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RESEARCH PAPER

Rarefied gas flow through a rough channel into a vacuum

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Abstract
Rarefied gas flow through a rough channel of finite length into a vacuum was investigated using the direct simulation Monte Carlo method. The non-equilibrium effects at the input and output of the channel were considered by including certain pre- and post-channel regions into the geometry under consideration. The mass flow rate through a short and long channel was computed in a wide range of gas rarefaction from the free molecular regime to the hydrodynamic near. It is shown that the noticeable effect of surface roughness on flow rate is manifested in the free molecular and transition regimes. The analysis is provided for the flow field inside the channel as well as in the upstream and downstream regions. The results obtained are consistent with the theoretical, numerical, and experimental data available in the open literature.

Keywords Rarefied gas · Surface roughness · Gas flow rate · Direct simulation Monte Carlo method

1 Introduction

An important area of the rarefied gas dynamics, both in terms of theory and practice, is the study of the internal rarefied flow (Sharipov and Seleznev 1998). Theoretically, this problem is particularly interesting since almost all flow regimes, from hydrodynamic to free molecular, can be present in a flow field, e.g. during an outflow of a sufficiently dense gas into a vacuum. Of practical interest is the research of gas flows in various micro- and nanoelectromechanical systems (MEMS/NEMS), such as microturbines, micro- and nanoseparators, micropumps, microgyroscopes, thermal flow micro-sensors, micro heat pipes, micro total analysis systems (or lab-on-a-chip systems), etc. (Gad-el-Hak 2001; Li 2008; Beeby et al. 2004).

Indeed, in some cases, the geometrical dimensions of operating components are such that the structure of the surface and interfaces of a device plays a decisive role in its operation. For example, the surface of micro- and nanoelectromechanical systems can range from sufficiently smooth, as in the case of a crystalline silicon surface, to technical with a fairly substantial roughness. In such devices, the rarefied gas flow through a straight channel is the most common configuration.

The effect of surface roughness on the free molecular gas flow in cylindrical channels (tubes) was first fully investigated in Davis et al. (1964) by setting up an experiment on channels with different surface structures as well as by numerical task modeling using a test particle Monte Carlo method. The authors specifically showed that, except for a very smooth surface, surface roughness reduces the conductance below that calculated with the assumption of a diffuse reflection from smooth walls. The increase in conductance for a smooth glass surface can perhaps be explained by a specular reflection.

Using a direct simulation Monte Carlo (DSMC) method, Usami et al. (1989) studied rarefied gas flow through a 2D channel with a surface roughness in the form of triangular grooves for different Knudsen numbers. The reduction of flow conductivity caused by surface roughness was obtained in the free molecular and transition regimes. The authors conclude that the surface roughness can hardly affect the mass flow when the inverse Knudsen number $1/\mathrm{Kn} > 100$ and the fineness of the grooves does not seem to affect the mass flow.

Sugiyama et al. (1996, 2001) investigated the effects of surface roughness on the rarefied gas flow through channels experimentally and numerically by the DSMC method. The surface roughness is modeled by two-dimensional triangular waves of constant amplitude (Sugiyama et al. 1996) and

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circular cones with the same base radius (Sugiyama et al. 2001). The obtained results well predict the measured reduction of free-molecule and near-free molecule flow conductance through the passages composed of two flat plates.

Inter alia, Sun and Faghri (2003) used the DSMC method to investigate the effects of surface roughness on nitrogen flow in a microchannel. The surface roughness was modeled by an array of rectangular modules placed on two surfaces of a parallel plate channel. The effects of relative surface roughness, roughness distribution and gas rarefaction on flow were studied. It was found that the effect of surface roughness is more pronounced at low Knudsen numbers.

For DSMC simulation, a conical surface roughness model has been developed and has been linked to the Cercignani–Lampis (CL) scattering kernel (Lilly et al. 2007). DSMC simulations using the smooth, fully diffuse model, the conical model, the CL scattering kernel and a direct geometric representation of a regular triangular surface were compared against their own experiments for smooth and rough channel flows using helium and nitrogen gases. As a result, the authors concluded that the direct geometric representation is most costly but more accurate compared to the experiment.

It should be noted that all the above-mentioned DSMC studies, except for Sun and Faghri (2003), are devoted to the outflow of rarefied gas through a rough channel into a vacuum. These works present almost no tabular data for comparison and a limited range of gas rarefaction.

Another well-known technique of direct numerical simulation is the molecular dynamics method. In Cao et al. (2006), non-equilibrium molecular dynamics simulation is applied to investigate the effect of surface roughness on slip flow of gaseous argon in submicron platinum channels. The roughness is, respectively, modeled as triangular, rectangular, sinusoidal and randomly triangular structures. The authors, in particular, have shown that the effects of roughness and rarefaction on the friction coefficient of gas microflows are strongly coupled.

Molecular dynamics simulations of force-driven (in particular, pressure-driven) nano-channel gas flows in a wide range of Knudsen numbers show two distinct flow regions: near-wall and bulk flow regions (Barisik and Beskok 2016). While the bulk flow region can be determined using kinetic theory, transport in the near-wall region is dominated by gas–wall interactions. While density and velocity distributions in the near-wall region are determined by the gas–wall interaction parameter, velocity distribution also depends on the local Knudsen number. Near-wall normalized density, velocity and mass flux distributions are independent of the local thermodynamic state, channel dimensions and the applied force. As a result, the authors proposed a procedure that can correct kinetic theory-based mass flow rate predictions in the literature for various nano-channel gas flows.

It should be noted that the molecular dynamics method never became a frequent practice in internal rarefied flow simulation. In our opinion, it should rather be used in a hybrid approach as "responsible" for modeling the gas–surface interaction.

The most common approach in fluid flow calculations is the conventional continuous approach which uses the Navier–Stokes equation. This approach considers fluid flow as a continuous flow with macroscopic state parameters continuously changing in space and time. Specifically, based on the Navier–Stokes equation with no-slip boundary conditions, friction factors of a single-phase microchannel with various roughness structures (rectangular, triangular, elliptical) were investigated numerically (Wang et al. 2005). The two-dimensional numerical solution shows significant influence of surface roughness including the height, size, and spacing of the roughness elements on the Poiseuille number.

The gas slip flow in microtubes is studied incorporating the effect of a three-dimensional (3D) random surface topography as characterized by the fractal geometry (Yan et al. 2015). The flow field in microtubes is numerically analyzed by solving the 3D Navier–Stokes equation with the extended first-order slip boundary conditions. The authors claim that the effect of surface topography, rarefaction and compressibility are coupled, in particular the effect of surface roughness increases with the increasing rarefaction (Knudsen number) and the increase in the fractal dimension makes the Poiseuille number more sensitive to the Mach number.

Ji et al. (2006) studied the influence of roughness in a slip flow regime using the Navier–Stokes equation with the second-order slip boundary conditions. Roughness was modeled as rectangular elements on two parallel plates with different spacings and heights to investigate the effect of wall roughness on friction factor and Nusselt number. The authors state that the effect of wall roughness is reduced with the increasing Knudsen number.

Using the Navier–Stokes equation with the high-order slip conditions, the effects of roughness on the rarefied gas flow in a long microtube were investigated (Li et al. 2002). Roughness was modeled as a porous film borrowed from the roughness modeling of a magnetic disk. It was shown that the mass flow rate and the nonlinearity of pressure distributions increase with the increasing porous film thickness and permeability.

Note that the correct application of the Navier–Stokes equation for fluid flow calculations has a limited range of gas rarefaction: at $\mathrm{Kn} < 10^{-3}$, the Navier–Stokes equation is correct with no-slip boundary conditions and at $10^{-3} < \mathrm{Kn} < 10^{-1}$ it is still true, yet under slip boundary conditions (Schaaf and Chambre 1961).

In recent years, the lattice Boltzmann method (LBM) has been widely used in tasks related to medium flow modeling. The LBM is based on the numerical solution of a time-,

space- and velocity-discrete Boltzmann equation. In particular, the LBM is applied to investigate the gaseous flow in a microchannel with surface roughness which is modeled by an array of rectangular modules (Chai et al. 2008). The effects of relative surface roughness, roughness distribution, and rarefaction on gaseous flow are studied in detail. It is shown that there is a very significant effect on the friction factor and mass flow rate. This effect becomes more significant with the decrease of the Knudsen number.

The fractal roughness effect of the two-dimensional micro Poiseuille gas flows of nitrogen and helium is studied by a modified lattice Boltzmann model at high Knudsen numbers (Liu and Ni 2009). It is shown that the effects of rarefaction, compressibility and roughness are strongly coupled, in particular the resistance coefficient decreases with increasing velocity due to gas compressibility, whereas resistance coefficient increases with the increasing roughness. Moreover, the resistance coefficient of nitrogen becomes larger than that of helium with the same roughness.

A lattice Boltzmann model of gas flow through rough surface microchannels as characterized by fractal geometry is developed and numerically analyzed to investigate the role of rough surface topography on gas slip flow (Zhang et al. 2012). Compared with a smooth microchannel, the presence of surface roughness reduces boundary slip for gas flow in a microchannel, thus increasing the Poiseuille number and decreasing the mass flow rate.

A multiple-relaxation-time lattice Boltzmann model of Couette flow was developed to investigate the rarefied gas flow through microchannels with roughness characterized by fractal geometry, especially to elucidate the coupled effects of roughness and rarefaction on microscale gas flow in the transition flow regime (Deng et al. 2016). The results indicate that surface roughness effect on gas flow behavior becomes more significant in rarefied gas flow with the increasing Knudsen number.

It should be noted that the lattice Boltzmann method works stably at velocities much lower than the speed of sound in the medium: the smaller the velocity, the better (Sukop and Thorne 2006). In particular, the authors He and Luo (1997) note that their model of the LBM is applicable at Mach < 0.15. Note that when a rarefied gas flows into a vacuum, the Mach number may significantly exceed unity (Varoutis et al. 2008).

There are studies on the effect of surface roughness on the internal rarefied flow, which are based on other models of the Boltzmann kinetic equation. Specifically, the mass flow rates of the Poiseuille and Couette flows in a rough microchannel are obtained by solving the linearized Bhatnager–Gross–Krook model of the Boltzmann equation (Su et al. 2017). The surface roughness is characterized by the Weierstrass-Mandelbrot fractal function. The results show that the surface roughness reduces the local mass flow rate as compared to the smooth channel, but the amount of reduction varies for Couette and Poiseuille flows of different Knudsen numbers.

The effect of surface roughness on the thermal and hydrodynamic flow characteristics has been investigated for the flow through a planar microchannel of finite length caused by a small pressure drop using the nonlinear S-model of the Boltzmann equation (Rovenskaya 2017). A surface roughness is configured with a series of triangular, semicircular, trapezoidal and rectangular obstructions. A competition between rarefaction and effects of roughness geometry is estimated by varying a rarefaction regime of flow from continuum up to the free molecular regime.

Guo et al. (2015) offered a 3D Gauss model to investigate the influence of wall roughness on the fluid flow and heat transfer in microchannels using the Fluent CFD software. The authors state that their model, rather than the fractal one, is an efficient and convenient method to build a rough wall.

The following works can be distinguished among the latest experimental studies in this field. In Yoshida et al. (2010), the effect of surface material and roughness on channel conductance at molecular flow was measured for N_2, Ar, He, and H_2. The experimental results were compared with the Monte Carlo calculations. It established that the conductance of the experimental channel depends on the slope of the surface roughness rather than the arithmetic surface roughness or the material.

The effects of relative roughness on flow behavior and heat transfer characteristics for circular microchannels are experimentally studied in Xing et al. (2016). Based on experimental results and theoretical analysis, the authors concluded that the flow behavior and heat transfer in circular microchannels are different from those in macrochannels and the roughness effect cannot be ignored in microchannels. It was found that the friction factor in microchannels increases markedly with the increasing surface roughness.

Unfortunately, there are very few experimental studies in this field and their conclusions may be contradicting. For example, Turner et al. (2004) claimed that in their experiments the influence of surface roughness on the friction factor is insignificant for both the continuum and slip flow regimes.

This paper is a natural continuation of a series of ours previous works (Sazhin 2008, 2009a,b, 2010, 2012, 2015b), where the DSMC method was used to obtain high-precision dependencies of the gas flow rate and detailed flow field in a slit and a smooth rectangular channel on a wide set of defining parameters: gas rarefaction, reduced length (length to channel height ratio), pressure ratio (output to input pressure of the channel), gas–surface scattering, and gas molecule–molecule interaction. F. Sharipov and his colleagues made a significant contribution to the application of the

DSMC method to slit, orifice, channel and tube flows (Varoutis et al. 2008, 2009, 2012; Sharipov 2004, 2012,2013, 2017; Sharipov and Kozak 2009, 2011; Sharipov and Barreto 2015).

The purpose of this work is to provide a detailed study of the gas flow through a rough channel into a vacuum, depending on the roughness model parameters in a wide range of rarefaction. A reliable basis for achieving this goal is provided by our recent papers (Sazhin 2015a, 2018, 2019), where we investigated the gas flow through a rough channel under free molecular conditions.

2 Problem definition and solution method

Consider the stationary flow of a monatomic gas in a system of two infinitely large containers connected by a rough channel of length l. In the upstream region, away from the channel, there is an equilibrium gas at pressure P_1 and temperature T_1. In the downstream region, pressure P_2 is so small compared with P_1 that one can assume $P_2=0$. Surface temperature in the entire simulated system is T_1. We assume that the channel width w considerably exceeds its height h, which allows passing onto a 2D geometry of the problem.

Figure 1 shows a computational domain and coordinate system, where the model channel has a rough surface in the form of a "saw" with identical teeth (Λ is the height of a sawtooth, α and β are the inclination angles). Notably, only half of the geometry of the problem is investigated, since symmetry is assumed in the flow field relative to the center line of the channel. For this purpose, a specular reflecting boundary is placed on the center line of the channel, coinciding with the y-axis.

The computational domain includes solid and open boundaries marked by solid and dot-dash lines, respectively. The solid boundary prescribes the gas–surface scattering law and the open boundaries prescribe the corresponding distribution function. Specifically, the left open boundary prescribes the distribution function of molecules crossing a certain plane in equilibrium gas at pressure P_1 and temperature T_1; on the right open boundary, in turn, the distribution function equals zero ($P_2=0$). Thus, the left open boundary demonstrates the injection of new particles into the computational domain and the right open boundary indicates their removal. In the steady state, the number of injected and removed particles is approximately the same, so the total of particles in the computational domain remains almost unchanged.

The main computed value in the gas flow simulation in the system under consideration is the gas mass flow rate Q through the channel. The results is given as $Q^*=Q/Q_0$, where Q_0 is the gas flow rate through a slit ($l=0$) in the free molecular flow regime. Q_0 has analytical formula $(h\cdot P_1)/(\sqrt{\pi}\cdot v_1)$, where v_1 is the most probable molecular speed at T_1. In practice, Q and Q_0 correspond to the mass flow rates through a channel and a slit with $w\gg h$ per unit width. Thus, to get the mass flow rate in [kg/s] through a specific channel, Q^* should be multiplied by $\frac{h[m]w[m]P_1[Pa]}{\sqrt{\pi}v_1[\frac{m}{s}]}$.

Gas rarefaction is characterized by parameter $\delta=(h\cdot P_1)/(\mu_1\cdot v_1)$, where μ_1 stands for gas viscosity at T_1. The hard sphere model is used to simulate gas molecule–molecule interaction and the complete diffuse scattering model is applied to simulate gas–surface scattering. The rarefaction parameter is inversely related to Knudsen number $Kn=\sqrt{\pi}/2\delta$ which is given as $Kn=\lambda_1/h$, where λ_1 is the mean free path of the gas molecules in equilibrium gas at P_1 and T_1.

As before, we will use the direct simulation Monte Carlo (DSMC) method (Bird 1994) based on the majorant frequency (MF) scheme (Ivanov and Rogasinsky 1988). The DSMC method is a stochastic approach for solving the exact

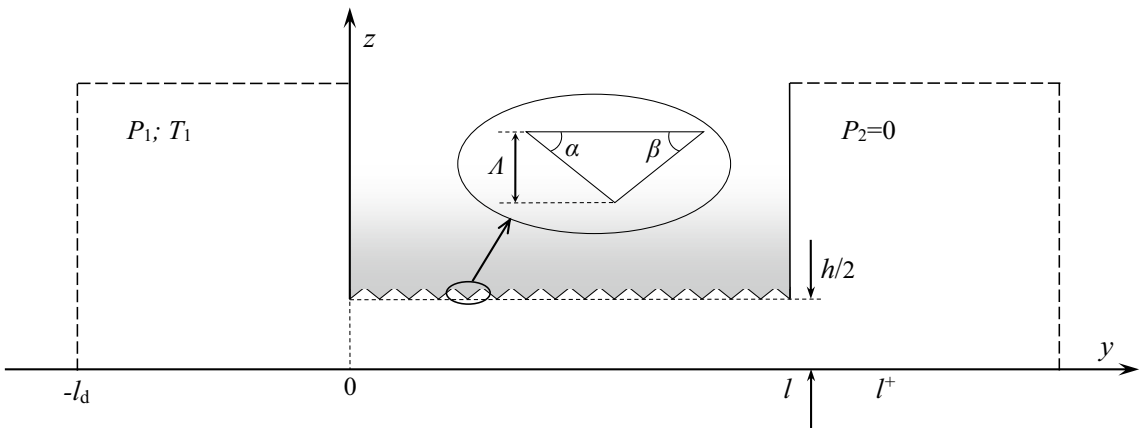

Fig. 1 Computational domain and coordinate system

Boltzmann equation and serves as an effective tool for solving the problems of gas dynamics for any gas flow regimes from free molecular to nearly hydrodynamic. The method allows considering numerous factors, such as a strong non-equilibrium and complex geometric configuration of the simulated system, as well as using various types of boundary conditions and models of surface structure and intermolecular interaction.

Direct statistic simulation by the Monte Carlo method is adequate to the physical nature of molecular transport and can be regarded as a numerical experiment. As with any physical experiment, there are two types of errors that can be distinguished in this method: a random error caused by the stochastic nature of the method and defined by the overall time of system monitoring, and a systematic error that depends on a set of simulation parameters. In our study, the overall system monitoring time was unlimited until the desired level of the random part of the computation error was achieved. The systematic part of the computation error can be reduced to values of the order of or significantly less than the random error by appropriately choosing the simulation parameters. For this purpose, a set of simulation parameters was chosen so that their further improvement (finer space grid, shorter time step, etc.) would not produce any changes in the simulation results beyond the random part of the computation error.

Table 1 shows the main simulation parameters, including the total number of model particles N, characteristic size of the computational domain l_d at the channel inlet and outlet, cell size Δx and time step Δt.

In all modeling cases, the number of model particles in the cell was not less than 50, time to reaching the stationary flow was $3\left(2l_d + l\right)/h$ in units of h/v_1. Additionally, to increase the reliability and accuracy of calculations, we used the following DSMC procedures: splitting the cell into sub-cells and using the weight planes.

In the DSMC method, the simulation of particle collisions is performed individually for each cell, i.e. only particles located in one and the same cell are assumed to participate in a collision as nearest neighbors. However, there is a possibility that a particle located near the border of its own cell is approached closer by particles of a different cell rather than by its "cellmates" which may be located at the opposite side of the same cell. To simulate a maximum credible collision of only the nearest neighbors, each cell was divided into sub-cells (nine in total), and, moreover, the two colliding particles had to be in one and the same sub-cell.

Table 1 Main simulation parameters: total number of model particles N, characteristic size of the computational domain l_d, cell size Δx and time step Δt

Rarefaction parameter, δ	$N, \times 10^6$	$\frac{l_d}{h}$	$\frac{\Delta x}{h}$	$\frac{\Delta t}{h/v_1}$
0.01	35	140	1/6	1/10
0.1	35	120	1/6	1/10
1	20	80	1/6	1/20
10	50	30	1/42	1/100
100	25	20	1/42	1/200

A serious problem for modeling the gas flow into a vacuum is a substantial drop of gas density in the downstream region. Since the number of particles in a cell is directly related to gas density, the number of particles in the cells down the stream may be insufficient for the modeling, unless special measures are taken.

To increase the number of particles in the range $0 \le y \le l^+$ (Fig. 1), the so-called weight planes were used. Particles that crossed these planes in the direction of the decreasing weight factor (in the flow direction) doubled in number, while in the direction of the increasing weight factor (against the flow direction) they were reduced in half. Obviously, crossing the weight plane changes the quantitative representation of gas molecules (i.e. the number of real gas molecules represented by one model particle) halved in the flow direction and doubled against the flow direction. Our computations used several of these planes, such as $y=0$; 5 h; 8 h; 10 h; 11 h for channel with $l/h = 10$ (Fig. 1). This is equivalent of an increase in the particle number by a factor of 32, if intersecting all the weight planes in the flow direction. Using multiple weight planes spaced apart with a weighting factor of 2, we can avoid an undesirable effect, i.e. prevent the appearance of a large number of particle clones (having the same coordinates and speeds). Instead, when using a single weight plane with a high weight factor, a great number of particle clones are generated, which does not allow for the reliable simulation of intermolecular collisions.

To increase the number of particles in the cells in the region of $y \ge l^+$(Fig. 1), we increased the linear dimensions of the cell in this region by a factor of 3. The value of l^+ was chosen on the assumption that the density dropped approximately by an order of magnitude and $l^+ = l + 2h$ in most calculations. Thus, the correct choice of simulation parameters and application of the special DSMC procedures made it possible to achieve the computation error not exceeding 1%. The details of choosing the simulation parameters and estimating the computation error can be found in our previous paper (Sazhin 2008).

3 Results and discussion

In this paper, we used a further improved program code which was tested in our recent studies (Sazhin 2015a, 2018) under free molecular conditions ($\delta=0$). We remind that we considered an isolated system consisting of two identical containers connected by a thin channel with a substantially

asymmetric surface roughness (asymmetrical "saw"), specifically with $\alpha = 90^0$ and $\beta = 10^0$ (Fig. 1). Since the system is isolated, then according to the second law of thermodynamics, the number of molecules in the first and second container should be the same. As a result, the transmission probabilities from the first volume to the second and backwards should be equal.

We computed the following values: the transmission probability through the rough channel, the mean path and mean passage time (for particles that passed the channel from the input to the output and at least once collided with the channel wall), as well as the mean number of particle's collisions with a rough surface when hitting the channel wall. The simulation is set up in such a way that, in one case, the particles start from one entrance section of the channel, and, in the other case, they start from the opposite. Setting different channel sizes and parameters of a non-symmetric "saw", we conducted enough tests to show that, for both simulation cases, the computed values are the same up to the computation error, even if the height of the "saw" is comparable with that of the channel, which gives hope for the correctness of future computations.

Early in this work, we verified the statement that we obtained earlier (Sazhin 2018, 2019) in a free molecular regime, regarding the independence of the channel transmission probability (and hence the mass flow rate) on the value of Λ, also for the transition and near hydrodynamic gas flow regimes. Table 2 shows the computation results for the gas mass flow rate Q^* through a modeled rough channel, as a function of the relative roughness height Λ/h and the length to height ratio l/h in the transition regime ($\delta = 1$). According to the table, as soon as roughness height Λ becomes negligibly small compared to channel height h, mass gas flow rate Q^* is no longer a function of Λ, as in the case of the free molecular regime. The same result was obtained in a near hydrodynamic regime ($\delta = 100$). Recall that this result agrees well with the conclusions of the classical theoretical work (Barantsev 1975). For further computations, we used the value of $\Lambda/h = 1 \times 10^{-4}$.

Table 3 shows the main computation results of our work, specifically the gas mass flow rate Q^* through a modeled rough channel into a vacuum, as a function of inclination angle α and rarefaction parameter δ for the length to height ratio $l/h = 1$ and 10. It should be noted that, in the case of a smooth channel, i.e. for $\alpha = \beta = 0°$, there is a corresponding reliable formula (Berman 1965, 1966) under the free molecular conditions and the corresponding reliable DSMC data (Varoutis et al. 2012) over the whole range of gas rarefaction. Our results in this case match within the computational error with both the formula and DSMC data.

For convenience of interpretation, we presented these results as Q/Q_{sm}, where Q_{sm} is a gas mass flow rate through a smooth channel (see Fig. 2). The figure shows that the effect of surface roughness on the gas flow rate depends on both the inclination angle and the channel length and gas rarefaction. A significant change in gas flow rate Q^* is observed at the inclination angle α ranging from 0 to 45 degrees. From Fig. 2a, it also follows that for channel $l/h = 1$ under free molecular conditions ($\delta = 0$), dependence the $Q^*(\alpha)$ has a minimum at $\alpha = 45^0$. This minimum was first recorded by Davis et al. (1964) when the transmission

Table 2 Gas mass flow rate Q^* through a modeled rough channel into a vacuum, as a function of relative roughness height Λ / h in the case of rarefaction parameter $\delta = 1$, inclination angle $\alpha = \beta = 45°$ and length to height ratio $l/h = 1$ and 10

Length to height ratio, l/h	Mass flow rate, Q^*				
	$\Lambda/h = 1 \times 10^{-1}$	1×10^{-2}	1×10^{-3}	1×10^{-4}	1×10^{-5}
1	0.700	0.688	0.686	0.687	0.685
10	0.171	0.165	0.165	0.164	0.165

Table 3 Gas mass flow rate Q^* through a modeled rough channel into a vacuum, as a function of inclination angle α and rarefaction parameter δ for the length to height ratio $l/h = 1$ and 10

l / h	δ	Mass flow rate, Q^*				
		$\alpha = \beta = 0°$	15°	30°	45°	60°
1	0	0.6844	0.6645	0.6315	0.6010	0.6057
	0.01	0.686	0.668	0.635	0.611	0.609
	0.1	0.697	0.677	0.646	0.622	0.619
	1	0.764	0.743	0.708	0.687	0.683
	10	1.024	1.008	0.983	0.967	0.965
	100	1.35	1.34	1.33	1.33	1.33
10	0	0.2406	0.2162	0.1939	0.1842	0.1823
	0.01	0.240	0.215	0.193	0.184	0.183
	0.1	0.237	0.212	0.190	0.181	0.179
	1	0.219	0.198	0.175	0.165	0.164
	10	0.290	0.274	0.256	0.245	0.244
	100	0.784	0.776	0.770	0.766	0.765

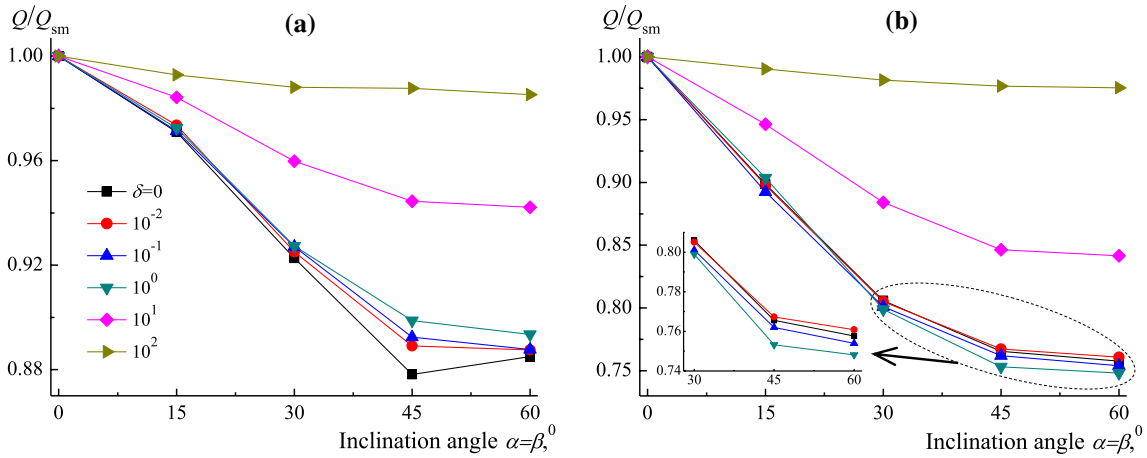

Fig. 2 **a**, **b**. The relative gas flow rate Q/Q_{sm} through a modeled rough channel into a vacuum, as a function of inclination angle α and rarefaction parameter δ for the length to height ratio $l/h=1$ (**a**) and 10 (**b**)

probability through a short rough tube was calculated by the Monte Carlo method. As soon as the channel became long enough, this minimum disappeared, which was demonstrated by Davis et al. (1964) and our Fig. 2b. The free molecular case is given a detailed analysis in our work (Sazhin 2019).

As expected, the effect of surface roughness on gas flow is more pronounced in a longer channel rather than in short one. Indeed, in short channels, a noticeable fraction of gas molecules pass through the channel without colliding with the surface. As seen in Fig. 2, when $l/h=1$, the value of Q/Q_{sm} may decrease to 88% and when $l/h=10$ to 75%.

As Fig. 2 shows, the surface roughness depends on gas rarefaction δ as follows: the denser the gas, the lesser the effect of the surface roughness on the gas flow in the channel. Indeed, in a dense gas, intermolecular collisions substantially exceed molecular collisions with the surface. As can be seen from the figure, the effect of the rough surface on the flow rate through the channel in a dense gas with $\delta=100$ is weak. In a rarefied gas, at $\delta<1$, the role of roughness remains almost unchanged; especially for a long channel $l/h=10$, as can be seen in Fig. 2b, the curves for $\delta=10^0$; 10^{-1}; 10^{-2} and 0 are not significantly different.

Apart from flow rate Q^*, of interest is the distribution of macroscopic gas parameters. However, as our studies have shown, the effect of surface roughness on the distribution of gas macro-parameters is insignificant. For example, Fig. 3 shows the distribution of dimensionless density n/n_1 (where $n_1=P_1/kT_1$ and k is the Boltzmann constant), temperature T/T_1 and longitudinal bulk velocity u_y/v_1 along the central line of the channel with $l/h=10$ at rarefaction $\delta=1$ for inclination angle $\alpha=\beta=15°$. Distributions for other inclination angles ($\alpha=\beta=30°$; 45°; 60°) and even in the smooth channel ($\alpha=\beta=0°$) do not differ much from the one presented. From the experience of our previous works (Varoutis et al. 2008; Sazhin 2009b), a

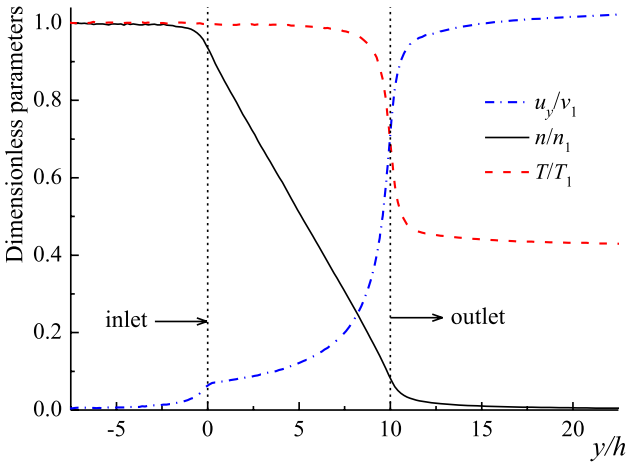

Fig. 3 The distribution of dimensionless density n/n_1, temperature T/T_1 and longitudinal bulk velocity u_y/v_1 along the central line of the channel with $l/h=10$ at rarefaction $\delta=1$ for inclination angle $\alpha=\beta=15°$

marked difference in the distribution of gas macro-parameters should be expected in case of a significant change in the flow rate under the effect of the control parameters. Apparently, the surface roughness does not have such an effect.

As is seen in Fig. 3, density n, temperature T and longitudinal bulk velocity u_y vary from the upstream values in equilibrium gas n_1, T_1 and zero, respectively, to the vacuum conditions in the downstream region. Gas density drops to zero, while the temperature and the longitudinal bulk velocity reach certain limits, i.e. they "freeze". Since gas temperature is a measure of the chaotic molecular motion, the longitudinal bulk velocity and the temperature show multidirectional behavior, i.e. as the longitudinal bulk velocity increases, the temperature decreases.

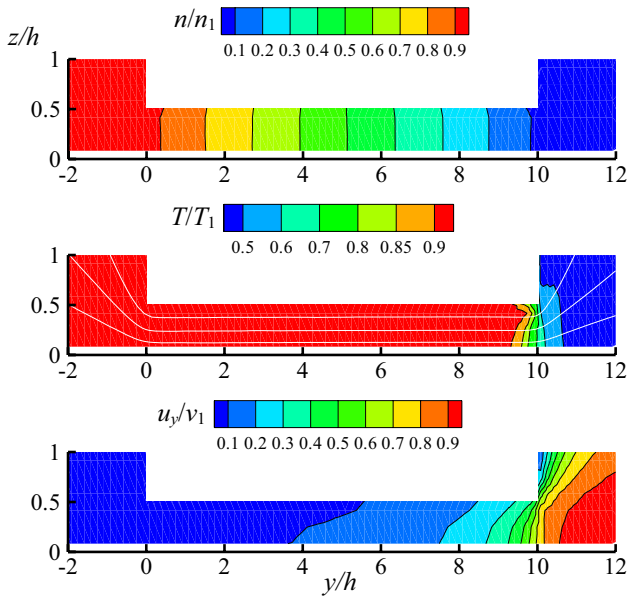

Fig. 4 The distribution of dimensionless density n/n_1, temperature T/T_1 and longitudinal bulk velocity u_y/v_1 in the yz-plane near and inside the channel with $l/h=10$ at rarefaction $\delta=1$ and inclination angle $\alpha=\beta=15°$. The figure showing the temperature also demonstrates the streamlines

Concluding the discussion, we present a complete picture of the flow field in Fig. 4, where density n/n_1, temperature T/T_1 and longitudinal bulk velocity u_y/v_1 are plotted in the yz-plane near and inside the channel with $l/h=10$ at rarefaction $\delta=1$ and inclination angle $\alpha=\beta=15°$. The figure showing the temperature also demonstrates the streamlines. The gas density decreases evenly along the length of the channel, while a noticeable change in temperature and longitudinal bulk velocity occurs near the end of the channel and beyond in the downstream region.

4 Conclusion

The rarefied gas flow through a two-dimensional rough channel of finite length into a vacuum was investigated using the direct simulation Monte Carlo method. Mass flow rate Q^* through a short and long channel was computed in a wide range gas rarefaction, from the free molecular regime to near hydrodynamic. Noticeable change in gas flow rate Q^* is observed when inclination angle α varies from 0 to 45 degrees. The effect of surface roughness on the gas flow is more pronounced in longer channels rather than in short ones. The role of roughness depends on gas rarefaction δ as follows: the denser the gas, the lesser the effect of surface roughness on the flow rate through the channel.

The analysis is provided for the distribution of macroscopic gas parameters and streamlines inside the channel as well as in the upstream and downstream regions. It has been established that the effect of surface roughness on the distribution of gas macro-parameters is insignificant.

The results obtained can be used for preliminary analysis in the development and optimization of micro- and nanofluidic devices, for space applications and vacuum systems engineering.

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