1}}{M_t} \\ &= \delta \left\{ \alpha \frac{\lambda_{s,t+1}}{\lambda_{s,t}} + (1 - \alpha) \frac{\lambda_{o,t+1}}{\lambda_{o,t}} \right\}. \tag{44} \end{aligned}$$

I assume the firms are symmetric. Then, I have the first-order conditions as below.

$$w_t = \Omega_t(1 - \xi) \frac{Y_t}{L_t}. \tag{45}$$

$$q_t = E_t \Lambda_t \left\{ \Omega_{t+1} \xi \frac{Y_{t+1}}{K_t} + q_{t+1}(1 - \psi) \right\}. \tag{46}$$

$$1 = q_t \exp(v_t) \left\{ 1 - \zeta^I \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} - \frac{\zeta^I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} + E_t \Lambda_t q_{t+1} \exp(v_{t+1}) \zeta^I \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2. \tag{47}$$

$$1 + \eta(\Omega_t - 1) - \zeta^P \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} + E_t \zeta^P \Lambda_t \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t} \frac{Y_{t+1}}{Y_t} = 0. \tag{48}$$

Inflation rate is given by

$$\pi_t = \frac{P_t}{P_{t-1}}. \tag{49}$$

I define the steady state markup  $\mu = \frac{\eta-1}{\eta}$  and add the markup shock to this equation as

$$1 + \frac{\mu \exp(\kappa u_t)}{\mu \exp(\kappa u_t) - 1} (\Omega_t - 1) - \zeta^P (\pi_t - 1) \pi_t + E_t \zeta^P \Lambda_t (\pi_{t+1} - 1) \pi_{t+1} \frac{Y_{t+1}}{Y_t} = 0, \tag{50}$$

with the shock process,

$$u_t = \rho_u u_{t-1} + \epsilon_t^u. \tag{51}$$

$u_t$  represents the process of the markup shock, and  $\kappa = \zeta^P(\mu - 1)$  is a constant to adjust the size of the markup shock so that a 1% shock to  $u_t$  becomes a one unit shock to inflation in (50) when linearized.<sup>10</sup> The markup shock process is formulated with the persistency parameter  $\rho_u$  and an i.i.d. disturbance  $\epsilon_t^u$ .

Aggregate capital evolution is

$$K_t = (1 - \psi) K_{t-1} + \exp(v_t) I_t \left( 1 - \frac{\zeta^I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right). \tag{52}$$

Aggregate real dividend is

$$d_t = Y_t - w_t L_t - I_t - \frac{\zeta^P}{2} (\pi_t - 1)^2 Y_t. \tag{53}$$

Aggregate output is

$$Y_t = \exp(A_t) K_{t-1}^\xi L_t^{1-\xi}. \tag{54}$$

<sup>10</sup> The linearized equation of (50) is given by  $\hat{\pi}_t = \delta E_t \hat{\pi}_{t+1} + \frac{1}{\zeta^P(\mu-1)} \hat{\Omega}_t + \frac{1}{\zeta^P(\mu-1)} \kappa u_t$ .

### 3.3 Market clearing and monetary policy

I assume a standard Taylor rule for monetary policy as

$$\frac{R_t^n}{R_{ss}^n} = \left( \frac{R_{t-1}^n}{R_{ss}^n} \right)^{\theta_M} \left\{ \left( \frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \left( \frac{Y_t}{Y_t^f} \right)^{\phi_y} \right\}^{(1-\theta_M)} \epsilon_t^M, \quad (55)$$

where  $Y_t^f$  is output under the flexible price equilibrium at time  $t$ ,  $\theta_M \in [0, 1)$  is the persistence parameter of monetary policy,  $\phi_\pi (> 1)$  is the reaction parameter of monetary policy to the inflation rate,  $\phi_y$  is the reaction parameter of monetary policy to the output gap, and  $\epsilon_t^M$  is an i.i.d. shock to the nominal interest rate (monetary policy shock). I define output under the flexible price equilibrium,  $Y_t^f$ , as that with no price adjustment costs ( $\zeta^P = 0$ ) at time  $t$ .

Aggregate labor supply is defined as

$$L_t = \alpha L_{s,t} + (1 - \alpha) L_{o,t}. \quad (56)$$

Aggregate consumption is

$$C_t = \alpha C_{s,t} + (1 - \alpha) C_{o,t}. \quad (57)$$

The resource constraint of the entire economy becomes

$$\begin{aligned} Y_t = C_t + I_t + \frac{\zeta^P}{2} (\pi_t - 1)^2 Y_t \\ + \alpha \left\{ p_t^s \frac{\zeta^S}{2} (S_{s,t} - S_{ss})^2 + \frac{\zeta^B}{2} (B_{s,t} - B_{ss})^2 \right\} \\ + (1 - \alpha) \left\{ p_t^s \frac{\zeta^S}{2} (S_{o,t} - S_{ss})^2 + \frac{\zeta^B}{2} (B_{o,t} - B_{ss})^2 \right\}. \end{aligned} \quad (58)$$

## 4 Quantitative analysis

In this section, I show the quantitative implications of the model economy. Using a calibrated model, I compute the theoretical moments and the impulse responses of macroeconomic variables to shocks. I assume time frequency is quarterly throughout this paper. The model is solved by the first-order perturbation method around the steady state.<sup>11</sup> By applying the first-order perturbation method to the simplified model setting of Adam et al. (2017), where dividends and wages are exogenous, I can generate volatility of price dividend ratio similar to Adam et al. (2017) as shown in Oshima (2019).

<sup>11</sup> Winkler (2019) uses the second-order perturbation method to solve his New Keynesian model. I use the first-order perturbation method because the excess return of the stock is out of my interest.



Theoretical moments with the presence of both of two agents show realistic stock price and risk-free rate volatilities at the same time. Homogenous models with subjective beliefs have difficulty in having this feature.

#### 4.1 Calibration

Table 1 lists the choice of parameter values for the baseline model. Parameter values are calibrated at quarterly rates and assume those of the U.S. economy. I set the basic parameters, the rate of relative risk aversion  $\gamma$ , habit formation parameter  $\phi$ , inverse of Frisch elasticity  $\nu$ , depreciation rate  $\psi$ , capital share  $\xi$ , and investment adjustment cost parameter  $\zeta^I$  following Christiano et al. (2005). The discount rate  $\beta$  is 0.99, which is within the conventional range. The relative utility weight of labor  $\chi$  is set so that the steady state labor amount becomes 0.3. In Christiano et al. (2005), the Philips curve is not formulated based on Rotemberg type price adjustment costs, which I use in the model. For the demand elasticity parameter  $\eta$  and price adjustment parameter  $\zeta^P$ , I follow Ireland (2001), who uses Rotemberg type price adjustment costs.

The monetary policy reaction parameter to the inflation rate  $\phi_\pi$  has a conventional value, 1.5. The monetary policy reaction parameter to the output gap  $\phi_y$  is also set at

**Table 1** Baseline parameters

Parameters	Value	Description
$\delta$	0.99	Discount rate
$\gamma$	1.0	Rate of relative risk aversion
$\phi$	0.65	Habit formation parameter
$g$	$\frac{1}{150}$	Constant Kalman gain
$\psi$	0.025	Depreciation rate
$\chi$	6.5	Relative utility weight of labor
$\nu$	1	Inverse of Frisch elasticity
$\xi$	0.36	Capital share
$\zeta^I$	2.5	Investment adjustment cost parameter
$\zeta^P$	77	Price adjustment parameter
$\eta$	6	Demand elasticity
$\phi_\pi$	1.5	Monetary policy reaction parameter to inflation
$\phi_y$	0.125	Monetary policy reaction parameter to output gap
$\alpha$	0.94	Population share of subjective belief households
$\theta_M$	0.8	Monetary policy persistence parameter
$\zeta^S$	10	Stock adjustment cost parameter
$\zeta^B$	0.0005	Bond adjustment cost parameter
$\rho_A$	0.95	Autoregressive parameter of productivity shock
$\rho_V$	0.71	Autoregressive parameter of investment specific shock
$\rho_Z$	0.94	Autoregressive parameter of preference shock
$\rho_u$	0.5	Autoregressive parameter of markup shock

a conventional value, 0.125.<sup>12</sup> The monetary policy persistence parameter  $\theta_M$  is 0.8 following Christiano et al. (2005) or Smets and Wouters (2007). The autoregressive parameters of productivity  $\rho_A$  and investment-specific shock  $\rho_V$  follow Smets and Wouters (2007). The autoregressive parameter of the preference shock  $\rho_Z$  follows Levin et al. (2005). The autoregressive parameter of the markup shock  $\rho_u$  is set at 0.5 so that the standard deviation of inflation becomes close to the data.

The model specific parameter is the Kalman gain  $g$ . In Adam et al. (2017), its estimated value is 0.02–0.03 depending on the assumptions. However, in my general equilibrium model, this range of  $g$  results in explosive paths. As seen in the fourth term of (33), a large value of  $g$  leads to an explosive path of stock price when this equation is solved backward. Adam et al. (2016) estimate this value at 0.007–0.008. I set the value of  $g$  at 0.007 so that the model can satisfy the Blanchard-Kahn condition and generate realistic moments of the stock price. The baseline population share of households with subjective beliefs,  $\alpha$ , is set to fit the theoretical moments of stock price to the data.

The stock adjustment cost parameter  $\zeta^S$  and the bond adjustment cost parameter  $\zeta^B$  are set so that the relative size of the stock adjustment cost becomes larger than that of the bond adjustment cost. The calibrated values of the stock and bond adjustment costs in Table 1 do not generate notable differences in allocation variables in impulse responses under any population share of households with subjective beliefs,  $\alpha$ . The reasons why the relative size of the stock adjustment cost parameter is much higher than that of the bond adjustment cost parameter are as follows.

In a heterogeneous model, when the households with subjective beliefs have a large capital gain expectation,  $m$ , they increase stock holdings  $S_{s,t}$  as implied in (18), ceteris paribus. If the stock adjustment cost  $\zeta^S$  is large, the increase in stock holdings by the households with subjective beliefs is limited, whereas the stock holdings need to increase largely if  $\zeta^S$  is small. To finance the increase in stock holdings, they need to decrease bond holdings. If the bond adjustment cost  $\zeta^B$  is small, they mainly decrease bond holdings. However, as the bond adjustment cost  $\zeta^B$  is larger, changing bond holdings is more costly, and the households are more inclined to reduce consumption as well as bond holdings. On the other hand, households with objective beliefs decrease stock holdings, increase bond holdings, and raise consumption. As I mentioned in Sect. 1, one of the purposes of this study is to describe the model so that the effects of the existence of subjective households are minimal. By setting  $\zeta^S$  at a large value and  $\zeta^B$  at a small value, the effects of the presence of heterogeneous beliefs on consumption and consequently on business cycles become sufficiently small.<sup>13</sup>

Intuitive interpretations of the difference in relative sizes of the two adjustment cost parameters are as follows. The first interpretation is a transaction cost for households to change asset holdings. Adjustment costs of risk-free bonds should be small because, in reality, households can easily adjust positions of assets such as bank deposits, which have a similar role to risk-free bonds for households in this model. By setting  $\zeta^B$  at a

<sup>12</sup> This quarterly value implies 0.5 at an annual rate.

<sup>13</sup> The effect on consumption of the sizes of two parameters does not appear in the homogenous cases with subjective beliefs,  $\alpha = 1$ . This is because  $S_{s,t}$  is always equal to 1 (or  $S_{ss}$ ) in Eq. (18) and  $B_{s,t}$  is always equal to 0 (or  $B_{ss}$ ) in Eq. (8) from the market clearing conditions of stock and bond markets, (27), (28), and (30). The same observation applies to the homogenous cases with objective beliefs,  $\alpha = 0$ .

**Table 2** Steady state values

Variables	Steady state value	Actual data
$C_{ss}/Y_{ss}$	0.79	0.80
$I_{ss}/Y_{ss}$	0.21	0.20
$K_{ss}/(Y_{ss} * 4)$	2.14	3.16
$p_{ss}^s/d_{ss}$	99.0	70.4

The subscript *ss* means steady state value. Actual data are the average of 1980–2017 quarterly data.  $Y_{ss}$  for  $K_{ss}/(Y_{ss} * 4)$  is real GDP data to be consistent with  $K_{ss}$  data.  $Y_{ss}$  data for  $C_{ss}/Y_{ss}$  and  $I_{ss}/Y_{ss}$  is explained in “Appendix”. Actual data for  $p_{ss}^s/d_{ss}$  is the price earnings ratio

sufficiently low level close to 0, Eq. (8) becomes close to a standard Euler equation for the risk-free bond because the term of bond adjustment costs becomes close to 0. On the other hand, costs to change stock positions, such as transaction fees, are generally higher than those to change deposit positions in reality.

The second interpretation is that adjustment cost parameters are closely related to the variance of expected returns in a simple mean-variance optimization framework. As mentioned in Footnote 8,  $\zeta^S$  and  $\zeta^B$  represent the variance of expected returns of stocks and bonds in spirit, respectively. The variance of the risk-free bond return is 0 in a two-period mean-variance optimization problem because the risk-free return is known at the timing of investing. I assume a non-zero small value for  $\zeta^B$  because it is necessary to describe heterogeneous demand for the bonds from the two agents. Though I assume a non-zero positive value for  $\zeta^B$ , it should be sufficiently small to reflect the size of the risk of the bonds. A large value for  $\zeta^S$  implies that the variance of stock returns is much larger than that of the bonds.

### 4.2 Steady state values

Table 2 shows the steady state values of our model. These values are in quarterly rates and the subscript *ss* indicates steady state value. These values do not depend on whether the model is of subjective or objective beliefs because I set the steady state *m* at 1. For comparison, this table shows actual data values averaged over 1980–2017.<sup>14</sup>  $C_{ss}/Y_{ss}$  and  $I_{ss}/Y_{ss}$  are close to the actual value. The actual data for *Y* in these two fractions is the sum of *C* and *I* with a deflator adjustment as explained in “Appendix A.1”.  $K_{ss}/(Y_{ss} * 4)$  is within a plausible range. The actual  $Y_{ss}$  for  $K_{ss}/(Y_{ss} * 4)$  is real GDP data. The price dividend ratio  $p_{ss}^s/d_{ss}$  is similar to the data.

### 4.3 Model moments

Table 3 compares the second moments of the baseline model with the actual data. Moments are calculated on a quarterly basis. I show the model moments with four different population shares of households with subjective beliefs ( $\alpha$  at 1, 0.94, 0.5, and

<sup>14</sup> The sample period almost corresponds to the periods during which the U.S. central bank targeted the interest rate rather than money growth.

**Table 3** Second moments

Variables	Model ( $\alpha = 1$ )	Model ( $\alpha = 0.94$ )	Model ( $\alpha = 0.5$ )	Model ( $\alpha = 0$ )	Actual data
$SD[\log R^f]$	0.020	0.020	0.020	0.020	0.024
$SD[\log C]$	0.026	0.026	0.026	0.026	0.026
$SD[\log P^s]$	0.886	0.197	0.051	0.031	0.204
$SD[\log d]$	0.060	0.060	0.060	0.060	0.045
$SD[\log Y]$	0.033	0.033	0.033	0.033	0.037
$SD[\log I]$	0.069	0.069	0.069	0.069	0.113
$SD[\log \pi]$	0.004	0.004	0.004	0.004	0.004
$Corr[\log P^s, \log Y]$	0.33	0.586	0.882	0.897	0.59
$Corr[\log P^s, \log d]$	-0.59	-0.17	0.11	0.15	-0.44
$Autocor[\log C(-1)]$	0.99	0.99	0.99	0.99	0.97
$Autocor[\log C(-2)]$	0.98	0.98	0.98	0.98	0.94
$Autocor[\log P^s(-1)]$	0.88	0.76	0.93	0.94	0.95
$Autocor[\log P^s(-2)]$	0.69	0.57	0.87	0.90	0.87

The actual data are 1980–2017 quarterly U.S. data de-trended by third-order time polynomial regressions except for interest rate data

0). Theoretical moments of the models are based on 0.7% productivity shocks, 0.3% monetary policy shocks, 0.3% preference shocks, 0.7% investment-specific shocks, and 0.2% markup shocks in standard deviation on a quarterly basis.<sup>15</sup> Actual data moments cover 1980–2017 of the U.S. economy. I take the natural log and de-trend it by the third-order time polynomial regression, cubic detrending, except interest rate data. Details about data source are given in “Appendix A.1”.

The standard deviation of the stock price of the data is 0.204. The cases with  $\alpha = 0.94$  show a realistic standard deviation of the stock price, 0.197. What to note is that the overall theoretical second moments match the data well without assuming a high risk aversion rate or high habit formation. In contrast, the standard deviation of stock price in  $\alpha = 0.5$  case is 0.051 and that in  $\alpha = 0$  case is 0.031. These values are too small compared to the data.

Assuming a small share of objective households in addition to subjective households provides quantitative benefits such that the model has realistic stock price and risk-free rate volatilities at the same time. In the case of homogenous subjective beliefs ( $\alpha = 1$ ), the standard deviation of the stock price is 0.886, which is much larger than the data, 0.204, with realistic risk-free rate volatility. This is because the reactions of stock prices to interest rate shocks are too large in this homogenous case. In the case of  $\alpha = 0.94$ , the standard deviation of the stock price is close to the data, with the risk-free rate volatilities close to the data as well. When  $\alpha$  is less than 1, future dividends and discount rates play the roles as seen in the last term of the right hand side of (33). This reduces the effect of the real interest rate at time  $t$  which has strong impacts on stock prices with subjective beliefs.

Several correlations in the model become close to the data when I assume positive  $\alpha$ . The correlation between stock prices and dividends is negative and close to the data in  $\alpha = 1$  and  $\alpha = 0.94$  cases. The data show a negative correlation between stock prices and dividends while  $\alpha = 0$  and  $\alpha = 0.5$  cases show positive correlations.<sup>16</sup> When output increases under negative interest rate shocks, investment increases under the lowered real interest rates in response to these shocks. Since investment is more volatile than output, dividends are squeezed to a certain extent as implied in Eq. (38). In cases with  $\alpha = 1$  and  $\alpha = 0.94$ , stock prices react strongly to decreases in real interest rates. Hence, stock prices and dividends show a negative correlation in these cases. In the case with  $\alpha = 0$  and  $\alpha = 0.5$ , since the stock price is mainly determined by the sum of future discounted dividends in contrast to the cases with  $\alpha = 1$  and  $\alpha = 0.94$ , dividends as well as real interest rates play major roles in affecting stock prices. Therefore, these cases do not show a negative correlation between stock prices and dividends.

<sup>15</sup> In Smets and Wouters (2007), the productivity shock is estimated as 0.45%, monetary policy shock is estimated as 0.24%, preference shock is estimated as 0.24%, and investment-specific shock is estimated as 0.45% in standard deviation on a quarterly basis. The sample period in their study is 1966–2004. Smets and Wouters (2007) use an ARMA(1,1) process for the markup shock process which is different from this study. Levin et al. (2005) estimate that standard deviation of markup shock is 0.2%. The sample period in their study is 1955–2001.

<sup>16</sup> The definition of dividend in the model shown in (53) is not the same as that in the statistics of U.S. Bureau of Economic Analysis because of my model structure. The data are constructed here to be consistent with the definition of dividend in the model.

The correlation between stock price and output in  $\alpha = 0.94$  case is closer to the data than in other cases.  $\alpha = 1$  case indicates a lower correlation than the data, and  $\alpha = 0$  or  $\alpha = 0.5$  cases shows a higher correlation than the data. When  $\alpha$  is high, near term real interest rates play an important role in stock pricing and momentum effects, by which the stock price reacts to past growths of itself, strongly affect stock prices as well. As shown in the next subsection, the momentum effects on stock prices in high  $\alpha$  cases generate oscillations that are not observed in output responses. These oscillations reduce the correlation between stock price and output. On the other hand, objective beliefs imply that stock prices are affected by future flows of real interest rates rather than near term real interest rates. In a New Keynesian model including this model, the future flow of real interest rates is a key determinant of output. Since future real interest rates matter to both stock prices and output,  $\alpha = 0$  or  $\alpha = 0.5$  case shows a higher correlation between stock prices and output than the data and  $\alpha = 1$  or  $\alpha = 0.94$  cases.

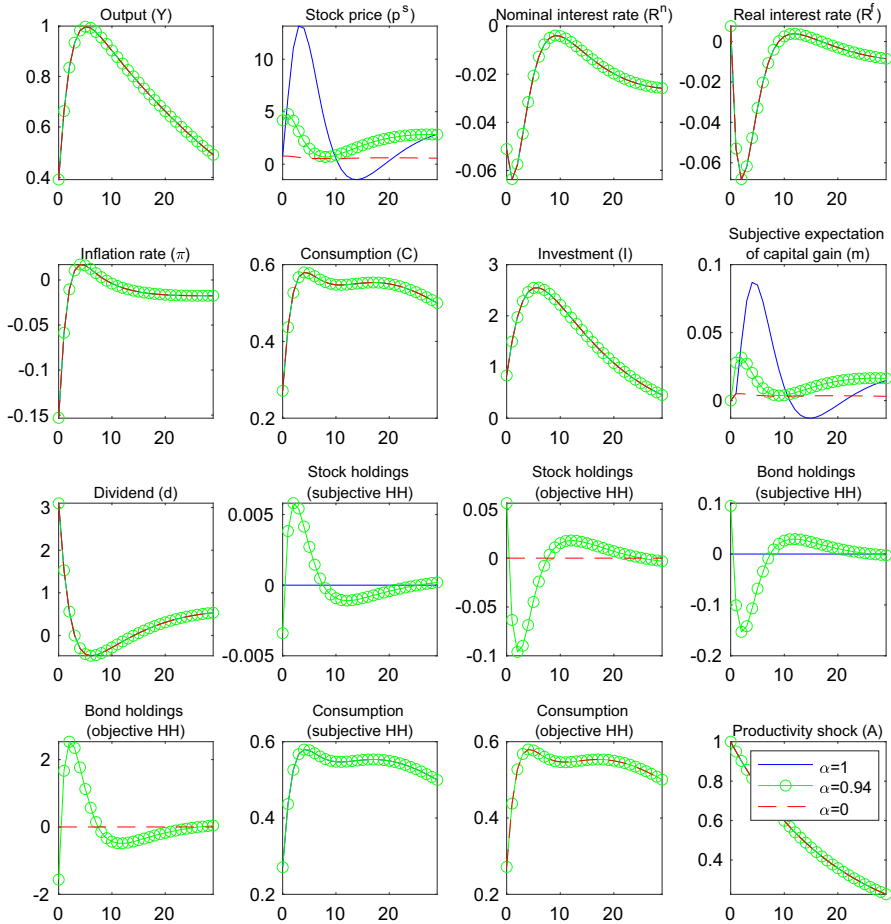
Adam et al. (2017) claim that survey measures of investors' expected return correlate positively with the price dividend ratio, whereas rational return expectations correlate negatively with the price dividend ratio.<sup>17</sup> The actual correlation between survey expectations of the stock price growth and price dividend ratio provided by Adam et al. (2017) based on 1946–2012 data is 0.79. When  $\alpha = 1$ , the correlation between price dividend ratio  $\log P^s/d$  and stock price growth expectation  $\log m$  in the model is 0.88, which is close to that in Adam et al. (2017). However, I find that the positive correlation between the price dividend ratio and the weighted average of stock price growth expectation,  $\alpha m_t + (1 - \alpha)E_t[p_{t+1}^s/p_t^s]$ , are limited only to the case of  $\alpha = 1$  or cases with  $\alpha$  very close to 1.

#### 4.4 Impulse response

This subsection shows the impulse responses to productivity, monetary policy, markup, preference, and investment-specific technology shocks. I show that heterogeneity in beliefs provides more realistic impulse responses of the stock price than homogenous cases. The time frequency is quarterly. Figure 1 indicates the impulse responses to a 1.0% positive productivity shock on a quarterly basis. Figure 2 indicates the impulse responses to a 0.25% positive monetary policy shock (shock to increase the nominal interest rate) on a quarterly basis (1.0% at an annualized rate). Figure 3 indicates the impulse responses to a 1.0% markup shock on a quarterly basis. Figures 4 and 5 show the impulse responses to a 1.0% preference shock and a 1.0% investment-specific technology shock on a quarterly basis, respectively.

In these figures, I compare the responses under different shares of households with subjective beliefs ( $\alpha=1, 0.94$ , and 0). The three cases do not show differences in aggregate variables except for the responses of stock price  $p^s$  and the subjective expectation of capital gain  $m$ . This can be considered as an advantage of this model because it can generate large stock price reactions without unrealistically increasing the responses of fundamental variables.

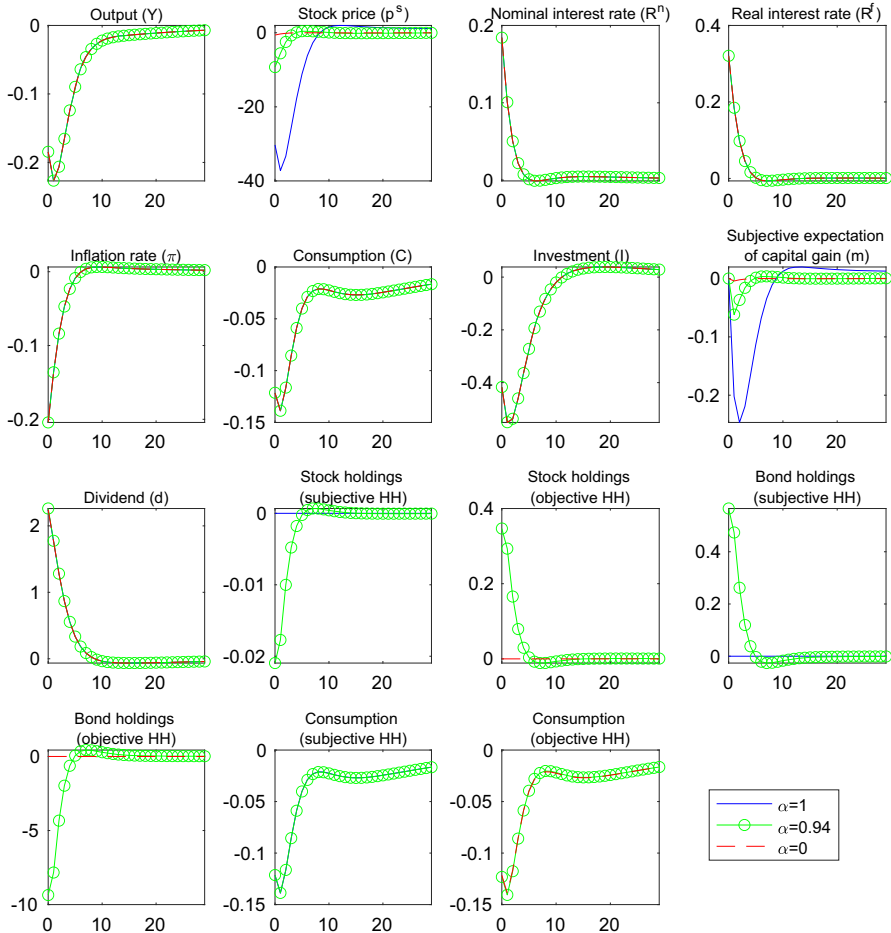
<sup>17</sup> Due to data accessibility to subjective stock price growth expectations to cover the same data periods with other variables, I did not show  $Corr[\log P^s/d, \log(\alpha m_t + (1 - \alpha)E_t[p_{t+1}^s/p_t^s])]$  in Table 3.



**Fig. 1** Impulse response to a 1.0% positive productivity shock. The solid line indicates the subjective expectation case. The dotted line indicates the objective expectation case. The line with the circle marker indicates the heterogeneous case,  $\alpha = 0.94$ . 1 on the vertical axis scale amounts to 1% except for bond holdings. Bond holdings are shown in amount differences from their steady state because their steady state value is 0. Return variables,  $R^n$  and  $R^f$  are in percentage point differences from their steady states. Otherwise, variables are shown in percentage deviation from their steady state levels

Under positive productivity shocks (Fig. 1), stock prices increase in all of the three cases of  $\alpha$ . The nominal interest rate decreases due to a decrease in inflation via a reaction of monetary policy. Dividends also increase by the shock. Consequently, the stock price increases. As  $\alpha$  increases, stock price reactions become large. Under positive markup shocks (Fig. 3), stock prices decrease in all of the three cases because the nominal interest rate increases to reduce inflation, though dividends increase against a backdrop of curbed investment caused by monetary tightening.

Since the model is heterogeneous, the charts include responses of stock and bond holdings and consumption of each agent. As the subjective expectation of capital gains increases under the positive productivity shock, households with subjective beliefs

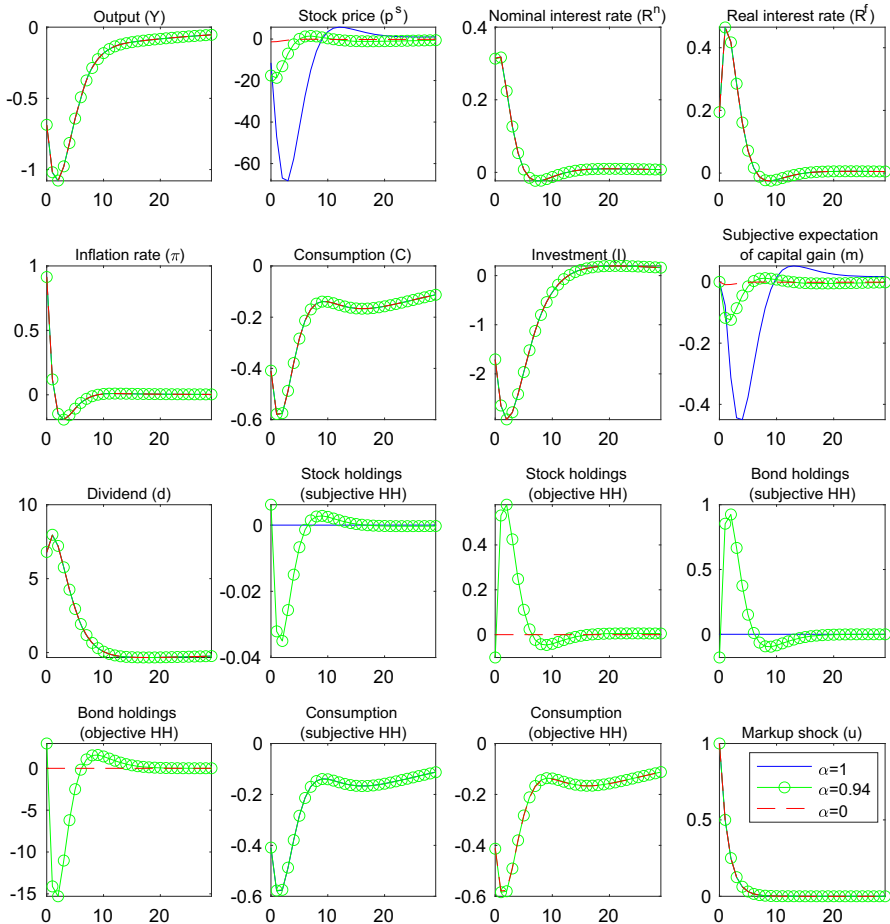


**Fig. 2** Impulse response to a 0.25% positive monetary policy shock (1.0% at an annualized rate). The solid line indicates the subjective expectation case. The dotted line indicates the objective expectation case. The line with the circle marker indicates the heterogeneous case,  $\alpha = 0.94$ . 1 on the vertical axis scale amounts to 1% except for bond holdings. Bond holdings are shown in amount differences from their steady state because their steady state value is 0. Return variables,  $R^h$  and  $R^f$  are in percentage point differences from their steady states. Otherwise, variables are shown in percentage deviation from their steady state levels

increase their stock holdings as implied in (18). Instead, they reduce their bond holdings to finance the purchase of the stocks. On the other hand, households with objective beliefs sell their stocks to households with subjective beliefs and increase bond holdings. Consumption of each agent behaves similarly because I calibrate  $\zeta^S$  and  $\zeta^B$ , such that the impact of subjective beliefs on the allocation variables is minimal across any values of  $\alpha$ , as discussed in Sect. 4.1.

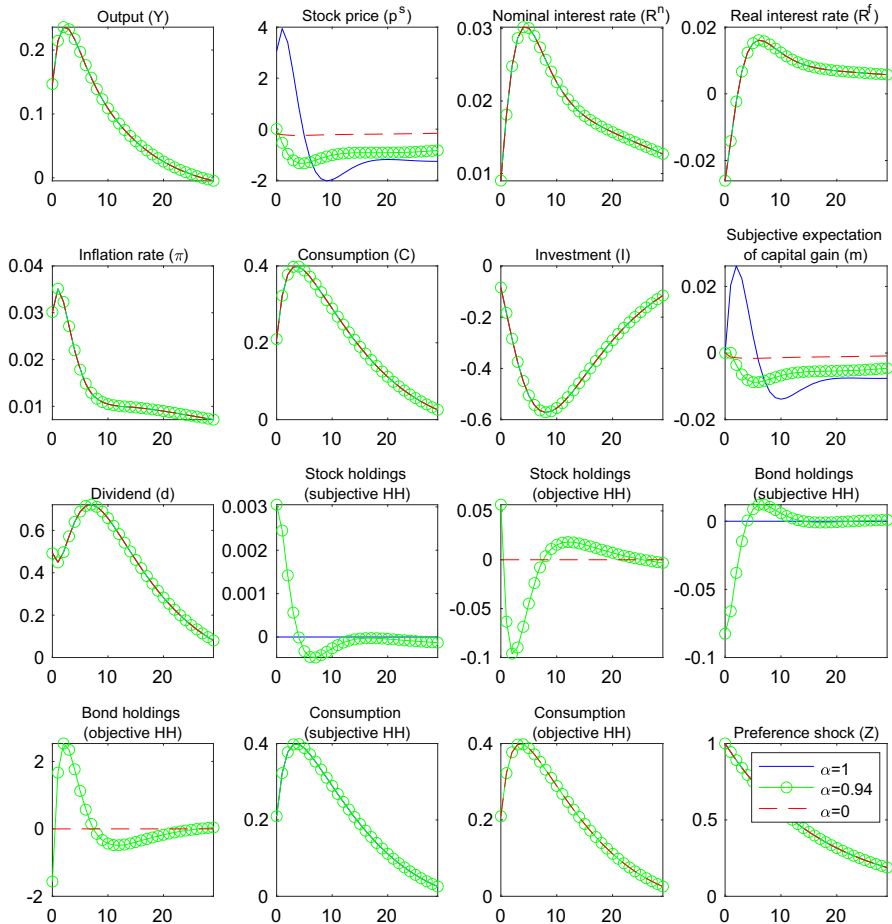
As summarized by Challe and Giannitsarou (2014), empirical studies suggest that a 100 basis point increase of the interest rate at an annual rate is associated with a 2.2–9% decrease in stock prices. In Fig. 2, the model with  $\alpha = 0.94$  shows a 9.3%





**Fig. 3** Impulse response to a 1% positive markup shock. The solid line indicates the subjective expectation case. The dotted line indicates the objective expectation case. The line with the circle marker indicates the heterogeneous case,  $\alpha = 0.94$ . 1 on the vertical axis scale amounts to 1% except for bond holdings. Bond holdings are shown in amount differences from their steady state because their steady state value is 0. Return variables,  $R^n$  and  $R^f$  are in percentage point differences from their steady states. Otherwise, variables are shown in percentage deviation from their steady state levels

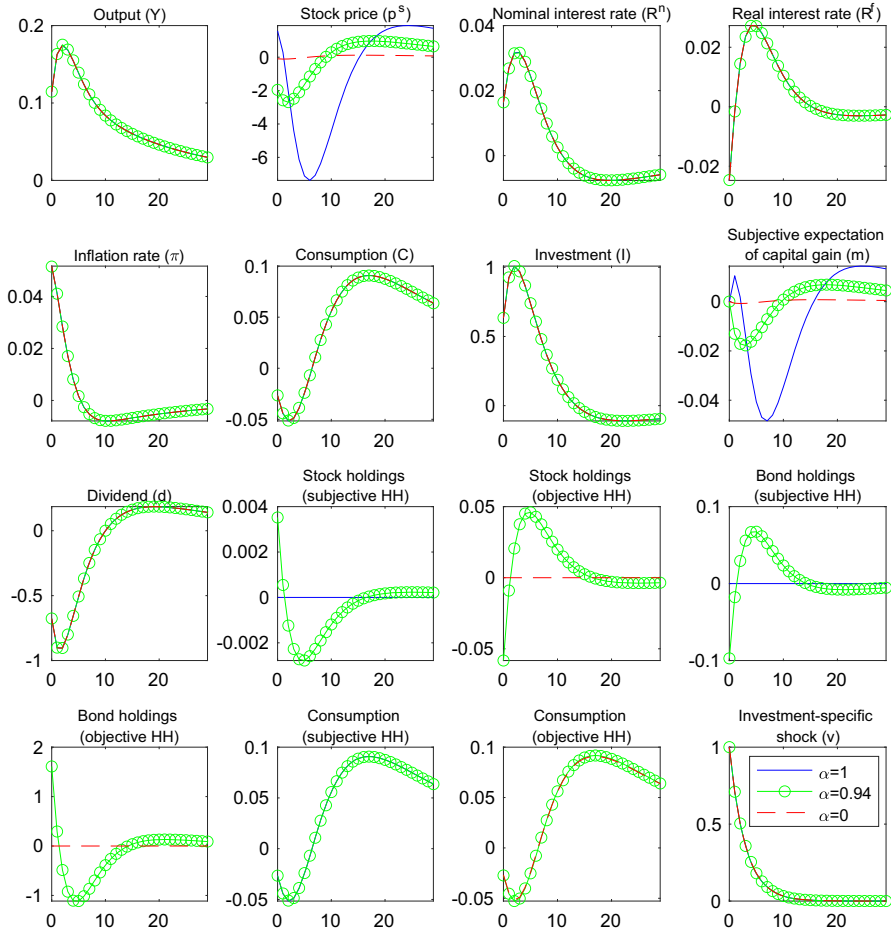
decrease as an initial reaction to a shock of 100 basis points of the nominal interest rate at an annual rate (25 basis points at a quarterly rate) and 4.5% decreases over four quarters on average at and after the shock in its impulse response, which is close to the estimates of empirical studies. In contrast, the stock price in the homogenous subjective belief case ( $\alpha = 1$ ) reacts to the monetary policy shock too strongly. The model with  $\alpha = 1$  shows a 30.3% decrease as an initial reaction and 31.6% decreases over four quarters on average at and after the shock. The case of  $\alpha = 0$  shows a 0.6% decrease as an initial reaction and 0.3% decreases over four quarters on average at and after the shock, which are much smaller than the estimates of empirical studies. Thus,



**Fig. 4** Impulse response to a 1% positive preference shock. The solid line indicates the subjective expectation case. The dotted line indicates the objective expectation case. The line with the circle marker indicates the heterogeneous case,  $\alpha = 0.94$ . 1 on the vertical axis scale amounts to 1% except for bond holdings. Bond holdings are shown in amount differences from their steady state because their steady state value is 0. Return variables,  $R^n$  and  $R^f$  are in percentage point differences from their steady states. Otherwise, variables are shown in percentage deviation from their steady state levels

assuming heterogeneity helps generate realistic stock price responses to interest rate shocks.

In response to the positive preference shock (Fig. 4), the stock price decreases in  $\alpha = 0$  case. This is because the real interest rate increases over time in response to the preference shock, and in the medium run, the increases in consumption crowd out investment, which consequently reduces capital accumulations and hence the capacity of production. Decreases in stock prices reacting to preference shocks are similarly observed in existing studies based on general equilibrium models. For example, Nisticò



**Fig. 5** Impulse response to a 1% positive investment specific technology shock. The solid line indicates the subjective expectation case. The dotted line indicates the objective expectation case. The line with the circle marker indicates the heterogeneous case,  $\alpha = 0.94$ . 1 on the vertical axis scale amounts to 1% except for bond holdings. Bond holdings are shown in amount differences from their steady state because their steady state value is 0. Return variables,  $R^n$  and  $R^f$  are in percentage point differences from their steady states. Otherwise, variables are shown in percentage deviation from their steady state levels

(2012) shows a decrease in the stock price in response to a preference shock.<sup>18</sup> Usually, existing studies of stock prices based on real business cycle models and new Keynesian models do not include preference shocks. They mainly consider productivity shocks as seen in Jermann (1998) or De Paoli et al. (2010). In this sense, the observations in

<sup>18</sup> Castelnuovo and Nistico (2010) claim that incorporating sticky wages helps generate the procyclicality of dividends and their study shows positive responses of stock prices to preference shocks. However, their model does not include investment and capital, which could be crowded out under positive preference shocks and reduce the long-run production capacity. I examined how sticky wages change properties of responses to preference shocks in the model. However, it does not change responses of stock prices qualitatively from what the model without sticky wages presents in this study.

my model do not contradict existing studies. However, in the cases with  $\alpha = 1$  and  $\alpha = 0.94$  in the model, the stock price reacts positively to the shock at least initially. This is because in these cases, the stock price reacts strongly to the current real interest rate, which decreases initially since the inflation rate increases in response to the shock given the nominal rigidity.

In Fig. 5, the stock price decreases in response to the investment-specific technology shock because of the increase in the real interest rate and decreases in dividends due to increased investment, as shown in (53). Decreases in stock prices in response to investment-specific technology shocks are similarly found in the existing studies, such as Christiano and Fisher (2003), who claim that stock prices are countercyclical relative to investment-specific shocks. Justiniano et al. (2010) discuss investment shocks and business cycles, and show that the real interest rate increases in response to the investment shock, which implies decreases in the stock price if the movement of the real interest rate in their model is applied to my model. In my model, even though dividends and stock prices decrease in the short run, increases in investment eventually expand the production capacity in the medium run. As the real interest rate goes back to the steady state over time, the impacts of the increase in the production capacity gradually lift the stock price.

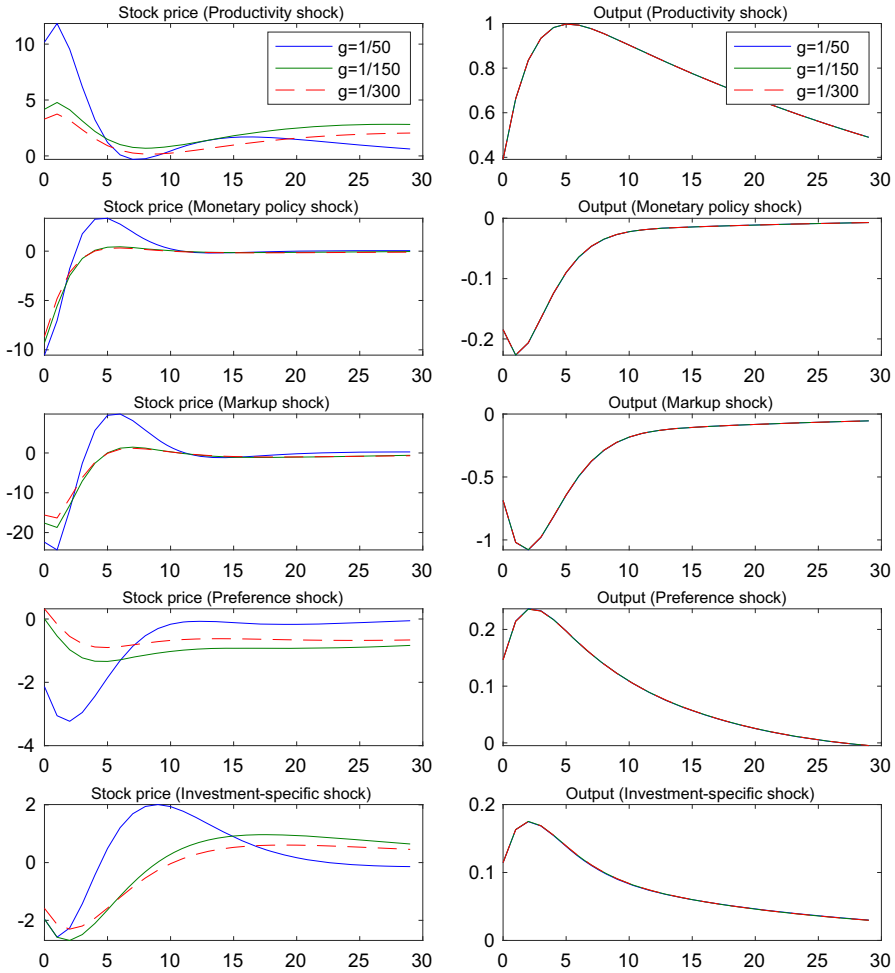
Why do levels of  $\alpha$  change the stock price responses? The value of  $\alpha$  determines to what degree future values of real interest rates and dividends affect stock prices. In (33), as the value of  $\alpha$  decreases, the impact of the current real interest rate ( $\hat{R}_t^f$ ) becomes smaller since the share of households with subjective beliefs decreases. On the other hand, as the value of  $\alpha$  decreases, the impacts of the expectation term ( $E_t \hat{p}_{t+1}^s$ ) become more relevant since the share of the households with objective beliefs increases.

#### 4.5 Benefits of assuming heterogeneity

This subsection discusses the benefit of assuming heterogeneity. A large value of the Kalman gain  $g$  leads to high volatility of the stock price. When  $g$  is large, households with subjective beliefs strongly react to observed stock price growths when they update their beliefs. The larger  $g$  becomes, the stronger the effect of momentum on stock prices becomes. So far, I have assumed that  $g$  is  $\frac{1}{150}$ . In Fig. 6, I show how the Kalman gain  $g$  with different values,  $\frac{1}{50}$ ,  $\frac{1}{150}$ , and  $\frac{1}{300}$ , affects stock price reactions to productivity, monetary policy, markup, preference, and investment-specific technology shocks, given the other parameters are kept at the baseline values. A large value of  $g$  increases stock price volatilities in all cases of five shocks as shown in the figure.

Based on this observation, a natural question would be whether it is possible to generate moments close to the data by adjusting the value of the Kalman gain without introducing heterogeneity, that is, under  $\alpha = 1$ . In Sect. 4.3, I briefly mentioned that in homogenous models the standard deviation of the stock price is too large with realistic interest rate volatilities when I set  $g$  at  $\frac{1}{150}$ . Here I explain the benefit of assuming heterogeneity in detail by experimenting with other values of  $g$  and other sizes of shocks.

The first column in Table 4 shows theoretical moments when the value of  $g$  is lowered to  $\frac{1}{1000}$  under  $\alpha = 1$  with keeping the same sizes of shocks as the benchmark



**Fig. 6** Impulse response of stock price under different values of Kalman gain  $g$  to a 1.0% positive productivity shock, a 0.25% (1.0% at an annualized rate) positive monetary policy shock, a 1.0% positive markup shock, a 1.0% positive preference shock, and a 1.0% positive investment-specific technology shock. 1 on the vertical axis scale amounts to 1% deviation from steady state levels

case. This leads to too large volatility of the stock price, 0.49, while the corresponding data is 0.204. To reduce the stock price volatility under  $g = \frac{1}{1000}$ , I set interest rate shocks at 0.07%, which is smaller than the benchmark, and sizes of other shocks are the same as the benchmark, as shown in the second column of the table. By this revision, the stock price volatility becomes close to the data. However, the model moment of the interest rate becomes too small, 0.008, compared to the data, 0.024. Besides, the standard deviation of inflation of the model, 0.002, deviates from the data, 0.004. Similar observations are also found in a case with  $g = \frac{1}{500}$ , which sets the size of interest rate shock at 0.05%. Even with different values of  $g$ , homogenous models

**Table 4** Second Moments of  $\alpha = 1$  case

Variables	$g = \frac{1}{1000}$ benchmark shock	$g = \frac{1}{1000}$ small rate shock	$g = \frac{1}{500}$ small rate shock	Actual data
$SD[\log R^f]$	0.020	0.008	0.007	0.024
$SD[\log C]$	0.026	0.026	0.026	0.026
$SD[\log P^S]$	0.491	0.198	0.199	0.204
$SD[\log d]$	0.060	0.046	0.046	0.045
$SD[\log Y]$	0.033	0.032	0.032	0.037
$SD[\log I]$	0.069	0.068	0.068	0.113
$SD[\log \pi]$	0.004	0.002	0.002	0.004
$Corr[\log P^S, \log Y]$	0.28	0.42	0.46	0.59
$Corr[\log P^S, \log d]$	-0.57	-0.01	0.04	-0.44
$Autocor[\log C(-1)]$	0.99	0.99	0.99	0.97
$Autocor[\log C(-2)]$	0.98	0.98	0.98	0.94
$Autocor[\log P^S(-1)]$	0.64	0.82	0.87	0.95
$Autocor[\log P^S(-2)]$	0.35	0.59	0.67	0.87

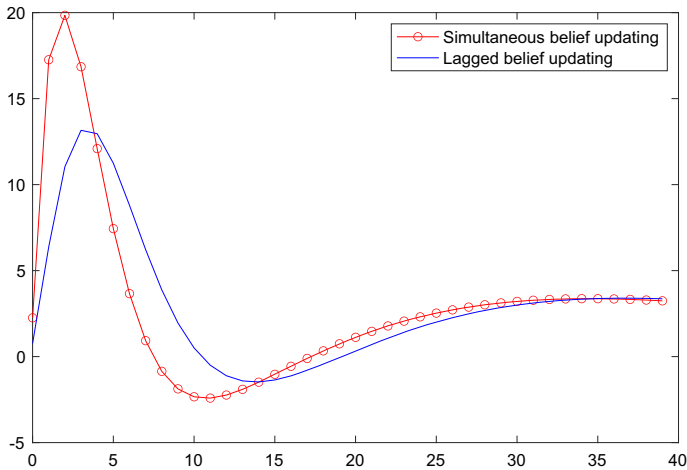
“ $g = \frac{1}{1000}$  benchmark shock” shows a case that shock sizes are the same as Table 3. “ $g = \frac{1}{1000}$  small rate shock” shows a case that shock sizes other than the interest rate shock are the same as in Table 3. The size of interest rate shock is 0.07%, which is calibrated to match the data of the standard deviation of the stock price. “ $g = \frac{1}{500}$  small rate shock” shows a case that shock sizes other than the interest rate shock are the same as Table 3. The size of interest rate shock is 0.05%, which is calibrated to match the data of the standard deviation of the stock price. The actual data are 1980–2017 quarterly U.S. data de-trended by third-order time polynomial regressions except for interest rate data

have difficulty with matching moments of the stock price, interest rate, and inflation at the same time.<sup>19</sup> This is one of the benefits of assuming heterogeneity.

Being related to this moment matching, in homogenous cases with  $\alpha = 1$ , the stock price reaction to a 1% interest rate shock deviates from what is claimed in the existing empirical studies even under different values of  $g$ . In the case of  $g = \frac{1}{1000}$ , the reaction of the stock price is a 30% decrease. Challe and Giannitsarou (2014) claim that empirical studies suggest that a 1% point increase in the interest rate at an annual rate is associated with a 2.2%–9% decrease in stock prices. In the case of  $g = \frac{1}{500}$ , the reaction is a 30% decrease, too. Assuming heterogeneity as in the baseline model shows reactions close to the studies, which homogenous cases cannot attain.

Another benefit of incorporating heterogeneity is to improve theoretical moments even when one assumes the simultaneous belief updating. In the quantitative analysis thus far, I have used the lagged belief updating shown in (19), instead of the simultaneous belief updating shown in (14), as I mentioned in Sect. 3.1.2. This is mainly because the model with the simultaneous belief updating shows an inferior fitting to the data in the homogenous case ( $\alpha = 1$ ). Figure 7 indicates the impulse responses of stock prices to a productivity shock in the homogenous case ( $\alpha = 1$ ) with the simulta-

<sup>19</sup> I experimented lower values for  $g$  than  $\frac{1}{1000}$ . However, results are similar to what I discuss here.



**Fig. 7** Impulse response of stock price to a 1.0% positive productivity shock ( $\alpha = 1$ ) with two belief updating rules: the simultaneous belief updating by Eq. (14) and the lagged belief updating by Eq. (19). 1 on the vertical axis scale amounts to 1% deviation from steady state levels

neous belief updating and the lagged belief updating. Clearly, the simultaneous belief updating generates larger volatility than the lagged belief updating.

Table 5 shows theoretical moments of the model with the simultaneous belief updating. The sizes of shocks are the same as those in Table 3 in Sect. 4.3. In the homogenous case ( $\alpha = 1$ ), the standard deviation of the stock price is much larger than the data. The correlation between the stock price and output is smaller than that in the lagged belief updating case shown in Table 3 mainly due to a large oscillation of the stock price as implied in the low values of the autocorrelation of the stock price in Table 5. However, even in the simultaneous belief updating, assuming heterogeneity helps mitigate this problem as indicated in the case of  $\alpha = 0.95$  in Table 5.<sup>20</sup> This is another benefit of introducing heterogeneity to the subjective belief model of stock price capital gains.

## 5 Monetary policy and stock price reaction

How does the stance of the monetary policy rule, one of the key fundamental parts of the economy, magnify or compress stock price volatilities via the presence of subjective beliefs? In this section, I show how the parameters in the monetary policy rule affect stock price volatilities. By introducing Bayesian learning and heterogeneity, one can relate the realistic volatilities of the stock price to parameters that construct monetary policy. Models with homogenous beliefs are not suitable for this analysis because they generate unrealistically large reactions of stock prices to movements of the interest rate, which is one of the most important instruments of monetary policy. This analysis contributes to discussions about “gradualism” of monetary policy.

<sup>20</sup> I set  $\alpha = 0.95$  instead of 0.94 in this study to match the moments.

**Table 5** Second Moments of the case with the simultaneous belief updating

Variables	Model ( $\alpha = 1$ )	Model ( $\alpha = 0.95$ )	Model ( $\alpha = 0.5$ )	Model ( $\alpha = 0$ )	Actual data
$SD[\log R^f]$	0.020	0.020	0.020	0.020	0.024
$SD[\log C]$	0.026	0.026	0.026	0.026	0.026
$SD[\log P^s]$	1.339	0.198	0.051	0.031	0.204
$SD[\log d]$	0.060	0.060	0.060	0.060	0.045
$SD[\log Y]$	0.033	0.033	0.033	0.033	0.037
$SD[\log I]$	0.069	0.069	0.069	0.069	0.113
$SD[\log \pi]$	0.004	0.004	0.004	0.004	0.004
$Corr[\log P^s, \log Y]$	0.23	0.576	0.882	0.897	0.59
$Corr[\log P^s, \log d]$	-0.58	-0.17	0.11	0.15	-0.44
$Autocor[\log C(-1)]$	0.99	0.99	0.99	0.99	0.97
$Autocor[\log C(-2)]$	0.98	0.98	0.98	0.98	0.94
$Autocor[\log P^s(-1)]$	0.57	0.72	0.92	0.94	0.95
$Autocor[\log P^s(-2)]$	0.29	0.53	0.87	0.90	0.87

The moments are based on the simultaneous belief updating shown in (14). The actual data are 1980–2017 quarterly U.S. data de-trended by third-order time polynomial regressions except for interest rate data



I examine three parameters in the monetary policy rule, the persistence parameter  $\theta_M$ , parameter of reaction to inflation  $\phi_\pi$ , and parameter of reaction to the output gap  $\phi_y$  in (55) as the monetary policy stance. I analyze the effects of the monetary policy stance on the stock price under five shocks: productivity, markup, preference, investment-specific technology, and monetary policy shocks. In this analysis, the shocks have the same sizes as those in Sect. 4.4: a 1.0% positive productivity shock, 1.0% positive markup shock, 1.0% positive preference shock, 1.0% investment-specific technology shock, and 0.25% (1.0% at an annualized rate) positive monetary policy shock (shock to increase the nominal interest rate). I set  $\alpha = 0.94$  in this section.<sup>21</sup>

A summary of this section is as follows. A change of the parameter in the monetary policy rule which strongly affects the volatility of “current” real interest rates has a large impact on the stock price under the presence of subjective beliefs because the presence limits forward-looking pricing of stocks. Among the three parameters in the monetary policy rule, the persistence parameter generally has large effects on stock prices. This is because the high persistence parameter reduces the volatility of “current” real interest rates regardless of the type of economic shocks originating from an economy, that is, four shocks other than the monetary policy shock. On the other hand, output is not necessarily affected as much as stock prices by a change in the persistence parameter against a backdrop of the forward-looking characteristic of a New Keynesian model, since the path of the real interest rate tends to more persistently deviate from the steady state under the high persistence parameter. Furthermore, in some cases, the stock price becomes less volatile whereas output becomes more volatile when increasing monetary policy inertia.

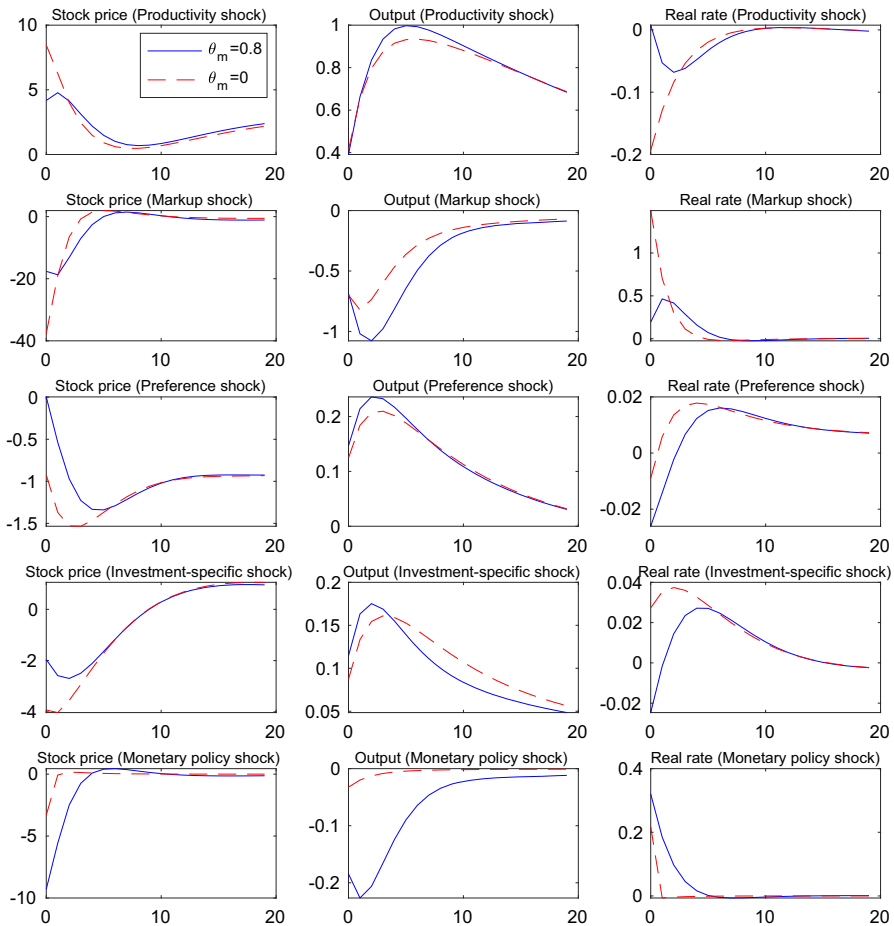
In contrast, changing the reaction parameter to inflation does not generate clearly twisted relations of stock price and output stabilities. A high reaction parameter to inflation simply increases the volatility of the real interest rate right after a shock and onward as well. A similar observation applies to the reaction parameter to the output gap.

## 5.1 Monetary policy persistence parameter and stock price

What are the impacts on stock prices of changing the level of the persistence parameter in the monetary policy rule? Figure 8 indicates the impulse responses of stock prices to positive productivity, markup, preference, investment-specific technology, and monetary policy shocks under two different persistence parameters of monetary policy,  $\theta_M = 0.8$  and  $\theta_M = 0$  in (55). Other parameters are the same as the baseline settings in Table 1.

In response to a 1% positive productivity shock, the initial reaction of the stock price becomes significantly smaller in the case with the high monetary policy persistence parameter ( $\theta_M = 0.8$ ), a 4.2% increase, than an 8.5% increase in the case with the low persistence parameter ( $\theta_M = 0$ ). The figure also includes corresponding reactions of output to see how financial and non-financial volatilities are related. Under the positive productivity shock, the strong monetary policy persistence implies that the

<sup>21</sup> In addition, in “Appendix A.2”, I investigate the effects on stock price volatilities under the habit formation parameter  $\phi$  with different values.



**Fig. 8** Stock price responses in  $\alpha = 0.94$  case with different monetary policy persistence parameters to a 1.0% positive productivity shock, a 1.0% positive markup shock, a 1.0% positive preference shock, a 1.0% positive investment specific technology shock, and a 0.25% positive monetary policy shock (1.0% at an annualized rate). These sizes are the same as those in Sect. 4.4. The value of  $\alpha$  is 0.94. 1 on the vertical axis scale amounts to 1% deviation from steady state levels

nominal interest rate decreases only gradually in response to decreases in inflation rates. Therefore, the nominal interest rate is higher in  $\theta_M = 0.8$  case than in  $\theta_M = 0$  case. The inflation expectation decreases less in  $\theta_M = 0.8$  case than in  $\theta_M = 0$  case because of the high persistence of the nominal interest rate with the forward-looking nature of the New Keynesian model. However, the decreases in the real interest rate are smaller in  $\theta_M = 0.8$  case than in  $\theta_M = 0$  case since the effects of the nominal interest rates well surpass those of the inflation expectations. As implied by (33), the stochastic discount factor (or real interest rate) at time  $t$  has a dominant impact on stock prices at time  $t$  under the presence of subjective expectations. Therefore, smaller decreases in the real interest rate in  $\theta_M = 0.8$  case generate smaller increases in the

real stock price than  $\theta_M = 0$  case. Because the initial stock price reaction is small, the momentum effect also becomes weak as implied in the last term of (33). Therefore, the overall impacts on the stock price are muted in  $\theta_M = 0.8$  case. These observations imply that the persistent monetary policy significantly stabilizes stock price volatility under the productivity shock, whereas the differences in output between two cases are not necessarily large.

A high persistence in the monetary policy rule under the positive markup shock also provides large impacts on stock prices. An initial decrease in the stock price in  $\theta_M = 0.8$  case,  $-18\%$ , is much smaller than that in  $\theta_M = 0$  case,  $-38\%$ . Increases in the nominal interest rate in  $\theta_M = 0.8$  case are smaller than those in  $\theta_M = 0$  case because of the high inertia of monetary policy. Even though the inflation expectation becomes lower in  $\theta_M = 0.8$  case than in  $\theta_M = 0$  case, the real interest rate is lower in  $\theta_M = 0.8$  case than in  $\theta_M = 0$  because the effects of the nominal interest rates dominate those of the inflation expectations. When households with subjective beliefs exist, the stock price strongly reacts to the current real interest rate, rather than to the future path of real interest rates. Therefore, the stock price shows small drops in  $\theta_M = 0.8$  case, whereas it indicates large decreases in  $\theta_M = 0$  case during the initial periods.

A high persistence in the monetary policy rule under the positive preference shock mitigates stock price drops. Decreases in the stock price during the initial periods in  $\theta_M = 0.8$  case are much smaller than those in  $\theta_M = 0$  case. The average reaction over the initial three periods in  $\theta_M = 0.8$  case is  $-0.5\%$  while that in  $\theta_M = 0$  case is  $-1.3\%$ . In  $\theta_M = 0.8$  case, the real interest rate is lower than in  $\theta_M = 0$  case because of the high persistence of the nominal interest rate. The persistent monetary policy stabilizes stock price volatility under the preference shock while the differences in output between the two cases are small. Similarly, high persistence in the monetary policy rule mitigates stock price drops under the positive investment-specific technology shock. The average decrease in the stock price during the initial two periods in  $\theta_M = 0.8$  case is  $-2.3\%$ , which is about half of that in  $\theta_M = 0$  case,  $-4.0\%$ . In  $\theta_M = 0.8$  case, the real interest rate decreases, whereas it increases in  $\theta_M = 0$  case because of the difference in the nominal interest rates under positive inflation expectations. While the differences in output between the two cases are small during early periods after the shock, stock prices show relatively large differences during the corresponding periods.

These analyses provide policy implications for “gradualism” of monetary policy. In Sect. 1, I mentioned that there were discussions in actual policymaking in the Federal Open Market Committee in February 1994 with regard to whether a 25 basis point policy tightening was preferable to a 50 basis point tightening because some members considered the larger move to have a very high probability of “cracking financial markets” (FOMC Secretariat (1994)). The analysis based on the model implies that inertia of the policy rate is important for the stock price stability and supports gradualism of monetary policy. Under structural shocks that originate from the economy, such as productivity, markup, preference, and investment-specific shocks, stock price reactions stabilize with highly persistent monetary policy. This is because the high persistence parameter reduces volatility of “current” real interest rates regardless of the type of economic shocks originating from an economy.

On the other hand, output is not necessarily affected as much as stock prices by the levels of the persistence parameter. Furthermore, in some cases, the impacts on the stabilities of the stock price and output become opposite when changing the level of the parameter. For example, in response to a 1% positive productivity shock, the average deviation of the stock price from its steady state during the first five periods after the shock is 3.7% in  $\theta_M = 0.8$  case and 4.5% in  $\theta_M = 0$  case, whereas the average deviation of output from its steady state during the corresponding periods is 0.8% in  $\theta_M = 0.8$  case and 0.7% in  $\theta_M = 0$  case. Under a 1% positive markup shock, the average deviation of the stock price from its steady state during the first five periods after the shock is -11.8% in  $\theta_M = 0.8$  case and -12.5% in  $\theta_M = 0$  case, whereas the average deviation of output from its steady state during the corresponding periods is -0.9% in  $\theta_M = 0.8$  case and -0.7% in  $\theta_M = 0$  case. This implies that a high persistence of monetary policy does not necessarily provide the stability of stock prices and output at the same time under the productivity and markup shocks when heterogeneous beliefs exist.

Why do these reversed relations of reactions between the stock price and output appear when subjective beliefs exist? Heterogeneity of beliefs limits forward-looking pricing of stocks, whereas output depends on the current and expected future path of the real interest rate onward by the forward-looking characteristic of a New Keynesian model. Under the high persistence parameter, the path of the real interest rates tends to more persistently deviate from the steady state while volatilities of the “current” real rates are subdued.

Figure 8 includes reactions of the real interest rate for each shock. For example, the top right box in the figure shows the real interest rate reactions under the productivity shock. During the first three periods, the real interest rate in  $\theta_M = 0$  case is lower than that in  $\theta_M = 0.8$  case. However, from time four to eleven, the real interest rate in  $\theta_M = 0$  case is higher than that in  $\theta_M = 0.8$  case. When the economy is located at time one, the real interest rate in  $\theta_M = 0$  case is clearly lower than in  $\theta_M = 0.8$  case, which leads to a higher stock price in  $\theta_M = 0$  case than in  $\theta_M = 0.8$  case. On the other hand, the discounted sums of the current and expected future real interest rates evaluated at time one are close between the two cases, which results in similar values of output under the two cases. At time five, the discounted sum of the real interest rates is lower in  $\theta_M = 0.8$  case, which leads to higher output in  $\theta_M = 0.8$  case, though stock prices in the two cases show close values. A similar discussion about the paths of the real interest rates holds under the markup shock as well. The analysis implies that monetary policy might need to consider this trade-off caused by monetary policy inertia under the existence of subjective beliefs.

Figure 8 also includes an analysis of the effects of the high persistence parameter under the monetary policy shock. In discussing gradualism, however, this needs to be carefully evaluated since the characteristic of the monetary policy shock is considered to be different from other four shocks. The discussions about gradualism target inertial policy responses to shocks originating from the economy, rather than an unexpected disturbance produced by monetary policy itself. Persistent monetary policy under this shock implies that the monetary authority does not revise the disturbance for long periods. Although this seems unrealistic, I show the results to study the consequences when the monetary authority takes this policy stance.

In a response to the positive monetary policy shock (shock to increase the nominal interest rate), stock prices drop further in  $\theta_M = 0.8$  case than in  $\theta_M = 0$  case. A strong persistence implies that the authority maintains an increase in the nominal interest rate after an unexpected positive nominal interest rate shock. This policy management continues to push up the “current” nominal interest rates at each point of time with low inflation expectations and leads to high real interest rates. At the same time, output decreases in  $\theta_M = 0.8$  case much larger than in  $\theta_M = 0$  case. The impacts of the shock on output become large in the high persistence case, whereas a one-period transitory shock to the nominal interest rate does not have strong effects on output. This analysis implies that if the authority generates a monetary policy shock, revising the disturbance clearly helps stabilize the stock market as well as the non-financial side of the economy.

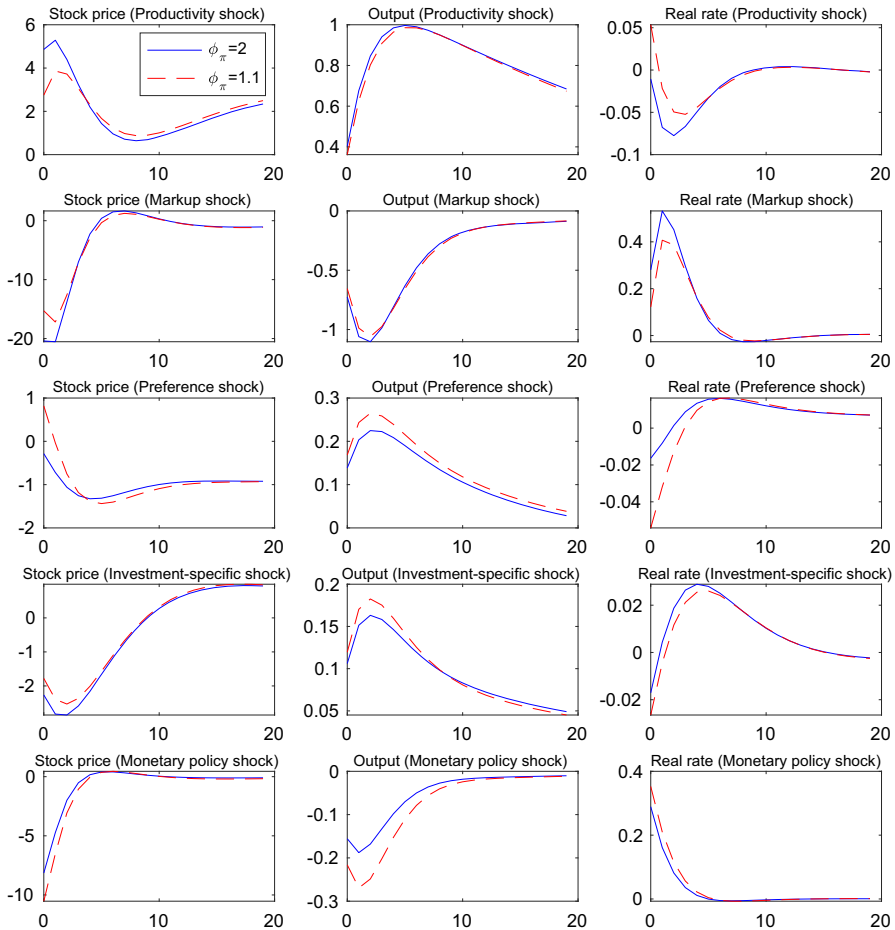
## 5.2 The parameter of monetary policy reaction to inflation and stock price

Figure 9 indicates the impulse responses of stock prices to positive productivity, markup, preference, investment-specific technology, and monetary policy shocks under two different reaction parameters of monetary policy to the inflation rate,  $\phi_\pi = 2$  and  $\phi_\pi = 1.1$  in (55). Other parameters are the same as the baseline settings in Table 1.

In the case of the high reaction parameter of monetary policy to inflation,  $\phi_\pi = 2$ , the stock price initially increases by 4.9% reacting to a 1% positive productivity shock while the reaction remains a 2.8% increase in  $\phi_\pi = 1.1$  case. While the stock prices show this difference, the responses of output in the two cases indicate similar paths. In  $\phi_\pi = 2$  case, monetary policy is more sensitive to decreases in inflation rates than in  $\phi_\pi = 1.1$  case. The nominal interest rate becomes lower, and the inflation expectation becomes higher in  $\phi_\pi = 2$  case than in  $\phi_\pi = 1.1$  case. These developments generate large decreases in the near-term real interest rates and result in a large increase in the stock price in  $\phi_\pi = 2$  case. Because output reacts to not only the current but the expected future path of the real interest rate, reactions of output in the two cases are similar.

Under a 1% positive markup shock, the initial response of the stock price in  $\phi_\pi = 2$  case is  $-20.4\%$  and that in  $\phi_\pi = 1.1$  case is  $-15.3\%$ . Output decreases by  $-0.72\%$  in  $\phi_\pi = 2$  case and it does by  $-0.65\%$  in  $\theta_M = 0$  case in the first period. The stock price indicates a relatively larger difference between the two cases than output does. The nominal interest rate increases in  $\phi_\pi = 2$  case more than in  $\phi_\pi = 1.1$  case to curb inflation, and consequently, the real interest rate becomes high. Because the stock price reacts strongly to the current real interest rate under the presence of subjective beliefs, the stock price shows a larger drop in  $\phi_\pi = 2$  case than in  $\phi_\pi = 1.1$  case. On the other hand, reactions of output under the two cases are similar.

In response to a 1% positive preference shock, the stock price increases by 0.8% in the initial period in  $\phi_\pi = 1.1$  case while it decreases by  $-0.3\%$  in  $\phi_\pi = 2$  case. In  $\phi_\pi = 1.1$  case, the increase in the nominal interest rate in response to the increase in inflation is subdued compared to that in  $\phi_\pi = 2$  case. Since the nominal interest rate is lower, inflation is higher in  $\phi_\pi = 1.1$  case, and this results in lower real interest



**Fig. 9** Stock price responses in  $\alpha = 0.94$  case with different monetary policy reaction parameters to inflation to a 1.0% positive productivity shock, a 1.0% positive markup shock, a 1.0% positive preference shock, a 1.0% positive investment specific technology shock, and a 0.25% positive monetary policy shock (1.0% at an annualized rate). These sizes are the same as those in Sect. 4.4. The value of  $\alpha$  is 0.94. 1 on the vertical axis scale amounts to a 1% deviation from steady state levels

rates. A low reaction of monetary policy to inflation supports the stock price under the preference shock.

Under a 1% positive investment-specific technology shock, the low reaction of monetary policy to inflation rate,  $\phi_\pi = 1.1$  case, mitigates stock price drops. However, the difference of stock prices between  $\phi_\pi = 1.1$  and  $\phi_\pi = 2$  cases is not large. The average response of stock prices over the initial three periods in  $\phi_\pi = 1.1$  case is  $-2.2\%$  and that in  $\phi_\pi = 2$  case is  $-2.6\%$ . This is because the shock does not generate large volatility of inflation in this setting. The difference in the nominal interest rates is limited, and consequently, the difference in the stock prices under the two cases is

small. The average reaction of output over the initial three periods in  $\phi_\pi = 1.1$  case is 0.16%, which is higher than 0.14% in  $\phi_\pi = 2$  case.

In response to a 0.25% positive monetary policy shock (1.0% at an annualized rate), the nominal rate hike decreases the inflation rate. In response to the decrease in inflation, the monetary policy rule simultaneously adjusts the nominal interest rate. In the case with  $\phi_\pi = 1.1$ , the monetary policy is less reactive than the case with  $\phi_\pi = 2$ . Hence, the real interest rate in the case with  $\phi_\pi = 1.1$  becomes relatively higher than that in the case with  $\phi_\pi = 2$ . The stock price shows larger drops,  $-10.5\%$ , in the initial period in  $\phi_\pi = 1.1$  case than it does in  $\phi_\pi = 2$  case,  $-8.1\%$ . Output reactions in the initial period are  $-0.22\%$  in  $\phi_\pi = 1.1$  case and  $-0.16\%$  in  $\phi_\pi = 2$  case, respectively.

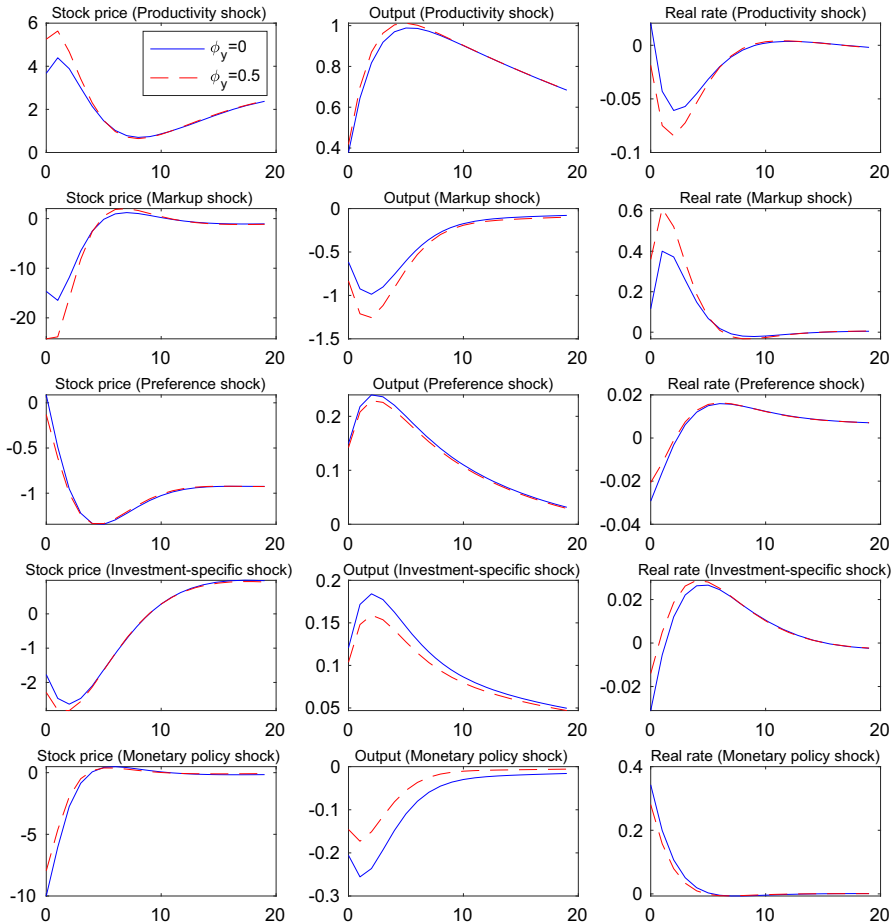
Under all five shocks, the stock price and output similarly increase or decrease in the same direction by changing  $\phi_\pi$ . That is, there are no notably twisted relations or trade-offs between stock price and output stabilities by changing the reaction parameter to inflation. The difference in the strength of the reaction of monetary policy to inflation does not generate the paths of the real interest rates which could lead to trade-offs between stock price and output stabilities in contrast to what one observes in the study of monetary policy inertia.

### 5.3 The parameter of monetary policy reaction to output gap and stock price

Figure 10 indicates the stock price responses to positive productivity, markup, preference, investment-specific technology, and monetary policy shocks under two different reaction parameters of monetary policy to the output gap,  $\phi_y = 0.5$  and  $\phi_y = 0$  in (55). Other parameters have the same values as the baseline settings in Table 1. As a major difference of reacting to the output gap from reacting to inflation, a reaction to inflation implies that the policy rate reacts to not only the current but also the expected flow of future output gaps, whereas large reactions to the output gap imply that it strongly responds to the current output gap rather than its expected future flow. Overall qualitative impacts on the stock price of changing the reaction parameter to the output gap are similar to those of changing the reaction parameter to inflation.

In response to a 1% positive productivity shock, the stock price increases by 5.3% in  $\phi_y = 0.5$  case in the first period, which is larger than 3.7% in  $\phi_y = 0$  case. The nominal interest rate is lower in  $\phi_y = 0.5$  case than in  $\phi_y = 0$  case because it reacts to decreases in the output gap more sensitively in  $\phi_y = 0.5$  case. The real interest rate in  $\phi_y = 0.5$  case is lower because of the nominal rate and the inflation rate supported by the nominal rate, which generates higher stock prices in  $\phi_y = 0.5$  case than in  $\phi_y = 0$  case. Increases in output in the first period are 0.42% in  $\phi_y = 0.5$  case and 0.38% in  $\phi_y = 0$  case, respectively. The stock prices show a large difference in their reactions between  $\phi_y = 0.5$  and  $\phi_y = 0$  cases, while output indicates similar paths between the two cases.

In response to a 1% positive markup shock, the initial reaction of the stock price in  $\phi_y = 0.5$  case is  $-24.2\%$  and that in  $\phi_y = 0$  case is  $-14.7\%$ , while output decreases by  $-0.83\%$  in  $\phi_y = 0.5$  case and decreases by  $-0.61\%$  in  $\phi_y = 0$  case. The stock price indicates a relatively large difference between the two cases compared to output.



**Fig. 10** Stock price responses in  $\alpha = 0.94$  case with different monetary policy reaction parameters to the output gap to a 1.0% positive productivity shock, a 1.0% positive markup shock, a 1.0% positive preference shock, a 1.0% positive investment specific technology shock, and a 0.25% positive monetary policy shock (1.0% at an annualized rate). These sizes are the same as those in Sect. 4.4. The value of  $\alpha$  is 0.94. 1 on the vertical axis scale amounts to a 1% deviation from steady state levels

The nominal interest rate rises to stabilize an increased output gap in  $\phi_y = 0.5$  case more than in  $\phi_y = 0$  case.

Under a 1% positive preference shock,  $\phi_y = 0$  case shows higher stock prices (−0.5% on average over the first three periods) than  $\phi_y = 0.5$  case does (−0.6% on average over the corresponding periods). This is because the strong reaction of monetary policy to the increase in the output gap lifts the nominal interest rate. However, the difference in stock prices between  $\phi_y = 0.5$  and  $\phi_y = 0$  cases is small. A strong reaction of monetary policy to the output gap does not generate large differences in the nominal rates. In this setting, deviations of the output gap from its steady state are not very large in response to the preference shock, and therefore, the difference in the



strength of monetary policy reaction to the output gap does not lead to large differences in the real interest rates. Similarly, the difference in output between  $\phi_y = 0.5$  and  $\phi_y = 0$  cases is small.

Under a 1% positive investment-specific technology shock,  $\phi_y = 0$  case shows higher stock prices than  $\phi_y = 0.5$  case does. The real interest rate level in  $\phi_y = 0$  case is lower than  $\phi_y = 0.5$  case because the monetary authority does not explicitly react to the increase in the output gap associated with the positive investment-specific technology shock in  $\phi_y = 0$  case. The difference in stock prices between  $\phi_y = 0.5$  and  $\phi_y = 0$  cases is not very large. The average deviation of the stock price from its steady state over the initial three periods in  $\phi_y = 0$  case is  $-2.3\%$  and that in  $\phi_y = 0.5$  case is  $-2.6\%$ . The average deviation of output from its steady state over the initial three periods is  $0.16\%$  in  $\phi_y = 0$  case and that is  $0.14\%$  in  $\phi_y = 0.5$  case. Since the movements of output gaps are limited under the positive investment-specific technology shock, the difference in the degrees of the reaction to the output gap in the monetary policy rule does not result in notable differences between the two cases.

In response to a 0.25% positive monetary policy shock (1.0% at an annualized rate), the average deviation of the stock price from its steady state over the initial three periods is  $-6.3\%$  in  $\phi_y = 0$  case, and it is  $-4.8\%$  in  $\phi_y = 0.5$  case. The average deviation of output from its steady state over the initial three periods is  $-0.23\%$  in  $\phi_y = 0$  case and  $-0.16\%$  in  $\phi_y = 0.5$  case. The nominal rate hike caused by the shock decreases the output gap. In response to the decrease in the output gap, monetary policy simultaneously adjusts the nominal interest rate. In the case with  $\phi_y = 0$ , the monetary policy is less reactive than the case with  $\phi_y = 0.5$ . Hence, the real interest rate in the case with  $\phi_y = 0$  becomes relatively higher than the case with  $\phi_y = 0.5$ . Because the stock price reacts strongly to the real interest rate in the model, the stock price shows larger drops in  $\phi_y = 0$  case than  $\phi_y = 0.5$  case. However, the stock prices do not indicate a large difference between the two cases compared to the difference in output. This is because the paths of the “current” real interest rates in the two cases do not deviate largely at each point of time.

Under all five shocks, the stock price and output similarly increase or decrease in the same direction by changing  $\phi_y$ . That is, there are no notable trade-offs between stabilizing the stock price and output when one changes the reaction parameter to the output gap. The difference in the strength of the reaction of monetary policy to the output gap does not clearly generate the paths of the real interest rates which could lead to the trade-off. This implication is similar to the study of the reaction parameter to inflation.

## 5.4 Implications for monetary policy

To summarize Sect. 5, findings are as follows. A large deviation of the current real interest rate from the steady state, even if that is a short-run phenomenon, is important for the stock price under the presence of subjective beliefs because it limits the forward-looking nature of stock pricing. This is the source of differences in volatilities between the stock price, a financial variable, and output, a non-financial variable. Among the parameters in the monetary policy rule, the persistence parameter affects behaviors

of stock prices under a wide range of shocks. This is because a high persistence parameter reduces the volatility of “current” real interest rates regardless of the type of economic shocks originating from an economy. On the other hand, the path of the real interest rate tends to more persistently deviate from the steady state under a high persistence parameter. This sometimes leads to observations that output is not necessarily affected as much as the stock price by changing the persistence parameter against a backdrop of the forward-looking characteristic of a New Keynesian model. Furthermore, depending on the paths of the real interest rates caused by the differences in monetary policy inertia, stabilities of the stock price and output could contradict each other in some cases.

This discrepancy raises an issue for monetary policy to stabilize the financial and non-financial sides of the economy at the same time under the presence of subjective beliefs. Even though the discussion about gradualism of monetary policy holds under subjective beliefs of capital gains of stock prices, it would be necessary to consider the stability of output if stabilizing the current real rate results in persistent deviations of the real interest rate from its steady state afterwards. This is because stock prices are strongly affected by the near-term rate under the existence of subjective beliefs, while output is rather affected by the path of the real rates.

The reaction parameter to inflation has strong impacts on stock prices when a structural shock significantly moves inflation because the parameter determines the volatility of the policy rate given the volatility of inflation. Another point to note about the reaction parameter to inflation is that changing the parameter does not generate clearly twisted relations between stock price and output stabilities. A high reaction parameter simply tends to increase the volatility of the real interest rate right after a shock and onward as well. Similar discussions apply to a change in the reaction parameter to the output gap.

## 6 Conclusion

In this study, I develop a New Keynesian model with two types of households, those with subjective and objective beliefs about capital gains from stock prices, as the first step to studying the effect of heterogeneity in beliefs on stock prices in a general equilibrium model. Households with subjective beliefs construct their expectations of capital gains by Bayesian learning from observed growth rates of stock prices.

Assuming two types of households provides several benefits. It enhances moment matching compared to a model with only subjective or objective beliefs without setting the relative risk aversion rate away from the conventional parameter values estimated in dynamic stochastic general equilibrium models. Incorporating two types of agents improves the quantitative performances of the model under monetary policy shocks. The model can generate plausible stock price drops in response to a positive interest rate shock, which cannot be attained in a homogenous agent model with subjective beliefs.

The quantitative improvement of stock price behaviors allows me to conduct a realistic analysis of how the stance of monetary policy affects stock price volatilities. Strong inertia of monetary policy provides the stability of stock prices. Similarly, weak

reactions of monetary policy to inflation or the output gap also help stabilize stock prices. These are because the volatility of the near-term real interest rate has dominant effects on stock prices under the presence of subjective beliefs since the presence limits the forward-looking nature in stock pricing. This analysis supports discussions about “gradualism” of monetary policy. Interestingly, however, the large inertia of monetary policy does not necessarily provide the stability of output because output depends on the current and future path of the real interest rate in the forward-looking manner. This discrepancy implies the necessity for monetary policy to consider a potential trade-off between stabilizing the financial and non-financial sides of the economy under heterogeneous beliefs.

Finally, I address directions to extend this study. The model is solved by the first-order perturbation method for the simplicity of calculation and abstracts higher-order terms. By including higher-order terms, it would be possible to argue excess returns and volatility at the same time. I assume that the population share of each household is fixed exogenously. Endogenous population share changes could further improve moment matching. Bayesian estimations of parameters including the Kalman gain would be also an interesting approach to examine the model validity. These considerations remain for further research.

## A Appendix

### A.1 Data Sources

The actual data formations are conducted by procedures as follows. The actual data used in this paper is of the United States. Real consumption, real investment, and real wage data are from the Federal Reserve Bank of St. Louis FRED economic database (<https://fred.stlouisfed.org/>). FRED series IDs for these variables are PCECC96, GPDIC1, and LES1252881600Q, respectively. These are seasonally adjusted quarterly data. For real output data, I took a sum of seasonally adjusted nominal consumption and investment, and divided it by the implicit price deflator for gross domestic purchases. The corresponding FRED series ID are PCEC, GPDI, and A712RD3A086NBEA, respectively. Real capital stock data is real net stock (private fixed assets) from the Bureau of Economic Analysis. The total nominal compensation of employee data is from FRED and its FRED series ID is A4102C1Q027SBEA. This is seasonally adjusted. To retrieve the labor amount, I divided the total nominal compensation data by real wage and implicit price deflator for gross domestic purchases. For dividend for which development is consistent with the model, I subtracted nominal total compensation and nominal investment (FRED series ID: GDPI) from the nominal output, which I define as the sum of nominal consumption (FRED series ID: PCEC) and nominal investment, and divided it by the implicit price deflator for gross domestic purchases. The stock price data is S&P 500 index data. Real stock price data are deflated by implicit GDP deflator (FRED series ID: GDPDEF). As the above dividend data covers all U.S. companies, including unlisted companies, I constructed dividend data, which is a product of the S&P stock index level and its dividend yield for the price dividend ratio in Sect. 4.2.

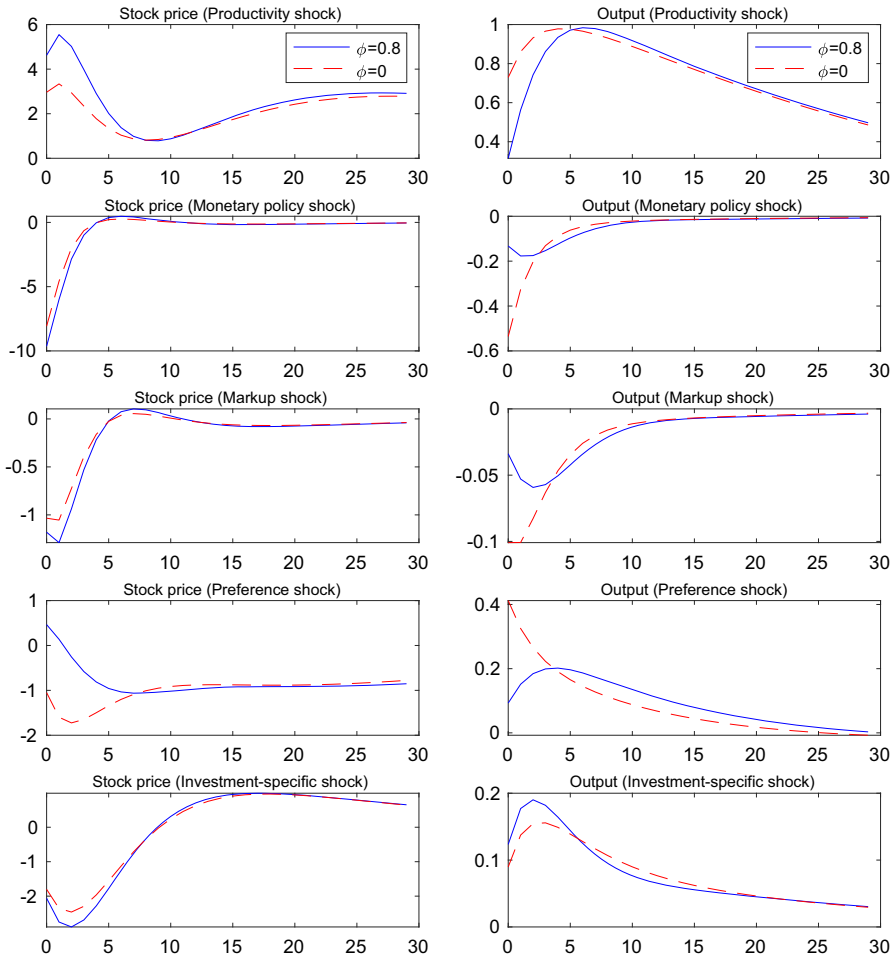
Interest rate data is based on the Federal Reserve Bank of New York's treasury term premia database ([https://www.newyorkfed.org/research/data\\_indicators/term\\_premia.html](https://www.newyorkfed.org/research/data_indicators/term_premia.html)). I use one-year fitted zero coupon market yield of U.S. treasury for the nominal interest rate. I deflated it by the actual inflation rates of the implicit gross domestic product deflator. In Sect. 4.2, I used gross yields data. Both rates in Sect. 4.3 are of the natural log of gross yields.

In Sect. 4.3, I take the natural log and de-trend by the third-order time polynomial regression, except interest rate data. I chose third-order because the Akaike information criterion shows the lowest value for output when I examined the fit up to the fourth order. For consistency, I used the same order in time polynomial regressions for other variables, too. I chose the time polynomial regression to detrend among other candidates of detrending methods for the following reasons. One candidate is to take the first difference. The distance of the stock price level from the long-run trend and persistence of the deviation of the stock price, which implies the cyclical momentum of stock prices, are what this study wants to explain using subjective beliefs. The first difference is not necessarily the best candidate for detecting the cyclical momentum of the stock price in terms of the distance of the stock price level from the long-run trend. Given this, a candidate to detect momentum or level deviation from the trend is the Hodrick-Prescott filter. However, this method is not necessarily a good candidate to extract momentum or level deviations from the long-run trend, either. Hence, I avoid using the Hodrick-Prescott filter and use the time polynomial regression in this study. Real capital data is real net private fixed assets provided on annual basis. I translated them to a quarterly basis by interpolation for de-trending.

## A.2 Habit formation parameter and stock price

This appendix section shows how parameters other than those of monetary policy affect stock price volatilities. I discuss the effects of the habit formation parameter on stock price volatility. I show the high habit formation case,  $\phi = 0.8$ , and the low habit formation case,  $\phi = 0$ , in Fig. 11. Under the productivity shock, a high habit formation parameter amplifies stock price volatilities. The intuition behind this is that the strong need for consumption smoothing by high habit formation generates large volatilities of real interest rates. In addition, when  $\alpha$  is close to 1, the shape of dividend flows over time matters because  $\hat{d}$  at time  $t + 1$  has a strong impact in (33). When consumption smoothing is strong and the stochastic discount factor increases in response to a positive productivity shock, firms are inclined to increase their dividends far into the future. Therefore,  $\hat{d}_{t+1}$  becomes small by firms' optimization in case of strong consumption smoothing. This implies small stock price reactions from the second term of the right hand side in (33). However, these dividend effects on stock price have a much smaller magnitude than the real interest rate effects, because the second term has less weight than the first term. Therefore, the stock price shows a larger increase in  $\phi = 0.8$  case than  $\phi = 0$  case.

Under a positive monetary policy shock, the high habit formation parameter case does not differ much from the low habit formation parameter case, as shown in the right-hand side chart in Fig. 11. A monetary policy shock directly changes the real



**Fig. 11** Stock price response with different habit formation parameters to a 1.0% positive productivity shock, a 0.25% (1.0% at an annualized rate) positive monetary policy shock, a 1.0% positive markup shock, a 1.0% positive preference shock, and a 1.0% positive investment-specific technology shock. These sizes are the same as those in Sect. 4.4. The value of  $\alpha$  is 0.94. 1 on the vertical axis scale amounts to 1% deviation from steady state levels

interest rate given the price rigidity. Therefore, stochastic discount factors that affect stock prices do not show notable differences between two habit formation parameter cases.

## References

Adam, K., Marcet, A.: Internal rationality, imperfect market knowledge and asset prices. *J Econ Theory* **146**(3), 1224–1252 (2011)  
 Adam, K., Merkel, S.: Stock price cycles and business cycles. *Working Paper* (2018)

- Adam, K., Marcet, A., Nicolini, J.P.: Stock market volatility and learning. *J Finance* **71**(1), 33–82 (2016)
- Adam, K., Marcet, A., Beutel, J.: Stock price booms and expected capital gains. *Am Econ Rev* **107**(8), 2352–2408 (2017)
- Bansal, R., Yaron, A.: Risks for the long run: a potential resolution of asset pricing puzzles. *J Finance* **59**(4), 1481–1509 (2004)
- Barberis, N., Greenwood, R., Jin, L., Shleifer, A.: X-capm: an extrapolative capital asset pricing model. *J Financ Econ* **115**(1), 1–24 (2015)
- Bernanke, B.: Gradualism (2004). <https://www.federalreserve.gov/boarddocs/speeches/2004/200405202/default.htm>. Remarks at an economics luncheon co-sponsored by the Federal Reserve Bank of San Francisco (Seattle Branch) and the University of Washington, Seattle, Washington, May 20, 2004
- Bernanke, B.S., Kuttner, K.N.: What explains the stock market's reaction to federal reserve policy? *J Finance* **60**(3), 1221–1257 (2005)
- Bilbiie, F.O.: Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *J Econ Theory* **140**(1), 162–196 (2008)
- Brock, W.A., Hommes, C.H.: Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *J Econ Dyn Control* **22**(8–9), 1235–1274 (1998)
- Caines, C., Winkler, F.: Asset price learning and optimal monetary policy. Working Paper (2018)
- Campbell, J.Y., Cochrane, J.H.: By force of habit: a consumption-based explanation of aggregate stock market behavior. *J Polit Econ* **107**(2), 205–251 (1999)
- Campbell, J.Y., Mankiw, N.G.: Permanent income, current income, and consumption. *J Bus Econ Stat* **8**(3), 265–279 (1990)
- Castelnuovo, E., Nistico, S.: Stock market conditions and monetary policy in a dsge model for the us. *J Econ Dyn Control* **34**(9), 1700–1731 (2010)
- Challe, E., Giannitsarou, C.: Stock prices and monetary policy shocks: a general equilibrium approach. *J Econ Dyn Control* **40**, 46–66 (2014)
- Choi, J.J., Mertens, T.: Extrapolative expectations and the equity premium. Working Paper (2013)
- Christiano, L.J., Fisher, J.D.M.: Stock market and investment goods prices: implications for macroeconomics. NBER Working Paper, No. 10031 (2003)
- Christiano, L.J., Eichenbaum, M., Evans, C.L.: Nominal rigidities and the dynamic effects of a shock to monetary policy. *J Polit Econ* **113**(1), 1–45 (2005)
- Collin-Dufresne, P., Johannes, M., Lochstoer, L.A.: Parameter learning in general equilibrium: the asset pricing implications. *Am Econ Rev* **106**(3), 664–98 (2016)
- Debortoli, D., Galí, J.: Monetary policy with heterogeneous agents: insights from tank models. Working Paper (2017)
- De Long, J.B., Shleifer, A., Summers, L.H., Waldmann, R.J.: Positive feedback investment strategies and destabilizing rational speculation. *J Finance* **45**(2), 379–395 (1990)
- De Paoli, B., Scott, A., Weeken, O.: Asset pricing implications of a new keynesian model. *J Econ Dyn Control* **34**(10), 2056–2073 (2010)
- Eusepi, S., Preston, B.: Expectations, learning, and business cycle fluctuations. *Am Econ Rev* **101**(6), 2844–72 (2011)
- Evans, G.W., Honkapohja, S.: Learning and expectations in macroeconomics. Princeton University Press, Princeton (2001)
- FOMC Secretariat. Transcript: federal open market committee meeting, February 3–4, 1994., 1994. <https://www.federalreserve.gov/monetarypolicy/files/FOMC19940204meeting.pdf>. Board of Governors of the Federal Reserve System
- Galí, J., López-Salido, J.D., Vallés, J.: Understanding the effects of government spending on consumption. *J Eur Econ Assoc* **5**(1), 227–270 (2007)
- González-Páramo, J.: Uncertainty and gradualism in monetary policy, 2006. <https://www.ecb.europa.eu/press/key/date/2006/html/sp060317.en.html>. Remarks at Member of the Executive Board and Governing Council of the European Central Bank Universidad Pablo de Olavide Sevilla (2006)
- Hanson, S.G., Stein, J.C.: Monetary policy and long-term real rates. *J Financ Econ* **115**(3), 429–448 (2015)
- Harrison, J.M., Kreps, D.M.: Speculative investor behavior in a stock market with heterogeneous expectations. *Q J Econ* **92**(2), 323–336 (1978)
- Hirshleifer, D., Li, J., Jianfeng, Yu.: Asset pricing in production economies with extrapolative expectations. *J Monet Econ* **76**, 87–106 (2015)
- Ireland, P.N.: Sticky-price models of the business cycle: specification and stability. *J Monet Econ* **47**(1), 3–18 (2001)

- Jermann, U.J.: Asset pricing in production economies. *J Monet Econ* **41**(2), 257–275 (1998)
- Justiniano, A., Primiceri, G.E., Tambalotti, A.: Investment shocks and business cycles. *J Monet Econ* **57**(2), 132–145 (2010)
- Kaplan, G., Moll, B., Violante, G.L.: Monetary policy according to hank. *Am Econ Rev* **108**(3), 697–743 (2018)
- Lansing, K.J.: Rational and near-rational bubbles without drift. *Econ J* **120**(549), 1149–1174 (2010)
- Laopodis, N.T.: Monetary policy and stock market dynamics across monetary regimes. *J Int Money Finance* **33**, 381–406 (2013)
- Levin, A.T., Onatski, A., Williams, J.C., Williams, N.: Monetary policy under uncertainty in micro-founded macroeconomic models. *NBER Macroecon Ann* **20**, 229–287 (2005)
- Nisticò, S.: Monetary policy and stock-price dynamics in a dsge framework. *J Macroecon* **34**(1), 126–146 (2012)
- Oshima, K.: Subjective beliefs, monetary policy, and stock price volatility. Working Paper (2019)
- Paul, P.: The time-varying effect of monetary policy on asset prices. *Rev Econ Stat* forthcoming (2019)
- Rotemberg, J.J.: Sticky prices in the united states. *J Polit Econ* **90**(6), 1187–1211 (1982)
- Rudebusch, G.D.: Monetary policy inertia: fact or fiction? *Int J Central Bank* **2**(4), 85–135 (2006)
- Scheinkman, J.A., Xiong, W.: Overconfidence and speculative bubbles. *J Polit Econ* **111**(6), 1183–1220 (2003)
- Shiller, R.J., Fischer, S., Friedman, B.M.: Stock prices and social dynamics. *Brook Pap Econ Act* **1984**(2), 457–510 (1984)
- Smets, F., Wouters, R.: Shocks and frictions in us business cycles: a bayesian dsge approach. *Am Econ Rev* **97**(3), 586–606 (2007)
- Srouf, G.: Why do central banks smooth interest rates? Working Paper (2001)
- Timmermann, A.G.: How learning in financial markets generates excess volatility and predictability in stock prices. *Q. J. Econ* **108**(4), 1135–1145 (1993)
- Wei, C.: A quartet of asset pricing models in nominal and real economies. *J Econ Dyn Control* **33**(1), 154–165 (2009)
- Winkler, F.: The role of learning for asset prices and business cycles. *J Monetary Econ* forthcoming (2019)

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