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RESEARCH PAPER

An experimental investigation of one- and two-degree of freedom VIV of cylinders

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Abstract In the paper, an experiment investigation was conducted for one- and two-degree of freedom vortex-induced vibration (VIV) of a horizontally-oriented cylinder with diameter of 11 cm and length of 120 cm. In the experiment, the spring constants in the cross-flow and in-line flow directions were regulated to change the natural vibration frequency of the model system. It was found that, in the one-degree of freedom VIV experiment, a "double peak" phenomenon was observed in its amplitude within the range of the reduced velocities tested, moreover, a "2T" wake appeared in the vicinity of the second peak. In the two-degree of freedom VIV experiment, the trajectory of cylinder exhibited a reverse "C" shape, i.e., a "new moon" shape. Through analysis of these data, it appears that, besides the non-dimensional in-line and cross-flow natural vibration frequency ratios, the absolute value of the natural vibration frequency of cylinder is also one of the important parameters affecting its VIV behavior.

Keywords One- and two-degree of freedom VIV · "2T" wake · "New moon" trajectory · Natural vibration frequency ratio

1 Introduction

Early investigations on vortex-induced vibration (VIV) mostly targeted the one-degree of freedom motion in the cross-flow direction normal to the incoming flow direction, while the in-line flow influence was neglected. Scientists such as Feng [1], Sarpkaya [2] and Williamson et al. [3] carried out experiment investigations on one-degree of freedom VIV of elastically-supported cylinders. They summarized the trends of such parameters as VIV amplitude, frequency and phase, and developed a detailed description and discussion on the properties of one-degree of freedom VIV. In addition, they conducted a systematic investigation on the influence of damping, mass ratio and Reynolds number (*Re*) on VIV.

In subsequent investigations, researchers found that the cylinder will generate periodical vibration in the in-line direction too, resulting from the oscillating towing force with a frequency equal to twice the frequency in cross-flow. Under the combined action of cross-flow and in-line vibrations, the cylinder will appear to have figure "8" motion trajectories. The participation of in-line vibration makes the cross-flow vibration response under two-degree of freedom VIV different from the experimental result of the traditional one-degree of freedom VIV. Researchers have carried out many investigations on the two-degree of freedom VIV of cylinders, including numerical and experimental studies. For instance, Fu et al. [4], Huang et al. [5], Guo et al. [6], Pan et al. [7], Williamson et al. [8], Tryantafylloul et al.[9], Vandiver et al. [10] and Lie et al. [11] carried out significant investigations on VIV phenomena and made predictions for VIV responses of rigid or flexible, elastically-supported cylinders.

Although further investigations have been carried out on VIV, there are still many matters and phenomena unclarified, even for the case of VIV response of a simple elastically-supported rigid cylinder. Therefore, in this paper, the authors conducted free VIV experiments for a horizontally-oriented rigid cylinder elastically supported on both ends and subject to various boundary conditions. The purpose is to explore differences in the variation of cylinder responses with reduced velocity between one- and two-

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degree of freedom VIV under different natural vibration frequency ratios.

2 Setup of the model test

The experiment was carried out in the Ship Model Towing Tank Lab of Harbin Engineering University, College of Shipbuilding. The tank is 108 m long and 7 m wide, and the water depth is 3.5 m.

The experimental model is composed of a cylinder model with a VIV test fixture. In the experiment conducted, the PVC plastic cylinder was oriented horizontally. The outer diameter was 11 cm, the length was 120 cm and the wall thickness was 4 mm, so the length-diameter ratio was $L/D = 10.91$. Due to the short model length, the change of horizontal deflection was neglected and the cylinder model can be considered as a rigid body.

The VIV test fixture consists of several components. The cylindrical model is attached to the cross-beam of the vertical motion system. This system constrains the motion to be vertical and the motion is restrained by a set of adjustable springs applied through a pulley system. The vertical motion system is attached to a carriage that allows horizontal motion and this motion is restrained by a separate spring system. Together, the vertical motion system and the horizontal carriage allow the model to have an arbitrary two-degree of freedom motion.

Adjusting the spring constants of vertical and horizontal spring systems allowed independent and separate changes in the vertical and longitudinal natural vibration frequencies. The model test fixture was attached to the side axle of a trailer to the main tow carriage using a fixed bracket. The model experiment setup and spring system are shown in Fig. 1.

a b

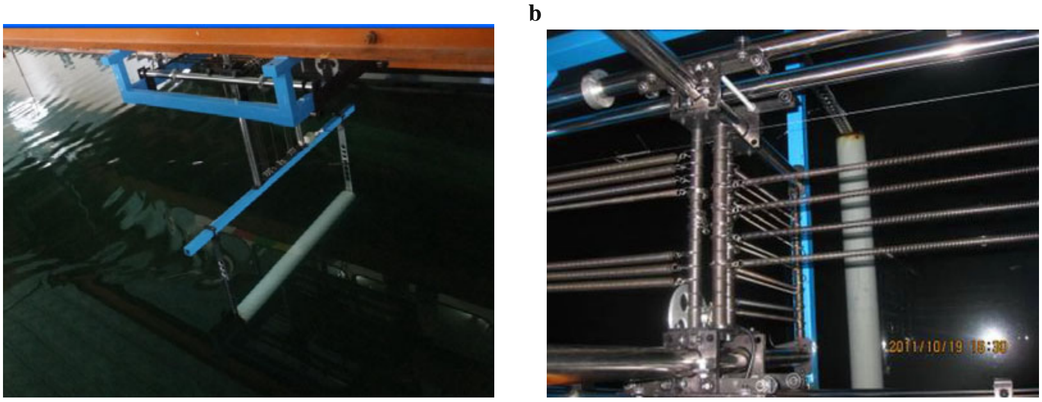

Fig. 1 Model experiment setup and spring system

In the design, processing and assembly of the model experiment system, care was taken to assure that the system had left and right symmetry in the direction of incoming flow, so that the cylinder was kept horizontal and the motion kept strictly two dimensional. Clamping blocks were mounted on the horizontal guide and vertical motion structure. These restraints were used to restrict the motion to be in either vertical or cross-flow directions so that the respective natural frequencies could be measured and the one-degree of freedom VIV experiments be conducted.

In experiment process, the spring stiffness and the initial length of the vertical spring were so selected that the extension capability of the spring was larger than the possible maximum vertical amplitude for all current velocities. Different vertical spring extension ranges were involved in the experiments. All the ranges were at least twice the diameter of cylinder to avoid spring compression. In designing longitudinal spring system, the front and rear spring were opposed to each other and adjustable, so that when acted upon by the maximum towing forces and with cylinder motions of one-half of the cylinder diameter, these springs would remain in tension.

In the current research, the vertical motion of cylinder was defined as cross-flow direction, and the horizontal motion defined as the in-line direction. The cross-flow and in-line vibration displacement of the cylinder were measured by a linear displacement measurement system, the vibration acceleration was measured by the acceleration sensor. In the experiment, the data sampling rate was set to 50 Hz, and the sampling time during the test segment in which the model's behavior was not significantly changed was not lower than 20 lateral vibration periods.

3 Test matrix

The purpose of the present experiment was to investigate the VIV properties of cylinder under different natural vibration frequencies in the cross-flow and in-line directions. These natural frequencies were adjusted by increasing or reducing the spring constants in the corresponding direction. Table 1 shows the natural vibration frequencies of the model system in the cross-flow and in-line directions, which are all actual measured values.

Table 1 Parameters in the cylinder VIV experiment

In-line direction of natural frequency f_x/Hz	Cross-flow direction of natural frequency f_y/Hz	Ratio between natural frequencies f_x/f_y	Cross-flow direction of mass ratio m_y^*	Cross-flow direction of damping ζ_y	In-line direction of mass ratio m_x^*	In-line direction of damping ζ_x
0.515 5		0.835				0.116 9
0.568 1		0.920				0.118 4
0.666 7		1.080				0.121 5
0.694 4	0.617 3	1.125	1.517	0.147 7	2.296	0.122 2
0.735 2		1.191				0.149 1
0.806 5		1.306				0.159 2
∞		∞				∞
0.515 5		0.959				0.116 9
0.568 1	0.537 6	1.057	1.517	0.129	2.296	0.118 4
0.666 7		1.240				0.121 5
0.694 4		1.292				0.122 2
0.735 2	0.537 6	1.368	1.517	0.129	2.296	0.149 1
0.806 5		1.500				0.159 2
∞		∞				∞
0.515 5		1.175				0.116 9
0.568 1		1.295				0.118 4
0.666 7		1.520				0.121 5
0.694 4	0.438 6	1.583	1.517	0.117 2	2.296	0.122 2
0.735 2		1.676				0.149 1
0.806 5		1.839				0.159 2
∞		∞				∞

The notation "∞" in the in-line direction of natural frequency means that the model system was fixed in the in-line direction and the model was only allowed to move in the cross-flow direction (one-degree of freedom vibration).

The vibrational mass of the model system in the cross-flow direction includes the cylinder model and part of the vertical motion mechanism. In the in-line direction, the vibrational mass includes the entire vertical motion mechanism and the pulley. Therefore, as noted in the figure, the vibration mass ratios of the model system in the cross-flow and in-line directions are different.

In the experiment, the water temperature was 17°C, the trailer velocity was 0.2 m/s–1.2 m/s, the interval was 0.1 m/s. Thus, the Reynolds number ranged in $3.3\times10^4 < Re < 1.3\times10^5$.

4 Definitions of the parameters

The reduced velocity U_r is defined as

$$U_\mathrm{r} = \frac{U}{f_\mathrm{n} D}, \tag{1}$$

where U is the velocity of incoming flow, D is the diameter of cylinder, f_n is the natural vibration frequency of vibration system in calm water. The cross-flow direction natural vibration frequency in calm water is used in this formula.

The amplitude is given as a non-dimensional relative amplitude, which is defined as

$$A^* = \frac{A_\mathrm{RMS}}{D}, \tag{2}$$

where A_RMS is the mean square root amplitude. The non-dimensional frequency is defined as the ratio between the vibration frequency of cylinder and its natural frequency in calm water, i.e.

$$f^* = \frac{f_\mathrm{osc}}{f_\mathrm{n}}. \tag{3}$$

The frequency ratio f_x/f_y is defined as the ratio between the in-line direction natural frequency and the cross-flow direction natural frequency.

5 Treatment on experiment data

The main properties obtained in the cylinder VIV experiment include vibration amplitude, vibration frequency and phase angle, etc. In order to assure the accuracy in the measurement and processing of experiment data, the data from linear displacement sensor was compared with the double integral of the acceleration sensor.

In data processing, system identification was applied for analyzing the motion trajectory of cylinder [12]. In the analysis, first the VIV trajectory of cylinder was fitted using a 5th-order Fourier series. It was assumed that the fundamental frequency, ω, of VIV trajectory of cylinder in the X and Y directions were the same. The fitted Fourier series of the cylinder vibration trajectory were as follows

$$\begin{aligned} x_c(t) &= a_{0,0} + \sum_{i=1}^{5} [a_{i,1}\cos(\mathrm{i}\omega t) + a_{i,2}\sin(\mathrm{i}\omega t)], \\ y_c(t) &= b_{0,0} + \sum_{i=1}^{5} [b_{i,1}\cos(\mathrm{i}\omega t) + b_{i,2}\sin(\mathrm{i}\omega t)], \end{aligned} \tag{4}$$

where $a_{0,0}$, $a_{i,1}$, $a_{i,2}$, $b_{0,0}$, $b_{i,1}$, $b_{i,2}$, $i = 1, 2, 3, 4, 5$ are relevant coefficients.

In determining the coefficients $a_{0,0}$, $a_{i,1}$, $a_{i,2}$, $b_{0,0}$, $b_{i,1}$, $b_{i,2}$, $i = 1, 2, 3, 4, 5$ and the basic frequency ω, a least-squares fit system identification method was applied for analysis and calculation.

In the least-squares fit, a measure ψ was defined as

$$\psi = \int_t^{t+\Delta t} [(X(t) - X_c(t))^2 + (Y(t) - Y_c(t))^2]\mathrm{d}t. \tag{5}$$

In this formula, $X(t)$, $Y(t)$ are respectively the vibration trajectory of cylinder in experiment measurement along the in-line and cross-flow directions; $X_c(t)$, $Y_c(t)$ are respectively the Fourier series fit of the vibration trajectory of cylinder in Formula (4) along the in-line and cross-flow directions. The values of ω, $a_{0,0}$, $a_{i,1}$, $a_{i,2}$, $b_{0,0}$, $b_{i,1}$, $b_{i,2}$, $i = 1, 2, 3, 4, 5$ that minimized ψ for this model test were selected as the best fit. For this purpose a data sampling rate of 200 Hz was used.

6 Experiment results

6.1 Result of the one-degree of freedom VIV in the cross-flow direction

Figure 2 compares the experiment results of cylinder amplitude for one-degree of freedom VIV with similar results of other researchers.

Due to the limits of the equipment in the early experiments, the reduced velocities were mostly restricted within the narrow-strip range of $U_r \leqslant 12$. It was found that, the variation of one-degree of freedom vibration response amplitude against the reduced velocity can be embodied as different response branches. When the reduced velocity exceeds the frequency lock-in section of $5 \leqslant U_r \leqslant 8$, the amplitude will gradually decrease, finally enter a non-resonant region. In the experiment carried out in the present paper, where the maximum reduced velocity was $U_r \approx 25$, the amplitudes exhibits a marked "double peak" property. As shown in Fig. 2, when the reduced velocity lies within the traditional narrow-strip section ($U_r \leqslant 12$), the amplitude shows a variation trend similar to the result of other researchers [13, 14]. However, when the reduced velocity increases beyond this section, the amplitude will not continue to decrease. Within the reduced velocity range of $14 \leqslant U_r \leqslant 22$, a new peak response would appear again. In addition, the secondary peak values varies with the natural vibration frequencies. When the natural vibration frequency $f_n = 0.438\,6$ Hz, the peak value is about 30% of the peak values under other natural frequencies.

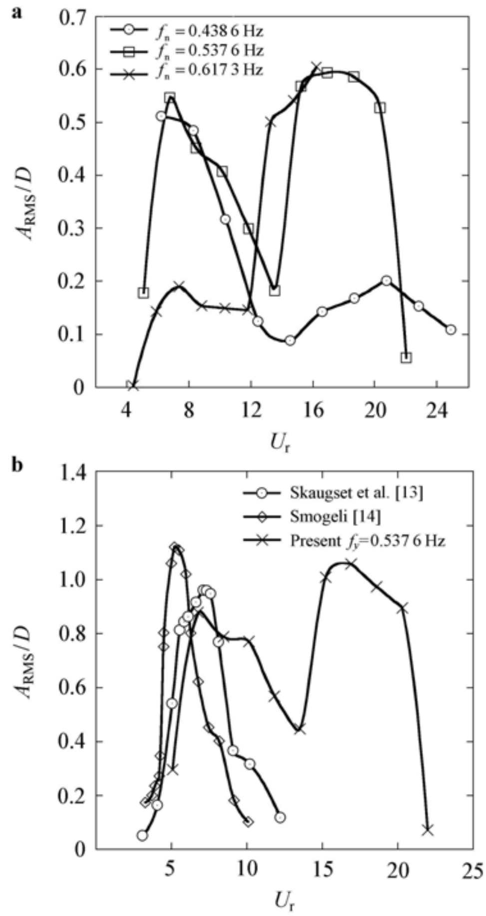

Fig. 2 Variation of amplitude of one-degree of freedom VIV versus reduced velocity. **a** Results of A_{RMS}/D; **b** Comparison with other test's results of amplitude

Figure 3 gives the experiment results of the cross-flow vibration frequency and added mass of cylinder.

Figure 3a shows that, when the secondary peak value appears, the non-dimensional vibration frequency f^* is about 2 or 3, and "Resonance" will occur. Figure 3a also shows that in this reduced velocity range, 3-order high-frequency lift occurs frequently, i.e., "2T" mode wake is generated. However, when the natural frequency f_n is equal to 0.438 6, the vortex-induced lift only takes the form of single frequency.

The "double peak" phenomenon embodied in the VIV of cylinder under high reduced velocity were reported in the wind tunnel experiments on cantilever cylinder [15, 16], Fujarra et al. [17] also drew similar conclusion from water tank experiments. In the VIV experiment on cantilever cylinder, they also found that the end amplitude of cylinder would show an apparent peak value ($A \approx 0.8D$) again within the reduced velocity range of $16 \leqslant U_r \leqslant 20$. Kitagawa et al. [18] attributed the "double peak" phenomenon, which appears in the VIV of cantilever cylinder, to "vortex-induced vibration at end (ECIV)". The reasons are summarized as follows. When the current speed increases, the effect of cylinder end appears gradually, and the vortex shedding frequency at the cylinder end approaches gradually to the natural vibration frequency of the cylinder. The frequency "lock in" occurs again, leading to the appearance of "double peak" under high reduced velocity.

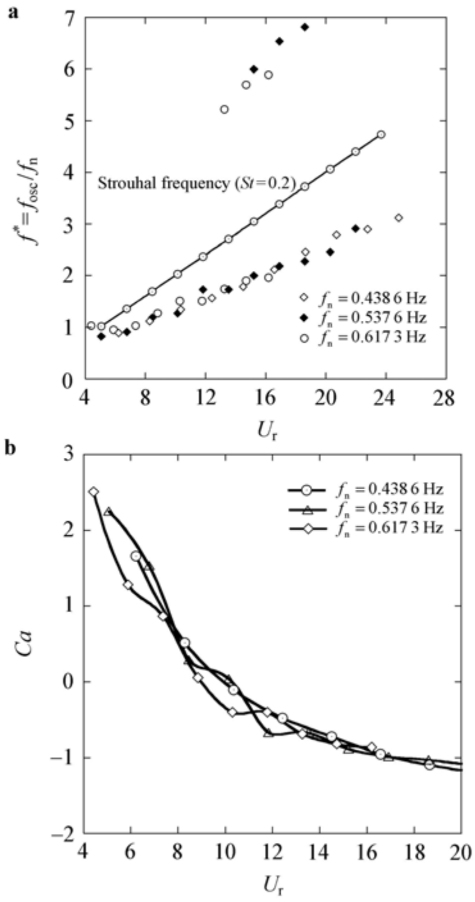

Fig. 3 Variation of cross-flow non-dimension vibration frequency and added mass coefficient against reduced velocity. **a** Results of non-dimension vibration frequency; **b** Results of added mass

At present, the appearance of "double peak" phenomenon in one-degree of freedom VIV of horizontally-oriented cylinder is seldom seen. In combination with the experimental observation and result analysis of the investigation conducted up to now, in this paper the author makes the following inference on the "double peak" phenomenon for the case of one-degree of freedom VIV of horizontally-oriented cylinders. When the frequency of vortex shedding (induced force) is of an integer times (2, 3, $\cdots$) the natural vibration frequency of the system, the shedding vortex will generate "super-resonance " response with the cylinder, resulting in an increase of its amplitude. The large cross-flow amplitude may promote the generation of "2T" wake, while the shedding of "2T" wake strengthens the cross-flow amplitude, they would supplement each other.

6.2 Results of typical two-degree of freedom VIV of cylinder

As an example, let us consider the cross-flow natural vibration frequency of a cylinder at f_y = 0.537 6 Hz, the in-line flow natural vibration frequencyf_x = 0.568 1 Hz, and the frequency ratio f_x/f_y = 1.057. We will make a detailed description of its two-degree of freedom VIV phenomenon. The variation of the root mean square amplitude $A_{\rm RMS}$ of the cylinder VIV versus the reduced velocity and the corresponding vibration trajectory are given in Fig. 4a.

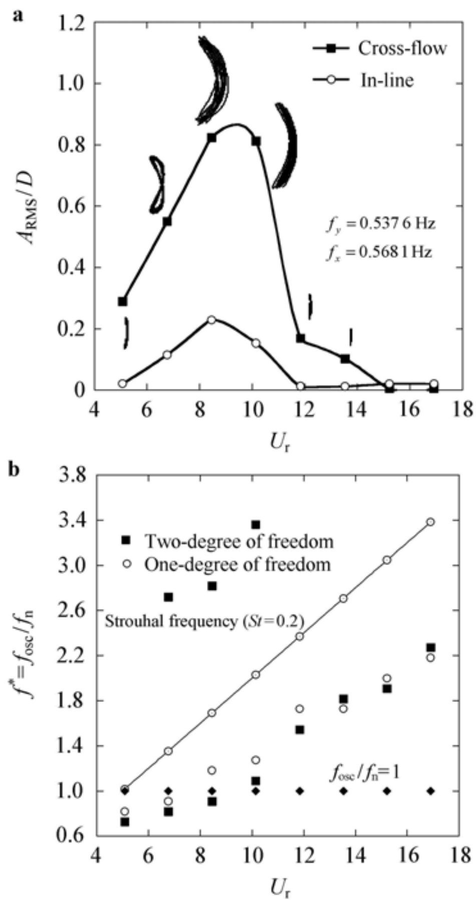

Fig. 4 Variation of root mean square amplitude and non-dimension vibration frequency versus the reduced velocity (f_y = 0.537 6 Hz, f_x = 0.568 1 Hz). **a** Results of amplitude; **b** Results of non-dimension vibration frequency in the cross-flow direction

From Fig. 4a, it may be seen that the cylinder amplitude increases when the reduced velocity U_r < 8.45. At U_r = 8.45, maximum amplitudes appear simultaneously in both the cross-flow and in-line flow directions. At this reduced frequency, the cross-flow maximum relative amplitude is $1.21D$ ($A_{RMS}/D \approx 0.8$). In comparison with the one-degree of freedom VIV experiment, the value is larger by 46%. In addition, the ratio of the in-line and cross-flow amplitudes is $A_x/A_y \approx 0.28$. When the reduced velocity lies within the range of $8.45 < U_r < 10.1$, the amplitude remains at a high level. It may be seen from the non-dimensional vibration frequency indicated in Fig. 4b that, at the point of $f^* \approx 1$, i.e., the vortex shedding frequency is locked in by the natural vibration frequency of cylinder. The vibration response lies within the resonance area. At higher reduced frequencies the vibration response suddenly decreases and gradually enters the non-resonance area where the amplitude is basically zero.

By comparing Figs. 3 and 4 it is found that within the range of the traditional narrow-strip reduced velocity ($U_r < 12$), the non-dimensional frequency for one-degree of freedom is higher than the result obtained for two-degree of freedom. In addition, at a relatively low reduced velocity ($U_r = 6.76$), the frequency lock-in phenomenon ($f^* \approx 1$) has already appeared. Therefore, in comparison with the two-degree of freedom vibration, the peak response of one-degree of freedom vibration appears earlier. With the increase of the reduced velocity, the variation of vibration frequencies of cylinder under different degree freedoms will not show apparent difference.

The non-dimensional vibration trajectory curves corresponding to the reduced velocity of 6.76–10.15 are given in Fig. 5.

It is found that, when the reduced velocity is 6.76, the cylinder VIV shows a counterclockwise figure “8” trajectory drifting to the downstream direction. With an increase of reduced velocity, the balance location of the in-line flow vibration moves towards the direction of the incoming flow, in addition, the vibration trajectory evolves into an overturned “C” (or “new moon” shape). However, the in-line vibration frequency is still 2 times the cross-flow value. At the maximum cross-flow amplitude, the in-line motion direction of cylinder faces toward the upstream of the incoming flow. In this condition, the phase angle between the in-line and the cross-flow motions is 92°. The motion changes when the reduced velocity increases to 10.15. At the maximum cross-flow amplitude, the in-line motion direction of cylinder is the same as the direction of the incoming flow, the trajectory shows a clockwise form, and the calculated phase angle at this time is 79.7°. Simultaneously, the balance location of the in-line vibration moves toward the initial balance location from the upstream direction.

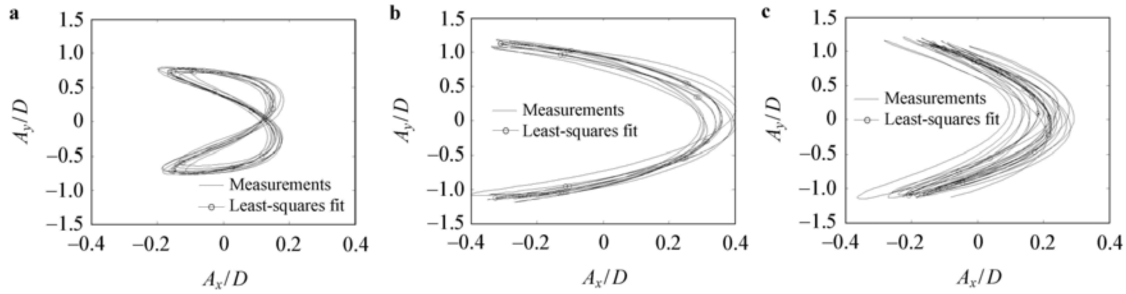

Fig. 5 Typical VIV trajectory curve ($f_y = 0.537\,6$ Hz, $f_x = 0.568\,1$ Hz, $U_r = 6.76$–10.15). **a** $U_r = 6.76$; **b** $U_r = 8.46$; **c** $U_r = 10.15$

6.3 Results of two-degree of freedom VIV of cylinder

We now discussed the whole test results of cylinder VIV with two-degree of freedom. The variation of cylinder amplitude against the reduced velocity under different natural frequency ratios is given in Fig. 6 for $f_y = 0.617\,3$ Hz. Figure 7 shows the results of the non-dimensional vibration frequency and cross-flow added mass in the cross-flow direction.

According to Fig. 6, when $f_x/f_y < 1$, the cross-flow and in-line amplitudes of cylinder and the variation under different natural frequencies do not show apparent difference. When $f_x/f_y = 0.835$, A_{RMS} peak value in the cross-flow direction is about $0.839D$, which is slightly less than $0.855D$ for the case of $f_x/f_y = 0.920$. In this case, the reduced velocity U_r is about 8.8. When the natural frequency ratio f_x/f_y increases, the amplitude decreases. When $f_x/f_y = 1.124$, the A_{RMS} in cross-flow is about $0.634D$. At higher values of f_x/f_y, i.e., when the in-line stiffness increases, the amplitude does not decrease correspondingly. When the natural frequency ratio $f_x/f_y = 1.191$, the cross-flow amplitude A_{RMS} increases to $0.727D$. In addition, the reduced velocity at which the peak amplitude appears is delayed ($U_r = 10.31$). When the natural frequency increases further to 1.306, the peak amplitude will still appear at $U_r = 10.31$, the value is a little lower than that for $f_x/f_y = 1.191$. When $f_x \approx f_y$, the cross-flow maximum amplitude A_{RMS} is about $0.707D$, which is 5.78 times the in-line peak amplitude. It was found by a power spectrum analysis that the cross-flow vortex-induced lift has a 3-order high harmonic component in the “lock in” section, the relevant phenomena have been analyzed previously and will not be repeated here.

The motion trajectories of cylinder under different conditions are shown in Fig. 8 for $f_y = 0.617\,3$ Hz.

According to Fig. 8, when the cylinder exhibits a VIV peak response, its trajectory is a “new moon” shape, not traditional figure “8”-shape. This phenomenon agrees with the experimental results of Williamson et al. [8] and Dahl et al. [9].

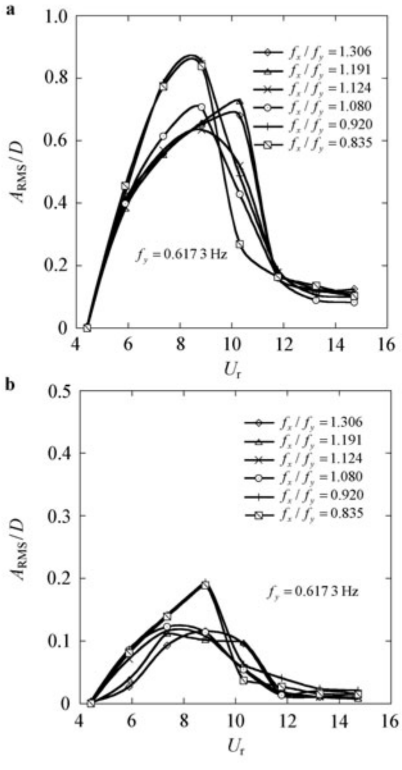

Fig. 6 Variation trend of mean square root amplitude against reduced velocity. **a** Cross-flow direction; **b** In-line direction

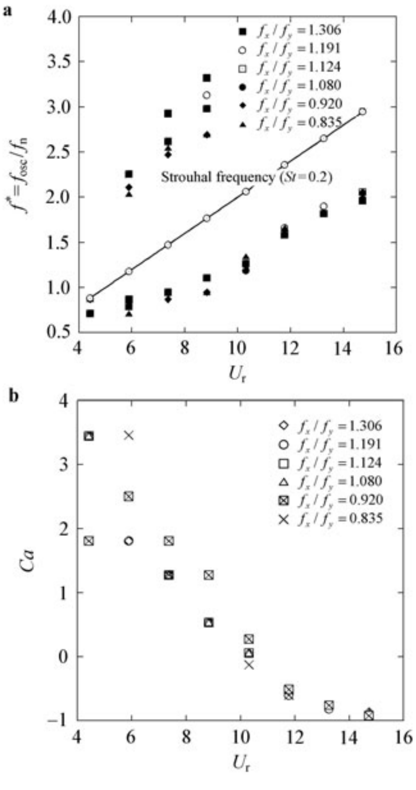

Fig. 7 Variation trends of non-dimension vibration frequency and cross-flow added mass against reduced velocity in the cross-flow direction (f_y = 0.617 3 Hz). **a** Results of non-dimension vibration frequency; **b** Results of added mass

When the reduced velocity is 7.36, by calculation and analysis, it is determined that, when f_x/f_y = 1.306, the phase angle between the in-line and cross-flow motions is about 180.98°. With the gradual reduction of f_x/f_y, the phase angle is reduced correspondingly. When f_x/f_y = 0.92 and 0.835, the motion trajectory of cylinder will change from counter-clockwise to clockwise, the phase angle is close to but less than 90°.

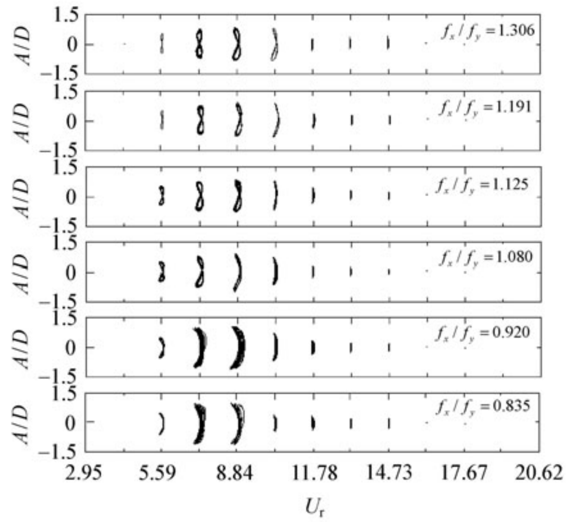

Fig. 8 VIV trajectories of cylinder under different natural frequency ratios (f_y = 0.617 3 Hz)

Figure 9 gives the variation of cylinder VIV amplitude versus reduced velocity under different natural frequency ratios for f_y = 0.537 6 Hz.

In this condition, the amplitude did not suddenly increase when the f_x/f_y increases to proper value. It may be clearly seen from Fig. 9 that, when the natural frequency ratio increases, the amplitude gradually decreases. In addition, the variation of the peak value in the curve is indicated in the figure, from which it is seen that with an increase of f_x/f_y, the peak amplitude will drift toward a higher reduced velocity. According to the curve in this figure, when $f_x/f_y \approx 1$, the peak amplitude A_{RMS} in the cross-flow direction is about $0.82D$, while the in-line peak amplitude A_{RMS} is about $0.23D$. In comparison with the result of one-degree of freedom experiments, the increase in the amplitude in the cross-flow direction is about 50.7%.

The motion trajectories of cylinder under different natural vibration frequency ratios and reduced velocities are shown in Fig. 10 for f_y = 0.537 6 Hz.

By observing the trajectories shown in Fig. 10 we see that when the peak amplitude appears, the corresponding motion trajectory is of "new moon" shape. With the increase in the natural frequency ratio, the "new moon" trajectory gradually evolves into figure "8"-shape.

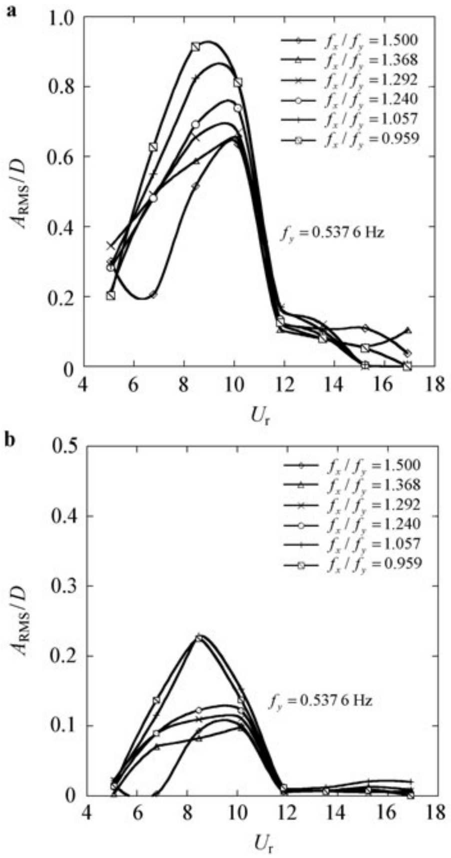

Fig. 9 Variation trend of the cylinder VIV amplitude against the reduced velocity U_r ($f_y = 0.537\,6$ Hz). **a** Cross-flow direction; **b** In-line direction

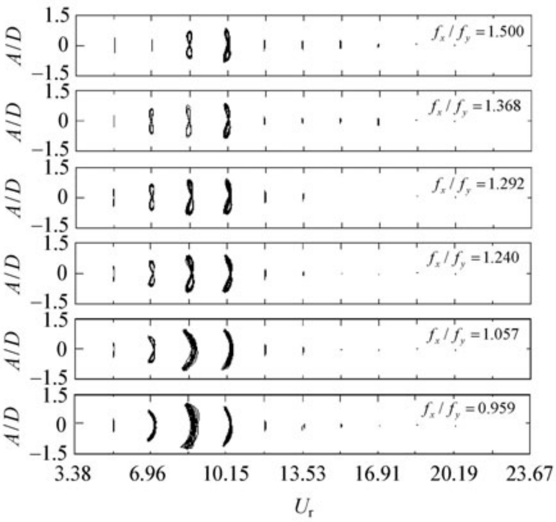

Fig. 10 VIV trajectories of cylinder under different natural frequency ratios ($f_y = 0.537\,6$ Hz)

Figure 11 gives the variation of VIV amplitude of cylinder versus reduced velocity under different natural frequency ratios for $f_y = 0.438\,6$ Hz.

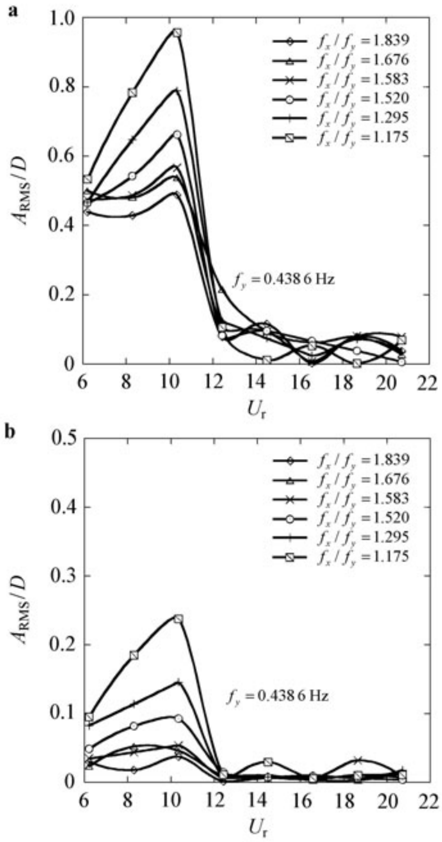

Fig. 11 Variation trend of the VIV amplitude against the reduced velocity ($f_y = 0.438\,6$ Hz). **a** Cross-flow direction; **b** In-line direction

In the experiments, when the peak amplitude appeared at the natural vibration frequency ratio of 1.175, the corresponding reduced velocity is $U_r = 10.4$. For this case, the cross-flow peak amplitude A_{RMS} is about $0.957D$ (the maximum amplitude is $1.31D$), while the in-line peak amplitude A_{RMS} is about $0.237D$ (the maximum amplitude is $0.51D$). As mentioned above, when the natural frequency ratio increases, the amplitude decreases. When f_x/f_y increases to 1.839, in comparison with the case where the frequency ratio is 1.175, the cross-flow amplitude A_{RMS} is decreased by about 49% and the in-line amplitude is reduced by about 84%.

Dahl et al. [9] carried out experiment investigation on the VIV response of cylinder for natural frequency ratio $f_x = f_y$–$2f_y$, and it was found that, when $f_x/f_y = 1.90$, the cross-flow amplitude would exhibit a "double peak" mode. When the reduced velocity $U_r \approx 5$, the first response peak appeared and the motion trajectory of cylinder showed that the drooping protrusion drifted to the upstream of the incoming flow. When $U_r \approx 8$, the response peak appeared again, at this time, the drooping protrusion of figure "8"-shape trajectory drifted to the downstream of the incoming flow. In the series of experiments conducted in this paper, the cross-flow natural vibration frequency is kept at 0.438 6 Hz and the maximum natural frequency ratio $f_x/f_y = 1.839$, which is close to the condition in the above-mentioned experiment.

In the present experiments, at the minimum current velocity the reduced velocity U_r exceeds 6, though no direct evidence indicates that a response peak does exist for $U_r < 6$. By analyzing the amplitude curves corresponding to the natural frequency ratio of 1.583, 1.676 and 1.839 it was found that, within the reduced velocity range of $6.2 < U_r < 10.4$, the amplitude firstly decreases and then increases. Therefore, it is reasonable to believe that the first peak value of the cylinder amplitude perhaps exists at some point within the range of $U_r < 6.2$, so that the peak at $U_r = 10.4$ is a "double peak" response.

The motion trajectories of cylinder under different conditions are given in Fig. 12.

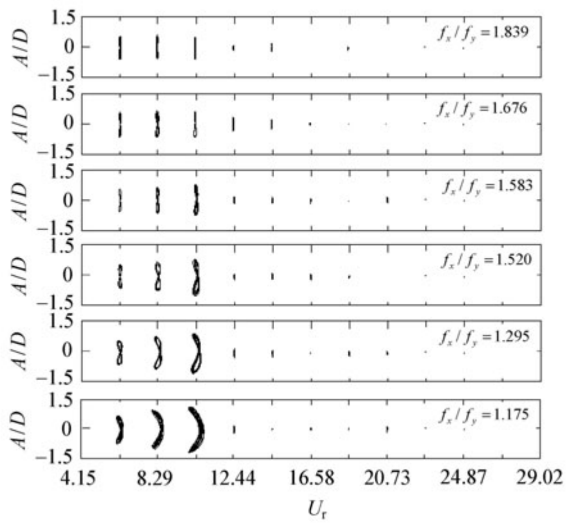

Fig. 12 VIV trajectory of cylinder under different natural frequency ratios ($f_y = 0.438\,6$ Hz)

When the natural vibration frequency ratio $f_x/f_y < 1.520$, the drooping protrusions of the motion trajectories of cylinder under various current velocities all drift to the downstream direction. In addition, within the "lock in" range, the drifting trend becomes intenser with the monotonic increase of reduced velocity. With monotonic increase of the natural frequency ratio, the motion phase angle between the in-line and the cross-flow directions becomes gradually smaller and the motion trajectories appear to be drifting to the upstream direction within an increasingly large flow velocity range. In addition, the balance location of the in-line vibration of cylinder gradually moves from the upstream of the initial location to the downstream direction.

As mentioned above in the discussion about the natural frequency, when the peak amplitude appears, the corresponding motion trajectory is of "new moon"-shape. With increases in the natural frequency ratio, the "new moon" trajectory gradually evolves into figure "8"-shape and the in-line amplitude gradually decreases. Under high natural vibration frequency ratio, the trajectory of cylinder appears to be slender "I"-shape.

From the above analysis, it may be deduced that the cylinder natural vibration frequency ratio affects the VIV. Figure 13 finally gives the curve of peak amplitude versus natural frequency ratio.

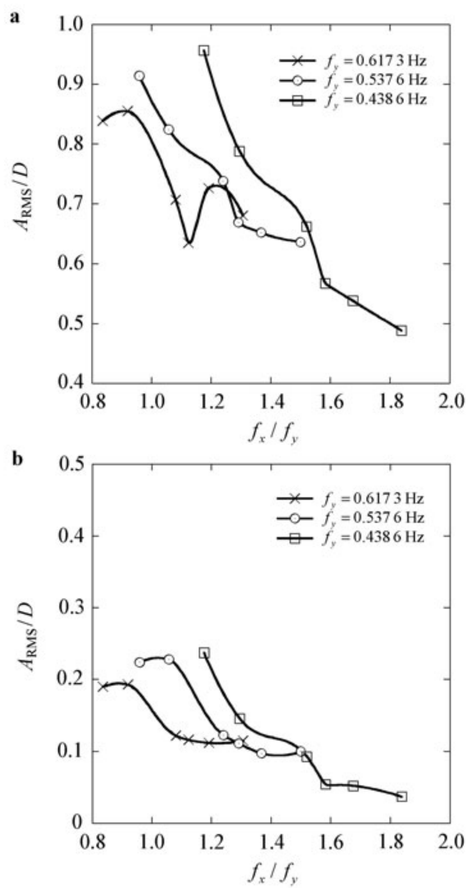

Fig. 13 Variation trend of the peak amplitude against the natural frequency. **a** Cross-flow direction; **b** In-line direction

This figure shows that, in both the cross-flow and in-line directions, the peak amplitude drops with the rise of the in-line and cross-flow vibration frequency ratio of cylinder. In addition, the smaller the cross-flow natural vibration frequency, the larger the amplitude.

In comparing the results of all amplitude experiments shown in Fig. 13, it can be seen that the amplitude peak values still differ greatly under different cross-flow natural vibration frequencies and when f_x/f_y changes a little. It is reasonable to speculate that, besides the non-dimensional in-line and cross-flow nature vibration frequency ratio f_x/f_y, the absolute value of the natural vibration frequency is also one of the important parameters that affect the vibration behavior.

7 Conclusion

In the paper, a set of horizontally-oriented cylinder VIV experiments were designed and conducted. In the experiment, by regulating the spring system, the natural vibration frequency of the model was changed systematically in both the vertical and horizontal directions. As a result, the following conclusions are drawn.

(1) In the model experiment investigation on the one-degree of freedom VIV of cylinder, it was found that, within the range of the reduced velocities studied, the amplitude of cylinder exhibited "double peak" behavior. In addition, in the vicinity of the second response peak, the cross-flow vortex-induced force widely showed 3-order high frequency components, i.e., a "2T" wake was generated. This is rarely reported in the VIV experiments on horizontally-oriented cylinders.
(2) In the model experiment investigation on the two-degree of freedom VIV of cylinder, it was found that, within some cross-flow and in-line natural vibration frequency range, the reverse "C"-shape, i.e., the "new moon" shape of trajectory would also appear in addition to the traditional "8"-shape trajectory.
(3) In the two-degree of freedom VIV experiments, it was found that, under different vibration frequencies and for different in-line and cross-flow vibration frequency ratios, the amplitude results of cylinder varied considerably. The results clearly showed interaction between the in-line and the cross-flow vibrations and the results are instructive for further investigations.
(4) Finally, it was found that the absolute value of the natural vibration frequency of cylinder has also important influences on the VIV results besides the non-dimensional in-line and cross-flow vibration frequency ratio.

References

1 Feng, C.C.: The measurements of vortex-induced effects in flow past a stationary and oscillating circular and D-section cylinders. [MS Thesis], University of British Columbia (1968)
2 Sarpkaya, T.: Vortex-induced oscillations. ASME J. Appl. Mech. **46**, 241–258 (1995)
3 Khalak, A., Williamson, C.H.K.: Motions, forces and mode transitions in vortex-induced vibrations at low mass-damping. Journal of Fluids and Structures **13**, 813–851 (1999)
4 Ren, T., Fu, S.X, Li, R.P, et al.: Full scale riser vortex-induced-vibration response prediction based on model test. Journal of Ship Mechanics **15**, 364–370 (2011)
5 Tang, S.Z, Huang, W.P., Liu, J.J, et al.: Vortex induced vibration and fatigue damage of risers under two-dimensional vortex excitation considering different frequency ratios. Journal of Vibration and Shock **30**, 124–128 (2011)
6 Zhang, Y.B., Guo, H.Y., Meng, F.S., et al.: Model tests for vortex-induced vibration of a top tension riser based on wavelet transformation. Journal of Vibration and Shock **30**, 149–154 (2010)
7 Pan, Z.Y., Cui, W.C, Liu, Y.Z.: A predicting model for self-excited VIV of a circular cylinder at low mass-damping. Journal of Ship Mechanics **9**, 115–124 (2005)
8 Jauvtis, N., Williamson, C.H.K.: Vortex-induced vibration of a cylinder. Journal of Fluids and Structures **17**, 1035–1042 (2003)
9 Dahl, J.M., Hover, F.S., Triantafyllou, M.S.: Two-degree-of-freedom vortex-induced vibrations using a force assisted apparatus. Journal of Fluids and Structures **22**, 807–818 (2006)
10 Vikestad, K., Larsen, C.M., Vandiver, J.K.: Experimental study of excited circular cylinder in current. In: Proceedings of the 16th International Conference on Offshore Mechanics and Arctic Engineering, ASME, Yokohama, Japan 231–240 (1997)
11 Wu, J., Larsen, C.M., Lie, H.: Estimation of hydrodynamic coefficients for VIV of slender beam at high mode orders. In: Proceedings of the International Conference on Offshore Mechanics and Arctic Engineering, OMAE2010, Shanghai, China (2010)
12 Kang, Z., Webster, W.C.: An application of system identification in the two-degree-freedom VIV experiments. Journal of Marine Science and Application **8**, 99–104 (2009)
13 Skaugset, K., Baarholm, R.: Effect of marine growth on an elastically mounted circular cylinder. OMAE2008-57586 (2008)
14 Smogeli, O.: Design and evaluation of a force-feedback control system for viv experiments. Sivilingenior, NTNU and MIT (2002)
15 Wootton, L.R.: The oscillations of large circular stacks in wind. Proceedings of Institution of Civil Engineers **43**, 573–598 (1969)
16 Kawai, H.: Vortex-induced vibration of tapered cylinder. Journal of Fluid Engineering **59**, 49–52 (1994)
17 Fujarra, A.L.C., Pesce, C.P.: Vortex-induced vibration of a flexible cantilever. Journal of Fluids and Structures **15**, 651–658 (2001)
18 Kitagawa, T., Fujino, Y.: Effects of free-end condition on end-cell-induced vibration. Journal of Fluids and Structures **13**, 499–518 (1999)