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ORIGINAL ARTICLE

Study on GPS–PPP precision for short observation sessions

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Abstract Precise point positioning is increasingly being used in geodetic applications that in many cases are based on static 24-hour RINEX files. Since there are many applications where sub-centimeter position accuracy is not required and users wish to use a single receiver and not be dependent on differential correction, we will evaluate PPP performance for static positioning with 12-, 6-, 3-, 1- and ½-h observations. We have, therefore, considered a dataset for the year 2013 from 14 European GNSS stations. The data were analyzed using GIPSY-OASIS II software package and evaluated in terms of repeatability of the coordinates and of coherence with the formal error indicated for each PPP solution. Particular attention was paid to solutions showing large discrepancies in coordinates. The test shows that PPP precision for the 24-h files is below 5 mm, but decreases slightly for the 12-, 6- and 3-h observation sets. For the 1-h and the ½-h RINEX files, precision is within 5 and 10 cm, respectively. The analysis is completed with a discussion on the impact of the ambiguity resolution that shows how it significantly improves only the easting component and moreover has a higher influence on the formal error rather than on the solutions. Lastly, the study contains an investigation into the reliability of the formal error associated with the PPP solutions. We show that the formal error can be used to identify incorrect solutions, but is not suitable to represent the real accuracy. For that reason, we propose to use the formal error given for the float solutions even for the ones with fixed ambiguities.

Keywords PPP · GIPSY-OASIS · Short observation period · GPS

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Introduction

Precise point positioning (PPP; Zumberge and Heflin 1997) has become a method for obtaining precise positioning using GNSS reaching centimeter accuracy. This approach means that the coordinates of a single GNSS receiver can be calculated with respect to the reference system of the satellite orbits used in the processing. Since the PPP solutions are independent of the relative distances, this approach is becoming increasingly popular: Many online web services that allow automatic GNSS data processing using PPP have been developed over recent years. These include Automated Precision Positioning Service (APPS by JPL—http://apps.gdgps.net/), Canadian Spatial Reference System-Precise Point Positioning (CSRS-PPP—http://webapp.geod.nrcan.gc.ca/), magicGNSS PPP (produced by GMV—http://magicgnss.gmv.com/ppp/), GNSS Analysis and Positioning Software (GAPS from the University of New Brunswick http://gaps.gge.unb.ca/). By analyzing a time series of static PPP solutions, it is possible to find a repeatability of less than 5 mm for the horizontal plane (Gandolfi et al. 2016; Kouba and Héroux 2001), but these results are obtained using 24-hour Receiver Independent Exchange Format (RINEX) files.

Having less accurate applications in mind, we have investigated the PPP performance in terms of repeatability of the solutions, considering observation sets with a time span of less than 24 h. The idea was to evaluate what one would obtain using a single stand-alone receiver, without any augmentation services or geodetic infrastructure. This issue has been addressed in previous papers; for example, Héroux et al. (2001) found that at least 90 min is needed for a PPP solution to converge to 1 dm. Gandolfi et al. (2005) have worked with a dataset composed of data gathered over 3 years from a single permanent GPS station located in Antarctica and data collected over six months from six stations located in Italy. These data were processed through GIPSY-OASIS II v.4 software, with no attempt to resolve ambiguity. In that particular study, a PPP solution repeatability of under 10 cm was found for the 1-h solutions, while for observation sets longer than 3 h, repeatability was under 2 cm. Ghoddousi-Fard and Dare (2006) found sub-decimeter level precisions for hourly position estimates in PPP, while reaching centimeter level precision for the 4-hourly position estimates.

Jet Propulsion Laboratory (JPL) has recently implemented a method to handle phase ambiguity resolution (wide-lane and phase bias—WLPB; Bertiger et al. 2010) using GIPSY version 6.1 software, which should influence the results in the case of shorter observation times, as demonstrated for a different software packages developed by Geng et al. (2009). There is evidence (Collins et al. 2008) that the impact on ambiguity resolution in PPP is higher for shorter observation times than for the 24-hour files.

A subject similar to one covered here is also discussed in Abdallah and Schwieger (2015), involving a single day for each of four German stations and different PPP software packages. In a comparison between PPP and the differenced approach using the Bernese software package, Soycan and Alta (2011) tested observation times from 1 to 24 h on a dataset collected over seven days from 60 stations, obtaining precisions at the centimeter level for observation times of at least 3 h.

Here, we used a similar approach to those mentioned above. In this case, we have considered a much larger dataset in terms of time and number of stations, in the form of daily observations for 1 year from 14 European Permanent Network (EPN) class A stations located in and around Italy (Fig. 1). This allowed us to obtain more reliable statistics and also find some particular aspects that can be properly evaluated only by using a very consistent dataset. We used GIPSY-OASIS II version 6.3 software, because of its ambiguity resolution capacity in the PPP calculation. We should bear in mind that not only has the ambiguity resolution recently been introduced to the GIPSY PPP processing procedures but that new models and new products, such as precise orbits, are now available even for calculations where ambiguity resolution is not included. A comparison between the two solutions, before and after the ambiguity resolution, will therefore be provided to evaluate the actual impact of WLPB, depending on the observation time window. For each station, we considered 1 year of 30-second data supplied as daily RINEX files for 2013. Data for less than 24 h were obtained by splitting the daily RINEX files into 12-, 6-, 3-, 1- and ½-h sections. In order to evaluate the PPP performance, we considered the times series scattering with respect to a reference model for each station, which represents an estimation of the repeatability of the measurements. We evaluated the formal error given by the software, keeping in mind that this will be the only tool available to the user for judging the quality of a single PPP solution.

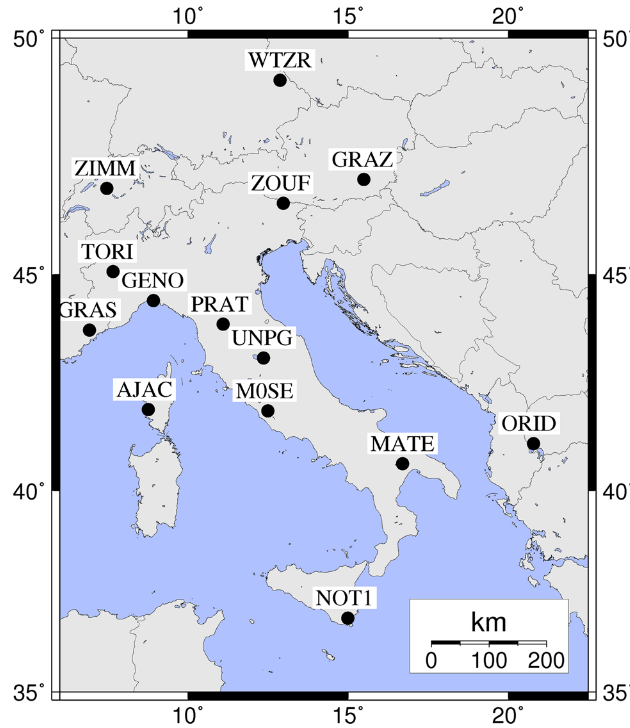

Fig. 1 Location of the EPN permanent stations considered in the test

Dataset and PPP calculation

Each daily 30-second RINEX file was split into several shorter sections using the TEQC software (Estey and Meertens 1999). This meant that, from every daily RINEX file, we obtained two 12-h files, four 6-h files, eight 3-h files, twenty-four 1-h files and forty-eight ½-h files. These files simulate the measuring sessions of the different observation times. A summary of the considered dataset is given in Table 1.

Table 1 Summary of the process to split RINEX files containing data for 1 year for each GNSS station

For each station	
RINEX time span (h)	No. of RINEX files
24	365
12	730
6	1460
3	2920
1	8760
½	17520

Each of the RINEX files in the dataset was processed through a PPP approach, using the GIPSY-OASIS II software package, version 6.3 (Webb and Zumberge 1997). Below are the options selected for the test, and most are the default parameters suggested by JPL for GIPSY users:

- Orbit and clock products: non-fiducial precise FlinnR orbits from JPL, including information to enable single receiver phase ambiguity resolution using GIPSY-OASIS software (WLPB).
- Antenna phase center variation: IGS absolute phase center calibration file (igs08.atx).
- Cutoff angle for observations: 7°.
- Data rate: 300 s.
- Smoother option: static solution.
- Number of iterations for ambiguity resolution: 1.
- Tide models: solid earth tide (WahrK1, FreqDepLove; Wahr, 1985), polar tide model (PolTid) and ocean tide model (OctTid)—GIPSY default option.
- Tropospheric model: VMF-1 (Kouba 2008).
- Troposphere estimation parameters: random walk, set to 3 [mm/sqrt(h)] with wet gradient set to 3.6 [mm/h]—GIPSY default option.

The aim of this work is to evaluate the precision of the solutions depending on the time of observation under normal field survey conditions. Typically, the receivers used in these surveys are dual-frequency geodetic receivers, equipped with standard clocks. Although the WTZR station features a hydrogen maser atomic clock, we processed data from this station using the same standard parameters, without introducing different weightings for the receiver clock errors. We used the EPN GNSS stations to ensure the use of a consistent dataset and reliable statistics.

The solutions obtained directly from the GIPSY data processing using JPL non-fiducial orbits are not strictly aligned to the IGb08. These solutions were, therefore, aligned to the IGb08 solution using the X-files provided by JPL.

Post-analysis of the solutions

A reference is needed to evaluate the repeatability of the solutions, and we assumed, for this purpose, both the regression line of the time series and also the model of seasonal movements. The post-processing procedure used to calculate the coordinate differences of each PPP solution with respect to the chosen reference is described below. Hereafter we call these coordinate differences simply "residuals," i.e., the differences between the PPP solutions and the calculated reference models in terms of coordinates.

First, the geocentric coordinates were transformed into a local geodetic coordinate system, in order to obtain results in terms of northing (n), easting (e) and up (u) (Leick et al. 2015), as these are easier to interpret. For each of the 14 GNSS stations, we denote $S^{*}_{kj}(t)$ to be the value of the geodetic component (k) of a solution (j) at the epoch t, where $k = n,\ e,\ u$; $j = 1\ldots m$ with m being number of solutions. To prevent the influence of discontinuities in the time series of the coordinate components, we removed the official EPN_A_IGb08.SNX reference solutions from our solutions. In order to achieve this, the reference solution was converted to the same local geodetic coordinate system used for the S^{*}_{kj} solution. The difference with respect to IGb08 is given by:

$$S_{kj}(t) = S^{*}_{kj}(t) - \mathrm{REF}_k(t) \quad (1)$$

where $\mathrm{REF}_k(t)$ is the reference value of the k component for each site at the epoch t.

The linear definition of the IGb08 reference solution is known to be a simplification of the point position measured over time, since a point may have a periodical motion due to local effects on the ground. The repeatability of the measures should be evaluated with respect to the point positions at the epoch of measurement. This means that if only the linear trend of the coordinates is considered, that may affect the evaluation of the repeatability, especially for the more precise measures. For that reason, a model of the movement of the point was calculated for each site, so that it could be used as reference position in the calculation of the residuals of the measured coordinates.

In order to define a reference model of the time series for each site, only the solutions derived from the 24-hour RINEX files were considered and even data from 2012 and 2014 were calculated and taken into account in addition to the ones of year 2013. Linear regressions on n, e and u coordinates were computed using a classical weighted least squares approach, with the weight being the inverse of the formal error derived from the data processing. We defined m_k and q_k as the slope and the intercept of the regression line. We used the Lomb–Scargle Periodogram (LSP; Lomb

1976; Scargle 1982) to estimate the most powerful frequency f_{k1} of the time series $S_{kj}(t)$ in a frequency domain between 1 month and 2 years. Let A_{k1} and B_{k1} be the coefficients of the sine wave associated with this frequency, as estimated by least squares. The sine wave was removed from the $S_{kj}(t)$ time series, and the procedure was iterated for the first five frequencies to obtain a reference model $\mathrm{mod}_k(t)$ calculated as:

$$\mathrm{mod}_k(t) = q_k + t(j) * m_k + \sum_{i=1}^{5} [A_{ki} \sin(2\pi f_{ki} * t) + B_{ki} \cos(2\pi f_{ki} * t)] \quad (2)$$

Our choice of using five frequencies came from the need to represent waves that may have different shapes than a sinusoid. The residuals v_{kj} of each PPP solution relative to the reference model, derived from the 24-hour solutions, were then computed as:

$$v_{kj} = S_{kj}(t) - \mathrm{mod}_k(t) \quad (3)$$

These residuals will be the subject of the later discussion and represent what we assume to be the "real" error affecting each PPP solution.

Figure 2 shows the models (2) obtained for each station. Evidence from the graphs shows that the amplitude range of the waves is within 2 mm in the horizontal plane and 4 mm on the height; these signals mostly cover a one-year period. The wave shapes have long periods and are different for each site, thus probably describing a movement of the points rather than the movements of the solutions obtained by means of the GNSS technique.

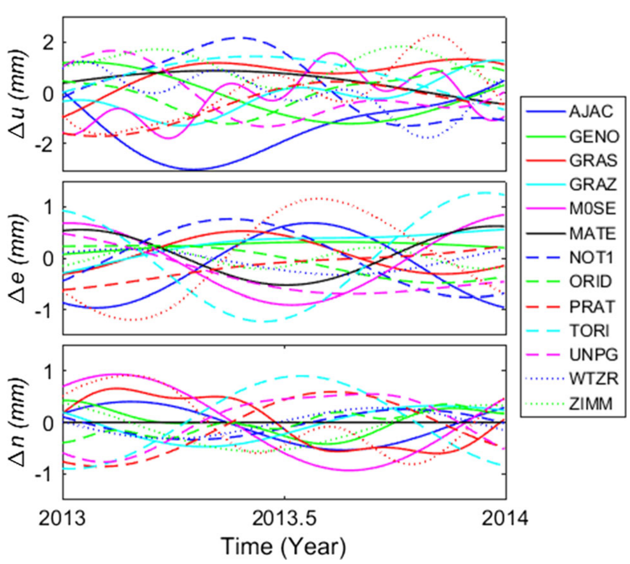

Fig. 2 Periodical signals estimated using the first five most powerful frequencies using LSP

Test results

First of all, the value of the mean of the differences (1) with respect to the IGb08 was calculated in order to evaluate the consistency with respect to the reference frame as a function of the observation period. Table 2 contains the mean values of the differences for the 14 stations included in the test. Looking at the values for the longer observation sessions, these can be considered quite small and coherent to what is reported in Gandolfi et al. (2016) for a different analyzed period. Of course, these values have to be further discussed after looking at the scattering of the solutions, which is expected to be higher for the shorter observation sessions. Nevertheless, even for observation sessions of 1 h, the solutions are not biased with respect to IGb08.

The time series of the residuals v_k were prepared for the three geodetic components of each station and the six different time spans defining the PPP solutions. Figure 3 shows an example of the time series of the residuals for station WTZR, with a superimposition of the solutions relating to the different observation times using color coding. The residuals derived from the shorter observation times are more scattered, as is expected, while the solutions from the 24-h observations confirm a repeatability of less than 1 cm.

For all the time series and, in particular the ones with shorter observation times, there are clearly some very large residuals. Our first question concerns the possibility of highlighting these outliers by looking at the formal error of each PPP solution. We then thought about the problems that could occur if an outlier cannot be recognized by looking at the formal error, since this is the only tool that a user has for estimating the quality of a single measurement. After rejecting the outliers, we evaluated the precision that is obtainable from the PPP approach relative to the observation time, and we then addressed the problem of actual reliability and suitability of the formal error to act as an estimator of the achieved precision of the solutions.

Table 2 Mean of the differences S_{kj} with respect to the IGb08 reference solutions for the different observation times

Obs. time (h)	Mean of the differences S_{kj} with respect to the IGb08 reference solutions (mm)		
	n	*e*	*u*
½	2.7	13.0	6.7
1	2.4	3.7	5.6
3	2.5	1.0	4.8
6	2.5	0.8	4.9
12	2.7	0.9	4.7
24	2.8	0.9	4.6

Values are average of the 14 GNSS stations of the test

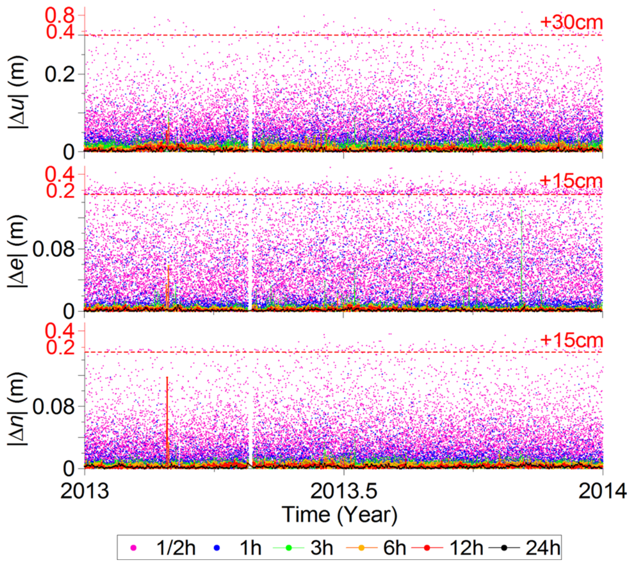

Fig. 3 Superimposition of the time series of the residuals (absolute value) for the WTZR station expressed in the local topocentric reference system. Different *colors* indicate different observation times. For each of the three components, the y-axis is divided into two parts where the bottom one (*black*) has a larger spacing than the top one (*red*) in order to better represent the less scattered solutions. The *red dashes* represent the separation line between the two areas of each graph

Outlier solution analysis

We have assumed that a solution with a residual relative to the reference solution greater than 30 cm is not suitable for any technical application. This is quite an arbitrary assumption based on experience, which is the only criterion we found to determine a threshold over which to reject the solutions without making other assumptions about the final purposes of the survey. We, therefore, considered 30 cm as the threshold for identifying outliers.

A user who can make use of the formal error given by GIPSY in the covariance matrix of the solutions, hereafter called σ_{ppp}, may consider taking $3\sigma_{ppp}$ as the limiting value below which the residuals will fall with roughly a 99 % probability. This should be true under the hypothesis of a normal distribution of the errors, if σ_{ppp} represent the real precision of the solution. We, therefore, sought all the solutions with a formal error of more than 10 cm in at least one geodetic component, meaning that there is the formal certainty of having a residual of less than 30 cm. The results are given in Table 3, divided according to the different observation times.

The percentage of the outliers is quite low for every kind of solution obtained with at least a 1-h observation time, but it is evident that the values increase greatly for solutions with an observation session of ½ an hour. For station NOT1, the percentage of outliers is significant even when considering the 1-h solutions; this will be later discussed. We removed these outliers from the time series, and then we looked for the percentage of solutions that still have a residual of greater than 30 cm. These solutions may be a major problem because they are not detectable by a surveyor who only the single measure has available. As shown in Table 4, among the solutions with a formal error of less than 10 cm, only 0.5 % have a residual relative to the reference of more than 30 cm, and that occurs only in the case when the observation times is 1 h and less. We also removed these outliers from the time series of the residuals before continuing with the evaluation of the results. Once again, station NOT1 had the worst results compared to the other stations, with a 1.2 % probability of having a wrong solution that cannot be recognized by looking at the formal error.

Regarding the detailed information about station NOT1 available on the EPN website (http://www.epncb.oma.be/), some features are evident by looking at the RMS graph of the L2 code multipath and, in particular, at the number of cycle slips compared to the other stations for the year 2013. In order to focus on this aspect, we calculated the number of daily cycle slips through TEQC software. Figure 4 shows the daily cycle slips together with the residuals of solutions for 1-h time span. There is a clear correlation between the presence of cycle slips and the high residuals of the solutions. Moreover, a high number of cycle slips have no effect on the solutions with long observation times, but do have a strong impact on the solutions of 1 h or less. This is clear when considering the convergence time

Table 3 Percentage of PPP solutions with a formal error greater than 10 cm in at least one geodetic component

Obs. time (h)	AJAC	GENO	GRAS	GRAZ	M0SE	MATE	NOT1	ORID	PRAT	TORI	UNPG	WTZR	ZIMM	ZOUF	Mean value
24	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.1	0.1	0.0	0.0	0.1
3	0.0	0.0	0.0	0.0	0.0	0.0	2.1	0.0	0.0	0.0	0.1	0.1	0.0	0.0	0.2
1	0.0	2.3	0.0	0.1	0.2	1.0	13.5	0.0	0.1	0.9	0.1	0.1	0.0	0.1	1.3
½	41.3	86.0	45.3	33.8	39.0	73.0	88.7	43.7	65.6	76.4	55.8	28.7	44.5	55.8	55.5

Table 4 Percentage of PPP solutions with a residual relative to the reference greater than 30 cm despite the formal error being within 10 cm

Obs. time (h)	AJAC	GENO	GRAS	GRAZ	M0SE	MATE	NOT1	ORID	PRAT	TORI	UNPG	WTZR	ZIMM	ZOUF	Mean value
1	0.2	1.0	0.1	0.1	0.2	0.6	1.2	0.1	0.5	0.4	0.3	0.0	0.2	0.3	0.4
½	0.4	0.8	0.4	0.3	0.4	0.3	0.3	0.3	0.8	0.7	0.3	0.4	0.2	0.9	0.4

The statistics concerning observation times longer than 1 h are not reported because no undetectable outliers were found

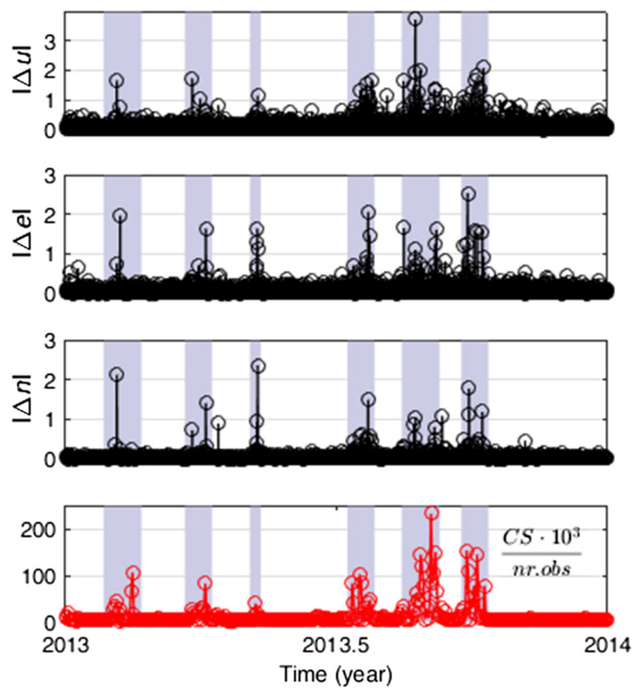

Fig. 4 Correlation between residual solutions and cycle slips. The *top three graphs* show the absolute values of the residuals for the 1-h solutions of station NOT1 expressed in meters. The *bottom graph* shows the daily number of cycle slips*10^3 normalized with respect to the number of observation

needed for the PPP to solve the phase ambiguities and the segmentation of the observing periods caused by cycle slips.

Precision compared to observation time

For each time series v_k of residuals relative to the reference, which were cleaned of identified outliers, we calculated the standard deviation σ. These results are given in Table 5, expressed in millimeters. The high precision of PPP is confirmed for the solutions where the observation time is longer than 3 h, and even in the case of 1-h observation solutions the scattering of the time series remains under 5 cm in the plane.

For solutions involving ½-h observation times, the repeatability is worse, but still remains under 10 cm for all components. We need to mention that this result has been derived from only a portion of all solutions, because almost half of the solutions were rejected during the previous phase.

Especially for the shorter observation times, it is evident that the easting component is weaker than the northing component, probably due to the geometry of GPS constellation and in agreement with Yigit et al. (2014). A summary is given in Fig. 5, showing the mean of the RMS of the 14 time series for each observation time.

Precision compared to the PPP formal error

In order to evaluate the reliability of the formal error given by GIPSY, we compared the values of the residuals v_{kj} with the PPP formal error σ_{ppp}. The percentage of solutions with an underrated formal error is given in Table 6, meaning that these are solutions with a residual greater than $3\sigma_{\text{ppp}}$. The values are then averaged over the whole network and divided according to the topocentric components.

The PPP formal error is underrated in about the 40 % of cases, and the error becomes more reliable for shorter observation times, where there are fewer epochs processed and the lower rate of redundancy in the equations is taken into account. By analyzing the solutions with an underrated error, there is evidence that the residual value may be ten times that of $3\sigma_{\text{ppp}}$ or even greater. This means that the formal error cannot be trusted as an estimation of the real precision of the solutions with anything like enough confidence. Figure 6 (left) shows the percentage of solutions that have a residual v_{kj} within the value of $n\sigma_{\text{ppp}}$, with $n = \frac{v}{\sigma}$. Under the hypothesis of a normal distribution of the errors, if σ_{ppp} represents the real RMS of the solution, the percentages of Fig. 6 should be close to 99.73 % when $\frac{v}{\sigma} \geq 3$, but this condition is verified only for the ½-h solutions. For the longer observation times, there are still 10–20 % of solutions with a residual greater than seven times the related formal error. We must remember that we are still considering only the solutions "cleaned" by the outliers, described above. We stress that the real precision of the PPP is shown above and that the formal errors are clearly underrated for the less scattered solutions, but less so for those with more scattering. Thus, for many cases of positioning applications where a positional precision of 10 or 20 cm is acceptable, this will not be a problem and the formal error can be used to detect the outliers as shown above.

Table 5 RMS of the time series of the residual relative to the models cleaned of the outliers

SITE	24 h			12 h			6 h			3 h			1 h			1/2 h		
	σn	σe	σu	σn	σe	σu	σn	σe	σu	σn	σe	σu	σn	σe	σu	σn	σe	σu
AJAC	2	2	6	4	3	7	4	3	10	5	7	15	17	39	48	28	57	60
GENO	2	2	6	4	3	8	5	4	12	7	8	19	24	49	68	31	69	75
GRAS	3	2	5	4	4	6	5	6	10	7	10	16	16	35	44	27	55	61
GRAZ	2	2	5	3	2	6	4	4	9	5	6	12	16	33	40	33	66	67
M0SE	2	2	5	4	4	8	4	5	11	5	8	15	16	36	42	31	60	64
MATE	2	2	5	3	2	7	4	3	9	5	8	16	17	38	55	26	53	66
NOT1	2	3	8	4	5	11	6	8	18	8	13	26	21	49	70	24	61	65
ORID	2	2	6	5	5	11	5	5	12	6	10	16	14	32	41	22	50	54
PRAT	2	2	5	4	3	7	4	3	10	6	7	15	22	48	61	34	67	76
TORI	2	2	6	3	3	8	4	4	11	6	8	17	20	40	56	32	67	75
UNPG	2	2	5	4	11	11	4	4	10	6	6	14	18	37	49	29	60	66
WTZR	3	2	5	4	3	8	5	4	11	6	6	16	18	37	41	41	71	75
ZIMM	4	3	5	5	4	6	5	5	9	6	6	12	15	30	37	27	53	57
ZOUF	3	2	6	4	3	7	5	4	10	7	9	17	24	43	57	40	74	80
Mean value	2	2	5	4	4	8	5	4	11	6	8	16	18	39	51	30	62	67

Values are in mm

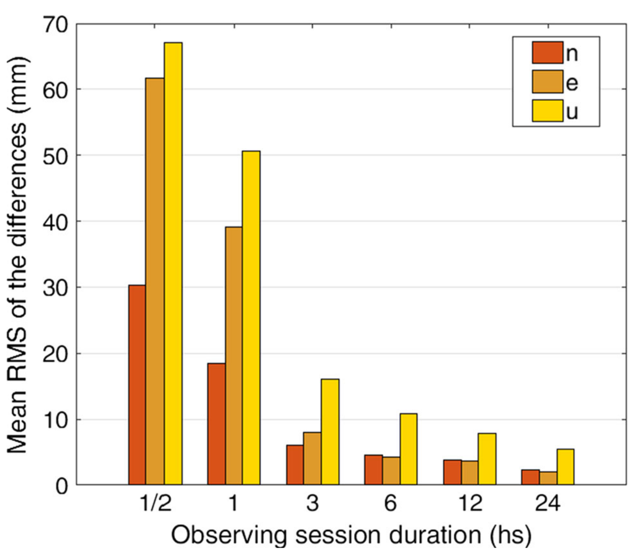

Fig. 5 RMS of the time series of the residuals relative to the reference models after rejection of the outliers. The x-axis shows the length of the observation set, and the y-axis shows the RMS expressed in mm

Impact of the phase ambiguity resolution

In order to produce a PPP solution with fixed phase ambiguity, the GIPSY software package in a previous instance produces a solution with a float phase ambiguity and related covariance matrix. This means that it was possible to repeat the same analysis process shown above considering the same PPP solutions but now without the ambiguity resolution. The main results are summarized in Table 7.

Table 6 Percentage of the PPP solutions with a residual value relative to the reference model that is greater than three times the formal error σ_{ppp}

Obs. time	Solution with a residual v_{kj} > than $3\sigma_{ppp}$		
	n (%)	*e* (%)	*u* (%)
0.5	1	1	1
1	22	24	19
3	31	28	24
6	41	35	28
12	51	40	31
24	47	44	36

In this case, the identification of the outliers produces results that are substantially equal to those described above. Looking at the precision of the time series of the residuals, it appears that the phase ambiguity resolution has a major impact on the easting component, while it is almost negligible on the northing and up components. For the easting component, the RMS improves from 1 mm for the 24- and 12-h solutions, to 3 mm for the 6-h solutions and to 5 mm for the 3- and 1-h solutions. For the ½-h solution, there is a 3 mm improvement to the RMS that depends on the ambiguity resolution, but it must be remembered that these results are calculated on a dataset cleaned of outliers, which are also in this case more than 50 % for this observation time span.

If we define $\sigma_{ppp\_amb\_float}$ the formal error given by GIPSY for the PPP solutions calculated with float ambiguities, the

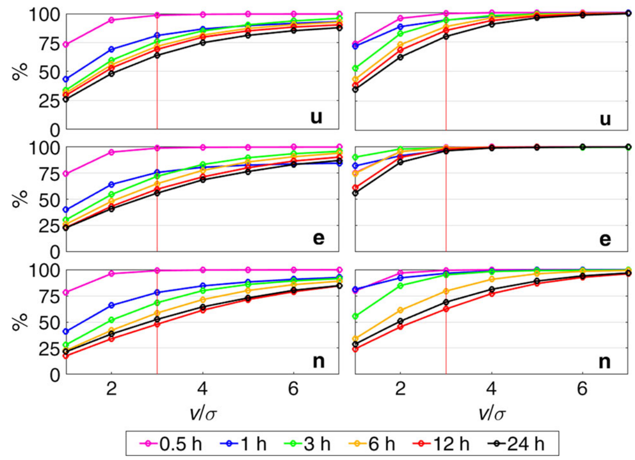

Fig. 6 Cumulative percentage of the solution having the v/σ ratio reported on the x-axis. All the residuals v are related to solutions calculated with fixed phase ambiguities, whereas in the *left panel* the σ values are related to σ_{ppp} but in the *right panel* such values are related to $\sigma_{ppp\_amb\_float}$

percentage of the solutions with a residual of more than $3\sigma_{ppp\_amb\_float}$ is reported in the last three columns of Table 7. In most cases, and especially for the easting component, these percentages are lower than those in Table 6 that is related to the solution with fixed phase ambiguity.

The ambiguity-fixed PPP solutions are, therefore, more precise, but the solutions with float ambiguities have a more reliable formal error. It seems that the ambiguity resolution could lead to overrating the formal precision. For this reason, it may be a better practice to associate the formal error $\sigma_{ppp\_amb\_float}$ also to the ambiguity-fixed solutions. Having carried this out, the results in terms of percentage of solutions with residual v_{kj} greater than $3\sigma_{ppp\_amb\_float}$ are reported in Table 8. These percentages show that the formal error of the solutions with float ambiguity is significantly more reliable in terms of estimating the precision of a ambiguity-fixed PPP solution calculated, especially when considering shorter observation times and, in particular for the Easting component.

Figure 6 (right) reports the percentage of solutions with a residual v_{kj} within the value of $n\sigma_{ppp\_amb\_float}$, where again $n = \frac{v}{\sigma}$. By comparing the two panels, it is evident that using the formal error $\sigma_{ppp\_amb\_float}$ instead of σ_{ppp} leads to a more reliable estimation of the real precision of the PPP solutions.

We are well aware that associating a covariance matrix of the ambiguity-float solution to the ambiguity-fixed solution is not a rigorous approach. Ambiguity resolution definitely improves the real accuracies of the solutions, but seems to have a high impact on the covariance matrix, thus leading this to being underrated.

Discussion and conclusions

Our goal was to evaluate the performance of PPP when using observations of less than 24 h. Different time spans were considered, decreasing observation length down to half an hour. The dataset was constituted by 1 year of observations from 14 permanent stations located in and around Italy, using for each station 1 year of 30-second data in daily RINEX files. Each daily RINEX was split into sections of 12, 6, 3, 1 and ½ h. Each was processed through a PPP approach by using the GIPSY-OASIS II software package.

First, we highlight the good consistency of the PPP solutions with respect to the reference frame for all the considered observation times. The mean of the residuals with respect to the formal solution IGb08 is at the millimeter level for all the time series and reaches the centimeter level only for observation sessions for which the scattering of the solutions is heavy.

Subsequently, for each station a reference model of the "real" position was calculated using the 24-h solutions, taking into account both trend and seasonality. The residual of the coordinates compared to the models was assumed to be the actual errors of the solutions. In terms of repeatability, which is given here in terms of the RMS of the time

Table 7 Summary of the test results obtained by analyzing only the PPP solutions with float ambiguity

Obs. time (h)	PPP solutions with a formal error >10 cm (%)	PPP solutions with $v > 30$ cm but sigma < 10 cm (%)	RMS of the time series of the residuals relative to the models (mm)			PPP solutions with a residual value $>3\sigma_{ppp}$		
			n	*e*	*u*	*n* (%)	*e* (%)	*u* (%)
½	55.5	0.2	31	65	71	1	1	1
1	2.8	0.2	20	44	48	3	3	4
3	0.2	0.1	7	13	18	16	6	16
6	0.1	0.0	5	7	12	32	11	22
12	0.0	0.0	4	5	8	47	17	27
24	0.0	0.0	2	3	6	44	25	33

Values are averaged over the 14 GNSS stations in the test

Table 8 Percentage of the PPP solutions with a residual value relative to the reference model greater than three times the formal error $\sigma_{ppp\_amb\_float}$

Obs. times	Solution with a residual v_{kj} > than $3\sigma_{ppp\_amb\_float}$		
	n (%)	*e* (%)	*u* (%)
0.5	1	1	1
1	3	3	6
3	5	1	6
6	20	1	12
12	37	2	15
24	31	4	20

In this case, the residual of each solution calculated with fixed ambiguity is compared to the formal error of the same solution with float ambiguity

series of the residuals, the PPP solutions confirmed a precision (1σ) of less than 1 cm in the horizontal, at least for observation times longer than 3 h. For the shorter observation times, the precision is worse, but still remains under 10 cm of RMS in the horizontal, even when considering ½-h datasets. Similar results were found when also considering PPP solutions of the same dataset obtained without phase ambiguity resolution, showing how it significantly improved precision only for the easting component. This is rather different to what had been found by Geng et al. (2009). In his work, based on a different software package, the ambiguity resolution seems to have a higher improvement of the solutions compared to what we found.

Next, solutions having a formal error higher than 10 cm were considered outliers and therefore removed. This step had an impact on the ½-h solutions only, showing that about 55 % were to be rejected. Another statistical data which was investigated are the percentage of solutions having a residual higher than 30 cm, but where the formal error was less than 10 cm. Luckily, this percentage is negligible for the observing sessions longer than 1 h and reaches 0.4 % for the 1-h and ½-h solutions. Not all the GNSS stations have shown a similar behavior; in particular NOT1 shows a higher percentage of outliers than others. This is true especially for the 1-h solutions, and it was demonstrated to be due to a high number of cycle slips.

The formal error given by PPP is shown to be an important tool for identifying incorrect solutions that could have an error in the coordinate estimation greater than 30 cm and, therefore, considered here as unusable for technical applications. When evaluating the more precise solutions, the GIPSY formal error underrates the real errors in about 20–40 % of the cases, sometimes significantly and is, therefore, not considered to be a suitable tool for evaluating the precision of the solutions. A more reliable estimation of the real precision of the solution can be obtained by taking into account the formal error given by GIPSY for the float solution. This value is always available because the software must calculate a float ambiguity solution and related error estimates before fixing the ambiguities.

Regarding the solutions derived for the ½-h observation time series, we note that all the results were obtained by considering solutions with a formal error less than 10 cm, which are only about half of the total number of observations. It follows that the values are not sufficiently reliable for use. For this reason, we recommend observation sets of at least 1 h. A similar test showed that, in the cases where a position accuracy of 1 m is sufficient and a threshold of 33 cm is used when considering the formal error, the percentage of outliers is about 6 %. Moreover, in that case, only 0.2 % of the solutions have a residual of more than 1 m, which means that PPP solutions obtained from ½-h observation sessions are still reliable for applications requiring 1 m position accuracy. The RMS of the residual time series after such outlier rejection is under 5, 9 and 12 for the northing, easting and up components, respectively.

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