Acta Mech. Sin. (2012) 28(1):34–40
DOI 10.1007/s10409-012-0014-x

RESEARCH PAPER

Boundary layer flow over a moving surface in a nanofluid with suction or injection

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Received: 28 August 2010 / Revised: 28 March 2011 / Accepted: 10 May 2011

Abstract An analysis is performed to study the heat transfer characteristics of steady two-dimensional boundary layer flow past a moving permeable flat plate in a nanofluid. The effects of uniform suction and injection on the flow field and heat transfer characteristics are numerically studied by using an implicit finite difference method. It is found that dual solutions exist when the plate and the free stream move in the opposite directions. The results indicate that suction delays the boundary layer separation, while injection accelerates it.

Keywords Nanofluid · Moving plate · Boundary layer · Suction/injection · Dual solutions

1 Introduction

It is well known that conventional heat transfer fluids such as water, mineral oil and ethylene glycol have, in general, poor heat transfer properties compared to those of most solids. An innovative way of improving the heat transfer of fluids is to suspend small solid particles in the fluids. This new kind of fluids named as "nanofluids" was introduced in 1995 by Choi [1]. The term nanofluid is used to describe a solid liquid mixture which consists of base liquid with low volume fraction of high conductivity solid nanoparticles. These fluids enhance enormously the thermal conductivity of the base fluid which is beyond the explanation of any existing theory. They are also very stable and have no additional problems, such as sedimentation, erosion, additional pressure drop and non-Newtonian behavior, due to the tiny size of nanoelements and the low volume fraction of nanoelements required for conductivity enhancement. These, with their various potential applications, have recently attracted intensive studies on nanofluids [2–13]. The use of particles of nanometer dimension was first continuously studied by a research group at the Argonne National Laboratory around a decade ago. These suspended nanoparticles can change the transport and thermal properties of the base fluid. The comprehensive references on nanofluids can be found in the recent book [14] and in review papers [15–21].

The aim of the present paper is to extend the work by Rohni et al. [22] to the case where the plate is permeable. The governing partial differential equations are first transformed into a system of ordinary differential equations before being solved numerically. The numerical results obtained are then compared with those reported by Rohni et al. [22] for an impermeable plate. Suction or injection of a fluid through the bounding surface, as, for example, in mass transfer cooling, can significantly change the flow field and, as a consequence, affect the heat transfer rate at the surface. In general, suction tends to increase the skin friction and heat transfer coefficients, whereas injection acts in the opposite manner [23]. Injection of fluid through a porous bounding heated or cooled surface is of general interest in practical application including film cooling, control of boundary

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layer etc. This can lead to enhanced heating (or cooling) of the system and can help to delay the transition from laminar flow [24]. We mention to this end that studies of the boundary layer flows of a Newtonian (or regular) fluid past a permeable static or moving flat plate have been done by Merkin [25], Weidman et al. [26], Ishak et al. [27], Zheng et al. [28], and Zhu et al. [29,30], while Bachok et al. [31,32] have considered the boundary layers over a vertical plate in the regular Newtonian fluid.

2 Problem formulation

Consider a steady two-dimensional boundary layer flow past a moving semi-infinite permeable flat plate in a water based nanofluid containing different type of nanoparticles: Cu, Al_2O_3 and TiO_2. The nanofluid is assumed incompressible and the flow is assumed laminar. It is also assumed that the base fluid (i.e. water) and the nanoparticles are in thermal equilibrium and no slip occurs between them. Further, it is assumed that the plate moves with a constant velocity $\bar{u}_w = \lambda U$, where λ is a constant and U is the constant free stream velocity or the velocity of the far field (inviscid) flow [26]. We consider a Cartesian coordinate system $(\bar{x}, \bar{y})$, where $\bar{x}$ and $\bar{y}$ are the coordinates measured along the plate and normal to it, respectively, and the flow takes place at $\bar{y} \geq 0$. It is also assumed that the constant temperature of the moving surface is T_w and that of the ambient nanofluid is T_∞, where $T_w > T_\infty$ (heated plate).

Using the nanofluid model proposed by Tiwari and Das [13], the basic steady conservation of mass, momentum and energy equations in the coordinates $\bar{x}$ and $\bar{y}$ can be written as

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho_{\mathrm{nf}}}\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\mu_{\mathrm{nf}}}{\rho_{\mathrm{nf}}}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right), \tag{2}$$

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{1}{\rho_{\mathrm{nf}}}\frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\mu_{\mathrm{nf}}}{\rho_{\mathrm{nf}}}\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right), \tag{3}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha_{\mathrm{nf}}\left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right), \tag{4}$$

where we assume that the boundary conditions are

$$\begin{aligned} &\bar{v} = \bar{v}_w, \quad \bar{u} = \bar{u}_w = \lambda U, \quad T = T_w, \quad \text{at} \quad \bar{y} = 0, \\ &\bar{u} = \bar{u}_e = U, \ \ \bar{v} = 0, \ \ T = T_\infty, \ \ \bar{p} = p_\infty, \ \ \text{as} \ \ y \to \infty. \end{aligned} \tag{5}$$

Here $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$ and $\bar{y}$ axes, respectively, T is the temperature of the nanofluid, $\bar{p}$ is the fluid pressure, μ_{nf} is the viscosity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid and ρ_{nf} is the density of the nanofluid, which are given [12]

$$\begin{aligned} &\alpha_{\mathrm{nf}} = \frac{k_{\mathrm{nf}}}{(\rho C_{\mathrm{p}})_{\mathrm{nf}}}, \\ &\rho_{\mathrm{nf}} = (1-\varphi)\rho_{\mathrm{f}} + \varphi\rho_{\mathrm{s}}, \\ &\mu_{\mathrm{nf}} = \frac{\mu_{\mathrm{f}}}{(1-\varphi)^{2.5}}, \\ &(\rho C_{\mathrm{p}})_{\mathrm{nf}} = (1-\varphi)(\rho C_{\mathrm{p}})_{\mathrm{f}} + \varphi(\rho C_{\mathrm{p}})_{\mathrm{s}}, \\ &\frac{k_{\mathrm{nf}}}{k_{\mathrm{f}}} = \frac{(k_{\mathrm{s}} + 2k_{\mathrm{f}}) - 2\varphi(k_{\mathrm{f}} - k_{\mathrm{s}})}{(k_{\mathrm{s}} + 2k_{\mathrm{f}}) + \varphi(k_{\mathrm{f}} - k_{\mathrm{s}})}, \end{aligned} \tag{6}$$

where φ is the nanoparticle volume fraction, $(\rho C_{\mathrm{p}})_{\mathrm{nf}}$ is the heat capacity of the nanofluid, k_{nf} is the thermal conductivity of the nanofluid, k_{f} and k_{s} are the thermal conductivities of the fluid and the solid fractions, respectively, and ρ_{f} and ρ_{s} are the densities of the fluid and the solid fractions, respectively. It should be mentioned that the use of the above expression for k_{nf} is restricted to spherical nanoparticles where it does not account for other shapes of nanoparticles [2]. Also, the viscosity of the nanofluid μ_{nf} has been approximated by Brinkman [33] as viscosity of a base fluid μ_{f} containing dilute suspension of fine spherical particles.

Further, we introduce the boundary layer variables, which are defined as

$$\begin{aligned} &x = \bar{x}/L, \quad y = Re^{1/2}(\bar{y}/L), \quad u = \bar{u}/U, \\ &v = Re^{1/2}(\bar{v}/L), \quad u_{\mathrm{e}} = \bar{u}_{\mathrm{e}}/U, \\ &\theta = (T - T_\infty)/(T_w - T_\infty), \\ &p = (\bar{p} - p_\infty)/(\rho_{\mathrm{f}} U^2), \quad u_w = \bar{u}_w/U, \end{aligned} \tag{7}$$

where L is a characteristic length of the plate, p_∞ is the pressure of the ambient nanofluid and $Re = UL/\nu_{\mathrm{f}}$ is the Reynolds number with ν_{f} being the kinematic viscosity of the nanofliud. Substituting Eq. (7) into Eqs. (1)–(4) and taking into account the boundary layer approximations, and the fact that the present is a flow under zero pressure gradient, we obtain the following dimensionless boundary layer equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{\mathrm{nf}}}{\rho_{\mathrm{nf}}\nu_{\mathrm{f}}}\frac{\partial^2 u}{\partial y^2}, \tag{9}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{\alpha_{\mathrm{nf}}}{\nu_{\mathrm{f}}}\frac{\partial^2 \theta}{\partial y^2}, \tag{10}$$

and the boundary condition equation (5) become

$$\begin{aligned} &u = u_w = \lambda, \quad v = v_w, \quad \theta = 1, \quad \text{at} \quad y = 0, \\ &u = u_{\mathrm{e}} = 1, \quad \theta = 0, \quad \text{as} \quad y \to \infty. \end{aligned} \tag{11}$$

It should also be mentioned that $v_w < 0$ is for suction, $v_w > 0$ is for injection and $v_w = 0$ is for impermeable surface.

We look for a similarity solution of Eqs. (8)–(11) of the following form

$$\eta = y/(2x)^{1/2}, \quad f(\eta) = \psi/(2x)^{1/2}, \quad g(\eta) = \theta, \tag{12}$$

where ψ is the stream function and is defined in the usually way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which identically satisfies Eq. (8). In order that similarity solutions of Eqs. (8)–(11) exist, we take

$$v_w(x) = -\frac{f_0}{(2x)^{1/2}}, \tag{13}$$

where $f_0 = f(0)$ is a non-dimensional constant which determines the transpiration rate, with $f_0 > 0$ for suction, $f_0 < 0$ for injection and $f_0 = 0$ for impermeable surface. It is worth mentioning that $x > 0$ in Eqs. (12) and (13), since the boundary layer does not start at the leading edge $x = 0$, but somewhere upstream of the plate $x > 0$ [34, 35]. Using Eqs. (12) and (13), Eqs. (9) and (10) are reduced to the following ordinary differential equations

$$\frac{f'''}{(1-\varphi)^{2.5}[1-\varphi+\varphi(\rho_s/\rho_f)]} + ff'' = 0, \tag{14}$$

$$\frac{1}{Pr}\frac{g''k_{nf}/k_f}{[1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f]} + fg' = 0, \tag{15}$$

where prime denotes differentiation with respect to η and $Pr = \nu_f/\alpha_f$ is the Prandtl number. The boundary conditions (11) become

$$\begin{aligned} &f(0) = f_0, \quad f'(0) = \lambda, \quad g(0) = 1, \\ &f'(\eta) = 1, \quad g(\eta) = 0, \quad \text{as} \quad \eta \to \infty. \end{aligned} \tag{16}$$

The physical quantities of interest are the skin friction coefficient and the Nusselt number, which are, respectively, defined as

$$C_f = \frac{\tau_w}{\rho_f u_w^2}, \qquad Nu = \frac{L q_w}{k_f(T_w - T_\infty)}, \tag{17}$$

where τ_w is the surface shear stress and q_w is the surface heat flux, which are given by

$$\tau_w = \mu_{nf}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}, \qquad q_w = -k_{nf}\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0}. \tag{18}$$

Substituting Eqs. (7) and (12) into Eq. (17) and using Eq. (18), we get

$$\begin{aligned} &(2Re_x)^{1/2}C_f = \frac{f''(0)}{(1-\varphi)^{2.5}}, \\ &(2/Re_x)^{1/2}Nu = -\frac{k_{nf}}{k_f}g'(0). \end{aligned} \tag{19}$$

Results for $f_0 = 0$ can be compared with those reported by Rohni et al. [19].

3 Results and discussion

Numerical solutions to the nonlinear ordinary differential equations (14) and (15) with the boundary conditions (16) were obtained using the Runge–Kutta–Fehlberg method with shooting technique. This method has been successfully used by the present authors to study various boundary value problems (see for example Ishak et al. [36,37]). We find the missing slopes $f''(0)$ and $g'(0)$, for some values of the governing parameters, namely the nanoparticle volume fraction φ, the moving parameter λ and the suction/injection parameter f_0. Three types of nanoparticles were considered, namely, copper Cu, alumina Al_2O_3, and titania TiO_2. Following Oztop and Abu–Nada [12], the value of Prandtl number Pr is taken as 6.2 (for water) and the volume fraction of nanoparticles is from 0 to 0.2 ($0 \le \varphi \le 0.2$) in which $\varphi = 0$ corresponds to the regular Newtonian fluid. It is worth mentioning that we have used data related to thermophysical properties of the fluid and nanoparticles as listed in Table 1 [12] to compute each case of the nanofluid. The numerical results are summarized in Table 2 and Figs. 1 to 10.

Table 1 Thermophysical properties of fluid and nanoparticles (Oztop and Abu–Nada [12])

Physical properties	Fluid phase (water)	Cu	Al_2O_3	TiO_2
$C_p/(\mathrm{J\cdot kg^{-1}\cdot K^{-1}})$	4 179	385	765	686.2
$\rho/\ (\mathrm{kg\cdot m^{-3}})$	997.1	8 933	3 970	4 250
$k/(\mathrm{W\cdot m^{-1}\cdot K^{-1}})$	0.613	400	40	8.953 8

Table 2 Values of λ_c for different nanoparticles and different values of f_0 when $Pr = 6.2$ and $\varphi = 0.1$

Nanoparticles	f_0	Rohni et al. [22]	Present
Cu	−0.3		−0.165 7
	0	−0.354 1	−0.354 1
	0.3		−0.599 7
Al_2O_3	−0.3		−0.190 3
	0	−0.354 1	−0.354 1
	0.3		−0.559 2
TiO_2	−0.3		−0.188 7
	0	−0.354 1	−0.354 1
	0.3		−0.561 7

Figures 1 and 2 show the variation of $f''(0)$ and $-g'(0)$ with λ for Cu-water nanofluid and different values of f_0 when $Pr = 6.2$ and $\varphi = 0.1$. It is seen that the solution is unique when $\lambda \ge 0$, while dual solutions are found to exist when $\lambda < 0$, i.e. when the plate and the free stream move in the opposite directions. The values of $f''(0)$ are positive when $\lambda < 1$, and they become negatives when the value of λ exceeds 1, for all values of the suction/injection parameter f_0. Physically, positive value of $f''(0)$ means that the fluid exerts a drag force on the plate, and negative value means the opposite. The zero value of $f''(0)$ when $\lambda = 1$ does not mean separation, but it corresponds to the equal velocity of the plate and the free stream. Figures 1 to 6 also show that

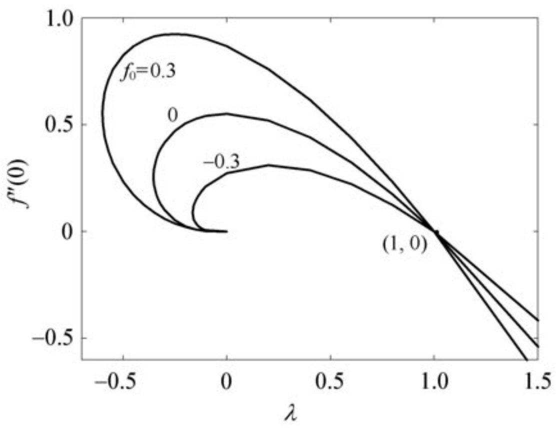

Fig. 1 Variation of $f''(0)$ with λ for Cu-water nanofluid and different values of f_0 when $Pr = 6.2$ and $\varphi = 0.1$

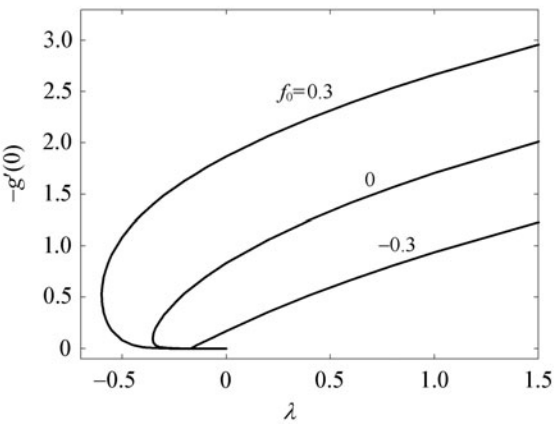

Fig. 2 Variation of $-g'(0)$ with λ for Cu-water nanofluid and different values of f_0 when $Pr = 6.2$ and $\varphi = 0.1$

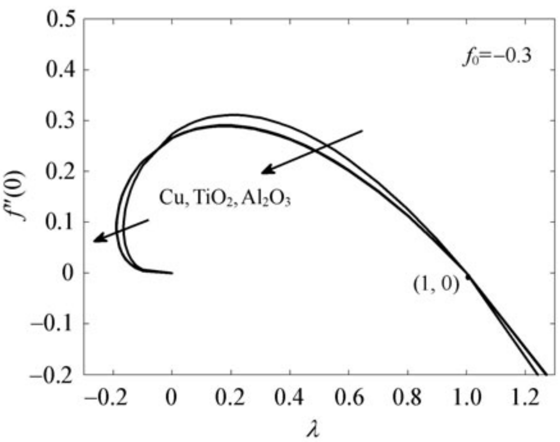

Fig. 3 Variation of $f''(0)$ with λ for different nanoparticles when $f_0 = -0.3$, $Pr = 6.2$ and $\varphi = 0.1$

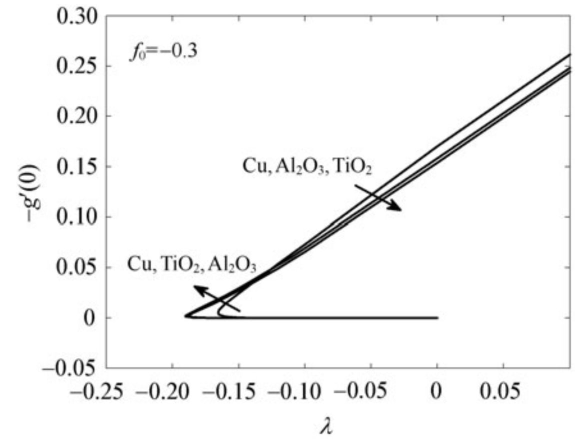

Fig. 4 Variation of $-g'(0)$ with λ for different nanoparticles when $f_0 = -0.3$, $Pr = 6.2$ and $\varphi = 0.1$

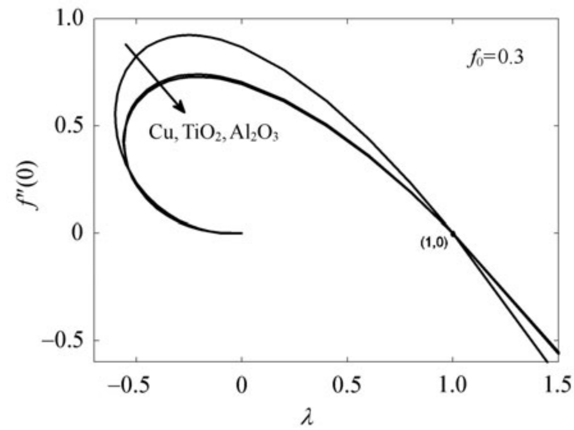

Fig. 5 Variation of $f''(0)$ with λ for different nanoparticles when $f_0 = 0.3$, $Pr = 6.2$ and $\varphi = 0.1$

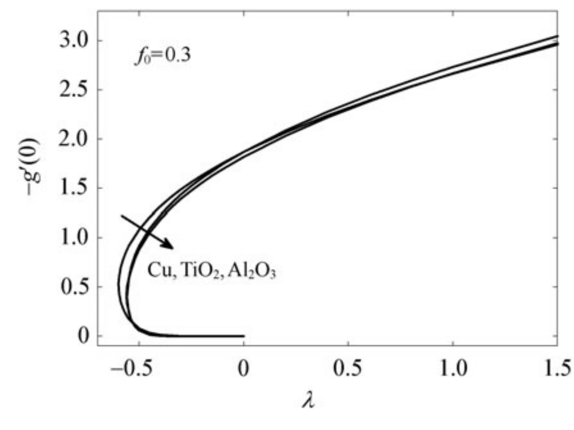

Fig. 6 Variation of $-g'(0)$ with λ for different nanoparticles when $f_0 = 0.3$, $Pr = 6.2$ and $\varphi = 0.1$

for a particular value of f_0, the solution exists up to a certain critical value of λ, say λ_c. Beyond this value, the boundary layer approximations breakdown, and thus the numerical solution can not be obtained. The boundary layer separates from the surface at $\lambda = \lambda_c$, where λ_c denotes a critical value of λ. Based on our computations, the critical value of λ, say λ_c are presented in Table 2, which show a favorable agreement with previous investigations for the case $f_0 = 0$. Moreover, from Table 2, we found that for all nanoparticles the values of $|\lambda_c|$ increase as f_0 increases. Hence, suction delays the boundary layer separation, while injection accelerates it.

Figures 7 and 8 illustrate the variations of the skin friction coefficient and the local Nusselt number, given by Eq. (19) with the nanoparticle volume fraction parameter φ for three different nanoparticles: copper Cu, alumina Al_2O_3, and titania TiO_2 with different values of f_0. One can see that these quantities increase almost monotonically with increasing φ. In addition, it is noted that the lowest heat transfer rate is obtained for nanoparticles TiO_2 due to domination of conduction mode of heat transfer. This is because TiO_2 has the lowest value of thermal conductivity compared to Cu and Al_2O_3, as can be seen from Table 1. This behavior of local Nusselt number is similar to that reported by Oztop and Abu–Nada [12]. However, the difference in the values for Cu and Al_2O_3 is negligible. The thermal conductivity of Al_2O_3 is approximately one tenth for Cu, as given in Table 1. However, a unique property of Al_2O_3 is its low thermal diffusivity. The reduced value of thermal diffusivity leads to higher temperature gradients and, therefore, higher enhancement in heat transfer. The Cu nanoparticles have high values of thermal diffusivity and, therefore, which reduces the temperature gradients and therefore affect the performance of Cu nanoparticles. As volume fraction of nanoparticles increases, the local Nusselt number becomes larger especially in the case of suction. The highest heat transfer rate is recorded for Cu-nanofluid for both cases, suction and injection.

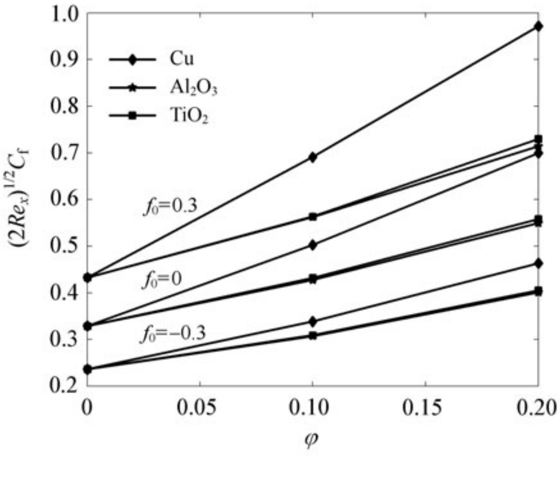

Fig. 7 Variation of skin friction coefficient with φ for different nanoparticles and f_0 when $Pr = 6.2$ and $\lambda = 0.5$

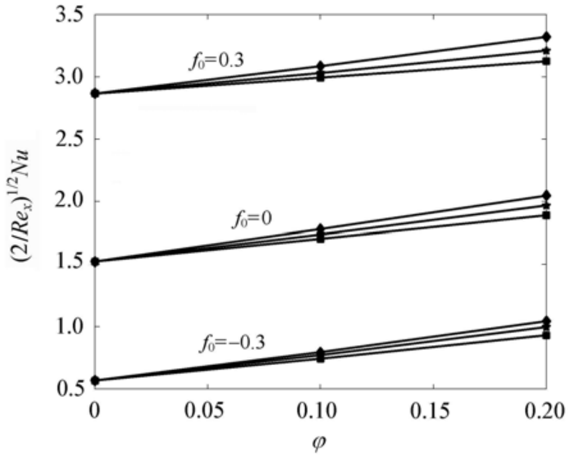

Fig. 8 Variation of the local Nusselt number with φ for different nanoparticles and f_0 when $Pr = 6.2$ and $\lambda = 0.5$

Figures 9 and 10 present the velocity profile $f'(\eta)$ and the temperature profile $g(\eta)$ for Cu-water nanofluid and various values of f_0 when $Pr = 6.2$ and $\lambda = -0.1$. It is seen that all these profiles satisfied asymptotically the far field boundary conditions equation (16). In these figures the solid lines and the dash lines are for the upper and lower branch solutions, respectively. These velocity and temperature profiles support the existence of dual nature of solutions presented in Figs. 1 and 2. The velocity profiles for the upper and lower branch solutions when $\lambda = -0.1$ (Fig. 9) show that, the velocity gradient at the surface is positive, which produces positive value of the skin friction coefficient. The temperature gradient at the surface as shown in Fig. 10 is in agreement with the curves of $-g'(0)$ shown in Fig. 2.

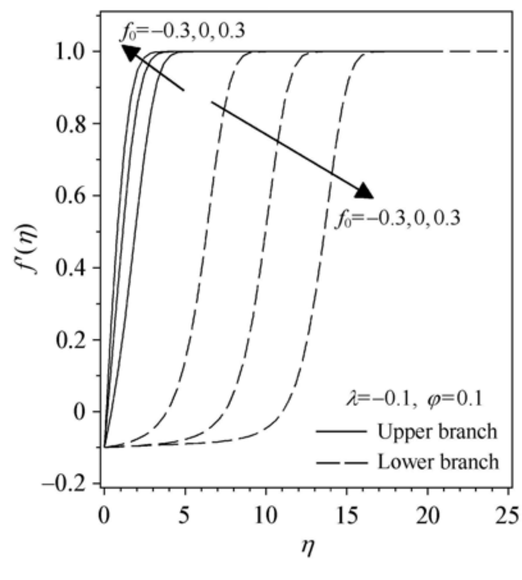

Fig. 9 Velocity profiles for Cu-water nanofluid and different values of f_0 when $Pr = 6.2$ and $\lambda = -0.1$

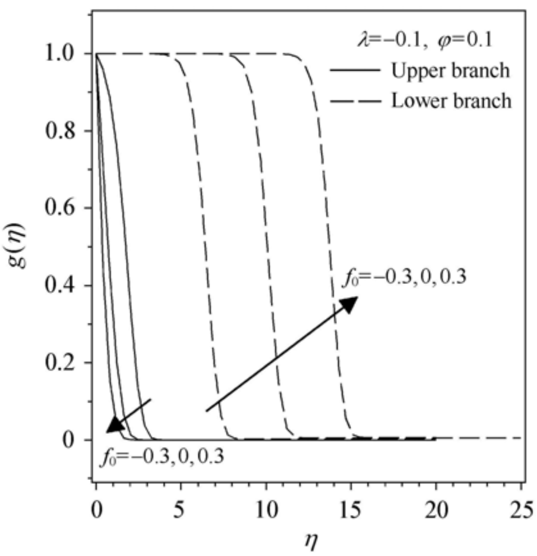

Fig. 10 Temperature profiles for Cu-water nanofluid and different values of f_0 when $Pr = 6.2$ and $\lambda = -0.1$

4 Conclusions

We have theoretically studied the existence of dual similarity solutions in boundary layer flow over a moving surface immersed in a nanofluid with suction and injection effects. The governing boundary layer equations were solved numerically using the Runge–Kutta–Fehlberg method with shooting technique. Discussion were carried out for the effects of suction/injection parameter f_0, the nanoparticle volume fraction φ and the moving parameter λ on the skin friction coefficient $f''(0)$ and the local Nusselt number $-g'(0)$. It was found that dual solutions exist when the plate and the free stream move in the opposite directions. It was also shown that introducing suction is to increase the range of λ for which the solution exists, and in consequence delays the boundary layer separation, while it was found that injection acts in the opposite manner.

Acknowledgements This work was supported by a research grant from the Universiti Kebangsaan Malaysia (Project Code: UKM-GGPM-NBT-080-2010). The authors thank the anonymous reviewers for their valuable comments which led to the improvement of this paper.

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