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A BDS‑3 B1C/B2a dual‑frequency joint tracking architecture based on adaptive Kalman flter and extended integration time

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Abstract

With the development of the third-generation BeiDou navigation satellite system (BDS-3), the availability of new civil B1C and B2a signals along with existing B1I and B3I signals will greatly beneft the design of BeiDou multi-frequency receivers. For instance, BDS-3 can provide signal observations at multiple frequencies to form fexible design strategies of advanced receiver baseband signal and a linear combination of observations used for diferent purposes. The B1C/B2a receivers have been proposed recently which, however, are limited by the integration time to improve the tracking sensitivity further. Also, the signal observations and characteristics of the dual-frequency tracking architecture have not been fully explored, which can be further designed to get mutual benefts and reach an optimal tracking performance. In order to improve the tracking performance, especially in challenging environments such as in the presence of a weak signal and selective frequency signal attenuation, a B1C/B2a joint tracking architecture for BDS-3 dual-frequency receivers is proposed based on an adaptive Kalman flter and an extended integration time. The experimental results show that compared to the modifed traditional tracking architectures, the proposed joint tracking architecture can improve the receiver's tracking sensitivity up to 3.29 dB and achieve an optimal availability, accuracy and robustness performance in both tracking and navigation solutions.

Keywords B1C/B2a · Dual-frequency · Joint tracking · Adaptive Kalman flter · Extended integration time

Introduction

With the development of the third-generation BeiDou system (BDS-3), the availability of the new BeiDou civil B1C and B2a signals along with existing B1I and B3I signals will greatly beneft the design of BeiDou multi-frequency receivers (Yang et al. [2019](#page-14-0)). For instance, multi-frequency signals can provide signal observations at multiple frequencies to form fexible design strategies of advanced receiver baseband signal processing such as inner-band aiding and joint tracking and form a linear combination of observations used for diferent purpose like ionospheric-free, wide-lane

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and narrow-lane combinations. Also, the availability, accuracy and robustness in both tracking and navigation solutions can be improved through redundancy of multiple signal frequencies from the same satellite. The new signals B1C and B2a can share two frequencies at 1575.42 MHz and 1176.45 MHz, respectively, with the modernized GPS L1C/L5 and Galileo E1/E5 signals, as well as some modern navigation signal features like binary offset carrier (BOC) modulation and pilot/data components (Lu et al. [2019](#page-14-1); CSNO [2017a,](#page-14-2) [b\)](#page-14-3), to improve the ranging performance and the interoperability (Yao and Lu [2016\)](#page-14-4). From the above, the use of B1C and B2a signals presents new opportunities to increase the performance of a receiver.

The ideas of collaboration across pilot and data channels from the same carrier frequency and even across diferent frequencies transmitted from the same satellite have been presented in many works of literature for tracking purposes. For signal tracking collaborations on the same carrier frequency (e.g., data and pilot channels), the output of individual channels has been combined using the optimal linear combined standard tracking loop (denoted here as OLC-STL) either at the correlator level (Spilker and Dierendonck

[1999](#page-14-5); Mongrédien et al. [2006](#page-14-6)) or at the discriminator level (Tran [2004](#page-14-7); Muthuraman et al. [2008\)](#page-14-8) for GPS L5 and L2C signals and at the discriminator level for BeiDou B2a and B1C signals (Li et al. [2018,](#page-14-9) [2019](#page-14-10); Wu et al. [2019\)](#page-14-11). But all research discussed to date uses the same integration time for signals coming from pilot and data channels at diferent frequency bands, which, however, is limited by navigation symbol bits in the data channel. Compared to the data channel, the pilot channel can use a longer integration time to track signals more robustly since it is not modulated by the navigation data and thus allowing satellite signals to be tracked in challenging environments (Kaplan and Hegarty [2017](#page-14-12); Yao and Lu [2016\)](#page-14-4).

Research on signal tracking collaborations across diferent carrier frequency bands from the same satellite can be divided into two categories, i.e., the inter-band Doppler-aided and Kalman flter technologies. The inter-band Doppler-aided dual-frequency tracking technologies are explored using the optimal linear combined Doppler-aided tracking loop, denoted here as OLC-DATL, for GPS L1 CA/L5 (Ries et al. [2002](#page-14-13); Bolla and Lohan [2018](#page-13-0); Bolla and Borre [2019\)](#page-13-1), L1 CA/L2C (Qaisar [2009\)](#page-14-14) and BeiDou B1C/B2a (Wu et al. [2019](#page-14-11)) signals which explore the inherent linear relationship among diferent signal frequencies from the same satellite and improve the tracking performance of the aided signal. However, the overall performance of the structure depends on the band to provide the Doppler information. Therefore, this structure is susceptible to the selective frequency signal attenuation present in the signal band that provides Doppler-aiding information. For selective frequency signal attenuation, this is the case when a single-frequency band is afected by interference and only one signal is severely attenuated, such as the L5 band attenuation. Kalman flter-based technologies are designed for GPS L1 CA/L5 (Megahed et al. [2009](#page-14-15)) and L1 CA/L1C (Macchi [2010\)](#page-14-16) signals at correlator-level measurements and GPS L1 CA/L2C (Gernot et al. [2011\)](#page-14-17) signals at discriminator-level measurements. Compared to the Doppler-aided method, the Kalman flter method can improve the tracking accuracy using advanced fltering theory and can fuse the measurements from diferent carrier frequency bands, which can provide benefts to the tracking of each band. Several problems, however, still exist. The frst one is the limitation of the integration time. The existing Kalman flter-based methods combine the traditional

GPS L1 C/A signal with one of the modernized pilot signals such as GPS L5, L2C and L1C, and the integration time of these combinations is still limited by the navigation symbol bit period of the GPS L1 C/A (20 ms) signal. Moreover, this problem will be more severe for BeiDou B1C and B2a signals (10 ms and 5 ms, respectively). Therefore, the tracking sensitivity will be greatly limited. Second, since the Kalman flter-based technology fuses the information from two frequency signals, it can enhance the tracking performance of each band and has the potential to better withstand the selective frequency signal attenuation problem. This, however, has not been discussed.

We propose an adaptive Kalman filter-based B1C/B2a joint tracking architecture for BDS-3 receivers. The non-coherent processing of the phase lock loop (PLL) proposed by Borio and Lachapelle ([2009\)](#page-13-2) is frst used to design the B2a data channel tracking loop to extend the integration time and to avoid the limitation of the navigation symbol bits, while the pure PLL discriminator is used to extend the integration time of the B1C and B2a pilot channels. Then, a joint Kalman flter is explored to combine the B1C and B2a signals in detail.

The signal properties of the B1C and B2a signals are frst reviewed, followed, by gives a detailed description of the design of an adaptive Kalman flter-based B1C/B2a joint tracking architecture for a BDS-3 dual-frequency receiver. We then address the feld experiments conducted using live BDS-3 signals to evaluate the proposed architecture. Finally, conclusions and future work are provided.

B1C and B2a civil signal specifcations

As shown in Table [1](#page-1-0), the $BOC(1, 1)$ and quadrature multiplexed binary offset carrier $(6, 1, 4/33)$ $(QMBOC(6, 1, 4/33))$ modulations are used for the B1C data and pilot components, respectively. The received B1C signal to be processed by baseband functions of the receiver can be expressed as (Gao et al. [2019\)](#page-14-18):

$$
r_{\rm BIC} = s_{\rm d,n}^{\rm BIC} + s_{\rm p61,n}^{\rm BIC} + j \cdot s_{\rm p11,n}^{\rm BIC} + \eta \tag{1}
$$

where $s_{d,n}^{BIC}$ is the data component, $s_{p61,n}^{BIC}$ and $s_{p11,n}^{BIC}$ denote the $BOC(6,1)$ and $BOC(1,1)$ components of the pilot channel, respectively. η is the additive white Gaussian noise. The

Table 1 BeiDou B1C and B2a civil signal specifcations

Signal component	Carrier frequency (MHz)	Primary code period (ms)	Secondary code period (ms)	Modulation	Symbol rate (sps)	Received power difference (dB)
B1C data	1575.42	10		BOC(1,1)	100	
B ₁ C pilot			18,000	QMBOC (6,1,4/33)	θ	$\approx +4.77$
B ₂ a data	1176.45			BPSK(10)	200	$\approx +6.02$
B ₂ a pilot			100			$\approx +6.02$

details of $s_{d,n}^{B1C}$, $s_{p61,n}^{B1C}$ and $s_{p11,n}^{B1C}$ can be referred to Gao et al. [\(2019\)](#page-14-18).

Also, a binary phase shift keying (10) (BPSK(10)) modulation is used for both B2a data and pilot components. The received B2a signal to be processed by baseband functions of the receiver is expressed as (Gao et al. [2019](#page-14-18)):

$$
r_{\rm B2a} = s_{\rm d,n}^{\rm B2a} + j \cdot s_{\rm p,n}^{\rm B2a} + \xi \tag{2}
$$

where $s_{d,n}^{B2a}$ is the data component, $s_{p,n}^{B2a}$ is the pilot component and ξ is the additive white Gaussian noise. For details of $s_{d,n}^{B2a}$ and $s_{p,n}^{B2a}$ refer to Gao et al. ([2019\)](#page-14-18). It is noted that the two noise terms on B1C and B2a are statistically independent with diferent variances.

It can be seen that the B2a signal has a higher transmit power, lower carrier frequency and higher primary code frequency compared to the B1C signal. This makes the B2a signal have potentially stronger signal carrier-to-noise power spectral density ratio (*C*/*N*₀), slower carrier Doppler dynamics and better signal tracking noises, especially for the code tracking accuracy. According to the interface control documents (ICDs) of B1C (CSNO [2017a](#page-14-2)) and B2a (CSNO [2017b](#page-14-3)), the ephemeris data of each satellite can be decoded from either B1C or B2a data channels. Again, the B1C data component has a much lower transmit power compared to other components of B1C and B2a. Therefore, as for the BDS-3 dual-frequency receiver, the B2a data channel is chosen here instead of the B1C data channel to receive the ephemeris data, and the B1C data channel can be ignored to reduce the complexity of the dual-frequency receiver and will not be discussed in the following sections.

Adaptive Kalman flter‑based B1C/B2a joint tracking architecture

In order to achieve more mutual benefts between B1C and B2a signals and overcome the integration time limitation caused by the data channel, an adaptive Kalman flterbased B1C/B2a joint tracking loop (KF-JTL) is proposed in this research.

As shown in Fig. [1,](#page-2-0) the B1C and B2a signals are received by the antenna, digitized and down-converted by the radio frequency (RF) front-end board, and sent to the acquisition block. The signal acquisition is implemented with joint process of data and pilot components of B1C and B2a signals, respectively, to get the coarse code phase and Doppler estimation. At the beginning of the tracking, we use shorter integrated time to help the tracking loop fnish the pull-in stage. The longer integrated time is used after the tracking loop becomes stable and keeps lock. The details of the acquisition block can be referred to Li et al. ([2019](#page-14-10)).

In the tracking loop, the B2a data channel is processed with non-coherent integrations as suggested by Borio and Lachapelle ([2009\)](#page-13-2), which can extend the integration time to avoid the limitation of the navigation symbol bits. Then, the KF-JTL fuses the data and pilot channels of B1C and B2a into a joint Kalman flter with the same but long integration time in order to explore the collaboration of B1C and B2a in the tracking loop. The details of the KF-JTL architecture are designed in the following subsections.

Fig. 1 Block diagram of the B1C/B2a KF-JTL. Non-coherent processing is applied to the B2a data channel and all B1C and B2a channels fuse into a joint Kalman flter

 \equiv

 $T₁$

Dynamic system model of the KF‑JTL architecture

As discussed above, the Doppler shift between B1C and B2a signals are correlated. Therefore, similar to Xie et al. [\(2017](#page-14-19)), the system state vector of the Kalman flter which includes both B1C and B2a state vectors is defned as:

$$
\mathbf{X} = \begin{bmatrix} \delta \tau_{\text{BIC}}, & \delta \tau_{B2a}, & \delta \phi_{0,\text{BIC}}, & \delta \phi_{0,B2a}, & \delta f_{0,\text{B2a}}, & \delta a_{\text{B2a}} \end{bmatrix} \tag{3}
$$

where $\delta \tau_{\text{B1C}}$ and $\delta \tau_{B2a}$ are the code phase errors between the local replica code and the incoming signals for B1C and B2a, respectively. $\delta\phi_{0,\text{B1C}}$ and $\delta\phi_{0,\text{B2a}}$ indicate the initial phase errors of B1C and B2a, respectively, at the start of the integration interval. In addition, the frequency error and acceleration error of B2a signal are chosen as the common carrier Doppler frequency and acceleration error states for B1C and B2a signals due to their inherent relations in carrier Doppler frequency.

B2a signals, respectively, and *T* is the update rate of the KF, which is equal to the update interval of the loop denoted as T_{loop} . In addition, the total integration time of all channels is denoted as T_{int} . As for the B2a data channel, T_{int} equals to the max coherent integration time, i.e., 5 ms multiplied by the number of non-coherent integrations.

The noise associated with the dynamic model can be written as (Macchi [2010\)](#page-14-16):

$$
\mathbf{w} = \begin{bmatrix} w_{\text{code,BIC}}, & w_{\text{code,B2a}}, & w_{\text{clk}}, & w_{\text{freq}}, & w_{\text{acc}} \end{bmatrix}^T \tag{6}
$$

where $w_{code,B1C}$ and $w_{code,B2a}$ are the ionospheric divergence efect, i.e., delay for the code and advance for the phase, for B1C and B2a signals, respectively. Further, w_{clk} and w_{freq} are the process noise of the clock bias and the clock drift caused by the oscillator, and w_{acc} is the process noise of the LOS acceleration noise which is related to the receiver's dynamics.

Hence, the covariance matrix Q of w transformed in the discrete domain can be written as (Brown and Hwang [2011](#page-14-20)):

$$
Q_{k} = E[\mathbf{w}_{k} \cdot \mathbf{w}_{k}^{T}]
$$
\n
$$
= \begin{bmatrix}\nS_{c1}T + \frac{S_{d}\beta_{1}^{2}T^{3}}{3} & \frac{S_{d}\beta_{1}\beta_{2}T^{3}}{3} & \frac{S_{d}\alpha\beta_{1}T^{3}}{3} & \frac{S_{d}\beta_{1}T^{3}}{3} & \frac{S_{d}\beta_{1}T^{2}}{2} & 0 \\
\frac{S_{d}\beta_{1}\beta_{2}T^{3}}{3} & S_{c2}T + \frac{S_{d}\beta_{2}^{2}T^{3}}{3} & S_{b1}T + \frac{S_{d}\alpha_{2}^{2}T^{3}}{3} + \frac{S_{d}\alpha^{2}T^{3}}{20} & \frac{S_{d}\alpha T^{3}}{3} + \frac{S_{d}\alpha T^{3}}{2} & \frac{S_{d}\beta_{2}T^{2}}{2} & 0 \\
\frac{S_{d}\beta_{1}T^{3}}{3} & \frac{S_{d}\beta_{2}T^{3}}{3} & S_{b1}T + \frac{S_{d}\alpha_{2}^{2}T^{3}}{3} + \frac{S_{d}\alpha T^{3}}{20} & \frac{S_{d}\alpha T^{3}}{3} + \frac{S_{d}\alpha T^{5}}{3} & \frac{S_{d}\alpha T^{2}}{2} + \frac{S_{d}\alpha T^{3}}{2} \\
\frac{S_{d}\beta_{1}T^{2}}{2} & \frac{S_{d}\beta_{2}T^{2}}{2} & \frac{S_{d}\alpha T^{2}}{2} + \frac{S_{d}\alpha T^{3}}{3} + \frac{S_{d}\alpha T^{4}}{3} & S_{b2}T + \frac{S_{d}T^{3}}{3} + \frac{S_{d}T^{2}}{3} & \frac{S_{d}T^{2}}{2} + \frac{S_{d}T^{4}}{3} & \frac{S_{d}T^{2}}{2} \\
0 & 0 & \frac{S_{d}\alpha_{1}T^{2}}{2} + \frac{S_{d}\alpha T^{3}}{3} & \frac{S_{d}T^{2}}{2} + \frac{S_{d}T^{4}}{3} & S_{d}T^{4} & S_{d}T^{2} \\
\frac{S_{d}\beta_{1}T^{2}}{2} & \frac{S_{d}\beta_{2}T^{2}}{2} & \frac{
$$

Then, the discrete system model for this implementation is written as follow (Kaplan and Hegarty [2017\)](#page-14-12):

$$
\mathbf{X}_{k+1} = \mathbf{\Phi}_k \cdot \mathbf{X}_k + \mathbf{w}_k \tag{4}
$$

where **w** is the additive white Gaussian noise sequences which represent the process noises with covariance values of \mathbf{Q} . $\mathbf{\Phi}_k$ is the state transition matrix which can be expressed as (Xie et al. [2017](#page-14-19)):

$$
\Phi_{k} = \begin{bmatrix} 1 & 0 & 0 & 0 & \beta_{1}T & 0 \\ 0 & 1 & 0 & 0 & \beta_{2}T & 0 \\ 0 & 0 & 1 & 0 & \alpha T & \alpha T^{2}/2 \\ 0 & 0 & 0 & 1 & T & T^{2}/2 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
$$
(5)

where $\beta_1 = f_{code, B1C}/f_{B2a}$, $\beta_2 = f_{code, B2a}/f_{B2a}$ and $\alpha = f_{\rm B1C}/f_{\rm B2a}$. Further, $f_{\rm B1C}$ and $f_{\rm B2a}$ are the nominalized carrier frequency for B1C and B2a signals, respectively, $f_{ca,B1C}$ and $f_{ca,B2a}$ are the nominalized code frequency for B1C and

where S_{c1} and S_{c2} correspond to the difference between the divergence of the ionospheric delay for the code and the phase over time for B1C and B2a signals, respectively. S_{b1} is the product of nominal B1C carrier frequency squared value f_{BIC}^2 and the power spectral density (PSD) of w_{clk} , S_{b2} is the product of nominal B2a carrier frequency squared value f_{B2a}^2 and the PSD of w_{clk} , S_d is the product of nominal B2a carrier frequency squared value f_{B2a}^2 and the PSD of w_{freq} , and S_a is the PSD of w_{acc} . The products S_{b1} , S_{b2} and S_d are calculated as follows (Brown and Hwang [2011](#page-14-20)):

Table 2 Typical *h*-parameters of TCXO and OCXO

Oscillator type	$h_0(s)$	h_{-2} (Hz)
TCXO	10^{-21}	2×10^{-20}
OCXO	2×10^{-25}	6×10^{-25}

$$
\begin{cases}\nS_{b1} = f_{B1C}^2 \cdot h_0/2 \\
S_{b2} = f_{B2a}^2 \cdot h_0/2 \\
S_d = f_{B2a}^2 \cdot 2 \cdot \pi^2 \cdot h_{-2}\n\end{cases}
$$
\n(8)

where h_0 and h_{-2} are the oscillator *h*-parameters. The typical *h*-parameters of temperature compensated crystal oscillator (TCXO) and oven-controlled crystal oscillator (OCXO) are listed in Table [2](#page-3-0) (Brown and Hwang [2011\)](#page-14-20).

Measurement model of the KF‑JTL architecture

As shown in Fig. [1,](#page-2-0) all the discriminator outputs are used as the measurement information of the Kalman flter. The measurement equation of the Kalman flter is modeled as (Kaplan and Hegarty [2017](#page-14-12)):

$$
\mathbf{Z}_{k} = \mathbf{H}_{k} \cdot \mathbf{X}_{k} + \boldsymbol{\nu}_{k}
$$
 (9)

where \mathbf{Z}_k is the input measurement, \mathbf{H}_k is the measurement matrix, v_k is the noise item in the input measurement. The details of the input measurement model are given below.

Measurement outputs of the B2a data channel

As shown in Fig. [2,](#page-4-0) the non-coherent integration processing of the JTL method suggested by Borio and Lachapelle [\(2009\)](#page-13-2) is used here for the B2a data channel.

The sampled B2a signal is frst passed through a bank of correlators to produce *I/Q* correlator outputs for the data component. Then, the navigation symbol bits are removed by the squaring operation. After that, the bit removal results are accumulated and sent to the non-coherent carrier discriminator and the modifed-normalized early-late magnitude (NELM) code discriminator to estimate the carrier phase error and code phase error, which can be described as follows (Borio and Lachapelle [2009](#page-13-2)):

$$
\delta_{\mathrm{d},\varphi}^{\mathrm{B2a}} = \frac{1}{2}ATAN2 \left[2 \sum_{k=0}^{K-1} I_{\mathrm{P},k} Q_{\mathrm{P},k} , \sum_{k=0}^{K-1} \left(I_{\mathrm{P},k}^2 - Q_{\mathrm{P},k}^2 \right) \right] \tag{10}
$$

$$
\delta_{\rm d,r}^{\rm B2a} = (1 - \Delta) \frac{\sqrt{\sum_{k=0}^{K-1} |E_k|^2} - \sqrt{\sum_{k=0}^{K-1} |L_k|^2}}{\sqrt{\sum_{k=0}^{K-1} |E_k|^2} + \sqrt{\sum_{k=0}^{K-1} |L_k|^2}}
$$
(11)

where $E_k = \sqrt{I_{E,k}^2 + Q_{E,k}^2}$ and $L_k = \sqrt{I_{L,k}^2 + Q_{L,k}^2}$, I_P , Q_P , I_E Q_{E} , I_{L} and Q_{L} denote the correlator outputs of in-phase and quadra-phase prompt, early and late. Δ is the code correlator spacing.

Measurement outputs of B2a and B1C pilot channels

Benefting from the data-less characteristic, both pilot channels of B2a and B1C use the pure PLL discriminator after secondary code wipe-off to acquire better noise mitigation performance due to its absence of squaring loss and larger linearity region compared to Costas discriminator. The carrier phase error of B2a and B1C can be estimated using pure PLL discriminator as (Kaplan and Hegarty [2017](#page-14-12)):

$$
\delta_{p,\varphi}^{\text{B2a}} \text{ or } \delta_{p,\varphi}^{\text{B1C}} = ATAN2(Q_p, I_p)
$$
\n
$$
\tag{12}
$$

Also, the NELM code discriminator is used in B2a pilot channel to estimate the code tracking error (Kaplan and Hegarty [2017](#page-14-12)):

$$
\delta_{p,\tau}^{B2a} = (1 - \Delta) \cdot \frac{E - L}{E + L} \tag{13}
$$

where $E = \sqrt{I_{\rm E}^2 + Q_{\rm E}^2}$ and $L = \sqrt{I_{\rm L}^2 + Q_{\rm L}^2}$.

As shown in Fig. [3,](#page-5-0) the mismatched tracking mode, also called narrowband (NB) tracking mode, is adopted for the

Fig. 2 Non-coherent processing of the B2a data channel

Fig. 3 Mismatched tracking of the B1C pilot channel

B1C pilot channel, which ignores the wideband $BOC(6,1)$ component, and only the BOC(1,1) component of B1C pilot component is tracked (Li et al. [2019](#page-14-10); Gao et al. [2019\)](#page-14-18).

Also, in order to cancel the ambiguous problem of the code tracking, the autocorrelation side-peak cancelation technique (ASPeCT)-based dot-product (DP) discriminator proposed in Julien et al. [\(2007\)](#page-14-21) for BOC signals is used for the B1C pilot channel. The code phase error of the B1C pilot channel can be obtained by:

$$
\delta_{p,r}^{\text{BIC}}(\tau) = \frac{\left(-\beta \left[\left(I_{E_{\text{B/P}}} - I_{L_{\text{B/P}}}\right) \cdot I_{P_{\text{B/P}}} + \left(Q_{E_{\text{B/P}}} - Q_{L_{\text{B/P}}}\right) \cdot Q_{P_{\text{B/P}}} \right]\right)}{(6 + \beta \cdot \Delta) \cdot \left(I_{P_{\text{B}}}^2 + Q_{P_{\text{B}}}^2\right)}
$$
(14)

where $I_{P_B}, Q_{P_B}, I_{E_B}, Q_{E_B}, I_{L_B}$ and Q_{L_B} denote the correlator outputs of in-phase and quadra-phase prompt, early and late for BOC(1,1) in B1C pilot channel with spreading code (pseudo random noise (PRN) code \times square wave subcarrier). $I_{P_{R,p}}$, $Q_{P_{BP}}$, $I_{E_{BP}}$, $Q_{E_{BP}}$, $I_{L_{BP}}$ and $Q_{L_{BP}}$ denote the correlator outputs of in-phase and quadra-phase prompt, early and late for BOC(1,1) in B1C pilot channel with PRN code only. β is a coefficient in the combination of the two squared correlation functions to eliminate any small remaining peak caused by the impact of the front-end filter. Δ is the code correlator spacing.

Measurement model and adaptive measurement noise covariance

According to the measurements of B1C and B2a described above, in analogy with the model shown in Xie et al. [\(2017](#page-14-19)),

the measurement model of the joint Kalman flter can be written as:

$$
\mathbf{Z}_{k} = \left[\delta_{p,\tau}^{B1C}; \delta_{p,\varphi}^{B1C}; \delta_{d,\tau}^{B2a}; \delta_{d,\varphi}^{B2a}; \delta_{p,\tau}^{B2a}; \delta_{p,\varphi}^{B2a} \right]
$$
(15)

$$
\mathbf{H}_{k} = \begin{bmatrix} 1 & 0 & 0 & 0 & \beta_{1}T/2 & 0 \\ 0 & 0 & 1 & 0 & \alpha T/2 & \alpha T^{2}/6 \\ 0 & 1 & 0 & 0 & \beta_{2}T/2 & 0 \\ 0 & 0 & 0 & 1 & T/2 & T^{2}/6 \\ 0 & 1 & 0 & 0 & \beta_{2}T/2 & 0 \\ 0 & 0 & 0 & 1 & T/2 & T^{2}/6 \end{bmatrix}
$$
(16)

$$
\mathbf{R}_{k} = \begin{bmatrix} R_{p,r}^{B1C} & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{p,\varphi}^{B1C} & 0 & 0 & 0 & 0 \\ 0 & 0 & R_{d,\tau}^{B2a} & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{d,\varphi}^{B2a} & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{p,r}^{B2a} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{p,\varphi}^{B2a} \end{bmatrix} \tag{17}
$$

where $\delta_{p,\tau}^{B1C}$, $\delta_{d,\tau}^{B2a}$ and $\delta_{p,\tau}^{B2a}$ are the code phase discriminator of B1C pilot, B2a data and B2a pilot channels, respectively. Further, $\delta_{p,\varphi}^{B1C}$, $\delta_{d,\varphi}^{B2a}$ and $\delta_{p,\varphi}^{B2a}$ are the carrier phase discriminator of the B1C pilot, B2a data and B2a pilot channels, respectively. The variables of $R_{p,\tau}^{B1C}$, $R_{p,\varphi}^{B1C}$, $R_{d,\tau}^{B2a}$, $R_{d,\varphi}^{B2a}$, $R_{p,\varphi}^{B2a}$ and $R_{p,\varphi}^{B2a}$ are co-variances of $\delta_{p,\tau}^{B1C}$, $\delta_{p,\varphi}^{B1C}$, $\delta_{d,\tau}^{B2a}$, $\delta_{d,\varphi}^{B2a}$, $\delta_{p,\tau}$ $\delta_{p,\varphi}^{B2a}$, respectively, which can be calculated directly by:

$$
R_{\text{ii,k}} = \sigma_{\text{ii,k}}^2 = \frac{1}{N_{\text{w}}} \sum_{n}^{N_{\text{w}}} \left(\delta_{k-n} - \overline{\delta}_{k} \right)^2 \tag{18}
$$

where δ_k denotes the discriminator output, $\sigma_{ii,k}^2$ is the cocovariance, N_w is the estimator window length, and $\overline{\delta}_k$ is the average value of the δ_k .

By using (18), the measurement noise covariance of the Kalman filter is calculated according to the signal qualification to get best measurement weights, especially to reduce and even block the attenuated signals.

NCO feedback and sliding average window

The numerically controlled oscillator (NCO) feedback values of B1C and B2a can be calculated as follows:

$$
\begin{cases}\n\text{NCO}^{\text{BIC}}_{f_{\text{IF}}} = \alpha \cdot \delta f_{0,\text{B2a}} + \alpha \cdot \delta a_{\text{B2a}} \cdot T \\
\text{NCO}^{\text{BIC}}_{\phi} = \delta \phi_{0,\text{BIC}} + \alpha \cdot \delta f_{0,\text{B2a}} \cdot T + \frac{1}{2} \cdot \alpha \cdot \delta a_{\text{B2a}} \cdot T^2 \\
\text{NCO}^{\text{BIC}}_{f_{\text{code}}} = \beta_1 \cdot \text{NCO}^{\text{B2a}}_{f_{\text{IF}}} \\
\text{NCO}^{\text{BIC}}_{\tau} = \delta \tau_{\text{BIC}}\n\end{cases} \tag{19}
$$

$$
\begin{cases}\n\text{NCO}^{B2a}_{f_{IF}} = \delta f_{0,B2a} + \delta a_{B2a} \cdot T \\
\text{NCO}^{B2a}_{\phi} = \delta \phi_{0,B2a} + \delta f_{0,B2a} \cdot T + \frac{1}{2} \cdot \delta a_{B2a} \cdot T^2 \\
\text{NCO}^{B2a}_{f_{\text{code}}} = \beta_2 \cdot NCO^{B2a}_{f_{IF}} \\
\text{NCO}^{B2a}_{\tau} = \delta \tau_{B2a}\n\end{cases} (20)
$$

where NCO $_{f_{IF}}^{B2a}$ and NCO $_{\phi}^{B2a}$ are the NCO values of B2a's carrier frequency and carrier phase, respectively, NCO^{B2a}_{fcode}

Fig. 4 Diagram of the sliding

average window

and NCO^{B2a} are the NCO values of B2a's code frequency and code phase, respectively, $NCO_{f_{IF}}^{BIC}$ and NCO_{ϕ}^{BIC} are the NCO values of B1C's carrier frequency and carrier phase, respectively, and NCO_f^{B1C} and NCO_{τ}^{B1C} are the NCO values of B1C's code frequency and code phase, respectively.

In addition, as discussed before, the total integration time of B1C pilot, B2a pilot, and data channels can be chosen to be the same and long time to improve the tracking sensitivity, especially for attenuated signals. But a longer integration time will reduce the update interval of the loop and increase the latency of the estimation, which would further reduce the dynamic and robustness of the loop. Therefore, a sliding average window is introduced to increase the update rate of the loop while keep the integration time of the loop (Borio and Lachapelle [2009;](#page-13-2) Borio et al. [2009](#page-13-3)). The diagram of the sliding average window is shown in Fig. [4.](#page-6-0)

As shown in Fig. $4, T_{int}$ is the total integration time of all channels, and T_{loop} is the update interval of the loop. At T_k time, the correlator outputs or their squaring values from $T_k - T_{int}$ to T_k will be used as the discriminators' inputs so as to generate the measurements of the joint Kalman flter. The NCOs will be updated in the following with the update interval of T_{loop} and wait for the next time T_{k+1} to be updated again, namely $T_{k+1} = T_k + T_{loop}$. This means that the measurements of the Kalman flter are generated with the correlator outputs or their squaring values and the length is always T_{int} , i.e., the sliding average window length. Meanwhile, the loop updates with a relatively faster interval of T_{loop} , i.e., the window's sliding step.

Fig. 5 Open-sky station of the static experiment (left) and the experiment platform of the dynamic vehicle experiment (right)

Table 3 Confgurations of the NUT4NT RF front-end

Table 4 The detail BWs of OLC-STL and OLC-DATL

Static test results and discussions

Results and discussions

Static and dynamic feld experiments were conducted at Harbin Engineering University (HEU), China. As shown in Fig. [5](#page-7-0), the BDS-3 signals of the static experiment were collected by a NUT4NT (Amungo [2017\)](#page-13-4) RF front-end board on the roof of the #16 building at HEU on December 19, 2018, while the BDS-3 signals of the dynamic vehicle experiment were also received by the NUT4NT at HEU campus on December 22, 2018. Also, the NovAtel SPAN-CPT navigation system is used as a reference system. The detailed confgurations of the NUT4NT are shown in Table [3](#page-7-1).

Figure [6](#page-7-2) shows the sky plot of the BDS-3 satellites and the C/N_0 values of PRN 23 in the static experiment. All C/N_0 values of B1C and B2a channels are of good quality.

Fig. 7 Block diagram of the AWGN noise with progressively increasing variance added to the open-sky feld static data

Fig. 6 Sky plot of the BDS-3 satellites (left) and the C/N_0 values of PRN 23 (right) of the static experiment

Fig. 8 Range rate and C/N_0 values of PRN 23 for B2a signal only under the continuously stronger AWGN noise

Table [4](#page-7-3) shows the details of the bandwidths (BWs) for OLC-STL and OLC-DATL methods in the static test. Also, the T_{int} is set as 50 ms and the T_{loop} is set as 20 ms for both B1C and B2a's tracking loop for all methods.

Weak signal condition test

As shown in Fig. [7,](#page-7-4) an additive white Gaussian noise (AWGN) with progressively increasing power was artifcially added using Matlab to the open-sky feld static data to further explore the weak signal tracking performance of the proposed algorithm. In this case, the signal C/N_0 of the feld static data is reduced after 2 s of the start of the experiment by a step value of about 2 dB-Hz/s. PRN 23 is chosen as an example here to analyze the performance.

Figure [8](#page-8-0) shows the range rate and C/N_0 values of PRN 23 for the B2a signal using diferent tracking strategies. Compared to the traditional B2a data channel with maximum integration time 5 ms, denoted as (pure data, 5 ms), the B2a data channel with the non-coherent processing method, denoted as (pure data, 50 ms), can extend the total integration time to 50 ms and improve the tracking sensitivity by

Table 5 Carrier loop tracking C/N_0 thresholds of PRN 23

Method	Tracking C/N_0 threshold (dB-Hz)		
	B ₁ C pilot	B ₂ a data	B ₂ a pilot
OLC-STL	14.20	14.42	13.67
OLC-DATL	10.11	14.42	13.67
KF-JTL	8.86	11.13	11.09

Fig. 10 Block diagram of the selective frequency signal with sudden attenuation with AWGN noise added to the open-sky feld static data

about 5.97 dB-Hz and reach closely to the performance of the B2a pilot channel using the pure PLL, denoted as (pure pilot, 50 ms). Finally, the B2a data and pilot channels' linear combination denoted as $(pilot + data, 50 \text{ ms})$ with the noncoherent processing method used in the B2a data channel can get the best tracking sensitivity up to 14.42 dB-Hz. The non-coherent processing method is proven to help extend the integration time and improve the tracking sensitivity, so the B2a data channel in OLC-STL and OLC-DATL is modifed hereinafter with the non-coherent processing in order to provide a fair comparison with the proposed KF-JTL.

Figure [9](#page-8-1) shows the range rate and C/N_0 values of PRN 23 for B1C and B2a signals using diferent tracking methods. The carrier loop tracking C/N_0 threshold statistics of PRN 23 for B1C and B2a signals using diferent methods are summarized in Table [5.](#page-9-0)

It can be seen that the OLC-DATL gets a better tracking sensitivity for B1C signal than the OLC-STL method due to the Doppler aiding information from the B2a tracking loop. But the B1C loop is still limited by the B2a tracking performance and is lost immediately when the B2a loop looses lock. However, in the KF-JTL method, both the B1C and B2a tracking sensitivity are improved distinctly, and the tracking errors of the KF-JTL method are much lower than other methods, especially when the C/N_0 decreases. The results indicate that the KF-JTL can fuse B1C and B2a tracking information together and achieve better tracking sensitivity and robustness compared to OLC-STL and OLC-DATL methods.

Selective frequency signal attenuation test

As shown in Fig. [10,](#page-9-1) in this test, either B1C signal or B2a signal is deeply attenuated in a selective frequency way. This is performed by adding considerable Matlab AWGN noise suddenly to the open-sky feld static data. This causes sudden C/N_0 drops in different periods of the test before signal recovery.

As shown in Fig. [11,](#page-10-0) the B1C and B2a signals are deeply attenuated during the period 3–8 s and 13–18 s, respectively. Also, the C/N_0 values of both B1C and B2a signals are attenuated by at least 34 dB during the attenuation period.

In the top panel of Fig. [11,](#page-10-0) the B1C loop looses lock in the OLC-STL method when the B1C signal attenuation occurs, and the B2a Doppler errors increase vastly during B2a signal attenuation. In the middle panel, the B1C loop keeps lock with the aiding of the B2a signal, although the B1C Doppler values are noisier than the un-attenuated signal during B1C signal attenuation. However, both B1C and B2a Doppler errors increase vastly during B2a signal attenuation. In the bottom panel, the Doppler values of B1C and B2a fuctuate slightly at the beginning of their attenuation, but the tracking loops recover rapidly and keep tracking with much smaller noise compared to the traditional methods.

The results indicate that the proposed KF-JTL gets the mutual beneft of B1C and B2a signals and improves the receiver's resistance to selective frequency signal attenuation in either L1 band or L5 band.

Dynamic vehicle test results and discussions

As shown in Fig. [12](#page-10-1), the vehicle starts from an open-sky location and then enters a foliage and residential environment. There are 6 visible satellites in the sky plot.

Table [6](#page-10-2) provides the BWs for OLC-STL and OLC-DATL methods in the dynamic experiment. Also, the T_{int} is set as 30 ms and the T_{loop} is set as 5 ms for all methods.

Figure [13](#page-11-0) shows C/N_0 values of B1C and B2a during the dynamic experiment using diferent methods. In the top panel, the B2a signal achieves better tracking robustness with the OLC-STL method compared to B1C, while the B1C signal gets better tracking by the aiding of the B2a signal using the OLC-DATL method as shown in the middle panel. In the bottom panel, the KF-JTL achieves the best C/N_0 values compared to OLC-STL and OLC-DATL, especially for PRN 29 and 35, which can keep tracking until the end of the dynamic experiment.

Figure [14](#page-11-1) shows carrier and code Doppler values of B1C and B2a using diferent methods for PRN 35. The B2a signals achieve better tracking robustness performance compared to B1C signals in both OLC-STL and OLC-DATL methods, while the B1C tracking accuracy gets better by **Fig. 11** Range rate (left) and Doppler error values (right) of PRN 23 for B1C and B2a under the selective frequency signal attenuation test using OLC-STL (top), OLC-DATL (middle) and KF-JTL (bottom) methods

Fig. 12 Trajectory (left) and the sky plot of the BDS-3 satellites (right) in the dynamic vehicle experiment

Table 6 Detail BWs of OLC-STL and OLC-DATL

the aiding of the B2a signal in the OLC-DATL method, especially for the code Doppler. Compared to OLC-STL and OLC-DATL methods, the KF-JTL method gets the best tracking performance, especially for the lower C/N_0 signals during the period of 75–100 s, which indicates that the KF-JTL achieves the best tracking accuracy and robustness performance, especially in the challenging environment.

Fig. 13 C/N_0 values of B1C and B2a signals during the dynamic experiment using OLC-STL (top), OLC-DATL (middle) and KF-JTL (bottom) methods

Fig. 14 Carrier Doppler (top) and code Doppler (bottom) values of B1C (left) and B2a (right) for PRN 35 during the dynamic experiment

Table [7](#page-12-0) gives the statistics of the number of times loss of locks occurred for all satellites. The instances of loss of B1C or B2a carrier loop lock are counted from the beginning of the experiment. The total tracking time is 100 s, and N/A indicates that the satellite keeps lock until the end of the test.

Figure [15](#page-12-1) shows the numbers of satellites generating measurements during the dynamic experiment. The start time is BDS time 523,170.1 s, and the total number of PRNs at the beginning is 6. When the vehicle enters the foliage and residential area, the number of available satellites in the OLC-STL method gradually decreases and becomes less than 4 satellites at about 66.3 s. The number of available satellites in the OLC-DATL method is less than 4 at about **Table 7** Statistics of the time of loss of locks for all satellites

L

Fig. 15 Number of satellites generating measurements during the dynamic experiment

Fig. 16 Position errors of the dynamic vehicle experiment

Fig. 17 Velocity errors of the dynamic vehicle experiment

79.8 s, while the proposed KF-JTL keeps at least 4 satellites most of the time.

The position and velocity errors using diferent methods are shown in Figs. [16](#page-12-2) and [17,](#page-12-3) respectively. It can be seen that during the frst 66.3 s which is with relatively good propagation conditions, the KF-JTL method gets smaller position and velocity errors compared to the OLC-STL and OLC-DATL methods. After 66.3 s, the number of satellites in the OLC-STL method is lower than 4 satellites; hence, no navigation solutions are generated. The navigation errors increase vastly, and the navigation solutions are outputted until about 79.8 s, in the OLC-DATL method. However, the KF-JTL method can work until the end of the test with a relatively slight increasing error compared to OLC-DATL.

The navigation error statistics shown in Table [8](#page-13-5) indicate that the KF-JTL method reduces the position errors about

Table 8 RMS position and velocity error statistics

RMS errors		Method			
			OLC-STL	OLC-DATL	KF-JTL
Position errors (m)	Before 66.3 s	δE	1.23	1.01	0.94
		δN	0.89	0.80	0.79
		δU	2.76	2.48	1.78
	After 66.3 s	δE		34.53	1.08
		δN		13.99	3.69
		δU		98.86	3.98
Velocity errors (m/s)	Before 66.3 s	δV_E	0.26	0.26	0.14
		δV_N	0.54	0.54	0.26
		δV_U	0.78	0.78	0.37
	After 66.3 s	δV_E		8.58	1.41
		δV_N		5.48	1.66
		δV_U		27.68	3.78

24%, 1% and 36% compared to the OLC-STL method, 7%, 0% and 28% compared to the OLC-DATL method, and reduces the velocity errors about 46%, 52% and 53% (OLC-STL method), 46%, 50% and 53% (OLC-DATL method) in east, north and up directions, respectively, during the frst 66.3 s. During 66.4–86 s, which covers the severe foliage and residential area, the KF-JTL method reduces the position errors about 94%, 74% and 96% and velocity errors around 84%, 70% and 86% compared to the OLC-DATL method in east, north and up directions, respectively.

The dynamic vehicle experiment results dedicate that the proposed KF-JTL method achieves an optimal availability, accuracy and robustness performance in both tracking and navigation compared to traditional OLC-STL and OLC-DATL methods.

Conclusions and future work

An adaptive Kalman filter-based B1C/B2a joint tracking architecture (KF-JTL) for the BDS-3 dual-frequency receiver is proposed to improve the tracking sensitivity and robustness, especially in challenging environments. The proposed KF-JTL uses a non-coherent processing architecture to extend the integration time for B2a data channel to avoid the limitation of the navigation symbol bits and compensates the diference of the tracking sensitivity between B2a data and pilot channels. A joint Kalman flter has been equipped with common carrier Doppler frequency and acceleration error states to fuse signals of B1C pilot channel and the B2a data/pilot channels using longer integration time, adaptive measurement noise covariance and a sliding average window to get the mutual beneft between these two signals. The feld experiment results show that, compared to the modifed traditional OLC-STL and OLC-DATL architectures, the proposed KF-JTL can achieve optimal availability, accuracy and robustness performance in both tracking and navigation solutions.

In the future, more feld experiments should be conducted given the rapid development of the BDS-3 system. Also, some other non-coherent processing technologies such as the power diference (PD) non-coherent discriminator can be used in the Kalman flter-based B1C/ B2a joint tracking architecture to further extend the tracking performance, especially in dense fading environments.

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