Acta Mech. Sin. (2011) 27(5):730–737
DOI 10.1007/s10409-011-0491-3

RESEARCH PAPER

Design of piezoelectric energy harvesting devices subjected to broadband random vibrations by applying topology optimization

Zhe-Qi Lin · Hae Chang Gea · Shu-Tian Liu

Received: 18 June 2010 / Revised: 29 April 2011 / Accepted: 18 May 2011

Abstract Converting ambient vibration energy into electrical energy by using piezoelectric energy harvester has attracted a lot of interest in the past few years. In this paper, a topology optimization based method is applied to simultaneously determine the optimal layout of the piezoelectric energy harvesting devices and the optimal position of the mass loading. The objective function is to maximize the energy harvesting performance over a range of vibration frequencies. Pseudo excitation method (PEM) is adopted to analyze structural stationary random responses, and sensitivity analysis is then performed by using the adjoint method. Numerical examples are presented to demonstrate the validity of the proposed approach.

Keywords Topology optimization · Energy harvesting · Piezoelectric material

The project was supported by the National Basic Research Program of China (2011CB610304), the National Science & Technology Major Project (2009ZX04014-034) and the Research Fund for the Doctoral Program of Higher Education of China (20090041110023).

Z.-Q. Lin · S.-T. Liu (✉)
Department of Engineering Mechanics,
Dalian University of Technology,
116024 Dalian, China
e-mail: stliu@dlut.edu.cn

H.C. Gea
Department of Mechanical and Aerospace Engineering,
Rutgers, The State University of New Jersey,
Piscataway, NJ, 08854, USA

1 Introduction

The quest for everlasting energy sources of micro devices has been intensified in the recent years due to the limitations on the applications and deployments of conventional electrochemical power sources arising from their short lifespan. A completely autonomous energy source is particularly advantageous in low power systems with restricted accessibility, such as biomedical implants, remote micro-sensors and wireless devices. The ultimate renewable energy source for micro devices should be equipped with an energy-harvesting mechanism capable of capturing ambient energy and converting it into useable energy. Although a number of harvestable ambient energy sources exist, including waste heat, vibration, wind and solar energy, vibration energy has gained much attention due to its widespread availability. As one of the methods used to convert mechanical vibrational energy to electrical energy, piezoelectric transduction, in contrast to electromagnetic and electrostatic transduction, has attracted much interest. Many theoretical and experimental works are available on modeling and applications of piezoelectric energy harvesters [1–4]. An overview of research in this field has been given by Anton and Sodano [5].

Piezoelectric materials can be configured in different ways to improve the efficiency of piezoelectric power harvesting devices. Better geometric designs may lead to a better utilization of the piezoelectric materials. Goldschmidt-boeing and Woias [6] studied a family of beam shapes ranging from rectangular beams to triangular beams in terms of efficiency and maximum tolerable excitation amplitude, and showed that triangular-shaped beams are more effective than rectangular-shaped ones. A parametric study was performed by Mo et al. [7] to investigate the effects of varied configurations on the produced energy of piezoelectric generators. Lee et al. [8] proposed a stochastic design optimization to determine the optimal configuration of the energy harvester in terms of energy efficiency and durability.

Topology optimization method has been used to determine successfully the optimal design layout in the past two decades. Applications of topology optimization method to the design of piezoelectric devices have been reported, such as piezoelectric transducers [9], piezoelectric actuators [10–12] and piezoelectric resonators [13]. Most recently, topology optimization method is applied to the design of piezoelectric energy harvesting devices. Zheng et al. [14] proposed a topological optimum design to maximize energy conversion of the energy harvesting devices. Topology optimization formulations are developed by Nakasone and Silva [15] to maximize energy conversion of the piezoelectric device and control resonance frequencies of the structure.

Most of previous investigation on the design of energy harvesters considered the systems subject to deterministic excitations. The vibration source is typically modeled by a single harmonic signal; consequently, the energy harvester obtained is tailored to operate only at a single resonant frequency of the driving source. However, the ambient vibration sources where the energy harvester is deployed are always random and broadband in many applications. The performance of the single-frequency energy harvesters will suffer greatly in such an environment.

To address this performance issue, we propose a topology optimization based approach to design piezoelectric energy harvesting devices subjected to stochastic excitations, a more practically available ambient source. The optimal configurations of the piezoelectric energy harvesting devices and the optimal distribution of mass loading are determined simultaneously to maximize the energy harvesting performance from a specific vibration input within a defined range of frequency. In such a situation, the ambient vibration should be described using the theory of random processes and the harvested power should be obtained by using stochastic analysis techniques. In this paper, for convenience in performing sensitivity analysis during the topology optimization process, the pseudo excitation method (PEM) developed by Lin [16] is used for structural stationary random response analysis and the adjoint method is used to evaluate the sensitivity analysis. This paper is organized as follows. Firstly, the energy harvesting optimization formulation is presented. After a brief review of the pseudo excitation method, sensitivity analysis is carried out. Finally, numerical examples are analyzed to validate the proposed method.

2 Problem formulation

In this section, problem formulation is presented for the design of piezoelectric energy harvesting devices under ambient random vibrations. First, a finite element model with coupled mechanical and electrical fields is introduced. Then, the optimization formulation is elucidated for maximizing the energy harvesting performance under a prescribed frequency range.

2.1 Finite element modeling

Applications of piezoelectric materials are always involved in both mechanical and electric fields. A coupled mechanical and electrical finite element formulation is used to model piezoelectric energy harvesting devices.

In the finite element formulation, piezoelectric systems under external excitations and charges can be modeled as

$$\begin{aligned} &\boldsymbol{M}_{uu}\ddot{\boldsymbol{u}} + \boldsymbol{K}_{uu}\boldsymbol{u} + \boldsymbol{K}_{u\varphi}\boldsymbol{\Phi} = \boldsymbol{F}, \\ &\boldsymbol{K}_{\varphi u}\boldsymbol{u} + \boldsymbol{K}_{\varphi\varphi}\boldsymbol{\Phi} = \boldsymbol{Q}, \end{aligned} \tag{1}$$

where $\boldsymbol{K}_{u\varphi} = (\boldsymbol{K}_{\varphi u})^{\mathrm{T}}$ represents the piezoelectric coupling matrix; $\boldsymbol{K}_{uu}$ and $\boldsymbol{K}_{\varphi\varphi}$ denote the structural stiffness and dielectric conductivity matrices; $\boldsymbol{M}_{uu}$ is the structural mass matrix; $\ddot{\boldsymbol{u}}$ denotes displacement acceleration vectors; $\boldsymbol{u}$ and $\boldsymbol{\Phi}$ denote displacement and electric potential vectors; $\boldsymbol{F}$ and $\boldsymbol{Q}$ are the applied force and electric charge vectors, respectively.

Since topology optimization method is used in the design of energy harvesting devices, a material model such as the homogenization method or the solid isotropic materials with penalization (SIMP), should be defined first. For simplicity of derivations, the SIMP model is used to describe the material properties in the design domain as

$$\begin{aligned} &c = \rho(x)^{p_1} c_0, \\ &m = \rho(x) m_0, \\ &e = \rho(x)^{p_2} e_0, \\ &\varepsilon = \rho(x)^{p_2} \varepsilon_0, \end{aligned} \tag{2}$$

where c, m, e and ε denote the stiffness, mass, piezoelectric and dielectric properties, respectively; $\rho(x)$ is the volume density of each element and $0 \le \rho(x) \le 1$; p_1 and p_2 are penalty coefficients to the material.

Note that there is often a mass placed on the piezoelectric beam harvesters in practice. In this paper, the distribution of a non-stiffening mass layer with certain geometry, as well as the layout of the piezoelectric material, is to be determined. We use a topology description function (TDF) to formulate the distribution of the mass layer on the piezoelectric device. Take a circle layer with radius r and center coordinate (x, y) for example, as shown in Fig. 1, the topology description function can be represented as

$$T(x', y') = r - \sqrt{(x'^2 - x^2) + (y'^2 - y^2)}. \tag{3}$$

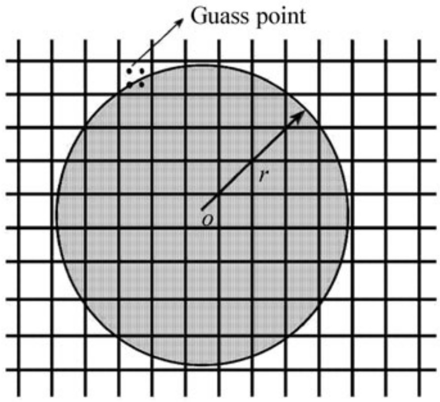

Fig. 1 Schematic diagram of mass loading distribution

Then, the density at an arbitrary point (x', y') can be expressed as

$$\rho_{\text{add}} = \rho_{\text{cir}} H(T(x', y')), \tag{4}$$

where ρ_{cir} is the density of the mass layer, $H(t)$ is modified Heaviside function defined by

$$H(t) = \begin{cases} 0, & t < -\Delta, \\ \dfrac{3}{4}\left(\dfrac{t}{\Delta} - \dfrac{t^3}{3\Delta^3}\right) + \dfrac{1}{2}, & -\Delta \le t < \Delta, \\ 1, & t \ge \Delta, \end{cases} \tag{5}$$

where Δ is a small positive number and 2Δ denotes the width of the "gray density" in boundary; $T(x', y') < -\Delta$ represents that point (x', y') is outside the mass layer; $-\Delta \le T(x', y') < \Delta$ represents that it is on the boundary; $T(x', y') > \Delta$ represents it is inside the mass layer.

2.2 Energy harvesting optimization

To measure the performance of an energy harvester under random vibration as shown in Fig. 2, we define an average harvested power in the time domain as

$$E[P(t)] = E\left[\frac{v^2(t)}{R}\right], \tag{6}$$

where $E[\cdot]$ is the expectation operator; $P(t)$ is the harvested power; $v(t)$ is the voltage; R is the resistance.

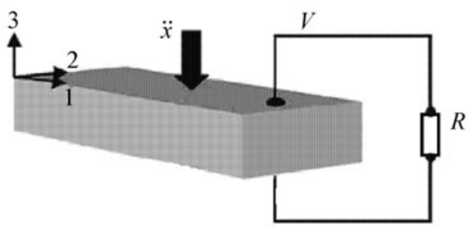

Fig. 2 Schematic diagram of a piezoelectric energy harvester

For a prescribed frequency range, $f_1 \le f \le f_2$, the average power in the frequency domain can be given as

$$E[P(t)] = \frac{E[v^2(t)]}{R} = \frac{1}{R}\int_{\omega_1}^{\omega_2} S_{\text{vv}}(\omega)\mathrm{d}\omega, \tag{7}$$

where $\omega_1 = 2\pi f_1$, $\omega_2 = 2\pi f_2$, $S_{\text{vv}}(\omega)$ represents auto power spectral density (auto-PSD) of output voltage.

Then, we can formulate the topology optimization problem as to maximize the average harvested power within the prescribed frequency range with a given amount of piezoelectric materials

$$\begin{aligned} &\underset{\rho, x}{\text{Max}} && E[P(t)] = \frac{1}{R}\int_{\omega_1}^{\omega_2} S_{\text{vv}}(\omega)\mathrm{d}\omega, \\ &\text{Subject to} && \sum_{i=1}^{N} \rho_i \Omega_i \le \overline{\Omega}, \\ &&& 0 \le \rho_i \le 1, \quad i = 1, 2, \cdots, N, \\ &&& \underline{x} \le x \le \overline{x}, \end{aligned} \tag{8}$$

where ρ is the volume density of each element; x is the center coordinate of the mass layer; Ω_i is the volume of the i-th element; $\overline{\Omega}$ is the upper bound of the material volume; N is total number of design variables; $\underline{x}$ and $\overline{x}$ are the lower bound and upper bound of the coordinate, respectively.

3 PEM for coupled piezoelectric system

In the proposed optimization problem, the structural random response analysis needs to be carried out for getting the objective function. Conventional methods including complete quadratic combination (CQC) method and square root of the sum of squares (SRSS) method are applicable for structural random response analysis. However, when these methods are used for the present problem, the sensitivity analysis becomes very cumbersome during the topology optimization process. Therefore, an efficient algorithm for random response analysis named pseudo excitation method (PEM) is used for the coupled piezoelectric system in this paper.

3.1 Structural response by using PEM

Consider a structure subjected to a single random excitation, its equation of motion is

$$\boldsymbol{M}\ddot{\boldsymbol{y}} + \boldsymbol{K}\boldsymbol{y} = \boldsymbol{p}x(t), \tag{9}$$

in which $x(t)$ is a stationary random process with a power spectrum density of $S_{xx}(\omega)$ and $\boldsymbol{p}$ is a given constant vector. The structural stationary random response can be obtained by using PEM. Let $x(t) = \sqrt{S_{xx}(\omega)}\,\mathrm{e}^{\mathrm{i}\omega t}$, then Eq. (9) becomes the following harmonic equation

$$\boldsymbol{M}\ddot{\boldsymbol{y}} + \boldsymbol{K}\boldsymbol{y} = \boldsymbol{p}\ \sqrt{S_{xx}(\omega)}\,\mathrm{e}^{\mathrm{i}\omega t}. \tag{10}$$

Its stationary solution is

$$\boldsymbol{y}(t) = \boldsymbol{Y}(\omega)\,\mathrm{e}^{\mathrm{i}\omega t}. \tag{11}$$

Using its first q modes for mode-superposition, then

$$\boldsymbol{Y}(\omega) = \sum_{j=1}^{q} \gamma_j H_j \boldsymbol{\psi}_j \sqrt{S_{xx}(\omega)}, \tag{12}$$

$$H_j = (\omega_j^2 - \omega^2)^{-1}, \quad \gamma_j = \boldsymbol{\psi}_j^{\mathrm{T}} \boldsymbol{p}\,, \tag{13}$$

in which ω_j and $\boldsymbol{\psi}_j$ are the j-th natural angular frequency and modal shape, respectively; γ_j is mode-participation factor. According to the PEM, the PSD matrix of $\boldsymbol{y}$ would be

$$\boldsymbol{S}_{yy}(\omega) = \boldsymbol{y}^* \boldsymbol{y}^{\mathrm{T}} = \sum_{j=1}^{q}\sum_{k=1}^{q} \gamma_j \gamma_k H_j^* H_k \boldsymbol{\psi}_j \boldsymbol{\psi}_k^{\mathrm{T}} S_{xx}(\omega). \tag{14}$$

This is well-known CQC formula [16,17]. It involves the cross-correlation terms between all participant modes. Therefore, it usually requires considerable computational efforts. The comparisons of efficiency for PEM and conventional CQC were detailed in Ref. [18].

3.2 PEM for coupled piezoelectric system

For a piezoelectric structure subjected to a base acceleration $\ddot{x}$ excitations with a power spectrum density of $S_{\ddot{x}}(\omega)$, the auto-PSD function of voltage $S_{\mathrm{vv}}(\omega)$ can be obtained by using pseudo excitation method. Form a pseudo acceleration excitation by using power spectrum density $S_{\ddot{x}}(\omega)$,

$$\ddot{x} = \sqrt{S_{\ddot{x}}(\omega)}\,\mathrm{e}^{\mathrm{i}\omega t}, \tag{15}$$

then the external force has a harmonic form of $\boldsymbol{F} = \boldsymbol{M}_{uu}\boldsymbol{E}\sqrt{S_{\ddot{x}}(\omega)}\,\mathrm{e}^{\mathrm{i}\omega t}$, $\boldsymbol{E}$ denotes the direction of the acceleration excitation. The structural responses can be expressed as $\boldsymbol{u} = \boldsymbol{u}_{\mathrm{e}}\,\mathrm{e}^{\mathrm{i}\omega t}$ and $\boldsymbol{\Phi} = \boldsymbol{\Phi}_{\mathrm{e}}\,\mathrm{e}^{\mathrm{i}\omega t}$, where ω is the excitation frequency. Introducing these expressions to Eq. (1) yields the following frequency response equations

$$\begin{aligned} &-\omega^2 \boldsymbol{M}_{uu}\boldsymbol{u}_{\mathrm{e}} + \boldsymbol{K}_{uu}\boldsymbol{u}_{\mathrm{e}} + \boldsymbol{K}_{u\varphi}\boldsymbol{\Phi}_{\mathrm{e}} = \boldsymbol{M}_{uu}\boldsymbol{E}\sqrt{S_{\ddot{x}}(\omega)},\\ &\boldsymbol{K}_{\varphi u}\boldsymbol{u}_{\mathrm{e}} + \boldsymbol{K}_{\varphi\varphi}\boldsymbol{\Phi}_{\mathrm{e}} = \boldsymbol{0}\,. \end{aligned} \tag{16}$$

The structural response can be solved from the above system by using the model superposition method. To extract the output voltage from the potential vector, a unity filter vector, Ψ, in the form of $[0 \;\; 0 \;\; \cdots \;\; 1 \;\; \cdots \;\; 0]^{\mathrm{T}}$ with the unity entry corresponding to the node of interests is used. In this way, the voltage generated by the device is expressed as

$$v_{\mathrm{e}} = \boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{\Phi}_{\mathrm{e}}. \tag{17}$$

The auto-PSD function of voltage can be obtained according to pseudo excitation method,

$$S_{\mathrm{vv}}(\omega) = v_{\mathrm{e}}^* v_{\mathrm{e}} = v_{\mathrm{e}}^2, \tag{18}$$

where the asterisk "$*$" represents complex conjugate.

4 Sensitivity analysis

The sensitivity of S_{vv} with respect to design variable x_i can be obtained by differentiating Eq. (18) as

$$\frac{\partial S_{\mathrm{vv}}}{\partial x_i} = 2 v_{\mathrm{e}} \frac{\partial v_{\mathrm{e}}}{\partial x_i}, \tag{19}$$

where v_{e} is the potential response when a pseudo excitation is applied to the structure. The sensitivity of potential response, $\partial v_{\mathrm{e}}/\partial x_i$, can be derived by using the adjoint method.

A new function is defined by adding the governing equations to the voltage response as follows

$$\begin{aligned} v_{\mathrm{e}} = {} & \boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{\Phi}_{\mathrm{e}} + \boldsymbol{\alpha}^{\mathrm{T}}[-\omega^2\boldsymbol{M}_{uu}\boldsymbol{u}_{\mathrm{e}} + \boldsymbol{K}_{uu}\boldsymbol{u}_{\mathrm{e}} + \boldsymbol{K}_{u\varphi}\boldsymbol{\Phi}_{\mathrm{e}} \\ & -\boldsymbol{M}_{uu}\boldsymbol{E}\sqrt{S_{\ddot{x}}(\omega)}] + \boldsymbol{\beta}^{\mathrm{T}}(\boldsymbol{K}_{\varphi u}\boldsymbol{u}_{\mathrm{e}} + \boldsymbol{K}_{\varphi\varphi}\boldsymbol{\Phi}_{\mathrm{e}}), \end{aligned} \tag{20}$$

where $\boldsymbol{\alpha}, \boldsymbol{\beta}$ are arbitrary adjoint displacement and potential vectors. Taking derivative of Eq. (20), the sensitivity of the potential response with respect to design variable x_i can be written as

$$\begin{aligned} \frac{\partial v_{\mathrm{e}}}{\partial x_i} = {} & \boldsymbol{\alpha}^{\mathrm{T}}\Big[-\omega^2\frac{\partial \boldsymbol{M}_{uu}}{\partial x_i}\boldsymbol{u}_{\mathrm{e}}^{+}\frac{\partial \boldsymbol{K}_{uu}}{\partial x_i}\boldsymbol{u}_{\mathrm{e}}^{+}\frac{\partial \boldsymbol{K}_{u\varphi}}{\partial x_i}\boldsymbol{\Phi}_{\mathrm{e}} \\ & -\frac{\partial \boldsymbol{M}_{uu}}{\partial x_i}\boldsymbol{E}\sqrt{S_{\ddot{x}}(\omega)}\Big] + \boldsymbol{\beta}^{\mathrm{T}}\Big(\frac{\partial \boldsymbol{K}_{\varphi u}}{\partial x_i}\boldsymbol{u}_{\mathrm{e}} + \frac{\partial \boldsymbol{K}_{\varphi\varphi}}{\partial x_i}\boldsymbol{\Phi}_{\mathrm{e}}\Big) \\ & +(-\boldsymbol{\alpha}^{\mathrm{T}}\omega^2\boldsymbol{M}_{uu} + \boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{K}_{uu} + \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{K}_{u\varphi})\frac{\partial \boldsymbol{u}_{\mathrm{e}}}{\partial x_i} \\ & +(\boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{K}_{u\varphi} + \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{K}_{\varphi\varphi} + \Psi^{\mathrm{T}})\frac{\partial \boldsymbol{\Phi}_{\mathrm{e}}}{\partial x_i}. \end{aligned} \tag{21}$$

By defining and solving the adjoint system as the following coupled system for the adjoint displacement and potential vectors, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$

$$\begin{aligned} &-\omega^2\boldsymbol{M}_{uu}\boldsymbol{\alpha} + \boldsymbol{K}_{uu}\boldsymbol{\alpha} + \boldsymbol{K}_{u\varphi}\boldsymbol{\beta} = \boldsymbol{0}\,,\\ &\boldsymbol{K}_{\varphi u}\boldsymbol{\alpha} + \boldsymbol{K}_{\varphi\varphi}\boldsymbol{\beta} = -\Psi, \end{aligned} \tag{22}$$

a very simple form of the sensitivity of the potential response can be obtained from Eq. (21) and it can be readily solved with the SIMP formulation.

5 Numerical example

Optimal designs of piezoelectric energy harvesters subjected to random acceleration excitations are presented in this section using the proposed topology optimization method. PZT is employed for the piezoelectric layers and they are bonded to the top and bottom of an elastic layer. The size of each layer is 20 mm × 10 mm × 0.25 mm. The piezoelectric layers are considered as design domain and their material properties are given in Table 1. The elastic layer is made of Aluminum with Young's modulus of 71 GPa, Poisson's ratio of 0.33 and density of 2 700 kg/m^3. A non-stiffening mass layer is placed on the device with radius of 0.002 m, thickness of 0.002 m and density of 7 800 kg/m^3. For the objective of maximizing the energy harvesting performance, the optimal layout of the piezoelectric material and location of the mass layer are to be determined simultaneously. The upper bound of volume constraint is set to be 40% of the total volume of the design domain. The resistance is set as 10 kΩ and keeps constant during the optimization process.

In the finite element model, 8-node brick element is used and equal-potential constraints are applied to the nodes on the electrodes. For the sake of manufacturability, the configurations of the structures at the top and bottom piezoelectric layers are assumed to be the same in the optimization process. It is assumed that the ambient base excitation is stationary white noise, which has a constant power spectral density across the frequency range considered.

Table 1 Material properties of PZT [19]

C_{11}	115.65 GPa	e_{13}	−12.31 C/m^2
C_{12}	64.89 GPa	e_{23}	−12.31 C/m^2
C_{13}	62.29 GPa	e_{33}	20.76 C/m^2
C_{33}	92.98 GPa	e_{52}	17.04 C/m^2
C_{44}	17.86 GPa	e_{61}	17.04 C/m^2
C_{55}	17.86 GPa	ε_{11}	8.93 nF/m
C_{66}	17.86 GPa	ε_{22}	8.93 nF/m
ρ	7 640 kg/m^3	ε_{33}	6.92 nF/m

5.1 Example 1

In the first case, a ground acceleration excitation is applied to the piezoelectric bimorph generator, as shown in Fig. 3. This model is modified from the piezoelectric bimorph generator by Roundy et al. [20]. The left end of the structure is fixed and the right end is free. A band of excitation frequencies ranged from 10 Hz to 400 Hz, which is lower than the first order natural frequency of the structure, as shown in Fig. 4, is applied to the piezoelectric bimorph generator.

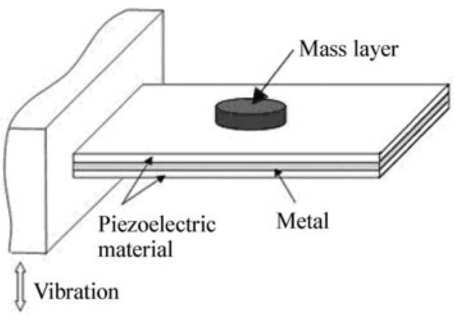

Fig. 3 Schematic of a piezoelectric bimorph generator mounted as a cantilever

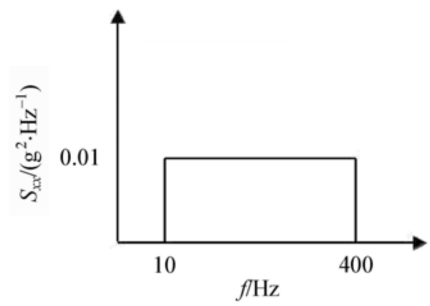

Fig. 4 Band limited white noise PDS loading

A 40 by 20 finite element model is built for each layer and the proposed methodology is used to design the piezoelectric energy harvester. Figure 5 shows the iteration history of the optimization process. The optimal location of the mass layer and the optimal layout of the piezoelectric material are showed in Figs. 6 and 7. We can see that the mass layer is placed at the free end of the device. That is because the distribution of the mass layer on the free end helps to tune the structure to the low driving frequencies. In addition, largest deformation can be generated when the mass loading distributes on the free end. From Fig. 7 we can see that the piezoelectric material tends to distribute to the supporting area and the free end. Intermediate design variable appear in the supporting area. This is because the strain in these areas is greater, as shown in Fig. 8, and thus piezoelectric materials should be placed at the supporting area to produce more electric energy. However, these additional piezoelectric materials also change the stiffness and mass of the structure thereby changing the structural dynamic response. Adding piezoelectric material to the supporting area increases the natural frequency of the structure and decreases the deformation. Since the excitation frequencies are lower, the piezoelectric materials tend to distribute at the free end and consequently decrease the natural frequency. Furthermore, the piezoelectric materials placed at the free end also increase the moment induced by the gravity, which help to produce more electric energy.

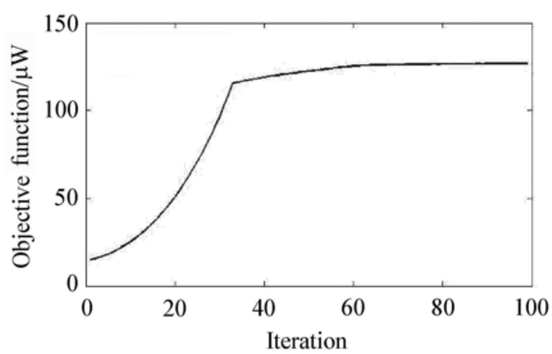

Fig. 5 Iteration history of the optimization design of the piezoelectric generator

Fig. 6 Optimal location of the mass layer under a band of excitation frequencies ranged from 10 Hz to 400 Hz

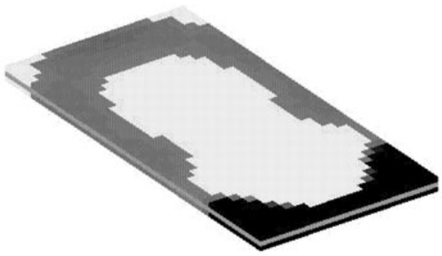

Fig. 7 Optimal layout of the piezoelectric material under a band of excitation frequencies ranged from 10 Hz to 400 Hz

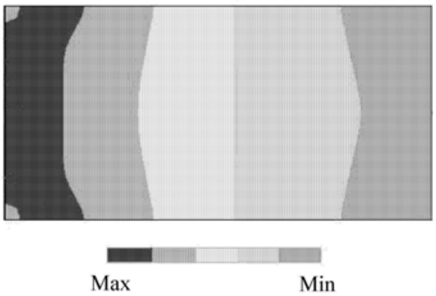

Fig. 8 Strain distribution of the structure without piezoelectric layer under gravity load

5.2 Example 2

In the second example, a band of higher excitation frequencies is applied to the same piezoelectric bimorph generator as the previous example. The frequencies of external excitations are chosen from 1 800 Hz to 2 200 Hz. The iteration history of simultaneously optimizing location of mass layer and distribution of piezoelectric material is showed in Fig. 9. The optimal results are showed in Figs. 10 and 11. In this example, the optimal location of mass layer shows a compromise between tuning the structure to the driving frequencies and changing the moment of the mass loading. A U-shape distribution of piezoelectric materials is generated, which is similar to part of the previous design in the same area. However, no piezoelectric material is placed at the free end. This is because the excitation frequencies are higher in this example and the optimized design should increase its natural frequency by moving piezoelectric materials to the support area rather than to the free end. As mentioned in the previous example, the strain is greater in the support area, so the piezoelectric materials distributed here is also helpful to generate more electric energy.

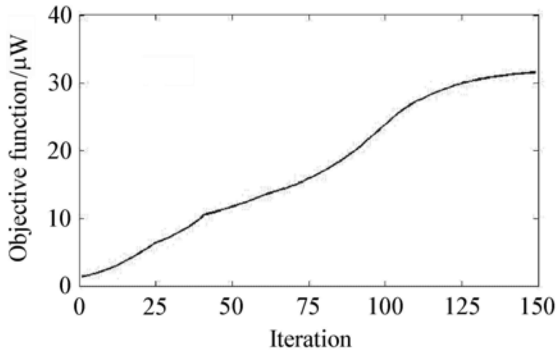

Fig. 9 Iteration history of the optimization design of the piezoelectric generator

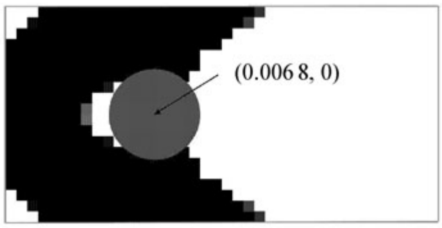

Fig. 10 Optimal location of the mass layer under a band of excitation frequencies ranged from 1 800 Hz to 2 200 Hz

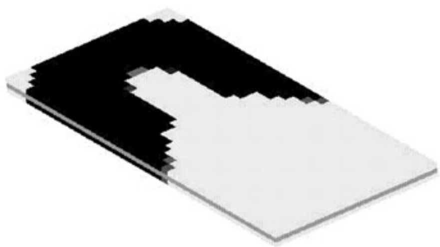

Fig. 11 Optimal layout of piezoelectric material under a band of excitation frequencies ranged from 1 800 Hz to 2 200 Hz

5.3 Example 3

An energy generator model modified from Roundy et al. [21] is taken as the third example. The piezoelectric bimorph is simply supported at four corners and a mass layer is placed on the structure as shown in Fig. 12. A ground acceleration excitation is applied to the piezoelectric bimorph generator with a band of excitation frequencies ranged from 10 Hz to 400 Hz. The optimal configuration of the piezoelectric material and the optimal location of the mass layer are determined by using the present method. The iteration history of the objective function is shown in Fig. 13. The optimal position of the mass layer is located at the center of the structure, as shown in Fig. 14. The final configuration of piezoelectric material is shown in Fig. 15. Figure 16 plots the strain distribution of the simply supported plate without piezoelectric layer under gravity load. Comparing Figs. 15 with 16, one can easily find that the piezoelectric materials tend to be placed at the area with higher strain.

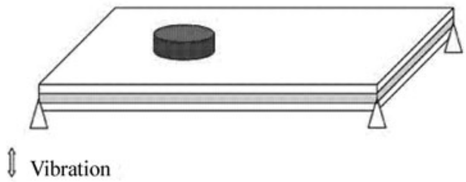

Fig. 12 Schematic of a piezoelectric bimorph generator simply supported

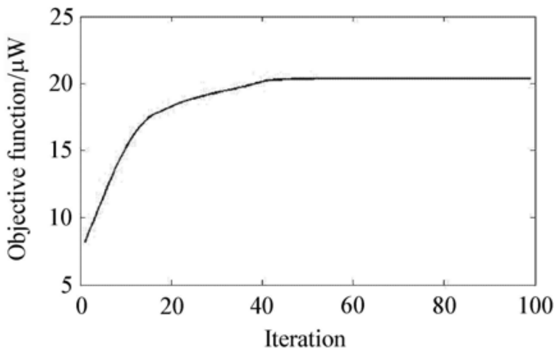

Fig. 13 Iteration history of the optimization design of the piezoelectric generator

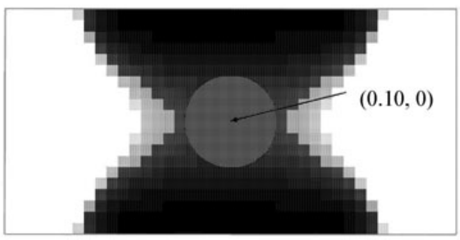

Fig. 14 Optimal location of the mass layer under a band of excitation frequencies ranged from 10 Hz to 400 Hz

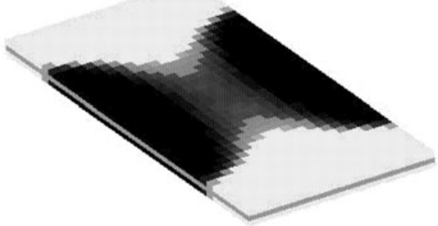

Fig. 15 Optimal layout of the piezoelectric material under a band of excitation frequencies ranged from 10 Hz to 400 Hz

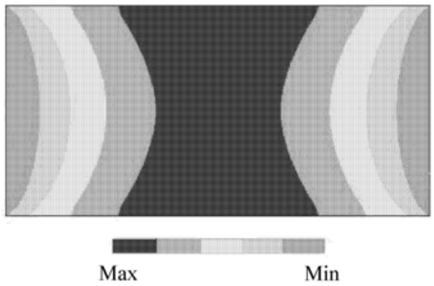

Fig. 16 Strain distribution of the simply supported plate without piezoelectric layer under gravity load

6 Conclusion

In this paper, the design of piezoelectric energy harvesting devices under stochastic excitation is studied. A topology optimization based method is proposed to simultaneously determine the layout of piezoelectric material and location of mass layer. The PEM is used to analyze structural stationary random responses. The energy harvester obtained is designed to maximize energy harvesting performance within a range of ambient excitations. The adjoint method is used to derive the sensitivity. A few numerical examples are given to demonstrate the effectiveness of the proposed method.

References

1 Roundy, S., Wright, P.K., Rabaey, J.: A study of low level vibrations as a power source for wireless sensor nodes. Computer Communications **26**(11), 1131–1144 (2003)
2 Lu, F., Lee, H. P., Lim, S.P.: Modeling and analysis of micro piezoelectric power generators for micro-electromechanical-systems applications. Smart Materials and Structures **13**(1), 57–63 (2004)
3 Sodano, H.A., Lloyd, J., Inman, D.J.: An experimental comparison between several active composite actuators for power generation. Smart Materials and Structures **15**(5), 1211–1216 (2006)
4 Erturk, A., Inman, D.J.: A distributed parameter electromechanical model for cantilevered piezoelectric energy harvesters. Journal of Vibration and Acoustics **130**(4), 041002 (2008)
5 Anton, S.R., Sodano, H.A.: A review of power harvesting using piezoelectric materials (2003–2006). Smart Materials and Structures **16**(3), R1–R21 (2007)
6 Goldschmidtboeing, F., Woias, P.: Characterization of different beam shapes for piezoelectric energy harvesting. Journal of Micromechanics and Microengineering **18**(10), 104013 (2008)
7 Mo, C., Kim, S., Clark, W.W.: Theoretical analysis of energy harvesting performance for unimorph piezoelectric benders with interdigitated electrodes. Smart Materials and Structures **18**(5), 055017 (2009)
8 Lee, S., Youn, B.D., Jung, B.C.: Robust segment-type energy harvester and its application to a wireless sensor. Smart Materials and Structures **18**(9), 095021 (2009)

9 Silva, E.C.N., Kikuchi, N.: Design of piezocomposite materials and piezoelectric transducers using topology optimization – part III. Archives of Computational Methods in Engineering **6**(4), 305–329 (1999)
10 Buehler, M.J., Bettig, B., Parker, G.G.: Topology optimization of smart structures using a homogenization approach. Journal of Intelligent Material Systems and Structure **15**(8), 655–667 (2004)
11 Kögl, M., Silva, E.C.N.: Topology optimization of smart structures: design of piezoelectric plate and shell actuators. Smart Materials and Structures **14**(2), 387–399 (2005)
12 Carbonari, C.C., Silva, E.C.N., Nishiwaki, S.: Optimum placement of piezoelectric material in piezoactuator design. Smart Materials and Structures **16**(1), 207–220 (2007)
13 Ha, Y., Cho, S.: Design sensitivity analysis and topology optimization of eigenvalue problems for piezoelectric resonators. Smart Materials and Structures **15**(6), 1513–1524 (2006)
14 Zheng, B., Chang, C.J., Gea, H.C.: Topology optimization of energy harvesting devices using piezoelectric materials. Struct Multidisc Optim. **38**(1), 17-23 (2009)
15 Nakasone P.H., Silva, E.C.N.: Design of piezoelectric energy harvesting devices and laminate structures by applying topology optimization. In: Proc. of SPIE **7286**, 728603 (2009)
16 Lin, J.H.: A fast CQC algorithm of PSD matrices for random seismic responses. Computers and Structure **44**(3), 683–687 (1992)
17 Clough, R.W., Penzien, J.: Dynamics of Structures. (2nd edn.) McGraw-Hill, New York (1994)
18 Lin, J., Zhao, Y., Zhang, Y.: Accurate and highly efficient algorithms for structural stationary/non-stationary random responses. Computer Methods in Applied Mechanics and Engineering **191**(1-2), 103–111 (2001)
19 Zhu, X.H., Yu, Z.X.: ANSYS Finite Element Analysis of Advanced Engineering Sample Selection. Publishing House of Electronics Industry, Beijing (2004) (in Chinese)
20 Roundy, S., Wright, P.K.: A piezoelectric vibration based generator for wireless electronics. Smart Materials and Structures **13**(5), 1131–1142 (2004)
21 Roundy, S., Leland, E.S., Baker, J., et al.: Improving power output for vibration-based energy scavengers. IEEE Pervasive Computing **4**(1), 28–36 (2005)