# **RESEARCH PAPER**



# **Multi-scale fatigue damage model for steel structures working under high temperature**

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Received: 30 July 2018 / Revised: 10 October 2018 / Accepted: 30 October 2018 / Published online: 27 February 2019 © The Chinese Society of Theoretical and Applied Mechanics and Springer-Verlag GmbH Germany, part of Springer Nature 2019

#### **Abstract**

In order to better understand the fatigue mechanisms of steel structures working under high temperature, a multi-scale fatigue damage model at high temperature is developed. In the developed model, the macroscopic fatigue damage of metallic materials due to the collective behavior of micro-cracks is quantified by using the generalized self-consistent method. The influence of temperature on fatigue damage of steel structures is quantified by using the previous creep damage model. In addition, the fatigue damage at room temperature and creep damage is coupled in the multi-scale fatigue damage model. The validity of the developed multi-scale damage model is verified by comparing the predicted damage evolution curve with the experimental data. It shows that the developed model is effectiveness. Finally, the fatigue analysis on steel crane runway girders (CRGs) of industrial steel melt shop is performed based on the developed model.

**Keywords** Steel structures · High temperature · Multi-scale damage model · Microcracks · Generalized self-consistent method

#### **1 Introduction**

Although steel structures are the main type of civil engineering due to its high strength and good structural integrity performance, their fatigue issue could be a major concern under repeated loading. The early researches of the American Society of Civil Engineers showed that 80–90% of the failures in steel structures are related to fatigue [\[1\]](#page-7-0).

Much effort has been made on fatigue problems of steel structures using a macro-phenomenological method. For example, Yue et al. [\[2\]](#page-7-1) analyzed the fatigue performance of carbon fiber reinforced polymer (CFRP) reinforced steel crane girders by experiments and studied the influence of CFRP on the stress concentration. Then, Tong et al. [\[3\]](#page-7-2) studied the fatigue problem of a series of end-coped steel CRGs severed for 14 years using the S–N method. In their study, Tominaga et al. [\[4\]](#page-7-3) studied the fatigue improvement effect of ultrasonic impact treatment method by analyzing both the strengthened in-plane gusset and out-of-plane gusset detail of steel crane runway girders (CRGs). The micro method also

 $\boxtimes$  Zhaoxia Li zhxli@seu.edu.cn has been used to analyze fatigue damage of steel structures by studying the nucleation and growth of short cracks [\[5](#page-7-4)[–8\]](#page-7-5).

However, the fatigue failure of steel structures is a macroscale problem, and the mechanism is the collective behaviors of short cracks in the micro-scale. As a result, the macrophenomenological method neglects micro-fatigue damage mechanism, which leads to loss of the accuracy to some degree, while the huge costs of the micro method makes it difficult to be applied into the practice  $[9-11]$  $[9-11]$ . Therefore, a multi-scale method should be developed to analyze the fatigue damage of steel structures.

On the other hand, steel structures often serve in a high temperature environment, such as the reactor pressure vessels in power plants [\[12\]](#page-7-8), the thin-walled structures in aircrafts [\[13\]](#page-8-0), and the CRGs in melt shops. Research has shown that creep damage affects the behavior of microcracks, which enables to accelerate the fatigue process of metallic materials, and the fatigue damage also can accelerate creep damage evolution [\[14,](#page-8-1) [15\]](#page-8-2). Therefore, the failure of steel structures is usually due to the coupling process between fatigue damage and creep damage. However, traditional research tends to analyze the fatigue behavior of steel structures without consideration of the effect of temperature. Taking the industrial steel melt shops as an example: Kuwanmura et al. [\[16\]](#page-8-3) analyzed the fatigue cracks in the weld joint between the bottom

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flanges and gusset plates of CRGs and developed a method to inspect and repair these fatigue cracks. Furthermore, Ávila et al. [\[17\]](#page-8-4) attempted to predict the fatigue life of the crane girder of a melt shop with over 30 years of operation by combining the S–N method with finite element method.

Therefore, although many studies have been conducted, the fatigue mechanisms of steel structures at high temperature is still unresolved. It is difficult to predict accurately the fatigue life of steel structures. In order to solve the two problems mentioned, a multi-scale fatigue damage model at high temperature is developed by considering the coupling of fatigue damage  $D_f$  and creep damage  $D_{cr}$ . The validity of the developed model is verified and the fatigue analysis of steel CRGs of industrial steel melt shop is performed based on the developed model.

#### **2 The definition of damage variable at high temperature**

The foundation of damage mechanics was laid by new concepts, such as the continuity put forward by Kachanov [\[18\]](#page-8-5) in 1958, damage factor by Rabotnov [\[19\]](#page-8-6) in 1963, and damage mechanics by Janson and Hultin [\[20\]](#page-8-7) in 1977. Damage accounts for the process in which the strength and stiffness of representative volume element (RVE) degenerates and RVE fractures ultimately. According to continuum damage mechanics theory, the damage variable *D* is defined as

$$
D = \frac{A - \bar{A}}{A},\tag{1}
$$

where  $A$  is the cross-sectional area of RVE,  $\overline{A}$  is the effective cross-sectional area.

Based on the strain equivalence principle [\[21\]](#page-8-8), the damage variable can be calculated by

$$
D = 1 - \frac{\bar{E}}{E},\tag{2}
$$

where *E* is the Young's modulus,  $\overline{E}$  is the effective Young's modulus.

The fatigue damage is divided into two categories, according to the fatigue life  $N_f$ , that are high cycle fatigue damage  $(N_f > 10^5)$  and low cycle fatigue damage  $(N_f < 10^5)$ , respectively [\[21\]](#page-8-8). The fatigue damage process of metals is consists of crack nucleation stage, short crack stage and long crack stage, and the second stage consumes the majority, above 90% for high cycle fatigue damage [\[22\]](#page-8-9). Short cracks grow within the grain domains and hardly overcome the grain boundary obstacle, as shown in Fig. [1.](#page-1-0) The research demonstrates that fatigue damage strongly depends on the collective behavior of short cracks [\[23,](#page-8-10) [24\]](#page-8-11).



<span id="page-1-0"></span>**Fig. 1** A micro-crack stopped at the grain boundary

The work by Hong et al. [\[25\]](#page-8-12) concluded that fatigue failure occurs only when the value of short crack density reaches its limitation. The distribution of micro structure is of highly random and the orientation and size of grain have great influence on the collective behavior of short cracks [\[26,](#page-8-13) [27\]](#page-8-14).

In the paper, the fatigue damage due to the collective behavior of short cracks is quantified by using general selfconsistent method. The multi-scale fatigue damage  $D_{f-cr}$ is related to both fatigue damage  $D_f$  at room temperature and creep damage  $D_{cr}$  at high temperature. On the basis of Ref.  $[28]$ , the multi-scale damage variable  $D_{f-cr}$  is defined as

<span id="page-1-3"></span>
$$
D_{\rm f-cr} = \frac{D_{\rm f}}{1 - a_1 D_{\rm cr}} + \frac{a_3 D_{\rm cr}}{1 - a_2 D_{\rm f}},\tag{3}
$$

where  $a_1$  stands for the influence parameter of creep damage on fatigue damage, *a*<sup>2</sup> stands for the influence parameter of fatigue damage on creep damage,  $a_3$  is the parameter related to the critical creep temperature  $T_c$ .

<span id="page-1-1"></span>
$$
a_3 = \begin{cases} 1, T \ge T_c, \\ 0, T < T_c. \end{cases} \tag{4}
$$

#### **3 Multi-scale fatigue damage model at high temperature**

#### <span id="page-1-2"></span>**3.1 Development of the multi-scale fatigue damage model at room temperature**

In this study, short cracks are assumed to be random distributed and grow within their grain domain in the material. Therefore, the fatigue damage  $D_f$  is caused by two parts under cyclic load (uniaxial tension) at room temperature: one is the



<span id="page-2-0"></span>**Fig. 2** Collective behavior of short cracks under cyclic loading

growth of existing short cracks and the other is their initiation, as shown in Fig. [2.](#page-2-0)

Similarly to Ref. [\[29\]](#page-8-16), the initiation and propagation law of short cracks of metallic materials can be expressed as Eqs. [\(5\)](#page-2-1) and [\(6\)](#page-2-2), respectively

$$
\dot{n}_N(a,N) = L(\Delta \sigma)^l \left(1 - \frac{a}{d}\right),\tag{5}
$$

$$
\frac{da}{dN} = A_0(\Delta \gamma)^{\alpha} (d_0 - a), \tag{6}
$$

where  $L$ ,  $l$ ,  $A_0$ ,  $\alpha$  are model parameters,  $d$  is the grain size, *a* is crack length,  $d_0$  is the same as that dominate length of the microstructure scale, whose value is about 0.1 mm when the applied stress levels below the fatigue limit,  $\Delta \sigma$  is stress range, and  $\Delta \gamma$  is the plastic shear strain range (for the condition of uniaxial stress range or the axial strain range,  $\Delta\sigma$  and  $\Delta\varepsilon$  are used, respectively [\[30\]](#page-8-17)).

Let  $\Delta(N/N_f) \rightarrow 0$  and  $\Delta a \rightarrow 0$ , the equilibrium evolution equation of the density of short cracks in the material can be expressed as

$$
\frac{\partial [n(a, N/N_{\rm f})]}{\partial (N/N_{\rm f})} = \dot{n}_N(a, N/N_{\rm f}) - \frac{\partial [n(a, N/N_{\rm f})\dot{a}(a, N/N_{\rm f})]}{\partial a}, \quad (7)
$$

where *N* is the number of cycles,  $N_f$  is the fatigue life,  $n_N(a,$  $N/N_f$ ) is the number of short cracks with length *a* in the area,  $\dot{n}_N(a, N)$  and  $\dot{a}(a, N/N_f)$  are the initiation rate and propagation rate, respectively.

The short crack nucleation rate per unit area  $n_N(a, N/N_f)$ can be written as

<span id="page-2-3"></span>
$$
\dot{n}_N(a, N/N_f) = \begin{cases} L(\Delta \sigma)^l N_f, \ a \le d, \\ 0, \ a > d. \end{cases}
$$
 (8)

The short crack growth rate  $\dot{a}(a, N/N_f)$  with crack length *a* can be modified as

<span id="page-2-4"></span>
$$
\dot{a}(a, N/N_{\rm f}) = A(\Delta \sigma)^{\alpha} (d - a) N_{\rm f}.
$$
\n(9)

Combining Eqs.  $(8)$  and  $(9)$  with the initial condition  $n(a, 1)$  $0 = 0$ , the function of short cracks density of metallic material, i.e. the solution of Eq. [\(7\)](#page-2-5), can be obtained as

<span id="page-2-7"></span>
$$
n(a, t_1) = \frac{L_{\rm ff}}{2A_{\alpha f}} \Big( e^{2A_{\alpha f}t_1} - 1 \Big),\tag{10}
$$

where  $L<sub>lf</sub>$  and  $A<sub>αf</sub>$  are model parameters and can be obtained from experiments, the time parameter  $t_1$  equals to  $N/N_f$ .

In fact, the fatigue damage caused by each short crack is dependent [\[31,](#page-8-18) [32\]](#page-8-19), while the interaction was neglected in Ref. [\[29\]](#page-8-16), which leads to a somewhat inaccurate damage model, especially at the late stage of fatigue damage. The general self-consistent method can be used to obtain the effective elastic modulus of the cracked material by considering the influence of single crack on the material with assumption that there are *n* cracks with absolutely random distribution. The potential energy equilibrium equation of the cracked material with uniform tension can be expressed as

<span id="page-2-2"></span><span id="page-2-1"></span>
$$
-\frac{1}{2}(\bar{C}_{ijkl})^{-1}\sigma_{ij}^{0}\sigma_{kl}^{0} = -\frac{1}{2}(C_{ijkl})^{-1}\sigma_{ij}^{0}\sigma_{kl}^{0} - \frac{\sum_{k}\Delta\phi^{(k)}}{V},
$$
\n(11)

where  $\bar{C}_{ijkl}$  and  $C_{ijkl}$  are the tensors of the elastic modulus of the microcracked solid and intact matrix material, respectively, *V* is volume of the material,  $\Delta \phi^{(k)}$  is the potential energy released from the *k*-th crack.

As for the plane problem, there are some tunnel cracks in the material subjected to uniaxial tension. The relationship between the effective elastic modulus *E* and plane-strain modulus  $\bar{E}$  can be expressed as [\[33\]](#page-8-20)

<span id="page-2-6"></span><span id="page-2-5"></span>
$$
\frac{\bar{E}(1-\mu^2)}{E} = \left(1 + \pi \rho + D_E^{2D} \rho^2\right)^{-1},\tag{12}
$$



<span id="page-3-2"></span>**Fig. 3** Creep curve of metallic materials

where  $\mu$  is Poisson's ratio,  $D_E^{2D}$  is a parameter dependent on Poisson's ratio and  $D_E^{2D}(0.3) = 1.12$ ,  $\rho$  is the crack parameter and can be expressed as Eq. [\(13\)](#page-3-0)

$$
\rho = \frac{1}{4}n\left\langle a^2 \right\rangle,\tag{13}
$$

where  $\langle a^2 \rangle$  is dimensionless and is defined as average of square the crack length *a* where the dimension of *a* is millimeter.

Therefore, the multi-scale fatigue damage variable  $D_f$  at room temperature can be obtained by combining Eqs. [\(12\)](#page-2-6) and [\(13\)](#page-3-0) with Eq. [\(2\)](#page-1-1)

$$
D_{\rm f} = 1 - \frac{1}{1 + \frac{1}{4}\pi n \langle a^2 \rangle + \frac{1.12}{16}n^2 \langle a^2 \rangle^2}.
$$
 (14)

Substituting Eq.  $(10)$  into Eq.  $(14)$ , the fatigue damage variable  $D_f$  can be rewritten as

$$
D_{\rm f} = 1 - \frac{1}{1 + \pi \frac{L_{\rm ff}}{8A_{\rm af}}} \left( e^{2A_{\rm erf}t_1} - 1 \right) \left\langle a^2 \right\rangle + \frac{1.12}{16} \left[ \frac{L_{\rm ff}}{2A_{\rm erf}} \left( e^{2A_{\rm erf}t_1} - 1 \right) \right]^2 \left\langle a^2 \right\rangle^2 \tag{15}
$$

#### <span id="page-3-6"></span>**3.2 Creep damage model for metallic materials**

According to creep mechanics [\[34\]](#page-8-21), creep is defined as the time-dependent deformation of the material exposed to hightemperature environment (above 40% of its melting limit), which consists of three main phases, as shown in Fig. [3.](#page-3-2)  $\varepsilon_0$  is the instantaneous strain caused by the applied load rather than creep strain. The creep rate of metallic materials decreases in the first stage, almost stays constant in the second stage and increases quickly in the third stage in which the creep damage occurs. The duration of the three creep stages is related to the material properties, stress level and environmental temperature.

In 1929, Norton [\[35\]](#page-8-22) developed the famous Norton creep model which utilizes a power law to represent the constitutive relation of metallic materials

<span id="page-3-9"></span>
$$
\dot{\varepsilon} = B\sigma^{n_0},\tag{16}
$$

where *B* is a constant creep parameter,  $n_0$  is the creep power exponent.

Creep damage of metallic materials is referred to the process in which degeneration of strength and stiffness of the materials occurs. In 1958, Kachanov [\[18\]](#page-8-5) proposed a classical model to describe the creep damage behavior of metals

<span id="page-3-3"></span>
$$
\dot{D} = \left[\frac{\sigma}{A_1(1 - D)}\right]^r,\tag{17}
$$

where  $A_1$  and  $r$  are material parameters which can be obtained from creep test under constant uniaxial tension condition.

Integrate for Eq. [\(17\)](#page-3-3) and let the value of *D* ranges from 0 to 1 and *t* ranges from 0 to *t.* One can obtain

<span id="page-3-4"></span><span id="page-3-0"></span>
$$
t = \frac{1}{r+1} \left[ 1 - (1-D)^{r+1} \right] \left( \frac{\sigma}{A_1} \right)^{-r}.
$$
 (18)

The failure time  $t_c$  can be obtained when the damage reaches its limitation  $(D = 1)$ 

<span id="page-3-5"></span>
$$
t_{\rm c} = \frac{1}{r+1} \left(\frac{\sigma}{A_1}\right)^{-r}.\tag{19}
$$

<span id="page-3-1"></span>Combining Eq.  $(18)$  with Eq.  $(19)$ , one can obtain the expression of creep damage variable as

<span id="page-3-8"></span>
$$
D_{\rm cr} = 1 - \left(1 - \frac{t}{t_{\rm c}}\right)^{\frac{1}{r+1}} = 1 - \left(1 - \frac{t_1 N_{\rm f} T_0}{t_{\rm c}}\right)^{\frac{1}{r+1}},\tag{20}
$$

where  $T_0$  is period of the cyclic loading.

#### <span id="page-3-7"></span>**3.3 Development of the multi-scale fatigue damage model at high temperature**

As described previously, the multi-scale fatigue damage for steel structures at high temperature is the nonlinear couple of fatigue damage at room temperature and creep damage at high temperature, which has been established in Sects. [3.1](#page-1-2) and [3.2,](#page-3-6) respectively. Therefore, substituting Eqs. [\(15\)](#page-3-7) and [\(20\)](#page-3-8) into Eq. [\(3\)](#page-1-3), one can obtain the expression of the multi-scale temperature dependent fatigue damage variable, expressed as

$$
D = \frac{1 - \frac{L_{lf}}{1 + \pi \frac{L_{lf}}{8A_{\alpha f}} \left(e^{2A_{\alpha f}t_1} - 1\right)(a^2) + \frac{112}{16} \left[\frac{L_{lf}}{2A_{\alpha f}} \left(e^{2A_{\alpha f}t_1} - 1\right)\right]^2 \left(a^2\right)^2}}{1 - a_1 \left[1 - \left(1 - \frac{t_1 N_f T_0}{t_c}\right)^{\frac{1}{r+1}}\right]}
$$



<span id="page-4-1"></span>**Fig. 4** Damage evolution curves of 16Mn steel. **a**  $\Delta \sigma = 353.0$  MPa. **b**  $\Delta \sigma = 366.8$  MPa. **c**  $\Delta \sigma = 372.7$  MPa

$$
+\frac{a_3\left[1-\left(1-\frac{t_1N_{\rm f}T_0}{t_{\rm c}}\right)^{\frac{1}{r+1}}\right]}{1-a_2\left\{1-\frac{L_{lf}}{1+\pi\frac{L_{lf}}{8A_{\rm cf}}\left(e^{2A_{\rm cf}t_1}-1\right)(a^2)+\frac{1.12}{16}\left[\frac{L_{lf}}{2A_{\rm cf}}\left(e^{2A_{\rm cf}t_1}-1\right)\right]^2\left(a^2\right)^2\right\}}\right\}}.(21)
$$

Let  $D = 1$  at  $N = N_f$ , the multi-scale temperature dependent fatigue life function  $F(N_f)$  can be expressed as

$$
F(N_{\rm f}) - 1 = 0. \tag{22}
$$

In general, these parameters can be supposed to  $a_1 = 1$ ,  $a_2 = 1, a_3 = 1$ , the function of the fatigue life (Eq. [\(22\)](#page-4-0)) can be rewritten as

$$
\frac{1 - \frac{1}{1 + \pi \frac{L_{\text{lf}}}{8A_{\text{arf}}}\left(e^{2A_{\text{aff}}t_1} - 1\right)\langle a^2\rangle + \frac{1.12}{16}\left[\frac{L_{\text{lf}}}{2A_{\text{arf}}}\left(e^{2A_{\text{arf}}t_1} - 1\right)\right]^2\langle a^2\rangle^2}}{\left(1 - \frac{t_1 N_{\text{f}} T_0}{t_{\text{c}}}\right)^{\frac{1}{r+1}}}
$$

$$
+\frac{1-\left(1-\frac{t_1N_fT_0}{t_c}\right)^{\frac{1}{r+1}}}{\frac{1}{1+\pi\frac{L_{lf}}{8A_{\alpha f}}(e^{2A_{\alpha f}t_1}-1)(a^2)+\frac{1.12}{16}\left[\frac{L_{lf}}{2A_{\alpha f}}(e^{2A_{\alpha f}t_1}-1)\right]^2(a^2)^2}-1=0. \tag{23}
$$

## **4 Determination of the developed model parameters**

#### <span id="page-4-0"></span>**4.1 Determination of the parameters of the multi-scale fatigue damage model at room temperature**

Note that the CRGs of melt shops with heavy work task are usually made of Q345 (16Mn) steel. In the paper, three groups of experimental damage evolution of 16Mn steel at different stress ranges ( $\Delta \sigma = 353.0 \text{ MPa}$ , 366.8 MPa, 372.7 MPa, respectively) are chosen from Ref. [\[36\]](#page-8-23). Substituting the parameters into Eq. [\(15\)](#page-3-7), the model parameters  $L_{\text{lf}}$  and  $A_{\alpha f}$ can be obtained by utilizing a nonlinear fitting method. The fitting results are shown in Fig. [4](#page-4-1) and Table [1.](#page-5-0)

<span id="page-5-0"></span>**Table 1** Parameters of the developed model Stress ranges (MPa) *L*<sub>If</sub> *A*<sub>αf</sub>  $353.0$  2.131 × 10<sup>-8</sup> 12.564  $366.8$  1.340 × 10<sup>-7</sup> 11.697 372.7 2.403 ×  $10^{-4}$  7.673



<span id="page-5-1"></span>**Fig. 5** Creep curves of Q345 steel at different stress level

#### **4.2 Determination of the parameters of the creep damage model**

In the section, the parameters of both the Norton creep model and the Kachanov creep damage model for Q345 steel are obtained based on the creep experimental data chosen from Ref. [\[37\]](#page-8-24). Three creep curves at different stress level under 500 °C temperature are shown in Fig. [5.](#page-5-1)

The Norton creep model is only used to analyze the second stage by ignoring the elastic strain. Therefore, substituting the corresponding value of stress and creep rate from Fig. [5](#page-5-1) into Eq. [\(16\)](#page-3-9), the creep constant *B* and creep power exponent  $n_0$  can be obtained by numerical fitting. As shown in Fig. [6,](#page-5-2) one can obtain that  $n_0 = 5.54$ ,  $B = 4.88 \times 10^{-15}$ MPa·min−1.

Similarly, substituting the corresponding value of  $(\sigma, t_c)$ from Fig. [5](#page-5-1) into Eq. [\(19\)](#page-3-5), the model parameters  $A_1$  and  $r$  can be obtained by numerical fitting. As shown in Fig. [7,](#page-5-3) one can obtain that  $A_1 = 8.70 \times 10^2$ ,  $r = 5.59$ .

## **5 Verification of the developed model**

In this section, the validity of the developed multi-scale fatigue damage model is verified. Although the damage  $D_{f-cr}$  is the joint effect of fatigue damage  $D_f$  at room temperature and creep damage  $D_{cr}$  at high temperature by



<span id="page-5-2"></span>**Fig. 6** Parameters of the Norton model



<span id="page-5-3"></span>**Fig. 7** Parameters of creep damage model

nonlinear superposition, the reliability of Kachanov creep damage model has been verified by many researchers [\[38,](#page-8-25) [39\]](#page-8-26). Therefore, the reliability of the developed damage model depends on the validity of the verification at room temperature, which is lower than the creep limit temperature  $T_c$ .

The fatigue experimental data of the damage evolution of Q345 (16Mn) steel is chosen from Ref. [\[40\]](#page-8-27). The fatigue damage model parameters at the stress range 337.1 MPa ( $\Delta \sigma$  = 337.1 MPa) are calculated based on the three groups of data and the multi-scale damage evolution curve of Q345 steel can be obtained accordingly. Comparison is made between the predicted curve and the corresponding experimental data, as shown in Fig. [8.](#page-6-0) It shows that the predicted curve agrees well with the experimental results except for somewhat discrepancy when  $N/N_f$  is in the regime 0.85–0.95, which demonstrates that the developed model is acceptable.



<span id="page-6-0"></span>**Fig. 8** Verification of the developed damage model

# Monitoring systems Stress history data Stress amplitudes  $\Delta \sigma$  Stress level  $\sigma$ Fatigue damage  $D_f$ *f Creep damage*  $D_c$ multi-scale temperature Multi-scale temperature dependent damage *Df-cr* dependent damage *Df-cr* Fatigue life of the CRGs

<span id="page-6-2"></span>**Fig. 10** Flow chart of the application of the developed damage model

#### **6 Application of the developed model**

In practice, fatigue cracks are usually detected in the weak area of the CRGs, such as the weld joint between the upper flange and web plate  $[41, 42]$  $[41, 42]$  $[41, 42]$ , as shown in Fig. [9.](#page-6-1)

Therefore, the section is to analyze the fatigue damage problem of steel CRGs of melt shops using the developed model. The stress amplitude  $\Delta \sigma$  and equivalent constant stress  $\sigma$  should be obtained based on the stress history data of the in-service steel CGRs. The flow chart of fatigue damage analysis of the CRGs at high temperature using the developed model is illustrated in Fig. [10.](#page-6-2)

The stress amplitude  $\Delta \sigma = 353.0$  MPa, the equivalent constant stress  $\sigma = 66$  MPa, the temperature  $T = 500$  °C. The crane runs about 88 times a day on the CRG. Note that the fatigue life  $N_f = 9.8 \times 10^5$  and the critical damage value  $D_c = 0.407$  [\[36\]](#page-8-23).

In practice, engineers tend to analyze the fatigue damage of steel structures using a macro-phenomenological method, such as Miner's law [\[43\]](#page-8-30) and Lemaitre's law [\[12\]](#page-7-8). In order to demonstrate the advantages of the developed multi-scale model, a comparison is made among the analysis result on the Miner model, the Lemaitre model and the developed model, shown in Fig. [11.](#page-7-9) The multi-scale temperature dependent fatigue damage evolution curve of steel CRGs of melt shops is obtained, as shown in Fig. [12.](#page-7-10)

The following conclusions can be drawn from Figs. [11](#page-7-9) and [12.](#page-7-10) (1) The analysis results based on the developed model are more confidence, the results on the Miner model are too conservative, while the results of the Lemaitre model may be slightly overestimated. (2) The multi-scale damage of the CRGs mainly depends on creep damage in the early phase. (3) The fatigue life  $N_f$  of the studied CRG is about 29.6 years. (4) The fatigue damage analysis results based on the developed

<span id="page-6-1"></span>

**Fig. 9** Fatigue crack being detected at the weld joint between the upper flange and web plate of the steel CRG



<span id="page-7-9"></span>**Fig. 11** Damage evolution curve of steel CRGs on different model



<span id="page-7-10"></span>**Fig. 12** Multi-scale fatigue damage evolution curve of steel CRGs

damage model are more accurate than that only on fatigue damage at room temperature or creep damage.

#### **7 Conclusions**

A multi-scale fatigue model at high temperature is developed to study the fatigue problem of steel structures, which are subjected to cyclic loading under high temperature environment. The influence of the micro-collective behavior of short cracks of metallic materials on its macro-damage variable is studied by considering their interaction using generalized self-consistent method. The parameters of Norton model for Q345 steel are obtained. The fatigue analysis on steel CRGs of industrial steel melt shops are performed based on the Miner model, the Lemaitre model and the developed model. The main works and conclusions are listed as following.

- (1) A multi-scale fatigue model for steel structures working under high temperature is developed by considering the nonlinear coupling of fatigue damage at room temperature and creep damage at high temperature.
- (2) The validity of the developed fatigue model is verified by comparing the numerical results with the experimental data which illustrates the developed model is acceptable.
- (3) The analysis results based on the developed model are more confidence, compared with the results on the Miner model and Lemaitre model.
- (4) The fatigue analysis on steel CRGs of melt plant is performed based on the developed multi-scale fatigue model, which shows that the fatigue analysis results are more accurate than that using only fatigue damage or creep damage.

**Acknowledgements** The work was financially supported by the National Program on Key Research Project (Grant 2016YFC0701301- 02) and Jiangsu Province Natural Sciences Fund Subsidization Project (Grant BK20170655), to which the authors are most grateful.

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