## **RESEARCH PAPER**



# **Electrokinetics in nanochannels grafted with poly‑zwitterionic brushes**

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## **Abstract**

In this paper, we compute the electrokinetic transport in soft nanochannels grafted with poly-zwitterionic (PZI) brushes. The transport is induced by an external pressure gradient, which drives the ionic cloud (in the form of an electric double layer or EDL) at the brush surfaces to induce an electric field that drives an induced electroosmotic transport. We characterize the overall transport by quantifying this electric field, overall flow velocity, and the energy conversion associated with the development of the electric field and a streaming current. We specially focus on how the ability of the PZI to ionize and demonstrate a significant charge at both large and small pH can be efficiently maneuvered to develop a liquid transport, an electric field, and an electrokinetically induced power across a wide range of pH values.

# **1 Introduction**

Functionalizing nanoscale interfaces (e.g., walls of nanochannels or the surfaces of nanoparticles) with polymer and polyelectrolyte (PE) brushes (Alexander [1977](#page-16-0); Gennes [1976,](#page-17-0) [1980;](#page-17-1) Netz and Andelman [2003](#page-17-2); Das et al. [2015](#page-16-1); Milner [1991\)](#page-17-3) has been extensively used for a myriad of applications such as targeted drug delivery (Knop et al. [2010](#page-17-4); Suk et al. [2015\)](#page-17-5), oil recovery (ShamsiJazeyi et al. [2014\)](#page-17-6), ion and biosensing (Groot et al. [2013;](#page-17-7) Yameen et al. [2009;](#page-17-8) Ali et al. [2008,](#page-16-2) [2010a](#page-16-3); Umehara et al. [2009\)](#page-17-9), current rectification (Ali et al. [2010b](#page-16-4)), fabrication of nanofluidic diodes (Vilozny et al. [2013;](#page-17-10) Ali et al. [2009\)](#page-16-5) and nanoactuators (Moya et al. [2005](#page-17-11)), and many more. The central idea that drives most of these applications is how the brushes respond to the environmental stimuli (e.g., local pH, salt concentration, temperature, etc.) and regulate the transport of different species. Under these conditions, there have been significant efforts in studying the ion and liquid transport in nanochannels or nanopores grafted with PE brushes that are pH-responsive (Groot et al. [2013](#page-17-7); Yameen et al. [2009](#page-17-8); Chen and Das [2015a,](#page-16-6) [b](#page-16-7), [2017a;](#page-16-8) Patwary et al. [2015](#page-17-12); Li et al. [2016](#page-17-13); Milne et al. [2014;](#page-17-14) Yameen et al. [2009;](#page-17-15) Tagliazucchi et al. [2010](#page-17-16); Lin et al. [2016;](#page-17-17) Ma et al. [2015](#page-17-18); Xue et al. [2014](#page-17-19); Zhou et al. [2016](#page-18-0); Ali et al. [2008](#page-16-9); Gilles et al. [2016](#page-17-20); Tagliazucchi and Szleifer [2012](#page-17-21)).

Poly-zwitterion (PZI) is a particular type of PE that contains both negative and positive sites (Lowe and McCormick [2006](#page-17-22)). These sites typically ionize as a function of the local pH; however, the extent of ionization of the positive site and that of the negative site are different at different pH. Therefore, at a given pH, the PZI is either negatively or positively charged. The PZI molecules have been extensively employed in a large number of applications, such as the fabrication of "smart" materials with environmental–stimuli-responsive switchable properties (Ilcikova et al. [2015](#page-17-23)), sub-surface imaging and oil recovery (Urena-Benavides et al. [2016](#page-17-24)), capturing chemical moieties (Saleh et al. [2017\)](#page-17-25), drug delivery (Xiao et al. [2012](#page-17-26)), biomacromolecular separation (Zhao et al. [2017](#page-18-1)), removal of organic pollutants (Monteil et al. [2016\)](#page-17-27), use as heterogeneous catalysts (Fidale et al. [2013](#page-17-28)), and many more. In this paper, we study the electrohydrodynamics in a nanochannel grafted with such PZI molecules existing in a "brush" like state. There have been significant previous efforts where interfaces grafted with such PZI brushes have been used for a variety of applications such as triggering extreme lubrication (Chen et al. [2009,](#page-16-10) [2011](#page-16-11)), reversible switching of the surface wettability (Azzaroni et al. [2006;](#page-16-12) Cheng et al. [2008\)](#page-16-13), inducing repeatable adhesion (Kobayashi and Takahara [2013](#page-17-29)), fabrication of antifouling surfaces (Higaki et al. [2016\)](#page-17-30), regulating ion selectivity in nanopores (Zeng et al. [2015\)](#page-18-2), etc. However, this is for the first time that its effect in electrohydrodynamics and

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electrokinetic energy conversion in a brush-grafted nanochannel is being probed.

Our paper provides detailed calculations of the pHresponsive electric double layer electrostatics and how that electrostatics regulates the flow and the overall electrokinetics in the presence of an externally imposed pressuredriven transport. We calculate the electric field induced by this pressure-driven transport and how this electric field and the induced streaming current couple to generate an electrokinetic power. This power generation is an example of electrochemomechanical energy conversion and has been touted as one of the key applications of nanochannel electrokinetic transport (Patwary et al. [2015;](#page-17-12) Chanda et al. [2014](#page-16-14); Chen and Das [2015d](#page-16-15); Chen et al. [2018](#page-16-16); Daiguji et al. [2004](#page-16-17); Nguyen et al. [2013](#page-17-31); Heyden et al. [2006a,](#page-17-32) [2007](#page-17-33)). Here, we establish that working with the PZI brush allows for the generation of the large electrokinetic power across a wide range of pH (i.e., for both large and small pH). In other words, this paper points to a new design information in the context of electrokinetic power generation in soft or PE brush-grafted nanochannels—a single design allows the flexibility of generating electrokinetic power across a wide spectrum of pH, which is not possible for brush-free nanochannels (Daiguji



et al. [2004](#page-16-17); Nguyen et al. [2013](#page-17-31); Heyden et al. [2006a,](#page-17-32) [2007\)](#page-17-33) or nanochannels grafted with the PE (and not PZI) brushes (Patwary et al. [2015](#page-17-12)).

## **2 Theory**

## **2.1 Electrostatics**

We consider a pressure-driven transport in a nanochannel of height 2*h* and grafted with a layer of PZI brushes of constant height  $d$  (with  $d < h$ ) (see Fig. [1](#page-1-0)). To obtain the overall transport, we would have to first get the electrostatics of the EDL induced by the brushes. Considering the bottom half of a nanochannel (i.e.,  $-h \le y \le 0$ ), the free-energy functional dictating the EDL electrostatics can be expressed as follows:

$$
\mathcal{F} = \int f\left[\psi, n_{\pm}, n_{\text{H}^+}, n_{\text{OH}^-}\right] \mathrm{d}^3 \mathbf{r},\tag{1}
$$

where  $d^3$ **r** represents the volumetric integration,  $\psi$  is the electrostatic potential,  $n_i$  is the number density of the ion  $i$  $(i = \pm, H^+, OH^-)$ , and *f* is the free-energy density expressed as follows:



<span id="page-1-0"></span>**Fig. 1** Schematic showing the pressure-driven transport and induced electric field in a PZI brush-grafted nanochannels. The PZI brush is positively charged for small pH ( $pH_{\infty}$  < 7) (a) and negatively charged for large pH (pH<sub>∞</sub> > 7) (**b**), leading to the generation of a positive streaming potential (**a**) and a negative streaming potential (**b**). In

this figure, the ionic charges of the PZI are represented in green (for  $pH_{\infty}$  < 7, the PZI is more positive, while, for  $pH_{\infty}$  > 7, the PZI is more negative), the cations (from the salt) in blue (dark), the anions (from the salt) in orange, the  $H^+$  ions in blue (light), and the OH ions in purple. (Color figure online)

$$
f = k_{\rm B} T \bigg[ n_{+} \bigg( \ln \bigg( \frac{n_{+}}{n_{+,\infty}} \bigg) - 1 \bigg) + n_{-} \bigg( \ln \bigg( \frac{n_{-}}{n_{-,\infty}} \bigg) - 1 \bigg) + n_{\rm H^{+}} \bigg( \ln \bigg( \frac{n_{\rm H^{+}}}{n_{\rm H^{+},\infty}} \bigg) - 1 \bigg) + n_{\rm OH^{-}} \bigg( \ln \bigg( \frac{n_{\rm OH^{-}}}{n_{\rm OH^{-},\infty}} \bigg) - 1 \bigg) \bigg] - \frac{\epsilon_{0} \epsilon_{r}}{2} |\nabla \psi|^{2} + e\psi \big(n_{+} - n_{-} + n_{\rm H^{+}} - n_{\rm OH^{-}} - \varphi n_{\rm A^{-}} + \varphi n_{\rm BH^{+}} \bigg) \quad \text{[for} \quad -h \le y \le -h + d],
$$
  

$$
f = k_{\rm B} T \bigg[ n_{+} \bigg( \ln \bigg( \frac{n_{+}}{n_{+,\infty}} \bigg) - 1 \bigg) + n_{-} \bigg( \ln \bigg( \frac{n_{-}}{n_{-,\infty}} \bigg) - 1 \bigg) + n_{\rm H^{+}} \bigg( \ln \bigg( \frac{n_{\rm H^{+}}}{n_{\rm H^{+},\infty}} \bigg) - 1 \bigg) + n_{\rm OH^{-}} \bigg( \ln \bigg( \frac{n_{\rm OH^{-}}}{n_{\rm OH^{-},\infty}} \bigg) - 1 \bigg) \bigg]
$$
  

$$
- \frac{\epsilon_{0} \epsilon_{r}}{2} |\nabla \psi|^{2} + e\psi \big(n_{+} - n_{-} + n_{\rm H^{+}} - n_{\rm OH^{-}} \bigg) \quad \text{[for} \quad -h + d \le y \le 0].
$$
 (2)

In Eq. ([2](#page-2-0)),  $k_B T$  is the thermal energy, *e* is the electronic charge,  $\epsilon_0$  is the permittivity of free space,  $\epsilon_r$  is the relative permittivity of the medium, *e* is the electronic charge,  $n_{i,\infty}$  is the bulk number density of the ions  $i$  ( $i = \pm, H^+, OH^-$ ), and  $\varphi$ is the dimensionless distribution of the PZI chargeable sites (PZICS) of a given brush molecule. Here, we consider identical relative permittivities both inside and outside the brush layer very much similar to the previous studies (see review article in Das et al. [2015\)](#page-16-1). Of course, one can consider an even more rigorous model by accounting for the fact that the relative permittivity inside the brush is different from that outside the brush necessitating the consideration of the ion-partitioning effect (Poddar et al. [2016\)](#page-17-34). Typically, such a consideration will become important for a very dense system of brushes. We do not consider such a dense system, where the penetration depth will be very small enforcing virtually no flow inside the brush layer. With the brush being a PZI brush, the PZICS will simultaneously consist of a negative charge centre and a positive charge centre. The formation of the negative charge centre can be attributed to the ionization

<span id="page-2-0"></span>of  $1/m<sup>3</sup>$ ) of the ionic groups of the PZI molecule (namely  $n_{A^-}$  and  $n_{BH^+}$ ) can be expressed as follows [very similar to the form proposed previously (Zeng et al. [2015\)](#page-18-2)]:

$$
n_{A^{-}} = \frac{K'_{a}\gamma_{a}}{K'_{a} + n_{H^{+}}},
$$
\n(3)

<span id="page-2-1"></span>
$$
n_{\rm BH^{+}} = \frac{K_{\rm b}' \gamma_{\rm b}}{K_{\rm b}' + n_{\rm OH^{-}}},\tag{4}
$$

where  $\gamma_a$  and  $\gamma_b$  are the maximum site densities of acidic and basic functional groups of the PZI,  $K'_a = 10^3 N_A K_a$ , and  $K'_{b} = 10^{3} N_{A} K_{b}$  ( $N_{A}$  is the Avogadro number). Of course, Eqs.  $(2)$  $(2)$ – $(4)$  $(4)$  $(4)$  show the dependence of the overall problem on the pH of the system.

The equilibrium electrostatic potential and the concentration distribution of different ions can be obtained by minimizing  $\mathcal F$ . Minimizing  $\mathcal F$  with respect to  $\psi$ , we get (considering only the bottom half of the nanochannel):

$$
\frac{\delta \mathcal{F}}{\delta \psi} = 0 \Rightarrow \frac{\partial f}{\partial \psi} - \frac{d}{dy} \left( \frac{\partial f}{\partial \psi'} \right) \Rightarrow \frac{d^2 \psi}{dy^2} = \frac{e(n_- - n_+ + n_{OH^-} - n_{H^+} + \varphi n_{A^-} - \varphi n_{BH^+})}{\epsilon_0 \epsilon_r} \quad \text{[for } -h \le y \le -h + d],
$$
\n
$$
\frac{\delta \mathcal{F}}{\delta \psi} = 0 \Rightarrow \frac{\partial f}{\partial \psi} - \frac{d}{dy} \left( \frac{\partial f}{\partial \psi'} \right) \Rightarrow \frac{d^2 \psi}{dy^2} = \frac{e(n_- - n_+ + n_{OH^-} - n_{H^+})}{\epsilon_0 \epsilon_r} \quad \text{[for } -h + d \le y \le 0].
$$
\n(5)

of an acidic functional group HA (HA  $\leftrightarrow$  H<sup>+</sup> + A<sup>-</sup>; ionization constant  $K_a$  having the units of moles/liter) yielding A<sup>−</sup> ions. On the other hand, the formation of the positive charge centre can be attributed to the ionization of a basic functional group B  $(B + H_2O \leftrightarrow BH^+ + OH^-$ ; ionization constant  $K<sub>b</sub>$  having the units of moles/liter) yielding  $BH<sup>+</sup>$ ions. Under these conditions, the number densities (in units

<span id="page-2-3"></span>Minimizing F, with respect to  $n_{+}$ ,  $n_{\text{H}^{+}}$  and  $n_{\text{OH}^{-}}$ , we get the expression of the ion distributions:

<span id="page-2-4"></span><span id="page-2-2"></span>
$$
\frac{\delta F}{\delta n_{\pm}} = 0 \Rightarrow n_{\pm} = (n_{\pm,\infty}) \exp\left(\mp \frac{e\psi}{k_{\text{B}}T}\right) \quad \text{[for } y \ge -h],\tag{6}
$$

$$
\frac{\delta F}{\delta n_{\text{OH}^{-}}} = 0 \Rightarrow n_{\text{OH}^{-}} = (n_{\text{OH}^{-},\infty}) \exp\left[\frac{e\psi}{k_{\text{B}}T} \left(1 + \varphi \frac{K_{\text{b}}' \gamma_{\text{b}}}{\left(K_{\text{b}}' + n_{\text{OH}^{-}}\right)^{2}}\right)\right] \quad \text{[for } -h \le y \le -h + d],
$$
\n
$$
\frac{\delta F}{\delta n_{\text{OH}^{-}}} = 0 \Rightarrow n_{\text{OH}^{-}} = (n_{\text{OH}^{-},\infty}) \exp\left(\frac{e\psi}{k_{\text{B}}T}\right) \quad \text{[for } -h + d \le y \le 0].
$$
\n(7)

and

$$
\frac{\delta F}{\delta n_{\text{H}^+}} = 0 \Rightarrow n_{\text{H}^+} = (n_{\text{H}^+,\infty}) \exp\left[-\frac{e\psi}{k_\text{B}T} \left(1 + \varphi \frac{K_a' \gamma_a}{\left(K_a' + n_{\text{H}^+}\right)^2}\right)\right] \quad \text{[for } -h \le y \le -h + d],
$$
\n
$$
\frac{\delta F}{\delta n_{\text{H}^+}} = 0 \Rightarrow n_{\text{H}^+} = (n_{\text{H}^+,\infty}) \exp\left(-\frac{e\psi}{k_\text{B}T}\right) \quad \text{[for } -h + d \le y \le 0].
$$
\n(8)

Here,  $n_{\pm,\infty}$  are the bulk number density of the electrolyte ions,  $n_{\text{H}^+,\infty} = 10^3 N_{\text{A}} 10^{-\text{pH}_{\infty}}$  is the bulk number density of hydrogen ions (pH<sub>∞</sub> is the bulk pH), and  $n_{OH^-,\infty} = 10^3 N_A 10^{-pOH_{\infty}}$  $(pOH<sub>∞</sub>)$  is the bulk pOH) is the bulk number density of the hydroxide ions and  $pH_{\infty} + pOH_{\infty} = 14$ . The bulk number densities are the number densities of the ions in the microchannel reservoirs (where  $\psi = 0$ ) connecting the nanochannel (Baldessari and Santiago [2008;](#page-16-18) Das et al. [2013,](#page-16-19) [2014](#page-16-20)). Solution of  $\psi$  can be obtained by first using Eqs. [\(6](#page-2-2))–([8\)](#page-3-0) to replace the ion number densities appearing in Eq. ([5](#page-2-3)), and then solving the resultant differential equation in  $\psi$  in the presence of the boundary conditions expressed in the following:

<span id="page-3-0"></span>equations of Eq. [9\)](#page-3-1), and symmetry at the nanochannel centreline (final equation of Eq. [9\)](#page-3-1). The critical thing to note here is that this differential equation in  $\psi$  will also contain the unresolved expression for  $n_{\text{H}^+}$  and  $n_{\text{OH}^-}$ ; this stems from the fact that, while the expressions for the number densities of  $n_+$ are explicit in  $\psi$  (see Eqs. [6](#page-2-2), [7\)](#page-2-4),  $n_{\text{H}^+}$  and  $n_{\text{OH}^-}$  are implicit in  $\psi$ (see Eqs. [7](#page-2-4), [8](#page-3-0)). Therefore, we shall have a set of equations for  $\psi$ ,  $n_{\text{H}^+}$ , and  $n_{\text{OH}^-}$  that will be needed to be solved simultaneously. Finally, we would like to point out that this coupled solution of  $\psi$  and  $n_{\text{H}+}$  as well as  $\psi$  and  $n_{\text{OH}^-}$  will require the information on the distribution of  $\varphi = \varphi(y)$ . We shall discuss this choice of  $\varphi(y)$  later. As already mentioned, the first equation of Eq. [\(9](#page-3-1)) describes the grafting wall as an uncharged

$$
\left(\frac{\mathrm{d}\psi}{\mathrm{d}y}\right)_{y=0} = 0, \quad (\psi)_{y=(-h+d)^+} = (\psi)_{y=(-h+d)^-}, \quad \left(\frac{\mathrm{d}\psi}{\mathrm{d}y}\right)_{y=(-h+d)^+} = \left(\frac{\mathrm{d}\psi}{\mathrm{d}y}\right)_{y=(-h+d)^-}, \quad \left(\frac{\mathrm{d}\psi}{\mathrm{d}y}\right)_{y=-h} = 0. \tag{9}
$$

The boundary conditions in Eq. ([9\)](#page-3-1), respectively, represent the condition of an uncharged nanochannel wall (first equation of Eq. [9](#page-3-1)), identical potential and potential gradient at the brush–liquid interface at  $y = -h + d$  (second and third

<span id="page-3-1"></span>wall. We intentionally considered such an uncharged wall and not a more typical silica wall. A silica wall would typically show a negative charge density for an acidic pH (Behrens and Grier [2001\)](#page-16-21). On the other hand, for such an acidic pH, the





<span id="page-3-2"></span>**Fig. 2** Transverse variation of **a** dimensionless electrostatic potential  $\bar{\psi}$  and **b** dimensionless velocity profile  $\bar{u}$  for different values of bulk salt concentration  $c_{\infty}$ . Other parameters for this figure are pH<sub>∞</sub> = 4 (or bulk pH),  $pK_a = 4$ ,  $pK_b = 4$ ,  $\bar{d} = 0.3$ ,  $\gamma_a = 10^{-4}$  M,  $\gamma_b = 10^{-4}$  M,

 $\bar{\alpha} = 1, u_r = 1, R_i = 1, \frac{N_p a^3 \sigma}{d} = 1, h = 100 \text{ nm}, k_B = 1.38 \times 10^{-23} \text{ J/K},$ *T* = 300 K,  $e = 1.6 \times 10^{-19}$  C,  $\epsilon_0 = 8.854 \times 10^{-12}$  F/m, a = 1 nm, and  $\epsilon_r = 79.8$ 

brushes are positively charged (see Fig. [2,](#page-3-2) later). Under such conditions, the brushes will get attracted to the wall forming a "mushroom"-like or "pancake"-like configuration (Das et al. [2015\)](#page-16-1), i.e., deviate from the brush-like configuration making our analysis invalid.

#### **PZI brush layer in an acidic solution**

We first consider the PZI brush layer dissociation in an acidic solution. We consider that the acid furnishes the same anion as the salt. As a consequence, the bulk number density of the salt anion will be  $n_{\infty} + n_{H^+,\infty}$ . Under this condition, we can non-dimensionalize equations  $(7, 8)$  $(7, 8)$  $(7, 8)$  $(7, 8)$  $(7, 8)$  as well as the equation that results from using Eqs.  $(6)$  $(6)$ – $(8)$  $(8)$  to replace the ion number densities in Eq. [\(5](#page-2-3)) to yield:

$$
\left(\frac{d\bar{\psi}}{d\bar{y}}\right)_{\bar{y}=0} = 0, \quad (\bar{\psi})_{\bar{y}=(-1+\bar{d})^{+}} = (\bar{\psi})_{\bar{y}=(-1+\bar{d})^{-}},
$$
\n
$$
\left(\frac{d\bar{\psi}}{d\bar{y}}\right)_{\bar{y}=(-1+\bar{d})^{+}} = \left(\frac{d\bar{\psi}}{d\bar{y}}\right)_{\bar{y}=(-1+\bar{d})^{-}}, \quad \left(\frac{d\bar{\psi}}{d\bar{y}}\right)_{\bar{y}=-1} = 0.
$$
\n(13)

<span id="page-4-5"></span>In the above equations,  $\bar{y} = y/h$ ,  $\bar{\lambda} = \lambda/h$  ( $\lambda = \sqrt{\frac{\epsilon_0 \epsilon_r k_B T}{2 \epsilon_0^2 \sum_{n=1}^{n} n_r^2}}$ 2*e*<sup>2</sup> ∑ *<sup>i</sup> ni*,<sup>∞</sup> is the EDL thickness),  $\bar{d} = d/h$ ,  $\bar{\psi} = e\psi/(k_B T)$ ,  $\bar{n}_{H^+} =$  $n_{\text{H}^+}/n_{\infty}$ ,  $\bar{n}_{\text{OH}^-} = n_{\text{OH}^-}/n_{\infty}$ ,  $\bar{n}_{\text{H}^+,\infty} = n_{\text{H}^+,\infty}/n_{\infty}$ ,  $\bar{n}_{\text{OH}^-, \infty} =$  $n_{\text{OH}^{-},\infty}/n_{\infty}$ ,  $\bar{K}'_a = K'_a/n_{\infty}$ , and  $\bar{\gamma}_a = \gamma_a/n_{\infty}$ . Here,  $n_{\infty} = 10^3 N_A c_{\infty} (c_{\infty}$  is the concentration in *M*, while  $n_{\infty}$  is the

$$
\frac{d^2 \bar{\psi}}{d\bar{y}^2} = \frac{1}{2\bar{\lambda}^2} \left[ \bar{n}_- - \bar{n}_+ + \bar{n}_{OH^-} - \bar{n}_{H^+} + \varphi \bar{n}_{A^-} - \varphi \bar{n}_{BH^+} \right] \quad [\text{for } -1 \le \bar{y} \le -1 + \bar{d}]
$$
\n
$$
= \frac{1}{2\bar{\lambda}^2} \left[ \left( 1 + \bar{n}_{H^+,\infty} \right) \exp\left(\bar{\psi}\right) - \exp\left(-\bar{\psi}\right) + \bar{n}_{OH^-} - \bar{n}_{H^+} + \varphi \frac{\bar{K}_a' \bar{\gamma}_a}{\bar{K}_a' + \bar{n}_{H^+}} - \varphi \frac{\bar{K}_b' \bar{\gamma}_b}{\bar{K}_b' + \bar{n}_{OH^-}} \right],
$$
\n
$$
\frac{d^2 \bar{\psi}}{d\bar{y}^2} = \frac{1}{2\bar{\lambda}^2} \left[ \bar{n}_- - \bar{n}_+ + \bar{n}_{OH^-} - \bar{n}_{H^+} \right] \quad [\text{for } -1 + \bar{d} \le \bar{y} \le 0]
$$
\n
$$
= \frac{1}{2\bar{\lambda}^2} \left[ \left( 1 + \bar{n}_{H^+,\infty} \right) \exp\left(\bar{\psi}\right) - \exp\left(-\bar{\psi}\right) + \left(\bar{n}_{OH^-,\infty} \right) \exp\left(\bar{\psi}\right) - \left(\bar{n}_{H^+,\infty} \right) \exp\left(-\bar{\psi}\right) \right].
$$
\n(10)

$$
\bar{\psi} = -\frac{\ln\left(\frac{\bar{n}_{\rm H^{+}}}{\bar{n}_{\rm H^{+},\infty}}\right)}{1 + \frac{\bar{K}_{\rm a}^{\gamma}\bar{r}_{\rm a}\varphi(\bar{y})}{\left(\bar{K}_{\rm a}^{\prime} + \bar{n}_{\rm H^{+}}\right)^{2}}}
$$
 [for  $-1 \le \bar{y} \le -1 + \bar{d}$ ], (11)  
( $\bar{n}_{\rm tot}$ )

$$
\bar{\psi} = -\ln\left(\frac{\bar{n}_{\mathrm{H}^+}}{\bar{n}_{\mathrm{H}^+,\infty}}\right) \quad \text{[for} \quad -1 + \bar{d} \le \bar{y} \le 0\text{]}.
$$

$$
\bar{\psi} = \frac{\ln\left(\frac{\bar{n}_{\text{OH}^{-}}}{\bar{n}_{\text{OH}^{-}}\right)}}{1 + \frac{\bar{K}_{\bar{b}}^{\prime}\bar{\gamma}_{\bar{b}}\varphi(\bar{y})}{\left(\bar{K}_{\bar{b}}^{\prime} + \bar{n}_{\text{OH}^{-}}\right)^{2}}}
$$
 [for  $-1 \le \bar{y} \le -1 + \bar{d}$ ],  
\n
$$
\bar{\psi} = \ln\left(\frac{\bar{n}_{\text{OH}^{-}}}{\bar{n}_{\text{OH}^{-},\infty}}\right) \quad \text{[for } -1 + \bar{d} \le \bar{y} \le 0].
$$
\n(12)

The corresponding dimensionless boundary conditions obtained by non-dimensionalizing equation ([9\)](#page-3-1) become:

<span id="page-4-1"></span>number density in  $1/m<sup>3</sup>$ ). In addition, as established in our previous studies (Chen and Das [2015b](#page-16-7), [c](#page-16-22)), we can consider a cubic profile for  $\varphi(y)$ , that is

<span id="page-4-6"></span><span id="page-4-2"></span>
$$
\varphi(\bar{y}) = \beta(\bar{y} + 1 - \bar{d})^2 \left(\bar{y} + 1 + \frac{\bar{d}}{2}\right),\tag{14}
$$

where  $\beta = 4/\overline{d}^3$ . We provide a detailed discussion later on this choice of the cubic profile later in Sect. [4](#page-15-0).

<span id="page-4-0"></span>Furthermore, as we are considering the PZI brush layer dissociation in an acidic solution, the concentration of the OH<sup> $-$ </sup> ions would be very small, so that we have  $\bar{K}^{\prime}_{b} \gg \bar{n}_{\text{OH}}$ , and consequently, Eq. [\(12](#page-4-0)) reduces to:

$$
\bar{n}_{\text{OH}^{-}} = (\bar{n}_{\text{OH}^{-},\infty}) \exp\left[\bar{\psi}\left(1 + \varphi \frac{\bar{r}_{\text{b}}}{\bar{K}_{\text{b}}'}\right)\right] \quad \text{[for } -1 \le \bar{y} \le -1 + \bar{d}\text{]},
$$
\n
$$
\bar{n}_{\text{OH}^{-}} = (\bar{n}_{\text{OH}^{-},\infty}) \exp(\bar{\psi}) \quad \text{[for } -1 + \bar{d} \le y \le 0].
$$
\n(15)

<span id="page-4-4"></span><span id="page-4-3"></span>Therefore, Eq.  $(10)$  $(10)$  can be simplified as follows:

$$
\frac{d^2 \bar{\psi}}{d\bar{y}^2} = \frac{1}{2\bar{\lambda}^2} \left[ \left( 1 + \bar{n}_{H^+,\infty} \right) \exp\left(\bar{\psi}\right) - \exp\left(-\bar{\psi}\right) + \left(\bar{n}_{OH^-,\infty} \right) \exp\left[\bar{\psi}\left( 1 + \varphi \frac{\bar{r}_b}{\bar{K}_b'}\right) \right] - \bar{n}_{H^+} + \varphi \frac{\bar{K}_a' \bar{r}_a}{\bar{K}_a' + \bar{n}_{H^+}} - \varphi \bar{r}_b \right]
$$
\n
$$
\text{[for } -1 \le \bar{y} \le -1 + \bar{d}],\tag{16}
$$
\n
$$
\frac{d^2 \bar{\psi}}{d\bar{y}^2} = \frac{1}{2\bar{\lambda}^2} \left[ \left( 1 + \bar{n}_{H^+,\infty} \right) \exp\left(\bar{\psi}\right) - \exp\left(-\bar{\psi}\right) + \left(\bar{n}_{OH^-,\infty} \right) \exp\left(\bar{\psi}\right) - \left(\bar{n}_{H^+,\infty} \right) \exp\left(-\bar{\psi}\right) \right] \text{[for } -1 + \bar{d} \le \bar{y} \le 0].
$$

The explicit equilibrium electrostatic potential,  $H^+$ , and  $OH^$ ion concentration distributions can be obtained by numerically solving the coupled equations (Eqs. [11,](#page-4-2) [15](#page-4-3), [16](#page-4-4)) in the presence of the boundary condition expressed in  $(13)$  $(13)$ .

#### **Poly-zwitterionic brush layer in basic solution**

We next consider the case where the PZI brush layer is dissociating in a basic solution. We consider that the base furnishes the same cation as the salt. As a consequence, the bulk number density of the salt cation will be  $n_{\infty} + n_{\text{OH}^{-},\infty}$ .

### **2.2 Velocity field**

The pressure-driven transport considered here would give rise to an electric field. This electric field will d rive an electroosmotic (EOS) flow, whose direction would always be opposite to the direction of the pressure-driven transport. Considering this overall velocity field (which is a combination of the pressure-driven transport and an EOS flow) to be steady, uni-directional, and hydrodynamically fully developed, we can express it for the channel bottom half as follows:

$$
\eta \frac{d^2 u}{dy^2} - \frac{dp}{dx} + eE_S(n_+ - n_- + n_{H^+} - n_{OH^-}) - \frac{\eta}{\kappa} u = 0 \quad [-h \le y \le -h + d],
$$
  
\n
$$
\eta \frac{d^2 u}{dy^2} - \frac{dp}{dx} + eE_S(n_+ - n_- + n_{H^+} - n_{OH^-}) = 0 \quad [-h + d \le y \le 0].
$$
\n(19)

Furthermore, the solution being basic, we would have  $\bar{K}^{\prime}_{a} \gg \bar{n}_{\text{H}^{+}}$ , and consequently, Eq. ([11\)](#page-4-2) reduces to:

$$
\bar{n}_{\mathrm{H}^+} = (\bar{n}_{\mathrm{H}^+,\infty}) \exp\left[-\bar{\psi}\left(1 + \varphi \frac{\bar{r}_a}{\bar{K}_a^{\prime}}\right)\right] \quad \text{[for } -1 \le \bar{y} \le -1 + \bar{d}\text{]},
$$
  

$$
\bar{n}_{\mathrm{H}^+} = (\bar{n}_{\mathrm{H}^+,\infty}) \exp(-\bar{\psi}) \quad \text{[for } -1 + \bar{d} \le y \le 0].
$$
 (17)

Under these conditions, Eq.  $(10)$  can be simplified as follows:

<span id="page-5-1"></span><span id="page-5-0"></span>In Eq. ([19](#page-5-0)),  $-dp/dx$  is the employed pressure gradient,  $\eta$  is the dynamic viscosity of the liquid, *e* is the electronic charge,  $n_i$  is the number density of the ionic species *i*, and  $\kappa = a_k^2$ ( *<sup>d</sup>*  $\sigma a_k^3 N_p \varphi$  $\int_a^2$  is the permeability and  $\frac{\sigma a_k^3 N_p \varphi}{d}$  is the volume fraction of the PZI brush layer. For the present study, we consider the cubic profile for  $\varphi$  (see Eq. [14\)](#page-4-6). Of course, the

$$
\frac{d^2 \bar{\psi}}{d\bar{y}^2} = \frac{1}{2\bar{\lambda}^2} \left[ \exp(\bar{\psi}) - \left( 1 + \bar{n}_{\text{OH}^-, \infty} \right) \exp(-\bar{\psi}) + \bar{n}_{\text{OH}^-} - \left( \bar{n}_{\text{H}^+, \infty} \right) \exp\left[ -\bar{\psi} \left( 1 + \varphi \frac{\bar{r}_a}{\bar{K}_a'} \right) \right] + \varphi \bar{r}_a - \varphi \frac{\bar{K}_b' \bar{r}_b}{\bar{K}_b' + \bar{n}_{\text{OH}^-}} \right]
$$
\n
$$
\text{[for } -1 \le \bar{y} \le -1 + \bar{d}],
$$
\n
$$
\frac{d^2 \bar{\psi}}{d\bar{y}^2} = \frac{1}{2\bar{\lambda}^2} \left[ \exp(\bar{\psi}) - \left( 1 + \bar{n}_{\text{OH}^-, \infty} \right) \exp(-\bar{\psi}) + \left( \bar{n}_{\text{OH}^-, \infty} \right) \exp(\bar{\psi}) - \left( \bar{n}_{\text{H}^+, \infty} \right) \exp(-\bar{\psi}) \right] \left[ \text{for } -1 + \bar{d} \le \bar{y} \le 0 \right].
$$
\n(18)

The explicit equilibrium electrostatic potential, H<sup>+</sup>, and OH<sup>−</sup> ion concentration distributions can be obtained by numerically solving the coupled equations (Eqs. [12,](#page-4-0) [17](#page-5-1), [18](#page-5-2)) in the

presence of the boundary condition expressed in  $(13)$  $(13)$ .

<span id="page-5-2"></span>solution of the velocity field *u* would be sought in the presence of the known distribution of  $\psi$ ,  $n_{+}$ ,  $n_{\text{H}^{+}}$ , and  $n_{\text{OH}^{-}}$ .

<span id="page-5-3"></span>Using the calculations provided in the previous section, Eq. [\(19](#page-5-0)) can be expressed in dimensionless form as follows. In acidic solution:

$$
0 = \frac{d^2 \bar{u}}{d\bar{y}^2} - 1 + \frac{\bar{E}_S}{2\bar{\lambda}^2} [\exp(-\bar{\psi}) - (1 + \bar{n}_{H^+,\infty}) \exp(\bar{\psi}) + \bar{n}_{H^+} - \bar{n}_{OH^-,\infty} \exp\left[\bar{\psi}\left(1 + \varphi \frac{\bar{\gamma}_b}{\bar{K}_b'}\right)\right] - \bar{\alpha}^2 \varphi^2 \bar{u} \n\left[-1 \le \bar{y} \le -1 + \bar{d}\right],
$$
\n
$$
0 = \frac{d^2 \bar{u}}{d\bar{y}^2} - 1 + \frac{\bar{E}_S}{2\bar{\lambda}^2} [\exp(-\bar{\psi}) - (1 + \bar{n}_{H^+,\infty}) \exp(\bar{\psi}) + \bar{n}_{H^+,\infty} \exp(-\bar{\psi}) - \bar{n}_{OH^-,\infty} \exp(\bar{\psi})] \left[-1 + \bar{d} \le \bar{y} \le 0\right].
$$
\n(20)

In basic solution:

$$
0 = \frac{d^2 \bar{u}}{d\bar{y}^2} - 1 + \frac{\bar{E}_S}{2\bar{\lambda}^2} [(1 + \bar{n}_{OH^-,\infty}) \exp(-\bar{\psi}) - \exp(\bar{\psi}) + \bar{n}_{H^+,\infty} \exp\left[-\bar{\psi}\left(1 + \varphi \frac{\bar{r}_a}{\bar{K}_a'}\right)\right] - \bar{n}_{OH^-}] - \bar{\alpha}^2 \varphi^2 \bar{u}
$$
  
\n
$$
[-1 \le \bar{y} \le -1 + \bar{d}],
$$
  
\n
$$
0 = \frac{d^2 \bar{u}}{d\bar{y}^2} - 1 + \frac{\bar{E}_S}{2\bar{\lambda}^2} [(1 + \bar{n}_{OH^-,\infty}) \exp(-\bar{\psi}) - \exp(\bar{\psi}) + \bar{n}_{H^+,\infty} \exp(-\bar{\psi}) - \bar{n}_{OH^-,\infty} \exp(\bar{\psi})] [-1 + \bar{d} \le \bar{y} \le 0].
$$
\n(21)

In Eqs. [\(20](#page-5-3)) and ([21\)](#page-6-0),  $\bar{u} = \frac{u}{u_{p,0}}$  (where  $u_{p,0} = \frac{h^2}{\eta}$ d*p*  $\frac{d\varphi}{dx}$  is pressuredriven velocity scale),  $\bar{E}_S = \frac{E_S}{E_0}$  (where  $E_0 = \frac{e\eta u_{p,0}}{\epsilon_0 \epsilon_r k_B T}$ d*p* d*x eh*<sup>2</sup>  $\frac{eh^2}{\epsilon_0 \epsilon_r k_B T}$  is the scale of the electric field), and  $\bar{\alpha} = \frac{\sigma a_k^2 N_p}{d}$ .  $u_r = \frac{u_{e,0}}{u_{p,0}}$  is taken to be unity, where  $u_{e,0} = \frac{k_B T}{e}$  $\frac{\epsilon_0 \epsilon_r E_0}{\eta}$  is the electroosmotic velocity scale. Solution of Eqs. [\(20](#page-5-3)) and ([21\)](#page-6-0) is sought in the presence of the following dimensionless boundary conditions:

<span id="page-6-2"></span><span id="page-6-0"></span>
$$
i = 2e \int_{-h}^{0} \left( u_{+}n_{+} - u_{-}n_{-} + u_{H^{+}}n_{H^{+}} - u_{OH^{-}}n_{OH^{-}} \right) dy = 0,
$$
\n(23)

where  $u_i$  ( $i = \pm, H^+, OH^-$ ) is the ion migration velocity, which is expressed as follows:

<span id="page-6-1"></span>
$$
u_i = u + \frac{ez_i E_S}{f_i}.\tag{24}
$$

$$
(\bar{u})_{\bar{y}=-1} = 0; \left(\frac{d\bar{u}}{d\bar{y}}\right)_{\bar{y}=0} = 0; \quad (\bar{u})_{\bar{y}=(-1+\bar{d})^{+}} = (\bar{u})_{\bar{y}=(-1+\bar{d})^{-}}; \quad \left(\frac{d\bar{u}}{d\bar{y}}\right)_{\bar{y}=(-1+\bar{d})^{+}} = \left(\frac{d\bar{u}}{d\bar{y}}\right)_{\bar{y}=(-1+\bar{d})^{-}}.
$$
\n
$$
(22)
$$

Of course, the solution of  $\bar{u}$  requires the value of the  $\bar{E}_S$ . Calculation of  $\bar{E}_\text{S}$  is discussed in the following subsection.

## **2.3 Streaming electric field**  $E_5$

Here,  $f_i$  is the ionic friction coefficient and  $z_i$  is the valence for ion *i*. Substituting Eq.  $(24)$  $(24)$  in Eq.  $(23)$  $(23)$ , we finally obtain the dimensionless streaming electric field as follows.

<span id="page-6-4"></span>In acidic solution:

To obtain  $E<sub>S</sub>$ , we consider that the net ionic current (per unit width) *i* is equal to zero, that is



<span id="page-6-3"></span>**Fig. 3** Transverse variation of **a**  $\bar{\psi}$  and **b**  $\bar{u}$  for different values of  $c_{\infty}$ . Here, we consider pH<sub>∞</sub> = 10 (bulk pH). All other parameters are identical to that used in Fig. [2](#page-3-2)



<span id="page-7-2"></span>**Fig.** 4 Transverse variation of **a**  $\bar{\psi}$  and **b**  $\bar{u}$  for different values of  $\bar{d}$ . Here, we consider  $c_{\infty} = 10^{-4}$  M. All other parameters are identical to that used in Fig. [2](#page-3-2)



<span id="page-7-3"></span>**Fig.** 5 Transverse variation of **a**  $\bar{\psi}$  and **b**  $\bar{u}$  for different values of  $\bar{d}$ . Here, we consider  $c_{\infty} = 10^{-4}$  M. All other parameters are identical to that used in Fig. [3](#page-6-3)

<span id="page-7-0"></span>
$$
\bar{E}_{\rm S} = \frac{\int_{-1}^{0} \bar{u} \left[ -\exp(-\bar{\psi}) + (1 + \bar{n}_{\rm H^{+},\infty}) \exp(\bar{\psi}) - \bar{n}_{\rm H^{+}} + \bar{n}_{\rm OH^{-}} \right] d\bar{y}}{\int_{-1}^{0} [R_{+} \exp(-\bar{\psi}) + R_{-}(1 + \bar{n}_{\rm H^{+},\infty}) \exp(\bar{\psi}) + R_{\rm H^{+}} \bar{n}_{\rm H^{+}} + R_{\rm OH^{-}} \bar{n}_{\rm OH^{-}}] d\bar{y}}.
$$
\n(25)

In basic solution:

<span id="page-7-1"></span>
$$
\bar{E}_{\rm S} = \frac{\int_{-1}^{0} \bar{u} \left[ -(1 + \bar{n}_{\rm OH^{-}}, \Delta) \exp(-\bar{\psi}) + \exp(\bar{\psi}) - \bar{n}_{\rm H^{+}} + \bar{n}_{\rm OH^{-}} \right] d\bar{y}}{\int_{-1}^{0} [R_{+}(1 + \bar{n}_{\rm OH^{-}}, \Delta) \exp(-\bar{\psi}) + R_{-} \exp(\bar{\psi}) + R_{\rm H^{+}} \bar{n}_{\rm H^{+}} + R_{\rm OH^{-}} \bar{n}_{\rm OH^{-}})] d\bar{y}},\tag{26}
$$



<span id="page-8-0"></span>**Fig.** 6 Transverse variation of **a**  $\bar{\psi}$  and **b**  $\bar{u}$  for different values of p $K_a$ . Here, we consider  $c_\infty = 10^{-4}$  M. All other parameters are identical to that used in Fig. [2](#page-3-2)



<span id="page-8-1"></span>**Fig. 7** Transverse variation of **a**  $\bar{\psi}$  and **b**  $\bar{u}$  for different values of pK<sub>b</sub>. Here, we consider  $c_{\infty} = 10^{-4}$  M. All other parameters are identical to that used in Fig. [3](#page-6-3)

where  $R_i = \frac{e^2 z_i^2 \eta}{\epsilon_0 \epsilon k_0}$  $\frac{1}{\epsilon_0 \epsilon_r k_B T_f}$  is a dimensionless parameter, often interpreted as the inverse of the ionic Peclet number. We take  $f = \frac{eE_0}{u_{p,0}} = \frac{e^2\eta}{\epsilon_0 \epsilon_r k_B T}$ . Of course, we would use Eq. ([25](#page-7-0)) in Eq. [\(20\)](#page-5-3) to obtain the integro-differential equation governing the velocity field  $\bar{u}$  within the PZI brush-grafted nanochannel in acidic condition; on the other hand, we would use Eq.  $(26)$  $(26)$ in Eq.  $(21)$  $(21)$  to obtain the integro-differential equation governing the velocity field  $\bar{u}$  within the PZI brush-grafted

nanochannel in basic condition. The integro-differential equations for both the cases are solved numerically in the presence of the BCs expressed in Eq. ([22](#page-6-4)). We were the first group to develop and solve such highly involved integrodifferential equations for obtaining the streaming electric field and the resulting electrokinetics in nanochannels grafted with charged polyelectrolyte brushes (Chen and Das [2015d;](#page-16-15) Patwary et al. [2015;](#page-17-12) Chen et al. [2018\)](#page-16-16)—in this study, we again use that theoretical framework to compute the



<span id="page-9-1"></span>**Fig.** 8 Transverse variation of **a**  $\bar{\psi}$  and **b**  $\bar{u}$  for different pH<sub>∞</sub> (bulk pH) values in an acidic solution. Here, we consider  $c_{\infty} = 10^{-4}$  M. All other parameters are identical to that used in Fig. [2](#page-3-2)



<span id="page-9-2"></span>**Fig. 9** Transverse variation of **a**  $\bar{\psi}$  and **b**  $\bar{u}$  for different pH<sub>∞</sub> (bulk pH) values in a basic solution. Here, we consider  $c_{\infty} = 10^{-4}$  M. All other parameters are identical to that used in Fig. [2](#page-3-2)

induced electrokinetics in nanochannels grafted with the PZI brushes.

# **2.4 Efficiency of the electrochemomechanical energy conversion**

Generation of the nanofluidic streaming current  $(i<sub>s</sub>)$  and the streaming electric field  $(E<sub>s</sub>)$  is a process of nanoscale electrochemomechanical energy conversion, since the mechanical energy of the pressure-driven flow and the chemical energy of the EDL are converted to the electrical energy associated with the generation of  $i_S$  and  $E_S$ . This efficiency  $\xi$ of this energy conversion can be expressed as follows:

<span id="page-9-0"></span>
$$
\xi = \frac{P_{\text{out}}}{P_{\text{in}}}.\tag{27}
$$

Here,  $P_{\text{in}}$  and  $P_{\text{out}}$  are the input and the output powers (per unit area), expressed as follows:

$$
P_{\text{out}} = \frac{1}{4} i_{\text{S}} E_{\text{S}}, \quad P_{\text{in}} = \left| -\frac{\mathrm{d}p}{\mathrm{d}x} \right| Q_{\text{in}}.\tag{28}
$$

Here, the streaming current is

$$
i_{\rm S} = 2e \int_{-h}^{0} u(n_{+} - n_{-} + n_{\rm H^{+}} - n_{\rm OH^{-}}) dy,
$$
 (29)

or in a dimensionless form:

$$
\bar{i}_{\rm S} = \int_{-1}^{0} \bar{u} \left( \bar{n}_{+} - \bar{n}_{-} + \bar{n}_{\rm H^{+}} - \bar{n}_{\rm OH^{-}} \right) d\bar{y},\tag{30}
$$

<span id="page-10-0"></span>where  $\bar{i}_S = \frac{i_S}{2h\epsilon n_\infty u_{p,0}}$  and  $Q_{in}$  is the input volume flow rate per unit width, expressed as follows:

<span id="page-10-2"></span>
$$
Q_{\rm in} = 2 \int_{-h}^{0} u_p \, \mathrm{d}y. \tag{31}
$$

Here,  $u_p$  is the pure pressure-driven velocity field governed by the following equations:

<span id="page-10-1"></span>
$$
\eta \frac{d^2 u_p}{dy^2} - \frac{dp}{dx} - \frac{\eta}{\kappa} u_p = 0, \qquad [-h \le y \le -h + d_0],
$$
  

$$
\eta \frac{d^2 u_p}{dy^2} - \frac{dp}{dx} = 0, \qquad [-h + d_0 \le y \le 0].
$$
 (32)





<span id="page-10-3"></span>**Fig. 10** Variation of **a** streaming electric field  $E_s$ , **b** steaming current  $i_S$ , **c** output power  $P_{\text{out}}$ , and **d** electrochemomechanical energy conversion efficiency  $\xi$  with pH<sub>∞</sub> for different values of  $c_{\infty}$ . To calculate the power, we use  $\frac{dp}{dx} = -5 \times 10^8 \text{ Pa/m}, \eta = 8.9 \times 10^{-4} \text{ Pa s, and consider}$ 

a nanofluidic chip that is 1 mm×10 cm×10 cm in dimensions (i.e., 1 mm in length and 10 cm in both breadth and width) with a porosity of 0.5. All other parameters are identical to that used in Fig. [2](#page-3-2)



<span id="page-11-2"></span>Fig. 11 Variation of a  $E_s$ , b  $i_s$ , c  $P_{out}$ , and d  $\xi$  with pH<sub>∞</sub> for different values of  $\overline{d}$ . Here we use  $c_\infty = 10^{-4}$  M. All other parameters are identical to that used in Fig. [10](#page-10-3)

For the acidic solution, we can, therefore, obtain (using Eqs. [25](#page-7-0), [27](#page-9-0), [28](#page-10-0), [30](#page-10-1), [31\)](#page-10-2):

Of course, both Eqs.  $(33)$  $(33)$  $(33)$  and  $(34)$  can be simplified to a unique form expressed as follows:

$$
\xi = \frac{1}{8\bar{\lambda}^2 \int_{-1}^{0} \bar{u}_p \mathrm{d}\bar{y}} \frac{\left[ \int_{-1}^{0} \bar{u} \left[ -\exp(-\bar{\psi}) + (1 + \bar{n}_{H^+,\infty}) \exp(\bar{\psi}) - \bar{n}_{H^+} + \bar{n}_{OH^-} \right] \mathrm{d}\bar{y} \right]^2}{\int_{-1}^{0} \left[ \exp(-\bar{\psi}) + (1 + \bar{n}_{H^+,\infty}) \exp(\bar{\psi}) + \bar{n}_{H^+} + \bar{n}_{OH^-} \right] \mathrm{d}\bar{y}}.
$$
(33)

On the other hand, for the basic solution, we can obtain the following (using Eqs. [26](#page-7-1), [27](#page-9-0), [28](#page-10-0), [30](#page-10-1), [31](#page-10-2)). In basic solution:

<span id="page-11-1"></span><span id="page-11-0"></span>
$$
\xi = \frac{1}{8\bar{\lambda}^2 \int_{-1}^{0} \bar{u}_p \, d\bar{y}} \frac{\bar{i}_S^2}{\bar{i}_C} = \frac{\bar{i}_C \bar{E}_S^2}{8\bar{\lambda}^2 \int_{-1}^{0} \bar{u}_p \, d\bar{y}}.
$$
\n(35)

$$
\xi = \frac{1}{8\bar{\lambda}^2 \int_{-1}^{0} \bar{u}_{p} d\bar{y}} \frac{\left[\int_{-1}^{0} \bar{u}\left[-(1 + \bar{n}_{OH^{-},\infty}) \exp(-\bar{\psi}) + \exp(\bar{\psi}) - \bar{n}_{H^{+}} + \bar{n}_{OH^{-}}\right] d\bar{y}\right]^2}{\int_{-1}^{0} [\exp(-(1 + \bar{n}_{OH^{-},\infty})\bar{\psi}) + \exp(\bar{\psi}) + \bar{n}_{H^{+}} + \bar{n}_{OH^{-}}\right] d\bar{y}}.
$$
(34)



<span id="page-12-0"></span>Fig. 12 Variation of a  $E_s$ , b  $i_s$ , (c  $P_{out}$ , and d  $\xi$  with pH<sub>∞</sub> for different values of pK<sub>a</sub>. Here, we use  $c_\infty = 10^{-4}$  M. All other parameters are identical to that used in Fig. [10](#page-10-3)

# **3 Results**

In Figs.  $2, 3, 4, 5, 6, 7, 8$  $2, 3, 4, 5, 6, 7, 8$  $2, 3, 4, 5, 6, 7, 8$  $2, 3, 4, 5, 6, 7, 8$  $2, 3, 4, 5, 6, 7, 8$  $2, 3, 4, 5, 6, 7, 8$  $2, 3, 4, 5, 6, 7, 8$  $2, 3, 4, 5, 6, 7, 8$  $2, 3, 4, 5, 6, 7, 8$  $2, 3, 4, 5, 6, 7, 8$  $2, 3, 4, 5, 6, 7, 8$  $2, 3, 4, 5, 6, 7, 8$  $2, 3, 4, 5, 6, 7, 8$ , and  $9$ , we provide the transverse variation of the dimensionless electrostatic potential  $(\bar{\psi})$  and the dimensionless velocity  $(\bar{u})$  for different combinations of the system parameters. An acidic solution (characterized by pH<sub>∞</sub> < 7) implies the presence of more H<sup>+</sup> ions than OH<sup>−</sup> ions. As a consequence, the ionization that produces the BH<sup>+</sup> charged group (this ionization produces more OH<sup>−</sup> ions) is more preferred than the ionization that produces the A<sup>−</sup> group (this ionization produces more  $H^+$  ions). Therefore, for such a pH  $(< 7)$ , the PZI attains a net positive charge under identical values of  $pK_a$  and  $pK_b$  leading to a positive value of the corresponding  $\bar{\psi}$ . This is evident in Figs. [2](#page-3-2)a, [4](#page-7-2)a, [6a](#page-8-0), and [8](#page-9-1)a. On the other hand, a basic solution (characterized by  $pH_{\infty} > 7$ ) has more OH<sup>-</sup> ions than H<sup>+</sup> ions. Accordingly, the ionization that produces  $H^+$  ions (i.e., the ionization that produces the A<sup>−</sup> group of the PZI) is much more preferred than the ionization that produces the OH<sup>−</sup> ions (i.e., the ionization that produces the  $BH<sup>+</sup>$  group of the PZI). As a consequence, for such a pH<sub>∞</sub> ( $>$  7), the PZI attains a net negative charge under identical values of  $pK_a$  and  $pK_b$  leading to a negative value of the corresponding  $\bar{\psi}$  (see Figs. [3](#page-6-3)a, [5a](#page-7-3), [7](#page-8-1)a, [9](#page-9-2)a). For both the cases of positive and negative  $\bar{\psi}$ , a decrease in the salt concentration ( $c_{\infty}$ ) increases the magnitude of  $\bar{\psi}$ . Smaller  $c_{\infty}$  leads to a larger EDL thickness ( $\lambda$ ), which would imply a larger  $\bar{\psi}$  for a given charge density ( $\sigma$ ), attributed to the fact that  $d\psi/dy \sim \sigma/(\epsilon_0 \epsilon_r)$  ⇒  $\psi \sim \sigma \lambda/(\epsilon_0 \epsilon_r)$ . This is evident in Figs. [2a](#page-3-2) and [3a](#page-6-3). Furthermore, an increase in the relative brush height (or larger *d*/*h* value) leads to a larger charge content of the system leading to a greater magnitude



<span id="page-13-0"></span>Fig. 13 Variation of a  $E_s$ , b  $i_s$ , c  $P_{\text{out}}$ , and d  $\xi$  with pH<sub>∞</sub> for different values of pK<sub>b</sub>. Here, we use  $c_\infty = 10^{-4}$  M. All other parameters are identical to that used in Fig. [10](#page-10-3)

(either positive or negative) of  $\bar{\psi}$  (see Figs. [4](#page-7-2)a, [5a](#page-7-3)). A larger value of  $pK_a$  for the case where the charging of the PZI is dominated by the formation of the positive sites (i.e., the situation that occurs at an acidic pH or  $pH_{\infty}$  < 7) implies that the ionization of the PZI to produce the negative sites is retarded and, therefore, leads to a large net positive charge on the PZI and a larger positive magnitude of  $\bar{\psi}$ . This is depicted in Fig. [6a](#page-8-0). Exactly reverse occurs for a basic solution ( $pH > 7$ ) and larger  $pK_b$ . For such a solution, the PZI charge is dominated by the formation of the negative sites and a larger  $pK_b$  implies a weaker ionization of the positive sites making the PZI more negative (and hence,  $\bar{\psi}$  more negative). This is depicted in Fig. [7a](#page-8-1). Finally, in Figs. [8](#page-9-1) and [9](#page-9-2), we show the effect of the variation in  $pH_{\infty}$ . In the acidic range, a progressive lowering of  $pH_{\infty}$  (or a progressive increase in the number of  $H^+$  ions) implies a more retarded

ionization of the negative group of the PZI (this ionization produced  $H^+$  ions) implying a larger manifestation of the positive charge of the PZI ensuring a larger positive value of  $\bar{\psi}$ . This is witnessed for pH<sub>∞</sub> values varying from 6 to 4. However, for pH<sub>∞</sub> 3, we find that the  $\bar{\psi}$  becomes smaller than that at  $pH_{\infty}$  4. The reason is that, since we operate at  $c_{\infty} = 10^{-4}$  M, at pH 3 (or  $c_{H^+,\infty} = 10^{-3}$  M), the hydrogen ion concentration dictates the EDL thickness causing a decrease in the EDL thickness as compared to the case when  $\rm{pH}_{\infty}$  4. This lowering of the EDL thickness reduces the overall  $\bar{\psi}$ . This behavior is witnessed in Fig. [8](#page-9-1)a. On the other hand, in the basic range, a progressive increase in  $pH_{\infty}$  implying a progressive lowering of  $pOH<sub>∞</sub>$  (or a progressive increase in the number of OH<sup>−</sup> ions) leads to a suppression of the ionization that generates positive charge of the PZI (this ionization also produces the OH<sup>−</sup> ion) enforcing a larger negative

charge of the PZI. Therefore, one witnesses a progressively larger negative magnitude of  $\bar{\psi}$  as pH<sub>∞</sub> increases from 8 to 10. However, at  $pH_{\infty}$  11 or  $pOH_{\infty}$  3, the concentration of the OH<sup>−</sup> ions dictates the EDL thickness making the EDL thickness smaller than that for  $pH_{\infty}$  10 (or  $pOH_{\infty}$  4) enforcing a reduction in  $\bar{\psi}$  (see Fig. [9](#page-9-2)a).

The part (b) of Figs. [2](#page-3-2), [3](#page-6-3), [4,](#page-7-2) [5,](#page-7-3) [6](#page-8-0), [7](#page-8-1), [8,](#page-9-1) and [9](#page-9-2) provide the variation of the overall velocity field for the different combination of the system parameters. The overall velocity field is a combination of the pressure-driven transport (caused by the employed pressure gradient) and the induced EOS transport caused by the induced streaming electric field (shown in Figs. [10](#page-10-3), [11](#page-11-2), [12](#page-12-0), [13](#page-13-0)a). Regardless of the value of  $pH_{\infty}$  (or the corresponding sign of the net charge on the PZI), the EOS transport always opposes the pressure-driven transport and hence reduces the overall transport. Please note that here both the pressure-driven transport and hence the overall transport are positive—however, the net transport appears negative as we non-dimensionalize the velocity field by a characteristic velocity that is negative (i.e.,  $u_{p,0} = \frac{h^2}{\eta}$  $\frac{dp}{dx}$  < 0).

The induced electric field  $(E<sub>s</sub>)$  driving the EOS transport is positive for the acidic pH and negative for the basic pH (see Figs.  $10$ ,  $11$ ,  $12$ ,  $13a$ ).  $E<sub>S</sub>$  is induced by the downstream migration of the non-zero charge density of the EDL. For the acidic pH, the PZI is positively charged (manifested by a positive magnitude of  $\bar{\psi}$ ); therefore, the counterions will be anions. Thus, the downstream migration of the EDL charge density would imply a net downstream migration of the negative charges, thereby leading to a larger downstream accumulation of the negative charges. Therefore, the net potential will be more positive on the upstream side than the downstream side, ensuring that the electric field is positive (i.e., from left to right). This electric field interacts with the net EDL charge density to induce the EOS transport. The per unit volume EOS body force is  $f_{EOS} = e(n_+ - n_-)E_S$ . Of course, a positive  $E_S$  occurs when  $n_+ > n_+$  (as already discussed above) ensuring  $f_{\text{EOS}} < 0$  and hence  $u_{\text{EOS}} < 0$ . For a basic pH, the net PZI charge is negative making the counterions positive, and therefore, the downstream advection of the EDL charge density leads to a downstream accumulation of the positive ions. This ensures that the net potential is more positive downstream, enforcing  $E<sub>S</sub> < 0$ . Of course, as  $n_+ > n_$  for this case,  $f_{\text{EOS}} = e(n_+ - n_-)E_{\text{S}} < 0$  and  $u_{\text{EOS}} < 0$ for this case, as well.

A larger magnitude of  $\bar{\psi}$  leads to a larger difference between the counterion and coion number density within an EDL, which, in turn, would enforce both a larger magnitude of  $E<sub>S</sub>$  and an even larger magnitude of  $f<sub>FOS</sub>$ . Therefore, cases with a larger magnitude of  $\bar{\psi}$  would result in a larger magnitude of  $u_{\text{EOS}}$  and, hence, a larger reduction in the overall velocity field. Therefore, we witness a lesser velocity for a weaker salt concentration (see Figs. [2b](#page-3-2), [3b](#page-6-3)), for a larger

brush height (see Figs. [4b](#page-7-2), [5](#page-7-3)b), for a larger  $pK_a$  for acidic solution (see Fig. [6b](#page-8-0)), for a larger  $pK<sub>b</sub>$  for basic solution (see Fig. [7](#page-8-1)b), for smaller  $pH_{\infty}$  for acidic solution (see Fig. [8b](#page-9-1)) (except for very small  $pH_{\infty}$ , where the hydrogen ion number density dictates the EDL thickness), and for larger  $pH_{\infty}$ for basic solution (see Fig. [9b](#page-9-2)) (except for very large  $\rm{pH}_{\infty}$ ) where the hydroxyl ion number density dictates the EDL thickness).

Figures [10](#page-10-3), [11](#page-11-2), [12](#page-12-0) and [13a](#page-13-0) provide the variation of the streaming electric field  $E_S$  with pH<sub>∞</sub> for different system parameters. We invariably find a positive (negative)  $E<sub>S</sub>$  for acidic (basic) pH. As we have already discussed above, such a behavior can be attributed to the net positive (negative) charge of the PZI leading to anions (cations) becoming the counterions at an acidic (basic) pH. In addition, all the factors that lead to an enhancement in the magnitude of  $\bar{\psi}$  (see Figs. [2,](#page-3-2) [3,](#page-6-3) [4](#page-7-2), [5](#page-7-3), [6,](#page-8-0) [7,](#page-8-1) [8](#page-9-1)[9](#page-9-2)a) would augment the magnitude of  $E<sub>S</sub>$ . Such a connection directly follows from the fact that a larger magnitude of  $\bar{\psi}$  would imply a larger difference in the number densities between the counterions and coions, and hence, a larger magnitude of the electrostatic potential difference (caused by the flow-driven downstream accumulation of the counterions) leads to a larger  $E<sub>s</sub>$ . Therefore, one witnesses a larger magnitude of  $E<sub>S</sub>$  for a weaker salt concentration (see Fig. [10](#page-10-3)a), for a larger brush height (see Fig. [11a](#page-11-2)), for a larger  $pK_a$  for an acidic solution (see Fig. [12a](#page-12-0)), for a larger p $K<sub>b</sub>$  for a basic solution (see Fig. [13](#page-13-0)a), for smaller pH<sub>∞</sub> for acidic solution (see Figs. [10,](#page-10-3) [11](#page-11-2), [12](#page-12-0), [13a](#page-13-0)) (except for very small  $pH_{\infty}$ , where the hydrogen ion number density dictates the EDL thickness and this ensures a maximum in the magnitude of  $E_S$  at an intermediate pH<sub>∞</sub>), and for larger pH<sub>∞</sub> for basic solution (see Figs. [10,](#page-10-3) [11,](#page-11-2) [12,](#page-12-0) [13](#page-13-0)a) (except for very large  $pH_{\infty}$  where the hydroxyl ion number density dictates the EDL thickness and this ensures a minimum or a negative maximum in the magnitude of  $E<sub>S</sub>$  at an intermediate pOH<sub>∞</sub>). A critical observation from all the  $E<sub>S</sub>$  plots is a remarkable symmetry (in magnitude) across the  $pH_{\infty}$  spectrum. In other words, we get the same magnitude (with different sign) for the same values of  $pH_{\infty}$  and  $pOH_{\infty}$  (i.e., at large and small  $pH_{\infty}$ ). This obviously stems from the fact that PZI becomes charged at these extreme  $pH_{\infty}$  values. Therefore, this study points to this unique opportunity where one can attain a large magnitude of  $E<sub>S</sub>$  for both large and small pH.

Figures [10](#page-10-3), [11,](#page-11-2) [12](#page-12-0) and [13](#page-13-0)b provide the variation of the streaming current  $i_S$  with pH<sub>∞</sub> for different system parameters. This streaming current when multiplied by the streaming electric field produces the net output power (see Figs. [10,](#page-10-3) [11](#page-11-2), [12,](#page-12-0) [13c](#page-13-0)), which follows the same trend with the different parameters as the electric field  $E<sub>S</sub>$ . Therefore, we witness an increase in power with weaker  $c_{\infty}$  (see Fig. [10](#page-10-3)c), for a larger brush height (see Fig. [11](#page-11-2)c), for a larger  $pK_a$  for an acidic pH (see Fig. [12c](#page-12-0)), for a larger  $pK<sub>b</sub>$  for a basic solu-tion (see Fig. [13c](#page-13-0)), for smaller  $pH_{\infty}$  for acidic solution (see Figs. [10](#page-10-3), [11,](#page-11-2) [12,](#page-12-0) [13a](#page-13-0)) (except for very small  $pH_{\infty}$ , where the hydrogen ion number density dictates the EDL thickness and this ensures a maximum in the magnitude of power at an intermediate  $pH_{\infty}$ ), and for larger  $pH_{\infty}$  for basic solution (see Figs. [10,](#page-10-3) [11,](#page-11-2) [12,](#page-12-0) [13](#page-13-0)a) (except for very large  $pH_{\infty}$  where the hydroxyl ion number density dictates the EDL thickness and this ensures a minimum or a negative maximum in the magnitude of power at an intermediate pOH<sub>∞</sub>). Very much like  $E<sub>S</sub>$ , here too, we ensure a large *P* for both large and small pH<sub>∞</sub>. Finally, in Figs. [10](#page-10-3), [11,](#page-11-2) [12](#page-12-0) and [13](#page-13-0)d, we show the variation in the efficiency  $\xi$  in the electrokinetic (or electrochemomechanical) energy conversion. The trend with respect to different parameters is exactly identical to that of the power variation. Most importantly, here too, we ensure a significant conversion efficiency for both large and small pH.

# <span id="page-15-0"></span>**4 Discussions**

## **4.1 Neglecting the PE brush configurational details**

In the development of our theoretical model, we have neglected the configurational details of the PE brush. In other words, we have assumed a constant salt-concentration-independent height of the PE brush while developing our model. As we have established in our previous papers (Chen and Das [2015a;](#page-16-6) Li et al. [2016\)](#page-17-13), such an assumption is only valid if the factors dictating the PE brush configuration [namely the elastic  $(F_{el})$  and the excluded volume  $(F_{EV})$  energies] are decoupled from the corresponding electrostatic effects [namely, the energy associated with the PE charge  $(F_{elec})$  and that associated with the induced EDL  $(F_{\text{EDL}})$ ]. Such decoupling is possible if either  $F_{el} + F_{EV} \gg F_{elec} + F_{EDL}$  (which occurs when  $\sigma \gg \sigma_c$ ) or  $F_{el} + F_{EV} \ll F_{elec} + F_{EDL}$  (which occurs when  $\sigma \ll \sigma_c$ ). Here,  $\sigma$  is the grafting density and  $\sigma_c \approx a^{-1}t^{-1}$  (where *a* is the Kuhn length and *t* is the thickness of the polymer brush molecule) is the critical grafting density. Here, we assume that either of these conditions ( $\sigma \gg \sigma_c$ or  $\sigma \ll \sigma_c$ ) has been satisfied. Of course, in addition to the above conditions, we need additional constraint on the value of  $\sigma$ . For example, we need to ensure that  $\sigma$  is always large enough to ensure that the grafted polymers may form the brushes, i.e.,  $\sigma \gg a^{-2}N^{-6/5}$  (Chen and Das [2015a](#page-16-6)). Furthermore,  $\sigma$  needs to be small enough to ensure that the grafted brushes on opposing nanochannel walls do not interpenetrate, i.e.,  $\sigma \ll h^3 a^{-4} N^{-3} t^{-1}$  (Chen and Das [2015a](#page-16-6)). Therefore, in summary, our model is valid for  $\sigma \ll \sigma_c$  or  $\sigma \gg \sigma_c$ and  $a^{-2}N^{-6/5}$  ≪  $\sigma$  ≪  $h^3a^{-4}N^{-3}t^{-1}$  (Chen and Das [2015a](#page-16-6)).

It is worthwhile to note here that most of the papers studying the liquid flows in nanochannels grafted with PE brushes have considered such salt-concentration-independent brush height (Chanda et al. [2014](#page-16-14); Chen and Das [2015d,](#page-16-15) [b](#page-16-7); Patwary et al. [2015](#page-17-12); Poddar et al. [2016](#page-17-34); Milne et al. [2014](#page-17-14); Zhou et al. [2016](#page-18-0); Yeh et al. [2012b](#page-18-3), [a](#page-18-4); Benson et al. [2013](#page-16-23); Zhou et al. [2016](#page-18-0); Li et al. [2016](#page-17-35); Cao and You [2016;](#page-16-24) Li et al. [2017\)](#page-17-36) (or the brush height in the decoupled regime). Our paper (Chen and Das [2015a\)](#page-16-6) unraveled, for the first time, the physical conditions under which such decoupling is allowed. In another paper (Li et al. [2016](#page-17-13)), we provided the examples of experimental studies (Hoffmann et al. [2009](#page-17-37); Guo and Ballauff [2000;](#page-17-38) Wang et al. [2010;](#page-17-39) Guo and Ballauff [2001\)](#page-17-40) where the above condition of decoupling can be safely employed while describing the PE brush electrostatics. In a recent couple of papers, we have considered a simplistic system (a nanochannel grafted with end-charged brushes) and have provided, for the first time, the calculations for the liquid flows in PE brush-grafted nanochannels where the brush configuration is obtained through a self-consistent thermodynamic analysis (Chen et al. [2018](#page-16-16); Chen and Das [2017b](#page-16-25)). In these papers, we employed the Alexander–de-Gennes model (Alexander [1977](#page-16-0); Gennes [1976,](#page-17-0) [1980](#page-17-1)) to describe the monomer configuration. Such a situation was afforded by the fact that the PE charge was localized at the non-grafted end of the brush. On the other hand, for the present case where we consider a backbonecharged pH-responsive brush, such simplistic modeling will not be possible and any thermodynamically self-consistent approach would necessitate an analysis that remains missing in the literature despite the significant efforts by the previous researchers (Zhulina and Borisov [2011](#page-18-5)). In one of our papers (Chen and Das [2015c](#page-16-22)), we pinpoint that this lacuna stems from considering a Boltzmann distribution description of the hydrogen ions even within the PE brush layer. Therefore, a self-consistent analysis for the present problem would first require a self-consistent thermodynamic analysis of the pHand pOH-responsive PZI brushes, which is beyond the scope of the present study.

## **4.2 Choice of the cubic monomer density distribution**

Despite considering a decoupled regime, we would still need to know the dimensionless distribution of the PZI chargeable sites  $\varphi$  along the height of the PE brush. In several of our previous papers, we have described the need for considering a *non-unique* cubic distribution of these chargeable sites to ensure a continuity in the hydrogen and hydroxyl ion concentration distribution (Chen and Das [2015b](#page-16-7), [c;](#page-16-22) Li et al. [2016](#page-17-13); Patwary et al. [2015](#page-17-12)). This continuity would have been achieved by default had we been able to obtain a fully selfconsistent thermodynamic description of the pH-responsive PE brushes. No study has been able to achieve that yet. Under these circumstances, the consideration of this cubic monomeric distribution is the best description of  $\varphi$  that one might achieve for a pH-responsive PE brush.

### **4.3 Calculation of maximum output power**

Figures [10](#page-10-3)c, [11](#page-11-2)c, [12c](#page-12-0), and [13](#page-13-0)c provide the variation of the maximum output power using the expression for  $P_{\text{out}}$  in Eq. ([28\)](#page-10-0). To obtain these output power values, we consider a nanofluidic chip consisting of several nanochannels (each of half height 100 nm and grafted with the PZI brushes). The nanofluidic chip is 1 mm in length and 10 cm in both height and width, with a porosity of 0.5. Consequently, the total number of nanochannels present in the chip is 10 cm×  $0.5/(2 \times 100 \text{ nm}) = 2.5 \times 10^5$ . The applied pressure gradient is considered to be  $5 \times 10^8$  Pa/m. This is a reasonable pressure gradient that can be achieved by applying a pressure drop of 5 bar across the chip of length 1 mm. The previous experiments on nanofluidic transport have achieved similar pressure drops across millimetric lengths (Heyden et al. [2005,](#page-17-41) [2006b\)](#page-17-42). In the caption of Fig. [10,](#page-10-3) we have reported these numbers. We repeat them here for the sake of clarity. We also note here that we have previously provided such estimates of power generation at the chip scale using nanofluidic energy conversion in nanochannels grafted with end-charged PE brushes with pH-independent charges in the presence of realistic pressure drops (Chen et al. [2018\)](#page-16-16).

## **5 Conclusions**

Here, for the first time, we propose a design that uses a PZI brush-grafted nanochannel for the electrokinetic energy conversion. The unique ability of the PZI to express significant (but opposite charges) at extreme ends of the pH spectrum has been leveraged in this design to generate electrokinetic power from a pressure-driven transport across a wide range of pH spectrum. Typically, the pH responsiveness of nanochannels (with and without the PE brush grafting) enforces a narrow operating pH window for the maximum power generation. Use of PZI brushes expands that window and allows a large power generation across wide ranges (both large and small) pH values.

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