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A method for slope stability analysis considering subsurface stratigraphic uncertainty

Abstract The accuracy of stability evaluation of a natural slope consisting of multiple soil and rock layers, regardless the adopted analysis methods, can be highly dependent upon a precise description of the subsurface soil/rock stratigraphy. However, in practice, due to the limitation of site investigation techniques and project budget, stratigraphy of the slope cannot be observed completely and directly; therefore, there remains a considerable degree of uncertainty in the interpreted subsurface soil/rock stratification. Therefore, estimating and minimizing the uncertainty of the computed factor of safety (FS) due to the uncertain site stratigraphy is an important issue in gaining confidence on the stability evaluation outcome. Presented in this paper is a practical analysis approach for evaluating the stability of slopes considering uncertain stratigraphic profiles by incorporating a recently developed stochastic stratigraphic modeling technique into a conventional finite element simulation approach. The stochastic modeling techniques employed for simulating the stratigraphic uncertainty will be briefly described. The main efforts are focused on elucidating the additional benefits from the proposed analysis approach, including a more reasonable probabilistic estimation of FS with consideration of stratigraphic uncertainty, as well as an effective approach for finding the optimum location of additional borehole logs to reduce the uncertainty of FS due to uncertain subsurface stratigraphy.

Keywords Probabilistic analysis · Slope stability · Stratigraphic uncertainty · Uncertainty quantification · Stochastic geological model

Introduction

Acquiring accurate geotechnical site characteristics is crucial and essential for stability evaluation of a slope. However, due to the strong heterogeneity of geomaterials and limited site investigation data constrained by exploration techniques and project budget, subsurface information used in subsequent slope stability analysis typically involves various degrees of uncertainty. Consequently, it is well recognized that such uncertainties can exert significant effects on the accuracy of the slope stability evaluation outcome (Cho 2007, Elkateb et al. 2003, Fenton and Griffiths 2008, Griffiths et al. 2009, Phoon and Kulhawy 1999, Tang et al. 2015).

To provide reasonable and confident estimates of slope stability, substantial efforts have been conducted for incorporating probability and statistic theories into the conventional slope stability evaluation methods, in order to take the subsurface uncertainty into account. Most of the existing probabilistic slope stability analysis approaches (Cho 2009, Griffiths and Fenton 2004, Hicks et al. 2014, Ji et al. 2012, Jiang et al. 2014, Li et al. 2015) focused on describing the inherent variability of soil properties within one statistically homogeneous soil layer (i.e., a soil mass belongs to the same material type); hence, they are applicable only for slopes consisting of the same or very similar soil formations. It is worth noting that, for a natural slope consisting of

multiple soil/rock layers, there exists another significant source of uncertainty—the uncertainty of the spatial distribution of the different soil/rock formations/strata and the location of their boundaries (Evans 1982, Hansen et al. 2007), referred to as stratigraphic uncertainty. In a sense, it seems that such stratigraphic uncertainty requires more attention, due to the fact that the differences in material properties among multiple soil/rock formations can be much larger than the variation of material properties within a single soil/rock formation.

However, studies of the effects of stratigraphic uncertainties on the evaluation outcome of slope stability problems are relatively limited in the past (Elkateb et al. 2003, Li et al. 2016a). In most practical cases, the soil stratification is determined according to engineering judgment grounded on local experience (Elkateb et al. 2003) or deterministic interpretation approaches based on geostatistical methods or interpolation methods (Auerbach and Schaeben 1990, Blanchin and Chilès 1993, Calcagno et al. 2008, Mallet 1989, Wellmann et al. 2010). Recently, Li et al. (2016a) proposed a probabilistic slope stability evaluation approach considering the stratigraphic uncertainty, in which the uncertain subsurface stratigraphic profiles were simulated using a coupled Markov chain (CMC) model (Elfeki and Dekking 2001). Although the CMC model has been proven to be able to generate multiple stratigraphic profiles required by the subsequent probability-based analysis, it may suffer certain limitations for simulating the stratigraphic profiles of natural slopes, in which the dip angles of soil formations are typically variable, due to various geological processes during the formation of the slopes. To be specific, in the CMC model, the correlation between spatial distributions of different soil formations, which are governed by the stationary transition probability matrix, is only defined along the horizontal and vertical directions. This makes the boundaries of soil formations generated using the CMC model tend to be linear (i.e., horizontal, vertical, or at a constant angle), thereby limiting the performance of the CMC model for describing the complex geological settings reflected by the stratigraphic profile of a natural slope. Therefore, it seems desirable to improve the description of the uncertain slope stratigraphic profile by employing more advanced stratification modeling techniques, so that the stratigraphic uncertainty can be quantified more reasonably and that the slope stability evaluation error caused by such uncertainty can be estimated more accurately.

In this paper, focusing on quantifying and reducing the effects from uncertain stratigraphy on the stability evaluation outcome, we present a finite element analysis (FEA)-based probabilistic approach for slope stability analysis. In the proposed approach, a stochastic geological modeling technique (Li et al. 2016b) recently developed by the authors, which is proven to be capable of simulating complex geological structures based on available site investigation data and geological knowledge, is employed to generate samples of the uncertain stratigraphic profile. Incorporating the employed stochastic geological modeling technique with the FEA

simulations allows for performing a Monte Carlo simulation of thousands of possible subsurface stratigraphic profiles in determining the probability distribution of computed FS of the slope. Moreover, in practice, once the uncertainty or error of the computed FS can be quantified, there may be a need for subsequent steps to reduce the uncertainty or error to a desired level by conducting additional site investigation. As will be shown later, the proposed approach is able to provide an optimized layout scheme for additional borehole logs or in situ tests, based on the existing probabilistic analysis and uncertainty quantification results. This paper will provide a brief description of the stochastic modeling technique. Emphasis will be given to demonstrate the practical benefits of the proposed analysis approach in the slope stability evaluation problems: (1) obtaining a reasonable estimate of factor of safety (FS) in a probability manner, taking into account the stratigraphic uncertainty and (2) finding optimum locations of additional boreholes to efficiently reduce the uncertainty (in terms of standard deviation) of the computed FS.

The remaining content of this paper is organized as follows. The “[Stochastic stratigraphic modeling based on Markov random field](#)” section provides a brief review of the employed stochastic stratigraphic modeling technique (Li et al. 2016b), which allows for generation of thousands of realizations of soil profiles conditional on the limited, spatially distributed observation data, such as borehole logs. In the “[Stratigraphic realizations and uncertainty quantification](#)” section, an example of the application of the stochastic modeling technique for a natural slope problem is presented to demonstrate the detailed process for generating stratigraphic realizations and subsequent quantification of stratigraphic uncertainty. The “[Slope stability evaluation](#)” section demonstrates the use of the modeling results of stratigraphic profiles in finite element analysis to evaluate the stability of the slope with the attendant statistical analysis of computed FS. Additional discussion was given to elucidate the use of the simulation results to allow engineers to make informed decisions on the optimum locations of additional boreholes to improve the confidence level of the computed FS. Concluding remarks are presented at the end of this paper.

[Stochastic stratigraphic modeling based on Markov random field](#)

As one of the most sophisticated spatial statistical models, the Markov random field (MRF) model provides a convenient and consistent way for analyzing the spatial dependencies of physical phenomena (Li 2009, Li et al. 2016b, Zhang et al. 2001). In geostatistics, Markov priors have been widely used to describe various geological processes, including synthesizing stratigraphic sequences and sediment layering structures (Carle and Fogg 1996, 1997, Dai et al. 2005, Elfeki and Dekking 2001, Li 2007, Li et al. 1999, Wang et al. 2017, Weissmann et al. 1999). However, the Markov priors in most existing geological modeling methods based on Markov models are isotropic or stationary; such assumptions may limit the performance of these models for reproducing real stratigraphy with complex geologic settings. In this section, a recently developed MRF-based stochastic stratigraphic modeling technique using an anisotropic and non-stationary Markov prior will be introduced, which can incorporate various types of site investigation data and thereby provide more informed and realistic modeling results for the stratigraphy of natural slopes.

Adequate stratigraphic realizations of a slope and overview of the modeling process

In most practical slope stability evaluation problems, the direct observations of the subsurface stratigraphy of a slope, provided by common geotechnical site investigation such as borehole drillings, are typically sparsely located. Therefore, before further stability analysis can be implemented, the unobserved regions in the stratigraphic profile need to be interpreted reasonably, according to the available observations and the knowledge of the slope structures. Under this condition, the interpreted slope stratigraphy can be regarded as realistic, only if the following two characteristics of the real subsurface stratigraphy are adequately reflected. The first aspect is the general trends or the global characteristics of the stratigraphy of a slope, including its anisotropic (i.e., sequenced soil/rock layer structures) and non-stationary (i.e., spatially variant stratigraphic dips) nature, which are generally derived from and constrained by the available borehole logs, geophysical survey, and relevant geological knowledge. The second aspect is the local characteristics regarding the shape of the local boundaries (i.e., small patches and fluctuations between different soil/rock formations); MRF prior is suitable for describing such spatial transition of soil/rock formations, as a result of stratification or sedimentary processes, in a probabilistic manner (Elfeki and Dekking 2001, Norberg et al. 2002, Li et al. 2016b). To ensure that the generated slope stratigraphic realizations fulfill both the global characteristics and the local characteristics of the real stratigraphy simultaneously, we design a stochastic modeling approach consisting of two stages as follows:

In the first stage, a Monte Carlo simulation (MCS) using the single-side Markov property (i.e., sampling is extrapolated from borehole locations to the unobserved regions) is employed to generate initial configurations of stratigraphy. This stage is intended to generate a sufficient number of rough estimates of the real slope stratigraphy, whose global shapes and trends are in accordance with the available borehole logs and knowledge of stratigraphic dips, but local shapes may vary. To be more specific, the general idea of such simulation is somehow similar to the one of the conventional interpretation processes followed by geologists, in which curves indicating the boundaries of different soil formations are extended from one borehole location as start points and then are connected with the curves from other borehole locations according to the stratigraphic dips. However, significant differences in the implementation processes need to be noticed: (1) instead of the use of individual experience, the spatial correlation between soil units in the employed modeling method is explicitly defined using conditional probability (i.e., see the “[Markov random field and equivalent Gibbs distribution](#)” section) and implemented through sampling processes; (2) in addition to the borehole logs, available information of local stratigraphic dips indicated by various geophysical surveys can be incorporated; (3) the stratigraphic uncertainty is spontaneously considered in the MCS process, due to its stochastic nature. During this stage, large-scale global characteristics are completely honored, since the borehole information and the known knowledge of stratigraphic dips are fully exploited. Whereas, the local characteristics, governed by the defined MRF prior, is only partially honored (i.e., sampling is conducted using an MRF prior, but it has not been fully optimized).

In the second stage, sites in the generated initial configuration, other than the borehole locations, are optimized using a Markov Chain Monte Carlo (MCMC) algorithm. During this stage, the defined MRF prior (see the “Prior energy” section) is fully exploited and the local characteristics derived from it can be well presented in the generated stratigraphic realizations, indicated by that local modeling defects in the initial configurations are corrected and eliminated. It is worth mentioning that overcorrection lead by MRF prior also damages the large-scale characteristics of the real slope stratigraphy in the initial configurations, as the large-scale trend is formed by complicated physical processes and hence may not be perfectly expressed by the employed MRF prior for describing local spatial correlation. One of the known negative impacts is oversmoothing (Tolpekin and Stein 2009). To avoid such overcorrection, the initial stratigraphic configuration generated in stage no. 1 should be considered as a constraint during the following optimization process in stage no. 2. Therefore, we design a likelihood function (see the “Likelihood energy” section) as a similarity measure between the initial configuration and the corresponding optimized configuration in terms of energy. Combining the two aforementioned modeling stages, the generated stratigraphic realizations can reasonably reflect both the global and local characteristics according to both available observations and our geological knowledge; thereby, they can be regarded as an adequate estimate of the real stratigraphy of a slope. Sampling of stratigraphic configurations in both modeling stages is implemented by using a Gibbs sampler according to the prior function (see the “Prior energy” section) and the likelihood function (see the “Likelihood energy” section) introduced in the following paragraphs. Detailed explanations of a Gibbs sampler and the detailed modeling procedure can be found in Casella and George (1992) and Wang et al. (2016), respectively.

Markov random field and equivalent Gibbs distribution

Herein, only the basic concepts necessary for understanding the employed modeling technique in the present work are introduced. A complete introduction to MRFs and the equivalence between an MRF and the corresponding Gibbs distribution can be found in existing literature (Geman and Geman 1984, Li 2009). To conduct the modeling under the framework of MRF, the stratigraphic domain of interest is discretized into a finite set of cells denoted by \mathbf{R} . Each cell $r \in \mathbf{R}$ has eight neighboring cells $s \in \mathbf{R}$, denoted as $s \in \hat{\partial}r$. Next, a label $x_r \in \mathbf{L}$ (representing the set of all soil/rock formations in the modeled subsurface stratigraphy) is associated with each cell $r \in \mathbf{R}$, and we let $\mathbf{x} = (x_r | r \in \mathbf{R}) \in \Omega$ represent a subsurface stratigraphic configuration, where Ω is the set of all possible configurations. Under this condition, \mathbf{x} is said to be an MRF with respect to a given neighborhood system if the prior joint probability $P(\mathbf{x}) > 0$ for all $\mathbf{x} \in \Omega$ and the conditional probability are fulfilled with the following term:

$$P(x_r | \mathbf{x}_{-r}) = P(x_r | \mathbf{x}_{\partial r}) \quad (1)$$

We further denote the likelihood for measuring the similarity between an initial profile \mathbf{x}^i and an updated configuration \mathbf{x} as $P(\mathbf{x}^i | \mathbf{x})$. Such a likelihood function is intended to make an updated configuration have a higher chance to occur if it agrees more with the large-scale characteristics; the posterior probability for stratigraphic configuration \mathbf{x} , given the initial estimate \mathbf{x}^i , is denoted as

$P(\mathbf{x} | \mathbf{x}^i)$. According to the Bayes theorem, there exists a relationship among these probabilities:

$$P(\mathbf{x} | \mathbf{x}^i) \propto P(\mathbf{x})P(\mathbf{x}^i | \mathbf{x}) \quad (2)$$

Due to the equivalence of MRF and Gibbs functions (Clifford 1990), $P(\mathbf{x})$ and $P(\mathbf{x}^i | \mathbf{x})$ can be further specified by means of energy functions:

$$P(\mathbf{x}) = \frac{1}{Z_p} \exp\{-U(\mathbf{x})\} \quad (3)$$

$$P(\mathbf{x}^i | \mathbf{x}) = \frac{1}{Z_l} \exp\{-U(\mathbf{x}^i | \mathbf{x})\} \quad (4)$$

in which Z_p and Z_l are normalizing constants and $U(\mathbf{x})$ and $U(\mathbf{x}^i | \mathbf{x})$ are referred as prior and likelihood energy, respectively.

Prior energy

We model the prior energy as the sum of pairwise interactions between cells:

$$U(\mathbf{x}) = \sum_{c \in \mathbf{C}} V_c(\mathbf{x}) \quad (5)$$

where c represents a clique containing soil labels (x_r, x_s) of a pair of neighboring cells (r, s) , \mathbf{C} is the set of all the cliques in the MRF, and V_c is referred to as prior potential for each clique c . A classical Potts model (Wu 1982) is then employed for V_c :

$$V_c(x_r, x_s) = \begin{cases} -\rho(r, s) & \text{if } x_r = x_s \\ 0 & \text{if } x_r \neq x_s \end{cases} \quad (6)$$

in which $\rho(r, s)$ is the spatial correlation between soil labels of a pair of neighboring cells (r, s) .

To reflect the anisotropic (i.e., strength of the spatial correlation is directionally dependent) and the non-stationary (i.e., local stratigraphic dips differ from one location to another) nature of the modeled stratigraphy, $\rho(r, s)$ is designed to be related to two features: (1) the soil label of the neighboring cell x_s and (2) the relative direction of cell s to cell r , represented by polar angle $\theta_{s \rightarrow r}$ from the centroid of cell r to the centroid of cell s in a given global polar coordinate system $O\text{-}XY$, as shown in Fig. 1. Specifically, $\rho(r, s)$ is defined as the radius length along the polar angle $\theta_{s \rightarrow r}$ of an ellipse centered at the centroid of cell r . The major axis of this ellipse has a polar angle $\phi_r \in (-\frac{\pi}{2}, \frac{\pi}{2}]$, which can be defined according to the available information of the stratigraphic dips, in the defined global polar coordinate system $O\text{-}XY$, as shown in Fig. 1. The length of the minor axis of the ellipse is set to 1.0; the length of its major axis a is defined as the correlation strength parameter. Hence, $\rho(r, s)$ can be calculated using the following equation:

$$\rho(r, s) = a / \sqrt{\cos^2(\theta_{s \rightarrow r} - \phi_r) + a^2 \sin^2(\theta_{s \rightarrow r} - \phi_r)} \quad (7)$$

The anisotropy of the developed spatial correlation model (i.e., in terms of MRF prior) is reflected by the directionally variable

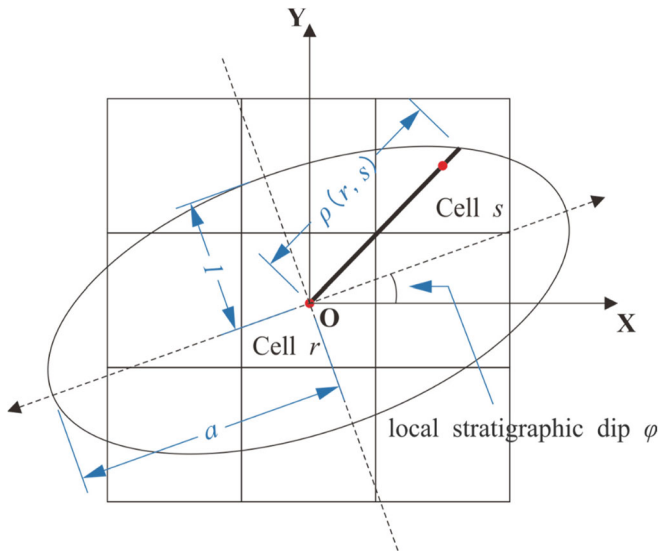


Fig. 1 Elliptical correlation model

radius length of the ellipse. The direction of the major axis indicates the tangential direction of the interface between soil formations (i.e., the local extension direction of the soil layer or the local stratigraphic dip). Meanwhile, the non-stationarity of the model is represented by the various polar angles ϕ_r of different cells, which allow the local stratigraphic dips varying from one location to another to represent complex geological structures.

Likelihood energy

As mentioned in the previous sections, MRF prior may lead to deterioration of the large-scale characteristics of the stratigraphy

in the initial configurations x^i . Therefore, we introduce the likelihood energy $U(x^i|x)$ to prevent such overcorrection. As a measure of similarity between the initial configuration x^i and its corresponding updated realization, $U(x^i|x)$ can be represented as the accumulation of the likelihood potential V_r , relating to the soil label assignment of each element r in the current configuration x and the initial estimate x^i :

$$U(x^i|x) = \sum_{r \in R} V_r(x, x^i) \tag{8}$$

Since the soil labels are discrete variables, the following form of V_r is employed in the proposed model:

$$V_r(x_r, x_r^i) = \begin{cases} -\mu & \text{if } x_r = x_r^i \\ 0 & \text{if } x_r \neq x_r^i \end{cases} \tag{9}$$

in which μ is referred to as a smoothing factor. In this study, μ is set as 0.2; detailed parametric study of μ can be found in the authors' previous study (Wang et al. 2016). It can be noted from Eqs. (3-4) and (8-9) that amendment of soil label assignments in the initial configuration x^i , which results in an increase of prior probability $P(x)$, accordingly leads a decrease of likelihood probability $P(x^i|x)$. In other words, during the MCMC process in the second modeling stage for maximizing the posterior probability $P(x|x^i)$ in Eq. (2), a balance is achieved between the prior part and the likelihood part. Such balance allows local amendments leading significant increase of prior probability with a reasonably smoothed boundary but rejects global changes that damage the overall shape of the slope stratigraphy determined by borehole logs and geological knowledge.

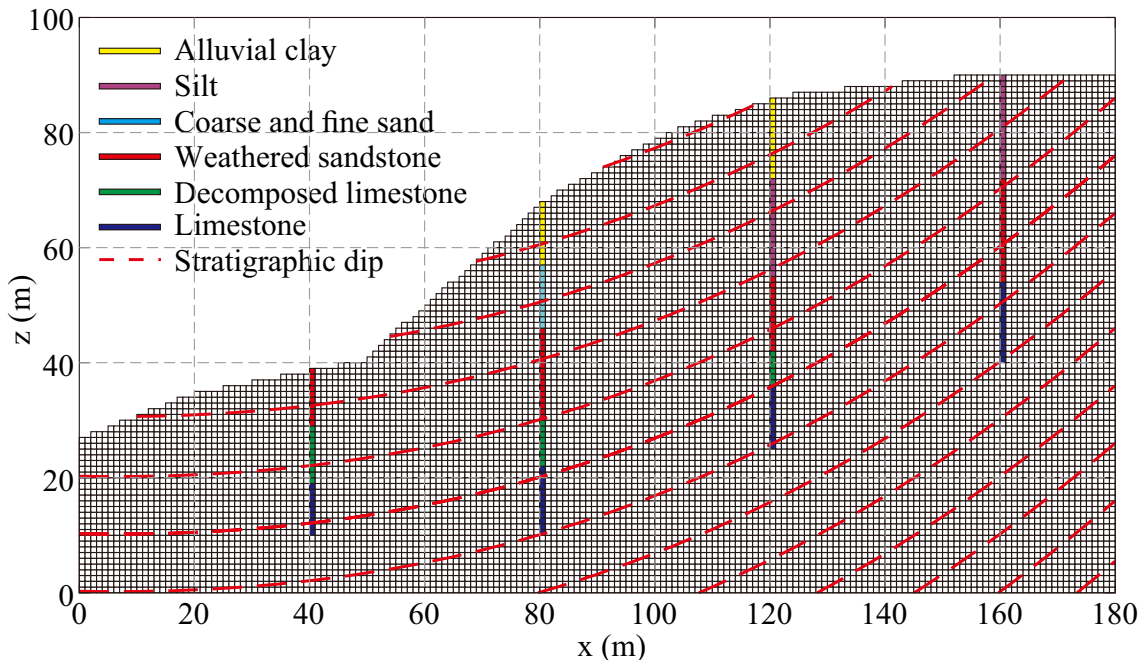


Fig. 2 Discretization of the slope body and the integrated site investigation data

Geology and available investigation data

In this section, we provide an example of stratigraphic modeling for a natural slope located near the city of Longyan in Fujian province, China. According to the geological map and the available borehole information, the slope of interest is located at an alluvial fan, the subsurface stratigraphy of which consists of depositional overburden Quaternary soils over Carboniferous limestone. There are four available borehole logs in the studied two-dimensional slope cross section as shown in Fig. 2, identifying a total of six major types of soil/rock formations in the slope. The bedrock consists of intact or decomposed limestone; the overburden

material is variable—it contains alluvial clay, silt, sand, and gravel of weathered sandstone. For conducting the employed geological modeling technique, the slope cross section of interest is discretized into an MRF using a square lattice with 11,793 cells, with the cell size as 1.0 m × 1.0 m.

The general trend of the stratigraphic dip of the slope is predefined using a set of curves given in Fig. 2, which is derived from borehole logs (i.e., checking the depths of each soil/rock layer in all the borehole logs) and the knowledge of the local geological settings. The polar angles ϕ_r , indicating local stratigraphic dips for all the cells in the random field, as an input of the employed modeling method to reflect and incorporate the available knowledge of the general trend of the stratigraphic dip of the studied

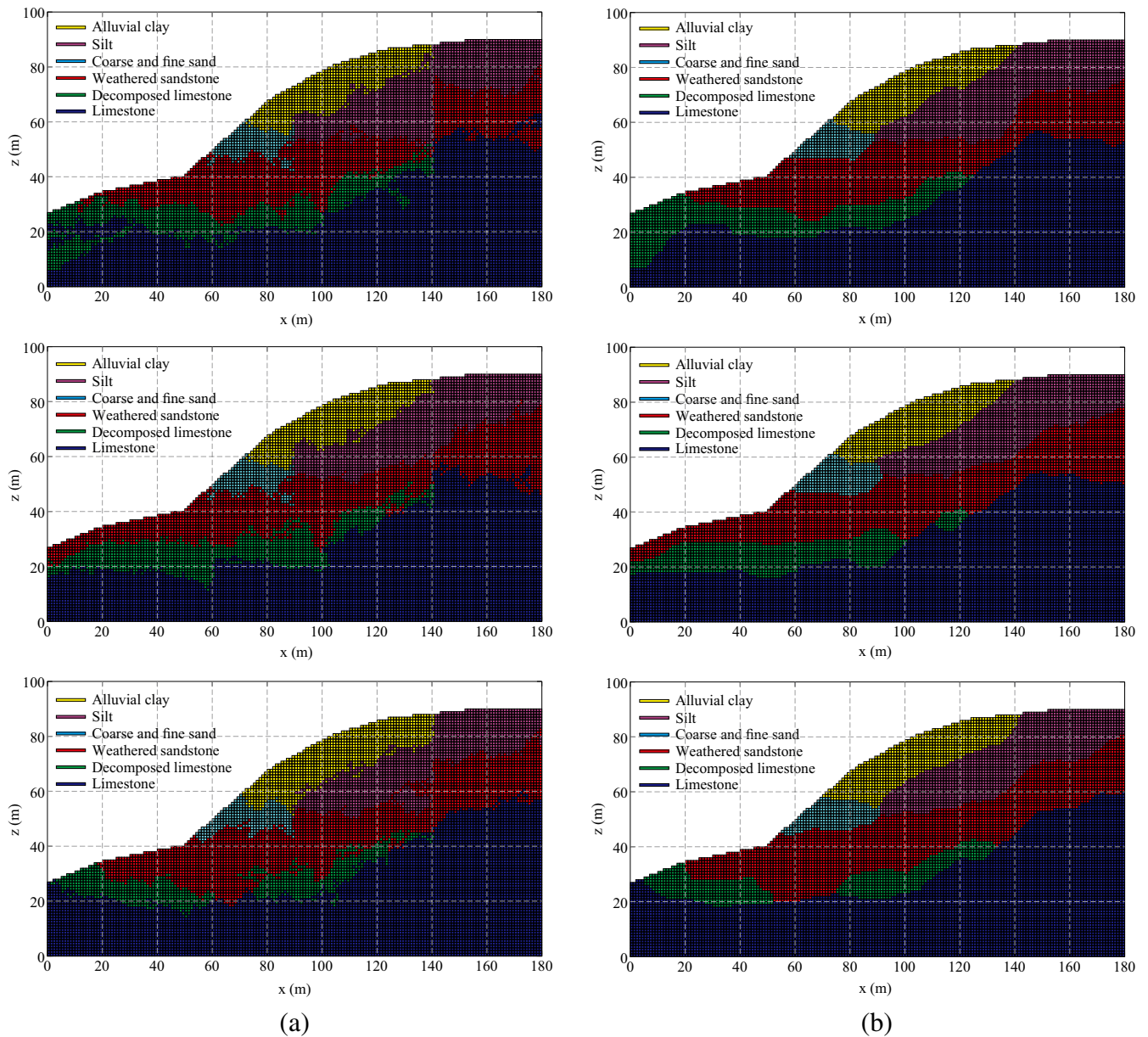


Fig. 3 a Initial configurations. b Corresponding stratigraphic realizations

slope, are then computed based on this set of curves through a deterministic kriging interpolation. The model parameter a , referred to as the correlation strength, is used to control the pattern of the generated stratigraphic profile. Methodology for selecting a proper value of a for a project site can be carried out by a Bayesian inferential framework, if sufficient borehole logs are provided (Li et al. 2016b). Alternatively, in the previous studies (Li et al. 2016b, Wang et al. 2016), the authors also provided an empirical and rough estimate of a that can be selected in accordance with one's judgment based on local geological settings. In this example, a model parameter a of 3.0 is selected, which typically gives a moderately layered pattern.

Generating subsurface stratigraphic realizations

Following the modeling procedure described previously in the "Stochastic stratigraphic modeling based on Markov random field" section, initial configurations of the stratification are firstly generated; three of them are shown in Fig. 3a. It can be noted that the overall shapes of all three slope stratigraphic profiles are in general agreement with the available borehole data and the predefined stratigraphic dips. However, there are some local modeling defects, such as sharp corners or discontinuous contact boundaries, which are due to those local characteristics governed by MRF prior. Such defects indicate that the energy function is far from optimized. To maintain the general shape as well as to eliminate the local defects, subsequent MRF evolutions are performed until a stable equilibrium state is reached. The corresponding stratigraphic realizations (i.e., which are the final outcomes of the employed stratigraphic modeling technique) of the three initial configurations are shown in Fig. 3b. As can be seen, most of the local defects have been eliminated in the final modeling outcome. Moreover, it can be noted that the predefined dip angles are kept in the obtained realizations of the stratigraphic profiles; the spatial

distributions of soil formations follow the predefined concave shape, which is the typical geomorphology of a depositional slope.

Quantification of stratigraphic uncertainty

After generation of a large number of stratigraphic profiles through the modeling process described above, we can measure the uncertainty of modeling results using the concept of information entropy, which was introduced previously by Wellmann and Regenauer-Lieb (2012) for uncertainty quantification in the context of geological simulation. For each cell in the MRF, its information entropy can be calculated as

$$H_i = -\sum_k^n p_k \log p_k \quad (10)$$

in which n is the total number of the possible soil labels the cell can have and p_k are the corresponding probabilities for each possible soil label, which are calculated according to the number of assigning this cell with soil label k for all the generated stratigraphic realizations. The definition of information entropy in Eq. (10) implies that the higher uncertainty in soil label a cell possesses (i.e., having equal possibility of being assigned with any of the possible soil labels), the higher information entropy value it will have, and vice versa. The map of the normalized information entropy calculated from 1000 stratigraphic realizations for this example is shown in Fig. 4, which provides a clear visualization of the stratigraphic uncertainty associated with each discretized cell in the slope. It can be noted that the stratigraphic uncertainty of the cells located close to the borehole locations is negligible, since the borehole data has a strong influence on the soil label assignment of the nearby cells. On the other hand, for those cells far from the boreholes, the contact boundaries between soil layers become more uncertain.

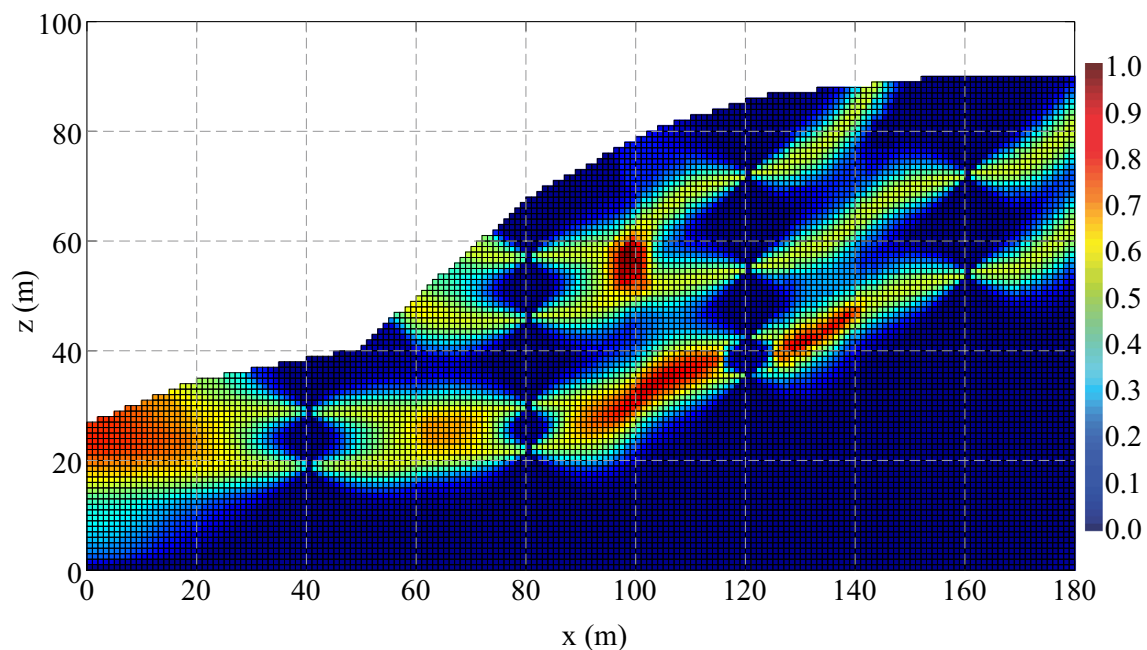


Fig. 4 Stratigraphic uncertainty represented by information entropy map (color legend represents the normalized information entropy)

Slope stability evaluation

The effects of uncertain stratification of the slope on the outcome of slope stability evaluation can be examined by performing stability analysis for each of the generated stratigraphic profiles, using either limit equilibrium-based analysis methods or finite element analysis (FEA). In this study, we use a commercial software ABAQUS (ABAQUS 2013) to compute the FS of the studied slope.

Probabilistic analysis of slope stability

An unstructured mesh consisting of 7212 triangular elements with unequal sizes is used to discretize the slope domain for the FEA of the slope stability, due to the convenience of triangular elements in depicting the irregular boundaries of the slope domain and the computational efficiency led by using locally refined mesh. Material type of each FEA element is assigned using the soil label of the

nearest MRF cell (i.e., according to the distance between the centroid of an FEA element and the centroid of an MRF cell). The dimension of FEA elements in the potential failure zone is smaller than the dimension of cells in the MRF; hence, the information loss in the potential failure zone during the material assignment can be avoided. In this way, each of the stratigraphic realization generated in the previous section can be converted to an FEA simulation case. The constructed FEA models corresponding to the three stratigraphic realizations in Fig. 3b are shown in Fig. 5a.

In FEA, the factor of safety of each simulation case is computed by using a shear strength reduction technique (Dawson et al. 1999). The Drucker-Prager (DP) model embedded in ABAQUS is adopted as the constitutive model for the six soil formations due to its advantage over the Mohr-Coulomb (MC) model in computational convergence for granular materials and that the required strength parameters of DP model can be computed according to the MC

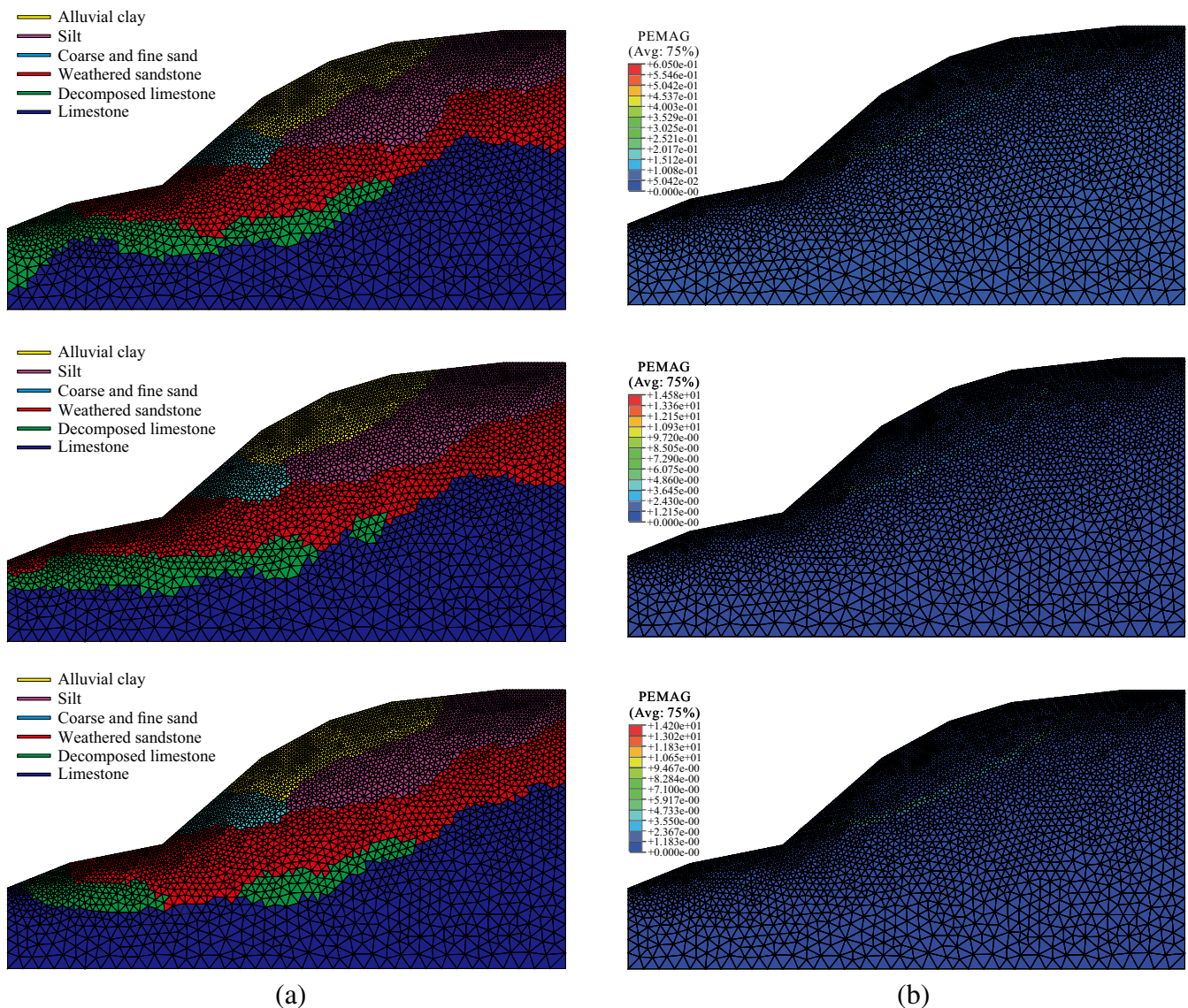


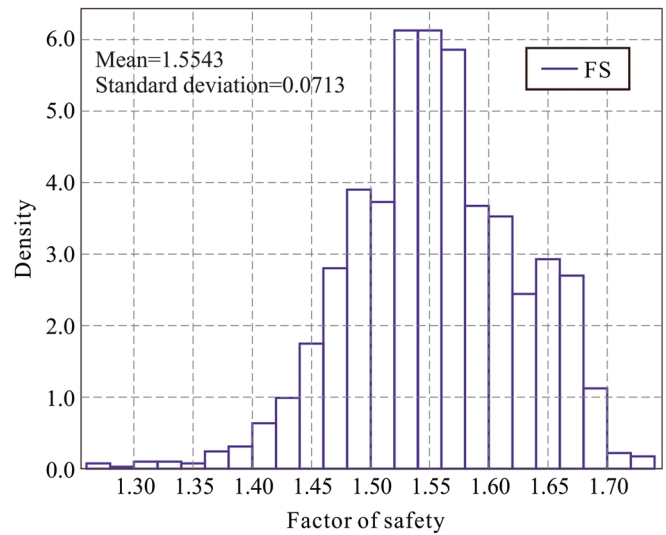
Fig. 5 a Converted FEA simulation cases. b Slip surface represented by equivalent plastic strain (PEMAG denotes the plastic strain magnitude, which is a scalar measure of the accumulated plastic strain)

parameters (ABAQUS 2013). Since this study focuses on the impact of stratigraphic uncertainty on the stability evaluation outcome, the spatial variability of material properties within a single soil/rock formation is not considered. Hence, the material properties of the six soil formations used in the FEAs are set as deterministic, as listed in Table 1.

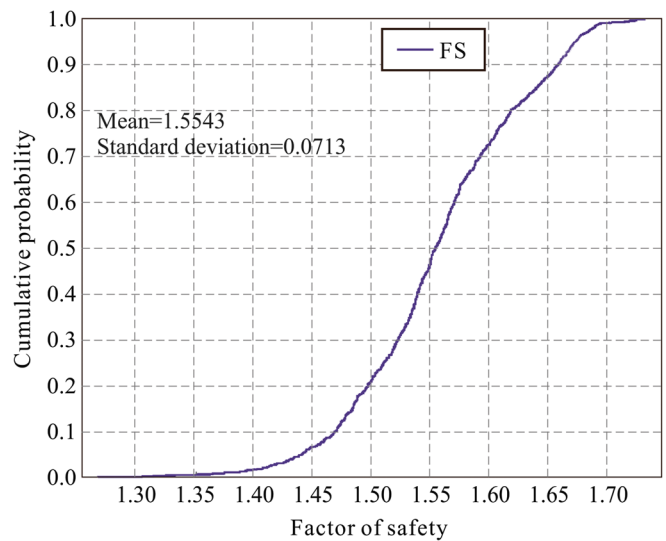
FEA for the 1000 simulation cases yields a total number of 1000 corresponding factor of safety (FS) values. Figure 6a presents the histogram formed by the computed FS values, which vary in a considerable wide range from 1.267 to 1.729. The mean and standard deviation of the computed FS values are 1.554 and 0.0713, respectively. The cumulative probability function of FS values is then plotted in Fig. 6b, which allows one to acquire the corresponding FS value for a certain confidence level. For each simulation case, the map of plastic strain magnitude obtained from FEA can be used to identify the slip surface, as shown in Fig. 5b for the three simulation cases shown in Fig. 5a. By normalizing the overlying maps of plastic strain magnitude of all the simulation cases, a new contour indicating the potential failure zone of the studied slope can be obtained, as shown in Fig. 7. According to the analysis results, it appears that the uncertain subsurface stratigraphic profile can exert significant effects on the slope stability evaluation results, in terms of both the computed FS and the location of the slip surface. Therefore, instead of computing a single FS by using a deterministic stratigraphic profile, it is necessary and highly recommended to obtain a more comprehensive understanding of the slope stability by taking stratigraphic uncertainty into consideration.

Identifying additional borehole location

In some slope stability analysis problems, the borehole information from existing boreholes may not be sufficient to limit the stratigraphic uncertainty to a desired level. Statistical analysis of computed FS showing large standard deviation indicates the need for additional borehole log information. Therefore, identifying the optimum locations of the additional boreholes, which lead to a most significant reduction of the uncertainty of computed FS, is an important issue. Since the uncertainty of FS is mainly caused by the uncertainty of stratigraphic profile in the potential failure region, as marked in Fig. 7, it seems reasonable to assume that the locations and depth of additional borehole drillings should be geared towards reducing the stratigraphic uncertainty in the potential failure zone to a maximum extent. To assist making informed decisions, we compute the sum of the information entropy values of all cells in each vertical column and then plot this against the horizontal coordinates in Fig. 8a. It can be noted



(a)



(b)

Fig. 6 a Histogram of the computed FS values obtained using four boreholes. b Cumulative probability function of the computed FS values obtained using four boreholes

that the curve of information entropy values has three peak points, which indicates that the stratigraphic uncertainty at

Table 1 Material properties of the six soil/rock formations

| | Young's modulus (MPa) | Poisson's ratio | Cohesion (kPa) | Friction angle (°) | Dilatancy angle (°) | Unit weight (kN/m ³) |
|----------------------|-----------------------|-----------------|----------------|--------------------|---------------------|----------------------------------|
| Alluvial clay | 30.0 | 0.3 | 18.3 | 13.9 | 0.0 | 20.0 |
| Silt | 30.0 | 0.3 | 9.0 | 20.2 | 0.0 | 20.0 |
| Coarse and fine sand | 50.0 | 0.3 | 3.2 | 24.5 | 0.0 | 20.0 |
| Weathered sandstone | 30.0 | 0.3 | 10.7 | 25.1 | 0.0 | 20.0 |
| Decomposed limestone | 80.0 | 0.3 | 33.2 | 32.6 | 0.0 | 25.0 |
| Limestone | 100.0 | 0.3 | 120.0 | 36.0 | 0.0 | 30.0 |

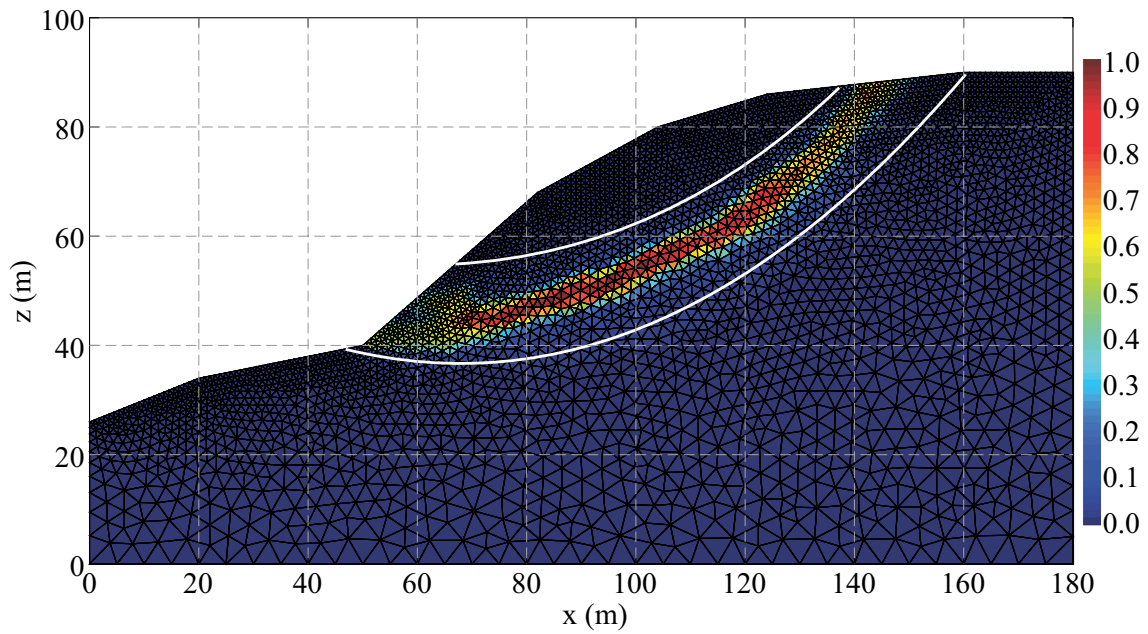


Fig. 7 Potential failure region obtained using four boreholes (color legend represents the normalized plastic strain magnitude)

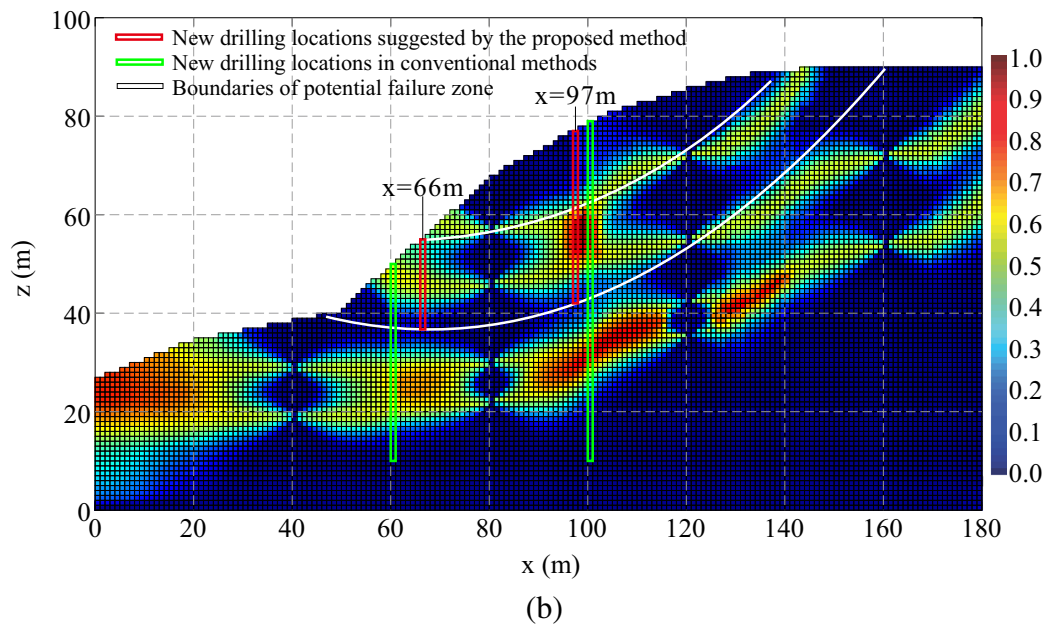
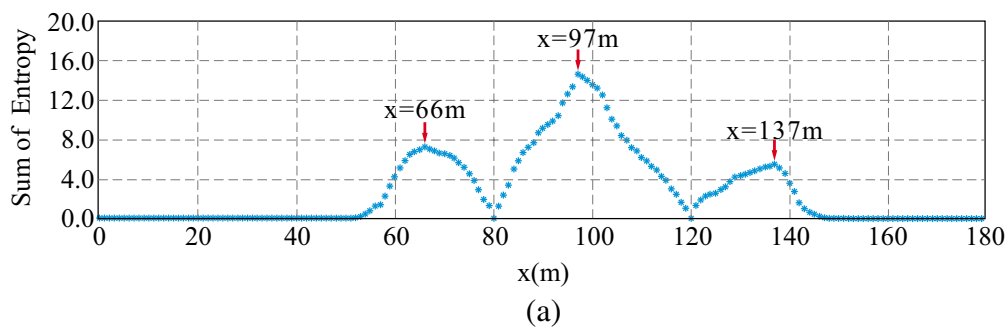


Fig. 8 a Stratigraphic uncertainty in the potential failure region. b Suggested new drilling locations aimed at decreasing stratigraphic uncertainty in the potential failure region (color legend represents the normalized information entropy)

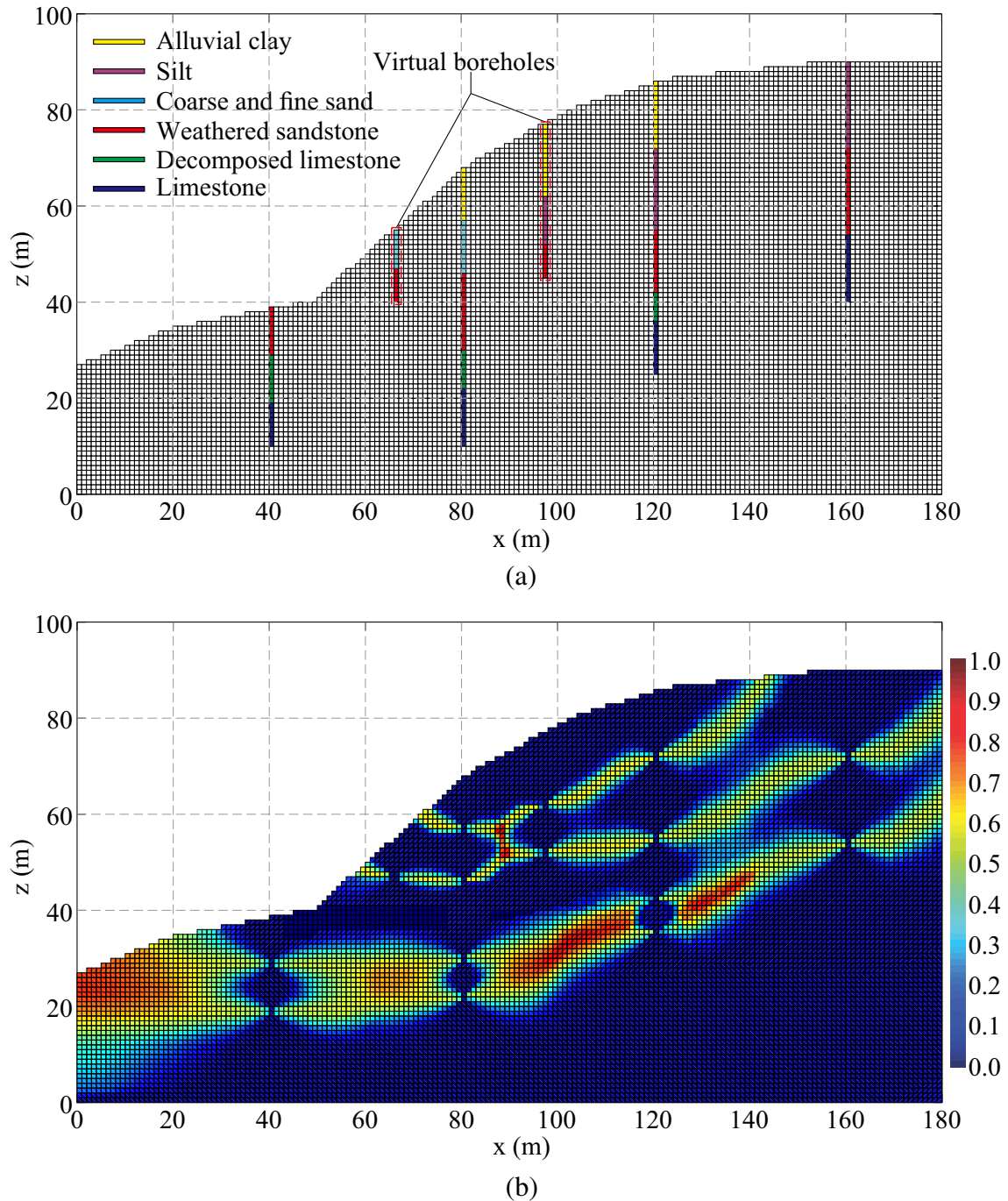


Fig. 9 a Additional virtual boreholes. b Corresponding information entropy map (color legend represents the normalized information entropy)

these three corresponding locations is the largest. Therefore, additional boreholes at these locations should be considered if one would like to reduce the stratigraphic uncertainty and improve the confidence level of computed FS.

Considering the magnitude of uncertainty at these three peaks and the previous analysis results shown in Fig. 7 that the potential starting points of the critical slip surface at the slope toe is more uncertain than the potential ending point of the critical slip surface at the slope crest (i.e., indicating that the strength of the soils near the slope toe may yield more significant impact on the

stability of the studied slope), we introduce two additional boreholes, shown in Fig. 9a, at the locations corresponding to the first two peak points in Fig. 8a. Stratigraphic profile modeling results, in terms of information entropy map, using both the original four boreholes and the two additional virtual boreholes, are presented in Fig. 9b. It can be noticed that by adopting two additional borehole log information, the extent of uncertain zones for soil type assignment has been greatly reduced. The histogram and cumulative probability function of the newly computed FS values for the case with two additional borehole information are shown

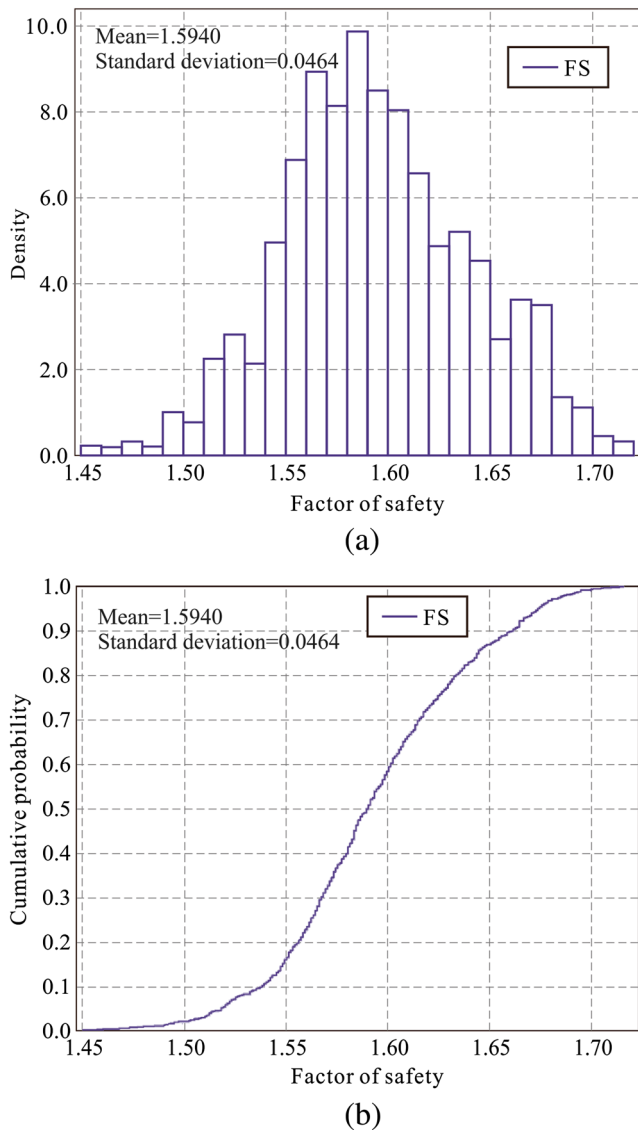


Fig. 10 a Histogram of the computed FS values obtained using additional virtual boreholes. b Cumulative probability function of the computed FS values obtained using additional virtual boreholes

in Fig. 10a, b, respectively. The mean value of the newly computed FS values is 1.594. The standard deviation of FS is 0.0464, which is significantly lower than the previously computed values. In contrast, it can be noted from Fig. 8 that drilling at the location at $x=60$ m leads to less decrease of stratigraphic uncertainty in the potential failure regions than the optimum drilling location ($x=66$ m). Therefore, it seems that the conventional way of selecting borehole drilling at the mid-way points of the existing borehole locations may not always yield the optimum results for reducing the uncertainty of the inferred stratigraphic profile and its effects on the computed FS.

The presented case demonstrates the use of the proposed analysis approach to suggest optimal locations of additional borehole drillings for slope stability evaluation, based on quantification of its stratigraphic uncertainty. It is worth to emphasize that since the main purpose of such borehole drillings is to minimize the uncertainty of the computed FS, focuses should be

given to the stratigraphic uncertainties associated with certain soil formations, the spatial distributions of which may exert significant impacts on the slope stability. Selection of these critical soil formations varies with different slopes and may require local experience. Specifically, in this study, instead of the stratigraphic uncertainty of the entire slope domain, only the stratigraphic uncertainty associated with the potential failure regions, which was estimated in previous FEAs using existing borehole logs, was taken into consideration for identifying the desired drilling locations. In practice, if such previous analysis of the potential failure regions of a slope (i.e., the possible location of the critical slip surface) is unavailable, local experience may be needed to predict the soil formations that the critical slip surface may pass through (i.e., very soft soils in a slope). Subsequently, while identifying new drilling locations, attentions should be given to the stratigraphic uncertainty associated with these soil formations, since their extensions may be very critical to the safety of a slope.

Concluding remarks

Effects of uncertainty in subsurface stratigraphic profile on the computed factor of safety of a natural slope can be significant and have not been quantified in the past. In this paper, we have demonstrated the use of a recently developed stochastic geological modeling technique, together with the conventional finite element analysis method, to compute the probability distribution of the factor of safety (FS) of a slope due to uncertain stratigraphic profile. The algorithm of the stochastic geological modeling technique based on the Markov random field was briefly explained, with a highlight of a novel correlation structure for assigning the soil labels for the unknown areas. Using the realizations of subsurface stratigraphic profiles generated from the stochastic geological modeling results, we further introduced the use of information entropy as a measure for quantifying the uncertainty of the generated subsurface stratigraphy.

As an illustrative example, a natural slope with four available borehole logs was used to generate 1000 stratigraphic profiles, which then were used in finite element analysis via strength reduction technique to compute the corresponding FS and the probability distribution. The example clearly demonstrated that both FS and the failure surface can vary considerably as a result of different subsurface soil stratigraphic realizations. Consequently, for slope stability evaluation, it is highly recommended that uncertainties related to the subsurface soil stratification be properly accounted for. To increase the confidence level or to reduce the standard deviation of the computed FS, the proposed probabilistic approach was further utilized to identify the optimum locations of additional borehole drilling. The presented example demonstrates that, compared with the conventional way of selecting borehole drilling at the mid-way points of the existing borehole locations, the proposed approach is able to provide a more efficient and targeted borehole layout for reducing the uncertainty of the inferred stratigraphic profile and its effects on the computed FS. As shown in the present example, the proposed computational methodology provides a practical and effective means to evaluate the stability of slopes with consideration of the influence of uncertain subsurface stratigraphy.

References

- ABAQUS V 6.13 (2013). Dassault Systemes Simulia Corp, Providence
- Auerbach S, Schaeben H (1990) Computer-aided geometric design of geologic surfaces and bodies. *Math Geol* 22(8):957–987. <https://doi.org/10.1007/BF00890119>
- Blanchin R, Chilès J-P (1993) The channel tunnel: geostatistical prediction of the geological conditions and its validation by the reality. *Math Geol* 25(7):963–974. <https://doi.org/10.1007/BF00891054>
- Calcagno P, Chilès J-P, Courrioux G, Guillen A (2008) Geological modeling from field data and geological knowledge: part i. Modelling method coupling 3d potential-field interpolation and geological rules. *Phys Earth Planet Inter* 171(1-4):147–157. <https://doi.org/10.1016/j.pepi.2008.06.013>
- Carle SF, Fogg GE (1996) Transition probability-based indicator geostatistics. *Math Geol* 28(4):453–476. <https://doi.org/10.1007/BF02083656>
- Carle SF, Fogg GE (1997) Modeling spatial variability with one and multidimensional continuous-lag Markov chains. *Math Geol* 29(7):891–918. <https://doi.org/10.1023/A:1022303706942>
- Casella G, George EI (1992) Explaining the Gibbs sampler. *Am Stat* 46:167–174
- Cho SE (2007) Effects of spatial variability of soil properties on slope stability. *Eng Geol* 92(3-4):97–109. <https://doi.org/10.1016/j.enggeo.2007.03.006>
- Cho SE (2009) Probabilistic assessment of slope stability that considers the spatial variability of soil properties. *J Geotech Geoenviron* 136:975–984
- Clifford P (1990) Markov random fields in statistics. *Disorder in Physical systems: A volume in honor of John M Hammersley*: 19–32
- Dai Z, Ritzi RW, Dominic DF (2005) Improving permeability semivariograms with transition probability models of hierarchical sedimentary architecture derived from outcrop analog studies. *Water Resour Res* 41:W07032
- Dawson E, Roth W, Drescher A (1999) Slope stability analysis by strength reduction. *Geotechnique* 49(6):835–840. <https://doi.org/10.1680/geot.1999.49.6.835>
- Elfeki A, Dekking M (2001) A Markov chain model for subsurface characterization: theory and applications. *Math Geol* 33(5):569–589. <https://doi.org/10.1023/A:1011044812133>
- Elkateb T, Chalaturnyk R, Robertson PK (2003) An overview of soil heterogeneity: quantification and implications on geotechnical field problems. *Can Geotech J* 40(1):1–15. <https://doi.org/10.1139/t02-090>
- Evans SG (1982) Landslides and surficial deposits in urban areas of British Columbia: a review. *Can Geotech J* 19(3):269–288. <https://doi.org/10.1139/t82-034>
- Fenton GA, Griffiths DV (2008) Risk assessment in geotechnical engineering. Wiley. <https://doi.org/10.1002/9780470284704>
- Geman S, Geman D (1984) Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images. *IEEE Trans Pattern Anal Mach Intell PAMI-6(6):721–741*. <https://doi.org/10.1109/TPAMI.1984.4767596>
- Griffiths D, Fenton GA (2004) Probabilistic slope stability analysis by finite elements. *J Geotech Geoenviron* 130(5):507–518. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2004\)130:5\(507\)](https://doi.org/10.1061/(ASCE)1090-0241(2004)130:5(507))
- Griffiths D, Huang J, Fenton GA (2009) Influence of spatial variability on slope reliability using 2-d random fields. *J Geotech Geoenviron* 135(10):1367–1378. [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0000099](https://doi.org/10.1061/(ASCE)GT.1943-5606.0000099)
- Hansen L, Eilertsen R, Solberg I-L, Rokoengen K (2007) Stratigraphic evaluation of a Holocene clay-slide in northern Norway. *Landslides* 4(3):233–244. <https://doi.org/10.1007/s10346-006-0078-4>
- Hicks MA, Nuttall JD, Chen J (2014) Influence of heterogeneity on 3d slope reliability and failure consequence. *Comput Geotech* 61:198–208. <https://doi.org/10.1016/j.compgeo.2014.05.004>
- Ji J, Liao H, Low BK (2012) Modeling 2-d spatial variation in slope reliability analysis using interpolated autocorrelations. *Comput Geotech* 40:135–146. <https://doi.org/10.1016/j.compgeo.2011.11.002>
- Jiang S-H, Li D-Q, Cao Z-J, Zhou C-B, Phoon K-K (2014) Efficient system reliability analysis of slope stability in spatially variable soils using Monte Carlo simulation. *J Geotech Geoenviron* 141:04014096
- Li W (2007) A fixed-path Markov chain algorithm for conditional simulation of discrete spatial variables. *Math Geol* 39(2):159–176. <https://doi.org/10.1007/s11004-006-9071-7>
- Li SZ (2009) Markov random field modeling in image analysis. **Springer Science & Business Media**
- Li W, Li B, Shi Y (1999) Markov-chain simulation of soil textural profiles. *Geoderma* 92(1-2):37–53. [https://doi.org/10.1016/S0016-7061\(99\)00024-5](https://doi.org/10.1016/S0016-7061(99)00024-5)
- Li D-Q, Jiang S-H, Cao Z-J, Zhou W, Zhou C-B, Zhang L-M (2015) A multiple response-surface method for slope reliability analysis considering spatial variability of soil properties. *Eng Geol* 187:60–72. <https://doi.org/10.1016/j.enggeo.2014.12.003>
- Li D-Q, Qi X-H, Cao Z-J, Tang X-S, Phoon K-K, Zhou C-B (2016a) Evaluating slope stability uncertainty using coupled Markov chain. *Comput Geotech* 73:72–82. <https://doi.org/10.1016/j.compgeo.2015.11.021>
- Li Z, Wang X, Wang H, Liang RY (2016b) Quantifying stratigraphic uncertainties by stochastic simulation techniques based on Markov random field. *Eng Geol* 201:106–122. <https://doi.org/10.1016/j.enggeo.2015.12.017>
- Mallet J-L (1989) Discrete smooth interpolation. *ACM Trans Graph(TOG)* 8(2):121–144. <https://doi.org/10.1145/62054.62057>
- Norberg T, Rosén L, Baran A, Baran S (2002) On modeling discrete geological structures as Markov random fields. *Math Geol* 34(1):63–77. <https://doi.org/10.1023/A:1014079411253>
- Phoon K-K, Kulhawy FH (1999) Characterization of geotechnical variability. *Can Geotech J* 36(4):612–624. <https://doi.org/10.1139/t99-038>
- Tang X-S, Li D-Q, Zhou C-B, Phoon K-K (2015) Copula-based approaches for evaluating slope reliability under incomplete probability information. *Struct Saf* 52:90–99. <https://doi.org/10.1016/j.strusafe.2014.09.007>
- Tolpekin VA, Stein A (2009) Quantification of the effects of land-cover-class spectral separability on the accuracy of Markov-random-field-based superresolution mapping. *IEEE Trans Geosci Remote Sens* 47(9):3283–3297. <https://doi.org/10.1109/TGRS.2009.2019126>
- Wang X, Li Z, Wang H, Rong Q, Liang RY (2016) Probabilistic analysis of shield-driven tunnel in multiple strata considering stratigraphic uncertainty. *Struct Saf* 62:88–100. <https://doi.org/10.1016/j.strusafe.2016.06.007>
- Wang H, Wellmann JF, Li Z, Wang X, Liang RY (2017) A segmentation approach for stochastic geological modeling using hidden Markov random fields. *Math Geosci* 49(2):145–177. <https://doi.org/10.1007/s11004-016-9663-9>
- Weissmann GS, Carle SF, Fogg GE (1999) Three-dimensional hydrofacies modeling based on soil surveys and transition probability geostatistics. *Water Resour Res* 35(6):1761–1770. <https://doi.org/10.1029/1999WR900048>
- Wellmann JF, Regenauer-Lieb K (2012) Uncertainties have a meaning: information entropy as a quality measure for 3-d geological models. *Tectonophysics* 526:207–216
- Wellmann JF, Horowitz FG, Schill E, Regenauer-Lieb K (2010) Towards incorporating uncertainty of structural data in 3d geological inversion. *Tectonophysics* 490(3-4):141–151. <https://doi.org/10.1016/j.tecto.2010.04.022>
- Wu F-Y (1982) The Potts model. *Rev Mod Phys* 54:235
- Zhang Y, Brady M, Smith S (2001) Segmentation of brain MRI images through a hidden Markov random field model and the expectation-maximization algorithm. *IEEE Trans Med Imaging* 20(1):45–57. <https://doi.org/10.1109/42.906424>

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