ORIGINAL ARTICLE

Estimation of GLONASS inter‑frequency clock bias considering the phase center ofset diferences on the L3 signal

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Abstract

GLONASS has been transmitting the code division multiple access navigation signals on the third band L3 for six GLO-NASS-M+and three GLONASS-K satellites, in addition to the legacy frequency division multiple access signals on L1 and L2. However, the signifcant inconsistency between the phase bias of diferent ionospheric-free (IF) combinations for GLONASS-M +satellites, known as the phase inter-frequency clock bias (IFCB), hinders the utilization of GLONASS triple-frequency positioning. We discuss the coupling relationship between GLONASS IFCB and the phase center ofset (PCO) and propose a new IFCB estimation model considering the PCO diferences between L3 and L2. GLONASS triplefrequency observations from 151 globally distributed IGS stations are employed to validate the proposed IFCB estimation model. The results show that the mean root mean square (RMS) value of IFCB estimates decreases from 0.097 m to 0.028 m when considering PCO diferences, suggesting the GLONASS IFCB is ignorable. Meanwhile, the L3 PCO estimates for GLONASS-M +satellites exhibit high stability and consistency, with standard deviations of 52, 113, and 13 mm, in *x*-, *y*-, and *z*-components, respectively. By correcting the estimated L3 PCO instead of the legacy IFCB, the GLONASS-only triple-frequency precise point positioning PPP achieves positioning accuracies of 1.8, 0.9, and 1.5 cm in east, north, and up components, with the improvement of 13%, 3%, and 33%, respectively. Moreover, the RMS value of L3 phase residuals reduces from 10.2 to 5.0 mm. Therefore, we recommend correcting the PCO on L3 for GLONASS-M+satellites and disregarding the IFCB for GLONASS triple-frequency positioning, which can signifcantly simplify the observation model and achieve higher accuracy.

Keywords GLONASS · Inter-frequency clock bias · Phase center offset · Triple-frequency precise point positioning · FDMA+CDMA

Introduction

As part of the GLONASS modernization program, the Russian satellite system commenced transmitting code division multiple access (CDMA) navigation signals on the third band L3 in 2011, in conjunction with the legacy frequency division multiple access (FDMA) signals on L1 and L2 (Urlichich et al. [2011\)](#page-10-0). As of February 2023, the GLONASS constellation comprises 24 operational satellites distributed across three orbital slots, consisting of 15

 \boxtimes Yongqiang Yuan yqyuan@sgg.whu.edu.cn GLONASS-M satellites, six GLONASS-M+satellites, and three GLONASS-K satellites. All GLONASS-M +(R04, R05, R12, R15, R21, and R24) and GLONASS-K satellites (R09, R11, and R22) are capable of transmitting the up-to-date L3 signals (Montenbruck et al. [2017](#page-10-1)). Extensive research has demonstrated that multi-frequency observations can expedite the initialization process of Global Navigation Satellite System (GNSS) precise positioning signifcantly and enhance accuracy (Geng et al. [2013](#page-9-0); Guo et al. [2016](#page-9-1); Li et al. [2019,](#page-10-2) [2020](#page-10-3)). Accordingly, the additional L3 signal is anticipated to bolster the performance of GLONASS navigation and positioning.

Zaminpardaz et al. ([2017](#page-10-4), [2021\)](#page-10-5) initially analyzed the GLONASS L3 signal and discovered that the new CDMA data has a lower noise level than that of GPS, and evaluated the triple-frequency GLONASS RTK performance using the FDMA+CDMA-integrated model, successfully achieving

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double-diferenced ambiguity resolution for a short baseline. However, in undiferenced triple-frequency GNSS data processing, such as precise point positioning (PPP), it is crucial to correct the inter-frequency clock bias (IFCB), caused by inconsistent and time-varying phase hardware delays between L1, L2, and L3. A lot of previous studies have reported that GPS Block IIF satellites sufer from severe IFCB, with inter-day amplitudes of up to 0.2 m (Montenbruck et al. [2010](#page-10-6); Li et al. [2013;](#page-10-7) Pan et al. [2017\)](#page-10-8). Zhang et al. [\(2022\)](#page-10-9) estimated the IFCB of GLONASS using the same method applied to GPS, based on epoch-diferenced geometry-free and ionospheric-free (GFIF) observations, and demonstrated that the IFCBs of GLONASS-K satellites are smaller than 0.01 m, while those of GLONASS-M+satellites reach 0.3 m. By correcting the estimated IFCB products, Zhang et al. [\(2023](#page-10-10)) accomplished GLONASS CDMA+FDMA-integrated PPP.

However, owing to the diferences in the characteristics of the satellite, orbit, and navigation signals between GPS and GLONASS, the GPS IFCB estimation method is not fully applicable to GLONASS satellites. GLONASS IFCB results derived from the legacy GPS estimation method are irregular and difficult to model, and experiments of GLO-NASS triple-frequency positioning are currently limited to regional stations or short baselines. Moreover, we observe that the IFCB time series are signifcantly diferent from each other between stations with large distances. One of the possible reasons is that the efect of satellite antenna phase center offsets (PCOs) has been overlooked in previous IFCB estimation models, which are diferent for L1, L2, and L3 signals. Although the time-varying part of PCO diferences in the radial component is ignorable for IFCB estimation due to the small variations of the satellite nadir angle, the horizontal PCO diferences change dramatically with the azimuth angle (Schmid et al. [2005](#page-10-11)). Currently, neither the GPS nor the GLONASS PCO at third frequencies is provided in the IGS antenna model. Zeng et al. [\(2021\)](#page-10-12) demonstrated that the diferences between GPS L5 and L2 PCOs are 0.2, 0.1, and 9.9 cm, in the along-track, cross-track, and radial components, respectively. Small horizontal PCO differences for GPS Block IIF satellites mean that the impact of PCO diferences can be ignored in GPS IFCB estimation. In contrast, we found that the horizontal PCO diferences for GLONASS-M+satellites can reach 0.7 m, which can lead to decimeter-level and time-varying errors. Therefore, the PCO diferences must be considered and carefully separated in GLONASS IFCB estimation.

In this contribution, we proposed a least-squared adjustment method to simultaneously estimate satellite phase

IFCBs and PCO differences using GFIF observations derived from a reference network. Triple-frequency observations from 151 stations are utilized to estimate GLONASS IFCB and validate the proposed model. The characteristics of GLONASS IFCB, decoupled from PCO diferences, are revealed and analyzed.

The GLONASS IFCB estimation model is demonstrated frst. Then, we introduce the data and processing strategies, present the experimental validation of the GLONASS-only triple-frequency PPP, and, fnally, the conclusions.

Method

We start with the GLONASS raw triple-frequency observation model. Then, we develop an FDMA+CDMA uncombined and undiferenced positioning model. Subsequently, we present a new GLONASS IFCB estimation method that accounts for the PCO diferences on the L3 signal.

GLONASS triple‑frequency observation model

Raw GLONASS observations of pseudorange *P* and carrier phase *L* can be expressed in units of meter as follows,

$$
\begin{cases}\nP_{\mathrm{r},n}^{\mathrm{s}} = \rho_{\mathrm{r}}^{\mathrm{s}} + t_{\mathrm{r}} - t^{\mathrm{s}} + \gamma_{n} \cdot I_{\mathrm{r},1}^{\mathrm{s}} + T_{\mathrm{r}}^{\mathrm{s}} + b_{\mathrm{r},n}^{R_{k}} - b_{n}^{\mathrm{s}} + e_{\mathrm{r},n}^{\mathrm{s}} \\
L_{\mathrm{r},n}^{\mathrm{s}} = \rho_{\mathrm{r}}^{\mathrm{s}} + t_{\mathrm{r}} - t^{\mathrm{s}} - \gamma_{n} \cdot I_{\mathrm{r},1}^{\mathrm{s}} + T_{\mathrm{r}}^{\mathrm{s}} + \lambda_{n}^{R_{k}} \cdot N_{\mathrm{r},n}^{\mathrm{s}} + B_{\mathrm{r},n}^{R_{k}} - B_{n}^{\mathrm{s}} + \varepsilon_{\mathrm{r},n}^{\mathrm{s}}\n\end{cases} \tag{1}
$$

where *s*, *r*, *n* are the satellite, receiver, and frequency identifiers; R_k refers to the GLONASS channel number; ρ_r^s is the geometric distance between the phase centers of satellite transmitter and receiver antenna; t_r and t^s denote the receiver and satellite clock offsets; f_{n,R_k} is the frequency, which are $f_{1,R_k} = 1602.0 + R_k \cdot 0.5625 \text{ MHz}, f_{2,R_k} = 1246.0 + R_k \cdot 0.4375 \text{ MHz},$ and $f_{3,R_k} = 1202.025 \text{ MHz}$; the $I_{r,1}^s$ refers to the slant ionospheric delay in L1; γ_n stands for the frequency-dependent multiplier factor, which is expressed as $\gamma_n = f_{1,R_k}^2 / f_{n,R_k}^2$; T_r^s is the tropospheric delay; $\lambda_n^{R_k}$ denotes the wavelength of carrier phase; $N_{r,n}^s$ is the integer phase ambiguity in cycles; *b* and *B* refer to the hard- or software delays associated with code and phase measurements, respectively; $e_{r,n}^{s,R_k}$ and $e_{r,n}^{s,R_k}$ are the sum of measurement noise and multipath errors.

For GLONASS FDMA L1 and L2 signals, the code and phase delays at the receiver end difer for satellites with diferent channel numbers. This leads to inconsistencies in receiver clock offsets, known as inter-frequency bias (IFB). Generally, the phase ambiguity can fully absorb IFB, whereas the code IFB is estimated as the diference relative to a particular satellite. Therefore, we reparametrize the raw observation equation

and derive the GLONASS FDMA+CDMA PPP model as follows:

$$
P_{r,1}^{s} = \rho_{r}^{s} + \hat{\tau}_{r}^{R_{0}} - \hat{r}^{s} + \hat{I}_{r,1}^{s} + T_{r}^{s} + \mu_{r}^{R_{k}} + e_{r,1}^{s}
$$
\n
$$
P_{r,2}^{s} = \rho_{r}^{s} + \hat{\tau}_{r}^{R_{0}} - \hat{r}^{s} + \gamma_{2} \cdot \hat{I}_{r,1}^{s} + T_{r}^{s} + \mu_{r}^{R_{k}} + e_{r,2}^{s}
$$
\n
$$
P_{r,3}^{s} = \rho_{r}^{s} + \hat{\tau}_{r}^{R_{0}} - \hat{r}^{s} + \gamma_{3} \cdot \hat{I}_{r,1}^{s} + T_{r}^{s} + \omega_{r}^{s} + e_{r,3}^{s}
$$
\n
$$
L_{r,1}^{s} = \rho_{r}^{s} + \hat{\tau}_{r}^{R_{0}} - \hat{r}^{s} - \hat{I}_{r,1}^{s} + T_{r}^{s} + \lambda_{1}^{R_{k}} \cdot \hat{N}_{r,1}^{s} + \epsilon_{r,1}^{s}
$$
\n
$$
L_{r,2}^{s} = \rho_{r}^{s} + \hat{\tau}_{r}^{R_{0}} - \hat{r}^{s} - \gamma_{2} \cdot \hat{I}_{r,1}^{s} + T_{r}^{s} + \lambda_{2}^{R_{k}} \cdot \hat{N}_{r,2}^{s} + \epsilon_{r,2}^{s}
$$
\n
$$
L_{r,3}^{s} = \rho_{r}^{s} + \hat{\tau}_{r}^{R_{0}} - \hat{r}^{s} - \gamma_{3} \cdot \hat{I}_{r,1}^{s} + T_{r}^{s} + \lambda_{3} \cdot \hat{N}_{r,3}^{s} - \Theta_{UC}^{s} + \epsilon_{r,2}^{s}
$$
\n(2)

with

 $\overline{}$

 \mathbf{I}

GLONASS IFCB estimation method

GFIF observations are commonly used for IFCB estimation (Pan et al. [2017;](#page-10-8) Zhang et al. [2022](#page-10-9)). By taking the diference between the IF combination phase observations of L1/L2 and L1/L3, the geometric distance, clock offsets, first-order ionospheric delay, and tropospheric delay can be eliminated. Hence, the GFIF observation $L_{r,\text{GFIF}}^s$ can be represented as the sum of the constant phase ambiguity, the time-varying IFCB, and the time-varying PCO errors in the line-of-sight direction:

$$
L_{\rm r, GFIF}^{\rm s} = L_{\rm r, IF12}^{\rm s} - L_{\rm r, IF13}^{\rm s}
$$

= $\hat{N}_{\rm r, GFIF}^{\rm s} + \Theta_{\rm IF}^{\rm s} + \mathbf{u}_{\rm r}^{\rm s} \cdot \mathbf{\Phi}^{\rm s} \cdot \mathbf{r}_{\rm GFIF}^{\rm s,PCO} + \varepsilon_{\rm r, GFIF}^{\rm s}$ (4)

(3)

$$
\hat{r}_{r}^{R_{0}} = t_{r} + (\alpha_{12}^{R_{k}} b_{r,1}^{R_{0}} + \beta_{12}^{R_{k}} b_{r,2}^{R_{0}})
$$
\n
$$
\kappa_{r}^{s} = \beta_{12}^{R_{k}} (b_{r,1}^{R_{k}} - b_{r,2}^{R_{k}}) - \beta_{12}^{R_{k}} (b_{1}^{s} - b_{2}^{s})
$$
\n
$$
\hat{r}_{r,1}^{s} = I_{r,1}^{s} + \kappa_{r}^{s}
$$
\n
$$
\mu_{r}^{R_{k}} = \alpha_{12}^{R_{k}} b_{r,1}^{R_{k}} + \beta_{12}^{R_{k}} b_{r,2}^{R_{k}} - (\alpha_{12}^{R_{k}} b_{r,1}^{R_{k}} + \beta_{12}^{R_{k}} b_{r,2}^{R_{0}})
$$
\n
$$
\omega_{r}^{s} = b_{r,3} - b_{3}^{s} - (\alpha_{12}^{R_{k}} b_{r,1}^{R_{k}} + \beta_{12}^{R_{k}} b_{r,2}^{R_{k}}) + (\alpha_{12}^{R_{k}} b_{1}^{s} + \beta_{12}^{R_{k}} b_{2}^{s}) - \gamma_{3} \kappa_{r}^{s}
$$
\n
$$
\hat{N}_{r,1}^{s} = N_{r,1}^{s} + B_{r,1}^{R_{k}} - B_{1}^{s} - (\alpha_{12}^{R_{k}} b_{r,1}^{R_{k}} + \beta_{12}^{R_{k}} b_{r,2}^{R_{k}}) / \lambda_{1}^{R_{k}} + (\alpha_{12}^{R_{k}} b_{1}^{s} + \beta_{12}^{R_{k}} b_{2}^{s}) / \lambda_{1}^{R_{k}} + \kappa_{r}^{s} / \lambda_{1}^{R_{k}}
$$
\n
$$
\hat{N}_{r,2}^{s} = N_{r,2}^{s} + B_{r,2}^{R_{k}} - B_{2}^{s} - (\alpha_{12}^{R_{k}} b_{r,1}^{R_{k}} + \beta_{12}^{R_{k}} b_{r,2}^{R_{k}}) / \lambda_{2}^{R_{k}} + (\alpha_{12}^{R_{k}} b_{1}^{s} + \beta_{12}^{R_{k}} b_{2}^{s}) / \lambda_{2}^{
$$

 $\hat{t}^s = t^s + (\alpha_{12}^{R_k} b_1^s + \beta_{12}^{R_k} b_2^s)$

where \hat{i} ^s is the satellite clock offsets that absorbed the ionospheric-free (IF) combination satellite hardware delays, commonly correcting with IGS precise clock products; $\hat{\tau}_r^{\mathcal{R}_0}$ denotes the receiver clock offsets that absorbed the IF combination receiver hardware delays of a reference satellite R_0 ; $\alpha_{12}^{R_k}$ and $\beta_{12}^{R_k}$ are coefficients of the IF combinations with $\alpha_{12}^{R_k} = f_{1,R_k}^2 / (f_{1,R_k}^2 - f_{2,R_k}^2)$ and $\beta_{12}^{R_k} = -f_{2,R_k}^2 / (f_{1,R_k}^2 - f_{2,R_k}^2)$; $\mu_r^{R_k}$ refers to the GLONASS IFB; ω_r^s is the third-frequency IFB, which have to be estimated for each receiver and satellite pair and can fully absorb the code IFCB (Li et al. [2018](#page-10-13)); \overline{B}_3^s 3 denote the constant part of B_3^s and can be absorbed into the phase ambiguity $\hat{N}^s_{r,3}$, while the time-varying part Θ^s_{UC} is the phase IFCB of the uncombined model. Pan et al. ([2019\)](#page-10-8) proved that the conversion between Θ_{UC}^s and the IF combination phase IFCB Θ_{IF}^s derived from GFIF observations is $\Theta_{\text{UC}}^s = \frac{f_{1,R_k}^2 - f_3^2}{f_3^2} \cdot \Theta_{\text{IF}}^s$.

with

$$
\begin{cases}\n\hat{N}_{\rm r, GFIF}^{s} = N_{\rm r, IF12}^{s} - N_{\rm r, IF13}^{s} + (\alpha_{12}B_{\rm r,1} + \beta_{12}B_{\rm r,2}) - (\alpha_{13}B_{\rm r,1} + \beta_{13}B_{\rm r,3}) \\
+ (\alpha_{12}B_{1}^{s} + \beta_{12}B_{2}^{s}) - (\alpha_{13}B_{1}^{s} + \beta_{13}\overline{B}_{3}^{s}) \\
\Theta_{\rm IF}^{s} = -\beta_{13} \cdot (B_{3} - \overline{B}_{3}) \\
\mathbf{r}_{\rm GFIF}^{s, \rm PCO} = \mathbf{r}_{\rm IF12}^{s, \rm PCO} - \mathbf{r}_{\rm IF13}^{s, \rm PO} \n\end{cases}
$$
\n(5)

where $\hat{N}^s_{\text{r,GFIF}}$ is the linear combination of phase ambiguities and the constant parts of phase hardware delays; Θ_{IF}^s denote the IF combination IFCB; $\mathbf{r}^{\text{s,PCO}}$ stands for the vector of satellite PCO correction in the satellite-fxed coordinate system, where the *z*-axis points to the Earth, the *y*-axis is the rotation axis of solar panel and *x*-axis follows the righthanded system (Schmid et al. 2005); Φ^s is the rotation matrix from satellite-fxed frame to ITRF, which can be obtained from IGS satellite attitude products; \mathbf{u}_r^s is the line-of-sight unit vector in ITRF; $\varepsilon_{\text{r,GFIF}}^s$ is the sum of measurement noise, multipath errors, and high-order ionospheric delays. It was reported that the time-varying part of receiver IFCB is small

enough to be ignored (Li et al. [2012](#page-10-14)). Therefore, we focus on the satellite IFCB in this contribution.

Then, the constant part $\hat{N}^s_{\rm r, GFFF}$ in GFIF observation can be eliminated by the epoch diference (ED) approach:

$$
\Delta L_{\text{r,GFIF}}^{\text{s}}(t, t-1) = L_{\text{r,GFIF}}^{\text{s}}(t) - L_{\text{r,GFIF}}^{\text{s}}(t-1)
$$

$$
= \Delta \Theta^{\text{s}}(t, t-1) + \Delta \mathbf{w}_{\text{r}}^{\text{s}}(t, t-1) \cdot \mathbf{r}_{\text{GFIF}}^{\text{s,PCO}} \tag{6}
$$

where $\Delta\Theta^s(t, t-1)$ denotes the ED IFCB; \mathbf{w}_r^s refers to the line-of-sight unit vector in the satellite-fxed coordinate system, which is expressed as $\mathbf{w}_{r}^{s} = [w_x, w_y, w_z]$; therefore, $\Delta \mathbf{w}_{r}^{s}(t, t-1)$ can be expressed as $\Delta \mathbf{w}_{r}^{s} = [\Delta w_{x}(t, t-1),$ $\Delta w_{y}(t, t-1), \Delta w_{z}(t, t-1)$]. Considering the standard deviation (STD) of GFIF observation is $\sigma_{\text{GFIF}} = \sqrt{(\alpha_{12}^2 - \alpha_{13}^2)\sigma_{L1}^2 + \beta_{12}^2\sigma_{L2}^2 + \beta_{13}^2\sigma_{L3}^2}$, the STD of $\Delta\Theta^{s}(t, t-1)$ can be expressed as $\sigma_{\Delta\Theta^{s}} = \sqrt{2}\sigma_{\text{GFIF}}$, where σ_{L1} , σ_{L2} , and σ_{L3} are all set as 0.02 cycle (Zaminparda et al. [2017](#page-10-4)). For quality control, ED GFIF observations that are larger than $3\sigma_{\Lambda\Theta^s}$ are removed (Zhang et al. [2022\)](#page-10-9). Hence, we can set up a least-square adjustment for each satellite using ED GFIF from a reference network, and the estimated parameters are:

$$
\mathbf{x} = [r_{\text{GFIF},x}^{\text{s,PCO}}, r_{\text{GFIF},y}^{\text{s,PCO}}, r_{\text{GFIF},z}^{\text{s,PCO}}, \Delta\Theta^{\text{s}}(t_1, t_0), \Delta\Theta^{\text{s}}(t_2, t_1), \cdots, \Delta\Theta^{\text{s}}(t_n, t_{n-1})]
$$
(7)

Afterward, the IFCB value at an arbitrary epoch t_n can be computed by the IFCB value at the reference epoch and ED IFCB values:

$$
\Theta^{s}(t_{n}) = \Theta^{s}(t_{0}) + \sum_{i=1}^{n} \Delta \Theta^{s}(t_{i}, t_{i-1})
$$
\n(8)

where $\Theta^{s}(t_0)$ can be set to an arbitrary value because it can be absorbed in phase ambiguities in parameter estimation.

Here, we set a zero-mean constraint for all IFCB values to guarantee the continuity of adjacent daily solutions, with

$$
\Theta^{s}(t_0) = \sum_{i=1}^{n} (i - n) \cdot \Delta \Theta^{s}(t_i, t_{i-1})
$$
\n(9)

Meanwhile, the satellite L3 PCO $(\mathbf{r}_3^{\text{s,PCO}})$ can be derived from the GFIF PCO estimates $(\mathbf{r}_{\text{GFF}}^{s,PCO})$ through the following equation:

$$
\mathbf{r}_{3}^{\text{s,PCO}} = \left[(\alpha_{12} - \alpha_{13}) \cdot \mathbf{r}_{1}^{\text{s,PCO}} + \beta_{12} \cdot \mathbf{r}_{2}^{\text{s,PCO}} - \mathbf{r}_{\text{GFIF}}^{\text{s,PCO}} \right] / \beta_{13}
$$
\n(10)

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Fig. 1 Distribution of IGS stations used for the GLONASS IFCB estimation (blue cycles) and triple-frequency PPP validation (red pentagrams)

where $\mathbf{r}_3^{\text{s,PCO}}$ and $\mathbf{r}_3^{\text{s,PCO}}$ refer to the satellite PCO of L1 and L2. And the STD of $\mathbf{r}_3^{\text{S,PCO}}$ can be computed by $\sigma_{\mathbf{r}_{3}^{\text{s,PCO}}} = \sigma_{\mathbf{r}_{GFIF}} / \beta_{13}$.

Data and processing strategy

As of February 2023, nine operational GLONASS satellites are transmitting L3 navigation signals, including six GLONASS-M+satellites (R04, R05, R12, R15, R21, and R24) and three GLONASS-K satellites (R09, R11, and R22). We collect the GLONASS triple-frequency observations for 100 days in DOY 210–310, 2022 to estimate the GLONASS IFCB and evaluate the performance of GLONASS-only triple-frequency PPP. Figure [1](#page-3-0) shows the distribution of the GLONASS reference network and user stations. We use 151 IGS stations in blue cycles for IFCB estimation and eight user stations in red pentagrams for GLONASS triple-frequency PPP. In addition, the precise GNSS orbit and attitude quaternion products from the German Research Center for Geosciences (GFZ) are utilized in IFCB estimation.

Table [1](#page-4-0) summarizes the processing strategy of GLO-NASS triple-frequency PPP. We utilize the undiferenced and uncombined PPP model. GLONASS IFBs are estimated as constant values for each satellite, and the third-frequency IFBs are estimated as the random walk for each station-satellite pair. In this contribution, the GREAT (GNSS + Research, Application, and Teaching) software developed by Wuhan University was employed to handle the IFCB estimation and multi-frequency PPP (Li et al. [2021\)](#page-10-15).

Items	Models
Observations	Undifferenced and uncombined code and phase observations
Sampling interval	30 s
Weighting	Elevation-dependent weighting with a 7° cutoff
Satellite antenna model	L1 and L2: use values from igs14.atx; L3: use the estimated values
Receiver antenna model	L1 and L2: use values from igs14.atx; L3: use values of L2
Satellite orbits and clocks	GFZ precise products
Tropospheric delay	Priori delay by Saastamoinen model (Saastamoinen 1973) with Global Mapping Function (Boehm et al. 2006); estimating zenith troposphere delays (ZTDs) as piecewise constants every 2 h
Ionospheric delay	Estimated as the random walk
Phase ambiguity	Constant over each continuous observation arc
Receiver clock offset	Estimated as white noise; estimate IFB for each GLONASS satellite as constants; estimate the third-frequency IFB for each satellite-receiver pair as the random walk

Table 2 Detailed information of the six stations used in singlereceiver IFCB estimation

Fig. 2 Single-receiver IFCB results for R09, R21, and G10 (from the left column to the right) in DOY 215–217, 2022. Six stations are divided into group A and group B with their results showing in the frst and second rows

Validation experiment

First, the single-receiver IFCB results are investigated to illustrate the coupling relationship between IFCB and PCO. Next, we perform the IFCB estimation with the new method, evaluate the IFCB results, analyze the PCO estimations, and provide an L3 PCO correction for GLONASS-M+satellites. Finally, we implement the GLONASS-only triple-frequency PPP using the estimated PCO corrections.

Analysis of single‑receiver IFCB results

Six IGS stations are selected and categorized into two groups, i.e., A and B, with their detailed information listed in Table [2](#page-4-1). Three stations in group A are equipped with different receivers and antennas, although their distances are relatively close (41 to 83 km). In contrast, the three stations in group B are outftted with identical receivers and antennas, but their distances are very far (211 to 539 km).

Figure [2](#page-4-2) depicts the IFCB time series of R09 (GLO-NASS-K), R21 (GLONASS-M+), and G10 (Block IIF, for comparison) obtained from the six selected stations in DOY 215–217, 2022. Notably, the time series of R09 exhibit minimal variation, with a scatter of less than 0.05 m, and demonstrate exceptional consistency across all six stations. Similar consistency can also be found for G10, albeit fuctuating over time. These fndings suggest that the IFCB estimates of GLONASS-K and GPS Block IIF satellites are solely satellite-dependent and are not infuenced by the type of receiver, antenna, or station location, as reported in previous studies (Pan et al. [2017;](#page-10-8) Zhang et al. [2022](#page-10-9)). However, this phenomenon is not observed for R21, as evidenced by the signifcant diferences in the three IFCB time series from group B with intersecting fuctuations.

In contrast, the R21 IFCB results from group A exhibit a high degree of similarity. This observation leads to the inference that the IFCB estimates of R21 are afected by the station location rather than the receiver type or antenna. Additionally, it is worth noting that the IFCB time series of each continuous observation arc exhibit V-shaped or invert-V-shaped patterns, indicating a possible association with the azimuth and the nadir of the satellite signals. We can further deduce that the inconsistency in the R21 IFCB estimates is due to the diference in satellite PCOs of L2 and L3.

Figure [3](#page-5-0) presents the time series of R21 IFCB in DOY 215–217 of 2022, and the line-of-sight unit vector in the *x*-, *y*-, and *z*-components in the satellite-fxed frame; these are also the coefficients of x -, y -, and z -offset in (7). It is observed that the variations of the *z*-coefficient, i.e., the cosine of nadir, are minimal (0.97–0.99), whereas the peakto-peak value of IFCB is approximately 0.2 m. Assuming that the *z*-ofset error causes the fuctuations of IFCB, the *z*-offset error would have to be up to 10 m, which is highly improbable. On the other hand, the variation of the *y*-coeffcient is signifcant; however, the time series of the *y*-coeffcient does not follow the V-shaped or invert-V-shaped patterns like IFCB. In contrast, the shape of the time series of the *x*-coefficient is strikingly similar to that of the IFCB, and the peak-to-peak value of the *x*-coefficient reaches 0.3 , suggesting that the *x*-offset is the main cause of the fluctuations of IFCB estimates.

Fig. 3 Time series of R21 IFCB derived from MAC1 (upper) in DOY 215–217, 2022, and the line-of-sight unit vector $\mathbf{w} = [w_x, w_y, w_z]$ in the satellite-fxed coordinate system (bottom)

To elucidate the connection between IFCB and *x*-offset, we present a scatter diagram of R21 IFCB estimates and the coefficients of x -offset in Fig. [4](#page-6-0). As can be observed, all IFCB outcomes from the six stations exhibit linear correlations with the *x*-coefficient, with similar slopes of approximately -0.7 . These results further confirm the hypothesis that the *x*-offset of R21 is responsible for the inconsistency in the IFCB estimates.

GLONASS IFCB results considering the PCO diferences

Based on the above analysis, the IFCB estimation considering PCO diferences is performed for GLONASS-M+satellites using the proposed method. Figure [5](#page-6-1) illustrates the GLONASS IFCB results obtained from 151 globally distributed stations on DOY 295, 2022. Additionally, the results of three GLONASS-K satellites (R09, R11, and R22) are included for comparison purposes. Intraday IFCB time series of GLONASS-K satellites exhibit very small variation, while the IFCB time series of GLONASS-M+satellites without PCO corrections display severe and precipitous fuctuations, consistent with previous fndings (Zhang et al. [2023\)](#page-10-10). In contrast, the IFCB results considering PCO differences exhibit values around zero with peak-to-peak values smaller than 0.05 m, indicating that the true IFCB of $GLONASS-M + is$ very small when the influence of PCO diferences is eliminated. It should be noted that the IFCB time series of R15 is intermittent, as only 33 stations are tracking the signal of R15.

Fig. 4 Scatter diagram of the R21 IFCB values derived from the six stations in DOY 215–217, 2022 vs. the corresponding coefficients of *x*-ofset

To further investigate the inter-day characteristics of GLONASS IFCB, we have depicted the IFCB time series for 100 days in Fig. [6.](#page-7-0) The results of GLONASS-K satellites are also included for comparison purposes. As can be observed, the IFCB time series derived without PCO correction exhibit both a medium-term variation of approximately 8 days and a long-term variation that varies with the orbital planes. The medium-term variation can be attributed to the orbit parameters of GLONASS satellites, which have a period of revolution of 11 h,15 m,44 s, and a ground track repeat cycle of 8 days. Furthermore, the IFCB time-series amplitude changes with the sun's elevation to the orbital plane, i.e., the beta angle. As the absolute value of the beta angle increases, the amplitude decreases slowly and smoothly. Notably, the *y*-axis of the satellite-fxed coordinate system corresponds to the rotation axis of the solar panel, which is associated with the beta angle.

Consequently, the satellite attitude changes with the beta angle, leading to the variation of the PCO diferences in GFIF observations. Conversely, the IFCB time series considering PCO diferences, exhibit excellent stability and concentrate around zero. This phenomenon serves as further evidence that the PCO diferences in IFCBs have been successfully separated, and the resulting clean IFCBs can be obtained.

Then, we summarize the root mean square (RMS) values of 100-day IFCB estimates of GLONASS-M +satellites, as depicted in Fig. [7.](#page-7-1) By separating the PCO errors from GFIF observations, the mean RMS value of IFCB estimates decreases from 0.097 m to 0.028 m, with an improvement of 71%. Moreover, all GLONASS-M +satellites, except satellite R15, exhibit RMS values of less than 0.03 m and mean values of less than 1e-8 m. The remaining IFCB can be attributed to the observation noise, higher-order ionospheric

Fig. 5 IFCB time series for GLONASS-K and GLONASS-M+satellites derived from 151 stations on DOY 295, 2022

delays, or other unmodeled errors. Based on these fndings, we recommend that the IFCB of GLONASS-M+satellites can be disregarded if the correct satellite PCO is applied for the L3 signal.

GLONASS PCO results

In this section, we evaluate the accuracy of PCO derived from the GFIF observations of 151 IGS stations and present the final L3 PCO estimates for GLONASS-M + satellites. Figure [8](#page-8-0) illustrates the *x*-offset, *y*-offset, and *z*-offset estimates for GLONASS-M+satellites from DOY 210 to DOY 310 in 2022. We observe that the *x*-ofset estimates for all satellites exhibit excellent stability and converge to approximately -0.7 m. The *z*-offset estimates are nearly zero, which may refect the low sensitivity of ED GFIF observations to *z*-offsets. The *y*-offset estimates demonstrate relatively less stability than the *x*- and *z*-offsets, particularly for R05 in DOY 210–220, R12 in DOY 300–310, and R21/R24 in DOY 275–285. This may stem from the error of the satellite attitude model during the low beta angle. R15 PCO estimates are erratic owing to the scarce tracking stations of R15 during the experimental period.

IGS antenna model has provided the $IF(1, 2)$ combination PCO for GLONASS-M+satellites, which is (-545.00, 0.00, 2450.00) mm, and is used for the L1 and L2 PCO in this study. Then, the L3 PCO estimates can be derived from GFIF PCO estimates as (11). Table [3](#page-8-1) presents the L3 PCO estimates and their corresponding STDs for GLONASS-M+satellites. The R15 computation employs only data from **Fig. 6** IFCB time series for GLONASS-K and GLONASS-M+satellites from DOY 210 to DOY 310 in 2022. Red lines refer to the sun's elevation to the orbital plane, i.e., the beta angle

DOY 275–310 due to poor estimates during the initial days. Mean STD values for *x*-, *y*-, and *z*-offsets are 52, 113, and 13 mm, respectively, which suggests that the PCO estimates exhibit high accuracy. Notably, no signifcant diferences exist among the PCO estimates of GLONASS-M+satellites, particularly in the *x*- and *z*-components, with maximum differences of 7.6 and 16.1 mm. Therefore, the fnal L3 PCO estimates for GLONASS-M+satellites can be obtained by averaging the weighted values of all satellites except R15, which is (−1072.39, 152.94, 2425.01) mm. However, the x-offset appears deviant, as it even reaches 1 m. This is due to the fact that the obtained L3 PCO is a relative value against the PCO of L1 and L2. In this study, we derive the L3 PCO from the IF(1, 3) PCO, which is (131.89, -195.94, 2482.09) mm, where the L1 and L2 PCOs are fxed to the value of IF(1, 2). The uncombined L3 PCO correction for each frequency requires further investigation.

GLONASS‑only triple‑frequency PPP

To validate the estimated L3 PCO for GLONASS-M+satellites, we design three strategies to perform the GLONASSonly triple-frequency PPP as follows: **S1**: use the L2 PCO corrections from igs14.atx for L3 and no IFCB correction; **S2**: use the L2 PCO corrections from igs14.atx for L3 and use IFCB correction derived from the standard model; **S3**: use the estimated L3 PCO and no IFCB correction.

Fig. 7 RMS values of the IFCB estimates for GLONASS-M+satellites from DOY 210 to DOY 310 in 2022

Raw triple-frequency GLONASS data from eight globaldistributed stations (as shown in Fig. [1\)](#page-3-0) on DOY 295, 2022 are employed for the PPP. Figure [9](#page-8-2) illustrates the positioning errors of BRUX, MIZU, and NICO under the three strategies. It is observed that S1 sufers from a bias of up to 0.2 m due to the inappropriate L3 PCO corrections. By applying the IFCB correction from the standard model, S2 partially mitigates the positioning errors. However, the PCO diferences are not eliminated by the IFCB correction, and the performance difers depending on the station's geographic

Fig. 8 Time series of *x*-offset (blue), *y*-offset (red), and *z*-offset (green) estimates for GLONASS-M+satellites from DOY 210 to DOY 310 in 2022

Fig. 9 Positioning errors of GLONASS-only triple-frequency PPP for BRUX **a**, MIZU **b**, and NICO **c** on DOY 295, 2022

location. In contrast, S3 exhibits the best positioning accuracy and convergence time, suggesting the high accuracy of the estimated L3 PCO.

Figure [10](#page-9-3) depicts the RMS values of the positioning errors for the eight stations. First-hour results during the convergence time are excluded. As can be observed, the S3 strategy manifests the most elevated accuracy in the east, north, and up components, with average RMS values of 1.8, 0.9, and 1.5 cm. In comparison to the outcomes of S2, the positioning errors are reduced by 13%, 3%, and 33% in the three components. Moreover, the improvement in NICO and ZAMB in the up component exceeds 50%.

We also examined the phase residuals for the three strategies, as shown in Fig. [11.](#page-9-4) RMS values of L3 residuals for S1 range from 10 to 16 mm, while those of S2 exhibit a more drastic variation from 4 to 18 mm, indicating that the IFCB from standard model fails to model the extra bias in L3 phase observation properly. In contrast, the RMS values of L3 residuals for S3 are consistently around 5 mm, with an improvement of 51% compared with those of S2.

Conclusions

Modernization of GLONASS has led to six GLONASS-M+and three GLONASS-K operational satellites transmitting the new CDMA signal at the third frequency and the legacy FDMA signals L1 and L2. However, the presence of extra phase bias on GLONASS L3 observations, previously known as IFCB, hinders the application of GLONASS triple-frequency positioning. In this study, we investigate the characteristics of the GLONASS IFCB estimates derived from single-receiver GFIF observations. The GLONASS-M + satellite IFCBs estimated by stations that are located far apart exhibit signifcant discrepancies, whereas they are highly consistent for nearby stations. Then, we demonstrate that the single-receiver IFCB estimates of GLONASS-M + satellites are independent of receiver or antenna type and are solely infuenced by the relative position between the satellite and the receiver. Furthermore, we discover that the IFCB estimates of GLONASS-M+satellites demonstrate a notable linear correlation with the satellite PCO in the *x*-component of the satellite-fxed coordinate system.

Fig. 10 RMS values of the positioning errors of GLONASS-only triple-frequency PPP for the selected stations on DOY 295, 2022. Firsthour results during the convergence time are excluded

Fig. 11 RMS values of L3 residuals in PPP for the selected stations on DOY 295, 2022

Based on the analysis of GLONASS IFCB, we propose a modifed IFCB estimation model that simultaneously estimates GFIF PCOs and IFCBs. GLONASS triple-frequency observations from 151 globally distributed IGS stations are utilized for the IFCB estimation. We analyze both the intraday and inter-day characteristics of the IFCB estimates derived from these 151 stations. The results show that the IFCB time series considering PCO diferences exhibit minimal variation, with the mean RMS value decreasing from 0.097 m to 0.028 m. The performance of the IFCB estimates suggests that the majority of the extra phase bias in L3 is caused by the PCO diferences rather than the GPS-like IFCB. Therefore, the IFCB can be ignored in GLONASS triple-frequency data processing if only the L3 PCO is precisely corrected.

Hence, we evaluate the accuracy of the estimated PCO for GLONASS-M +satellites. The estimated *x*-, *y*-,

and *z*-offsets for each satellite exhibit high stability and consistency, with mean STD values of 52, 113, and 13 mm, respectively. Therefore, we obtain the final L3 PCO corrections for GLONASS-M+satellites by averaging the weighted values of PCO estimates for each satellite, which equates to (-1072.39, 152.94, 2425.01) mm. With the precise L3 PCO correction, we carry out the GLONASSonly triple-frequency PPP using the uncombined and undiferenced observation model. Raw triple-frequency GLONASS data from eight globally distributed stations on DOY 295, 2022 are employed for the PPP. By comparing the new strategy that employs the estimated L3 PCO value and ignores IFCB with the traditional strategy that utilizes the L2 PCO value for L3 and corrects the IFCB, we observe a significant improvement in positioning accuracy. Specifically, the new strategy results in an improvement of 13%, 3%, and 33% in the east, north, and up components, respectively. Moreover, the RMS value of L3 phase residuals reduces from 10.2 to 5.0 mm, further validating that the new strategy has appropriately modeled the extra bias on GLONASS L3.

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Data availability All data in this article are publicly accessible. The GNSS data and products can be obtained at [https://cddis.nasa.gov/](https://cddis.nasa.gov/archive) [archive](https://cddis.nasa.gov/archive).

Declarations

Competing interests The authors declare no competing interests.

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