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Comparison of selected statistical distributions for modelling the diameter distributions in near-natural *Abies–Fagus* forests in the Świętokrzyski National Park (Poland)

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Abstract The aim of the study is to compare selected theoretical distributions (normal, lognormal, Weibull, gamma, logistic, and exponential) in describing the tree diameter (DBH) distributions of mixed near-natural forests consisting of fir *Abies alba* Mill. and beech *Fagus sylvatica* L. growing in various vertical structures. Tree DBH data were collected between 1997 and 2008 from 51 sample plots established in the Świętokrzyski National Park in Poland. The empirical data represent differentiated DBH distributions, ranging from almost symmetric to extremely asymmetric ones. The chi-square test and the modified Kolmogorov–Smirnov test were chosen for the goodness-of-fit testing. In addition to the test statistics, the bias (B), the root mean square error (RMSE) and the graphical method (quantile–quantile plots) were used. In one-storied stands, the most suitable distributions were the normal and logistic distributions; in two-storied and multilayered stands, the Weibull and gamma distribution were the most suitable; and in selection stands, the exponential distribution was the most appropriate to describe the DBH distribution. The order of precision of the tested distributions (from the highest to the lowest) was Weibull, gamma, logistic, normal, exponential, and lognormal. The normal and exponential distribution should be applied only to one-storied and selection forests, respectively. The least suitable distribution for DBH distribution modelling was the lognormal one.

Keywords Stand structure · DBH distribution · Goodness-of-fit tests · *Abies alba* · *Fagus sylvatica* · Świętokrzyski National Park

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Introduction

The approximation of empirical diameter (DBH) distributions with the help of selected theoretical distribution functions plays an important role in forest research and practice. Diameter distributions are used in many cases as parts of growth and yield models. They also serve as models of the forest structure, which play a vital role in sustainable forest management.

There have been numerous papers on the DBH modelling published in the scientific forestry literature, starting from the paper by de Liocourt (1898), describing the use of the exponential distribution for DBH distribution modelling in stands having a complex structure. There have also been results of various studies published during the last decade, which proves the importance of the topic. For example Pretzsch (1995, 1997, 2001) provided the methodology for the stand structure modelling based on data on mixed beech *Fagus sylvatica* L.—larch *Larix decidua* Mill. stands and described various options for modelling growth and structure of stands having various species composition. The author also assessed the influence of silvicultural treatments on forest stand structure, including DBH

distribution (Pretzsch 1998). Examples of the most recent studies published in Poland include, e.g. papers by Poznański (1997), who described the relationship between the DBH distribution and the developmental phase of forests with a complex structure, and Zasada (2000), who searched for the best theoretical distribution for the DBH modelling in birch *Betula pendula* Roth stands.

For shade-tolerant species such as fir *Abies alba* Mill. and beech *Fagus sylvatica* L. in Central Europe, growth and yield models and models of the stand structure should be flexible enough to cover various stand structures, such as unimodal distributions of one-storied (Zasada 1995), more complex two-storied (Bernadzki and Żybura 1989), and the most challenging multilayered and selection stands (Pretzsch 2001). The structure of multilayered and selection *Abies* forests have been frequently described in the literature, especially in the context of various silvicultural regimes (e.g. Leibundgut 1945; Köstler 1956; Trepp 1974; Schütz 1981; Ammon 1995). Publications on mixed *Abies–Fagus* forests with a complex structure are much scarcer (e.g. Dittmar 1990, 1992; Gerold and Biehl 1992; Jaworski et al. 2000).

Historically, the very first theoretical distribution function used for DBH modelling in *Abies* stands with a complex structure was the exponential distribution (de Liocourt 1898). Subsequent functions used for the DBH distribution approximation in pure *Abies* and *Fagus* stands have included, among others, Weibull, beta, and gamma distributions (e.g. Maltamo et al. 1995; Merganič and Sterba 2006). In near-natural *Abies* and mixed *Abies–Fagus* stands of Central Europe the form of the DBH distribution heavily depends on the developmental stage of the stand, and it has usually a shape lying in between the exponential and symmetric normal distributions. This fact has been confirmed in numerous descriptions of natural and primeval forests, growing especially in mountainous areas (e.g. Korpel' 1995; Jaworski et al. 1999, 2000). Similar conclusions apply also to managed *Abies* and *Abies–Fagus* stands. Because DBH distributions of near-natural mixed *Abies* and *Fagus* stands with a diverse vertical structure can take various forms, from extremely skewed to almost symmetric, including, in many cases, bi- and multimodality (e.g. Zucchini et al. 2001), almost no results have been published so far on the mathematical diameter distribution model development for such complex stands.

The DBH models based on a single distribution function are in many cases not sufficient to provide a satisfactory fit to the data. In such situations a finite mixture of usually two distributions for all species together or for each species separately can be created, as described by Maltamo and Kangas (1998), Zhang et al. (2001), Liu et al. (2002) or Zasada and Cieszewski (2005).

The knowledge of theoretical distributions describing data from mixed forests of *Abies* and *Fagus* would be advantageous in dendrometry, silviculture, forest management, and ecology. It would provide a valuable information on the complex stands' structure and would allow to generate and model DBH distributions of such stands as a part of growth models. Thus, the aim of this study is to compare the selected theoretical distributions in describing DBH distributions of mixed near-natural forests, comprised of *Abies* and *Fagus* of various vertical structures, located in the Świętokrzyski National Park in Poland.

Material and methods

Characteristics of the study area

Research described in this paper was conducted in the Święta Katarzyna and Święty Krzyż forest sections of the Świętokrzyski National Park in the Świętokrzyskie Mountains. Forests in the investigated region have been under strict protection since 1922 or 1950. Before reservations and the national park were created, the majority of sample stands were commercially harvested only occasionally due to limited access.

Soils are Distric Cambisols and Haplic Luvisols [Kowalkowski 2000; the types and sub-types are given according to FAO ISRIC (1998)]. Based on the long-term (1955–1994) observations from the meteorological station located at the Święty Krzyż (575 m n.p.m.), in the analysed period the average observed yearly temperature is +5.9°C (the average temperature for January is −5.2°C; the average temperature for July is +15.9°C); the average annual rainfall is 923 mm; and the vegetation period length is about 182 days. The following plant communities have formed in such conditions: *Dentario glandulosae-Fagetum*, *Abietetum polonicum*, and *Querco roboris-Pinetum* (names after Matuszkiewicz 2002).

Field work

Fifty-one sample points were chosen between 1997 and 2008 in the Święta Katarzyna and Święty Krzyż forest sections close to Bodzentyn village (Fig. 1). These were circular sample plots of 0.1–0.4 ha. The DBH of all trees 7.0 cm and thicker was measured, and the vertical structure of the stand was assessed in each sample plot (Podlaski 2003). The vertical structure of the forest was analysed using the IUFRO stand's layer classification (100, 200 and 300, which denotes upper, middle and lower stand's level, respectively). The average age was calculated for each stand's layer based on the increment cores extracted from a few trees per layer. The stand was classified as a selection one when the age of trees

Fig. 1 Study area. *A* Święta Katarzyna forest section, *B* Święty Krzyż forest section
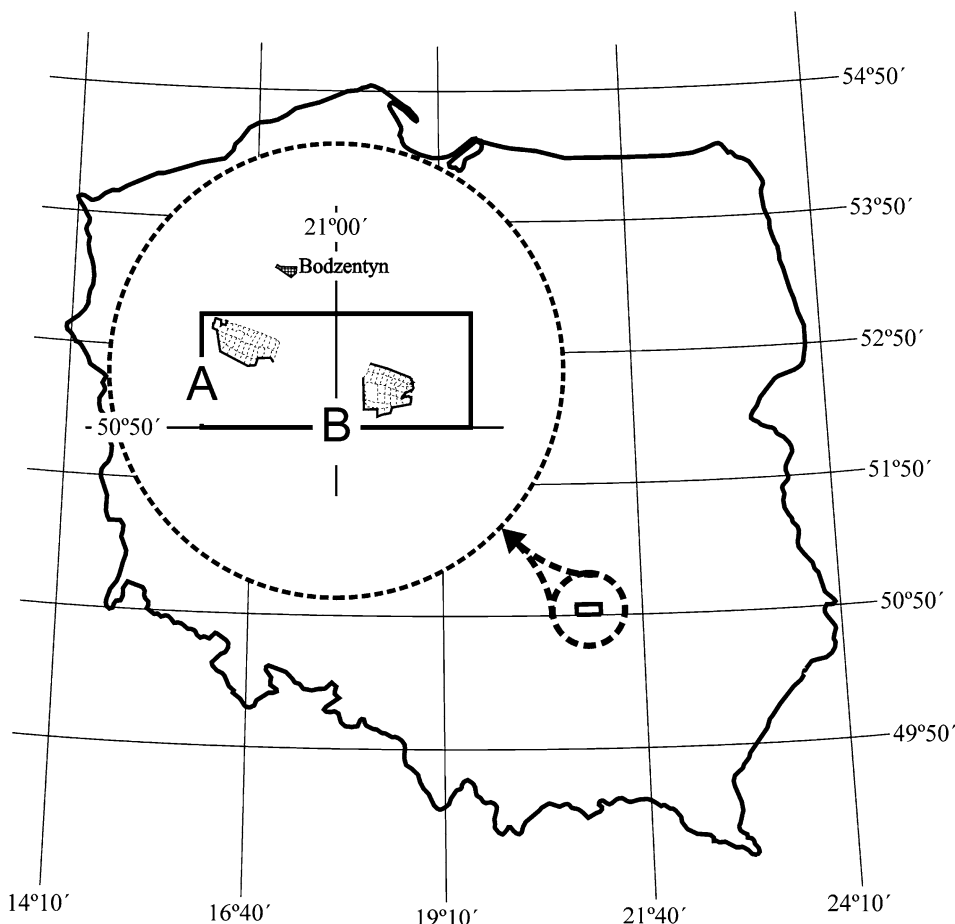
was strongly diversified, and trees having different age are mixed together spatially and vertically (Assmann 1970).

Data analysis

In this study six theoretical distributions (normal, lognormal, Weibull, gamma, logistic, and exponential) were used to fit the DBH data of 51 sample plots, representing stands of various vertical structures (Table 1). These theoretical distributions have the probability density functions (PDF) given by:

normal

$$f_X(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad (1)$$

lognormal

$$f_X(x; \mu, \sigma, \gamma) = \frac{1}{(x-\gamma)\sqrt{2\pi}\sigma} e^{-\frac{(\ln(x-\gamma)-\mu)^2}{2\sigma^2}} \qquad (2)$$

Weibull

$$f_X(x; \alpha, \beta, \gamma) = \frac{\alpha}{\beta} \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x-\gamma}{\beta}\right)^\alpha} \qquad (3)$$

gamma

$$f_X(x; \alpha, \beta, \gamma) = \frac{\beta^\alpha (x-\gamma)^{\alpha-1}}{\Gamma(\alpha)} e^{-\beta(x-\gamma)} \qquad (4)$$

logistic

$$f_X(x; \mu, \sigma) = \frac{1}{\sigma\left(1 + e^{\frac{x-\mu}{\sigma}}\right)^2} e^{-\frac{x-\mu}{\sigma}} \qquad (5)$$

exponential

$$f_X(x; \alpha, \gamma) = \alpha e^{-\alpha(x-\gamma)} \qquad (6)$$

with $x >$ location parameter γ, shape parameter $\alpha > 0$, scale parameter $\beta > 0$; μ is the mean, σ is the standard deviation, $\Gamma(\cdot)$ is the gamma function.

Normal, lognormal, Weibull, gamma, logistic, and exponential distribution suitably approximated empirical DBH distributions in stands with *Abies* and *Fagus* in Central Poland (Zasada 1995; Rymer-Dudzińska and Dudzińska 1999, 2001; Podlaski 2006) and their parameters are relatively easy to calculate (e.g. Zarnoch and Dell 1985; Borders et al. 1987; Borders and Patterson 1990). Distribution

parameters were estimated using the maximum likelihood method (MLE); they were calculated by the Statgraphics Plus ver. 5.1 software (MANUGISTICS 2001).

The chi-square test (Reynolds et al. 1988) and the modified Kolmogorov–Smirnov test (modified K–S) (D'Agostino and Stephens 1986) were chosen for testing the goodness-of-fit. The chi-square test divides the range of the data into a set of equiprobable classes (Law and Kelton 1999). It has been determined that with a minor adjustment in the definition of the empirical distribution function (EDF) for the Kolmogorov–Smirnov test (K–S), the statistical power of the K–S test can be significantly enhanced (D'Agostino and Stephens 1986). In fact, the modified K–S is uniformly at least as powerful as the K–S test. Furthermore, the conduct of the modified K–S test is no more complicated (i.e. it involves no more steps) than the K–S test. The modified Kolmogorov–Smirnov statistic D was compared to a table of critical values obtained through Monte Carlo studies, which are specific to the distribution being fit (D'Agostino and Stephens 1986). Details and tables of critical values may be found in D'Agostino and Stephens (1986). In addition to the test statistics, the bias (B), the root mean square error (RMSE) and the graphical method (quantile–quantile plots) were used (Reynolds et al. 1988; Liu et al. 2002):

$$B = \frac{\sum_{c=1}^{nc} (n_c - \hat{n}_c)}{nc} \tag{7}$$

$$\text{RMSE} = \sqrt{\frac{\sum_{c=1}^{nc} (n_c - \hat{n}_c)^2}{nc}} \tag{8}$$

where n_c and $\hat{n}_c$ are the observed and predicted number of trees, respectively, in the cth DBH class, nc is the number of DBH classes. The quantile–quantile plot is a graphical technique for determining if two data sets come from populations with a common distribution (NIST/SEMATECH 2008). The plot shows the fraction of observations at or below a certain DBH plotted versus the equivalent percentiles of the fitted distributions. The distribution is used to define the X-axis and is represented by the diagonal line. The fact that the points lie close to the diagonal line confirms the fact that the distribution provides a good model for the empirical data. The quantile–quantile plots were used to assign ranks allowing assessment of the accuracy of approximation. Rank '1' was assigned to the given theoretical distribution that best fit the empirical DBH distributions. Calculations were performed using Mathematica 5 software (Wolfram 2003).

The mean values of χ^2, D, RMSE and ranks revealed the versatility and usefulness of a given theoretical distribution to approximate empirical DBH distributions.

Results

The average basal area (BA) fraction per species of the investigated stands varied from 6.02 m^2 ha^{-1} (selection stands) to 14.28 m^2 ha^{-1} (multilayered stands—slightly skewed distribution) for *Abies*, and from 14.18 m^2 ha^{-1} (multilayered stands—extremely skewed distribution) to 19.52 m^2 ha^{-1} (one-storied stands) for *Fagus* (Table 1). In all stands both species accounted for more than 60% of the total basal area. The sample plots covered a broad spectrum of structures: 11 plots represented one-storied, 8 plots—two-storied, 25 plots—multilayered, and 7 plots—selection stands (Table 1, Figs. 2, 3, 4, 5, 6). The largest mean DBH was observed in the one-storied stands (25.2–39.2 cm), and the smallest—in multilayered stands with extremely skewed diameter distribution (16.5–27.8 cm) (Table 2). Extreme diameters (exceeding 100 cm) were observed in stands representing the complex structure (Table 2).

The DBH distributions of the investigated plots revealed usually positive skewness (left skewed). The largest skewness revealed diameter distributions of multilayered stands with extremely skewed diameter distribution (mean skewness was 2.6324), and the lowest—one-storied stands (mean skewness was 0.2006) (Table 2).

Good fit ($P > 0.05$) of the theoretical models to empirical DBH distributions was observed for at least 90% of plots in a case of the following distributions (the chi-square and the modified K–S test; Table 3):

1) one-storied stands—normal, logistic;
2) two-storied stands—Weibull;
3) multilayered stands—Weibull;
4) selection stands—exponential.

In a case of the lognormal distribution, fractions of plots with $P > 0.05$ were less or equal than 90% in all distinguished groups of stands (Table 3). The bias of the investigated plots was negligible and insignificant (for all plots and for all theoretical distributions it was from -1.9332×10^{-5} to 4.4912×10^{-4}).

Based on the mean values of χ^2 (the chi-square test), D (the modified K–S test), RMSE and ranks (quantile–quantile plots) the most accurate distributions were (Tables 3, 4, Figs. 2, 3, 4, 5, 6):

1) one-storied stands—normal, logistic, Weibull, gamma;
2) two-storied stands—Weibull, gamma;
3) multilayered stands (with slightly skewed distribution)—Weibull, gamma, logistic;
4) multilayered stands (with extremely skewed distribution)—Weibull, gamma;
5) selection stands—exponential, Weibull, gamma.

Table 1 Stand structure and basal area (BA) for the investigated sample plots

Stand structure	Number of sample plots	Abies alba (m² ha⁻¹)			Fagus sylvatica (m² ha⁻¹)			Other species (m² ha⁻¹)		
		Mean	Min	Max	Mean	Min	Max	Mean	Min	Max
One-storied	11	7.63	1.28	10.36	19.52	10.02	32.66	0.90	0.00	4.62
Two-storied	8	11.66	1.11	20.16	16.29	3.15	33.11	3.02	0.00	8.55
Multilayered (slightly skewed distribution)	14	14.28	3.87	18.66	16.19	2.82	28.11	4.71	0.00	11.16
Multilayered (extremely skewed distribution)	11	7.51	2.64	19.21	14.18	2.88	25.66	4.52	0.00	12.50
Selection	7	6.02	1.69	13.71	19.06	7.96	25.94	2.92	0.00	8.72

Fig. 2 Empirical and theoretical DBH distributions in one exemplary one-storied stand (sample plot No. 3); $P > 0.05$ for four theoretical distributions (the chi-square test and the modified Kolmogorov–Smirnov test). A Quantile–quantile plot
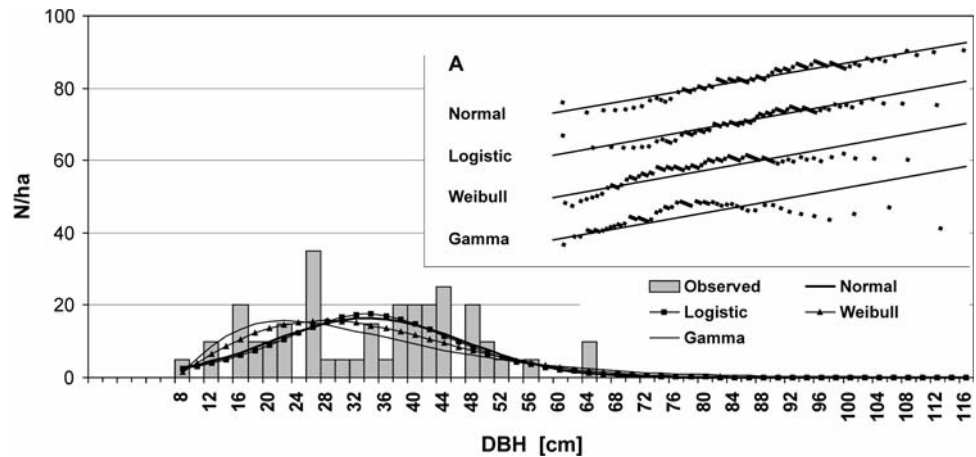
Fig. 3 Empirical and theoretical DBH distributions in one exemplary two-storied stand (sample plot No. 7); $P > 0.05$ for the Weibull distribution (the chi-square test and the modified Kolmogorov–Smirnov test). A Quantile–quantile plot
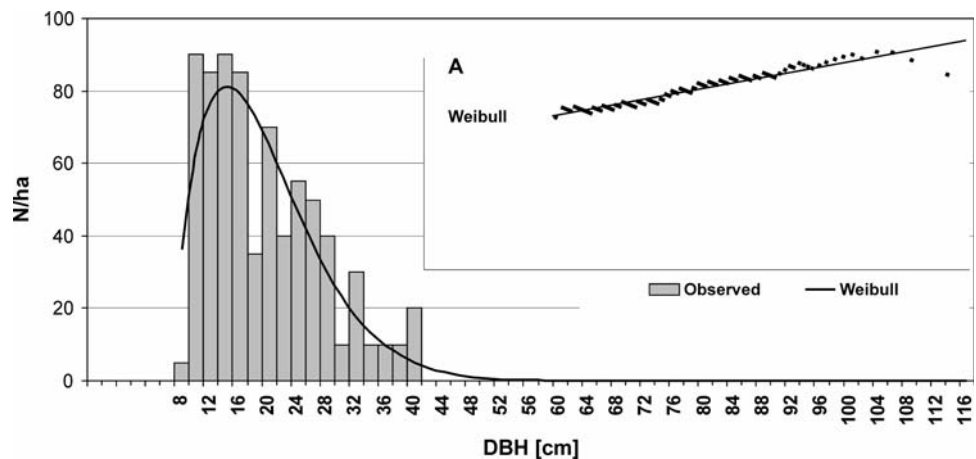
The Weibull and gamma were the most versatile distributions. They allowed for the accurate approximation of the empirical DBH distributions in almost all investigated *Abies* and *Fagus* stands. The least generality revealed the lognormal distribution.

Discussion

Similar results, giving a proof of suitability of Weibull and gamma distributions for describing the DBH distribution of natural mixed *Abies–Fagus* stands in various developmental phases, were reported by Podlaski (2006). Other authors have obtained the best results for S_B Johnson, Weibull, and beta distributions in *Abies* stands (Zasada 1995), and for lognormal, Weibull, and beta distributions in *Fagus* stands (Rymer-Dudzińska and Dudzińska 1999, 2001). However, the above-mentioned results, except Podlaski (2006), refer to one-storied, managed stands.

Apart from typical one-storied stands growing in managed forests, one-storied near-natural stands had more oblate and skewed DBH distributions. Such stands are scarce and have quite interesting DBH structures that have yet to be described in literature. Two-storied stands had a

Fig. 4 Empirical and theoretical DBH distributions in one exemplary multilayered stand (slightly skewed distribution on sample plot No. 1; skewness was 0.4862); $P > 0.05$ for three theoretical distributions (the chi-square test and the modified Kolmogorov–Smirnov test). *A* Quantile-quantile plot
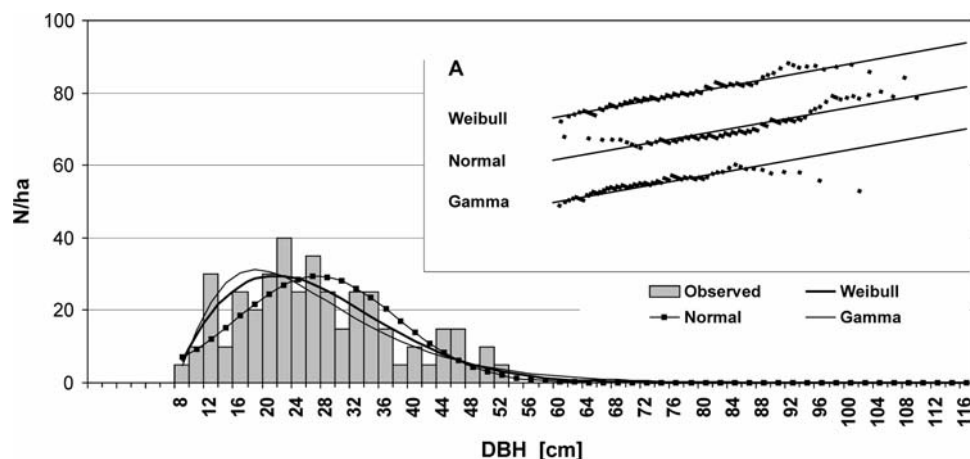
Fig. 5 Empirical and theoretical DBH distributions in one untypical exemplary multilayered stand (extremely skewed distribution on sample plot No. 4); $P > 0.05$ for two theoretical distributions (the chi-square test and the modified Kolmogorov–Smirnov test). *A* Quantile–quantile plot
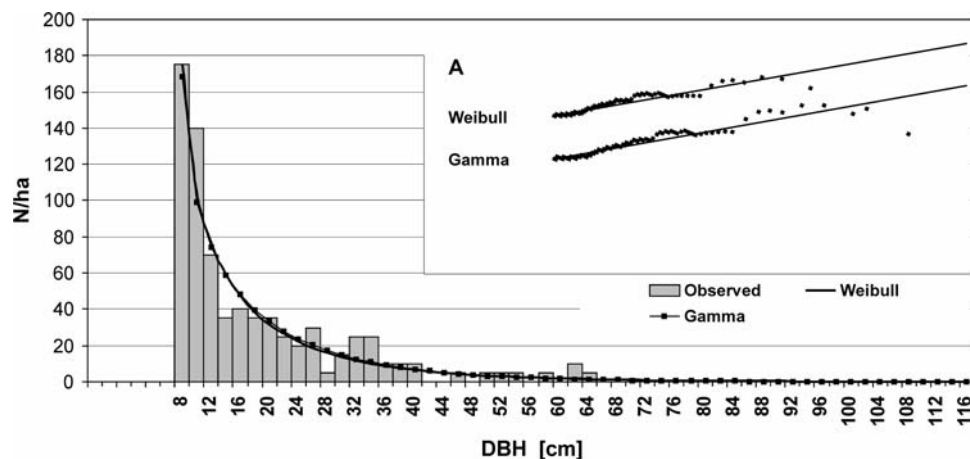
Fig. 6 Empirical and theoretical DBH distributions in one exemplary selection stand (sample plot No. 15); $P > 0.05$ for the exponential distribution (the chi-square test and the modified Kolmogorov–Smirnov test). *A* Quantile–quantile plot
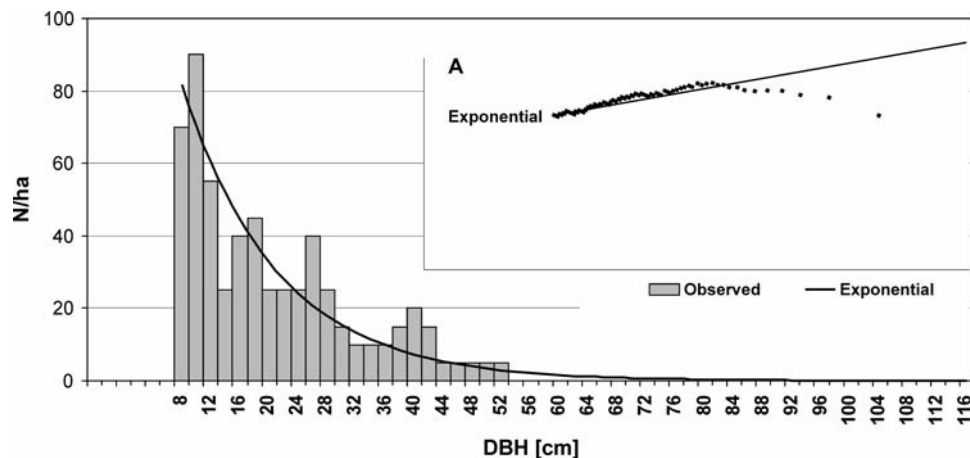
very high share of thin trees (DBH below 20 cm), creating the lower story. The height differences between upper and lower layers were not too large. Similar DBH distributions observed in primeval two-storied stands were described by Korpel' (1995).

The DBH distributions observed in the mixed multi-layered and selection forests differed between each other first of all because of the existence of the oldest and the thickest trees with DBH above 100 cm, and because of the admixture of the thinnest trees with DBH below 20 cm. Similar DBH distributions were described by Jaworski (1997) in primeval *Abies* and *Fagus* stands and by Karcz-marski (2005) in spruce *Picea abies* (L.) H. Karst. stands.

The approximation of DBH distributions using a single theoretical distribution is employed first of all in pure, even-aged stands (e.g. Zhang et al. 2003), but certain

Table 2 Statistics of DBH

Stand structure	Mean DBH (cm)		Max DBH (cm)	Skewness			Kurtosis		
	Min	Max		Mean	Min	Max	Mean	Min	Max
One-storied	25.2	39.2	76.0	0.2006	−0.1863	0.8441	−0.1421	−0.6722	1.9200
Two-storied	19.2	32.8	98.0	1.1911	0.6122	2.9888	2.4381	−0.1322	10.7792
Multilayered (slightly skewed distribution)	19.9	38.2	101.0	0.7475	0.1922	1.5928	1.1286	−0.8921	3.8664
Multilayered (extremely skewed distribution)	16.5	27.8	102.0	2.6324	1.4614	4.0192	8.1137	2.0113	19.2114
Selection	18.4	27.5	103.0	1.6281	0.8679	3.1239	3.9261	−0.1368	12.4688

Table 3 The chi-square test (χ^2 statistic) and the modified Kolmogorov–Smirnov test (D statistic) for testing goodness-of-fit

Stand structure	Distribution					
	Normal	Lognormal	Weibull	Gamma	Logistic	Exponential
Mean χ^2 statistics over all stands of a group						
One-storied	6.142*	32.381	11.241	14.579	8.002*	59.628
Two-storied	30.018	25.642	5.981*	12.641	39.277	30.111
Multilayered (slightly skewed distribution)	26.821	26.222	9.681*	12.844	20.822	55.628
Multilayered (extremely skewed distribution)	121.629	29.165	12.987*	15.666	102.873	38.117
Selection	69.747	27.555	9.778	10.321	49.521	9.244*
Mean D statistics over all stands of a group						
One-storied	0.612*	1.510	0.914	1.513	0.741*	2.921
Two-storied	1.619	1.299	0.516*	0.622*	1.084	1.998
Multilayered (slightly skewed distribution)	1.669	1.578	0.812*	0.813*	0.979	3.001
Multilayered (extremely skewed distribution)	3.211	2.001	1.329*	1.287*	2.965	1.780
Selection	2.022	1.622	0.791	0.782	1.961	0.749*

* fraction of plots where $P > 0.05$ is grater than 90%

Table 4 The mean root mean square error (RMSE) and the mean ranks (quantile–quantile plots) of the six investigated distributions

Stand structure	Distribution					
	Normal	Lognormal	Weibull	Gamma	Logistic	Exponential
Mean RMSE over all stands of a group						
One-storied	1.828	5.999	3.821	3.941	2.651	8.491
Two-storied	6.983	5.778	2.841	3.922	7.012	6.981
Multilayered (slightly skewed distribution)	6.121	5.122	3.311	3.428	5.792	7.700
Multilayered (extremely skewed distribution)	14.692	7.162	4.591	4.021	12.999	6.910
Selection	10.049	6.128	3.612	3.628	8.995	3.016
Mean ranks over all stands of a group (quantile–quantile plots)						
One-storied	1.6	5.0	3.0	3.4	2.1	5.9
Two-storied	4.1	3.9	1.1	1.9	5.5	4.5
Multilayered (slightly skewed distribution)	4.5	4.2	1.5	2.6	2.6	5.5
Multilayered (extremely skewed distribution)	5.5	3.7	1.4	2.3	3.9	4.3
Selection	5.6	4.0	2.1	2.6	5.4	1.3

research reveals similar possibilities in mixed stands (e.g. Maltamo 1997; Siipilehto 1999; Liu et al. 2002; Zhang et al. 2003). The use of a single function to model the empirical DBH distribution in mixed stands of *Abies* and *Fagus* is not always possible (e.g. in some parts with two distinct layers). Moreover, employment of a single

distribution is relatively simple from a computational point of view. In cases where it is not possible to use a single distribution, it is necessary to adapt more complex models, utilising mixture distributions (e.g. Siekierski 1991; Zhang et al. 2001; Zucchini et al. 2001; Liu et al. 2002; Zasada and Cieszewski 2005), a segmented approach (e.g. Goelz and Leduc 2002), or non-parametric methods (e.g. Maltamo and Kangas 1998).

The Weibull function appeared to be suitable for describing the wide variety of DBH distributions because of its well-known and documented flexibility (e.g. Merganič and Sterba 2006). The presented study confirmed the usability of the Weibull function to approximate the DBH distributions observed in forest with a share of *Abies* and *Fagus* growing in various vertical structures. The Weibull distribution is sufficient to characterise rather regular distributions. However, it fails to adequately describe irregular, multimodal stand structures. In such the cases it is necessary to use the modified Weibull and the mixture Weibull models (Zhang and Liu 2006). When the measurement of DBH starts from a pre-defined recording limit, the use of the truncated Weibull function is recommended (Merganič and Sterba 2006). The truncated Weibull function can be very useful e.g. in the case of the estimation of the data below the point of truncation (Merganič and Sterba 2006).

In the present study we applied a few validation methods. Goodness-of-fit tests have basic limitations related to the class width (the chi-square test) and to the parameter estimation method from the data (the K–S test). These limitations were lessened by using proper modifications (equiprobable classes and the modified statistics D) (see Law and Kelton 1999; D'Agostino and Stephens 1986). One main general problem has been identified when matching theoretical distributions to approximate empirical DBH distributions: tests may not give much useful information about the model (Reynolds et al. 1988). This is why we decided to introduce additional means of the goodness-of-fit assessment: the RMSE and the graphical procedure (quantile–quantile plots). Using a few validation methods, including first of all the chi-square test and the RMSE, is a standard procedure in studies on the diameter distribution models (e.g. Liu et al. 2002; Zhang and Liu 2006; Palahí et al. 2007). The results obtained from all the above-mentioned methods, especially the chi-square test, RMSE and the graphical procedure, were very similar (see Tables 3, 4), which allows us to draw more trustworthy conclusions.

Recapitulation of results and conclusions

1. The investigated sample plots represent mixed, near-natural stands with *Abies* and *Fagus*, characterised by various vertical structures (one-, two-, multilayered and selection).

2. In one-storied stands, the most suitable distributions were the normal and logistic distribution; in two-storied stands, the Weibull and gamma distribution were the most suitable; in multilayered stands, the Weibull and gamma distribution; and in selection stands, the exponential distribution was the most appropriate.

3. The most versatile distributions for DBH distributions modelling in the investigated near-natural stands were the Weibull and gamma distribution; the least suitable was the lognormal.

4. Using a single theoretical distribution to model empirical DBH distributions in near-natural mixed stands of *Abies* and *Fagus* gave satisfactory results in most analysed cases, allowing the use of simple computational methods.

5. Research on DBH distributions in near-natural mixed stands of *Abies* and *Fagus* should be continued. Including the relationship between vertical stand structure and parameters of Weibull and gamma distributions, as well as comparing DBH distributions in near-natural and managed stands of the same species composition, would be helpful.

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