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RESEARCH ARTICLE

Amplification and asymmetry in crashes and frenzies

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Abstract We often observe disproportionate reactions to tangible information in large stock price movements. Moreover these movements feature an asymmetry: the number of crashes is more than that of frenzies in the S&P 500 index. This paper offers an explanation for these two characteristics of large movements in which hedging (portfolio insurance) causes amplified price reactions to news and liquidity shocks as well as an asymmetry biased towards crashes. Risk aversion of traders is shown to be essential for the asymmetry of price movements. Also, we show that differential information can enhance both amplification and asymmetry delivered by hedging.

Keywords Amplification · Asymmetry · Crash · Frenzy · Hedging · Portfolio insurance

JEL Classification G11 · G12

1 Introduction

Sudden and large movements in stock prices have always drawn economists' attention. We see them in the form of frenzies, when the price movement is in the positive

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direction, and crashes, when the direction is negative. This paper focuses on two characteristics of crashes and frenzies: *amplification* and *asymmetry*.

In many cases, there seems to be no significant events prior to large price movements. Cutler et al. (1989) document that for the postwar movements in the S&P 500 index. This empirical fact suggests that large price movements are most often *amplified price reactions* to comparatively insignificant information or liquidity shocks. In addition, it is a stylized fact that the number of crashes is more than that of frenzies in the S&P 500 index: the histogram of de-trended security price levels does not form a symmetric distribution. In this vein, for instance, Hong and Stein (2003) report that of the ten largest 1-day movements in the S&P 500 since 1947, nine were declines. The asymmetry observed in the histogram of de-trended security price levels is referred to as *level asymmetry* in the literature.[1]

This paper offers an explanation for observed amplified price reactions to relatively insignificant shocks and observed (level) asymmetry biased towards crashes. Our explanation involves the use of *hedging (portfolio insurance) strategies* in the stock market. Hedgers, who use these strategies, sell after the market has declined and buy after the market rises. Therefore portfolio insurance is negatively price sensitive since conventional supply schedules are increasing functions of price. Brady Commission Report (1988) provides evidence for the use of portfolio insurance strategies during the crash of 1987 and furthermore blames these negatively price sensitive strategies for deepening the decline hence perhaps causing the crash. The studies of Chicago Mercantile Exchange, Miller, Hawke, Malkiel, and Scholes (1987), Commodity Futures Trading Commission (1987), Securities and Exchange Commission (1987) also highlight the important role of these strategies in the 1987 crash.[2] As a possible contributing factor to the crash of 1929, we see arguments focusing on the use of stop-loss orders which are primitive portfolio insurance strategies. Gennotte and Leland (1990) explain the 1987 crash in concordance with the findings of Brady Report by incorporating hedging (portfolio insurance) into a conventional noisy rational expectations model.

Following Gennotte and Leland (1990) we develop a static noisy rational expectations equilibrium (REE) model with hedgers using negatively price sensitive strategies in a CARA-Gaussian environment. Our results show that hedging strategies amplify the effect of news and liquidity shocks on price deviations. Convex hedging strategies cause overreaction to negative news and liquidity shocks, hence they create an asymmetry biased towards crashes. An important class of hedging functions (put-option replication strategies[3]) satisfies the convexity condition in a highly volatile market. We also examine the roles of risk aversion and asymmetric information in our analysis. We find that (1) risk aversion is necessary for asymmetry of price deviations, and (2) information asymmetry can enhance price amplification and asymmetry delivered by hedging.

[1] Van Nieuwerburgh and Veldkamp (2006) provide detailed descriptions for various types of asymmetry, including level asymmetry.

[2] Shiller (1989).

[3] Put-option replication is formally defined in Sect. 4. See Rubinstein and Leland (1981) for a detailed exposition of the subject.

The focus of our paper is characteristics of certain dynamic phenomena, namely crashes and frenzies. This might seem puzzling since we employ a static model for the analysis. However, in our static framework we can interpret comparative statics results on price as dynamic changes over time. In particular, the equilibrium price reactions to changes in the information or liquidity parameters are viewed as fluctuations over time. In the same fashion, crashes and frenzies are interpreted as high sensitivity to changes in information or liquidity parameters. That is, if we see a substantial fall in equilibrium price as a reaction to comparatively insignificant news, we call it a crash (or a frenzy in the case of a price increase) in our setup. Note that, by this interpretation, we also incorporate an observed characteristic, namely amplification, into our definition of crashes and frenzies.

As mentioned above, in our setup, hedging (portfolio insurance) is the cause of amplification and asymmetry in large price movements. Hedging strategies are naturally dynamic strategies dependent on the price trend. Before explaining how hedging strategies fit into our static environment, let us discuss why they would cause amplified and asymmetric deviations. For intuition, we can first look at stop-loss orders. With stop-loss orders we see sales after the market has fallen under some exercise value. The aim is to protect one's portfolio against future potential losses. Here it is easy to see how a crash can be the result of an amplified price reaction, because stop-loss itself puts a downward pressure on the price once the price begins to fall. Moreover since there is no accompanying upward pressure, we are likely to observe an asymmetry biased towards crashes in an environment where stop-loss orders prevail. In modern hedging strategies, such as put-option replication, the idea is the same, but now we have both upward and downward pressures on the price. That is, we see a buying spree from hedgers in a bull market, and sales in a bearish one; hence comes the amplified price reactions. If the downward pressure of the strategy were to be stronger than the upward one, we would observe asymmetry biased towards crashes. This summarizes most of what we are trying to formalize in Sect. 3.

Now we can return to the interpretation of hedging in our static environment. All hedging activity is aggregated into a deterministic supply function of price p, say $h(p)$. As we have only one trading period in our model, let us take p^* as our (hypothetical) initial price, and let $h(p^*) = 0$. A fall in the security price leads to positive hedging supply, thus for $p < p^*$, $h(p) > 0$. Similarly, we have positive hedging demand (or negative supply) with increasing price, thus $h(p) < 0$ for $p > p^*$. The more the price increases, the higher the hedging demand (and vice versa); thus we want h to be a *decreasing function* of p. In summary, we will view hedging as the change of a deterministic supply with respect to the change in price p compared to a hypothetical initial price p^*.

Given that our model is based on that of Gennotte and Leland (1990, henceforth GL in this section),[4] it is important to point out how our analysis differs from theirs and what this paper adds to their results:

[4] GL is the first paper to theoretically examine the role of hedging strategies during the 1987 crash. Jacklin et al. (1992) also attribute the 1987 crash to hedging strategies. Following Glosten and Milgrom (1985), they model a market with sequential trading. What delivers crash is the underestimation of the extent of hedging activities. This might cause a rise in the security price due to imperfect information aggregation, and ultimately learning leads to a price correction, in this case to a sudden decline in price.

1. GL defines crash as a discontinuity in the price function (or correspondence). We define crash simply as a large price decline triggered by a relatively insignificant information or liquidity shock.
2. GL does not analyze level asymmetry observed in large price movements. The main contribution of this paper is essentially the analysis of level asymmetry and how the model ingredients such as risk aversion, asymmetric information and hedging activity affect the extent of asymmetry that emerges.

Although there is an extensive theoretical literature on the amplification observed in crashes and frenzies, the asymmetric feature of these large movements has not been addressed until recent years. Moreover, most studies that analyze asymmetry actually focus on a type different from level asymmetry.[5] Hong and Stein (2003) is among the few papers that can account for the level asymmetry observed in stock prices. They use differences of opinion among investors (a trait of bounded rationality) and short-sales constraints to obtain the desired outcome. In our paper, the necessary ingredients for level asymmetry is risk averse rational traders and the presence of hedgers who employ portfolio insurance strategies.

Our paper is organized as follows. Section 2 develops a noisy REE model with hedgers and derive the unique equilibrium. Section 3 provides the results on amplification and asymmetry in price deviations. Sections 4 and 5 check whether the conditions for asymmetry derived in Sect. 3 are satisfied in practice. Finally, Sect. 6 examines the roles of risk aversion and asymmetric information in our analysis.

2 CARA-Gaussian economy

We employ a static REE model, which is a simplified version of Gennotte and Leland (1990) with one informed trader instead of many disparately informed traders. We mimic the approach of Demange and Laroque (19995) to compute the equilibrium price.

2.1 The model

We assume two periods of time in our model. Economic agents, whom we will specify later, competitively trade in the first period and consume in the second. There is only one good in the economy, and there are two securities (i.e. two claims on the good): a risk-free security and a risky security with a future random payoff X, which realizes in the second period. The price and the payoff of the risk-free security are normalized to 1.

There are two types of agents who are characterized by the information they possess:

[5] Most studies analyze *growth rate asymmetry* (see, e.g., Boldrin and Levine 2001; Chalkley and Lee 1998; Veldkamp 2005; Veronesi 1999; Zeira 1999). This type of asymmetry refers to the fact that the histogram of log changes in security prices do not form a symmetric distribution.

1. *Insider*[6] observes the risky security price p and a private random signal S on the risky security payoff X.
2. *Rational outsiders* observe the risky security price p.

All agents, namely the insider and outsiders, maximize expected utility of final wealth over the first period. Agents' utilities exhibit constant absolute risk aversion (CARA).

The net supply of the risky security is determined by two factors:

1 *Liquidity supply* is an exogenously determined random net supply by "liquidity traders". This random supply, L, is distributed with $N(0, \sigma_L)$, and neither insider nor outsiders know its realization.

2 *Hedging supply* is a deterministic net supply by "hedgers". Hedgers employ portfolio insurance strategies that are designed to cap losses on portfolio value by sacrificing upside gains. These strategies are not outcomes of an optimizing behavior,[7] but they are rather fixed rules dictating to buy when the market is going up and sell when it is going down. Therefore the deterministic supply from portfolio insurance strategies, h, is a decreasing function of the risky security price p.[8]

The aggregate net supply of the risky security is simply the sum of liquidity supply and hedging supply. The aggregate net supply is random due to liquidity supply, and this prevents rational outsiders from fully inferring insider's private signal after observing price.

All random variables in our model are Gaussian. The future payoff of the risky security, X, is a normal random variable with non-zero variance. Insider's signal on X is of the form $S = X + \Omega$, where Ω is distributed with $N(0, \sigma_\Omega)$. The random variables X, Ω, and L are jointly normally distributed and independent from each other. Note that, throughout the paper, the random variables are denoted by capital letters, and realizations of them are denoted by the corresponding small letters. The joint distribution of X, Ω and L is common knowledge. The hedging supply function h is also known to both insider and outsiders.

The CARA-Gaussian setup allows us to aggregate outsiders into a single agent, as all outsiders share the same information. From now on we denote the insider by i, and the outsider by o. The constant Arrow-Pratt measure of absolute risk aversion of insider is a_i, and that of outsider is a_o. To be more precise, $\frac{1}{a_o}$ is the sum of all rational outsiders' measures of risk tolerance (as we are aggregating all outsiders into a single agent). We define the *aggregate Arrow-Pratt measure of absolute risk aversion* A by setting $\frac{1}{A} = \frac{1}{a_i} + \frac{1}{a_o}$. Utility functions of insider and outsider are of the form, $u^j(W_j) = -\mathrm{e}^{-a_j W_j}$, $j = i, o$, where W_j is agent j's random final wealth (which realizes in the second period). Both agents maximize expected utility of final wealth over the first period and their expectations depend on their Gaussian information.

[6] We can justify the price-taking behavior of the single insider by assuming that she represents a continuum of mass one of insiders who act competitively.

[7] At least in the context of this model.

[8] Naturally, the particular functional form of the hedging supply h depends on the set of portfolio insurance strategies employed by hedgers. We need not specify or restrict these strategies for the equilibrium analysis conducted in Sects. 2.2 and 3.

Insider and outsider are endowed with deterministic wealth (holdings of risk-free claim on the good) e_i and e_o, respectively.

In the first period, the risky security is traded on the market against the risk-free security. If agent j, $j = i, o$, purchases D_j units of the risky security at price p, j's random final wealth would be $W_j = D_j X + (e_j - pD_j)$. As the rational agent j maximizes her expected utility of consumption in the second period, she solves

$$\begin{aligned} &\max_{D_j} \mathrm{E}[-\mathrm{e}^{-a_j W_j} | I_j] \\ \text{s. to} \quad &D_j X + (e_j - pD_j) = W_j, \end{aligned} \tag{1}$$

where D_j is j's net excess demand of the risky security and I_j is j's Gaussian information. In the first period, total net supply of the risky security at price p is the sum of liquidity supply and hedging supply, i.e. it is

$$l + h(p),$$

where l is the realization of random liquidity supply L.

In the second period, all uncertainty is resolved, and consumption takes place without any further trade.

2.2 Equilibrium

Next we define the equilibrium price in the fashion of rational expectations equilibrium: a rational expectations equilibrium price of the risky security is a function $P(s, l)$ such that, for any realization of signal and liquidity supply (s, l),

$$D_i(p|s) + D_o(p|P(s, l) = p) = l + h(p),$$

where $D_i(p|s)$ solves insider's maximization problem given in (1), conditional on the observation of the price p and the signal s,[9] and $D_o(p|P(s, l) = p)$ solves outsider's maximization problem given in (1), conditional on the observation of p and the knowledge about the price function $P(s, l)$ to update the beliefs on s.

Note that as insider is the only informed trader in the economy, observation of risky security's price does not add any information on top of what he already has. We let Σ denote outsider's Gaussian information. From the definition above we already know Σ coincides to the knowledge of $P(s, l) = p$; however we would like to express outsider's information explicitly as a function of s and l in the equilibrium, hence we introduce this new notation. The excess demand functions of insider and outsider are given by[10]

[9] The random variables are denoted by capital letters and realizations of them are denoted by the corresponding small letters.

[10] Expressions of excess demand functions in CARA-Gaussian environments are well-known, however we still provide the derivations in (B1) of Appendix B.

$$D_i(p|S=s) = \frac{\mathrm{E}[X|s]-p}{a_i \mathrm{var}(X|S)}, \quad D_o(p|\Sigma=\sigma) = \frac{\mathrm{E}[X|\sigma]-p}{a_o \mathrm{var}(X|\Sigma)}. \tag{2}$$

The following notation is introduced:[11]

$$a_i^* = a_i \mathrm{var}(X|S), \quad a_o^* = a_o \mathrm{var}(X|\Sigma), \quad \frac{1}{A^*} = \frac{1}{a_i^*} + \frac{1}{a_o^*}.$$

Given joint distributions of X, S, and L, A^* is only a function of insider's risk aversion a_i, and outsider's risk aversion a_o. That is, the value of A^* does not depend on the realization of insider's signal and liquidity supply (since normal conditional variances are independent of realizations). We further assume the following:

S1. $I + A^*h$ is strictly monotone (i.e. either strictly increasing or strictly decreasing).[12]

This assumption guarantees a continuous equilibrium price function that can be used for comparative statics. Without assuming S1, the proof of the existence of an equilibrium still holds, but it leads to a price correspondence which may not be single-valued. One now has the following:

Proposition 1 (Equilibrium) *Assume S*1. *Then the unique rational expectations equilibrium price is given by*

$$\begin{aligned} P(s,l) &= f^{-1}\left(\frac{A^*}{a_i^*}\mathrm{E}[X|s] + \frac{A^*}{a_o^*}\mathrm{E}[X|\sigma] - A^* l\right) \\ &= f^{-1}\left(\mathrm{E}[X|\sigma] + \frac{A^*}{a_i^*}(\sigma - \mathrm{E}[X|\sigma])\right), \end{aligned}$$

where f^{-1} *is the inverse of* $f \equiv I + A^*h$, *and* $\sigma = \mathrm{E}[X|s] - a_i \,\mathrm{var}(X|S)\, l$ *is the* (*realization of*) *outsider's information.*

Proof S1 guarantees that f^{-1} is a well-defined continuous function. Excess demand functions of insider and outsider are also well-defined since var Ω and var X are non-zero. Hence market clearing yields

$$\left(\frac{1}{a_i \mathrm{var}(X|S)} + \frac{1}{a_o \mathrm{var}(X|\Sigma)}\right) p + h(p) = \frac{\mathrm{E}[X|s]}{a_i \mathrm{var}(X|S)} + \frac{\mathrm{E}[X|\sigma]}{a_o \mathrm{var}(X|\Sigma)} - l.$$

Outsider's information σ is revealed by the observation of price and the knowledge of price function. The price function is essentially derived from the market clearing condition above, thus outsider's information coincides with the knowledge of market clearing condition. Since the hedging function h and distributions of S and L are

[11] Note that we abuse the notation here by writing $\mathrm{var}(X|S)$ instead of $\mathrm{var}(X|s)$, i.e. we condition the variance of X on the distribution of signal rather than its realization. However normal conditional variances do not depend on realizations, thus our notation for the variance fits to this characteristic of the Gaussian environment.

[12] I denotes the identity function, i.e. $I(x) = x \;\; \forall x \in \mathbb{R}$.

common knowledge, and values of conditional normal variances are independent from realizations,[13] outsider can induce the following information from market clearing: $\frac{\mathrm{E}[X|s]}{a_i \operatorname{var}(X|S)} - l$. Multiplying this argument by a known constant (namely $a_i \operatorname{var}(X|S)$) would not matter for the informational content, therefore outsider's information is equivalent to the knowledge of the realization $\sigma = \mathrm{E}[X|s] - a_i \operatorname{var}(X|S)\, l$. Recall that S and L are jointly normally distributed. So Σ (the random distribution σ belongs to) is also normally distributed, and outsider's demand as given in (2) holds. Rewriting market clearing condition we have

$$p + A^* h(p) = \frac{A^*}{a_i^*}\mathrm{E}[X|s] + \frac{A^*}{a_o^*}\mathrm{E}[X|\sigma] - A^* l,$$

where A^*, a_i^*, and a_o^* are as defined above. Writing $\frac{A^*}{a_o^*} = 1 - \frac{A^*}{a_i^*}$, and using definition of f; the result follows. □

Note that the equilibrium price of risky security given by Proposition 1 is a function of insider's private signal s and liquidity supply l. In the Gaussian framework, $\mathrm{E}[X|s]$ is a linear increasing function of s, and given s the assessment of conditional expectation does not put a burden on the agents from the informational perspective since all the parameters necessary to extract its functional form are common knowledge. Therefore the comparative statics results in this paper do not change qualitatively if the equilibrium price is taken as a function of the vector $(\mathrm{E}[X|s], l)$ rather than (s, l). Using $(\mathrm{E}[X|s], l)$ as the underlying parameters, the equilibrium price function takes the form[14]

$$P(\mathrm{E}[X|s], l) = f^{-1}\left(Q(\mathrm{E}[X|s], l)\right), \quad \text{where} \tag{3a}$$

$$\begin{aligned} Q(\mathrm{E}[X|s], l) = &-\frac{A^*}{a_o^*}\left\{1 - \frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma}\right\}\mathrm{E}X \\ &+ \left\{\frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma} + \frac{A^*}{a_i^*}\left(1 - \frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma}\right)\right\}\mathrm{E}[X|s] \\ &- \left\{a_i^* \frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma} + A^*\left(1 - \frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma}\right)\right\} l. \end{aligned} \tag{3b}$$

2.3 Trading behavior in the presence of hedgers

In this section, we will analyze the effect of hedging on rational agents' trading behaviors. However, before embarking on this analysis, it is important to identify how hedging affects the direction of price deviations. As it follows from (3a) to (3b), the equilibrium price[15] is a strictly increasing function of $\mathrm{E}[X|s]$ and a strictly decreasing function of l when there is no hedging supply. This is quite plausible since

13 See (A1) in Appendix A.

14 See (B2) in Appendix B for the derivation.

15 From now on, the term "price" stands for the risky security price unless otherwise stated.

security prices tend to increase in the presence of good news and they tend to fall when liquidity of the security increases. Theoretically, presence of hedgers may pervert this observed characteristic of security prices, that is, prices may fall with good news and increase with liquidity supply. This may happen because after good news or a fall in liquidity supply hedging demand may be so intense that rational insider and outsiders may find it profitable to sell the security, which may in turn decrease the price. However, in reality, such intense hedging supply (or demand) is not observed. The following lemma provides the necessary and sufficient condition for hedging to lead to price reactions in accord with reality.

Lemma 1 *Let f^{-1} be differentiable. Then $P(\mathrm{E}[X|s], l)$ is strictly increasing in $\mathrm{E}[X|s]$ and strictly decreasing in l if and only if*

(S1′) *$I + A^* h$ is strictly increasing.*

Note that assumption S1 necessarily holds when condition S1′ holds.

Having identified the condition under which hedging activity has no perverse effect on price deviations, we now investigate how rational traders (i.e. insider and outsider) and hedgers interact in our economy. Recall that equilibrium demand function of a rational trader is of the form

$$D_j(P(\mathrm{E}[X|s], l)|I_j) = \frac{\mathrm{E}[X|I_j] - P(\mathrm{E}[X|s], l)}{a_j \mathrm{var}(X|I_j)}, \tag{4}$$

where I_j stands for the Gaussian information of agent $j = i, o$. We can partition the rational demand into the *information effect* $\frac{\mathrm{E}[X|I_j]}{a_j \mathrm{var}(X|I_j)}$, and the *substitution effect* $-\frac{P(\mathrm{E}[X|s],l)}{a_j \mathrm{var}(X|I_j)}$. The overcoming effect among these two determines the direction of the rational demand reaction whenever price deviates. To further our analysis, we also incorporate the size of hedging activity as a parameter into the hedging supply by letting

$$h(p) = \alpha\, \Pi(p), \quad \forall p. \tag{5}$$

In the expression above, Π is a decreasing function of p, and α denotes the fraction of assets protected by hedging (portfolio insurance). We have the following result:

Proposition 2 (Trading behavior) *Assume that condition S1′ stated in Lemma 1 holds and f^{-1} is differentiable. Then*

(a) *$D_o(P(\mathrm{E}[X|s], l)|\sigma)$ is decreasing in $\mathrm{E}[X|s]$ and increasing in l.*
(b) *$D_i(P(\mathrm{E}[X|s], l)|\mathrm{E}[X|s])$ is increasing in l.*
(c) *If the fraction α of assets protected by hedging (portfolio insurance) is sufficiently small, then $D_i(P(\mathrm{E}[X|s], l)|\mathrm{E}[X|s])$ is increasing in $\mathrm{E}[X|s]$. If α is sufficiently large, then $D_i(P(\mathrm{E}[X|s], l)|\mathrm{E}[X|s])$ is decreasing in $\mathrm{E}[X|s]$.*

Part (a) of Proposition 2 shows that, regardless of the size of hedging activity, outsider's demand of risky security tends to decrease as insider's expectation of risky payoff increases and as liquidity supply decreases. The negative relationship between outsider's demand and insider's risky payoff expectation might seem puzzling at first, because outsider is able to partially infer insider's private information through price. However, all that is inferred by outsider is already compounded into price which prevents her from enjoying any informational rent. This is for instance the case in Grossman (1976): there is no random supply in Grossman's (1976) model and the information and substitution effects cancel each other exactly so that demands stay unresponsive to information shocks. In models with random supply, as is the case in our model, the substitution effect is relatively stronger for an outsider which renders a decrease in outsider's demand with good news. The presence of hedging only makes the substitution effect more pronounced as hedgers' demand follows the price trend. Therefore with or without hedging, the direction of outsider's demand reaction stays the same and moves in the opposite direction from insider's expectation.[16]

Part (c) of Proposition 2 shows that presence of hedging can alter the direction of insider's demand reaction to information shocks. Without any hedging activity, insider demands more of the risky security when good news arrives. Although this is most plausible, it may not be straightforward to observe this from the interaction between information and substitution effects. Good news not only implies a more pronounced information effect but also a more pronounced substitution effect as price increases after good news. Since the two effects move insider's demand in opposite directions, we need to identify which of the two overcomes the other. Given that insider has private information which is never fully incorporated into price due to random liquidity shocks, she can enjoy positive informational rent. This in turn makes information effect relatively stronger. However, in the presence of hedging, insider's demand reaction may change direction. If the size of hedging is large enough, the price sensitivity to information shocks is amplified excessively by hedgers who follow the price trend. As a consequence, substitution effect can overcome information effect, and insider's demand begins to decrease with good news.

Unsurprisingly the presence of hedging affects trading behaviors of both insider and outsider, however, as the discussion above shows, hedging has a more profound effect on insider's trading behavior compared to that of outsider since it can alter the direction of insider's demand reaction to information shocks.

2.4 Asymmetric price deviations and non-linear prices

As it follows from (3a), the equilibrium price is a function of insider's expectation of the risky payoff ($\mathrm{E}[X|s]$) and the liquidity supply (l). Next we discuss how the asymmetry between crashes and frenzies emerges in our setup. If the equilibrium price function P were linear in ($\mathrm{E}[X|s], l$), then negative and positive shocks (on insider's information or liquidity supply) of the same magnitude would create price deviations of the same size. Thus we could only attribute the asymmetry in favor of crashes to more frequent and significant negative shocks. As there is no evidence of more frequent negative information or liquidity shocks in the history of S&P 500, we are interested in asymmetric price deviations triggered by symmetric shocks. Formally, we have the

[16] Note that the result and the underlying intuition do not carry over to a model with many risky securities. See Admati (1985) for a detailed discussion of this issue.

following: given $(\mathrm{E}[X|s_0], l_0)$, we say that there is an *asymmetry in deviations* at the equilibrium price $P(\mathrm{E}[X|s_0], l_0)$ if for some $(\Delta_1, \Delta_2) > 0$[17]

$$P(\mathrm{E}[X|s_0], l_0) - P(\mathrm{E}[X|s_0] - \Delta_1, l_0) \neq P(\mathrm{E}[X|s_0] + \Delta_1, l_0) - P(\mathrm{E}[X|s_0], l_0), \text{ or}$$
$$P(\mathrm{E}[X|s_0], l_0) - P(\mathrm{E}[X|s_0], l_0 - \Delta_2) \neq P(\mathrm{E}[X|s_0], l_0 + \Delta_2) - P(\mathrm{E}[X|s_0], l_0).$$

Clearly, *non-linearity* of the equilibrium price function in $\mathrm{E}[X|s]$ or l is necessary and sufficient for asymmetry in price deviations. Recall that $f \equiv I + A^*h$. When there is no hedging supply, $f = I$ and $P(\mathrm{E}[X|s], l)$ is linear in $(\mathrm{E}[X|s], l)$ by (3a) and (3b). So asymmetric information by itself can not create asymmetric deviations in price. With non-zero A^*, non-linearity of hedging supply h becomes a necessary and sufficient condition for a non-linear equilibrium price function, and consequently for asymmetric deviations in price. Given $(\mathrm{E}[X|s_0], l_0)$ we say information and liquidity shocks cause a *bias towards negative price deviations* within the set $U_{s_0} \times U_{l_0}$ if for all $(\Delta_1, \Delta_2) > 0$ such that $\mathrm{E}[X|s_0] - \Delta_1 \in U_{s_0}$, $\mathrm{E}[X|s_0] + \Delta_1 \in U_{s_0}$, $l_0 - \Delta_2 \in U_{l_0}$, $l_0 + \Delta_2 \in U_{l_0}$, the following holds:

$$P(\mathrm{E}[X|s_0], l_0) - P(\mathrm{E}[X|s_0] - \Delta_1, l_0) > P(\mathrm{E}[X|s_0] + \Delta_1, l_0) - P(\mathrm{E}[X|s_0], l_0),$$
$$P(\mathrm{E}[X|s_0], l_0) - P(\mathrm{E}[X|s_0], l_0 - \Delta_2) > P(\mathrm{E}[X|s_0], l_0 + \Delta_2) - P(\mathrm{E}[X|s_0], l_0).$$

Suppose equilibrium price function P is continuously differentiable. Then there exists a bias towards negative price deviations within $U_{s_0} \times U_{l_0}$ if and only if $P(\mathrm{E}[X|s], l)$ is *strictly concave* in $\mathrm{E}[X|s]$ and l within $U_{s_0} \times U_{l_0}$. This is due to the fact that for a strictly concave and continuously differentiable function g

$$g(x_1) < g(x_0) + g'(x_0)(x_1 - x_0),$$

and letting x_1 equal to first $x_0 - \Delta_x$ and then $x_0 + \Delta_x$ one gets

$$g(x_0) - g(x_0 - \Delta_x) > g(x_0 + \Delta_x) - g(x_0).$$

Note the following obvious that whenever $P(\mathrm{E}[X|s], l)$ is globally concave in $\mathrm{E}[X|s]$ and l, all shocks will cause a bias towards negative price deviations in the economy. One can also interpret the strict concavity of equilibrium price P as overreaction to negative shocks.

3 Amplification and asymmetry

In this section we present comparative statics of the equilibrium price P. The first-order partial derivatives of $P(\mathrm{E}[X|s], l)$ with respect to $\mathrm{E}[X|s]$ and l determine the sensitivity of price to changes in insider's information and liquidity supply, respectively. The second-order partial derivatives determine the concavity of price function, hence it reveals the nature of bias within the asymmetric price deviations. Our purpose is

[17] $(\Delta_1, \Delta_2) > 0$ if and only if both Δ_1 and Δ_2 are strictly positive.

to see how hedging supply affects the first and second-order partial derivatives of the equilibrium price. If, in the presence of hedging supply, there is higher price sensitivity to changes in information and liquidity, we will be able to conclude that hedging amplifies price reactions. Also, if, with hedging supply, the equilibrium price becomes a concave function of the underlying parameters, this will imply that hedging creates a level asymmetry biased towards negative deviations.

Our first result is on amplification:

Proposition 3 (Amplification) *Assume that condition $S1'$ stated in Lemma 1 holds and f^{-1} is differentiable. As the fraction α of assets protected by hedging increases,*[18] *the equilibrium price function becomes more sensitive to information and liquidity shocks. That is,* $\left|\frac{\partial P(\mathrm{E}[X|s],l)}{\partial \mathrm{E}[X|s]}\right|$ *and* $\left|\frac{\partial P(\mathrm{E}[X|s],l)}{\partial l}\right|$ *are increasing functions of* α.

Proposition 3 reveals the *amplifying effect* of hedging activity on price movements. The intuition is easy to see: once the price begins to fall (due to bad news or increasing liquidity supply), there will be more hedging supply of the security which will further push the prices to much lower levels. So, in the presence of hedging, one will see amplified price reactions to the triggering events (such as bad news or higher liquidity). Naturally, the bigger the size of hedging activity is, the larger the price reactions will be. Of course, the same argument works for the price hikes.

Next we analyze the second characteristic of large price movements: the asymmetry in favor of crashes. To that end, we need to check the concavity of equilibrium price with respect to the parameters $\mathrm{E}[X|s]$ and l (see earlier discussion in Sect. 2.4). We have the following result:

Proposition 4 (Asymmetry) *Assume that condition $S1'$ stated in Lemma 1 holds and f^{-1} is twice-differentiable. If hedging supply h is a strictly convex function within the set*

$$P\left(U_{s_0}, U_{l_0}\right) = \{p : p = P(\mathrm{E}[X|s], l) \quad s.t. \quad (\mathrm{E}[X|s], l) \in U_{s_0} \times U_{l_0}\},$$

then:

(a) *information and liquidity shocks cause a bias towards negative price deviations within $U_{s_0} \times U_{l_0}$, i.e. $P(\mathrm{E}[X|s], l)$ is strictly concave in $\mathrm{E}[X|s]$ and strictly concave in l for $(\mathrm{E}[X|s], l) \in U_{s_0} \times U_{l_0}$,*

(b) *the extent of bias towards negative price deviations increases within $U_{s_0} \times U_{l_0}$ as the fraction α of assets protected by hedging increases, i.e.* $\frac{\partial^2 P(\mathrm{E}[X|s],l)}{\partial \mathrm{E}[X|s]^2}$ *and* $\frac{\partial^2 P(\mathrm{E}[X|s],l)}{\partial l^2}$ *are decreasing functions of α within $U_{s_0} \times U_{l_0}$.*

It is easy to check that even if condition $S1'$ does not hold, the results above (on amplification and asymmetry) will hold within the domain

$$\left\{(\mathrm{E}[X|s], l) \in \mathbb{R}^2 : (I + A^* h)' \left(P(\mathrm{E}[X|s], l)\right) > 0\right\}.$$

[18] See Eq. (5) to recall how α has been incorporated into the hedging supply function.

To sum up, under plausible conditions, whenever a shock (either of informational nature or liquidity based) occurs in the economy, the deviation in price is amplified due to hedging supply. Hence in the presence of hedging, the deviations are more likely to be significant, that is they are more likely to be a crash or a frenzy. Moreover if the hedging supply function is (globally) strictly convex, then a bias towards negative deviations is observed. We can summarize these results as follows:

Corollary 1 (Main result) *Assume that condition* $S1'$ *stated in Lemma* 1 *holds and* f^{-1} *is twice-differentiable. If hedging supply h is a strictly convex function, then:*

(a) *the equilibrium price function becomes more sensitive to information and liquidity shocks as the fraction* α *of assets protected by hedging increases,*
(b) *information and liquidity shocks cause a bias towards negative price deviations,*
(c) *the extent of bias towards negative price deviations increases as the fraction* α *of assets protected by hedging increases.*

One criticism towards the results of this section might be the extent of their dependence on hedging activity. After all, having lots of irrational agents, programmed to behave in ways to create amplification and asymmetry, would not be much of an explanation for the characteristics we are examining. Therefore we would like to show that our results do not stem from an imposed environment with a lot of irrational hedgers accompanied by just enough rational traders to equate supply and demand. The main difference between rational traders (insider, outsider) and hedgers is that their demands react differently to price deviations. That is, the demand of rational traders is a decreasing function of price whereas the demand of hedgers is increasing in price. So we can determine the dominance of a group (namely rational traders or hedgers) in the market by checking the sensitivity of their aggregate demand with respect to price.

Proposition 5 (Market demand) *Let* f^{-1} *be differentiable. Condition* $S1'$ *stated in Lemma* 1 *holds if and only if the aggregate demand Z of the risky security is strictly decreasing in p, where*

$$Z(p) = D_i(p|\mathrm{E}[X|s]) + D_o(p|\sigma) - h(p).$$

This proposition shows that demand of rational traders (insider and outsiders) prevail over that of hedgers if and only if condition S1$'$ holds. Since we get the results of this section with practically one condition, namely S1$'$, we can say that our results hold within an environment where rationality prevails.

4 Put-option replication

In this section, we examine a specific hedging (portfolio insurance) strategy: the put-option replication. Put-option replication was the most popular hedging strategy during 1980's, in particular, during the October 1987 crash. The formula for the put-option replication is taken from Gennotte and Leland (1990).[19] The hedging strategy is assumed to be applied to a fraction α of risky securities. The incremental hedging supply when new price is p, relative to the supply at the hypothetical initial price ($p^* = 1$), is given by

$$\hat{h}(p) = \alpha \left(\Phi(d(1)) - \Phi(d(p))\right),$$

where $\Phi(.)$ is the standard cumulative normal distribution function, and $d(.)$ is derived from the Black–Scholes formula

$$d(p) = \frac{\ln\left(\frac{p}{K}\right) + \frac{1}{2}\mathrm{var}(X|\Sigma)}{\sqrt{\mathrm{var}(X|\Sigma)}}.$$

with K as the striking price for the option (or the protection level in the replication case).[20]

Possibility of negative security prices is a caveat of the CARA-Gaussian framework. Naturally we focus on strictly positive prices for the analysis of put-option replication. Note that

$$\hat{h}'(p) = -\frac{\alpha\phi(d(p))}{p\sqrt{\mathrm{var}(X|\Sigma)}},$$

where $\phi(.)$ is the standard normal density function. Clearly, $\hat{h}$ is decreasing in the domain of strictly positive prices. Extracting $\hat{h}'(p)$, we get

$$\hat{h}'(p) = -\frac{\alpha}{p}\frac{\exp\left(-\frac{1}{2}\left(\frac{\ln\frac{p}{K}+\frac{1}{2}\mathrm{var}(X|\Sigma)}{\sqrt{\mathrm{var}(X|\Sigma)}}\right)^2\right)}{\sqrt{2\pi\,\mathrm{var}(X|\Sigma)}}.$$

Now it is easy to see the following:

Lemma 2 *Given* $p_0 > 0$, *if* $\alpha \leq \frac{p_0\sqrt{2\pi\,\mathrm{var}(X|\Sigma)}}{A^*}$, *then* $\hat{h}'(p) > -\frac{1}{A^*}$ *for all* $p \in [p_0, \infty)$. *Moreover, as* α *tends to* 0, *the set* $\left\{p : \hat{h}'(p) > -\frac{1}{A^*}\right\}$ *will converge to the domain of strictly positive prices* $(0, \infty)$.

So if the fraction α of assets protected by hedging is sufficiently small, condition S1$'$ stated in Lemma 1 holds for $\hat{h}$ over a strict subset of positive prices. It is easy to check that all our proofs will work over this strict subset. To be more precise, our

[19] Gennotte and Leland (1990) point out the differences in their formula compared to Black and Scholes (1973). They assume that interest rate has been normalized to zero, and assume a 1-year time horizon. Moreover in theirs payoff is normally distributed (as in our model), whereas in Black and Scholes (1973) payoff follows a log-normal process.

[20] In the actual Black–Scholes formula, we would have $d(p) = \left(\ln\left(\frac{p}{K}\right) + \frac{1}{2}\mathrm{var}(X|P)\right)\left(\sqrt{\mathrm{var}(X|P)}\right)^{-1}$. However as we elaborated before, observing P is equivalent to observing Σ (see Proposition 1).

results on amplification and asymmetry (stated in Propositions 3 and 4) still hold over the domain

$$\left\{(\mathrm{E}[X|s], l) \in \mathbb{R}^2 : \hat{h}'\left(P(\mathrm{E}[X|s], l)\right) > -\frac{1}{A^*}\right\},$$

and this domain converges to $\{(\mathrm{E}[X|s], l) \in \mathbb{R}^2 : P(\mathrm{E}[X|s], l) > 0\}$ as α tends to 0.

For the convexity of $\hat{h}$, we need to check the second-order partial derivative:

$$\begin{aligned}\hat{h}''(p) &= -\frac{\alpha}{\sqrt{\mathrm{var}(X|\Sigma)}} \frac{\phi'(d(p))d'(p)p - \phi(d(p))}{p^2} \\ &= \frac{\alpha\phi(d(p))}{p^2\sqrt{\mathrm{var}(X|\Sigma)}}\left(\frac{d(p)}{\sqrt{\mathrm{var}(X|\Sigma)}} + 1\right) \\ &= \frac{\alpha \exp(-d(p)^2)}{p^2\sqrt{2\pi\,\mathrm{var}(X|\Sigma)}}\left(\frac{\ln\left(\frac{p}{K}\right)}{\mathrm{var}(X|\Sigma)} + 2\right).\end{aligned}$$

The following result can be easily proved using this equation:

Lemma 3 $\hat{h}$ *is strictly convex over the domain* $\left\{p : p > \frac{K}{e^{2\mathrm{var}(X|\Sigma)}}\right\}$. *As* $\mathrm{var}(X|\Sigma)$ *tends to* ∞, *the domain where* $\hat{h}$ *is strictly convex will converge to the set of strictly positive prices.*

Now using Lemmas 2, 3, and our results from Sect. 3 (namely Propositions 3 and 4), we obtain the following:

Proposition 6 (Put-option replication) *If*

(i) *insider's risky payoff expectation and liquidity supply are in the domain*

$$\left\{(\mathrm{E}[X|s], l) \in \mathbb{R}^2 : P(\mathrm{E}[X|s], l) > \frac{K}{e^{2\mathrm{var}(X|\Sigma)}}\right\},$$

(ii) α *is less than* $\frac{K\sqrt{2\pi\,\mathrm{var}(X|\Sigma)}}{A^* e^{2\mathrm{var}(X|\Sigma)}}$, *and*

(iii) *hedgers employ put-option replication so that* $\hat{h}$ *is the hedging supply function,*

then:

(a) *the equilibrium price function becomes more sensitive to information and liquidity shocks as the fraction* α *of assets protected by hedging increases, i.e.* $\left|\frac{\partial P(\mathrm{E}[X|s],l)}{\partial \mathrm{E}[X|s]}\right|$ *and* $\left|\frac{\partial P(\mathrm{E}[X|s],l)}{\partial l}\right|$ *are increasing functions of* α,

(b) *information and liquidity shocks cause a bias towards negative price deviations, i.e.* $P(\mathrm{E}[X|s], l)$ *is strictly concave in* $\mathrm{E}[X|s]$ *and strictly concave in* l,

(c) *the extent of bias towards negative price deviations increases as the fraction* α *of assets protected by hedging increases, i.e.* $\frac{\partial^2 P(\mathrm{E}[X|s],l)}{\partial \mathrm{E}[X|s]^2}$ *and* $\frac{\partial^2 P(\mathrm{E}[X|s],l)}{\partial l^2}$ *are decreasing functions of* α.

Moreover as $\text{var}(X|\Sigma)$ *tends to* ∞*, the domain where the bias towards negative price deviations is observed will converge to* $\left\{(\mathrm{E}[X|s], l) \in \mathbb{R}^2 : P(\mathrm{E}[X|s], l) > 0\right\}$.

Proposition 6 reveals that price deviations are more likely to be significant, that is, they are more likely to be a crash or a frenzy, provided risky payoff is highly volatile and hedgers employ put-option replication as their portfolio insurance strategy. Moreover, for a large domain of positive security prices a bias towards negative deviations is observed under the same conditions. Thus we can conclude that for a large domain of positive security prices there would be a bias towards crashes when risky payoff is highly volatile and put-option replication is the prevalent hedging strategy.

Though put-option replication and other portfolio insurance strategies played an important role in modern times, it is hard to use the same argument for the first half of the century. The sophisticated portfolio insurance strategies did not even exist then. However there is a hedging strategy which has been in use arguably as long as stock markets existed: stop-loss. In its most primitive form, hedgers sell their risky securities when the price falls below a predetermined level, say K. Use of this primitive hedging form clearly creates the asymmetry we want: there is an additional downward pressure on sales once price falls below K whereas there is no pressure when market goes up. Hence we get price asymmetry biased towards crashes with stop-loss as well.

5 A numerical example: back to 1980s

The levels of risk aversion, hedging and risky payoff volatility necessary for substantial sizes of amplification and asymmetry are, of course, matters of concern. In other words, we do not want to generate amplification and asymmetry using implausible values for the parameters of our model. So we examine the following numerical example:

Let us take put-option replication, the most popular portfolio insurance strategy of 1980s, as the hedging function. We assume α to be 0.05, which is not far from the hedging size in the 1987 crash. The protection level K is assumed to be 85% of initial price. Let us fix the initial equilibrium price to be 1 so that K becomes 0.85. Assuming an expected 6% return on the risky security compared to a risk-free asset is reasonable for U.S. markets, thus we let $\mathrm{E}[X] = 1.06$. Outsider is assumed to be more risk averse than insider by letting $a_i = 0.70$ and $a_o = 1.40$. Take $\text{var}(X|S)$, $\text{var}(X|\Sigma)$ and $\frac{\text{cov}(X,\Sigma)}{\text{var}\,\Sigma}$ to be 200, 400 and 0.5, respectively.[21] Note that these values illustrate the informational advantage of insider through $\frac{\text{var}(X|\Sigma)}{\text{var}(X|S)} = 2$. We assume l to be 0 as liquidity supply is not biased.

Then to create a 20% price deviation in the negative direction it takes a 2.9% fall in the insider's expectation on risky payoff ($\mathrm{E}[X|s]$) whereas a positive price deviation of the same magnitude requires a 8.5% increase in $\mathrm{E}[X|s]$. This example clearly depicts the asymmetry.

Moreover if there were no hedging in the market, a 20% price movement in any direction would require a 18.1% change in the information parameter $\mathrm{E}[X|s]$. Clearly

21 $\frac{\text{cov}(X,\Sigma)}{\text{var}\,\Sigma}$ always takes values between 0 and 1. See Lemma B (C1) in Appendix C.

in the case with put-option replication, price is more sensitive to the parameter changes, which illustrates the amplification brought by hedging.

6 Roles of risk aversion and asymmetric information

Lee (1998) makes the following conjecture in the conclusion of his paper: "under risk aversion it is more difficult to trigger a frenzy than a crash because a surprise of the same degree in the direction of the good state induces a smaller response than the one in the direction of the bad state." Granted Lee's model exploits a totally different mechanism, his conjecture actually pinpoints the role of risk aversion in our analysis. The following proposition demonstrates this:

Proposition 7 *Assume that condition $S1'$ stated in Lemma 1 holds. As insider or outsider tends to be risk neutral, asymmetry vanishes in the equilibrium price deviations.*

Proposition 7 simply follows from Proposition 1 and Eqs. (3a)–(3b): f^{-1} converges to the identity function as either of the risk aversion parameters a_i or a_o converges to 0, which then implies $P(\mathrm{E}[X|s], l)$ converging to a linear function. If the equilibrium price converges to a linear function, it simply means that asymmetry in price deviations vanishes.

In our model, risk aversion allows hedging to be incorporated to the price function. If traders are risk neutral, then hedging does not affect price function at all, which consequently means there are no asymmetric deviations.

Next we discuss the role of asymmetric information in our analysis. For convenience, we first define a measure for the level of asymmetry regarding information. Notice that the ratio $\frac{\mathrm{var}(X|\Sigma)}{\mathrm{var}(X|S)}$ gives the imprecision of the information of outsider relative to that of the insider, i.e. given the gaussian nature of our framework this ratio delivers insider's informational advantage over outsider. So we let

$$\mu := \frac{\mathrm{var}(X|\Sigma)}{\mathrm{var}(X|S)},$$

and call the ratio μ, $\mu > 1$, the *measure of asymmetric information*.[22] The bigger the measure μ gets, the larger the asymmetry between insider and outsider is. Now we can easily see how asymmetric information affects our analysis:

Proposition 8 (Asymmetric information) *Assume that condition $S1'$ stated in Lemma 1 holds and $h'(.) < -\frac{1}{a_i^*}$. Also suppose that f^{-1} is continuously twice-differentiable and hedging supply h is strictly convex. There exists $\bar{\mu} > 1$ such that within the domain $(\bar{\mu}, \infty)$ of the asymmetric information measure μ*

(a) *the equilibrium price function becomes more sensitive to information and liquidity shocks as μ increases, i.e. $\left|\frac{\partial P(\mathrm{E}[X|s],l)}{\partial \mathrm{E}[X|s]}\right|$ and $\left|\frac{\partial P(\mathrm{E}[X|s],l)}{\partial l}\right|$ are increasing functions of μ,*

[22] Since outsider's information is more imprecise compared to that of insider's, the measure of asymmetric information $\mu \equiv \frac{\mathrm{var}(X|\Sigma)}{\mathrm{var}(X|S)}$ is always strictly greater than 1.

(b) *the extent of bias towards negative price deviations increases as μ increases, i.e.* $\frac{\partial^2 P(\mathrm{E}[X|s],l)}{\partial \mathrm{E}[X|s]^2}$ *and* $\frac{\partial^2 P(\mathrm{E}[X|s],l)}{\partial l^2}$ *are decreasing functions of μ.*

The only new assumption in this proposition, which has not been employed before, is

$$h'(.) < -\frac{1}{a_i^*} \equiv -\frac{1}{a_i \operatorname{var}(X|S)},$$

and this may be justified if the information of insider is sufficiently imprecise (i.e. if $\operatorname{var}(X|S)$ is sufficiently large). The proposition states that, with sufficiently large asymmetry between insider and outsider in terms of information owned, both amplification and asymmetry of price deviations will be more significant as the measure of asymmetric information μ increases.

So how can information asymmetry create the effects described above? Consider, to begin with, an economy in which outsider is sufficiently less informed compared to insider.[23] In this economy, outsider essentially plays the role of a buffer against information and liquidity shocks: due to information asymmetry, reactions of outsider's demand to such shocks are less pronounced than the reactions of insider's demand. The relative intensity between the demands of outsider and insider determines the relative weights of these demands in the price formation process, which in turn determines the level of price sensitivity to shocks. As the information asymmetry increases, insider's demand becomes more intense relative to that of outsider, and, as a consequence, price sensitivity to information and liquidity shocks become more pronounced. When hedging supply is added into this picture, price sensitivity is enhanced even further due to hedging's amplifying nature. Also, since price movements following shocks become more pronounced with increased information asymmetry, these movements trigger more aggressive reactions from hedgers who employ convex supply functions. This, in turn, leads to an enhanced asymmetry in price deviations.

Unlike risk aversion, asymmetric information is not a necessary ingredient in our model to create asymmetry in price deviations. However, its presence allows us to generate significant amplification and asymmetry with plausible risk aversion coefficients in numerical computations, as has been illustrated in Sect. 5.

Appendix A: Mathematical preliminaries

A1 Projection theorem

For jointly normally distributed random variables X and Θ, the following hold:

$$\mathrm{E}[X|\Theta = \theta] = \mathrm{E}[X] + \frac{\operatorname{cov}(X, \Theta)}{\operatorname{var}\Theta}(\theta - \mathrm{E}\Theta),$$
$$\operatorname{var}(X|\Theta) = \operatorname{var}(X) - \frac{(\operatorname{cov}(X, \Theta))^2}{\operatorname{var}\Theta}.$$

[23] Note that this is as well the case in Proposition 8 since the asymmetric information measure μ is within the domain $(\bar{\mu}, \infty)$ where $\bar{\mu} > 1$.

A2 Rao's formula

For a normal random variable X, the following holds:

$$\mathrm{E}[\mathrm{e}^X] = \mathrm{e}^{\left(\mathrm{E}X+\frac{\mathrm{var}X}{2}\right)}.$$

Appendix B: Derivations

B1 Derivation of excess demand functions

Since X is normal, W_j is also normal for $j = i, o$. By Rao's formula (A2) we have

$$\mathrm{E}[u^j(W_j)|I_j] = -\exp\left(-a_j D_j \mathrm{E}[X|I_j] - a_j(e_j - pD_j) + a_j^2 D_j^2 \frac{\mathrm{var}(X|I_j)}{2}\right).$$

Agent $j \in \{i, o\}$ solves the maximization problem, given in (1). The solution to this problem is

$$D_j(p) = \frac{\mathrm{E}[X|I_j] - p}{a_j \mathrm{var}(X|I_j)}.$$

B2 Derivation of (3b)

Recall that $\sigma = \mathrm{E}[X|s] - a_i^* l$. Projection theorem (A1) implies

$$\mathrm{E}[X|\sigma] = \mathrm{E}X + \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma}(\mathrm{E}[X|s] - a_i^* l - \mathrm{E}X).$$

Therefore

$$\begin{aligned} \mathrm{E}[X|\sigma] + \frac{A^*}{a_i^*}(\sigma - \mathrm{E}[X|\sigma]) = &\left\{\frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma} + \frac{A^*}{a_i^*}(1 - \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma})\right\} \mathrm{E}[X|s] \\ &- \left\{\frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma} + A^*(1 - \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma})\right\} l \\ &- \frac{A^*}{a_o^*}\left\{1 - \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma}\right\} \mathrm{E}X. \end{aligned}$$

B3 Derivatives of the function f^{-1}

Let $y = f(x)$. Then as $f \equiv I + A^* h$, we have

$$(f^{-1})'(y) = \frac{1}{1 + A^* h'(x)}, \quad (f^{-1})''(y) = -\frac{A^* h''(x)}{(1 + A^* h'(x))^3}.$$

B4 Price sensitivity and asymmetry

The sensitivity of price is determined by the partial derivatives of $P(\mathrm{E}[X|s], l)$ with respect to $\mathrm{E}[X|s]$ and l:

$$\frac{\partial P(\mathrm{E}[X|s], l)}{\partial \mathrm{E}[X|s]} = (f^{-1})' \left(Q(\mathrm{E}[X|s], l)\right) \left[\frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma} + \frac{A^*}{a_i^*}\left(1 - \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma}\right)\right], \tag{6a}$$

$$\frac{\partial P(\mathrm{E}[X|s], l)}{\partial l} = -(f^{-1})' \left(Q(\mathrm{E}[X|s], l)\right) \left[a_i^* \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma} + A^*\left(1 - \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma}\right)\right]. \tag{6b}$$

In the case of twice-differentiable price functions, concavity is determined by the second-order partial derivatives:

$$\frac{\partial^2 P(\mathrm{E}[X|s], l)}{\partial \mathrm{E}[X|s]^2} = (f^{-1})'' \left(Q(\mathrm{E}[X|s], l)\right) \left\{\frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma} + \frac{A^*}{a_i^*}\left(1 - \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma}\right)\right\}^2, \tag{7a}$$

$$\frac{\partial^2 P(\mathrm{E}[X|s], l)}{\partial l^2} = (f^{-1})'' \left(Q(\mathrm{E}[X|s], l)\right) \left\{a_i^* \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma} + A^*\left(1 - \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma}\right)\right\}^2. \tag{7b}$$

Appendix C: Proofs

C1 Lemma B. $\frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma} \leq 1$

Proof Note that

$$\begin{aligned}\mathrm{cov}(X, \Sigma) &= \mathrm{cov}\left(X, \mathrm{E}[X|S] - a_i \mathrm{var}(X|S) L\right) = \mathrm{cov}(X, \mathrm{E}[X, S]) \\ &= \mathrm{cov}\left(X, \mathrm{E}X + \frac{\mathrm{cov}(X, S)}{\mathrm{var}\,S}(S - \mathrm{E}X)\right) = \frac{(\mathrm{cov}(X, S))^2}{\mathrm{var}\,S},\end{aligned}$$

$$\begin{aligned}\mathrm{var}\,\Sigma &= \mathrm{var}(\mathrm{E}[X|S]) + {a_i}^2 (\mathrm{var}(X|S))^2 \mathrm{var}\,L \\ &= \mathrm{var}\left(\mathrm{E}X + \frac{\mathrm{cov}(X, S)}{\mathrm{var}\,S}(S - \mathrm{E}X)\right) + {a_i}^2 (\mathrm{var}(X|S))^2 \mathrm{var}\,L \\ &= \frac{(\mathrm{cov}(X, S))^2}{\mathrm{var}\,S} + {a_i}^2 (\mathrm{var}(X|S))^2 \mathrm{var}\,L.\end{aligned}$$

Hence the result follows. □

C2 Proof of Lemma 1

From Lemma B and Eqs. (6a) and (6b), it follows that

$$\operatorname{sign}\left(\frac{\partial P(\mathrm{E}[X|s], l)}{\partial \mathrm{E}[X|s]}\right) = \operatorname{sign}\left((f^{-1})'(Q(\mathrm{E}[X|s], l))\right),$$
$$\operatorname{sign}\left(\frac{\partial P(\mathrm{E}[X|s], l)}{\partial l}\right) = -\operatorname{sign}\left((f^{-1})'(Q(\mathrm{E}[X|s], l))\right).$$

The proof then simply follows from the fact that $(f^{-1})'(y) = \frac{1}{1+A^*h'(x)}$, given $y = f(x)$. □

C3 Proof of Proposition 2

(a) From the extraction of $\mathrm{E}[X|\sigma]$ it follows that $\frac{\partial \mathrm{E}[X|\sigma]}{\partial \mathrm{E}[X|s]} = \frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma}$ and $\frac{\partial \mathrm{E}[X|\sigma]}{\partial l} = -a_i{}^* \frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma}$. Recall from (6a) and (6b) that

$$\frac{\partial P(\mathrm{E}[X|s], l)}{\partial \mathrm{E}[X|s]} = (f^{-1})'(Q(\mathrm{E}[X|s], l)) \left\{\frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma} + \frac{A^*}{a_i^*}\left(1 - \frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma}\right)\right\},$$
$$\frac{\partial P(\mathrm{E}[X|s], l)}{\partial l} = -(f^{-1})'(Q(\mathrm{E}[X|s], l)) \times \left\{a_i^* \frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma} + A^*\left(1 - \frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma}\right)\right\}.$$

Following Lemma B (C1), $\frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma} + \frac{A^*}{a_i^*}\left(1 - \frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma}\right) \geq \frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma}$. Moreover h is a strictly decreasing function, hence $h'(.) < 0$. Then it follows from (B3) that

$$(f^{-1})'(Q(\mathrm{E}[X|s], l)) = \frac{1}{1 + A^*h'(P(\mathrm{E}[X|s], l))} \geq 1$$

under condition S1′. So $\frac{\partial P(\mathrm{E}[X|s],l)}{\partial \mathrm{E}[X|s]} \geq \frac{\partial \mathrm{E}[X|\sigma]}{\partial \mathrm{E}[X|s]}$ and $\frac{\partial P(\mathrm{E}[X|s],l)}{\partial l} \leq \frac{\partial \mathrm{E}[X|\sigma]}{\partial l}$. Therefore it follows from (4) that $D_o(P(\mathrm{E}[X|s], l)|\sigma)$ is decreasing in $\mathrm{E}[X|s]$ and increasing in l.

(b) We have $\frac{\partial \mathrm{E}[X|s]}{\partial l} = 0$. On the other hand, following Lemma 1, $\frac{\partial P(\mathrm{E}[X|s],l)}{\partial l} < 0$. Thus from (6) one observes that $D_i(P(\mathrm{E}[X|s], l)|\mathrm{E}[X|s])$ is increasing in l.

(c) We have $\frac{\partial \mathrm{E}[X|s]}{\partial \mathrm{E}[X|s]} = 1$. Recall from (6a) that

$$\frac{\partial P}{\partial \mathrm{E}[X|s]} = (f^{-1})'(Q(\mathrm{E}[X|s], l)) \left\{\frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma} + \frac{A^*}{a_i^*}\left(1 - \frac{\operatorname{cov}(X,\Sigma)}{\operatorname{var}\Sigma}\right)\right\}.$$

Note that $\frac{A^*}{a_i^*} = \frac{a_o^*}{a_i^*+a_o^*} \leq 1$. We also have $\frac{\text{cov}(X,\Sigma)}{\text{var}\,\Sigma} \leq 1$ from Lemma B (C1). So

$$\frac{\text{cov}(X,\,\Sigma)}{\text{var}\,\Sigma} + \frac{A^*}{a_i^*}\left(1 - \frac{\text{cov}(X,\,\Sigma)}{\text{var}\,\Sigma}\right) \leq 1.$$

On the other hand, we have shown in part (a) that $(f^{-1})'\,(Q(\text{E}[X|s], l)) \geq 1$. Therefore $\frac{\partial P(\text{E}[X|s],l)}{\partial \text{E}[X|s]}$ can be greater or less than 1 depending on the exact value of $(f^{-1})'\,(Q(\text{E}[X|s], l))$. Following from (B3),

$$(f^{-1})'\,(Q(\text{E}[X|s], l)) \to \begin{cases} 1 & \text{as } \alpha \to 0 \\ \infty & \text{as } \alpha \to -\frac{1}{A^*\Pi'\left(f^{-1}(Q(\text{E}[X|s],l))\right)}. \end{cases}$$

Note that under condition S1$'$, α cannot take values larger than $-\frac{1}{A^*\Pi'\left(f^{-1}(.)\right)}$, and also note that $-\frac{1}{A^*\Pi'\left(f^{-1}(.)\right)} \geq 0$ as Π is a decreasing function. Therefore $\frac{\partial P(\text{E}[X|s],l)}{\partial \text{E}[X|s]}$ is less than or equal to 1 if α is sufficiently small and it is greater than 1 if α is sufficiently large. Then it follows from (4) that $D_i\,(P(\text{E}[X|s], l)|\text{E}[X|s])$ is increasing in $\text{E}[X|s]$ for sufficiently small α, and it is decreasing in $\text{E}[X|s]$ for sufficiently large α. □

C4 Proof of Proposition 3

Note that $h' = \alpha\Pi' > -\frac{1}{A^*}$ due to condition S1$'$. We also know from Lemma B (C1) that $\frac{\text{cov}(X,\Sigma)}{\text{var}\,\Sigma} \leq 1$. Thus from (6a) and (6b) and (B3), given $p = P(\text{E}[X|s], l)$, one has

$$\left|\frac{\partial P(\text{E}[X|s], l)}{\partial \text{E}[X|s]}\right| = \frac{1}{1+\alpha A^*\Pi'(p)}\left\{\frac{\text{cov}(X,\Sigma)}{\text{var}\,\Sigma} + \frac{A^*}{a_i^*}\left(1 - \frac{\text{cov}(X,\Sigma)}{\text{var}\,\Sigma}\right)\right\},$$
$$\left|\frac{\partial P(\text{E}[X|s], l)}{\partial l}\right| = \frac{1}{1+\alpha A^*\Pi'(p)}\left\{a_i^*\frac{\text{cov}(X,\Sigma)}{\text{var}\,\Sigma} + A^*\left(1 - \frac{\text{cov}(X,\Sigma)}{\text{var}\,\Sigma}\right)\right\}.$$

Since Π is a decreasing function, it is straightforward to see that $\left|\frac{\partial P(\text{E}[X|s],l)}{\partial \text{E}[X|s]}\right|$ and $\left|\frac{\partial P(\text{E}[X|s],l)}{\partial l}\right|$ are increasing functions of α. □

C5 Proof of Proposition 4

Due to condition S1$'$, $h' > -\frac{1}{A^*}$. Hence from (B3) it follows that if h is strictly convex within the set $P(U_{s_0}, U_{l_0})$, f^{-1} is strictly concave within $P(U_{s_0}, U_{l_0})$, consequently $P(\text{E}[X|s], l)$ is strictly concave in $\text{E}[X|s]$ and strictly concave in l within $U_{s_0} \times U_{l_0}$. This proves (a).

From (7a), (7b) and (B3), given $p = P(\mathrm{E}[X|s], l)$ we have

$$\frac{\partial^2 P(\mathrm{E}[X|s], l)}{\partial \mathrm{E}[X|s]^2} = -\frac{\alpha A^* \Pi''(p)}{(1+\alpha A^* \Pi'(p))^3} \left\{ \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma} + \frac{A^*}{a_i^*} \left(1 - \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma}\right) \right\}^2,$$
$$\frac{\partial^2 P(\mathrm{E}[X|s], l)}{\partial l^2} = -\frac{\alpha A^* \Pi''(p)}{(1+\alpha A^* \Pi'(p))^3} \left\{ a_i^* \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma} + A^* \left(1 - \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma}\right) \right\}^2.$$

Recall that Π is a strictly decreasing function. If h (and hence Π) is strictly convex in $P\left(U_{s_0}, U_{l_0}\right)$, one has $\Pi''(p) > 0$ for $p \in P\left(U_{s_0}, U_{l_0}\right)$, thus $\frac{\partial^2 P(\mathrm{E}[X|s],l)}{\partial \mathrm{E}[X|s]^2}$ and $\frac{\partial^2 P(\mathrm{E}[X|s],l)}{\partial l^2}$ are decreasing functions of α for $(\mathrm{E}[X|s], l) \in U_{s_0} \times U_{l_0}$. Hence (b) is proved. □

C6 Proof of Proposition 5

We have $Z(p) = -\frac{p}{A^*} - h(p) + \frac{\mathrm{E}[X|s]}{a_i \mathrm{var}(X|S)} + \frac{\mathrm{E}[X|\sigma]}{a_o \mathrm{var}(X|\Sigma)}$ and $Z'(p) = -\frac{1}{A^*} - h'(p)$. Therefore condition S1′ holds if and only if Z is strictly decreasing in p. □

C7 Proof of Proposition 8

First, note that the assumptions employed in the statement of the proposition impose $-\frac{1}{A^*} < h'(.) < -\frac{1}{a_i^*}$. From (B3) and (B4), we have the following for $p = P(\mathrm{E}[X|s], l)$:

$$\left|\frac{\partial P(\mathrm{E}[X|s], l)}{\partial \mathrm{E}[X|s]}\right| = \frac{1}{1+A^* h'(p)} \left\{ \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma} + \frac{A^*}{a_i^*} \left(1 - \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma}\right) \right\},$$
$$\left|\frac{\partial P(\mathrm{E}[X|s], l)}{\partial l}\right| = \frac{1}{1+A^* h'(p)} \left\{ a_i^* \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma} + A^* \left(1 - \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma}\right) \right\};$$
$$\frac{\partial^2 P(\mathrm{E}[X|s], l)}{\partial \mathrm{E}[X|s]^2} = -\frac{A^* h''(p)}{(1+A^* h'(p))^3} \left\{ \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma} + \frac{A^*}{a_i^*} \left(1 - \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma}\right) \right\}^2,$$
$$\frac{\partial^2 P(\mathrm{E}[X|s], l)}{\partial l^2} = -\frac{A^* h''(p)}{(1+A^* h'(p))^3} \left\{ a_i^* \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma} + A^* \left(1 - \frac{\mathrm{cov}(X, \Sigma)}{\mathrm{var}\,\Sigma}\right) \right\}^2.$$

Observe that as $\mu \equiv \frac{\mathrm{var}(X|\Sigma)}{\mathrm{var}(X|S)} \to -\frac{1}{a_o \mathrm{var}(X|S)\left(\frac{1}{a_i^*} + h'(p)\right)}$ we have $1 + A^* h'(p)$ tending to 0. Consequently, as $\mu \to -\frac{1}{a_o \mathrm{var}(X|S)\left(\frac{1}{a_i^*} + h'(p)\right)}$

$$\left|\frac{\partial P(\mathrm{E}[X|s], l)}{\partial \mathrm{E}[X|s]}\right|, \left|\frac{\partial P(\mathrm{E}[X|s], l)}{\partial l}\right| \to \infty,$$
$$\frac{\partial^2 P(\mathrm{E}[X|s], l)}{\partial \mathrm{E}[X|s]^2}, \frac{\partial^2 P(\mathrm{E}[X|s], l)}{\partial l^2} \to -\infty. \tag{8}$$

At this point, we need to check that $-\frac{1}{a_o \mathrm{var}(X|S)\left(\frac{1}{a_i^*}+h'(p)\right)} > 1$ as $\mathrm{var}(X|\Sigma) > \mathrm{var}(X|S)$. Suppose not. Given $h'(.) < -\frac{1}{a_i^*}$, it must hold that $-a_o \mathrm{var}(X|S) \left(\frac{1}{a_i^*} + h'(p)\right) > 1$. Then

$$h'(p) < -\frac{1}{a_o \mathrm{var}(X|S)} - \frac{1}{a_i^*} < -\frac{1}{a_o \mathrm{var}(X|\Sigma)} - \frac{1}{a_i^*} = -\frac{1}{A^*},$$

which violates condition S1$'$ since it implies $h'(.) > -\frac{1}{A}$. This proves $-\frac{1}{a_o \mathrm{var}(X|S)\left(\frac{1}{a_i^*}+h'(p)\right)} > 1$.

Following the limit results derived in (8), there exists $\bar{\mu} > 1$ such that within the domain $\left(\bar{\mu}, -\frac{1}{a_o \mathrm{var}(X|S)\left(\frac{1}{a_i^*}+h'(p)\right)}\right)$ we have $\left|\frac{\partial P(\mathrm{E}[X|s],l)}{\partial \mathrm{E}[X|s]}\right|, \left|\frac{\partial P(\mathrm{E}[X|s],l)}{\partial l}\right|$ increasing in μ and $\frac{\partial^2 P(\mathrm{E}[X|s],l)}{\partial \mathrm{E}[X|s]^2}, \frac{\partial^2 P(\mathrm{E}[X|s],l)}{\partial l^2}$ decreasing in μ. Also, observe that condition S1$'$ is violated when

$$\mu \equiv \frac{\mathrm{var}(X|\Sigma)}{\mathrm{var}(X|S)} \geq -\frac{1}{a_o \mathrm{var}(X|S)\left(\frac{1}{a_i^*} + h'(p)\right)}$$

because then $1 + A^* h'(p) \leq 0$. Therefore if condition S1$'$ holds then within the domain $(\bar{\mu}, \infty)$ it holds that $\left|\frac{\partial P(\mathrm{E}[X|s],l)}{\partial \mathrm{E}[X|s]}\right|, \left|\frac{\partial P(\mathrm{E}[X|s],l)}{\partial l}\right|$ are increasing in μ and $\frac{\partial^2 P(\mathrm{E}[X|s],l)}{\partial \mathrm{E}[X|s]^2}$, $\frac{\partial^2 P(\mathrm{E}[X|s],l)}{\partial l^2}$ are decreasing in μ. □

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