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A THREE-DIMENSIONAL SOLUTION FOR LAMINATED ORTHOTROPIC RECTANGULAR PLATES WITH VISCOELASTIC INTERFACES *

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ABSTRACT When a body consists completely or even partly of viscoelastic materials, its response under static loading will be time-dependent. The adhesives used to glue together single plies in laminates usually exhibit a certain viscoelastic characteristic in a high temperature environment. In this paper, a laminated orthotropic rectangular plate with viscoelastic interfaces, described by the Kelvin-Voigt model, is considered. A power series expansion technique is adopted to approximate the time-variation of various field quantities. Results indicate that the response of the laminated plate with viscoelastic interfaces changes remarkably with time, and is much different from that of a plate with spring-like or viscous interfaces.

KEY WORDS laminated orthotropic plate, state-space method, viscoelastic interfaces, Kelvin-Voigt model

I. INTRODUCTION

In a series of papers treating problems pertinent to the mechanical response of composite laminates, Pagano[1−−3] proposed a method for deriving exact elasticity solutions based on the assumption of perfect bonding between any two adjacent layers in a laminate. These solutions are now well-known and have been frequently quoted in the community of mechanics of composite materials. However, owing to the complexity inherent in the fabrication process of composite laminates, various flaws, such as microcracks, inhomogeneities, and cavities, will be introduced into the adhesives, which in turn affect the behavior of laminated structures. Recently, many researchers have paid their attention to laminates with imperfect interfaces[4−−16].

One of the key problems associated with the research on imperfect laminates is how to depict the interfacial properties. There are certain simplified interfacial models that have been proposed. The most widely used one is the linear spring-like model[6−−9]. A dislocation-like model was recently suggested to consider the effect of an imperfect interface on the load transfer[10, 11]. In the above-mentioned works [6−−11], the responses of laminates under static loading are independent of the time variable. On the other hand, if a laminate has viscous interfaces, it will behave like a fluid, i.e. the deformation changes with time. He and Jiang[12] and Chen and Lee[13] showed that, when time approaches infinity, viscous interfaces would lose the ability of transferring shear stress totally.

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According to Hashin[14] and Fan and Wang[15], the viscoelastic model will be more appropriate for describing the behavior of interlaminar adhesives under a high temperature condition. Yan and Chen[16] derived an analytical solution for a layered isotropic plate with viscoelastic interfaces in cylindrical bending. But the analysis is very complicated because it is necessary to compute material eigenvalues to obtain real form solutions through a lengthy discussion, as was shown in Pagano[1−−3]. The analysis becomes more complicated when a three-dimensional plate problem is considered since more eigenvalues will be involved.

In this paper, we study the time-dependent response of a simply supported laminated orthotropic rectangular plate with viscoelastic interfaces using the state-space method, which is particularly effective in analyzing laminated structures[6−−8, 17]. For simplicity, we assume that the deformation is relatively slow so that the inertia effect is neglected in our analysis. The power series expansion technique is then adopted to approximate the variation of elastic fields with time, which was originally suggested by Chen et al.[13] for laminates with viscous interfaces. Numerical results are presented and discussed.

II. STATE-SPACE FORMULATIONS

Consider a laminate composed of N orthotropic layers such that the various axes of material symmetry are parallel to the plate axes x, y, z. The plate, as shown in Fig.1, simply supported on the ends $x = 0$, $x = a$ and $y = 0$, $y = b$, is subjected to a sinusoidal pressure at its top surface. Thus, we have the following boundary conditions

$$\begin{aligned} &\sigma_z = -p_0 \sin(r\pi\xi)\sin(n\pi\eta), \quad \tau_{xz} = 0, \quad \tau_{yz} = 0 \quad \text{at } z = h \\ &\sigma_z = 0, \quad \tau_{xz} = 0, \quad \tau_{yz} = 0 \quad \text{at } z = 0 \end{aligned} \tag{1a}$$

$$\begin{aligned} &\sigma_x = 0,\ w = v = 0 \quad \text{at } x = 0 \text{ and } x = a \\ &\sigma_y = 0,\ w = u = 0 \quad \text{at } y = 0 \text{ and } y = b \end{aligned} \tag{1b}$$

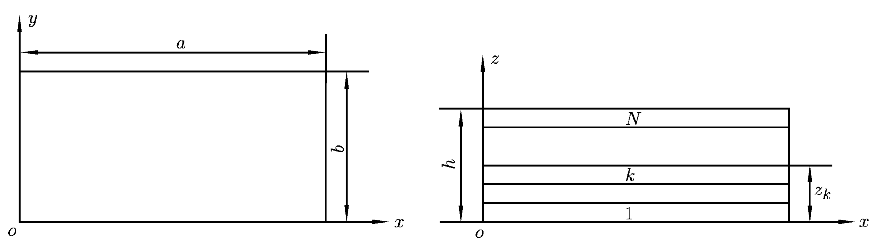

Fig. 1. Sketch of a rectangular bidirectional laminate.

From the well-known constitutive relations and the equilibrium equations, the state equation can be easily derived[7, 18, 19]. To satisfy the conditions on the four edges, as in Eq.(1b), the state variables are assumed to be in the form of

$$\begin{Bmatrix} \sigma_z \\ u \\ v \\ w \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{Bmatrix} -c_{44}^{(1)}\bar{\sigma}_z(\zeta,t)\sin(r\pi\xi)\sin(n\pi\eta) \\ h\bar{u}(\zeta,t)\cos(r\pi\xi)\sin(n\pi\eta) \\ h\bar{v}(\zeta,t)\sin(r\pi\xi)\cos(n\pi\eta) \\ h\bar{w}(\zeta,t)\sin(r\pi\xi)\sin(n\pi\eta) \\ c_{44}^{(1)}\bar{\tau}_{xz}(\zeta,t)\cos(r\pi\xi)\sin(n\pi\eta) \\ c_{44}^{(1)}\bar{\tau}_{yz}(\zeta,t)\sin(r\pi\xi)\cos(n\pi\eta) \end{Bmatrix} \tag{2}$$

where $\xi = x/a$, $\eta = y/b$ and $\zeta = z/h$, c_{ij} are the elastic constants, r and n are wave numbers, and the superscript (1) denotes the layer number. We then obtain from the state equation

$$\frac{\partial}{\partial\zeta}\boldsymbol{V}(\zeta,t) = \boldsymbol{A}\boldsymbol{V}(\zeta,t) \tag{3}$$

in which $\boldsymbol{V}(\zeta,t)=\left\{\bar{\sigma}_z(\zeta,t)\ \bar{u}(\zeta,t)\ \bar{v}(\zeta,t)\ \bar{w}(\zeta,t)\ \bar{\tau}_{xz}(\zeta,t)\ \bar{\tau}_{yz}(\zeta,t)\right\}^{\mathrm{T}}$, and

$$\boldsymbol{A}=\begin{bmatrix} 0 & 0 & 0 & 0 & -t_1 & -t_2 \\ 0 & 0 & 0 & -t_1 & \dfrac{c_{44}^{(1)}}{c_{55}} & 0 \\ 0 & 0 & 0 & -t_2 & 0 & \dfrac{c_{44}^{(1)}}{c_{44}} \\ -\dfrac{c_{44}^{(1)}}{c_{33}} & \dfrac{c_{13}}{c_{33}}t_1 & \dfrac{c_{23}}{c_{33}}t_2 & 0 & 0 & 0 \\ \dfrac{c_{13}}{c_{33}}t_1 & \dfrac{\beta_1}{c_{44}^{(1)}}t_1^2+\dfrac{c_{66}}{c_{44}^{(1)}}t_2^2 & \dfrac{\beta_2}{c_{44}^{(1)}}t_1t_2 & 0 & 0 & 0 \\ \dfrac{c_{23}}{c_{33}}t_2 & \dfrac{\beta_2}{c_{44}^{(1)}}t_1t_2 & \dfrac{\beta_3}{c_{44}^{(1)}}t_2^2+\dfrac{c_{66}}{c_{44}^{(1)}}t_1^2 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

where $t_1=r\pi h/a$ and $t_2=n\pi h/b$, c_{ij} are the elastic constants, and

$$\beta_1=c_{11}-c_{13}^2/c_{33},\quad \beta_2=c_{12}+c_{66}-c_{13}c_{23}/c_{33},\quad \beta_3=c_{22}-c_{23}^2/c_{33},\quad \beta_4=\beta_2-c_{66} \tag{5}$$

The solution to Eq.(3) is well known[7, 18, 19], from which we can derive

$$\boldsymbol{V}_1^{(k)}=\boldsymbol{M}_k\boldsymbol{V}_0^{(k)}\quad (k=1,2,\cdots,\ N) \tag{6}$$

where the subscript 1 denotes the upper surface, and 0 the lower surface, and the square matrix $\boldsymbol{M}_k=\exp[\boldsymbol{A}(\zeta_k-\zeta_{k-1})]$ is known as the transfer matrix of the k-th layer, with $\zeta_0=0$ and $\zeta_k=z_k/h=\sum_{j=1}^{k}h_j/h$, as dimensionless coordinates, where h_k is the thickness of the k-th layer.

In addition to the state variables, the other three secondary variables are expressed as

$$\sigma_x=\frac{c_{13}}{c_{33}}\sigma_z+\beta_1\frac{\partial u}{\partial x}+\beta_4\frac{\partial v}{\partial y},\quad \sigma_y=\frac{c_{23}}{c_{33}}\sigma_z+\beta_4\frac{\partial u}{\partial x}+\beta_3\frac{\partial v}{\partial y},\quad \tau_{xy}=c_{66}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \tag{7}$$

III. KELVIN-VOIGT VISCOELASTIC INTERFACIAL MODEL

The two plies contacting a viscoelastic adhesive/interphase (at $z=z_k$ for example) can slide with respect to each other, yielding an in-plane relative displacement. Since the adhesive layer is very thin, the mechanical forces on the two sides can be regarded as in a state of equilibrium. Thus, we have

$$\begin{aligned} &\sigma_z^{(k+1)}=\sigma_z^{(k)},\quad \tau_{xz}^{(k+1)}=\tau_{xz}^{(k)} \\ &u^{(k+1)}=u^{(k)}+\delta_x^{(k)},\ v^{(k+1)}=v^{(k)}+\delta_y^{(k)},\ w^{(k+1)}=w^{(k)}\quad \text{at } z=z_k \end{aligned} \tag{8}$$

where $\delta_x^{(k)}$ and $\delta_y^{(k)}$ are the relative sliding displacements at the kth interface. In this paper, we assume that the shear stress and the sliding displacement obey the Kelvin-Voigt viscoelastic law, i.e.

$$\tau_{xz}^{(k)}=\eta_{0x}^{(k)}\delta_x^{(k)}+\eta_{1x}^{(k)}\dot{\delta}_x^{(k)},\quad \tau_{yz}^{(k)}=\eta_{0y}^{(k)}\delta_x^{(k)}+\eta_{1y}^{(k)}\dot{\delta}_y^{(k)}\quad \text{at } z=z_k \tag{9}$$

where the dot over a quantity denotes differentiation with respect to time, and $\eta_{0x}^{(k)}$, $\eta_{0y}^{(k)}$ and $\eta_{1x}^{(k)}$, $\eta_{1y}^{(k)}$ are the elastic constants and viscous coefficients in the x- and y-directions, respectively. Setting $\eta_{0x}^{(k)}=\eta_{0y}^{(k)}=0$, we get the viscous model[12, 13], while when $\eta_{1x}^{(k)}=\eta_{1y}^{(k)}=0$, the constitutive relation in Eq.(9) degenerates to the linear spring model[6−−9]. It is natural to assume

$$\delta_x^{(k)}=h\bar{\delta}_x^{(k)}(t)\cos(r\pi\xi)\sin(n\pi\eta),\quad \delta_y^{(k)}=h\bar{\delta}_y^{(k)}(t)\sin(r\pi\xi)\cos(n\pi\eta) \tag{10}$$

Thus, Eq.(8) can be rewritten as

$$\boldsymbol{V}_0^{(k+1)}=\boldsymbol{V}_1^{(k)}+\boldsymbol{Q}^{(k)} \tag{11}$$

where $\boldsymbol{Q}^{(k)}=\{0\ \ \bar{\delta}_x^{(k)}\ \ \bar{\delta}_y^{(k)}\ \ 0\ \ 0\ \ 0\}^{\mathrm{T}}$. Meanwhile, Eq.(9) takes the following form:

$$\bar{\tau}_{xz}^{(k)}=\bar{\eta}_{0x}^{(k)}\bar{\delta}_x^{(k)}+\bar{\eta}_{1x}^{(k)}\frac{\mathrm{d}\overline{\delta}_x^{(k)}}{\mathrm{d}\tau},\quad \bar{\tau}_{yz}^{(k)}=\bar{\eta}_{0y}^{(k)}\bar{\delta}_y^{(k)}+\bar{\eta}_{1y}^{(k)}\frac{\mathrm{d}\overline{\delta}_y^{(k)}}{\mathrm{d}\tau}\quad \text{at } z=z_k \tag{12}$$

where $\bar{\eta}_{0i}^{(k)}=\eta_{0i}^{(k)}h/c_{44}^{(1)}$ $(i=x,y)$ are the dimensionless elastic constants, $\tau=c_{44}^{(1)}t/(\eta_{1x}^{(1)}h)$ is the dimensionless time, and $\bar{\eta}_{1i}^{(k)}=\eta_{1i}^{(k)}/\eta_{1x}^{(1)}$ $(i=x,y)$ are the viscosity ratios.

IV. ANALYSIS PROCEDURE

Since it seems difficult to obtain an exact solution based on the above state-space formulations, we adopt the power series expansion technique proposed by Chen and Lee [13]. First, we divide the time domain into a series of equal intervals $[0, \Delta\tau]$, $[\Delta\tau, 2\Delta\tau]$, $[2\Delta\tau, 3\Delta\tau]$, $\cdots$, each with a small time step of $\Delta\tau$. For a typical interval $[m\Delta\tau, (m+1)\Delta\tau]$, $(m = 0,\ 1,\ 2, \cdots)$, it is assumed that

$$
\begin{aligned}
\bar{\delta}_x^{(k)} &= \bar{\delta}_{xm,0}^{(k)} + (\tau - m\Delta\tau)\bar{\delta}_{xm,1}^{(k)} + (\tau - m\Delta\tau)^2\bar{\delta}_{xm,2}^{(k)} + (\tau - m\Delta\tau)^3\bar{\delta}_{xm,3}^{(k)} + \cdots \\
\bar{\delta}_y^{(k)} &= \bar{\delta}_{ym,0}^{(k)} + (\tau - m\Delta\tau)\bar{\delta}_{ym,1}^{(k)} + (\tau - m\Delta\tau)^2\bar{\delta}_{ym,2}^{(k)} + (\tau - m\Delta\tau)^3\bar{\delta}_{ym,3}^{(k)} + \cdots \\
\bar{\sigma}_z^{(k)} &= \bar{\sigma}_{m,0}^{(k)} + (\tau - m\Delta\tau)\bar{\sigma}_{m,1}^{(k)} + (\tau - m\Delta\tau)^2\bar{\sigma}_{m,2}^{(k)} + (\tau - m\Delta\tau)^3\bar{\sigma}_{m,3}^{(k)} + \cdots \\
\bar{u}^{(k)} &= \bar{u}_{m,0}^{(k)} + (\tau - m\Delta\tau)\bar{u}_{m,1}^{(k)} + (\tau - m\Delta\tau)^2\bar{u}_{m,2}^{(k)} + (\tau - m\Delta\tau)^3\bar{u}_{m,3}^{(k)} + \cdots \\
\bar{v}^{(k)} &= \bar{v}_{m,0}^{(k)} + (\tau - m\Delta\tau)\bar{v}_{m,1}^{(k)} + (\tau - m\Delta\tau)^2\bar{v}_{m,2}^{(k)} + (\tau - m\Delta\tau)^3\bar{v}_{m,3}^{(k)} + \cdots \\
\bar{w}^{(k)} &= \bar{w}_{m,0}^{(k)} + (\tau - m\Delta\tau)\bar{w}_{m,1}^{(k)} + (\tau - m\Delta\tau)^2\bar{w}_{m,2}^{(k)} + (\tau - m\Delta\tau)^3\bar{w}_{m,3}^{(k)} + \cdots \\
\bar{\tau}_{xz}^{(k)} &= \bar{\tau}_{xm,0}^{(k)} + (\tau - m\Delta\tau)\bar{\tau}_{xm,1}^{(k)} + (\tau - m\Delta\tau)^2\bar{\tau}_{xm,2}^{(k)} + (\tau - m\Delta\tau)^3\bar{\tau}_{xm,3}^{(k)} + \cdots \\
\bar{\tau}_{yz}^{(k)} &= \bar{\tau}_{ym,0}^{(k)} + (\tau - m\Delta\tau)\bar{\tau}_{ym,1}^{(k)} + (\tau - m\Delta\tau)^2\bar{\tau}_{ym,2}^{(k)} + (\tau - m\Delta\tau)^3\bar{\tau}_{ym,3}^{(k)} + \cdots
\end{aligned}
\tag{13}
$$

Obviously, we have $\bar{\delta}_{x0,0}^{(k)} = \bar{\delta}_{y0,0}^{(k)} = 0$ because of the zero initial condition at $t = \tau = 0$. In view of Eqs.(13), by equating the coefficients of the same order of $\tau - m\Delta\tau$ on the two sides of Eq.(11), we obtain

$$
\boldsymbol{V}_{0,m,i}^{(k+1)} = \boldsymbol{V}_{1,m,i}^{(k)} + \boldsymbol{Q}_{m,i}^{(k)} \qquad (m,\ i = 0, 1, 2, \cdots) \tag{14}
$$

where $\boldsymbol{Q}_{m,i}^{(k)} = \left\{ 0 \quad \bar{\delta}_{xm,i}^{(k)} \quad \bar{\delta}_{ym,i}^{(k)} \quad 0 \quad 0 \quad 0 \quad \right\}^{\mathrm{T}}$. Also, we obtain from Eq.(12)

$$
\bar{\delta}_{xm,i}^{(k)} = (\bar{\tau}_{xm,i-1}^{(k)} - \bar{\eta}_{0x}^{(k)}\bar{\delta}_{xm,i-1}^{(k)})/(\bar{\eta}_{1x}^{(k)} \cdot i), \quad \bar{\delta}_{ym,i}^{(k)} = (\bar{\tau}_{ym,i-1}^{(k)} - \bar{\eta}_{0y}^{(k)}\bar{\delta}_{ym,i-1}^{(k)})/(\bar{\eta}_{1y}^{(k)} \cdot i) \quad (i = 1, 2, 3, ...) \tag{15}
$$

From Eqs.(6), (13) and (14), we get the relations

$$
\boldsymbol{V}_{1,m,i}^{(k+1)} = \boldsymbol{M}_{k+1}\boldsymbol{V}_{1,m,i}^{(k)} + \boldsymbol{M}_{k+1}\boldsymbol{Q}_{m,i}^{(k)} \qquad (m,\ i = 0, 1, 2, \cdots) \tag{16}
$$

Continuing the above procedure layer by layer, we finally obtain

$$
\boldsymbol{V}_{1,m,i}^{(n)} = \boldsymbol{T}\boldsymbol{V}_{0,m,i}^{(1)} + \boldsymbol{S}_{m,i} \qquad (m, i = 0, 1, 2, \cdots) \tag{17}
$$

where $\boldsymbol{T} = \prod_{j=N}^{1} \boldsymbol{M}_j$ is the global transfer matrix and

$$
\boldsymbol{S}_{m,i} = \boldsymbol{M}_N\boldsymbol{Q}_{m,i}^{(N-1)} + \boldsymbol{M}_N\boldsymbol{M}_{N-1}\boldsymbol{Q}_{m,i}^{(N-2)} + \cdots + \prod_{j=N}^{2} \boldsymbol{M}_j\boldsymbol{Q}_{m,i}^{(1)} \tag{18}
$$

are the inhomogeneous terms associated with viscoelastic interfaces, which vanish in the case of a perfectly bonded laminate. Considering the boundary conditions in Eq.(1a), we have

$$
\begin{aligned}
\boldsymbol{V}_1^{(N)} &= \left\{ p_0/c_{44}^{(1)} \quad \bar{u}_1^{(N)} \quad \bar{v}_1^{(N)} \quad \bar{w}_1^{(N)} \quad 0 \quad 0 \right\}^{\mathrm{T}} \\
\boldsymbol{V}_0^{(1)} &= \left\{ 0 \quad \bar{u}_0^{(1)} \quad \bar{v}_0^{(1)} \quad \bar{w}_0^{(1)} \quad 0 \quad 0 \right\}^{\mathrm{T}}
\end{aligned}
\tag{19}
$$

Substituting this equation into Eq.(17), we obtain

$$\begin{Bmatrix} p_0/c_{44}^{(1)} \\ \bar{u}_1^{(N)} \\ \bar{v}_1^{(N)} \\ \bar{w}_1^{(N)} \\ 0 \\ 0 \end{Bmatrix}_{m,0} = \boldsymbol{T} \begin{Bmatrix} 0 \\ \bar{u}_0^{(1)} \\ \bar{v}_0^{(1)} \\ \bar{w}_0^{(1)} \\ 0 \\ 0 \end{Bmatrix}_{m,0} + \boldsymbol{S}_{m,0} \qquad (m = 0, 1, 2, \cdots) \tag{20}$$

$$\begin{Bmatrix} 0 \\ \bar{u}_1^{(N)} \\ \bar{v}_1^{(N)} \\ \bar{w}_1^{(N)} \\ 0 \\ 0 \end{Bmatrix}_{m,i} = \boldsymbol{T} \begin{Bmatrix} 0 \\ \bar{u}_0^{(1)} \\ \bar{v}_0^{(1)} \\ \bar{w}_0^{(1)} \\ 0 \\ 0 \end{Bmatrix}_{m,i} + \boldsymbol{S}_{m,i} \qquad (m = 0, 1, 2, \cdots; \, i = 1, 2, 3, \cdots) \tag{21}$$

When $\tau = 0$, we have $\boldsymbol{S}_{0,0} = \boldsymbol{0}$, and the state variables at the top and bottom surfaces of the plate can be solved from Eq.(20). Consequently, all physical variables in the plate can be determined from the following equation as well as Eq.(7)

$$\begin{aligned} \boldsymbol{V}_{m,i}^{(k)}(\zeta) = {} & \exp[\boldsymbol{A}(\zeta - \zeta_{k-1})] \\ & \times \left(\prod_{j=k-1}^{1} \boldsymbol{M}_j \boldsymbol{V}_{0,m,i}^{(1)} + \boldsymbol{Q}_{m,i}^{(k-1)} + \boldsymbol{M}_{k-1} \mathbf{Q}_{m,i}^{(k-2)} + \mathbf{M}_{k-1} \mathbf{M}_{k-2} \boldsymbol{Q}_{m,i}^{(k-3)} \right. \\ & \left. + \cdots + \prod_{j=k-1}^{2} \boldsymbol{M}_j \boldsymbol{Q}_{m,i}^{(1)} \right) \qquad (\zeta_{k-1} \le \zeta \le \zeta_k; \, m, i = 0, 1, 2, \cdots) \end{aligned} \tag{22}$$

This in turn gives $\bar{\delta}_{x0,1}^{(k)}$ and $\bar{\delta}_{y0,1}^{(k)}$, i.e. $\boldsymbol{S}_{0,1}$ by virtue of Eq.(15). Thus, all variables for $i = 1$ can be computed from Eq.(21). Continuing this procedure, we eventually obtain all the coefficients $(i = 1, 2, 3, \cdots,)$ in Eq.(13) for $m = 0$. Setting $\tau = \Delta\tau$ in Eq.(13) yields $\bar{\delta}_{x1,0}^{(k)}$ and $\bar{\delta}_{y1,0}^{(k)}$, which give the value of $\boldsymbol{S}_{1,0}$. By applying this analysis step by step, all physical variables at any time can be determined.

V. NUMERICAL COMPUTATION

For numerical examination, a normal sinusoidal pressure $p = -p_0 \sin(\pi\xi) \sin(\pi\eta)$, i.e. $r = n = 1$, is considered. The following dimensionless quantities are introduced:

$$\begin{aligned} & \sigma = -\frac{\sigma_z(a/2, b/2, z, t)}{p_0}, && \tau_{0x} = \frac{\tau_{xz}(0, b/2, z, t)}{p_0}, && \tau_{0y} = \frac{\tau_{yz}(a/2, 0, z, t)}{p_0} \\ & w_0 = \frac{c_{44}^{(1)}}{p_0} \frac{w(a/2, b/2, z, t)}{h}, && u_0 = \frac{c_{44}^{(1)}}{p_0} \frac{u(0, b/2, z, t)}{h} && \\ & v_0 = \frac{c_{44}^{(1)}}{p_0} \frac{v(a/2, 0, z, t)}{h}, && \delta_1 = \frac{c_{44}^{(1)}}{p_0} \frac{\delta_x(0, b/2, z, t)}{h}, && \delta_2 = \frac{c_{44}^{(1)}}{p_0} \frac{\delta_y(a/2, 0, z, t)}{h} \end{aligned} \tag{23}$$

First, to check the present method, the exact solution derived by Yan and Chen[16] for a laminated isotropic strip is employed for comparison. When $b \to \infty$, the three-dimensional solution of a rectangular laminate derived here degenerates to the plane-strain solution of a laminated strip. Thus, it is very easy to use the present method to analyze the problem of a three-layered isotropic strip considered by Yan and Chen[16]. All parameters of the present problem are the same as those in Ref.[16], and the comparison given in Table 1 indicates a good agreement.

Next, we consider a symmetric five-layered cross-ply laminated plate with imperfect interfaces. We assume that the first and fourth interfaces are viscoelastic with $\bar{\eta}_{0i}^{(1)} = 2\bar{\eta}_{0i}^{(4)} = 1$ and $\eta_{1i}^{(4)} = 2\eta_{1i}^{(1)}$, $(i =$

Table 1. **Comparison for a three-layered isotropic strip** ($\zeta = 1/3$)

$(M, \Delta\tau)$	$\tau = 0$				$\tau = 5$			
	σ	τ_{0x}	w_0	u_0	σ	τ_{0x}	w_0	u_0
(3,0.1)	0.240747	−3.94049	−1427.66	−73.9291	0.239863	−3.85315	−1577.85	−62.7945
(4,0.1)	0.240747	−3.94049	−1427.66	−73.9291	0.239863	−3.85315	−1577.85	−62.7945
(3,0.05)	0.240756	−3.94139	−1426.10	−74.0446	0.239872	−3.85399	−1576.40	−62.9019
(4,0.05)	0.240756	−3.94139	−1426.10	−74.0446	0.239872	−3.85399	−1576.40	−62.9019
Ref.[16]	0.2408	−3.9423	−1424.5	−74.160	0.2399	−3.8539	−1576.6	−62.885

$^*M$ represents the total terms adopted in the series expansions in Eq.(13).

x, y), the second is perfect, while the third one is viscous with $\eta_{1i}^{(3)} = \eta_{1i}^{(1)}$, $(i = x, y)$. Each layer in the plate is of the same thickness and the following parameters are adopted

$$\frac{E_L}{E_T} = 25, \quad \frac{G_{TT}}{E_T} = 0.2, \quad \frac{G_{TL}}{E_T} = 0.5, \quad \mu_{LT} = \mu_{TT} = 0.25, \quad \frac{a}{b} = 1.5, \quad b = 5h \tag{24}$$

where E is the Young's modulus, G the shear modulus, L signifies the direction parallel to the fibers, T the transverse direction, and μ_{LT} is the Poisson's ratio. The convergence study for this five-layered plate is given in Table 2. From Tables 1 and 2, it can be seen that the numerical results are of high precision when $M = 4$ and $\Delta\tau = 0.05$. So, $M = 4$ and $\Delta\tau = 0.05$ will always be assumed in the following calculation.

Table 2. **Convergence study of the present method for the five-layered plate** ($\tau = 50$, $\zeta = 0.4$)

$(M, \Delta\tau)$	$\tau = 50$					
	σ	τ_{0x}	τ_{0y}	u_0	v_0	w_0
(3,0.1)	0.4513256	−0.2174809	−3.856030	0.0794813	0.00012829	−20.12622
(3,0.05)	0.4513253	−0.2174798	−3.856027	0.0794935	0.00012804	−20.12621
(4,0.05)	0.4513253	−0.2174798	−3.856027	0.0794935	0.00012804	−20.12621

The through-thickness distributions of various field variables for the five-layered plate are depicted in Fig.2. It is noted that the response of the plate at $t = 0$ is identical to that of the corresponding perfect plate. With time elapsing, however, the distributions in the thickness direction change remarkably. Discontinuities of u_0 and v_0 at the imperfect interfaces are clearly shown in Figs.2(b) and 2(c), respectively when $\tau \neq 0$. The difference between u_0 or v_0 on the two sides of an interface is just the sliding displacement, i.e. $\delta_x^{(k)}$ or $\delta_y^{(k)}$, which has a strong effect on the response of the laminate. Owing to this sliding, the global bending stiffness is reduced, and the transverse displacement w_0 increases with time. This indicates that the plate may lose its function in virtue of the excessive deformation. We can also see the time dependency of the shear stresses from Figs.2(e) and (f). At the perfect interfaces, i.e. $\zeta = 0.4$, the curves of shear stresses always remain smooth, while the variation is very abrupt when the curves cross the imperfect interfaces, i.e. $\zeta = 0.2$, $\zeta = 0.6$ and $\zeta = 0.8$. The difference between the viscoelastic interface and the viscous interface is very visible as indicated in these two figures. In the viscous case, the interfaces will lose the capability of transferring shear stress gradually. On the contrary, the viscoelastic interfaces always hold the function of transferring shear stress. Furthermore, it is observed that, with time elapsing, the shear stresses at $\zeta = 0.4$ (perfect interface) become larger and larger as shown in Figs.2(e) and (f). This implies that even if an interface in a laminate is perfect, it may also become more dangerous owing to the existence of other imperfect interfaces in the same laminate. Since shear failure due to high interlaminar transverse shear stresses is the most common failure mode in the application of composite laminates, the effect of imperfect interfaces should be considered in the design of laminated structures, especially those of practical importance.

VI. CONCLUSIONS

A hybrid analysis combining the state-space method and the power series expansion technique is employed to investigate the response of a simply-supported laminated orthotropic plate with imperfect

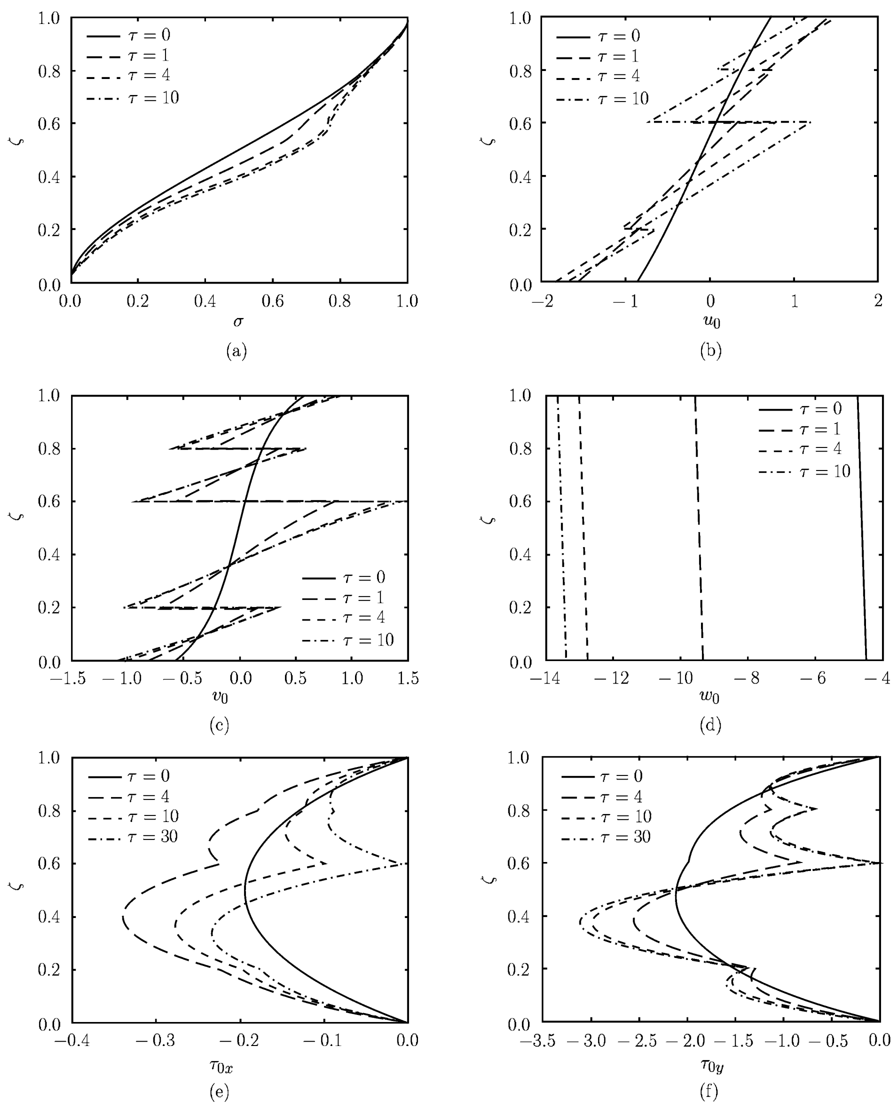

Fig. 2. Distributions of field variables along the thickness in the five-layered plate.

viscoelastic interfaces. Numerical investigation shows that the present analysis predicts accurate results when compared with the exact solution for a three-layered isotropic strip. A distinguished characteristic of the viscoelastic interface, which is different from the viscous interface, is the ability of transferring shear stress even when $t \to \infty$.

The numerical calculation indicates that the present analysis is effective, particularly for laminates with a large number of plies. This is mostly due to the small-scale final solving equations, which always take the same form as shown in Eqs.(20) and (21) regardless of the layer number. Further, the analysis is based on the three-dimensional elasticity formulations without introduction of any assumption of distribution on deformations and stresses. Thus, it can serve as a useful means of comparison for future studies using 2D plate theories or numerical methods.

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