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RESEARCH PAPER

Assessment of structural damage using natural frequency changes

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Abstract The present paper develops a new method for damage localization and severity estimation based on the employment of modal strain energy. This method is able to determine the damage locations and estimate their severities, requiring only the information about the changes of a few lower natural frequencies. First, a damage quantification method is formulated and iterative approach is adopted for determining the damage extent. Then a damage localization algorithm is proposed, in which a damage indicator is formulated where unity value corresponds to the true damage scenario. Finally, numerical studies and model tests are conducted to demonstrate the effectiveness of the developed algorithm.

Keywords Damage assessment · Damage detection · Strain energy · Modal analysis · Natural frequency

1 Introduction

Engineering structures will inevitably suffer different damages in their service life. Therefore structural damage identification is of great importance for structural safety and integrity. Owing to disadvantages of the localized-damage detection, global vibration-based structural damage detection has been paid much attention in recent years. The basic idea of this type of methods is that the modal parameters (including modal frequency, mode shapes, etc.) are functions of the structural properties. When damage occurs in the structure, the structural parameters will change, and the modal parameters of the structural system will also change. Therefore changes of modal parameters can be used to detect, locate and quantify the damage. Based on changes in frequencies, mode shapes, or their combination, various damage identification methods have been developed during the past decades [1–4].

Among the vibration-based structural damage detection methods, one type of damage identification methods is based on changes of natural frequencies. In practice, the natural frequency is easy to measure and is independent of the measured position. Generally, the measurement accuracy of natural frequency is higher than that of mode shape or modal damping. Many studies have used structural natural frequency to indicate structural damage. Cawley and Adams [5] were among the first to use an incomplete set of measured natural frequencies to identify the location and provide a rough damage estimates. Hassiotis and Jeong [6] introduced an optimization algorithm to identify both the location and severity using changes in natural frequencies. A detailed discussion on the use of natural frequency as a diagnostic parameter in structural assessment procedure could be found in the review paper by Salawu [7]. Early researches focused on indicating the existence and location of damage. Recently, Kim et al. [8] have proposed a methodology to non-destructively locate and estimate the size of damage in structures for which a few natural frequencies or a few mode shapes are available. A damage-localization algorithm to locate damage from changes in natural frequencies and a damage-sizing algorithm to estimate crack-size from natural frequency perturbation are formulated. However, damage at a single location is assumed during the formulation.

An alternative way of damage assessment utilizes mode shapes from measurements. Among the damage identification methods using mode shapes, the modal strain energy based methods seem to be promising for damage evaluation.

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Stubbs et al. [9] developed an algorithm for damage detection, which requires that the mode shapes before and after damage be known, but the modes do not need to be mass normalized and only a few modes are required. It has been found to be the most accurate algorithm in comparison with several other algorithms being investigated [10]. More recently, Li et al. [11] develops an improved damage localization method termed the modal strain energy decomposition (MSED) method. The MSED method defines two damage indicators, axial damage indicator and transverse damage indicator, for each member. Analyzing the joint information of the two damage indicators can greatly improve the accuracy in localizing damage elements. However, both these two methods are damage localization method. When the degree of the damage was estimated based on these methods, it was found that these estimates all underestimate the true damage level significantly.

As for the damage quantification, Hu et al. [12] developed cross modal strain energy (CMSE) method for damage severity estimation. The method involves solving a set of linear simultaneous equations for determining the damage severity, in which each equation is formulated based on the product terms from two same/different modes associated with the mathematical and experimental models, respectively. It was demonstrated that the CMSE method was capable of accurately estimating the damage degree of multiple damaged members after damage members were localized. Wang et al. [13] extended the CMSE method for both damage localization and severity estimation. Based on the employment of the cross modal strain energy, Li et al. [14] newly developed a method for the damage localization and severity estimate for three-dimensional frame structures. A three-dimensional five-story frame structures was used to numerically demonstrate its effectiveness for both single-damage and multiple-damage scenarios. However, the modes used are required to be spatially complete, i.e., mode shapes at all degrees of freedom must be available.

The primary objective of the present paper is to develop a new algorithm that can effectively localize the damaged members, as well as accurately estimate their severities, using changes of a few lower natural frequencies. Since frequency measurements can be easily acquired and are more reliable, the approach could provide an inexpensive structural assessment technique. The newly developed damage assessment method, named as iterative modal strain energy (IMSE) method, is able to effectively identify the geometric locations of the damaged members, and accurately quantify their severities at the same time, requiring only a small number of modal frequencies identified from the damaged structure. To demonstrate the effectiveness of the developed algorithm, numerical studies are conducted for a clamped-free beam structure and a plane frame structure based on data generated from the finite element models. Meanwhile, experimental data from a tested clamped-free beam are utilized to validate the new method. Numerical simulation and experimental study demonstrate that excellent results can be achieved for both single and double damage scenarios.

2 Iterative modal strain energy (IMSE) method

For a baseline structure model denoted by mass matrix $\boldsymbol{M}$ and stiffness matrix $\boldsymbol{K}$, the eigen-analysis for the structure was written as

$$\boldsymbol{K}\boldsymbol{\Phi}_i = \lambda_i \boldsymbol{M}\boldsymbol{\Phi}_i, \tag{1}$$

where λ_i and $\boldsymbol{\Phi}_i$ denote the i-th eigen value and eigen vector, respectively. Likewise, one writes the corresponding expression for the damaged structure as

$$\boldsymbol{K}^*\boldsymbol{\Phi}_i^* = \lambda_i^* \boldsymbol{M}^*\boldsymbol{\Phi}_i^*, \tag{2}$$

where $\boldsymbol{M}^*$ and $\boldsymbol{K}^*$ are the mass and stiffness matrices for the damaged structure, and λ_i^* and $\boldsymbol{\Phi}_i^*$ denote the associated i-th eigen value and eigen vector. Throughout the paper, superscript "$*$" is used to indicate values associated with the damaged structure. In Eqs. (1) and (2), one can treat $\boldsymbol{\Phi}_i$ and λ_i as analytical modal information for the baseline structure, and $\boldsymbol{\Phi}_i^*$ and λ_i^* the measured modal information from the damaged structure.

Generally, local damages will lead to loss of structural stiffness, with the mass unchanged. Therefore, $\boldsymbol{M}^* = \boldsymbol{M}$. In the following derivation, as $\boldsymbol{\Phi}_i$, λ_i, and λ_i^* are presumably known, the unknown terms are $\boldsymbol{K}^*$ and $\boldsymbol{\Phi}_i^*$. From Eqs. (1) and (2), the first step is to eliminate the mass matrix $\boldsymbol{M}$. Premultiplying Eq. (1) by $(\boldsymbol{\Phi}_i^*)^{\mathrm{T}}$ and Eq. (2) by $(\boldsymbol{\Phi}_i)^{\mathrm{T}}$ yields

$$(\boldsymbol{\Phi}_i^*)^{\mathrm{T}}\boldsymbol{K}\boldsymbol{\Phi}_i = \lambda_i(\boldsymbol{\Phi}_i^*)^{\mathrm{T}}\boldsymbol{M}\boldsymbol{\Phi}_i, \tag{3}$$

$$(\boldsymbol{\Phi}_i)^{\mathrm{T}}\boldsymbol{K}^*\boldsymbol{\Phi}_i^* = \lambda_i^*(\boldsymbol{\Phi}_i)^{\mathrm{T}}\boldsymbol{M}\boldsymbol{\Phi}_i^*. \tag{4}$$

Since $\boldsymbol{M}$ and $\boldsymbol{K}$ are symmetric matrices, one can show that

$$(\boldsymbol{\Phi}_i^*)^{\mathrm{T}}\boldsymbol{M}\boldsymbol{\Phi}_i = (\boldsymbol{\Phi}_i)^{\mathrm{T}}\boldsymbol{M}\boldsymbol{\Phi}_i^*, \tag{5}$$

$$(\boldsymbol{\Phi}_i^*)^{\mathrm{T}}\boldsymbol{K}\boldsymbol{\Phi}_i = (\boldsymbol{\Phi}_i)^{\mathrm{T}}\boldsymbol{K}\boldsymbol{\Phi}_i^*. \tag{6}$$

Using the scalar identities of Eqs. (5) and (6), combination of Eqs. (4) and (3) yields

$$(\boldsymbol{\Phi}_i)^{\mathrm{T}}\boldsymbol{K}^*\boldsymbol{\Phi}_i^* = \frac{\lambda_i^*}{\lambda_i}(\boldsymbol{\Phi}_i)^{\mathrm{T}}\boldsymbol{K}\boldsymbol{\Phi}_i^*. \tag{7}$$

It is assumed that the damaged locations are known a priori. Therefore the stiffness matrix of the damaged structure could be written as

$$\boldsymbol{K}^* = \boldsymbol{K} + \sum_{n=1}^{N_{\mathrm{d}}} \alpha_n \boldsymbol{K}_{\ell_n}, \tag{8}$$

where N_{d} is the total number of the damaged members; α_n and ℓ_n are the damage extent and the element number of the n-th damaged element, respectively. And the damage extent satisfies $-1 \le \alpha_n \le 0$, in which $\alpha_n = 0$ means no damage and $\alpha_n = -1$ totally damaged.

Substituting Eq. (8) into Eq. (7), one obtains

$$\sum_{n=1}^{N_\mathrm{d}} \alpha_n (\boldsymbol{\Phi}_i)^\mathrm{T} \boldsymbol{K}_{\ell_n} \boldsymbol{\Phi}_i^* = \left(\frac{\lambda_i^*}{\lambda_i} - 1\right) (\boldsymbol{\Phi}_i)^\mathrm{T} \boldsymbol{K} \boldsymbol{\Phi}_i^*. \tag{9}$$

Define the structural modal strain energy between the baseline structure and the damaged structure for the i-th mode, as

$$C_i = (\boldsymbol{\Phi}_i)^\mathrm{T} \boldsymbol{K} \boldsymbol{\Phi}_i^*, \tag{10}$$

and the corresponding elemental modal strain energy for the stiffness matrix $\boldsymbol{K}_{\ell_n}$ as

$$C_{n,i} = (\boldsymbol{\Phi}_i)^\mathrm{T} \boldsymbol{K}_{\ell_n} \boldsymbol{\Phi}_i^*. \tag{11}$$

Then Eq. (9) can be simplified as

$$\sum_{n=1}^{N_\mathrm{d}} \alpha_n C_{n,i} = b_i, \tag{12}$$

where

$$b_i = \left(\frac{\lambda_i^*}{\lambda_i} - 1\right) C_i. \tag{13}$$

When m modes are available for the baseline structure and damaged structure, totally m equations can be formed from Eq. (12). Written in a matrix form, one has

$$\boldsymbol{C}\boldsymbol{\alpha} = \boldsymbol{b}, \tag{14}$$

in which $\boldsymbol{C}$ is an m-by-N_d matrix, $\boldsymbol{\alpha}$ and $\boldsymbol{b}$ are column vectors of size N_d and m, respectively. When m is greater than or equal to N_d, a least-squares approach can be taken to solve for $\boldsymbol{\alpha}$. The estimate of $\boldsymbol{\alpha}$, denoted as $\hat{\boldsymbol{\alpha}}$, is written as

$$\hat{\boldsymbol{\alpha}} = (\boldsymbol{C}^\mathrm{T}\boldsymbol{C})^{-1}\boldsymbol{C}^\mathrm{T}\boldsymbol{b}. \tag{15}$$

It should be mentioned that only the damaged eigenvalues are known a priori from modal identification of measured responses from damaged structure in this paper. However, solving Eq. (15) requires the mode shapes at full coordinates of the damaged structures. Generally, it is very difficult to obtain the mode shapes at full coordinates, especially for complex structures. Therefore one should try to avoid utilizing the measured mode shapes for damage assessment, especially the mode shapes at full coordinates. An iterative approach is adopted here for solving Eq. (15) when the damaged mode shapes at full coordinates can not be acquired. One knows that the mode shapes for the damaged structure are associated with the damage severity as

$$\boldsymbol{\Phi}_i^* = \boldsymbol{\Phi}_i^*(\boldsymbol{K}^*, \boldsymbol{M}) = \boldsymbol{\Phi}_i^*\left(\boldsymbol{K} + \sum_{n=1}^{N_\mathrm{d}} \alpha_n \boldsymbol{K}_{\ell_n}, \boldsymbol{M}\right). \tag{16}$$

The damage severity can be estimated iteratively as follows.

Step 1: Assume the damage severity to be zeros initially, i.e., $\boldsymbol{\alpha}^{(0)} = \boldsymbol{0}$, where superscript "0" denotes initial values. Then compute the damaged mode shapes using Eq. (16), resulting in $\boldsymbol{\Phi}_i^{*(0)} = \boldsymbol{\Phi}_i^*(\boldsymbol{K}, \boldsymbol{M})$.

Step 2: Solve Eq. (15) for the estimated damage severity $\boldsymbol{\alpha}^{(1)}$ using the computed mode shapes $\boldsymbol{\Phi}_i^{*(0)}$. The first iteration for estimating the damage severity is finished.

Step 3: Compute the damaged mode shapes $\boldsymbol{\Phi}_i^{*(j-1)}$ via Eq. (16) by using the estimated damage severity $\boldsymbol{\alpha}^{(j-1)}$, and estimate the damage severity $\boldsymbol{\alpha}^{(j)}$ via Eq. (15) by using $\boldsymbol{\Phi}_i^{*(j-1)}$ for $j = 2, 3, \cdots$, sequentially.

Step 4: Set the condition of iteration termination. Repeat Step 3 until $\max\{|\ \boldsymbol{\alpha}^{(j)} - \boldsymbol{\alpha}^{(j-1)}|\} \leq tol$, where tol is a pre-determinated threshold. For example, one can set tol to be 0.001 or 0.005, up to the precision of the severity estimation.

2.1 Damage localization

In the above derivation, using Eq. (8) implies that prior knowledge of the locations of the damage members must be given. When the damage locations are not known, one should first find a way for damage localization. Since modal frequencies are a global property of the structure, the frequencies generally can not provide spatial information about structural changes. However, multiple frequency shifts can provide spatial information about structural damage because changes in the structure at different locations will cause different combinations of changes in the modal frequencies. In other words, only the true damage scenario can lead to the combinations of frequency changes measured from the damaged structure. Based on this idea, it is suggested to perform a residual analysis for each suspicious scenario of the true damage locations. For each suspicious scenario, one follows the above procedure to estimate $\boldsymbol{\alpha}$ using different combinations of modal frequency changes. Here, symbol $\hat{\boldsymbol{\alpha}}_k^i$ is denoted as the severity estimate corresponding to the k-th suspicious scenario associated with the i-th combination of frequency changes. If totally N_m combinations are used for estimation, then there are N_m damage severity estimates for each suspicious scenario, i.e., $\hat{\boldsymbol{\alpha}}_k^i$, $i = 1, 2, \cdots, N_m$. One writes the residual of any two estimates associated with each suspicious scenario as $e_k^{i,j} = |\hat{\boldsymbol{\alpha}}_k^i - \hat{\boldsymbol{\alpha}}_k^j|$. When there exist N_m estimates, one writes the residual as

$$e_k = \sum_{i=1}^{N_m} \sum_{j=i+1}^{N_m} |\hat{\boldsymbol{\alpha}}_k^i - \hat{\boldsymbol{\alpha}}_k^j|, \tag{17}$$

where the subscript "k" should be looped for all suspicious scenarios. Taking a structure with N_e members as an example, the suspicious damage location might be any element for a single damage case. Hence there are totally N_e suspicious scenarios. One should compute e_k from $k = 1$ to $k = N_e$, that is, residual $\boldsymbol{e}$ is a N_e-by-1 vector. For double damage scenario, any two members should be assumed to be the damaged elements. Therefore there are totally $N_e(N_e - 1)/2$ suspicious scenarios, which means residual $\boldsymbol{e}$ is a vector of size $N_e(N_e - 1)/2$. Alternatively, $\boldsymbol{e}$ can be re-arranged into the upper triangle of a N_e-by-N_e matrix. In principle, if the examined damage scenario is the true damage scenario, the residual e_k for this damage scenario should be the smallest, since the estimate $\hat{\boldsymbol{\alpha}}^i$ for any frequency combination should be equal to or close to each other. However, if the examined damage scenario is not the true damage scenario, the residual e_k would be large, since the estimate $\hat{\alpha}^i$ for any combination of modal frequencies may differentiate significantly from each other. So the quantity e_k can be employed as a damage indicator to quantify the goodness of the "fitting" among all the suspicious scenarios. For conciseness, the damage indicator is re-defined to judge the existence of damage as follows

$$e_k = \frac{\min(e_k)}{e_k}. \tag{18}$$

Thus, a simple damage localization algorithm is based on finding among all suspicious scenarios the particular one that corresponds to the case in which the damage indicator takes the value of 1.

2.2 Damage quantification

After the damage location(s) has(have) been localized, the damage severity can be determined by the accompanying severity estimates for the corresponding suspicious damage scenario. Alternatively, the damage severity can also be estimated using Eq. (15).

2.3 Merits of the IMSE method

In comparison to other diagnosis method, a number of advantages of the present IMSE method deserve to be mentioned: (1) The IMSE method employs the modal frequency information only, avoiding using the mode shapes from the damaged structure. As we know, it is very difficult to obtain the mode shapes at full coordinates, especially for complicated three-dimensional structures. (2) Only minimal information of modal frequencies from measurements is required. For single damage scenarios, only two modal frequencies (eg., the first two) are needed for damage localization and one for damage quantification. For double damage scenarios, the first three modal frequencies are enough for damage localization and severity estimation. Frequency combinations (1,2), (1,3), (2,3) and (1,2,3) could be used for damage localization and any one combination could be used for damage severity estimation. A frequency combination (1, 2) means utilization of both the first and the second modal frequencies when employing the IMSE method, because at least two frequencies are required for double-damage scenario. And the rest may be deduced by analogy.

3 Numerical studies

To evaluate the performance of the proposed method for damage localization and severity estimation, two problems are simulated. The first example makes use of a clamped-free beam structure which will also be experimentally tested in the next section. The second example studies a relatively complicated steel plane frame structure where different double damage scenarios are considered.

3.1 Clamped-free beam structure

The first example is a clamped-free beam of 200 cm long, 5.0 cm wide and 2.8 cm thick, as shown in Fig. 1. The beam was made of steel with Young's modulus of $E = 210$ GPa and mass density of $\rho = 7\,850$ kg/m^3. The beam is modeled by 20 equal Euler–Bernoulli beam elements.

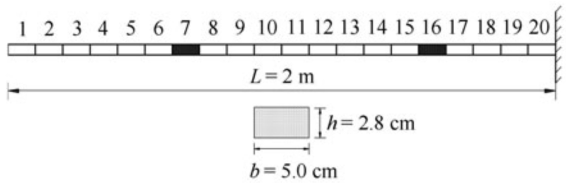

Fig. 1 The sketch of a clamped-free beam

Modal analysis is carried out to get the modal parameters. To get the assumed experimental modal frequencies of the damaged structure, it is assumed that the elastic modulus of the associated element is reduced. The modal analysis is again carried out in this damaged beam to get the assumed experimental modal parameters.

In this numerical study, both single and double damage scenarios are considered and the damage cases are listed in Table 1. Meanwhile, the first three modal frequencies of the beam before and after damage are also listed in Table 1. For the single damage scenario, it is assumed that Element 16 is damaged with Young's modulus reduced by 10%. For the double damage scenario, both Elements 7 and 16 are damaged with the corresponding elastic modulus reduced by 10% in both of them. Throughout this numerical study, the tolerance of iteration termination is set to be 0.005.

Table 1 Damage cases simulated in the beam structure

Structure	Damage location	Damage extent/%	Frequency/Hz 1st	2nd	3rd
Baseline	None	0	5.834	36.559	102.37
Case 1	16	−10	5.803	36.557	102.09
Case 2	7, 16	−10, −10	5.801	36.424	101.49

For the single damage scenario, implementing the IMSE method for damage localization needs at least two natural frequencies. After the damage location has been determined, any one frequency could be used for damage severity estimation. Of course, one can utilize the average value from multi-frequency shifts to reduce errors of estimation from one single frequency shift. When the first two natural frequencies are used in the algorithm, the damage indicator is shown in Fig. 2 with the damage indicator plotted against the element number. One can obviously see that Element 16 is damaged since the damage index at Element 16 equals to 1. The associated severity estimates corresponding to the first and second frequency shifts are -10.01% and -10.01%, respectively. Results from the first and third, the second and third frequency shifts are similar to that from the first two natural frequencies, not shown here for space limitation. The present IMSE method could localize the damage and estimate the severity accurately.

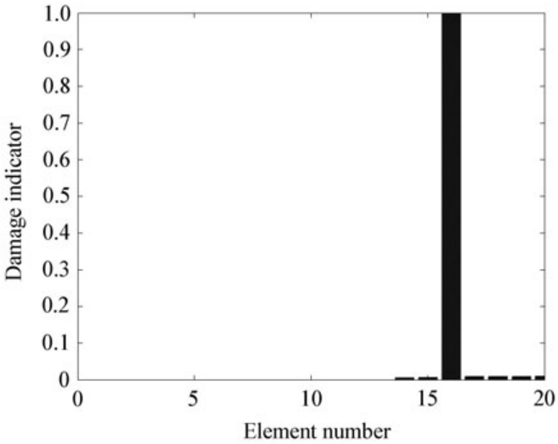

Fig. 2 The damage indicator for damage case 1 of beam structure when frequency combinations 1 and 2 are used

For the double damage scenario, implementing the IMSE method for damage severity estimation needs at least two natural frequencies and for damage localization at least three natural frequency changes. When the first three modal frequencies are identified from the damaged beam, one has four frequency combinations as (1,2), (1,3), (2,3) and (1,2,3), where (1,2) means the combination of the first and second modal frequencies. Any single frequency combination could be used for damage severity estimation and any two, three or all the combinations for damage localization. Performing the damage localization procedure using frequency combinations (1,2) and (1,3), one obtains the damage indicator, as shown in Fig. 3. The advantage of using graphic approach is that it provides a visual result. In this figure, each value of $\boldsymbol{e}$ corresponds to a trial of two presumed damage elements. For the present 20-element structure, there are 190 possible two-damage combinations. The underlined damage localization algorithm is searching for an e_k equivalent to unity among all 190 suspicious scenarios. From Fig. 3, one can clearly observe that the present method correctly points out the damage locations at Elements 7 and 16. When other frequency combinations are employed, a similar result as Fig. 3 has been observed (not shown here). While the correct damage locations are identified, the corresponding damage severity are also estimated accurately.

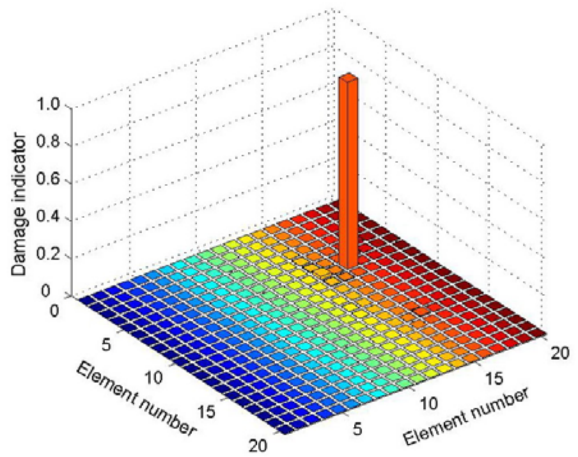

Fig. 3 The damage indicator for damage case 2 of beam structure when combinations (1,2) and (1,3) are used

3.2 Plane frame structure

The second example to demonstrate the proposed damage localization and severity estimation method is a two dimensional frame structure, as shown in Fig. 4. The structure consists of 25 two dimensional beam members, 14 nodes and 36 DOFs. Each node has three DOFs with motion confined in the plane of the structure. The circled numbers given in the figure are element numbers. The structure is fixed at both nodes 1 and 14. The geometrical dimensions are as follows: lengthes of vertical and horizontal members $l = 1$ m, cross section area $A = 1.0 \times 10^{-3}\,\mathrm{m}^2$ and inertial moment $I = 8.0 \times 10^{-8}\,\mathrm{m}^4$. The material properties of the structure include: elastic modulus $E = 207$ GPa, mass density $\rho = 7\,800\,\mathrm{kg/m}^3$ and Poisson's ratio $\nu = 0.3$.

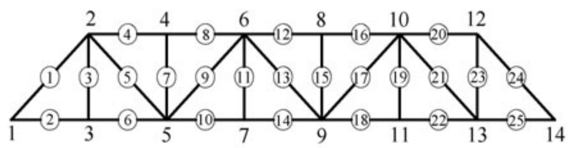

Fig. 4 The two dimensional plane frame structure

Using the finite-element method and performing an eigen analysis, one obtains that the undamaged model of the structure has the first three modal frequencies of 59.595, 78.913 and 86.49 Hz, respectively. Three damage cases are assumed: (1) Single damage in the 9th element in the inclined member with the stiffness reduced by 20%; (2) Double damages in the 9th element in the inclined member with the stiffness reduced by 20% and in the 18th element in the lower chord with the stiffness decreased by 10%, respectively; (3) Double damages in the 9th element in the inclined member with the stiffness reduced by 20% and in the 19th element in the vertical member with the stiffness decreased by 10%, respectively.

For the single damage scenario, the damage indicator is shown in Fig. 5 when the first two modal frequencies are used in the procedure. It is obvious that Element 9 is damaged since the damage indicator at Element 9 is equal to unity. And the damage severity is estimated to be 19.96%. Performing the IMSE method using other modal frequencies yields similar results. It can be seen that the proposed method can locate the damage and estimate the severity excellently.

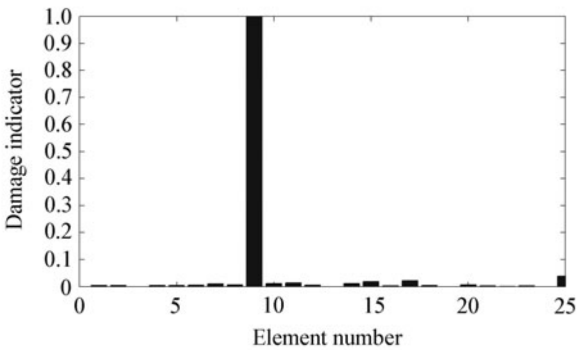

Fig. 5 The damage indicator for damage case 1 of frame structure when frequency combinations 1 and 2 are used

For the double damage scenarios, in damage cases 2 and 3 the damage detection is carried out for an inclined member combined with a horizontal or vertical member, respectively. In damage case 2, the damaged elements are an inclined member (Element 9) with 20% stiffness loss and a horizontal member (lower chord 18) with 10% stiffness loss, respectively. Damage members in case 3 include an identical inclined member as in case 2 with 20% damage and a vertical member (Element 19) with 10% damage, respectively. Figures 6 and 7 show the results of damage locations when

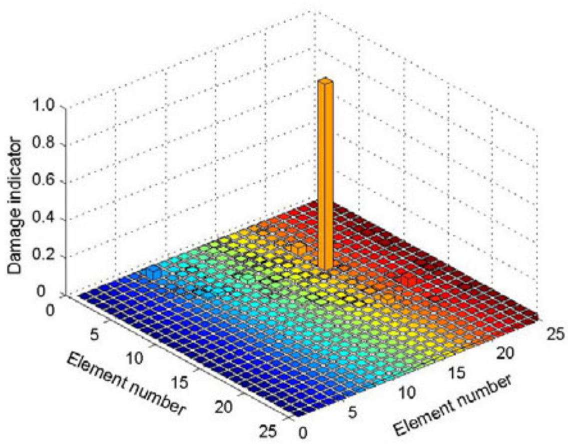

Fig. 6 The damage indicator for damage case 2 of frame structure when frequency combinations (1,2) and (1,3) are used

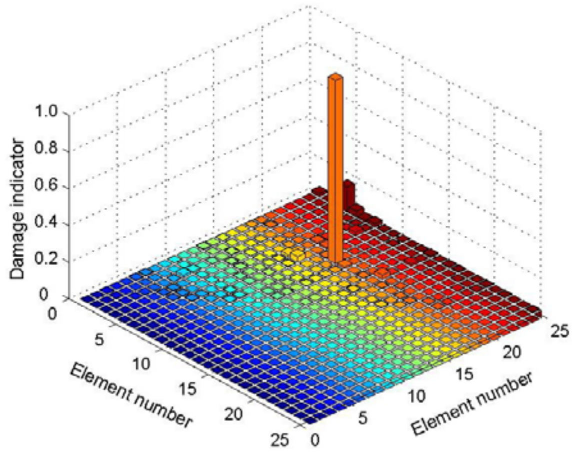

Fig. 7 The damage indicator for damage case 3 of frame structure when frequency combinations (1,2) and (1,3) are used

the first three modal frequencies are applied. One can clearly see that the damaged locations are at Elements 9 and 18 for damage case 2 and at Elements 9 and 19 for damage case 3, respectively. And the damage extents are estimated to be 20.03% and 9.98% for damage case 2 and 20.02% and 9.99% for damage case 3, respectively. Excellent results can be achieved for damage locations and damage extents by using the proposed method.

4 Experimental validation

The developed approach is further evaluated and validated using modal testing data. The tested structure is a clamped-free steel beam with its geometrical properties identical to those described in Sect. 3.1. The beam is welded to base foundation at the bottom end with the other end free, as shown in Fig. 8.

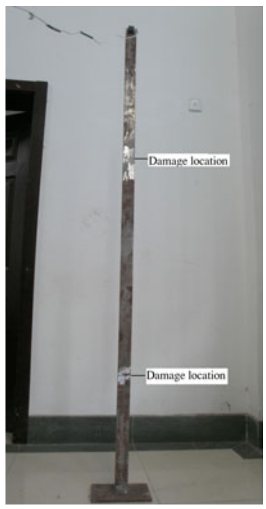

Fig. 8 The tested beam structure used in the experiment

Acceleration response was transversely measured at the free end of the beam with an accelerometer. Sensor used for the test is Model 2220-005 of SILICON DESIGNS with an operating frequency from 0 to 600 Hz, and an amplitude rate of $5g$. A dynamic force was generated by means of an impulse hammer. But the input was not measured. First dynamic measurements were performed for the undamaged beam. Then, cracks were generated to the desired depth using a saw cut (about 1 mm thick) on the desired locations on one side of beam along the width direction. Again, the dynamic responses for the cracked beam were measured in sequence for modal identification and damage assessment. All the vibration signals are collected with the dynamic data acquisition system of CRONOS PL16-DCB8 and stored in computer for analysis. During the dynamic testing, the measurement data were sampled at sampling frequency of 200 Hz. A section of typical dynamic response for the undamage beam is illustrated in Fig. 9. The Eigensystem Realization Algorithm [15,16] is used for the modal identification. The first three modes are identified for the undamaged and damaged structure.

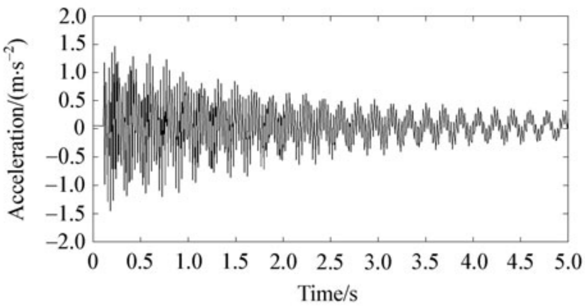

Fig. 9 A section of dynamic acceleration of the tested beam

The tested beam is analytically modeled with 20 beam elements, as shown in Fig. 1. The elastic modulus is assumed to be $E = 210$ GPa and mass density $\rho = 7\,850\,\mathrm{kg/m^3}$. The model with $E = 210$ GPa is named as initial FE model. The first three modal frequencies for the initial FE model and the undamaged state of the beam structure are shown in Table 2. One can see that there exists discrepancy of the modal frequencies between this initial FE model and the undamaged test structure. If Young's modulus E is assumed to be uniformly updated to match the undamaged target, then the Young's modulus E of the initial FE model should be reduced to $E = 190$ GPa and the modal frequencies for the initial FE model after updating match the undamaged counterparts very well, as listed in the fourth row of Table 2. For not confusing, the initial FE model after updating is named as updated FE model. It should be noted that this updated FE model is utilized as the baseline model hereafter for the damage assessment.

Table 2 The first three modal frequencies of the tested beam for the undamaged case

(Unit: Hz)

Structure	1st Freq.	2nd Freq.	3rd Freq.
Initial FE model	5.834	36.559	102.37
Undamaged beam	5.524	34.711	97.200
Baseline model	5.549	34.775	97.371

Three damage cases were simulated in the experimental test by thin saw crack(s). For damage case 1 in the experimental test, the cut location is 44.2 cm away from the clamped end, almost in the middle of Element 16, as shown in Fig. 1. And the cut depth was 7 mm on one single side, about 1/4 of the beam thickness. In damage case 2, the cut location is identical to damage case 1 and the cut depth was 14 mm, about 1/2 of the beam thickness. Damage case 3 simulates double damage case. In addition to the half depth cut in damage case 2, there was another half depth cut on the middle of Element 7, about 65.8 cm away from the free end of the beam.

First, the damage assessment for damage case 1 is investigated. The cut depth is about 1/4 of the beam thickness in damage case 1 and it is a relatively small damage. The first three identified modal frequencies for damage case 1 are listed as the second column in Table 3. When implementing the proposed method for damage location with different frequency combinations, the damage indicators are shown in Fig. 10, from which one can clearly observe that the present method correctly points out the damage locations at Elements 16. The estimated damage severities using frequency changes of the first two modes are −21.35% and −17.11%, respectively. The first three updated modal frequencies using the average damage severity estimates (−19.23%) are also listed in column 3 of Table 3 for comparison. The average damage severities corresponding to frequency combinations (1,3), (2,3) and (1,2,3) are −19.14%, −17.02%, −18.47%, respectively. And the updated frequencies are listed in columns 4–6 of Table 3. It could be seen that the updated frequencies match very well with the measured ones. It is demonstrated that the proposed IMSE method could localize the damage and estimate the severity for small damage.

Table 3 The first three modal frequencies of the tested beam for damage case 1

(Unit: Hz)

Modes	Measured	Updated			
		(1,2)	(1,3)	(2,3)	(1,2,3)
1st	5.478	5.487	5.487	5.495	5.490
2nd	34.771	34.771	34.771	34.771	34.771
3rd	96.885	96.805	96.809	96.882	96.832

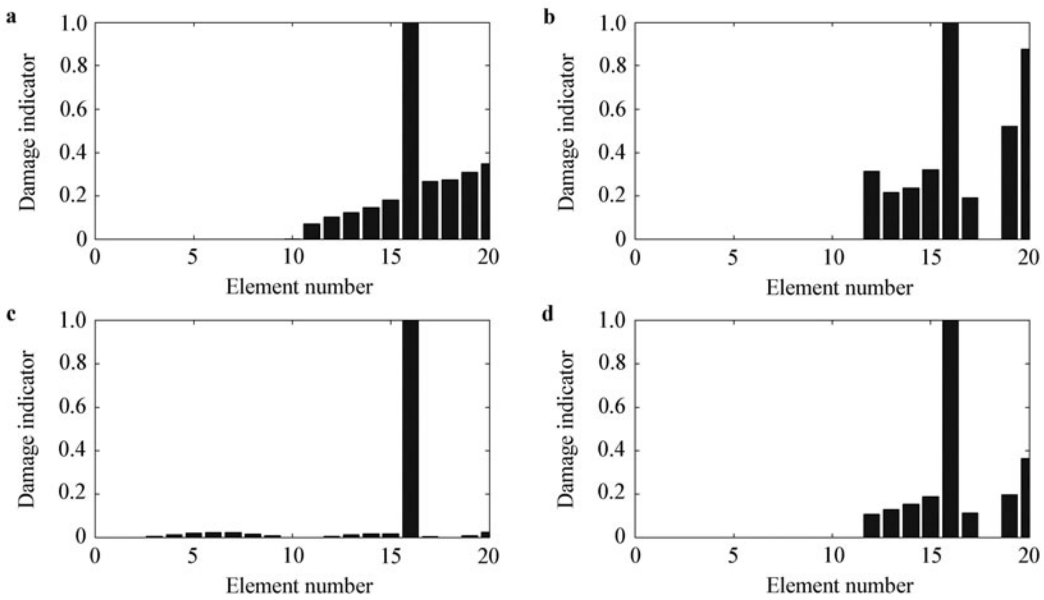

Fig. 10 The damage indicators for damage case 1 when different frequency changes from modal testing are used. **a** From combination (1, 2); **b** From combination (1, 3); **c** From combination (2, 3); **d** From combination (1, 2, 3)

For damage case 2, the cut depth is about 1/2 of the beam thickness and it represents a severe damage case. When implementing the proposed method for damage localization and severity estimation with different frequency combinations, similar results could be obtained, as demonstrated in Fig. 11, from which one can clearly observe that the present method correctly points out the damage locations at Elements 16. The estimated average damage severities corresponding to the frequency combinations (1,2), (1,3), (2,3) and (1,2,3) are −46.66%, −52.62%, −45.98% and −48.42%, respectively. The first three updated modal frequencies using the corresponding average damage severity estimates and the measured values for damage case 2 are listed in Table 4 for comparison. It could be seen that the updated frequencies match well with the measured ones, demonstrating that the estimated damage severities are accurate.

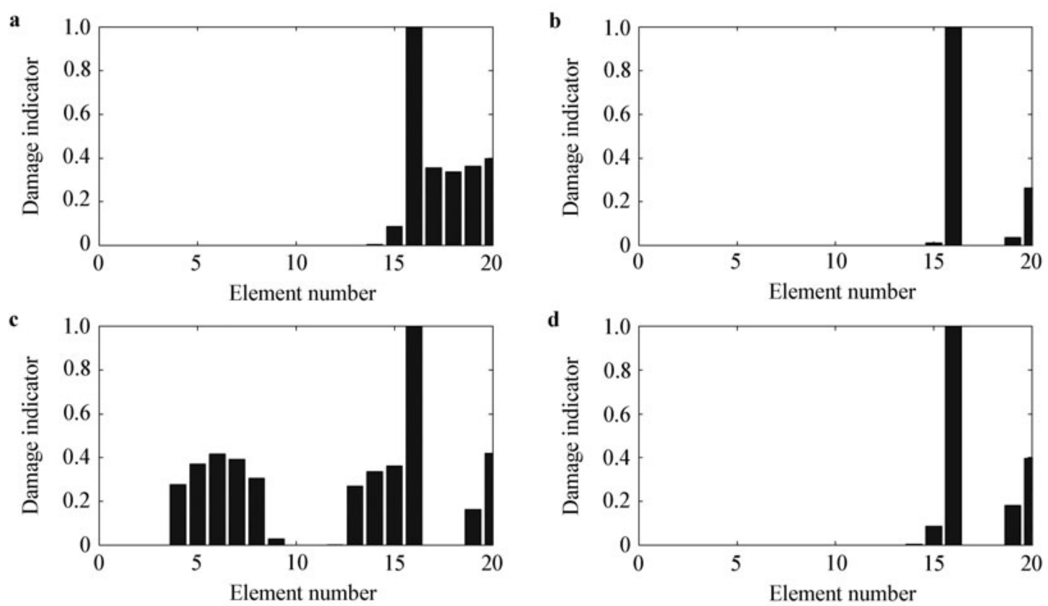

Fig. 11 The damage indicators for damage case 2 when different frequency changes from modal testing are used. **a** From combination (1, 2); **b** From combination (1, 3); **c** From combination (2, 3); **d** From combination (1,2, 3)

Table 4 The first three modal frequencies of the tested beam for damage case 2

(Unit: Hz)

Modes	Measured	Updated			
		(1,2)	(1,3)	(2,3)	(1,2,3)
1st	5.299	5.330	5.276	5.336	5.315
2nd	34.764	34.761	34.757	34.761	34.760
3rd	95.145	95.411	94.936	95.460	95.280

For double damage case 3, the first three identified modal frequencies are 5.289, 33.837 and 91.485 Hz, respectively. Implementing the proposed method for damage severity estimation requires two modal frequency changes, as described in the numerical study. Similar to damage case 2 in the numerical study, one can form four frequency combinations (1,2), (1,3), (2,3) and (1,2,3). By performing the damage localization procedure using frequency combinations (1,2) and (1,3), the damage indicator is found, and shown in Fig. 12. From Fig. 12, it could be clearly observed that the present method correctly point out the damage locations at Elements 7 and 16. When other frequency combinations are employed, a result similar to Fig. 12 has been observed (not shown here). While the correct damage locations are identified, the corresponding damage severity are also accurately estimated. The severity estimates associated with Elements 7 and 16 are, respectively, −45.43% and −50.45% for frequency combination (1,2), and −45.30% and −50.47% for frequency combination (1,3). Again, the first three updated modal frequencies using the damage severity estimates are listed in Table 5 for comparison. Obviously the updated frequencies match very well with the measured counterparts, demonstrating that the estimated damage severities are correct for double damages.

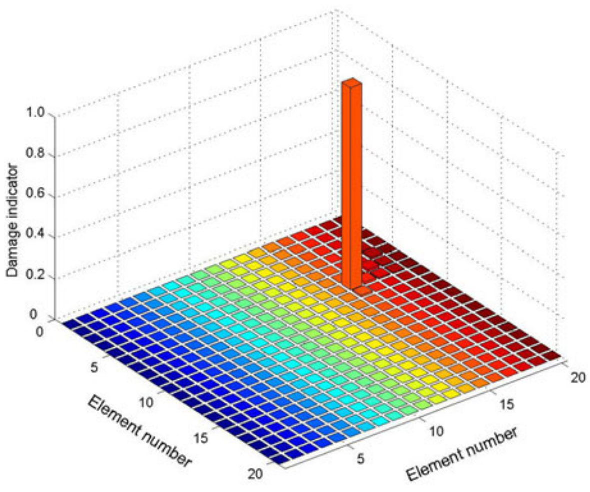

Fig. 12 The damage indicator for damage case 3 when frequency combinations (1,2) and (1,3) from modal testing are used

For investigating the robustness of IMSE method using different combinations of modal frequencies, the damage severity estimation using modal frequency combinations (1, 2), (1, 3), (2, 3) and (1, 2, 3) are conducted for the double damage case. The estimated damage magnitudes are listed in Table 5 and the first three updated modal frequencies with the estimated damage severity are list in Table 6. From Tables 5 and 6, it can obviously be seen that the results are very stable, agreeing well with each other.

Table 5 Severity estimates for different frequency combinations in damage case 3

Damage location	Combination			
	(1, 2)	(1, 3)	(2, 3)	(1, 2, 3)
Element 7	−0.454 3	−0.453 0	−0.454 6	−0.454 5
Element 16	−0.504 5	−0.504 7	−0.499 3	−0.499 6

Table 6 The first three modal frequencies for damage case 3 with different frequency combinations

(Unit: Hz)

Modes	Measured	Updated			
		(1,2)	(1,3)	(2,3)	(1,2,3)
1st	5.288 5	5.290 6	5.290 9	5.287 2	5.287 2
2nd	33.837	33.838	33.838	33.839	33.839
3rd	91.485	91.494	91.496	91.471	91.473

5 Conclusion

A newly developed damage localization and severity estimation method, termed as iterative modal strain energy (IMSE) method, is presented. This method is capable of accurately localizing the damage and estimating the severity of single and double damage scenarios, requiring only the information of a few lower natural frequency changes. First, assuming the damage locations to be known, a damage severity estimation algorithm based on modal strain energy is formulated. To avoid using the mode shapes from the damaged structure, an iterative method is adopted for solving the severity estimates. Second, a damage-localization algorithm is developed based on the fact that only the true damage scenario can lead to the combinations of frequency changes measured from the damaged structure. A damage indicator is formulated, where unity value corresponds to the true damage scenario. Finally, the effectiveness of the developed algorithm is illustrated by numerical study and experimental test data. Both single-damage and multiple-damage scenarios are considered. Numerical simulation and experimental validation demonstrate that excellent results can be achieved when the proposed approach is employed.

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