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RESEARCH REPORT

Training the equidistant principle of number line spacing

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Abstract The characteristics of effective numerical trainings are still under scientific debate. Given the importance of number line estimation due to the strong relation between task performance and arithmetic abilities, the current study aimed at training one important number line characteristic: the equidistant spacing of adjacent numbers. Following an embodied training approach, second-graders were trained using a randomized crossover design to divide a presented line into different numbers of equal segments by walking the line with equally spaced steps. Performance was recorded, and feedback as to the correct equidistant spacing was provided using the Kinect sensor system. Training effects were compared to a control training with no involvement of task-specific whole-body movements. Results indicated more pronounced specific training effects after the embodied training. Moreover, transfer effects to number line estimation and arithmetic performance were partially observed. In particular, differential training effects for bounded versus unbounded number line estimation corroborate the assumption that not only bodily experiences but also the need for a flexible adaption of the perspective on the training material might influence training success. Hence, more pronounced training effects of the embodied training might stem from different cognitive processes involved.

Keywords Mental number line · Equidistance · Number line estimation · Bounded/unbounded number line estimation task · Mathematical skills

Introduction

Mathematics is a vital part of our everyday life. A minimal knowledge of mathematics is necessary for almost any job, which is why it has been a milestone educational subject in schools for a long time. It is therefore not surprising that deficits in understanding of mathematical concepts have been found associated with less vocational success and a lower income (Butterworth et al. 2011; Parsons and Bynner 2005). When scholastic education is not sufficient to help an individual gain the necessary mathematical understanding for everyday life, remedial measures need to be provided. To be effective, these should not simply consist of a repetition of school lessons, but should attempt to convey the content in an alternative, more customized way (Dowker 2005). However, an important aspect is the to-be-trained content: It has repeatedly been shown that basic

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numerical skills are fundamental for children's future arithmetic development (e.g., Booth and Siegler 2008; Butterworth et al. 2011; Jordan et al. 2009; Moeller et al. 2011), and thus, trainings focusing on their improvement rather than on curricular learning material are among the most effective (Kaufmann et al. 2003; Kroesbergen and Van Luit 2003). An interesting new option to support children's mathematical abilities is embodied training scenarios which concentrate on enhancing basic numerical skills such as number line estimation (e.g., Link et al. 2013).

Training the number line concept

There already are a number of empirically evaluated trainings addressing number line estimation (e.g., Kucian et al. 2011; Ramani et al. 2012; Ramani and Siegler 2011; Siegler and Ramani 2008, 2009; Siegler 2009). In these training studies the mental number line concept is considered a building block for further numerical development because there is compelling evidence suggesting a close link between accurate number line estimation and present as well as future basic numerical and arithmetic abilities (Booth and Siegler 2006, 2008; Geary et al. 2012; Laski and Siegler 2007; Siegler and Booth 2004). An index for children's magnitude representation is their performance in the number line estimation task, in which they have to estimate the position of a target number (e.g., 26) on an otherwise empty number line with only the start and end point given (e.g., 0 and 100, e.g., Siegler and Opfer 2003; Siegler and Booth 2004). Booth and Siegler (2006), for example, observed more accurate number line estimations, meaning less deviation from estimated to actual target position, to be linked to higher mathematics achievement. Furthermore, children with mathematical disabilities were found to be present with impaired number line performances (e.g., Von Aster and Shalev 2007; Geary et al. 2012). This emphasizes the importance of the concept for children's numerical development. Based on such findings, Siegler and Ramani (2008, 2009) investigated the potential of training the number line concept in a series of intervention studies employing simple board games, in which children had to move tokens along numbered fields (see also Ramani et al. 2012; Ramani and Siegler 2011; Siegler 2009). The authors repeatedly observed that playing such linear number board games enhanced preschoolers' numerical understanding even beyond the directly trained number line estimation performance. Comparable results were found by Kucian et al. (2011), who took a first step toward a computerized and more game-like version of the number line estimation task in which children had to land rockets on the number line.

Importantly, all number line trainings rely on a number of characteristics that have been ascribed to the mental number line concept: (1) Numbers are represented in ascending order from left to right—at least in Western countries (e.g., Dehaene et al. 1993; Shaki et al. 2009); (2) it starts to develop early in life with its accuracy increasing with age and experience (de Hevia et al. 2014; de Hevia and Spelke 2010; Siegler and Booth 2004; Slusser et al. 2013; for a review see Patro et al. 2014); (3) numbers within a familiar range are proposed to be represented in a linear equidistant manner, meaning that the difference between adjacent numbers is constant[1] (e.g., Gallistel and Gelman 1992; Siegler and Booth 2004; but see Dehaene and Mehler 1992); and (4) it is hypothesized to result from systematic associations between physical and number space (e.g., Dehaene et al. 1993; Walsh 2003; Bueti and Walsh 2009).

From this spatial association, the hypothesis arose that our interactions with the physical world may shape the characteristics of the mental number line suggesting embodied representations of numbers (e.g., Domahs et al. 2010; Moeller et al. 2012; see Fischer and Brugger 2011; Fischer 2012; Myachykov et al. 2013 for theoretical elaborations).

Embodied numerical representations and trainings

The suggestion of embodied numerical representations picks up on the concept of embodied cognition (cf., Barsalou 2008, 2010; Wilson 2002). For instance, it has been found that even adults unconsciously activate finger-based representations of numbers in numerical tasks (Domahs et al. 2010; Klein et al. 2011). However, associations between bodily experiences and number processing do not seem to be limited to finger-based representations. For instance, Schwarz and Müller (2006) observed spatial–numerical associations for pedal responses (i.e., faster reaction with the left foot to small and the right foot to large numbers). Furthermore, recent studies suggest that not only active head movements (Loetscher et al. 2008), but also even the perception of passive self-

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[1] Please note that there is an ongoing debate on whether the scaling of the mental number line is linear or rather logarithmically compressed (e.g., Gallistel and Gelman 1992; Dehaene et al. 1990). In fact, prominent findings in numerical cognition research such as the problem size as well as the numerical distance effect can be accounted for by both the assumption of a linear scaling and scalar variability (e.g., Feigenson et al. 2004) of the representations of single numbers but also by the assumption of a logarithmically compressed scaling with fixed variability (e.g., Pica et al. 2004). However, the debate on the underlying scaling of the mental number line is not at the heart of the current study. Therefore, the interested reader is referred to the excellent discussion of this issue in Dehaene (2003) and Leslie et al. (2008).

motion influences number processing reliably (Hartmann et al. 2012a, b). More importantly, however, full-body movement has been shown to not only influence the random generation of numbers (Shaki and Fischer 2014), but also mental arithmetic (Anelli et al. 2014). These authors observed a congruency effect for motion direction and arithmetic operations: Performance was better when participants performed subtractions/additions after a lateral turn to the left/right, respectively.

These findings provide a theoretical and empirical basis for the idea of training spatial–numerical associations using full-body movements. A first study in this vein was conducted by Fischer et al. (2011), who trained children to associate small numbers with leftward jumps and large numbers with rightward jumps in a magnitude comparison task on a digital dance mat. Interestingly, this simple training led to significantly more improvements than a non-embodied control training in kindergarten children. This affected not only number line estimations, but also transferred to counting abilities, which were not trained directly. Taking the concept of embodied numerical trainings one step further, Link et al. (2013) trained first-graders in an embodied version of the number line estimation task including full-body movement as response. Children experienced number magnitude by walking on a spatially presented number line of up to 3 m to the estimated target position. Thereby, they could physically experience number magnitude as the to-be-walked distance from the starting point to the actual target position on the number line. The authors compared their embodied training to a control condition in which children were trained with the number line estimation task on a tablet PC. Results indicated both trainings to be beneficial enhancing children's number line estimation performance. However, only the embodied training led to transfer effects on children's addition skills. In a recently published pilot study, Fischer et al. (2015) observed similar effects for a number line estimation training on an interactive whiteboard.

In sum, previous embodied training studies that focused on improving children's number line estimation seem promising as they indicated beneficial effects not only with regard to the trained content but also to numerical tasks and competencies that were not part of the training. However, the number line concept itself captures different characteristics that have to be understood before displaying an accurate, linear estimation pattern. In this vein, it has been observed repeatedly that first-graders overestimate the position of relatively smaller numbers in the range from 0 to 100 meaning that they estimate 9 to be at about the position of 40 (Moeller et al. 2009; Siegler and Booth 2004). This overestimation of the relative spatial position of smaller numbers indicates that an understanding of the equal linear spacing between the numbers, or the understanding of equidistance, respectively, is not yet established. Accordingly, increasing linearity of children's estimation patterns was repeatedly observed to be associated with better math achievement (e.g., Siegler and Booth 2004). In the current study, a unique attempt was made to train children the equidistant spacing of numbers by training them to divide spatial distances into equal intervals. This should help them to grasp the general concept of equidistant spacing and transfer it to numbers on the number line. In accordance with the results of prior training studies (e.g., Fischer et al. 2011, 2015; Link et al. 2013), we expected such a training to be even more beneficial when associated with the whole-body experience of these spacings in an embodied training format. The current study pursued this idea.

The present study

To train second-graders' basic understanding of equidistant spacing, we created a task in which children had to generate different numbers of equal segments. To realize an embodied experience of equidistance we used the Kinect^{TM} sensor system. In the embodied training, children were trained to divide a physically presented line into a specific number of equidistant segments by walking along this line with equally spaced steps. The Kinect^{TM} sensor recorded children's segmentation and allowed us to provide feedback about the equidistance of their steps. The embodied training condition was compared to a control training, in which children performed the same task without the physical experience of taking steps. Instead children had to draw dashes onto a line presented on a tablet PC to indicate their segmentation.

Based on prior reports of children's estimation patterns in the number line estimation task, we expected children to violate the concept of equidistance specifically with regard to the extension of the first segment followed by increasing compression of later segments to compensate for their overstretched first estimate. By providing feedback about the correct equidistant segmentation, we tried to specifically train children's understanding of the concept of equidistance.

Furthermore, as understanding of the equidistance principle is important for number line estimation, we expected the training to transfer to and thus, improve children's number line estimation performance as well. Given the fact that number line estimation is closely linked to other numerical performances (e.g., Booth and Siegler 2006), we also hypothesized to observe training effects on other, not directly trained arithmetic tasks. Finally, as previous studies showed that an embodied spatial and bodily experience increased training benefit, we expected improvements to be more pronounced after the embodied as compared to the control training.

Methods

Participants Twenty second-graders (11 boys, mean age 7; 6 years, SD = 4.84 months) from a German primary school participated in the study after parents gave their written informed consent. Children did not show apparent signs of cognitive deficits or motor/bodily constraints.

Training procedure

In a randomized crossover design all children completed three sessions of an embodied and another three sessions of a computerized control training condition. The order of training condition was counter balanced across children (i.e., half of the children started with the embodied training condition, half of the children started with the control training condition; see also Fig. 1). Over the course of about 6 weeks, training sessions took place during school hours in one-on-one sessions of approximately 20 min. On average, each child attended approximately two training sessions per week. In both training conditions, the children's task was to divide a given line with a predefined number of steps (embodied condition) or dashes (control condition) into the same number of equidistant segments. For example, when presented with the number 3, children were supposed to either walk to the end of a physically presented line with three equal steps in the embodied condition or segment a line presented on a tablet PC with an electronic pen by drawing three dashes in the control condition. In both conditions, the last step or the last dash had to be placed at the end of the line. Each training session started with a practice item to ensure children's understanding of the task. Training items were determined by foot size and height of an average second-grader. The same training items were used in both conditions, with 3, 4, 5, 6 and 7 presented along with a long line (embodied: 2 m, control: 782 px) and 2, 3, 4 and 5 along with a short line (embodied: 1.5 m, control: 464 px). Item order was randomized for each training session with order of line lengths counter balanced across children. Each item was presented twice per training session.

Children started each training item from a fixed start point. To control this unspecific movement across training conditions, a similar training setup was chosen for the control training on the tablet PC (see Fig. 2, panel A and B for a schematic illustration). All training trials began at the same point (the calibration point for the Kinect^{TM}), from where children looked at a central screen on which the target numbers were displayed. Red and green rectangles in the lower left and right corner indicated the starting point (left or right) for the respective item. Children were instructed to walk to the side indicated by the green rectangle and perform the task starting from this side (embodied condition) or on the tablet PC placed on the respective side (control condition). For both training conditions feedback was provided as a video showing children a ball that jumped along the line marking where children positioned their estimates. Simultaneously, the actual equidistant segment was displayed together with an arrow to indicate children's deviation from equidistant spacing (cf. Fig. 2c, d).

Embodied training condition In the embodied training condition, the line was taped on the floor. To start, children placed the tips of both feet at the beginning of the line. They then had to walk in small/medium/big steps toward the other end of the line trying to separate it into the requested number of equidistant segments. Thereby, the step of one foot indicated one segment. The spatial positions of children's heels were recorded by the Kinect^{TM} sensor in 3D space. Children returned to the start point to watch the feedback video.

Control training condition Training items of the control training condition were presented on two tablet PCs

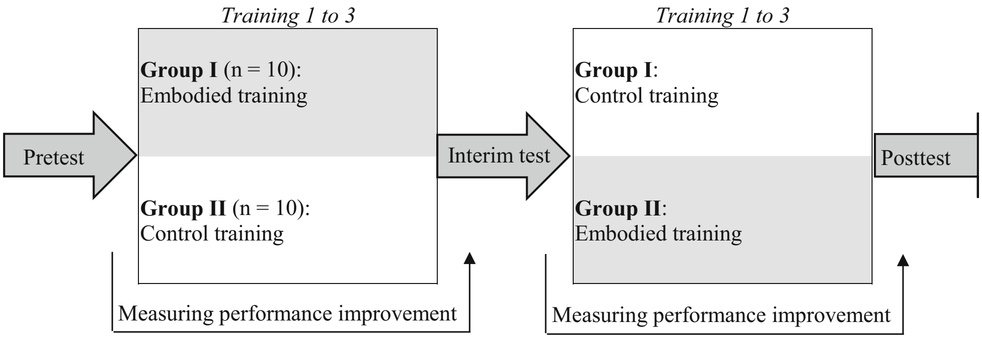

Fig. 1 Schematic overview of the applied crossover training design. Half of the children started with the embodied and the other half with the control training. After the interim test, training conditions changed. Performance improvements were calculated separately for the training conditions

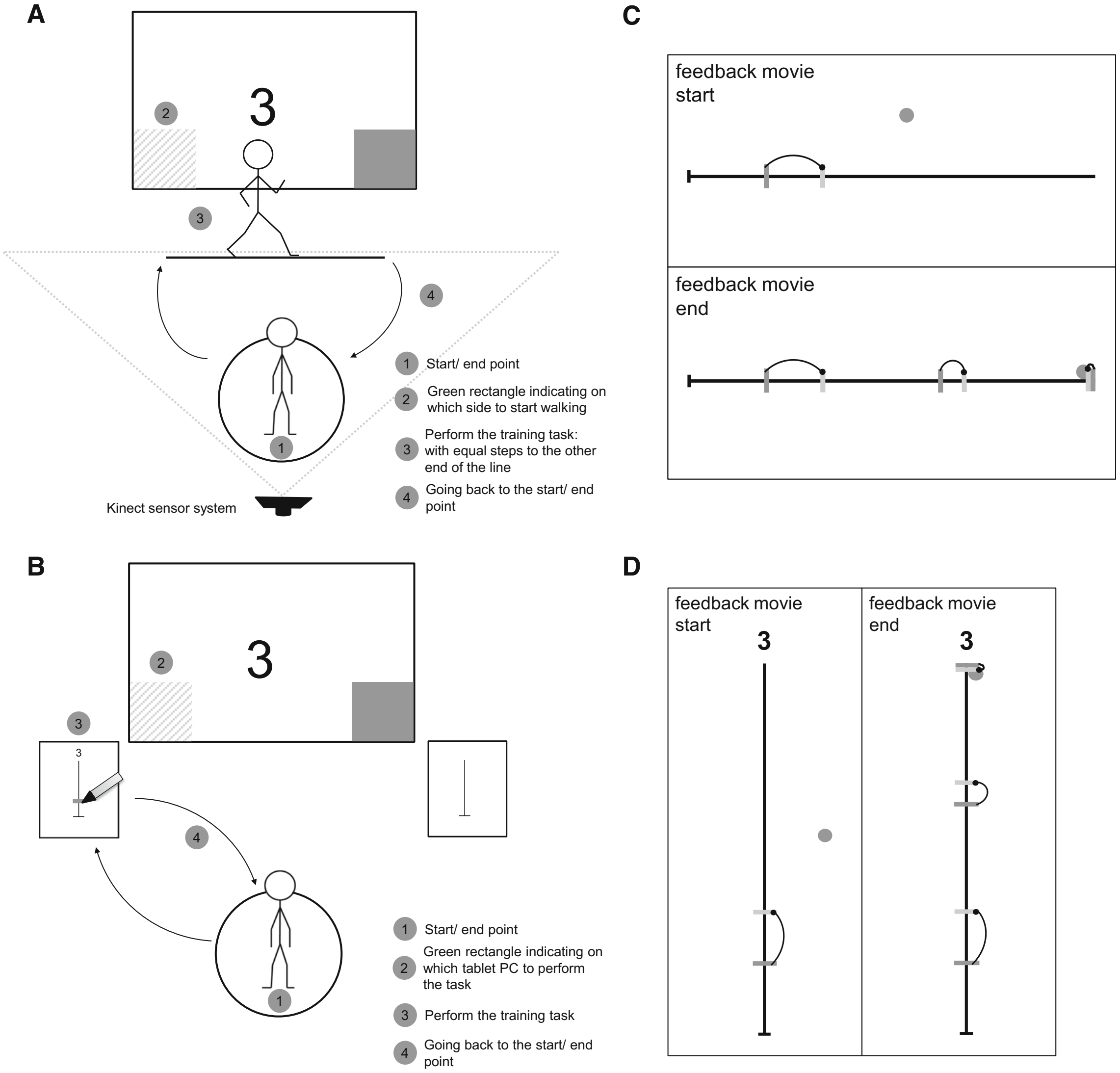

Fig. 2 Schematic illustration of the training conditions. *Panel* **a** depicts the embodied training condition where children had to solve the training task by walking with equal steps on the physically presented line. *Panel* **b** depicts the setup of the control training condition with the task being trained on a tablet PC on different tables positioned on the right and left side of the screen. *Panels* **c**, **d** depict presentation of the feedback: A jumping *red ball* (here: *dark gray*) indicated children's estimates of equidistant spacing which was also depicted by a *small red* (here: *dark gray*) line. Feedback as to the correct equidistant segment was provided by a *green* (here: *light gray*) line connected to children's estimates as is shown in both panels. While feedback presentation in the embodied training condition was provided on the screen (*panel* **c**), it was shown on the respective tablet PCs in the control training condition (*panel* **d**)

placed on tables at a distance of approximately 1.75 m. This balanced the unspecific movement between training conditions, as children had to walk to the left or right table as compared to walking to the left or right end of the line in the embodied condition. During the training on the screen of the tablet PC children had to separate the presented vertical line into the indicated number of segments with an electronic pen. The target number was displayed on top of the line. Children returned to the starting point after watching the feedback video on the tablet PC.

Pre- and posttest measures

Prior to children's first training session, before the first session of the other training condition and after their last training session a battery of performance measures was assessed to control for children's learning gains (see Fig. 1). The respective pre- and interim test directly preceded the first training session of each training condition to keep children's appointments at a minimum.

Equidistance understanding was measured with a paper pencil version of the computerized training task. Nineteen items, consisting of two practice trials and 17 critical items, were each presented on a separate page in each testing session. Items consisted of a vertical line with a marked start point on the lower end of the line and the number of segments placed above the upper end of the line. Children were instructed to separate the line into the indicated number of equal segments and place the last dash at the upper end of the line. The number 4 served as practice item, which was first solved by the experimenter to illustrate the task, and then by the child to ensure task understanding. Critical items were 2, 3, 4, 5, 6 and 7, each depicted along with three different line lengths (short: 135 mm, medium: 170 mm, long: 200 mm). Because 4 also served as practice item, it was only administered twice among the critical trials. This resulted in 17 (5 items *3 lengths + 2* item 4 = 17) items for the interim and posttest and 18 items for the pretest because item 3 for the middle length was accidently administered twice. Task performance was evaluated considering children's *equidistance estimation* of the first segment. We specifically focused on the first segment as we expected it to be indicative of children's (lacking) understanding of the equidistant principle comparable to their overestimation of small numbers in number line estimation (e.g., Moeller et al. 2009). Additionally, misestimations of the first segment should impact on the estimates of the other segments as those need to be adapted to the remaining space. Finally, evaluating the first segment allowed us to include performance on item "2" which required the estimation of one segment only since the last dash was predetermined by the end of the line. Equidistance estimation was analyzed by calculating children's mean deviation from equidistant segmentation for the first segment (MD) and the variability of deviation from equidistant segmentation for the first segment (VD). Prior to the analysis, deviations were standardized per length of the line (small/medium/long) and both variables, MD and VD, were z-transformed to standardize variances.

Number line estimation performance was assessed in two ways with a *bounded* and an *unbounded* version of the number line estimation task (cf. Cohen and Blanc-Goldhammer 2011). In the bounded number line task, children were presented with a 20-cm number line with the end points 0 and 100, on which they had to estimate target positions of 20 different numbers (1, 2, 4, 7, 9, 11, 16, 27, 35, 46, 52, 57, 68, 74, 82, 89, 92, 94, 97 and 99) presented in randomized order for each testing session. The target number was placed above the middle of the empty number line, and the numbers 50 and 10 were administered as practice trials.

In the unbounded task version, target numbers were placed above the start point of the number line (also 20 cm in length), and no end point was provided. Instead, a unit indicating the distance from 0 to 1 was depicted below the start point to help children position their estimates. The numerical range of this number line was 29, with only numbers smaller than 20 assessed to prevent strategies considering the end point of the physical line as the end of the to be estimated interval. The 16 critical items (all numbers <20 except 1, 5, 10 and 11) were presented in randomized order for each testing session.

For both tasks *percent absolute error* (*PAE* calculated as |estimate—target number|/scale *100; cf. Siegler and Booth 2004) was calculated as dependent variable.

Arithmetic performance was also assessed in two ways using the two subtests *addition* and *subtraction* of the Heidelberger Rechentest 1-4 (HRT 1-4, Haffner et al. 2005). Following standardized instructions, children had to solve as many additions/subtractions as possible within a time limit of 2 min. Three simple practice problems were presented on a separate page prior to the 40 critical items per subtest. *T-scores* of correctly solved problems served as dependent variables.

Control variables

In a group test the following control variables were assessed to obtain a rough estimate of children's general cognitive functioning: general cognitive ability, processing speed, as well as visual and verbal working memory capacity.

General cognitive ability was assessed by the three subtests *continuing series*, *classifications* and *matrices* of the Culture Fair Intelligence Scale (CFT 1-R, Weiß and Osterland 2013). *T-scores* served as dependent variable.

Writing speed as a measure of children's processing speed was assessed using the subtest *writing speed* of the HRT 1-4 (Haffner et al. 2005) in which children are supposed to copy as many numbers as possible within a time limit of 30 s. *T-scores* served as dependent variable.

To assess children's *verbal working memory capacity,* a list of 21 consonants was read aloud to them at a rate of one letter per second. Following the presentation, children had to write down all the letters they remembered irrespective of presentation order. The *sum of correctly reproduced letters* was considered the dependent variable.

Visual working memory capacity was measured by presenting children with 30 symbols on a screen at a rate of one symbol per second. After presentation, children had to identify as many target symbols as possible on an answer sheet with 30 symbols, 15 of which were target items presented before. The *sum of correct responses* (both correctly identified symbols and correctly rejected distractors, max. sum = 30) was used as the dependent variable.

Results

Prior to all analyses we evaluated possible effects of training order on children's performance improvement by running a 2 (training condition) × 2 (training order) repeated-measures ANOVA for each evaluation task separately. Since these analyses did neither indicate a significant main effect of training order (all $p > .05$) nor a significant interaction between training order and training condition (all $p > .05$), we did not include the factor training order in the subsequent analyses.

Our analyses followed a two-step procedure. In a first step we evaluated training effects separately for each training condition by evaluating whether performance changes between pre- and posttests differed significantly from zero. Because we postulated directed hypotheses that independent of the training condition children should improve their numerical performance, we evaluated those using one-sided *t* tests. In the second step, we evaluated differential training effects by comparing performance changes due to the embodied and the control training separately for the three tasks that were assessed (i.e., equidistance understanding, number line estimation, arithmetic performance). For the task assessing equidistance understanding we contrasted performance changes in MD and VD after the training conditions using repeated-measures ANCOVAs controlling for the covariates general cognitive abilities, writing speed as well as visual and verbal working memory capacity, which were centralized prior to analyses. Performance in transfer tasks was analyzed using 2 × 2 repeated-measures ANCOVAs with factors training condition (embodied vs. control training) and the dependent variables of each task (i.e., PAE in number line estimation with the two levels bounded vs. unbounded task version; *T*-scores for arithmetic performance with the two levels addition vs. subtraction) while simultaneously controlling for the covariates.

In the following, we first report on results for the equidistance segmentation task before the results for the transfer tasks are described.

Equidistance understanding

Training effects The *t* tests against zero did not reveal significant improvements for children's MD (mean deviation from equidistant segmentation) after both training conditions (see Table 1). Instead, after the control training children seemed to worsen descriptively. After the embodied training, a significant reduction of the variability of deviation for the first estimate was observed (see also Fig. 3). In contrast, regarding changes for VD after the control training, children again seemed to worsen descriptively.

Differential training effects The ANCOVA comparing pre- to posttest performance changes for MD revealed a significant main effect of training condition [$F(1, 15) = 7.37$, $p < .05$, $\eta_p^2 = .33$] which indicated that children improved their performances after the embodied training ($M = 0.50$)[2] more strongly compared to their impaired performances after the control training ($M = -1.43$)[2]. Considering main effects of the covariates, we only observed a significant main effect of the covariate writing speed [$F(1, 15) = 4.61$, $p < .05$, $\eta_p^2 = .24$], whereas all other main effects were not significant [all $F < 1$]. The main effect of writing speed indicated that children with slower writing speed capacity tended to benefit more strongly from the training ($r = -.17$). The ANCOVA also revealed a significant interaction between training condition and writing speed [$F(1, 15) = 5.87$, $p < .05$]. Inspection of the beta weights revealed a significant influence of writing speed in the control training condition [$B = -0.07$, $t(15) = 2.80$, $p < .05$], but not in the embodied training condition [$B = 0.03$, $t(15) = 1.44$, $p = .17$]. Thus, children with slower writing speed benefitted more from the control training. The ANCOVA also revealed a significant interaction between training condition and verbal working memory capacity [$F(1, 15) = 7.01$, $p < .05$]. Inspection of the beta weights revealed a significant influence of verbal working memory capacity in both training conditions, however, in different directions [embodied: $B = .22$, $t(15) = 2.37$, $p < .05$; control: $B = -0.28$, $t(15) = 2.43$, $p < .05$]. While children with higher verbal working memory capacity seemed to benefit more from the embodied training, children with lower verbal working memory capacity seemed to benefit more from the control training. All other interactions were not significant [all $F < 2.79$, all $p > .11$].

Results of the ANCOVA on VD changes also revealed a significant main effect of training [$F(1, 15) = 11.07$, $p < .01$, $\eta_p^2 = .43$] indicating children to improve their performance more strongly after the embodied ($M = 1.14$)[2] than after the control training ($M = -0.80$)[2]. The ANCOVA did not reveal any significant main effects of the covariates [all $F < 1.88$, all $p > .19$] or significant interactions between training conditions and the covariates [all $F < 1.79$, all $p > .20$].

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[2] Please note: While analyses were run using z-standardized values of MD and VD raw values are given here for reasons of better understandability.

Table 1 Pre- and posttest performance changes for the dependent variables of all evaluation tasks (z-transformed values in parentheses)

	Pre	Post	Performance changes			
			Mean	SD	$t(19)$	p_c
Equidistance understanding						
MD: mean deviation from equidistant segmentation						
Embodied	5.92 (−0.01)	5.43 (−0.21)	0.50 (0.19)	1.69 (0.66)	1.31	.14
Control	5.30 (−0.26)	6.74 (0.14)	−1.43 (−0.56)[a]	2.42 (0.95)	–	–
VD: variability of equidistant segmentation						
Embodied	4.97 (0.21)	3.83 (−0.30)	1.14 (0.51)	1.67 (0.75)	2.90	**<.05***
Control	4.00 (−0.22)	4.81 (0.14)	−0.80 (−0.36)[a]	1.97 (0.88)	–	–
Number line estimation						
Bounded PAE[b]						
Embodied	8.54	8.35	0.19	2.92	0.28	.39
Control	9.00	7.34	1.65	2.46	2.93	**<.05***
Unbounded PAE						
Embodied	15.45	13.56	1.89	3.72	2.27	**<.05***
Control	12.27	14.54	−2.27[a]	6.97	1.46	–
Arithmetic performance						
Addition						
Embodied	49.80	52.50	2.70	6.71	1.80	.08
Control	50.10	51.20	1.10	5.84	0.84	.22
Subtraction						
Embodied	48.50	49.70	1.20	5.45	0.98	.20
Control	48.70	50.75	2.05	4.16	2.20	**<.05***

p values were corrected for multiple testing according to Benjamini and Hochberg (1995)

* Significant at $\alpha = .05$

[a] Please note that evaluating children's performance changes without the a priori hypothesis of positive training effects by two-sided testing indicated no significant change for children's equidistance performance change of MD [$t(19) = 2.65$, $p_c = .06$] or VD [$t(19) = 1.82$, $p_c = .15$] after the control training. Similarly, children's performances in unbounded number line estimation did not change significantly after the control training [$t(19) = 1.46$, $p_c = .12$]

[b] Please not that due to missing data of one child degrees of freedom in the bounded number line estimation task were reduced to $df = 18$

Transfer tasks

Number line estimation

Training effects For bounded number line estimation children showed no significant performance improvement after the embodied training (see Table 1; Fig. 3), but improved significantly after the control training. An opposite result pattern was observed for changes in unbounded number line estimation performance: Children improved their performances significantly after the embodied training but tended to worsen after the control training (see Table 1).

Differential training effects Analyzing differential training effects for changes in bounded and unbounded number line estimation performance, an ANCOVA revealed no significant main effect of training condition or task variable [both $F(1, 14) < 1.94$, both $p > .18$] nor of any of the covariates [all $F(1, 14) < 4.21$, all $p > .05$]. However, the interaction between training condition and task variable was significant [$F(1, 14) = 6.40$, $p < .05$, $\eta_p^2 = .31$], indicating a significantly different influence of the trainings on the two tasks. Post hoc analysis of simple effects yielded a significant differential influence of the training conditions in the unbounded task [$t(19) = 2.25$, $p < .05$] but not in the bounded task [$t(18) = 1.64$, $p = .12$]. Thus, children improved their performances in unbounded number line estimation more strongly after the embodied ($M = 1.89$) than after the control training ($M = -2.27$).

All other interactions of the ANCOVA were not significant [all $F(1, 14) < 3.15$, all $p > .09$].

Arithmetic performance

Training effects Evaluating embodied training effects on children's arithmetic performance did only reveal a

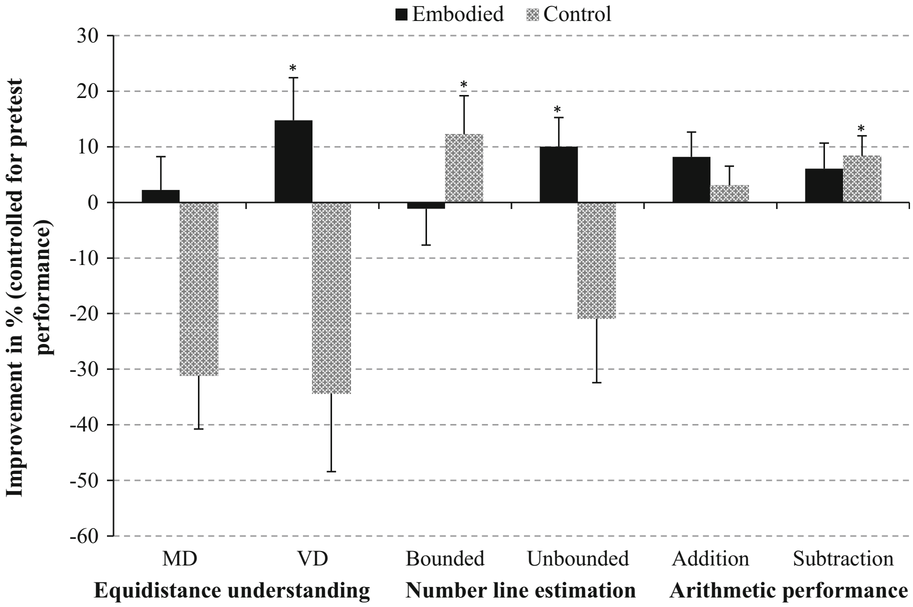

Fig. 3 Overview of children's mean performance improvements (%) in all evaluation tasks (controlled for pretest performance). Asterisks indicate significant performance improvements after the respective training condition. *MD* mean deviation from equidistant segmentation; *VD* variability of deviation from equidistant segmentation

marginal significant improvement of children's performance in the addition task (see Table 1). After the control training children improved significantly in their subtraction performance but did not show significant changes in their addition performance (see Table 1; Fig. 3).

Differential training effects Results of the conducted ANCOVA did not reveal significant main effects of training condition or task variable [both $F(1, 15) < 0.16$, both $p > .70$] nor of any of the covariates [all $F(1, 15) < 0.78$, all $p > .39$]. Interactions between training condition and/or task variable and the covariates were not significant except for the interaction between task variable and verbal working memory capacity [$F(1, 15) = 5.05$, $p < .05$]. Inspection of the beta weights indicated that higher verbal working memory capacity was related to worse addition performance [$B = -0.71$, $t(15) = 1.54$, $p = .14$] but to better subtraction performance [$B = 0.37$, $t(15) = 1.14$, $p = .27$]. Note that the slopes were not significantly different from zero.

Discussion

The current study set off to train a specific aspect of the mental number line: the equidistant spacing of adjacent numbers. Considering previous studies revealing beneficial effects of an embodied experience of the training content (Fischer et al. 2011; Link et al. 2013) an embodied training of equidistance was developed. Children had to separate a physically presented line into a specific amount of segments by walking the line with equally spaced steps. In a control condition, children completed the same task on a tablet PC. Training effects were evaluated with respect to both the trained concept of equidistance but also regarding transfer effects on number line estimation as well as arithmetic performance. In line with recent evidence on embodied numerical trainings (Fischer et al. 2011, 2015; Link et al. 2013), more pronounced training benefits for the embodied training were expected. However, the analyses revealed quite inconsistent results for the different evaluation tasks, some suggesting more beneficial effects of the embodied training and others indicating performance improvement after the control training only. In total, the results give rise to the assumption that different cognitive mechanisms might have been involved in the respective training conditions which are of differential importance for the respective outcome variables. In the following, we will discuss these differential aspects in more detail. Thereby, we will focus on the specific training effects referring to understanding of equidistance itself before we will elaborate on the transfer effects.

Training effects on children's equidistance understanding

Children's performance changes in the equidistance task were analyzed with regard to children's deviation from the correct segmentation and to the variability of their estimate of the first dash as these represent their initial scaling. Considering children's estimation error no general improvements were observed. Interestingly, however, results rather revealed a tendency that children's equidistance performance deteriorated after the control training. Yet, after the embodied training, children showed a significant improvement regarding their estimation variability which was also significantly different from the effect of the control training. This indicated at least higher consistency in children's decisions on the length of the first segment. Because children in particular have the tendency to systematically overestimate the spatial position of small numbers—as found for number line estimation (e.g., Booth and Siegler 2006; Laski and Siegler 2007; Moeller et al. 2009; Siegler and Booth 2004; Siegler and Opfer 2003)—this provides first evidence for the effectiveness of our embodied training for corroborating children's equidistance understanding.

Nevertheless, the question why we observed a reliable improvement of children's equidistance understanding only after the embodied training remains. Trying to answer this question we strongly suppose the embodied training components to having corroborated a deeper understanding of the equidistance concept in a very specific way. First, children experienced equidistance in a physical bodily way during the embodied training which may have led to a more comprehensive representation of the training content (i.e., incorporating proprioceptive aspects). According to previous studies (Fischer et al. 2011; Link et al. 2013) it is this bodily experience realizing a congruency between physical movements and the spatial dimension of the trained concept which should be particularly beneficial to enhance learning. More specifically, one might speculate about the underlying processes driving this by considering the necessary adaption of perspectives on the training material. In this context, different spatial reference frames as proposed by research on spatial cognition in general and with respect to the coding of target locations in particular might provide a framework for our explanation. Keulen, Adam, Fischer, Kuipers and Jolles (2002; see also Tipper et al. 1992) differentiated (1) a viewer-based reference frame referring to the coding of objects relative to a person's own view, (2) an environment-based reference frame referring to the coding of objects relative to other objects and (3) "an action-based frame of reference according to which objects are coded relative to the start position of the responding effector" (Keulen et al. 2002, p. 516). Given the different setups of the two training conditions, the embodied training involved all three reference frames and required flexible adjustment of children's perspective on the trained task (see also Huber et al. 2014, for a similar argument on the mental number line representation).

At the start of each training trial, children saw the to-be-segmented line in a horizontal layout and as a whole allowing an egocentric viewer-based frame of reference (cf. Fig. 2). During the training task, however, children had to leave this overview perspective and virtually become part of the task, thereby changing the perspective on the task to environment-based and action-based reference frames. Moreover, while performing the task, the horizontal layout changed and became rather vertically oriented toward the children. Moreover, with every step into the task their perspective changed again because the distance taken by previous steps laid behind the children. Accordingly, this required new calibrations and adjustments for the segmentation of the remaining line length with every step taken. Furthermore, during feedback presentation children saw the length of their own steps depicted on the screen. They were supposed to associate the first line with their first step, the second line with their second step and so on. Thereby, this way of depicting the feedback required them to change back from the previously in task perspective during the actual walking on the line perceived vertically stretched out before them to the overview perspective in the horizontal layout and associate the depicted correct segments to their own steps taken previously. In contrast, neither the control training task nor the depiction of feedback in the control training condition required any adjustments to the spatial orientation or changes of children's perspectives on the task. Children completed the task on the tablet PC being in an overview position and watched the feedback movie afterward from exactly the same overview perspective. This meant they had a constant viewer-based frame of reference. Accordingly, the specific training effects observed after the embodied training may stem from the combined physical experience of equidistance together with the required adaption of perspectives while performing the task. However, spatial transformations necessary to correctly interpret the feedback in the embodied training condition (integrating vertical and horizontal layouts) may even have worked against these beneficial processes resulting in the overall small—but nevertheless significant—specific training effect.

Considering the reliable influence of the covariates writing speed and verbal working memory capacity fits nicely to this interpretation. In both cases, lower scores on the covariate were associated with better improvements after the control training. In this context, writing speed may reflect a rough estimate for children's processing speed. Importantly, the finding that children with lower processing speed as well as lower verbal working memory capacity profited more from the control training corroborates our interpretation that this training condition seemed to be less demanding in terms of general cognitive abilities than the embodied training condition. In the control training, training trials and feedback were presented directly after one another with participants taking an overview perspective on the task. Therefore, less processing was necessary for children to relate their performance to the actual feedback. The same account can be applied successfully to the significant interaction with verbal working memory capacity. Comparable to writing speed, children with lower verbal working memory capacity also profited more from the control training. In contrast, children with higher verbal working memory capacity profited more from the embodied training to enhance their equidistance understanding. Thus, higher verbal working memory capacity was beneficial for the training condition which was more demanding by requiring the integration of overview and subjective perspective while actually walking along the line. Therefore, the interaction pattern with the control variables nicely underlines our suggestion that the embodied training was more demanding on children's domain general cognitive abilities than was the control training.

These differences in training conditions may account for why children improved their equidistance performance after the embodied training. Yet, this does not provide an obvious explanation for why children showed—even though only descriptively—worse performance after the control training. In particular as the training task and the evaluation task were practically identical. One possible reason for this might be that it was hard for children to differentiate between training trials, in which they were supposed to learn, and the evaluation task, in which they should have applied the learned content. The only difference between the training and the evaluation task was the different way the task was administered (tablet PC vs. paper pencil task) and the absence of feedback in the evaluation task. However, there is considerable evidence on the importance of feedback in computerized contexts (e.g., Kulik and Kulik 1991; see Li and Ma 2010, for a meta-analysis). In the latter, feedback not only provides information on one's performance but also has a motivating effect on student's learning. Because the equidistance training task was the first to be assessed within the battery of evaluation tasks at pre-, interim and posttest, it might have been difficult for children to differentiate between the training and the evaluation task. However, as children were used to receiving feedback on their performance during the three training sessions, their performance dropped in the evaluation task because no feedback was given in this task. Therefore, the strong resemblance between the two tasks with the absence of the motivating feedback in the evaluation task might account for the descriptively worse performance in the equidistant task after the control training.

Transfer effects

Number line estimation

Closer inspection of the transfer effects indicated that children's performance in bounded number line estimation improved only after the control training condition. This is of particular interest as it revealed that—even though the control training did not lead to an improvement of children's performance in the equidistance task—our control training nevertheless induced a differential training effect as there was no effect of the embodied training on bounded number line estimation.

In contrast to bounded number line estimation, training effects on unbounded number line estimation revealed an opposing pattern. Children improved after the embodied training but showed no performance changes after the control training.

Recent research clearly indicates that the small perceptual difference between the bounded and unbounded number line estimation (i.e., providing an end point to the presented number line in the bounded version) seems to be crucial (e.g., Cohen and Blanc-Goldhammer 2011; Cohen and Sarnecka 2014; Link et al. 2014b). There is accumulating evidence suggesting that estimation performance in the bounded number line estimation task is driven by the use of specific estimation strategies such as proportion judgment (i.e., the use of reference points, cf. Barth and Paladino 2011; Cohen and Blanc-Goldhammer 2011; Slusser et al. 2013). In contrast, the unbounded number line estimation task was argued to provide a purer and unbiased measure of the mental number line with no indications on the use of specific strategies other than magnitude estimation (Cohen and Blanc-Goldhammer 2011; Link et al. 2014a, b).

Thus, given the differential training effects on bounded and unbounded number line estimation it may be informative to also reconsider this pattern in the light of the differing frames of references required in the training conditions which might have led to the use of different solution strategies. Due to the constant environment-based perspective the control training task resembled to some extent a proportion-judgment task: Children could always relate the to-be-estimated segments to the start and end point as well as previously estimated segments by applying proportion-judgment strategies. This is backed by developmental studies which showed that children start using reference points in bounded number line estimation between the age of 6–8, which directly corresponds to the age of the children in the current study (see Slusser et al. 2013; Link et al. 2014a; Schneider et al. 2008). Such a proceeding, however, was not possible in the embodied training condition because the actual perspective on the task differed during several stages of the task (i.e., start, while on the task, when watching the feedback), making it impossible to use previous estimates as an anchor or use relative judgments (relating start and end point of the line). Thus, improvements in the bounded number line task after the control training might not (only) stem from an improved understanding of equidistant spacing but rather from proportion-based solution strategies which were trained implicitly. In contrast, the embodied training may have fostered equidistance understanding more explicitly as it did not allow children to rely on proportion-based strategies. This account nicely integrates the observation of children's improved understanding of equidistant spacing and unbounded number line estimation.

Arithmetic performances

Importantly, the specific training effects found on addition and subtraction further corroborate this account. Recently, Cohen and Sarnecka (2014) proposed number line estimation to be highly influenced by mensuration skills as both tasks require relating of line length and quantity information. They assumed that bounded number line estimation should be associated with subtraction and division skills (related to proportion-based strategies), whereas unbounded number line estimation should be associated with addition and multiplication skills (to repeat the given unit distance until the target is reached, see also Reinert et al. 2014 for a similar approach). Thus, our results are partially in line with this rationale: Children tended to improve their addition performance only after the embodied training after which we observed a significant improvement of children's unbounded number line estimation performance. In contrast, an improvement of subtraction performance was observed after the control training, which also improved children's bounded number line estimation performance. Thus, the pattern of associated performance improvements in our study seems to support the theoretical suggestions of Cohen and Sarnecka (2014). However, it is worth noting that such a dissociation of arithmetic skills and bounded/unbounded number line estimation was not found in a recent study with fourth-graders (Link et al. 2014c). In fact, Link et al. (2014b) found no association at all between unbounded number line estimation and arithmetic or numerical abilities, whereas bounded number line estimation performance was significantly associated with the assessed competencies. These differing observations might be due to the different age groups investigated: In younger children unbounded number line estimation might be solved by counting strategies or simply adding unit distances until reaching the final target position. In contrast, adults may rather solve unbounded number line estimation by multiplication strategies (cf. Reinert et al. 2014). However, fourth-graders, as investigated by Link et al. (2014b), might be too old to use counting but still too young to apply systematic multiplication strategies for solving unbounded number line estimation. Further research is needed to clarify the association of number line estimation and arithmetic competencies.

Considering that the present study provides further evidence suggesting a specific influence of embodied experiences in numerical training the question arises how these beneficial effects arise. In the following, we aim at briefly discussing a possible account.

Integration into a neural account of spatial–numerical associations

When discussing the beneficial effects of embodied numerical trainings it is important to consider recent theoretical ideas on how spatial–numerical associations emerge. In a theory of magnitude Walsh (2003, see also Bueti and Walsh 2009) suggested a general system dedicated to represent all kinds of magnitudes such as time, physical distance and numbers. Following this idea, spatial–numerical associations originate from the fact that the representations of all these different kinds of magnitudes draw on the same underlying representational system. Interestingly, this proposition is supported not only by the observation of numerous effects reflecting spatial–numerical associations in participants' behavior (see Fischer and Shaki 2015 for a recent review) but also by evidence on the neural level. For instance, Simon et al. (2002, see also Simon et al. 2004; Pinel et al. 2004) observed neighboring or even overlapping neural activation in the intraparietal cortex for the processing of number magnitude and tasks requiring the processing of physical spatial information such as making saccades, finger pointing or even grasping.

Based on this and further evidence (see Dehaene et al. 2004; Nieder and Dehaene 2009 for reviews) Dehaene (2005, see also Dehaene and Cohen 2007) suggested what they called the neural recycling hypothesis. The neural recycling hypothesis proposes that the acquisition of cultural abilities such as the understanding of numbers draws on evolutionally older brain circuitries and recycles them for the new purpose. For the case of numbers these seem to be intraparietal areas which were associated with a core representation of non-symbolic numerical magnitudes in primates (e.g., Nieder 2015 for a review). Interestingly, there is convincing evidence that homologues of these areas in the human brain are also crucially recruited for numerical tasks (e.g., Dehaene and Cohen 2007). Importantly, however, Hubbard et al. (2005) even take it a step further and propose that spatial–numerical associations arise from common parietal circuits for attention to as well as motoric responses in external space and the representations of numbers within the intraparietal sulcus, which we share with non-human primates.

Apart from neuro-imaging data, the hypothesis of common neural circuitries for the representation of number and attention as well as motor processes is also corroborated by computational modeling evidence. In their Spatial

Number Network (SpaN) Grossberg and Repin (2003) explain how numerical capabilities shared between humans and non-human species build on a spatial representation of number magnitudes in the Where cortical processing stream, in particular the inferior parietal cortex. More particularly, these authors suggest and show that representations "of numerical magnitude use specializations of more primitive neural mechanisms that have evolved in the Where cortical processing stream for purposes of motion perception, spatial attention, and target tracking" (Grossberg and Repin 2003, p. 1108). These specializations are argued to form a spatial–numerical map; this means an analogue mental number line.

Considering these ideas of neural recycling and its implementation in the SpaN model, our training can be interpreted in terms of helping children to calibrate their spatial–numerical map by actually moving in space. The beneficial effects of different embodied numerical trainings (e.g., Fischer et al. 2011; Link et al. 2013; Fischer et al. 2015) substantiate the notion that physical movements corroborate this process. However, our data also indicate that it may be full-body movement with the necessity to adapt to varying perspectives on the task at hand (see above) that drives the effect. Simpler movement of, for instance, only one hand (as required in the control training) may not be sufficient to elicit the beneficial effect of embodied numerical trainings. This is in line with the observation of Fischer et al. (in press) who found spatial–numerical associations to be more pronounced in a full body as compared to other response modes requiring less physical movements. In terms of the SpaN model this might be explained by the fact that full-body movements may trigger the Where path more strongly, which in turn leads to more specific positioning of numbers on the spatial–numerical map.

Interestingly, the SpaN model also accounts for specificities of the processing of multi-digit numbers (see Nuerk et al. 2015 for a review). In a combination of the spatial code from the Where and a specifying verbal code from the What stream (i.e., number words) the model even reflects the place-value structure of the Arabic number system. In particular, "learned semantic categories that symbolize separate digits, as well as place markers like 'ty,' 'hundred,' and 'thousand,' are associated […] with the corresponding spatial locations of the Where representation" (Grossberg and Repin 2003, p. 1107). Following this argument, children should benefit from an experimental training in which the place-value structure of the Arabic number system is made more explicit as it should specify the location of, for instance, tens and units on the spatial–numerical map. In line with this, a previous training study by Link et al. (2014c) provided first results on the specific training effects of an embodied training focusing on children's place-value understanding. In this study children had to solve a bounded number line estimation task by letting a bar grow from zero until it reached the spatial position of the target number. To let the bar grow children had to step on different fields on a dance mat of which three fields lied on the floor, while another three fields were put on a staircase. Children were told that the fields lying on the staircase which required more spatial and physical movement to step on them increased (right field) or decreased (left field) the length of the bar by 10, respectively, whereas stepping on the lower fields increased (right field) or decreased (left field) the length of the bar by only 1. In an intervention study, the authors observed that German-speaking children improved their number line estimation performance more strongly through this embodied training condition than through a control training employing exactly the same task but requiring input on a standard keyboard. Most important, however, was the finding that children specifically improved their estimation performance for target numbers for which inversion-related errors (i.e., mixing up tens and units) would have led to large estimation errors (e.g., 81 → 18).

Taken together, this suggests that the neural recycling hypothesis (Dehaene 2005; see Hubbard et al. 2005 for spatial–numerical associations) and its implementation in the SpaN model of Grossberg and Repin (2003) may indeed be a helpful theoretical framework for interpreting outcomes of embodied numerical trainings. In particular, both suggest that the mental number line shares "the neural circuitry involved in the development of multisensory, world-centred representations of space" (Hubbard et al. 2005, p. 445). This implies that the same computational transformations that support spatial updating are employed for internal navigation along an internal number line. Nevertheless, future studies are needed to substantiate this argument. In the next paragraph, we will discuss some avenues for future research.

Limitations and perspectives

So far, the current study focused on the comparison of the effects the two training conditions had on the performance indicators that were assessed prior and after the training conditions—as this is the standard procedure to evaluate the efficiency of interventions in general (cf. Melnyk and Morrison-Beedy 2012) and embodied numerical trainings in particular (e.g., Fischer et al. 2011; Link et al. 2013). Thus, training effects on children's equidistance understanding were assessed by a paper pencil version of the equidistance task closely resembling the task children had to perform during the training. However, for future studies it might also be a valuable approach to evaluate how children perform within one and across different training sessions. Although we cannot propose specific hypotheses about how children should improve differently in the control and the embodied training condition in the respective training tasks, the comparison of within training performances might add to the understanding of differential processes involved in the training conditions.

In this vein, it is also reasonable to think about assessing number line estimation performance in a similar spatial layout as equidistance understanding. In the current study, number line estimation was assessed following the standard procedure (i.e., the number line was presented horizontally, cf. Link et al. 2013; Siegler and Opfer 2003). However, given the fact that children were trained to indicate their segmentation on a line extending vertically in front of them to enhance their equidistance understanding, it would be interesting to evaluate possible transfer effects on a vertically oriented number line estimation task as well. Although there is first evidence (Mihulowicz et al. 2015) indicating that estimation performance on a vertical number line yielded results comparable to those for horizontal presentation, this question needs further empirical evaluation.

Conclusions

Taken together, the current study aimed at training the principle of equidistant spacing of adjacent numbers on the number line by using a new embodied training approach to enhance training effects. Children had to divide a presented line in specific numbers of equal segments by either walking the line with equally spaced steps in the embodied or separating it into equally spaced segments on a tablet PC in the control training.

Importantly, consideration of the evaluation tasks indicated a differential influence of the training conditions. On the one hand, we only observed specific training effects on the understanding of equidistance and transfer effects on unbounded number line estimation performance after the embodied training. In contrast, children only improved their bounded number line estimation and subtraction performance after the control training. Given the differences in the setup of our training conditions with respect to the involvement of bodily movements and the need for changes of the perspective on the task, these data indicate that it may not solely be the whole-body movement which is crucial for training success of embodied numerical trainings. Instead, flexible adaption of and transfer between different spatial perspectives during the training task and thus recruitment of different frames of references seem to be important for the training outcome as well. Following this argument, training tasks as we used in the control training might not be conducive on the concept of equidistance which we aimed to train. Instead, it seemed to also foster the maintenance of a constant perspective and integration of available visual cues using proportion-based strategies as also found for bounded number line estimation. In sum, this study is the first to provide systematic evidence that embodied trainings, besides the beneficial embodied experiences, might also differ from non-embodied trainings with respect to other cognitive processes. Further research is needed to better understand these concomitant processes with the aim to consider them specifically when further developing embodied numerical trainings.

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